California State Science Fair

Working Model for Model Rocket Altitude Prediction

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This is a complete model of all the forces acting on a model rocket in flight. It calculates altitude, velocity, Mach number, and dynamic pressure as functions of time. It properly accounts for all of the physics with the following assumptions:

1) The rocket is in vertical flight.

2) The rocket's altitude is less that 1000 m and the atmospheric conditions are given by the Standard Atmosphere.

3) There is no variation in gravity with height at these altitudes.

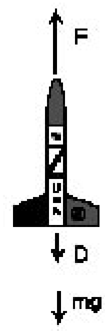


Fig. 1 -- Forces Acting on a Model Rocket in Flight.

This document assumes that the student has some basic understanding of the dynamics of atmospheric flight and rocket propulsion and a working knowledge of model rockets. A list of references are given at the end to assist in increased understanding.

A test case is set up for a 0.05 kg gross lift off (GLO) mass rocket using an Estes B5-8 motor. Aerodynamic drag coefficients were obtained using the USAF MISSILE DATCOM computer code for this model configuration. The student can modify the input parameters for different birds and motors. Program can be modified by student to be valid at higher altitudes if necessary.

Control parameters for Ordinary Differential Equation (ODE) solver:

Number of points -> $N := 150$

Start time -> $tstart := 0$

End time -> $tend := 15$

Physical parameters and models:

Gravity field (m/sec^2) -> $g := 9.8$

Atmosphere model: US 1976 Standard Atmosphere (z < 1000 m):

Temperature (K) -> $T(z) := 288.149 - .00649 \cdot z$

Pressure (Pa) -> $p(z) := 101325 \cdot \left(1 - 2.26 \cdot 10^{-5} \cdot z\right)^{5.25}$

Density (kg/m^3) -> $\rho(z) := \dfrac{p(z)}{T(z) \cdot 287}$

Speed of Sound (m/sec) -> $a(z) := \sqrt{401.8 \cdot T(z)}$

Rocket geometry:

Body diameter (m) -> $d := .0254$

Reference area (m^2) -> $S := \pi \cdot \left(\dfrac{d}{2}\right)^2$ $\quad S = 5.07 \cdot 10^{-4}$

Motor model -- Estes B8-5:

Propellant mass (kg) -> $m_p := 6.24 \cdot 10^{-3}$

Case mass (kg) -> $m_{case} := 13.06 \cdot 10^{-3}$

Burn time (sec) -> $t_b := .6$

Thrust (N) vs Time (sec) Model => $tdata := \begin{bmatrix} 0 \\ .1 \\ .2 \\ .3 \\ .4 \\ .5 \\ .6 \\ .7 \\ .8 \\ .9 \end{bmatrix}$ $fdata := \begin{bmatrix} 0. \\ 6 \\ 18 \\ 22 \\ 17 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

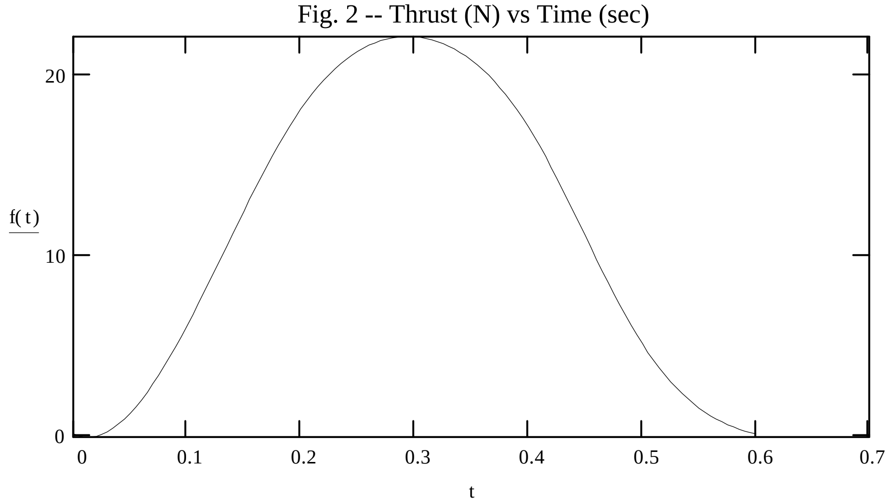
$t := 0, .005 .. t_b$

Interpolate Model for Thrust -> $f(t) := if\left(t \le t_b, interp(cspline(tdata, fdata), tdata, fdata, t), 0\right)$

