UNIVERSITY OF SOUTHERN CALIFORNIA

STRUCTURAL SYSTEM IDENTIFICATION AND HEALTH MONITORING OF BUILDINGS BY THE WAVE METHOD BASED ON THE TIMOSHENKO BEAM MODEL

by

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Copyright 2015 Mahdi Ebrahimian
To my wife, Maryam,
My parents and my family,
for their endless support.
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Abstract

This dissertation presents a new development of the wave method for structural health monitoring (SHM) of buildings. Robust and reliable SHM methods help save lives and reduce economic losses caused by earthquakes and other extreme events. Previously, in system identification and health monitoring, it was assumed that waves of different frequency propagate with constant velocity and the identification was based on the non-dispersive shear beam model of the structure. This study presents the first effort to consider dispersive wave propagation in system identification and health monitoring by the wave method. To consider dispersion due to bending deformation in buildings a Timoshenko beam model is used. Although buildings as a whole deform primarily in shear, bending deformation is always present to some degree especially for shear wall buildings. To identify allowable ranges of important parameters of the model parametric studies are performed. The model is further generalized to a non-uniform Timoshenko beam model which can take into account variation of properties with height and be used for higher resolution structural health monitoring. The models together with the suggested method to estimate initial values were validated on three full scale buildings. They were used to identify two full scale building from earthquake records and also to monitor the changes in a full-scale 7-story slice of shear wall building which was progressively damaged on UCSD-NEES shake table. It was shown that
the model is robust for structural identification and health monitoring of a wide range of building systems and can successfully model dispersion due to bending deformation.

**Keywords:** Wave propagation in buildings; Wave dispersion; Timoshenko beam; Nonuniform Timoshenko beam; Seismic interferometry; Propagator matrix; Structural health monitoring; Structural system identification; Earthquake damage detection; Impulse response; Millikan Library; Los Angeles 54-story office building; Shake table tests; UCSD-NEES Shake Table.
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A.1 Phase and group velocity
Chapter 1

Introduction

This chapter presents a brief introduction to structural health monitoring and its objectives, followed by the specific objectives and scope of the current study. The organization of this dissertation is presented at the end.

1.1 Background

Structural system identification and health monitoring is a broad topic spanning multiple disciplines e.g. mechanical, aerospace and civil engineering. This study focuses on civil engineering structures, in particular buildings.

Structural Health Monitoring (SHM) aims to detect whether damage has occurred, its location, time of occurrence and extent. Checking the safety of buildings soon after an earthquakes (or other disaster) has many potential benefits. For example: (i) hospitals and other emergency response facilities can work without interruption if assessed to be safe; (ii) weakened structures that may collapse during aftershocks can be evacuated; (iii) safe structures can be used as shelters for post-earthquake events like tsunami, when commute is disrupted and overcrowded streets obstruct emergency response (Hisada et al., 2012). Research in damage detection and health monitoring started in 1970’s for potential applications in the offshore oil industry and since then it has been an important research topic in civil engineering with applications to all civil infrastructures (Doebling et al., 1998).
In practice, the existence of damage in civil engineering infrastructure is assessed through periodic maintenance and post-event visual inspection (ATC-20, 1989). This procedure is time-consuming and inconvenient for the owner and residents. Non-structural members such as ceiling tiles and fire proofing should be removed to have access to the structural elements. It is also inaccurate because some damages may not be visible. Furthermore, after an extreme event like an earthquake number of qualified personnel available for inspection is very limited. After the Northridge 1994 earthquake, 130,000 buildings in Los Angeles area needed inspection (comprising only 11% of all buildings in Los Angeles area at the time) while in the two weeks following the earthquake an average 176 volunteers inspectors were available (City of LA Earthquake Plan, 2014).

To be practically useful SHM systems need to satisfy a number of challenging requirements. They should be: (i) sensitive enough to detect structural damages that can affect the structure serviceability; (ii) accurate to avoid false alarms and (iii) robust to work for real buildings and large amplitude response (Chang et al., 2003; Todorovska, 2009a).

Regardless of the method used, SHM involves the following steps (Todorovska, 2009a):

- The first step is to record the structural response, which is input to the method.
- The second step is to select the damage sensitive parameter(s); e.g. the natural frequencies of vibration and mode shapes, the wave velocities, the inter-story drifts etc. are some of the parameters that are currently being used. Choice of damage sensitive parameter is very important; it should be sensitive to damage but not to the other factors such as e.g. the environmental and operating conditions. The changes in the damage sensitive
parameter(s) are used to infer about location and extent of the damage. Reference data should be available to establish changes and relate them to the extent of damage.

- The last step is to make a decision on evacuation. In addition, in condition monitoring, the remaining life of the structure is also of interest.

SHM methodologies can be classified from different points of view. From spatial resolution point of view they are classified as global and local. The global methods can only determine if damage has occurred but local methods can detect location of damage. From another point of view, SHM methods can be classified into model-based (parametric) and non-parametric. Model-based methods assume a predefined model and adjust the model properties to match closely the recorded response. Non-parametric methods extract the dynamic properties of the system without assuming a model. From another point of view, SHM methods can be classified as vibration-based and wave propagation based methods. In the vibration-based methods, the damage sensitive parameters are vibrational characteristics of the structure such as the natural frequencies of vibration and the mode shapes while in the wave propagation methods, the damage sensitive parameters are the wave propagation characteristics such as wave travel times, wave velocities and wave numbers.

Structural damage detection and health monitoring in civil engineering structures is more challenging compared to mechanical and aerospace engineering structures. In the latter, although the parts may have complex shapes, each part can be manufactured and tested in detail beforehand to observe its behaviour at different levels of damage. In the case of civil engineering structures, the materials are more complex (especially concrete) and each project has its unique design specifications and site conditions. Although testing structural members and scaled or
full-scale models are possible, building a structure in real size on the actual site conditions for test purposes is prohibitively expensive (Chang et al., 2003). Civil engineering structures are systems consisting of the super structure, foundation and soil. Changes in the soil, affect change the system properties but they are mostly recoverable (see Udwadia and Trifunac (1974) and more examples in Trifunac and Ebrahimian (2014)). Ideally, the SHM the methodology should be able to distinguish between each part of the system. Many other challenges in system identification of buildings have been recognized since the 1970’s, including the amplitude dependency and nonlinearity of the response (Udwadia and Trifunac, 1974; Udwadia and Marmarelis, 1976; Marmarelis and Udwadia, 1976), uniqueness of the identification results (Udwadia and Sharma, 1978; Udwadia et al., 1978) and optimal sensor locations (Shah and Udwadia, 1978; Unwadia, 1994).

Monitoring the dynamic properties of buildings from full scale measurements goes back to the 1930’s (Carder, 1936). Udwadia and Trifunac (1974) studied changes in the apparent fundamental frequency of two buildings based on available ambient and forced vibration and also earthquake data. They showed that the frequency drops during large amplitude motions and recovers afterwards. Trifunac (1972a) compared results of ambient and force vibration tests on a twenty two story steel moment frame and a nine story reinforced concrete (RC) building. Many different methods have been suggested for structural system identification and health monitoring over the years. Detailed discussion of these methods can be found in review articles that have been published periodically (e.g. Doebling et al. (1998); Farrar et al. (2001); Chang et al. (2003); Carden and Fanning (2004); Todorovska and Trifunac (2009); Brownjohn et al. (2011)). Examples of a few of the methods with references to more recent articles include: (i) finite element model updating which involves matching selected parameters of a predefined finite element model
to observed response (e.g. Moaveni et al. (2010); Skolnik et al. (2006); Jafarkhani and Masri (2011)); fragility curves (e.g. Naeim et al. (2006)); (ii) methods based on image processing (e.g. Jahanshahi and Masri (2012)); (iii) methods based on changes in system frequency and damping (e.g. Trifunac et al. (2001a,b); Mikael et al. (2013)); (iv) methods based on neural networks (e.g. Saadat et al. (2004)); (v) methods based on substructure approach DeVore and Johnson (2014).

The system identification methodology in this study is model-based using wave propagation approach. Detailed literature review of wave propagation approach is presented in chapter 2. Wave propagation approach is robust when applied to large amplitude earthquake data and real buildings (Todorovska and Trifunac, 2008a,b) and also not sensitive to the effects soil-structure interaction (Todorovska, 2009b,c), the latter being the major advantage over the SHM methods that are based on detecting changes in the fundamental frequency of vibration (Clinton et al., 2006; Chang et al., 2003; Doebling et al., 1998).

1.2 Motivation and Objectives

In the wave propagation approach for system identification and health monitoring of buildings, the damage sensitive parameter is the wave velocity. The simplest way to calculate wave velocity is to measure wave travel time by cross correlation or impulse response analysis (Ivanović et al., 1999; Snieder and Şafak, 2006; Todorovska and Trifunac, 2008a,b). Because of the limited resolution in time, travel times read from peaks of impulse response functions or cross correlation have limited reliability (Todorovska and Rahmani, 2013). Furthermore, wave propagation in a buildings is dispersive meaning that the phase velocity is function of frequency (see appendix A for definition of phase velocity). Because of dispersion, the shape
of a propagating pulse will be distorted with distance, making reading the travel time from peaks ambiguous.

In this dissertation, that problem is solved by fitting a model of the structure in which the wave propagation is dispersive. The model should be physically useful to describe the dispersive wave propagation in the structure. Shear deformation is believed to be the most important contributor to a building deformation, consequently, initially shear beam models were used to identify and monitor buildings (Rahmani and Todorovska, 2013, 2014; Rahmani et al., 2015a). Wave propagation in shear beam models is not dispersive, meaning that the phase and group velocity are constant for all frequencies (see appendix A, B). Fitting a non-dispersive model to recorded data that is dispersive will introduce some artifacts in the identified shear wave velocities, discussed in chapter 4. To satisfy the non-dispersive assumption, shear beam models can be used over a narrower frequency band. However, that limits the spatial resolution and accuracy of the identification (Todorovska and Rahmani, 2013).

Dispersion can be caused by different phenomena. In buildings, one of the main causes of dispersion is the flexural deformation. The building deformation is a combination of shear and bending, the balance of which depends on the building system, construction materials, etc. In general, shear wall buildings are better described by models that take into account the bending deformation (see chapter 5). Shear-bending models are believed to be better physical models even for moment frame buildings that deform primarily in shear (Blume, 1968; Minami et al., 2013; Miranda and Reyes, 2002). Dispersion in the vertically propagating waves may be also caused by reflection form the lateral boundaries and discontinuities in distribution of stiffness (wave guide effect) (Todorovska and Lee, 1989;
Todorovska and Trifunac, 1989). Scattering from the slabs also introduces dispersion (Fukuwa and Matsushima, 1994; Safak, 1999).

The objective of this study is to extend the method of deconvolution interferometry to structures with dispersive wave propagation due to flexural deformation. Uniform and layered Timoshenko beam models of the building are used to take into account dispersion caused by flexural deformation. The TB model is convenient because it allows for analytical solution which enables better understanding of the physics of the problem. The model is then used for system identification and health monitoring of buildings. An algorithm is proposed to determine model parameters. The model is also tested on earthquake response and shake table test data of full-scale buildings. Another objective is to suggest simple methods to extract dispersion characteristics of the building from available data. The previously introduced dispersive models have been used to test the methods.

1.3 Organization of Dissertation

This dissertation is organized in six chapters:

Chapter 2 presents a detailed literature review of the wave propagation approach in the field of civil engineering in structural dynamics, identification and health monitoring.

Chapter 3 investigates wave propagation in a uniform viscoelastic Timoshenko beam. Timoshenko beam model is a simple model which accounts for bending deformation, shear deformation and rotatory inertia of the cross section. Important dimensionless parameters are introduced and analytical expressions are derived for the wave numbers, and phase and group velocities in terms of dimensionless parameters. Their asymptotic behavior is also discussed. Analytic expressions
are derived for the transfer function of a uniform cantilever Timoshenko beam representing a building. Pulse propagation in building is studied for different values of dimensionless parameters in ranges relevant for buildings. The model is validated on a full-scale building.

Chapter 4 presents two simple methods to extract the wave dispersion characteristics of a building from recorded data. The methods are tested on Timoshenko beam models of a real building (Millikan library) for which the dispersion is known.

Chapter 5 presents system identification of a 9 story RC building based on a uniform Timoshenko beam model. Recordings of a small magnitude earthquake are used. Identification is carried out by fitting model impulse response functions to observed ones in least squares sense. A procedure is described to approximate some of the model parameters from building geometry while the rest are found by fitting. A simple method to find suitable initial values for the fitting parameters is also suggested.

In chapter 6, the model is generalized to a nonuniform Timoshenko beam model with piecewise continuous properties along the height. This model enables to take into account the building variability with height which is common for high-rise buildings. It also enables to localize the damage in the building. The model is successfully used to identify a 54-story steel moment frame building as a 4 layer Timoshenko beam from its earthquake response.

In chapter 7, the model is used to detect damages in a 7-story full-scale slice of a shear wall building tested on UCSD-NEES shake table. It is shown that the method can successfully identify progressive damage in the test structure for different levels of shaking.

Finally, chapter 8 presents the summary and conclusions of the current study.
Chapter 2

Wave Propagation Approach to Analysis of Structural Response - Literature Review

This chapter presents a detailed literature review of the wave propagation approach to solve problems in structural dynamics, system identification and health monitoring of buildings.