Total Impulse (N*sec) -> $I_t := \int_0^{t_b} f(\xi)\, d\xi$ $I_t = 6.79$

Specific Impulse (sec) -> $I_{sp} := \frac{I_t}{m_p \cdot g}$ $I_{sp} = 111$

Fig. 2 -- Thrust (N) vs Time (sec)



Rocket mass model:

Body mass (kg) -> $m_{body} := .0307$

Dry mass (kg) -> $m_0 := m_{body} + m_{case}$ $m_0 = 0.04376$

Gross lift-off mass (kg) -> $m_{glo} := m_0 + m_p$ $m_{glo} = 0.05$

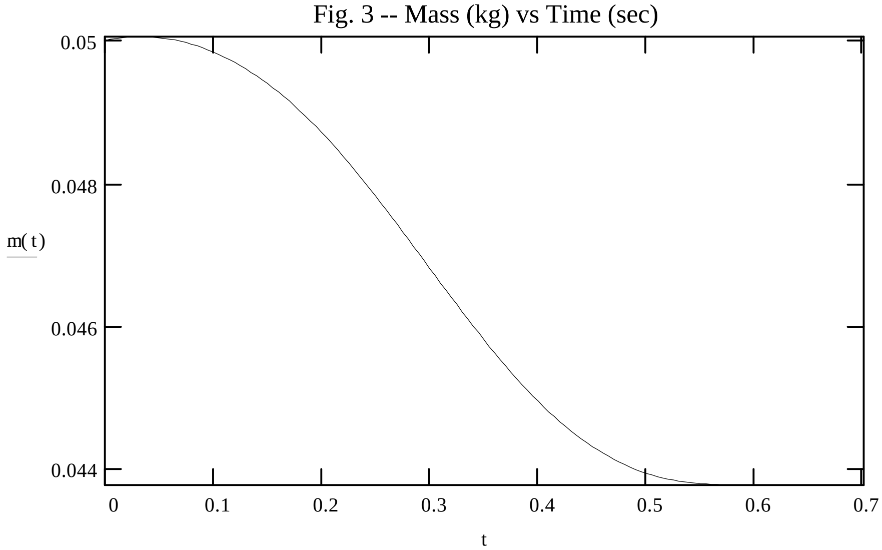
Use thrust vs time data to calculate mass as a function of time =>

$j := 1 .. N \quad \theta_0 := tstart \quad mass_0 := m_{glo} \quad \theta_j := tstart + \frac{tend - tstart}{N} \cdot j$

$$mass_j := if\left(\theta_j \le t_b, mass_{j-1} - \frac{m_p}{I_t} \cdot \int_{\theta_{j-1}}^{\theta_j} f(\tau)\, d\tau, m_0\right)$$

Interpolate to find mass as function of time -> $m(t) := interp(cspline(\theta, mass), \theta, mass, t)$

Fig. 3 -- Mass (kg) vs Time (sec)



Drag model:

Inport drag data from MISSILE DATCOM ->

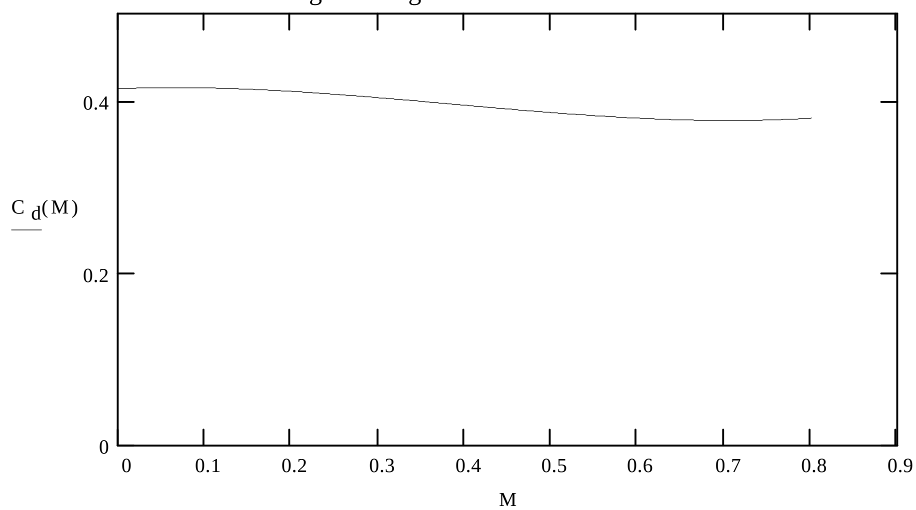
$$Mach := \begin{bmatrix} .2 \\ .4 \\ .6 \\ .8 \end{bmatrix} \qquad C_D := \begin{bmatrix} .41 \\ .394 \\ .379 \\ .379 \end{bmatrix}$$