To the knowledge of the author, wave propagation approach to calculate the seismic response of the buildings was first proposed by Kanai and Yoshizawa in a series of papers in 1964 (Kanai and Yoshizawa, 1963, 1964; Kanai, 1964). They viewed the seismic vibration of a building as multiple reflections of a wave in an elastic layer. In the first paper (Kanai and Yoshizawa, 1963), they derived a simple formula to calculate the base motion from the roof motion using ray theory. They tested the formula on the actual recording in buildings 2-12 stories high, and observed that the calculated motions at base are surprisingly close to recorded motions. Since there was no consideration of wave attenuation in the suggested formula, they concluded that good agreement is an indication that the most significant vibrational damping at the time of the earthquake was due to the energy dissipated into the ground. In the second paper (Kanai and Yoshizawa, 1964), they used the formula to calculate motion at the base of three dams (one gravity dam and two arch dams) from the top motion and showed that the calculated motion
agreed with the recorded motion. In the third paper, Kanai (1964) discussed application of the wave method to design of structures. He showed that the strain in the structure caused by an earthquake is proportional to the velocity amplitude of the seismic waves. Further, he showed that on soft ground the greatest damage to tall buildings is likely to take place at some considerable height but on hard ground it is likely to occur near the base.

The wave propagation approach to study the dynamic response of buildings was advanced to 2D models by Todorovska et al. (1988). They used isotropic 2D shear plate building models to study different phenomena concerning 2D wave propagation in structures. They considered three different refinements of the model: (i) homogeneous model; (ii) 2D shear plate with horizontal discontinuities; (iii) 2D shear plate with vertical discontinuities. The finite size of the building model and wave reflections from boundaries makes the wave propagation dispersive (frequency dependent phase velocity) for all three models but the dispersion curves are different because of the different boundary and continuity equations for each case.

Todorovska and Trifunac (1989) considered a 2D anisotropic shear plate building model to study the effects of traveling seismic waves on extended structures. They obtained analytical closed form solutions for the response of the model to incident plane SH waves. They suggested a simple way to calculate the shear wave velocities of the model based on properties of the building. They obtained analytical solutions for the displacement response of the model to incident plane SH waves for different values of the dimensionless parameters (height to length ratio and ratio of the incident wave phase velocity to the shear wave velocity in the building model). They concluded that the transfer of energy from the ground into the building is highly dependent on the ratio of the phase velocity of the ground
motion to the equivalent shear wave velocity of the building in the longitudinal direction. Todorovska and Trifunac (1990) used a 2D shear plate model consisting of two horizontal layers with different material properties to study the response of a building with a soft first floor. They showed that soft first floor can act as an isolation layer for the upper floors at the expense of large deformations. Todorovska and Lee (1989) used a 2D shear plate model consisting of vertical layers with different material properties to study response of the buildings with stiff shear elements e.g. shear wall at ends or central elevator core. Todorovska et al. (2001a) considered different simple 2D shear plate building models and investigated their dispersion characteristics. They considered homogeneous and layered anisotropic shear plate models. In the latter, the slabs and interstory space were represented as separate layers with hard and soft material types, respectively. They estimate model parameters for a seven-story concrete building and study dispersion curves for the models.

Safak (1998a,b, 1999) modeled a multistory building founded on layered soil by considering each story as another layer in the wave propagation path. The author calculated the seismic response under vertically propagating shear waves. The system was fully described in terms of the wave travel times in the layers and the reflection and transmission coefficients at the layer boundaries. The response was calculated in terms of aforementioned parameters. The author took into account the damping in the building and also the dissipation of energy due to waves going back into the soil. The foundation rocking was neglected and the building was assumed to deform only in shear. Safak (1998a,b, 1999) also proposed for the first time, the idea of using wave propagation approach for system identification and damage detection in multistory buildings.
Ivanović et al. (1999) calculated wave travel times from strong motion recordings in a seven-story RC building severely damaged by the 1994 Northridge earthquake. To calculate wave travel times, they found time delay corresponding to maximum cross correlation between windowed time histories of pairs of stations. The analysis was carried out for five earthquakes from 1971 to 1994. It was observed that large reduction in wave velocity occurs only during the damaging earthquake (1994 Northridge) and the location is consistent with the observed damage.

Todorovska et al. (2001b) estimated horizontal and vertical wave numbers in a seven-story RC building from earthquake records. They used recordings from four different earthquakes to estimate the wave numbers and the corresponding phase velocities between different pairs of instruments. They used data from the instruments located in the undamaged part of the building to study qualitative agreement of the phase velocities (inferred from wave numbers) between different events and also with previous independent estimates. For the instrument pair on the ground floor, the horizontal phase velocities inferred from wave numbers were of the same order of magnitude as the phase velocity of Love waves obtained from the soil profile supporting the building. For the instrument pairs along the height of the building, the vertical phase velocities inferred from wave numbers were of the same magnitude as the vertical shear wave velocity based on collective stiffness of columns and shear walls and based on observed apparent system periods during ambient vibration tests. The authors suggested that monitoring local changes in phase velocity is potentially useful for structural health monitoring and damage detection purposes.

Trifunac et al. (2003) continued the study done by Todorovska et al. (2001b) on the empirical wave numbers of a seven-story RC building. They used recordings of 11 earthquakes including one damaging earthquake (Northridge 1994) and
two of its aftershocks to compare changes in wavenumbers in undamaged and
damaged parts of the building. The authors observed significant increase in the
empirical wavenumbers calculated from the sensors located in damaged part of
the building during the damaging earthquake and its aftershocks. While no such
change was observed for undamaged parts in all events and for damaged parts for
events preceding the damaging earthquake. The authors concluded that empirical
wavenumbers can indicate location of damage.

Many researchers have used uniform shear beam model as a simple continuous
model of buildings. The compact and simple wave propagation formulation is
much more convenient for analysis compared to its discrete equivalent, a multiple
degree of freedom lumped mass model. Hall et al. (1995) used a uniform shear
beam model to study response of the building to ground displacement pulses.
They neglected the wave propagating into the ground and assumed full reflection
of the waves from the base. Iwan (1997) introduced a drift spectrum for pulse like
excitations based on a uniform damped shear beam model. The author wrote the
expression for shear strain in the beam based on multiple reflections of the seismic
wave in the beam assuming full reflection of the wave from the base. A more
detailed discussion of where maximum drift happens in the building in based on the
relationship between the dominant period of the ground motion and the period of
the shear beam can be found in Kanai (1964). Wave propagation approach has been
widely used in different soil-structure interaction studies e.g. structures on elastic
soil for incident SH waves (Luco, 1969; Trifunac, 1972b; Wong and Trifunac, 1974),
incident P and SV waves (Todorovska, 1993a,b), poroelastic soil (Todorovska and
Al Rjoub, 2006a,b; Todorovska and Al Rjoub, 2009) and analysis of nonlinear
waves in structure and soil (Gićev and Trifunac, 2007a,b, 2009a,b, 2010).
Haddadi and Kawakami (1998) and Kawakami and Haddadi (1998) suggested a method to extract simple waveforms from geotechnical borehole array data recorded during earthquakes. The method assumes a linear time-invariant system and is based on minimizing sum of the mean square values of the input and output motion in the frequency domain subjected to the constraint that the amplitude of the input wave at time zero is equal to 1. The simplified input and output motions in the frequency domain are in terms of the transfer function which can be calculated from recorded earthquake data. The simplified waveforms in the time domain were calculated by taking inverse Fourier transform. They called the suggested method normalized input-output minimization (NIOM). The results were compared with the results of impulse response and cross correlation analysis and were shown to be less sensitive to noise. The fact that impulse response functions presented in their study being are so sensitive to noise is likely due to the lack of regularization to avoid division by zero in the frequency domain (Snieder and Şafak, 2006). They successfully estimated S and P wave velocity in soil profiles from horizontal and vertical earthquake recordings, respectively.

Kawakami and Oyunchimeg (2003) used analytical models of soil layers and buildings to demonstrate the applicability of the NIOM method to different systems. For the soil layers, the NIOM method could closely obtain the assumed values of shear wave velocity. For the building models, the NIOM method could demonstrate the wave propagation. The authors also applied the NIOM method on earthquake response of four buildings to measure shear wave velocities in the buildings. They mentioned that the wave velocities are for the building as a whole and do not correspond to any of the construction materials. Oyunchimeg and Kawakami (2003) applied the NIOM method over narrow windows in response time history of buildings to monitor changes by measuring the wave travel times.
On a 7-degree of freedom building model, they showed that wave travel times measured from waveforms by the NIOM can closely approximate the fundamental period of the model. They also used earthquake recordings of 5 buildings, 6 to 20 stories high, three of which experienced some level of damage. For the damaged buildings, they observed an increase in the travel time and could approximate the time of damage from jumps in the travel time. For undamaged buildings, they showed that the wave travel times have only small fluctuations and are fairly constant. Kawakami and Oyunchimeg (2004) apply the NIOM method to analyze six analytical models of shear buildings 3 to 41 stories high, and 41 earthquake recordings on actual buildings. They showed that, for a uniform shear building model, the fundamental period can be approximated from the wave travel time via the simple relationship $T = 4\tau$ which corresponds to a uniform shear beam, where $T$ is the fundamental period and $\tau$ is the wave travel time. Assuming a non uniform building, $T = 4\tau$ deviates from the true value, overestimating (underestimating) for decreasing (increasing) stiffness toward the top. For real buildings, they show that in spite of the nonuniformities, the calculated values of $T = 4\tau$ agree well with reported period values from the previous studies.

Snieder and Şafak (2006) identified shear wave velocity and damping of Millikan Library, a 9-story RC building, based on deconvolved waveforms extracted from its response to a small earthquake. In contrast with previous works which measured wave travel time directly from accelerograms (Şafak, 1999) or from cross correlation (Ivanović et al., 1999), they based their work on deconvolution of recorded waves at different locations with respect to anyone of the sensors chosen as reference. In this approach, the transfer function (TF) is calculated by dividing the motion at all levels by the motion at the reference point in the frequency domain. Then,
impulse response functions (IRF) are calculated by taking inverse Fourier transform of the transfer functions. The waveforms extracted by the NIOM method (Kawakami and Oyunchimeg, 2004) also look very similar to IRFs extracted by deconvolution interferometry and physically represent the same idea just with a different regularization. TF and IRF are system functions in the frequency and time domains, respectively. For a 1D building model with only the horizontal motion and neglecting the base rocking, the authors analytically proved that IRFs are independent of both coupling of the building to the subsurface and excitation. The authors measured wave travel time by following pulse peaks in the IRFs. They showed that IRFs with respect to roof only consist of only two pulses one acausal and the other one causal and there is no further reflection from the base because of the artificial boundary condition at roof (dividing by roof response is like assuming that roof is fixed except for the time when virtual pulse is applied). They also estimated fixed based frequency of the building from amplitude spectrum of the IRFs and used the simple relationship between period of the first mode and shear wave velocity in a uniform shear beam \( (T = 4H/c_s) \) to calculate velocity of shear waves in the building.

Kohler et al. (2007) used earthquake recordings of a densely instrumented seventeen-story steel moment frame building to calculate impulse response functions following Snieder and Şafak (2006). They used data from 20 local and regional earthquakes and then stacked (averaged) the calculated impulse response functions to improve signal to noise ratio. They also developed a 3D finite element model of the building in ETABS based on structural drawings. They fitted Gaussian curves to stacked impulse response functions at subbasement level and used that as the input ground acceleration to the finite element model. They performed a linear elastic dynamic analysis on the model and showed that the propagating pulses
in the model agree closely with impulse response functions calculated from earthquake data. They also calculated impulse response functions for torsional response of the building for both the observed earthquake data and the finite element model.

Exploratory analysis by Todorovska and Trifunac (2008a,b) on two damaged buildings demonstrated that measuring wave travel times from impulse response analysis is both sensitive to damage and robust enough to be applicable on real data recorded on the buildings. Todorovska and Trifunac (2008a) calculated wave travel times from impulse response functions for a severely damaged 6-story reinforced concrete building. They used response of the building during the damaging earthquake in three time windows, before, during and after the occurrence of the severe damage (estimated from wavelet analysis Todorovska and Trifunac (2010)). They calculated the wave velocities from wave travel times and observed close agreement between the calculated values before damage and the stiffness distribution of the structure from structural drawings. They also estimated stiffness reduction from wave travel times and showed that it agrees with the observed location of damage. Assuming that the building deformed only in shear, they estimated the fixed based frequency of the building from the wave travel time using the corresponding relationship for a uniform shear beam $f_1 = 1/(4\tau)$. Todorovska and Trifunac (2008b) performed the same analysis for the East-West response of a damaged 7-story reinforced concrete building over a period of 24 years from recordings of 11 earthquakes. They showed that significant increase in wave travel was observed only during the damaging earthquakes, and that the detected changes in the corresponding shear wave velocities are consistent with the location and degree of the observed damage. They also compared instantaneous system frequency, $f_{sys}$, with the estimated fixed based frequency from wave travel times, $f_1$. They showed that while $f_{sys}$ changes
even when no structural damage was observed, the approximated $f_1$ agreed with the observed damage.

Todorovska (2009b) used seismic interferometry to identify a simple 2D soil-structure system with coupled horizontal and rocking response. The structure was modeled as a uniform shear beam supported by a circular rigid foundation embedded in an elastic homogeneous half-space. The author showed that because of the base rocking, the transfer functions and impulse response functions found from horizontal responses are not independent of the soil properties. The amplitude of impulse response functions are affected by the soil-structure-interaction and the reduction of amplitude reflects the system damping and not necessarily only the structural damping. However, if the data is sufficiently broadband, the pulse travel time found from impulse response functions is not affected by soil-structure-interaction effects and can be used to approximate the fixed-base frequency of the structure. If the travel time is measured from impulse response functions calculated on a frequency band containing only the first mode, the corresponding $f_1 = 1/(4\tau)$ is closer to the system frequency. The author also estimated the foundation rocking frequency from its relationship to the apparent system and fixed-based frequency.

Todorovska (2009c) followed the procedure suggested in Todorovska (2009b) to estimate the apparent system, fixed-base and rigid-body rocking frequency of a nine-story RC moment frame and shear wall building from four earthquake response data between 1970 and 2002. The apparent system frequency was read from peaks of the transfer function between roof and base horizontal response. A proxy of the fixed-base frequency was calculated from the wave travel time assuming that building deforms predominantly in shear. The wave travel times were obtained from the pulses in the impulse response functions. The rigid-body rocking frequency was estimated based on its relationship to the apparent system.
and fixed-based frequency. The author observed that both fixed-base and rigid-body rocking frequency are amplitude dependent and explained to what degree the changes in system frequency are caused by changes in rigid-body rocking and fixed-base frequency for different events.

Researchers have also used ambient vibrations recorded for long durations of time to perform impulse response analysis. Having long recordings enabled them to use averaging to improve the results. Prieto et al. (2010) extracted impulse response functions from 10 min ambient vibration data series and averaged the results over 1, 14, 30 and 50 day durations. Nakata and Snieder (2014) recorded ambient vibration of an eight story building for two weeks, analyzed the data in 30s intervals and average the extracted impulse response functions over four day intervals for monitoring purposes.

Nakata et al. (2013) provide a detailed comparison of impulse response analysis (referred to as deconvolution interferometry) with cross correlation and cross coherence analysis. Based on the presented discussion the latter two methods have the following undesirable properties: (i) cross correlated waveforms depend on the incoming wave and reflection coefficient from the ground; (ii) the waveforms in cross-coherence interferometry contain pseudo-events that propagate at velocities slower than the actual velocity of the building. Since impulse response analysis is independent of the soil properties (ignoring rocking at base see Todorovska (2009b)) and the incoming wave, they conclude that it is the most suitable method to estimate wave propagation velocity in the building. The authors also provide an extended version of impulse response functions calculated by Snieder and Şafak (2006) for the case of an arbitrary reference point. They calculate impulse response functions for records of 17 earthquakes, over a two week period, to monitor an 8-story building in Japan. They observed a negative correlation.
between wave velocities and maximum acceleration. In two parts of their experimental study they show that wave propagation in the building is dispersive: (i) for the earthquake with the largest observed acceleration in the building, they show that the peaks of impulse response functions from top to bottom are not located on a line; (ii) when they plot wave velocities calculated from traveling wave travel times and the ones from coda-wave interferometry versus maximum acceleration, the slopes of reduction are different because the former is dominated by first mode while the latter contains higher frequencies.

Todorovska and Rahmani (2013) addressed the spatial resolution and accuracy of identified shear wave velocities from wave arrival times from time shifts of peaks of impulse response functions. They used a multi-layer shear beam model to represent the building and found the shear wave velocity in each layer based on the travel times measured from time shifts of peaks of impulse response functions. They showed that as a consequence of Heisenberg-Gabor uncertainty principle, time of peaks of IRFs does not necessarily coincide with the true, physical pulse arrival time. To measure this error they used time localization of the source pulse. They suggested that for the IRFs found on the frequency interval \([0, \omega_{\text{max}}]\), the localization of the source pulse (a sinc function) can be measured by the half width of its main lobe, equal to \(\Delta t = \frac{\pi}{\omega_{\text{max}}} = \frac{1}{2f_{\text{max}}}\). Assuming that the two sensors are at distance \(z\) and the wave travel time is equal to \(\tau\), and the shear wave velocity is \(\beta = \frac{z}{\tau}\), they showed that \(\beta\) is an estimate of shear wave velocity in the layer and the error of this estimation is \(\Delta \beta = \frac{\pi}{\omega_{\text{max}}\tau}\beta\). So to decrease \(\Delta \beta\) one can increase \(\omega_{\text{max}}\) and \(\tau\) or decrease \(\beta\). From the equation of \(\Delta \beta\), they concluded that: (a) the error in identified \(\beta\) is smaller in more flexible buildings (where \(\beta\) is smaller), and (b) the error is smaller if \(\beta\) is identified from larger wave travel times, \(\tau\), i.e. wave propagated over larger distances. Note that there is a trade-off between detail and
accuracy of the identified velocity profile. In more detailed models \( \tau \) is smaller, and so error will be larger. (c) The error is smaller for larger signal bandwidth \( \omega_{\text{max}} \). It was also shown that \( \beta \) is not the exact shear wave velocity for the layer extending from 0 to \( z \), it is a weighted average value over \([z - \Delta z \ z + \Delta z]\) where \( \Delta z = \beta \Delta t = \frac{\pi \beta}{\omega_{\text{max}}} = \frac{\beta}{2f_{\text{max}}} \). The recorded response of buildings is small after some frequency which is much less than the capability of the recording instruments (25-50 Hz). So in real cases both \( \omega_{\text{max}} \) and \( \Delta z \) are limited. They also showed that the pulse amplitudes in IRFs are affected by both attenuation in building and foundation rocking. So, in general, it is not possible to identify damping of fixed-base structure from IRFs.

Rahmani and Todorovska (2013) improved the method in Todorovska and Rahmani (2013) by which the properties of the equivalent layered shear beam were identified. Previously, the identification was performed by picking the pulse arrival time in impulse response functions and shown to have limited accuracy due to finite bandwidth of the signal. They introduced two methods to improve fitting of the layered shear beam: (i) an iterative method to exactly match the model pulse arrival times to the ones in IRFs computed from observed response, (ii) least square fit of IRFs as waveforms over predefined time windows containing the main lobes of the pulses in IRFs for virtual source at top. They applied the two algorithms to the response of a 9-story RC building during the 2002 Yorba Linda earthquake and argued that fitting IRFs as waveforms in least square sense is more accurate than matching pulse arrival times because it uses information on amplitudes in addition to time shifts. They note that choosing the optimal value for cut-off frequency, \( f_{\text{max}} \), in low-pass filtering the data is important because it controls the spatial resolution and also the effects of dispersion. While a higher
value of $f_{\text{max}}$ increases the spatial resolution, but over a wide frequency band the pulses are more distorted due to dispersion and shear beam is not a valid model.

Rahmani and Todorovska (2014) used the previously introduced waveform inversion algorithm, which fits IRFs in the least squares sense, to identify a 54-story steel moment frame building. They used low-pass filtered data of the EW, NS and torsional responses of the building during the 1994 Northridge earthquake for identification. Considering only shear deformations, they used a shear beam model to represent the building in the lateral direction and a torsional shaft for torsional response. They identified the building using both uniform and 4-layer models. They showed that the identified models match closely the fitted IRFs and also the frequencies of vibration in the TFs.

Rahmani and Todorovska (2015) used records of six earthquakes from 1992 to 2010 to identify the wave velocities on the same 54-story building. None of the earthquakes caused any reported damage. The purpose of the study was to detect variations in wave velocities due to factors other than damage and detect possible changes due to degradation of the structure. Both uniform and 4-layer shear beam models were fitted to the observed IRFs to identify the wave velocities. The results showed a permanent reduction of 10-12% in the overall structural stiffness despite of no reported damage.

Rahmani et al. (2015a) further extended the waveform inversion method to perform time-velocity analysis which involved fitting impulse response functions calculated over a sliding time window. The changes in the wave velocity were measured with respect to the value at the beginning of the shaking where the response is smaller. Therefore, no pre-event measurement is necessary. The case study is a 12-story RC building lightly damaged by the 1971 San Fernando earthquake. To monitor changes, both uniform and two layer shear beam models were
fitted in IRFs calculated in a moving time window. The authors showed that the changes in the wave velocity (24%, 27% for upper half and 31%, 37% for lower half in NS and EW respectively) were consistent with the higher intensity of damage observed in the lower half of the building after the earthquake. They also compared time-wave velocity results with changes in instantaneous frequency and observed that the drop in fundamental system frequency is larger possibly due soil-structure interaction effects.

In conclusion, the advantages and limitations of the wave propagation approach to structural response of buildings can be summarized as follows:

i The boundary condition at the base is more realistic compared to the common fixed base assumption. This allows for seismic waves to propagate back into the ground and provides a better representation of damping (Kanai and Yoshizawa, 1963; Safak, 1999).

ii The soil layers supporting the structure can be easily incorporated in the model (Safak, 1999).

iii In case of a pulse like ground motion, wave propagation is a superior approach because the maximum response may occur before reflections produce standing waves (mode shapes) (Iwan, 1997). Also, for pulse like excitations, one may need many modes to capture the response (Gičev and Trifunac, 2009b).

iv Wave propagation studies often use continuous models for analysis. The biggest disadvantage of continuous models is that only limited number of problems with simple geometries can be solved analytically (Todorovska and Trifunac, 1989). Moreover, continuous modeling of structures is only valid for long wave lengths (lower frequency range) of the response. Nevertheless, exact solutions are desirable because they are convenient for parametric studies to understand
physics of the problem. They can also provide a basis for testing approximate methods (Todorovska and Trifunac, 1989).

v Another advantage of continuous models is that they provide the response at any desired level. So, there is no need for interpolation to calculate values corresponding to different floors which is the case for reduced order lumped mass models.

In particular, using impulse response functions calculated from deconvolution for system identification of the buildings has the following advantages over vibrational methods:

i Considering a model with only horizontal motion, impulse response functions are independent from the soil properties (Snieder and Şafak, 2006). For a more realistic case when base rocking is taken into account, the wave travel time calculated from impulse response functions over a broad band response is less affected by soil-structure interaction (see Todorovska (2009b), Chapter 4) than the fundamental frequency of vibration.

ii Impulse response functions are more informative than point values of the natural frequencies. They contain information about both amplitude and phase and are calculated over a frequency range.

iii Even when the natural frequencies are not obvious to read from the transfer function, one can easily fit impulse response functions a frequency band.

iv Assuming that a reasonable physical model is available, fitting impulse response functions is more reliable than reading travel time from peaks of impulse response functions because the peak value does not exactly correspond to the
true value of wave travel time (Todorovska and Rahmani, 2013; Rahmani and Todorovska, 2013).
Chapter 3

Wave Propagation in a Uniform Timoshenko Beam

This chapter is based on the article “wave propagation in a Timoshenko beam building model” published in *Journal of Engineering Mechanics* (Ebrahimian and Todorovska, 2014c).

Wave propagation in a Timoshenko beam model of a high rise building, excited by base motion, is analyzed. This model accounts for wave dispersion due to bending and is more realistic for analysis of wave propagation in high rise buildings than shear beam and other discrete nondispersive models. The model transfer-functions and impulse response functions are obtained from the analytical solution for forced vibration response of the beam excited by base motion. The impulse response functions represent the model response to a virtual input pulse; a set of such functions at different levels is used to study pulse propagation in the beam. Parametric study of dispersion and its effects on the model transfer-functions and impulse response functions is presented, in terms of dimensionless parameters, in ranges corresponding to buildings. The model is validated for a 9-story full-scale RC building and smaller earthquake excitation. The results provide insight into dispersive wave propagation in buildings caused by bending, which is useful for the interpretation of impulse responses of buildings obtained from earthquake records and for the development and testing of wave methods for structural health.
monitoring. The results can also be used for testing numerical models for pulse propagation in buildings.