Interpolate for drag vs Mach # ->

$$C_d(M) := interp\left(cspline\left(Mach, C_D\right), Mach, C_D, M\right)$$

$$M := 0, .001 .. .8$$

Fig. 4 - Drag Coefficient vs Mach Number



Solve ODE with Runge-Kutta Method:

Initial conditions -> $u^{\langle 0\rangle} := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Derivative vector -> $F(t,u) := \begin{bmatrix} u_1 \\ \dfrac{f(t)}{m(t)} - g - \dfrac{\rho(u_0)\cdot S\cdot C_d\left(\left|\dfrac{u_1}{a(u_0)}\right|\right)}{2\cdot m(t)}\cdot u_1\cdot |u_1| \end{bmatrix}$

Set up the Runge-Kutta equations ->

$K1(t,u,F,h) \equiv F(t,u)$

$K2(t,u,F,h) \equiv F\left(t+\dfrac{h}{2}, u+\dfrac{h}{2}\cdot K1(t,u,F,h)\right)$

$K3(t,u,F,h) \equiv F\left(t+\dfrac{h}{2}, u+\dfrac{h}{2}\cdot K2(t,u,F,h)\right)$

$K4(t,u,F,h) \equiv F(t+h, u+h\cdot K3(t,u,F,h))$ $\quad j := 0..N \quad i := 1..N$

$h := \left(\dfrac{tend - tstart}{N}\right)$ $\quad RK(t,u,F,h) \equiv \dfrac{h}{6}\cdot\left(K1(t,u,F,h) + 2\cdot K2(t,u,F,h) \ldots + 2\cdot K3(t,u,F,h) + K4(t,u,F,h)\right)$

Time (sec) -> $t_i := tstart + i\cdot h \quad \Delta t := h$

Solve with Runge-Kutta -> $u^{\langle i\rangle} := u^{\langle i-1\rangle} + RK\left(t_{i-1}, u^{\langle i-1\rangle}, F, h\right)$

Display Results and Find Max Values:

Altitude (m) ->	$h_j := u_{0,j}$	$\max(h) = 346.63$
Velocity (m/sec) ->	$v_j := u_{1,j}$	$\max(v) = 128.96$
Mach ->	$M_j := \frac{v_j}{a(h_j)}$	$\max(M) = 0.38$
Dynamic Pressure (Pa) ->	$q_j := .5 \cdot 1.4 \cdot p(h_j) \cdot (M_j)^2$	$\max(q) = 1.02 \cdot 10^4$
Drag (N) ->	$D_j := S \cdot q_j \cdot C_d(M_j)$	$\max(D) = 2.04$

Note: for this motor the ejection charge would have fired at 5 sec. This would be before the rocket would have reached its peak altitude.

Fig. 5 -- Altitude (m) vs Time (sec)

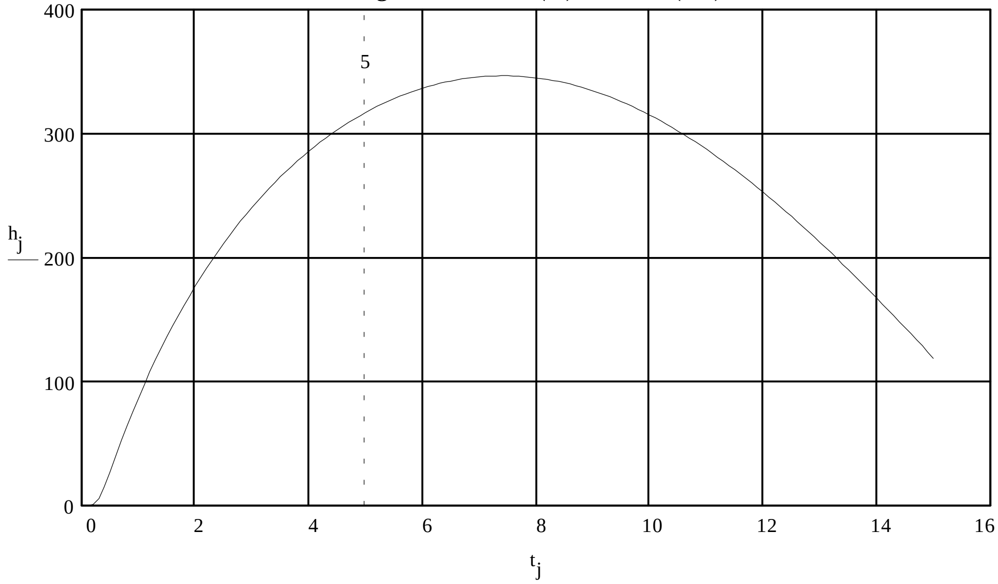


Fig. 6 -- Velocity (m/sec) vs Time (sec)