3.1 Introduction

Simple structural models and their analytical solutions are invaluable for the interpretation of observed seismic response of full-scale structures, calibration of numerical models and testing of structural system identification methods. The study was motivated by the evidence of dispersion found in a structural system identification study of a 9-story reinforced concrete (RC) building by a wave method (Rahmani and Todorovska, 2013). That study, which presented a new algorithm for identification by least squares fit of pulses in impulse response functions, also showed that fitting more realistic building models is needed to increase the spatial resolution of the method and enable detecting more localized damage. Towards this goal, it is beneficial first to understand the phenomena involved using simple analytical models. In this paper, we use a Timoshenko beam model to get insight into dispersion in seismic wave propagation in high rise buildings that is caused by bending deformation, and its effect on pulse propagation through the structure. This study is an extension of studies on pulse propagation in uniform and layered shear beam models (Kanai and Yoshizawa, 1963; Snieder and Şafak, 2006; Todorovska and Rahmani, 2013). The analysis is in terms of the governing dimensionless parameters, for a range of these parameters for buildings. Also, a model of an actual building is presented and compared with earthquake observations. The wave dispersion is derived from the uncoupled equation of motion of the beam (Timoshenko, 1921), while pulse propagation is analyzed using impulse response
functions, for virtual source at base and at top Snieder and Şafak (2006). The discussion of the results is from the structural identification perspective, and focuses on the nature, degree and implications of the effects of bending on the identification of the wave velocities. The results will be useful for interpretation of observed impulse response functions in buildings during earthquakes (Kohler et al., 2007; Rahmani and Todorovska, 2013; Todorovska and Trifunac, 2008a,b; Rahmani and Todorovska, 2013), and for testing wave methods for structural system identification. They can also be used for testing numerical models for simulation of response of buildings to impulsive ground motion, such as, e.g., the finite difference models in (Gičev and Trifunac, 2009b,a). Although the elements of a building, such as beams and columns, deform predominantly in bending, the seismic response of a building as whole is closer to that of shear beam (SB) than of Euler-Bernoulli (EB) beam, as evidenced by the ratios of observed frequencies of vibration of full-scale buildings (1:3:5 ...for SB and 1:6.27:17.5 ...for EB; Boutin et al., 2005). Consequently, shear beams and plates and other discrete nondispersive models have been used to study wave propagation in buildings (e.g. Kanai and Yoshizawa, 1963; Todorovska and Lee, 1989; Safak, 1999; Kawakami and Oyunchimeg, 2004; Gičev and Trifunac, 2009a,b; Todorovska, 2009b; Todorovska and Rahmani, 2013). Dispersion, however, is present to some degree in the response of buildings, and should be considered in more realistic models. One recent example is the dispersion evidenced in impulse response functions of Millikan library (9-story reinforced concrete frame building with side shear walls in the NS directions and a central core in the EW direction; Rahmani and Todorovska, 2013). The causes of dispersion in buildings are multiple. One cause is the contribution from bending to the total deformation of the structure, which is larger, e.g., if shear walls are present, addressed in this paper. Other possible causes include reflections from the lateral
boundaries and lateral discontinuities in the distribution of the stiffness (Todorovska and Lee, 1989; Todorovska and Trifunac, 1989). Timoshenko beam models of building have been used in modal analyses, and to estimate the nature of the building response from experimental data by matching the frequencies of vibration (e.g., Boutin et al., 2005). Such analyses, however, ignore the frequency shift of the fundamental mode due to soil-structure interaction, which introduces errors in such assessments, discussed in this paper. To the knowledge of the authors, Timoshenko beam models have not been previously used for analysis of wave propagation in high rise buildings. Wave propagation in Timoshenko beam models has been analyzed for use in nondestructive testing (NDT) of structural elements and other mechanical engineering applications (e.g. Park, 2005; Mead, 1985). The results of such studies, however, are not directly applicable to buildings, because of differences in the range of parameters and wavelengths of interest. E.g., in such applications, the beam models are assumed to deflect primarily in bending, the shear being a correction, while buildings deform primarily in shear, with the bending being the correction. Further, the shorter wavelengths are of greater interest in such applications, while the longer wavelengths, where the energy of response is concentrated, are of greater interest for building. As seen later in this paper, there are different regimes of the response of Timoshenko beam, for low vs. high frequencies, and for low vs. high ratio of bending to shear stiffness. This paper is organized as follows. In the Methodology section, the dispersion relation and analytic solution for forced vibration of the beam, excited by harmonic base motion, are derived, as well as the system functions in frequency and time domains. The effects of soil-structure interaction are ignored. The Results and Analysis section presents parametric studies of the dispersion and of the system functions, in terms of the governing dimensionless parameters in a range typical for buildings and
also an example of applying the suggested model on a real building. Finally, the
discussion and conclusions are presented.

### 3.2 Methodology

#### 3.2.1 Model and Solution for Forced Vibration Response

The building is modeled as a uniform viscoelastic Timoshenko beam (TB), of
height $H$, stress free at the top, cantilevered at the base, and excited by base motion
(Fig. 3.1a). The material is characterized by mass density $\rho$, Young’s modulus $E$
and shear modulus $G$, which implies longitudinal and shear wave velocities in the
material $c_L = \sqrt{E/\rho}$ and $c_S = \sqrt{G/\rho}$. The beam cross-section is characterized by
area $A$, radius of gyration $r_g = \sqrt{I/A}$, where $I$ is its second moment w.r.t. $x$, and
shear factor $k_G$ (Timoshenko, 1921).

![Figure 3.1: a) Cantilevered Timoshenko beam model, b) deflected element, and c) its free body diagram.](image)

Timoshenko beam theory accounts for both shear and flexural deformation, and
also for rotary inertia, and is essentially a combination of a shear and Rayleigh
beam connected in series. It is a linear theory, which assumes small deformations and that plane sections normal to the neutral axis remain plane but not normal. The additional rotation $\gamma(z; t)$ is considered as shear deformation, caused by uniform shear stress on the section. In reality, however, the shear stress on the section, which must be zero on the lateral boundaries, is nonuniform, and the plane sections become, in general, curved. The shear factor $k_G$ is such that the actual shear force on the section $V = k_G AG \gamma$. It depends on the shape of cross section, material properties and frequency (Mindlin and Deresiewicz, 1953; Cowper, 1966; Hutchinson, 2001). The limits of validity of the theory are discussed in the Results and Analysis section.

Let $\theta(z; t)$ and $\gamma(z; t)$ be angles representing the rotation due to bending and the shear angle of an infinitesimal beam element (Fig. 3.1b), and let $u(z; t)$ be the absolute horizontal displacement of the center of gravity of the element. For small deformations, $u$, $\theta$ and $\gamma$ are related by

$$\frac{\partial u}{\partial z} = \theta + \gamma \quad (3.1)$$

the shear force $V$ and bending moment $M$ being

$$V = k_G AG \gamma \quad (3.2)$$

$$M = EI \frac{\partial \theta}{\partial z} \quad (3.3)$$

Further, Kelvin-Voigt material damping is introduced by replacing $E$ and $G$ by $E \left(1 + \mu \frac{\partial}{\partial t}\right)$ and $G \left(1 + \mu \frac{\partial}{\partial t}\right)$, where $\mu$ is the viscosity constant, assumed to be the same for both types of deformation (Prasad, 1965).
Dynamic equilibrium of the beam element (Fig. 3.1c) implies the following decoupled equation of motion for $u(z; t)$

$$c_L^2 c_S^2 k_G \left(1 + \mu \frac{\partial}{\partial t}\right)^2 \frac{\partial^4 u}{\partial z^4} - \left(c_L^2 + k_G c_S^2\right) \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^4 u}{\partial z^2 \partial t^2}$$

$$+ \frac{k_G c_S^2}{r_g^2} \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial t^4} = 0 \quad (3.4)$$

the solution of which should satisfy the following boundary conditions

\begin{align*}
\text{at } z = 0 & : \ V(0; t) = 0, \ M(0; t) = 0 \\
\text{at } z = H & : \ \theta(H; t) = 0, \ u(H; t) = u_g(t) \quad (3.5) \quad (3.6)
\end{align*}

Analytical solution of (7.1) can be obtained in the frequency domain. Assuming harmonic base motion $u_g(t) = e^{-i\omega t}$, where $\omega$ is circular frequency, and trying a propagating wave solution of the form

$$u(z; t) = e^{i(kz - \omega t)} \quad (3.7)$$

gives the dispersion relation

$$c_L^2 c_S^2 k_G (1 - i\omega \mu)^2 k^4 - (c_L^2 + k_G c_S^2) (1 - i\omega \mu) k^2 \omega^2 - \omega^2 \frac{k_G c_S^2}{r_g^2} (1 - i\omega \mu) + \omega^4 = 0 \quad (3.8)$$

The roots of (5.7) give the dependence of the phase velocities/wave numbers on frequency, i.e. the dispersion curves. Among the many parameters, the dispersion is governed by two independent parameters, the dimensionless frequency $\Omega$ and the moduli ratio $R$ defined as

$$\Omega = \frac{\omega r_g}{c_S} \quad (3.9)$$
\[ R = \frac{G}{E} = \frac{c_S^2}{c_L^2} \]  
\quad (3.10)
(Mead, 1985). We also define dimensionless damping constant
\[ M = \frac{\mu c_S}{r_g} \]  
\quad (3.11)
In terms of these parameters, the roots of (5.7) give dimensionless wavenumbers
\[
K = k r_g = \pm \frac{\Omega}{\sqrt{2}} \sqrt{\left( \frac{1}{\alpha} \right) \left( \frac{1}{k G} + R \right) \pm \sqrt{\left( \frac{1}{\alpha^2} \right) \left( \frac{1}{k G} - R \right)^2 + \frac{4 R}{\alpha \Omega^2}}} \quad (3.12)
\]
where
\[
\alpha = 1 - i \omega M \quad (3.13)
\]
The dispersion is analyzed in the Results and Analysis section of this chapter.

Knowing the wave numbers
\[
U(z) = C_1 e^{i k_1 z} + C_2 e^{-i k_1 z} + C_3 e^{i k_2 z} + C_4 e^{-i k_2 z} \quad (3.14)
\]
where \( C_i \), \( i = 1, \ldots, 4 \) are constants. Alternatively, \( U(z) \) can be expressed as
\[
U(z) = D_1 \cos k_1 z + D_2 \sin k_1 z + D_3 \cos k_2 z + D_4 \sin k_2 z \quad (3.15)
\]
with
\[
D_1 = C_1 + C_2; \quad D_2 = i(C_1 - C_2) \quad (3.16)
\]
\[
D_3 = C_3 + C_4; \quad D_4 = i(C_3 - C_4)
\]
The boundary conditions imply

\[
\begin{pmatrix}
\frac{C_2}{C_1} \\
\frac{C_3}{C_1} \\
\frac{C_4}{C_1}
\end{pmatrix} = A^{-1} \begin{pmatrix}
\left(\frac{\Omega^2}{\alpha_k G K_1} - K_1\right) e^{iK_1(H/r_g)} \\
- \left(\frac{\Omega^2}{\alpha_k G} - K_1^2\right) \\
\frac{1}{K_1}
\end{pmatrix}
\]  

(3.17)

where

\[
A = \begin{bmatrix}
\left(\frac{\Omega^2}{\alpha_k G K_1} - K_1\right) e^{-iK_1(H/r_g)} & - \left(\frac{\Omega^2}{\alpha_k G K_2} - K_2\right) e^{iK_2(H/r_g)} & \left(\frac{\Omega^2}{\alpha_k G K_2} - K_2\right) e^{-iK_2(H/r_g)} \\
\left(\frac{\Omega^2}{\alpha_k G} - K_1^2\right) & \left(\frac{\Omega^2}{\alpha_k G} - K_2^2\right) & \frac{1}{K_2}
\end{bmatrix}
\]  

(3.18)

### 3.2.2 Transfer-Functions and Impulse Response Functions

Known the solution in the frequency domain, the beam transfer-functions and impulse responses can easily be computed. E.g., let \(\hat{u}(z; \omega)\) and \(\hat{u}_{\text{ref}}(z_{\text{ref}}; \omega)\) be the Fourier Transforms of displacement at height \(z\) and at some reference height \(z_{\text{ref}}\). Then the transfer-function (TF) \(\hat{h}(z, z_{\text{ref}}; \omega)\) of the response at \(z\) with respect to that at \(z_{\text{ref}}\) is

\[
\hat{h}(z, z_{\text{ref}}; \omega) = \frac{\hat{u}(z; \omega)}{\hat{u}(z_{\text{ref}}; \omega)} = \frac{U(z)}{U(z_{\text{ref}})}
\]  

(3.19)

where \(U(z)\) is as in eqn (5.11) or (3.15).

The impulse response function (IRF) at level \(z\) with respect to \(z_{\text{ref}}\) is the inverse Fourier transform of \(\hat{h}(z, z_{\text{ref}}; \omega)\), and, for damped beams, exists only over a finite frequency band \(|\omega| < \omega_{\text{max}}\). Then, the band limited impulse response is

\[
h(z, z_{\text{ref}}, \omega_{\text{max}}; t) = \frac{1}{2\pi} \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} \hat{h}(z, 0; \omega) e^{-i\omega t} d\omega
\]  

(3.20)
In practice, regularized transfer functions can be used (Snieder and Şafak, 2006). The IRF represents the response of the beam to a virtual pulse applied at the reference point at time $t = 0$. It represents the Green’s function of the beam, which, in addition to the natural boundary conditions, is also fixed at the reference point at all times except when the load is applied (Snieder and Şafak, 2006).

The IRF at the reference point gives the virtual source, which is a box function in the frequency domain and a sinc function in the time domain

$$\hat{h}(z_{ref}, z_{ref}, \omega) = \begin{cases} 1, & |\omega| < \omega_{\text{max}} \\ 0, & \text{otherwise} \end{cases}$$

$\Leftrightarrow$ $h(z_{ref}, z_{ref}; t) = \frac{\omega_{\text{max}} \sin \omega_{\text{max}} t}{\pi \omega_{\text{max}} t}$ \hspace{1cm} (3.21)

The main lobe of the sinc function has half-width $\Delta t = \pi/\omega_{\text{max}}$, which affects the resolving power of the IRFs, as shown in Rahmani and Todorovska (2013), and also seen in the examples.

### 3.3 Results and Analysis

As buildings are much stiffer in the axial than in the lateral direction, we focus in this paper on cases with $R = G/E \leq 1$. Also, without loss of generality, we show results for rectangular cross-section, for which $k_G = 5/6 \approx 0.83$ is used (Weaver et al., 1990), and for which $r_g = W/\sqrt{12}$, where $W$ is the width.
3.3.1 Dispersion

The dispersion curves have two branches, with undamped dimensionless wave numbers

\[ K_{1,2} = \frac{\Omega}{\sqrt{2}} \pm \sqrt{\left( \frac{1}{k_G} + R \right)^2 + 4R \Omega^2 - \frac{4R}{\sqrt{2}} \left( \frac{1}{k_G} - R \right) + \sqrt{\left( \frac{1}{k_G} - R \right)^2 + 4R \Omega^2}} \]  

(3.22)

corresponding to propagating waves if real. Analysis of Eqn (7.2) shows that \( K_1 \) is real for all \( \Omega \) and, therefore, transports energy through the beam, while \( K_2 \) becomes real only for \( \Omega > \Omega_{cr} = \sqrt{k_G} \). For \( \Omega < \Omega_{cr} \), \( K_2 \) is imaginary, and is associated with spatially decaying waves from the location where they are generated, referred to as near field or evanescent waves (Mead, 1985). (Fig. 3.2) shows \(|K_{1,2}| \) vs. \( \Omega \) for \( R = 0.01, 0.1, 0.5 \) and 1. It is seen that \( K_2 \) is greater for larger \( R \), and increases initially with frequency but then drops to zero as \( \Omega \to \Omega_{cr} \). Frequency \( \Omega_{cr} \) is the natural frequency of a thickness-shear mode (Reis, 1978; Mead, 1985). This mode is characterized by rotary motion of the cross-section and zero deflection of the neutral axis.

The corresponding phase and group velocities are (see appendix A for physical meaning of phase and group velocity)

\[ C_{1,2}^{ph} = \frac{c_{1,2}^{ph}}{c_S} = \frac{\Omega}{K_{1,2}} \pm \sqrt{2} \sqrt{\left( \frac{1}{k_G} + R \right)^2 + 4R \Omega^2} \]  

(3.23)

\[ C_{1,2}^{gr} = \frac{c_{1,2}^{gr}}{c_S} = 2K_{1,2} \frac{1}{\Omega} \left( \frac{1}{k_G} + R \right) \pm \sqrt{\left( \frac{1}{k_G} - R \right)^2 + 4R \Omega^2 + \frac{2R \Omega^2}{\left( \frac{1}{k_G} - R \right)^2 + 4R \Omega^2}} \]  

(3.24)

Eqns (4.4) and (4.5) imply that, for \( R \leq 1/k_G \approx 1 \), \( C_1^{ph} \to \sqrt{k_G} = C_T^B \) and \( C_2^{ph} \to 1/\sqrt{R} = C_L \) as \( \Omega \to \infty \) and \( K \to \infty \), where \( C_T^B = c_T^B / c_S \) and \( C_L = \)
Figure 3.2: Dimensionless wave numbers $K_{1,2} = k_{1,2}r_g$ vs. dimensionless frequency $\Omega = \omega r_g/c_S$ for rectangular cross-section ($k_G = 5/6$) and $R = 0.01, 0.1, 0.5$ and $1$. $c_L/c_S$ are the dimensionless shear and longitudinal wave velocities in the beam. Hence, $C_{1ph}^p$ approaches the same limit for all $R \leq 1/k_G$, while $C_{2ph}^p$ approaches different limits, depending on $R$. Because $\sqrt{k_G} < 1$, asymptotically $C_{1ph}^p < C_{2ph}^p$ for all $R \leq 1/k_G$. However, for $R > 1/k_G$, the limits reverse, $C_{1ph}^p \to 1/\sqrt{R}$ and $C_{2ph}^p \to \sqrt{k_G}$, but still $C_{1ph}^p < C_{2ph}^p$ in the limit. The group velocities have same limit as the corresponding phase velocities. Typically for buildings, the phase velocities follow regime $R \leq 1/k_G$, and, therefore, we show results only for $R \leq 1/k_G$.

Fig. 3.3 (a-d) shows phase and group velocities of the propagating waves versus $\Omega$ [Fig. 3.3 (a and c)] and versus $K$ [Fig. 3.3 (b and d)], for $R = 0.01, 0.1, 0.5$ and $1$ (same as in Fig. 3.2). The results are plotted up to higher value of $\Omega$ to illustrate...
the asymptotic trends of Eqns (4.4) and (4.5), discussed in the previous paragraph. However, they are not physically meaningful for very large $\Omega$, because of violations of the assumptions of the theory. The limitations of the Timoshenko beam theory have been tested by comparison of dispersion curves with those obtained from truncated series solutions of exact elasticity theory, and comparison of predicted and experimentally measured natural frequencies (Hutchinson, 1981; Mead, 1985; Stephen and Puchegger, 2006). Comparison of dispersion curves has shown that the first branch follows exactly the elasticity solution while the second branch starts to deviate from the elasticity solution at higher frequencies and has a different asymptote (Mead, 1985). For a free-free rod of circular cross-section, Hutchinson (1981) found $\Omega = 1.3$ to be the limit of validity of Timoshenko beam theory.

Region $\Omega < \Omega_{cr} < 1.3$ is of interest for this study, because difficulties in resolving the two propagating modes are avoided. As $\omega_{cr} = \sqrt{k_G c_s / r_g}$, this region is more broadband when $c_S$ is larger and $r_g$ is smaller. It is interesting that this region does not depend on $R$. Also of interest is the degree of variation of the first branch. Fig. 3.3 shows that the deviation of $c^{ph}_1$ from its asymptotic value $\sqrt{k_G c_s}$ is greater for larger $R$ and smaller $\Omega = \omega r_g / c_s$. The degree of variation of $c^{ph}_1$ near the modal frequencies will depend on $R$ and the mode position on the $\Omega = \omega r_g / c_s$ axis, and is the most for the first mode.

### 3.3.2 Transfer-Functions and Impulse Responses

It is also of interest how many modes of vibration are contained in the interval, and how the energy is partitioned between the propagating and evanescent waves. Fig. 3.4 shows the coefficients of expansion $D_i$, $i = 1, \ldots, 4$, normalized by $C_1$ (parts a), c) and e)), and transfer-functions (parts b), d) and f)) vs. $\Omega$ for fixed value of $H/r_g = 6$ and different values of $R = 0.01$, 0.5 and 1 ($H/r_g = 6$ and $R = 0.5$
Figure 3.3: (a) and (b) Dimensionless phase velocities $C_{\text{ph}}^{1,2} = c_{\text{ph}}^{1,2}/c_S$; (c) and (d) group velocities $C_{\text{gr}}^{1,2} = c_{\text{gr}}^{1,2}/c_S$ of propagating waves versus dimensionless frequency $\Omega = \omega r_g/c_S$ (a and c) and dimensionless wave number $K_{\text{1,2}} = k_{\text{1,2}} r_g$ (b and d), for same parameters as in Fig. 3.2

correspond roughly to Millikan library. Fig. 3.5 shows the same quantities but for a fixed value of $R = 0.5$ and different $H/r_g = 4, 8$ and 16. The coefficients were computed for zero damping, while the transfer-functions were computed for $M = 0.02$. Readings of the modal frequencies and their ratios are listed in Tables 3.1 and 3.2. It is seen that, for larger $R$ and $H/r_g$, the modes start to occur at lower frequencies, and therefore are more affected by dispersion. Also, more
modes fall in the interval $\Omega < \Omega_{cr}$. The degree of dispersion is also reflected in the degree of deviation of the frequency ratios from those for a uniform shear beam. The plots of the coefficients show that, for small $R$, the displacement is essentially $\cos k_1z$ for all $\Omega$. When $R$ is not small, the $\cos k_1z$ term dominates for low $\Omega$, but the $\sin k_1z$ term starts to dominate for high enough omega, up to $\Omega_{cr}$. The amplitudes of the evanescent waves decrease with frequency and practically diminish near $\Omega_{cr}$. Beyond $\Omega_{cr}$, these plots suggest complex interference patterns of the two propagating waves, and that first branch dominates the response also beyond $\Omega_{cr}$ for the smaller and moderate values of $R$ (0.01 and 0.5).

Figs. 3.6 and 3.7 show impulse responses, obtained from the transfer-functions in Figs. 3.4 and 3.5, versus dimensionless time $\bar{t} = t c_s / r_g$, at several levels in the beam, for virtual source at base [Figs. 3.6 and 3.7 (a, c, and e)] and at roof [Figs. 3.6 and 3.7 (b, d, and f)], and low pass filtered at $\Omega = 0.85 \approx \Omega_{cr}$, so that the second propagating wave is filtered out. The virtual source is marked as well as some of the ray paths. The propagation of the source pulse can clearly be seen, as well as its progressive deformation due to dispersion for larger $R$, larger $H/r_g$, and further from the source. For smaller $R$ and $H/r_g$, the pulse shape remains close to a sinc function. The plots for virtual source at base [Figs. 3.6 and 3.7 (a, c, and e)], show multiple arrivals of the pulse at the roof with alternating sign, following reflection from the base. In contrast, the plots for virtual source at roof [Figs. 3.6 and 3.7 (b, d, and f)], show only two propagating pulses, causal and acausal, the reflections from the base being suppressed by the condition that the roof motion must be zero at all times except during the virtual source pulse (Snieder and Safak, 2006). Therefore, they are more convenient for identification. Despite the presence of dispersion, dominant downward propagating causal and acausal pulses can be clearly identified and followed for all the values of the parameters considered,
except for the case $H/r_g = 16$ and $R = 0.5$ [Fig. 3.7 (f)], and where the causal and acausal pulses could not be resolved. Even for that same case, by reducing the cut-off frequency, impulse responses with dominant pulses can be obtained.

Following the derivation of the spatial resolution of the wave method proposed by Rahmani and Todorovska (2013) for the shear beam model, the minimum height
of a Timoshenko beam that can be resolved for $\omega < \omega_{cr}$ is derived from the condition that the separation of the causal and acausal pulses has to be at least half of the width of the source pulse. This implies that the minimum beam height that can be resolved is

$$H_{\text{min}} = \frac{1}{4f_{\text{max}}} \max_{f<f_{\text{max}}} (c_{1}^{ph}, c_{1}^{aq}) \approx \frac{1}{4} \lambda_{TB}^{S}(f_{\text{max}})$$

(3.25)
Figure 3.6: Impulse response functions at different levels in the beam for virtual source (a), (c), and (e) at the base and (b), (d), and (f) at the top, versus dimensionless time $\bar{t} = t c_s / r_g$, for fixed $H / r_g = 6$ and different $R = 0.01$, 0.5 and 1.

where $f_{\text{max}}$ is the bandwidth of the impulse response functions, and $\lambda_{TB}^S(f_{\text{max}})$ is the corresponding wavelength of shear waves in the Timoshenko beam. In a similar
Figure 3.7: Same as Fig. 3.6 for fixed $R = 0.5$ and different $H/r_g = 4, 8, \text{ and } 16$

... fashion, the minimum beam height to resolve the two branches of propagating waves can be derived.
Table 3.1: Dimensionless frequencies of vibration of TB for $R$ = 0.01, 0.5 and 1, and fixed $H/r_g = 6$

<table>
<thead>
<tr>
<th></th>
<th>$R = 0.01$</th>
<th></th>
<th>$R = 0.5$</th>
<th></th>
<th>$R = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega_i$</td>
<td>$\Omega_i/\Omega_1$</td>
<td>$\Omega_i$</td>
<td>$\Omega_i/\Omega_1$</td>
<td>$\Omega_i$</td>
</tr>
<tr>
<td>0.2324</td>
<td>1.0</td>
<td></td>
<td>0.1152</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.6992</td>
<td>3.0</td>
<td></td>
<td>0.4258</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>1.1880</td>
<td>5.1</td>
<td></td>
<td>0.8848</td>
<td>7.7</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Dimensionless frequencies of vibration of TB for $H/r_g = 4$, 8 and 16, and fixed $R = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>$H/r_g = 4$</th>
<th></th>
<th>$H/r_g = 8$</th>
<th></th>
<th>$H/r_g = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega_i$</td>
<td>$\Omega_i/\Omega_1$</td>
<td>$\Omega_i$</td>
<td>$\Omega_i/\Omega_1$</td>
<td>$\Omega_i$</td>
</tr>
<tr>
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<td>1.0</td>
<td></td>
</tr>
<tr>
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<td>0.2949</td>
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<td></td>
</tr>
<tr>
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<td>5.6</td>
<td></td>
<td>0.6328</td>
<td>9.0</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>0.9102</td>
<td>12.9</td>
<td></td>
</tr>
<tr>
<td>2.072</td>
<td>9.2</td>
<td></td>
<td>1.139</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>2.366</td>
<td>10.5</td>
<td></td>
<td>1.253</td>
<td>18.0</td>
<td></td>
</tr>
</tbody>
</table>

3.3.3 Model Validation

The model is validated on Millikan Library, a 9 story RC building in Pasadena, California, and records of the Yorba Linda, 2002 earthquake (M=4.8, R=40 km), studied earlier by Snieder and Şafak (2006) and Rahmani and Todorovska (2013) (Fig. 3.8). It has plan dimensions 21×23 m, and height 43.9 m above the ground and 48.2 meters above basement level. Lateral resistance to seismic forces is provided by RC moment resisting fame, and shear walls (external for the NS response, and internal for the EW response, forming a central core). The added shear stiffness of these walls increases the relative contribution of bending to the total response.
A clip of the photo of the building is shown in Fig. 3.8(b), and a sketch of a typical floor layout, with marked sensor locations, is shown in Fig. 3.8(j).