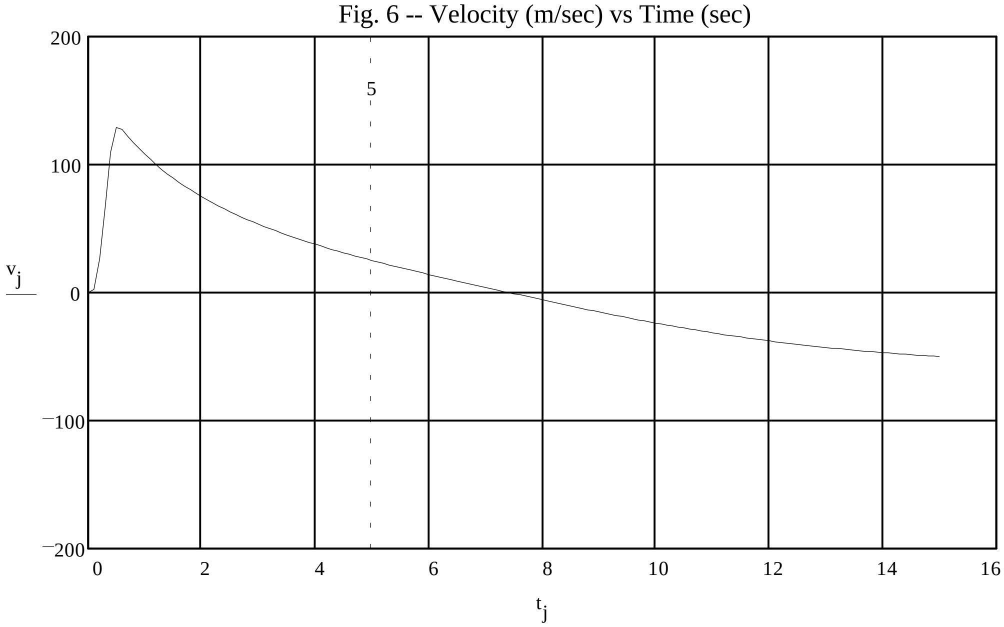


Fig. 7 -- Mach Number vs Time (sec)

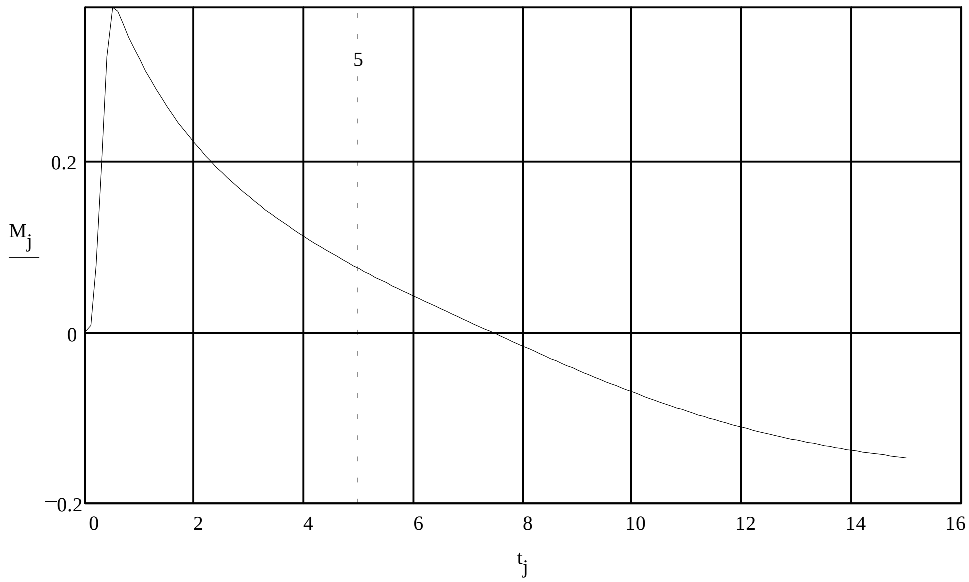
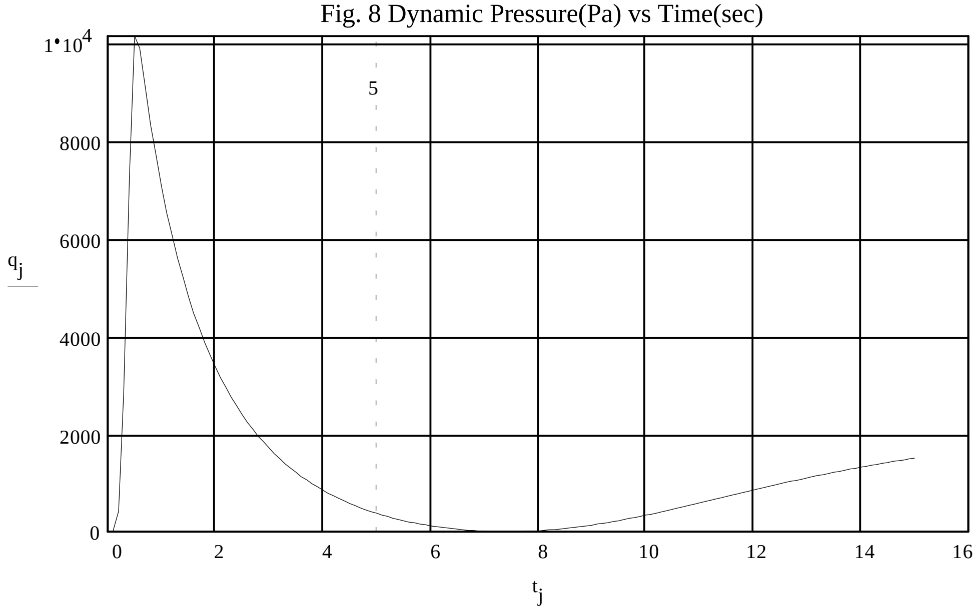