The input parameters of the model, $H$, $r_g$, $c_S$, $R$ and $\mu$, are determined as follows. We determined $H$ and $r_g$ from the building geometry, assuming that the mass and stiffness are uniformly distributed over a rectangular cross-section, and $\mu$, $c_S$, and $R = G/E$ by matching approximately (by trial and error) the model and observed transfer-functions (TF) between roof and ground level accelerations. We note that $R = G/E = 1/[2(1 + \nu)]$ does not corresponds to any of the individual construction materials, but to the building as a whole. Because the observed TFs are those of the soil-structure system while the model is fixed-based, the observed fundamental mode frequencies were corrected for shift due to soil-structure interaction, based on values from soil-structure identification study of forced vibration test data (Luco et al., 1988). The input parameters are listed in Table 3.3. The critical frequency for the model is $f_{cr} = 17.2$ Hz for the NS and 11.5 Hz for the EW response, and the velocity of shear waves in the beam, $c_{TB} = \sqrt{\frac{k_G}{c_S}}$, is $c_{TB} = 653$ m/s for the NS and 480 m/s for the EW response (also listed in Table 3.3). The model and observed system functions are compared in Fig. 3.8. Fig. 3.8(a and b) show comparison of the transfer-functions for the NS and EW responses, and Fig. 3.8(e-i) show comparison of the corresponding impulse response functions in different bands, containing one and two modes of vibration. Very good qualitative agreement of the impulse responses can be seen, which demonstrates that the model represents well pulse propagation in the actual building. Fig. 3.8 (c and d) show the model phase, group and shear wave velocities ($c_{ph1,2}, c_{gr1,2}$ and $c_{TB}^S$), which are compared with the estimates of pulse velocities, obtained by fitting a shear beam in low-pass filtered impulse response functions (Rahmani and Todorovska, 2013). These estimates are: 405 m/s for NS response, in band (0-15 Hz), and
243 m/s and 347 m/s for the EW response, in bands (0-7.5 Hz) and (7-15 Hz), shown by horizontal lines over the respective intervals. The estimate by Snieder and Şafak (2006) of 322 m/s for the NS response, obtained essentially from the first apparent frequency and referred to as shear wave velocity, is also shown by a dot. It can be seen that the empirical estimates fall between the model group and phase velocities over the respective intervals, and are considerably lower than the beam shear wave velocity, \( c_{SB} \), the high frequency asymptote. While this comparison suggests rough agreement of the single value estimates with theoretical curves, more detailed analyses are required to understand how to relate quantitatively such estimates to Timoshenko beam parameters.

Table 3.3: Properties of the uniform Timoshenko beam models of Millikan library

<table>
<thead>
<tr>
<th>TB model</th>
<th>W (m)</th>
<th>H (m)</th>
<th>( r_g ) (m)</th>
<th>( c_s ) (m/s)</th>
<th>( R = G/E )</th>
<th>( \mu ) (s)</th>
<th>( f_{cr} ) (Hz)</th>
<th>( c_{SB}^T ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>21</td>
<td>39</td>
<td>6.06</td>
<td>716</td>
<td>0.42</td>
<td>0.002</td>
<td>17.2</td>
<td>653</td>
</tr>
<tr>
<td>EW</td>
<td>23</td>
<td>39</td>
<td>6.64</td>
<td>526</td>
<td>0.82</td>
<td>0.003</td>
<td>11.5</td>
<td>480</td>
</tr>
</tbody>
</table>

3.4 Discussion and Conclusions

Timoshenko beam was used to understand how the presence of bending in the deformation of high rise buildings during seismic response may affect its identification from impulse responses in terms of velocities of wave propagation. In homogeneous or layered shear beam models, fitted previously in building response to estimate its wave velocities, there is only one propagating wave and dispersion only due to damping. A significant difference in the wave properties of a Timoshenko beam is the existence of a second wave, which is evanescent for frequencies below critical frequency, \( \omega_{cr} = \sqrt{k_G} c_s/r_g \), and a propagating wave otherwise,
adding complexity to the wave propagation. The first wave in the band $\omega < \omega_{cr}$ is of greater interest for wave based structural system identification and health monitoring, because it requires simpler estimation. This band is broader if the building is stiffer in shear and has smaller radius of gyration. The number of modes it contains further depends on the building axial stiffness and height, and is larger for larger $R$ and $H/r_g$. The further discussion focuses on the first wave, and applied to $R \leq 1$, which is typical for buildings.

Another significant difference in the wave properties is the existence of dispersion even for zero damping. The dispersion is controlled by two independent parameters, the dimensionless frequency $\Omega = \omega r_g/c_s$ and $R$. The phase velocities deviate more from their high frequency asymptote, the velocity of shear waves in the beam $c_s^{TB} = \sqrt{k_G c_s}$, for smaller $\Omega$ and larger $R$. Consequently, the modal frequencies are affected more by dispersion for lower modal number, larger $R$, and larger $H/r_g$ (for which they start occurring at lower $\Omega$). This is illustrated in 3.9 showing $\Omega_{n+1}/\Omega_n$ for the first few modes based on values in Table 3.2. The ratio of the second to first modal frequencies is most affected by dispersion, and the degree of its deviation from 3 (the value for a shear beam) is often used to assess the nature of a building from measured frequencies of vibration (e.g. Boutin et al., 2005). However, this ratio is also most affected by soil-structure interaction (SSI), which also lowers the fundamental frequency (Luco et al., 1988; Todorovska, 2009b). Consequently, ignoring the effects of SSI in assessing the nature of the beam from the ratio of the first two observed (apparent) frequencies would overestimate the degree of bending, and, likewise, ignoring dispersion in system identification would overestimate the SSI effects. Separating the two effects without additional information would be difficult. Similar conclusion was reached
earlier by Rahmani and Todorovska (2013) in a system identification analysis of Millikan library involving fitting a nondispersive model.

The presented results for impulse response functions for a wider range of dimensionless parameters, corresponding to buildings, showed that, despite the dispersion, prominent pulses can be identified in the impulse responses in selected frequency bands, which could be used for SHM. Further investigation is needed of the relationship between the measured pulse velocities and the phase and group velocities, and the reliability of monitoring the pulse velocity.

Based on the presented parametric study and the full-scale building example, we conclude that Timoshenko beam is a realistic model to study wave propagation in high rise buildings, which can provided valuable insight in dispersive wave propagation in buildings, useful for interpretation of recorded earthquake response of full-scale buildings, development of wave based SHM methods, as well as testing numerical models for pulse propagation in buildings.

3.5 Acknowledgments

The strong motion data for Millikan library were provided by the National Strong Motion Program of the U.S. Geological Survey, and are accessible from the Engineering Center for Strong Motion Data (www.strongmotioncenter.org/). The authors appreciate insightful discussions on the behavior of Millikan library with F. Udwadia, T. Heaton and M. Trifunac, and also the constructive comments of two of the anonymous reviewers which significantly improved the manuscript.
Figure 3.8: Model validation on Millikan library, Yorba Linda, 2002 earthquake: (a), (b) model and observed TFs; (c) model phase and (d) group velocities, (e)-(i) model and observed impulse response functions in different bands; (j) sketch of a typical floor layout [adapted from Snieder and Safak (2006)]; a photo of the building is shown in the corner of (b) (courtesy of M. Trifunac)
Figure 3.9: Frequency ratios $\Omega_{n+1}/\Omega_n$ based on Table 3.2
Chapter 4

Non-parametric Estimation of Wave Dispersion

This chapter is based on the article “nonparametric estimation of wave dispersion in high-rise buildings by seismic interferometry” published in *Earthquake Engineering & Structural Dynamics* (Ebrahimian et al., 2014).

Interferometric identification and health monitoring of high-rise buildings has been gaining increasing interest in recent years. The wave dispersion in the structure has been largely ignored in these efforts, but needs to be considered to further develop these methods. In this paper, (i) the goodness of estimation of vertical wave velocity in buildings, as function of frequency, by two nonparametric interferometric techniques is examined, using realistic fixed-base Timoshenko beam benchmark models. Such models are convenient because the variation of phase and group velocities with frequency can be derived theoretically. The models are those of the NS and EW responses of Millikan library. One of the techniques, deconvolution interferometry, estimates the phase velocity on a frequency band from phase difference between motions at two locations in the structure, while the other one estimates it approximately at the resonant frequencies based on standing wave patterns. The paper also (ii) examines the modeling error in wave velocity profiles identified by fitting layered shear beam in broader band impulse response functions of buildings with significant bending flexibility. This error may affect
inferences on the spatial distribution of damage from detected changes in such velocity profiles.

4.1 Introduction

The first objective of this paper is to examine the opportunities and limitations of nonparametric identification of vertical wave velocities in high rise buildings, in which wave propagation is dispersive, by seismic interferometry (Curtis et al., 2006; Snieder and Şafak, 2006). The wave velocities and modes of wave propagation are fundamental properties characterizing the structure, as are the natural frequencies of vibration and mode shapes. The focus is on dispersion in the lower frequency range, such as one caused by bending deformation, which may occur e.g. in buildings with shear-wall or moment-frame/shear-wall lateral force resisting system. The estimation method, which does not require an assumption about the nature of the dispersion, is first presented, followed by its application to simulated response by fixed-base Timoshenko beam (TB) models of the NS and EW responses of Millikan library, for which the dispersion curves can be computed analytically (Timoshenko, 1921; Mead, 1985; Ebrahimian and Todorovska, 2014c). The TB model accounts for deformation in shear and bending as well as for the effects of rotatory inertia. The errors and biases in the nonparametric estimation are assessed by comparison of the measured velocities in different frequency bands with the theoretical dispersion curves. Knowledge of the goodness of nonparametric estimation of wave dispersion in buildings is important for its potential use for structural health monitoring (SHM). To the knowledge of the authors, it has not been addressed previously.
The second objective of this paper is to investigate the modeling error in identified vertical wave velocities of a building by fitting nondispersive models over broader frequency bands. For this purpose, simulated response by the same uniform Timoshenko beam models is used as input in a recently proposed interferometric method, which fits a layered shear beam by matching, in the least squares sense, traveling pulses in the impulse response functions (Rahmani and Todorovska, 2013). The distortions in the identified velocity profiles are analyzed. This modeling error has not been addressed previously.

In the companion paper (Rahmani et al., 2015b), the same nonparametric estimation is applied to a simple soil-structure interaction (SSI) model with coupled horizontal and rocking response (Todorovska, 2009b). The study finds wave dispersion in the soil-structure system response, and explains why identification by deconvolution interferometry is not sensitive to the SSI effects. The latter is one of the most important features of this identification method for its use for SHM.

Wave dispersion is characterized, in general, by frequency dependent phase velocity and multiple modes of propagation. Evidence of frequency dependent vertical wave velocity in a building was revealed in our earlier interferometric analysis of Millikan Library EW response (Rahmani and Todorovska, 2013). However, detailed investigation of techniques for measuring dispersion and of the modeling error in the identified profiles was difficult at that time without an appropriate benchmark. Subsequently, we proposed to use Timoshenko beam as a simple analytical model to study dispersive wave propagation in buildings (Ebrahimian and Todorovska, 2014c). We identified the model parameters that control the dispersive behavior, and studied the wave and vibrational response for a range of values of the model parameters. Measuring the dispersion, however, was out of the scope of that study. In this paper, we use the TB model as a convenient benchmark to
investigate the accuracy of nonparametric interferometric techniques for measuring dispersion.

The linear seismic response of a building can be alternatively represented as superposition of its modes of vibration, characterized by their frequency, or as superposition of waves, characterized by their velocity of propagation (Kanai and Yoshizawa, 1963). The latter approach has some advantages for prediction of response to impulsive loads as well as for structural health monitoring and has been gaining increasing interest (Snieder and Şafak, 2006; Ebrahimian and Todorovska, 2014c; Rahmani and Todorovska, 2013; Rahmani et al., 2015b; Todorovska, 2009b; Safak, 1999; Kawakami and Oyunchimeg, 2004; Kohler et al., 2007; Todorovska and Trifunac, 2008a,b; Trifunac et al., 2010; Gičev and Trifunac, 2009b,a; Prieto et al., 2010; Todorovska, 2009c; Michel et al., 2011; Picozzi et al., 2011; Gičev and Trifunac, 2012; Todorovska and Rahmani, 2013; Rahmani and Todorovska, 2014, 2015; Nakata et al., 2013; Nakata and Snieder, 2014). In both the forward and inverse analyses, with the exception of (Ebrahimian and Todorovska, 2014c), it was assumed that the building deforms in shear only, and that the wave propagation is predominantly one-dimensional, vertically through the structure. While these assumptions may be appropriate for structures that behave like shear beam, and for exploratory data analyses and initial method development, in general, they may be a source of significant modeling error. It should also be mentioned that bending deformation is only one of the possible causes of dispersion in buildings. Other possible causes are, e.g., lateral reflections from the building boundaries and nonhomogeneities, as studied previously on models (Todorovska and Trifunac, 1989; Todorovska and Lee, 1989). Wave dispersion in a real building during different earthquakes, including one that caused damage, has been studied in (Todorovska et al., 2001b; Trifunac et al., 2003).
In this paper, on a realistic Timoshenko beam model of a specific building, we assess: (i) the degree of variation of the phase and group velocities with frequency over the recording range of strong motion instruments, (ii) the frequency bands where a single and multiple wave propagation modes exist, (iii) how the nonparametric estimate of wave velocity are related to the actual dispersion curves, and (iv) the modeling error in fitting a layered shear beam (SB) model into recorded response of buildings with shear walls. Resolving these issues is useful for further improvement of the existing wave methods for structural health monitoring (SHM) (Rahmani and Todorovska, 2013; Safak, 1999; Todorovska and Trifunac, 2008a,b). This paper is organized as follows. In the Methodology section, a brief review of Timoshenko beam theory is presented, followed by a summary of two interferometric techniques for measurement of wave velocity. In the results section, the TB models are described and validated. Then, results are presented of the identified wave velocities and their comparison with the theoretical dispersion curves. Subsequently, a 3-layer shear beam model is fitted in the impulse responses of the TB models, and artifacts in the identified velocity profile are analyzed. Finally, the conclusions drawn based on this study are presented.

4.2 Methodology

4.2.1 Theoretical dispersion relations for a Timoshenko beam building model

The building model we consider is a uniform, cantilever Timoshenko beam (TB), stress free at the top and excited by horizontal motion at the base (Ebrahimian and Todorovska, 2014c). The beam has height $H$, and its cross-section is characterized by area $A$, second moment of inertia $I$ and shear factor $k_G$. Its material is
characterized by mass density $\rho$, Young’s modulus $E$ and shear modulus $G$, which implies longitudinal and shear wave velocities in the material $c_L = \sqrt{E/\rho}$ and $c_S = \sqrt{G/\rho}$. Kelvin-Voight damping is assumed, with viscosity constant $\mu$ for both shear and bending deformation. This section briefly summarizes the model dispersion characteristics and frequency domain solution. More details, following the same notation, can be found in (Mead, 1985; Ebrahimian and Todorovska, 2014c; Reis, 1978). The shear factor $k_G$ is correction for the fact that the shear stress is not uniform on the cross-sectional area $A$. It depends on the shape of cross-section, material properties and frequency (Reis, 1978; Cowper, 1966).

The horizontal displacement of the neutral axis, $u(z; t)$, due to shear and bending deformation and considering the rotatory inertia, satisfies the following differential equation

$$c_L^2 c_S^2 k_G \left(1 + \mu \frac{\partial}{\partial t}\right)^2 \frac{\partial^4 u}{\partial z^4} - \left(c_L^2 + k_G c_S^2\right) \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^4 u}{\partial z^2 \partial t^2} + \frac{k_G c_S^2}{r_g^2} \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial t^4} = 0 \quad (4.1)$$

Substituting a trial propagating wave solution of the form $e^{i(kz - \omega t)}$ leads to the dispersion relation of the beam

$$c_L^2 c_S^2 k_G(1 - i\omega \mu)^2 k^4 - (c_L^2 + k_G c_S^2)(1 - i\omega \mu)k^2 \omega^2 - \omega^2 \frac{k_G c_S^2}{r_g^2} (1 - i\omega \mu) + \omega^4 = 0 \quad (4.2)$$

where $\omega$ is the circular frequency and $k$ is the wave number ($k = \omega/c$, where $c$ is the phase velocity). The dispersion relation has four roots
\[ K = \pm K_{1,2} = \pm \frac{\Omega}{\sqrt{2}} \sqrt{\left( \frac{1}{\alpha} \right) \left( \frac{1}{k_G} + R \right) \pm \sqrt{\left( \frac{1}{\alpha^2} \right) \left( \frac{1}{k_G} - R \right)^2 + \frac{4R}{\alpha^2}}} \] (4.3)

where \( K = k r_g \) is dimensionless wave number, \( r_g = \sqrt{I/A} \) is radius of gyration, \( \Omega = \omega r_g / c_s \) is dimensionless frequency, \( R = G/E = c_S^2 / c_L^2 \) is the moduli ratio, and \( \alpha = 1 - i\Omega M \), where \( M = \mu c_s / r_g \), is dimensionless damping constant. The dimensionless phase and group velocities corresponding to \( K_{1,2}, C_{1,2}^{ph} \) and \( C_{1,2}^{gr} \), can then be computed

\[ C_{1,2}^{ph} = \frac{c_{1,2}^{ph}}{c_S} = \frac{\Omega}{K_{1,2}} \] (4.4)

\[ C_{1,2}^{gr} = \frac{c_{1,2}^{gr}}{c_S} = \frac{1}{\frac{dK_{1,2}}{d\Omega}} \] (4.5)

It can be seen from eqn (7.2) that, for an undamped beam, \( K_1 \) is always real and corresponds to a propagating wave, while \( K_2 \) is an evanescent wave up to some frequency and a propagating wave beyond that frequency. The critical frequency is \( \Omega_{cr} = \sqrt{k_G} \).

TB theory is a linear theory, which assumes small deformations and that plane sections normal to the neutral axis remain plane but not necessarily normal. The latter assumption is violated at higher frequencies, which leads to errors. Studies of the limits of TB theory suggest that it is valid up to about \( \Omega = 1.3 \) (Mead, 1985; Hutchinson, 1981; Stephen and Puchegger, 2006). For a rectangular cross-section, which is assumed in this paper, \( r_g = W / \sqrt{12} \), where \( W \) is the width, and value \( k_G = 5/6 \approx 0.83 \) has been recommended (Weaver et al., 1990). This implies
\[ \Omega_{cr} = \sqrt{kG} = 0.91. \] The analysis in this paper focuses on the region \( \Omega < \Omega_{cr}, \) in which only one propagating wave exists.

### 4.2.2 Measurement of Wave Velocity from Time Delay

Interferometry refers to the study of interference patterns of waves aiming to obtain information about the waves themselves or about their path. A review of various techniques and applications of seismic interferometry can be found in (Curtis et al., 2006).

The phase velocity, \( c, \) can be obtained from the time lag, \( \tau, \) between motions recorded at two locations, which can be obtained from their cross-correlation. Let \( x(t) \) and \( y(t) \) be the recorded motions, \( H \) be the separation distance, and \( \phi_{x,y}(t) \) be the cross-correlation between \( x(t) \) and \( y(t). \) Then

\[
c = \frac{H}{\tau}, \quad \text{(4.6)}
\]

where \( \tau \) is the time for which \( \phi_{x,y}(t) \) is maximum. The cross-correlation, \( \phi_{x,y}(t), \) and its Fourier transform, \( \hat{\phi}_{x,y}(\omega), \) are

\[
\phi_{x,y}(t) = \int_{-\infty}^{\infty} x(u)y(u+t)du \quad \text{FT} \quad \hat{\phi}_{x,y}(\omega) = \hat{x}*(\omega)\hat{y}(\omega) \quad \text{(4.7)}
\]

Let \( \hat{h}(\omega) \) be the transfer-function between the two signals and \( h(t) \) be its inverse Fourier transform

\[
\hat{h}(\omega) = \frac{\hat{y}(\omega)}{\hat{x}(\omega)} \quad \text{FT} \quad h(t) = FT^{-1}\{\hat{h}(\omega)\} \quad \text{(4.8)}
\]

Substitution of \( \hat{y}(\omega) \) from eqn. (7.3) into eqn. (4.7) gives

\[
\hat{\phi}_{x,y}(\omega) = |\hat{x}(\omega)|^2 \hat{h}(\omega) \quad \text{(4.9)}
\]
from where

$$\hat{h}(\omega) = \frac{\hat{\phi}_{x,y}(\omega)}{[\hat{x}(\omega)]^2}$$  \hspace{1cm} (4.10)

Then, the time lag $\tau$ can also be measured from the impulse response function, $h(t)$, as the argument that maximizes $h(t)$, which has some advantages because the measurement will not be affected by spectral fluctuations of the input signal (Curtis et al., 2006; Snieder and Şafak, 2006). Another rationale for measuring $\tau$ from impulse response function is that $h(t)$ represents the response of the system, at the location where $y(t)$ was recorded, to a unit impulse applied at the location where $x(t)$ was recorded.

In this study, we measure the time delay $\tau$ between the motion at level $z$ and the motion at a reference level $z_{ref}$ in a frequency band $(\omega_c \pm \Delta \omega_s)$ from band-pass filtered impulse response functions (IRF) obtained as

$$h(z, z_{ref}, \omega_c, \Delta \omega_s; t) = FT^{-1} \left\{ \hat{h}(z, z_{ref}; \omega) \hat{S}(\omega_c, \Delta \omega_s; \omega) \right\}$$  \hspace{1cm} (4.11)

where the filter $\hat{S}(\omega_c, \Delta \omega_s; \omega)$, $\omega_c, \Delta \omega_s \geq 0$ is the shifted box function

$$\hat{S}(\omega_c, \Delta \omega_s; \omega) = \begin{cases} 1, & -\omega_c - \Delta \omega_s < \omega < -\omega_c + \Delta \omega_s \\ 1, & \omega_c - \Delta \omega_s < \omega < \omega_c + \Delta \omega_s \\ 0, & \text{otherwise} \end{cases}$$

$$\overset{\text{FT}}{\longleftrightarrow} S(\omega_c, \Delta \omega_s; t) = \frac{2\Delta \omega_s \sin \Delta \omega_s t}{\pi} \frac{1}{\Delta \omega_s} \cos \omega_c t$$  \hspace{1cm} (4.12)

For the TB models in this study, we compute $\hat{h}(\omega)$ from the frequency domain solution of the response of the beam (Ebrahimian and Todorovska, 2014c). The
virtual source is the IRF at the reference point \( z_{\text{ref}} \). As it can be seen from eqns (4.11) and (4.12), the virtual source function is the filter \( S(\omega_c, \Delta \omega_s; t) \)

\[
h(z_{\text{ref}}, z_{\text{ref}}, \omega_c, \Delta \omega_s; t) = S(\omega_c, \Delta \omega_s; t) \tag{4.13}
\]

In the limit when \( \Delta \omega_s \to \omega_c \), \( S(t) \) is the box function

\[
\hat{S}(\omega_c, \Delta \omega_s; \omega) \to_{\Delta \omega_s \to \omega_c} \begin{cases} 1, & |\omega| \leq 2\omega_c \\ 0, & \text{otherwise} \end{cases} \quad \text{FT} \quad S(\omega_c, \Delta \omega_s; t) \to_{\Delta \omega_s \to \omega_c} \frac{2\omega_c \sin 2\omega_c t}{\pi \cdot 2\omega_c t} \tag{4.14}
\]

As shown elsewhere, if the medium is homogeneous and there are no internal reflections, a virtual source at the top radiates causal and acausal pulses that propagate downward (Curtis et al., 2006; Snieder and Şafak, 2006; Trampert et al., 1993). A key issue in measuring \( \tau \) is that the incident and reflected pulses, as well as pulses corresponding to different wave modes can be resolved. The resolving power of the IRFs can be assessed roughly by analyzing the spread of the source function. The spread is illustrated in Fig. 4.1, which shows \( S(t) \) and \( \hat{S}(f) \) for both low-pass and band-pass source, where \( f = \omega/(2\pi) \). As it can be seen, the low-pass \( S(t) \) is a sinc function, with half-width of the central lobe \( \Delta t_s = \pi/(2\omega_c) = 1/(4\Delta f_s) \), which is approximately localized on the phase plane in the rectangle \((t \pm \Delta t_s) \times (\omega \pm \Delta \omega_s)\). If there is only one propagating mode in the band, a rough estimate of the minimum travel distance to resolve the incident and reflected pulse in a layer is \( h_{\text{min}} = \lambda_{\text{min}}/4 \), where \( \lambda_{\text{min}} = c/f_{\text{max}} = c/(2f_c) \) is the shortest wavelength in the pulse Rahmani and Todorovska (2013); Todorovska and Rahmani (2013). The band-pass \( S(t) \) is a harmonic that is amplitude modulated by a sinc function. The half-width of the central lobe of \( S(t) \) is \( \Delta t_c = \pi/(2\omega_c) = 1/(4f_c) \) while the
half-width of the envelope is $\Delta t_e = \pi/\Delta \omega_s = 1/(2\Delta f_s)$. When $\Delta f_s/f_c \ll 1$, $S(t)$ is such that the envelope contains many oscillations, and is almost flat. However, as $\Delta f_s/f_c$ increases, the side lobes becomes progressively smaller, $S(t)$ resembling a sinc function with few of its side lobes larger than regular, and, in the limit as $\Delta f_s/f_c \to 1^-$, the central pulse of the harmonic finally transforms into the main lobe of the low-pass $S(t)$.