Fig. 8 Dynamic Pressure(Pa) vs Time(sec)



Rocket Performance Loss Due to Drag and Gravity:

Theoretical Maximum Velocity Change (no drag or gravity) -> $\Delta v := g \cdot I_{sp} \cdot \ln\left(\frac{m_{glo}}{m_0}\right)$ $\Delta v = 145$

Gravity Delta V Loss (m/sec) -> $\Delta v_{loss\_grav} := t_b \cdot g$ $\Delta v_{loss\_grav} = 5.88$

Index for Burnout -> $N_b := \frac{t_b}{\Delta t}$ $N_b = 6$

Drag Delta V Loss (m/sec) ->

$$\delta_j := \frac{D_j}{m\left(t_j\right)} \qquad \Delta v_{loss\_drag} := \Delta t \cdot \left[\left(\delta_0 + \delta_{N_b}\right) \cdot .5 + \sum_{k=1}^{N_b - 1} \delta_k\right] \qquad \Delta v_{loss\_drag} = 11.67$$

Total Delta V Loss (m/sec) -> $\Delta v_{loss} := \Delta v_{loss\_grav} + \Delta v_{loss\_drag}$ $\Delta v_{loss} = 17.55$

So we have $\Delta v - \Delta v_{loss} = 127.45$ compare with $\max(v) = 128.96$

Suggested Further Reading

1) Stine, G. Harry, *Handbook of Model Rocketry*, ARCO, 1983. -- Best introductory book on model rocketry by the man who invented model rocketry.

2) *Model Rocket Engines*, Estes Technical Note TN-1, 1972. -- This is where I got the thrust vs. time curve for the B8-5 motor. (Estes calls them rocket engines but technically rocket engines use liquid fuels. Solid rockets are called rocket motors.)

3) Gregorek, Gerald M., *Aerodynamic Drag of Model Rockets*, Estes TR-11, 1970. -- This little book gives a very good overview of the causes of aerodynamic drag (good enough that when I studied aerodynamics under Prof. Gregorek at Ohio State we used this book in our introductory aero class). It includes a method for calculating model rocket drag that is basically the subsonic prediction methodology incorporated in the USAF MISSILE DATCOM computer code. Despite this level of sophistication the book is intended for high school students and is easy to read.

4) Allen, John E., *Aerodynamics: The Science of Air in Motion*, McGraw-Hill, 1982. -- An earlier edition was the first book on aerodynamics I read in high school. Still a great introduction to aero.

5) Chow, Chuen-Yen, *An Introduction to Computational Fluid Mechanics*, John-Wiley and Sons, 1979. -- Chapter 1 of this book covers the dynamics of a body moving through a fluid and introduces the 4^{th} order Runge-Kutta method. A college text but fairly readable.

6) Mandell, Gordon K., et al, *Topics in Advanced Model Rocketry*, MIT Press, 1973. -- Read this book if you are at all serious about model rockets.

7) *U.S. Standard Atmosphere, 1976*, National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, NOAA-S/T 76-1562, 1976.