![Figure 4.1: Virtual source function in the frequency (left) and time (right) domains, for a box function (top) and shifted box function (bottom).](image)

In the case of a band-pass source, measuring $\tau$ is most robust from the envelope, which can be obtained from the associated analytic signal to the IRF. The propagation of the peak of the envelope can be easily traced by some automatic algorithm. If the wave propagation is dispersive, the travel time of the envelope, $\tau_e$, will give the group velocity $c^{gr} = h/\tau_e$, from which the phase velocity can be calculated subsequently, and, if it is not dispersive, $c^{ph} = c^{gr}$. Let us first consider
a region in which there is only one propagating wave in the band. Then, the minimum travel distance to resolve the envelopes of the causal and acausal pulses is
\[ h_{\text{min}} \approx c^{gr}(f_c)/(4\Delta f_s). \]
The travel distance, however, cannot be larger than the building height, \( H \), and the width of the subband, \( \Delta f_s \), cannot be larger than the central frequency, \( f_c \). Then, \( c^{gr}(f_c) \) can be measured only if \( \Delta f_s \) is large enough, satisfying the following condition
\[
c^{gr}(f_c)/(4H) < \Delta f_s < f_c \tag{4.15}
\]
There is, however, a practical limit on how large \( \Delta f_s < f_c \) can be, imposed by the existence of higher propagation modes, which may also need to be resolved. In this paper, we choose bands that do not extend beyond the critical frequency. The aforementioned discussion implies that, for RC buildings with shear walls, which are stiff structures, characterized by larger wave velocities and shorter propagation time, measuring the group velocity by this nonparametric technique may not be always possible. Therefore, in the Results section, we attempt to measure the phase velocity by tracing a point with the same phase (the peak of the central lobe in wider subbands).

### 4.2.3 Measurement of Wave Velocity from Resonant Frequencies

The phase velocities and natural frequencies of vibration are closely related. For a uniform shear beam, the relationship is
\[
\frac{\omega_n}{c} H = \frac{(2n - 1)\pi}{2}, \quad n = 1, 2, \ldots \tag{4.16}
\]
Because of dispersion and differences in mode shapes, this relationship is not strictly correct even for a TB, but could be used to get rough estimates of phase velocity at the resonant frequencies, by fitting a uniform shear beam in each of the modes. Then, for the \( n \)-th mode

\[
c(f_n) = \frac{4Hf_n}{2n-1}, \quad n = 1, 2, \ldots
\]  \hspace{1cm} (4.17)

where \( c(f_n) \) is the velocity of a uniform fixed-base shear beam that has the same \( n \)-th mode frequency as the building. Because of dispersion, these estimates will vary with frequency, and will give approximately the shape of the dispersion curve.

### 4.3 Results and Analysis

#### 4.3.1 Approximate Timoshenko Beam Models of Millikan Library and Their Validation

Millikan Library is a densely instrumented 9-story reinforced concrete (RC) building in Pasadena, California (Snieder and Şafak, 2006; Rahmani and Todorovska, 2013; Todorovska and Rahmani, 2013; Luco et al., 1988). It has plan dimensions 21×23 m, and height 43.9 m above the ground and 48.2 meters above basement level. Its lateral force resisting system is a combination of a moment resisting frames and shear walls. The shear walls providing resistance to NS motions are located at the east and west ends, while those providing resistance to EW motions form a central core where the elevators are located. Because of the added shear stiffness by these walls, the relative contribution of bending deformation to the total response is larger than what it would have been otherwise. Fig. 4.2 shows a clip of a photo of the building (courtesy of M. Trifunac), and a sketch of a typical
Table 4.1 shows the parameters of approximate, fixed-base, uniform Timoshenko beam models of the NS and EW responses, obtained based on the building geometry and acceleration records of Yorba Linda earthquake of 2002 ($M=4.8$, $R=40$ km). The beam height, $H$, was chosen based on the roof height above ground level, where the highest sensor is located. The radius of gyration $r_g$ was estimated based on the dimensions of the building cross-section, assuming uniform distribution of rigidity. Value $k_G = 5/6$ was assumed, which is normally used for rectangular cross-section (Weaver et al., 1990). For the damping constant, $\mu$, small reasonable values were assumed. The material constants, $c_S$ and $R$, were determined by matching the frequencies of the first two modes, after correcting the observed (apparent) frequency of the first mode for SSI effects, based on findings of (Luco et al., 1988). The target fixed-base fundamental mode frequency of the TB model was assigned value that is higher than the observed apparent frequency by 22% for the NS and 12% for the EW responses. No adjustment was made to the observed second mode frequency, as it is known from studies of mathematical models that the frequency of the second and higher modes is not affected much by soil-structure interaction (Todorovska, 2009b; Bielak, 1971; Todorovska, 1993a; Fukuwa and Ghannad, 1996). Table 1 also shows the critical frequency, $f_{cr} = (1/2\pi)\sqrt{k_G c_S / r_g}$, derived from the model parameters, and the shear wave velocity in the beam, $c_S^{TB} = \sqrt{k_G c_S}$ (see section 4.2.1). We refer to these models as TB model NS and TB model EW.

Fig. 4.2 shows the model transfer-functions (part a), top), phase and group velocities (part a), bottom), and impulse response functions at four levels (ground floor, third floor, sixth floor and roof) in different frequency bands (part b)). The
Figure 4.2: Validation of the TB Models of Millikan library (Table 1) with the Yorba Linda, 2002 earthquake data. a) Model and observed transfer-function (top) and model phase and group velocities (bottom). A photo of the building is also shown (courtesy of M. Trifunac). b) Model and observed impulse response functions in different bands for the NS (top) and EW (bottom) responses. A typical floor layout is also shown in the corner (redrawn from Snieder and Şafak (2006))
Table 4.1: Properties of approximate uniform TB models of Millikan library NS and EW fixed-base responses

<table>
<thead>
<tr>
<th>TB Model</th>
<th>W (m)</th>
<th>H (m)</th>
<th>$r_g$ (m)</th>
<th>$c_S$ (m)</th>
<th>$R = \frac{G}{E}$ (s)</th>
<th>$\mu$ (s)</th>
<th>$f_{cr}$ (Hz)</th>
<th>$(c_s)_{TB}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>21</td>
<td>39</td>
<td>6.06</td>
<td>716</td>
<td>0.42</td>
<td>0.002</td>
<td>17.2</td>
<td>653</td>
</tr>
<tr>
<td>EW</td>
<td>23</td>
<td>39</td>
<td>6.64</td>
<td>526</td>
<td>0.82</td>
<td>0.003</td>
<td>11.5</td>
<td>480</td>
</tr>
</tbody>
</table>

observed transfer-functions and IRFs are also shown on the same graphs for comparison. Good qualitative agreement of the IRFs can be seen, which indicates that Timoshenko beam is a good physical model for this building. Further comparison is presented between the model phase and group velocities and published estimates for this building. The horizontal lines represent estimates from travel time measured from IRFs over selected frequency bands, reported in Rahmani and Todorovska (2013) (see Section 4.2.2), and the dot represents an estimate obtained from the fundamental apparent frequency, reported in Snieder and Šafák (2006) (see Section 4.2.3). It can be seen that the measured velocities are within the range of the beam phase and group velocities, and that dispersion is one of the reasons for the significant difference in the two estimates (Rahmani and Todorovska, 2013).

Table 4.2 shows the frequencies of vibration, $f_i$, where $i$ is the mode number, of the first three vibrational modes of the models, measured from transfer-functions, and the values of the phase and the group velocities at the modal frequencies, $c_{1ph}(f_i)$ and $c_{1gr}(f_i)$ (see eqns (4.4) and (4.5)). The ratios $f_i/f_1$, $c_{1ph}(f_i)/(c_S)_{TB}$ and $c_{1gr}(f_i)/(c_S)_{TB}$ area also listed, where $(c_S)_{TB}$ is the velocity of shear waves in the beam, and the high frequency asymptote for the first propagation mode. It can be seen that the frequency ratios for the TB models (1:3.7:7.7 for TB model NS, and 1:3.9:8.3 for TB model EW) are larger than the corresponding ratios for a pure shear beam (1:3.5:7). It can also be seen that the phase and group velocities
increase with frequency, but at progressively smaller rate for the higher modes, approaching a horizontal asymptote.

Table 4.2: Theoretical modal frequencies, \( f_i \), and phase and group velocities at modal frequencies, \( c_{ph}^i(f_i) \) and \( c_{gr}^i(f_i) \), for TB models NS and EW

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>( f_i ) (Hz)</th>
<th>( \frac{f_i}{f_1} )</th>
<th>( c_{ph}^i(f_i) ) (m/s)</th>
<th>( \frac{c_{ph}^i(f_i)}{c_s} )</th>
<th>( c_{gr}^i(f_i) ) (m/s)</th>
<th>( \frac{c_{gr}^i(f_i)}{c_s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS TB Model</td>
<td>1</td>
<td>2.07</td>
<td>1.0</td>
<td>276</td>
<td>0.42</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.67</td>
<td>3.7</td>
<td>458</td>
<td>0.70</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15.95</td>
<td>7.7</td>
<td>554</td>
<td>0.85</td>
<td>2.0</td>
</tr>
<tr>
<td>EW TB Model</td>
<td>1</td>
<td>1.24</td>
<td>1.0</td>
<td>165</td>
<td>0.34</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.85</td>
<td>3.9</td>
<td>286</td>
<td>0.60</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.24</td>
<td>8.3</td>
<td>359</td>
<td>0.75</td>
<td>2.2</td>
</tr>
</tbody>
</table>

4.3.2 What is the Meaning of the Pulse Velocity in Low-Pass Filtered IRFs?

In view of the fact that the phase velocity in a TB changes with frequency, a fundamental question that arises is how the single value estimates obtained from pulse travel time in broader-band IRFs are related to the actual velocities. Are they representative of the central value over the band or biased towards either part of the spectrum? We address this question graphically, first by comparing low-pass filtered (broader band) IRFs with IRFs in its subbands, all below critical frequency (Fig. ??a), top), and then comparing the identified wave velocities from pulse travel time in the low-pass filtered IRFs with the theoretical dispersion curves (Fig. 4.3b)). Table 4.3 (left) shows identified wave velocities \( c_{eq}^P \) in three low-pass bands, containing one, two and three modes of vibration, and their comparison
with the shear wave velocity of the TB, $c_{TB}^S$. The superscript $D$ indicates identification directly from the pulse time shifts (the Direct Algorithm (Todorovska and Rahmani, 2013)). The average of the travel times of the causal and acausal pulses over the height of the beam, $\tau$, is also listed. For the purpose of this numerical experiment, TB models with smaller damping were used ($\mu = 0.0005$ s) than the values listed in Table 4.1 in order to make visible the third mode in the TF. The transfer-functions of the actual models are shown in the bottom of part b). The measured pulse velocities are plotted in Fig. 4.3b as horizontal bars spanning the band.

Table 4.3: Measured velocities, $c_{eq}^D$, of TB models NS (top) and EW (bottom) from pulse travel time, $\tau$, in different frequency bands.

<table>
<thead>
<tr>
<th>Band (Hz)</th>
<th>$\tau$ (s)</th>
<th>$c_{eq}^D$ (m/s)</th>
<th>$c_{eq}^D/c_{TB}^S$</th>
<th>Band (Hz)</th>
<th>$\tau$ (s)</th>
<th>$c_{eq}^D$ (m/s)</th>
<th>$c_{eq}^D/c_{TB}^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NS TB Model:</strong> $c_{TB}^S = 653$ m/s, $H = 39$ m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-5</td>
<td>0.12</td>
<td>325</td>
<td>0.50</td>
<td>0-5</td>
<td>0.12</td>
<td>325</td>
<td>0.50</td>
</tr>
<tr>
<td>0-13</td>
<td>0.09</td>
<td>433</td>
<td>0.66</td>
<td>5-13</td>
<td>0.09</td>
<td>433</td>
<td>0.66</td>
</tr>
<tr>
<td>0-17</td>
<td>0.085</td>
<td>459</td>
<td>0.70</td>
<td>13-17</td>
<td>0.0725</td>
<td>538</td>
<td>0.82</td>
</tr>
<tr>
<td><strong>EW TB Model:</strong> $c_{TB}^S = 480$ m/s, $H = 39$ m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-3</td>
<td>0.205</td>
<td>190</td>
<td>0.40</td>
<td>0-3</td>
<td>0.205</td>
<td>190</td>
<td>0.40</td>
</tr>
<tr>
<td>0-7.5</td>
<td>0.155</td>
<td>252</td>
<td>0.52</td>
<td>3-7.5</td>
<td>0.150</td>
<td>260</td>
<td>0.54</td>
</tr>
<tr>
<td>0-11</td>
<td>0.133</td>
<td>294</td>
<td>0.61</td>
<td>7.5-11</td>
<td>0.118</td>
<td>332</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Visual comparison of peaks of the broader band IRF with the peaks of the IRFs over the subbands (Fig. 4.3a) shows that they occur in time much closer to the peaks corresponding to the higher frequency subbands. Therefore, for this particular shape of dispersion curves (characterized by phase velocity that increases with frequency but at a progressively smaller rate and eventually levels off), the measured velocity from pulse time shifts is biased towards the higher values of
Figure 4.3: Identification of the fixed-based TB models of Millikan library from pulse travel time in low-pass filtered IRFs. (a) Comparison of low-pass and band-pass filtered IRFs. (b) Comparison of the identified equivalent shear-beam velocity $c_{eq}^D$ (Table 4.3, left) with the theoretical dispersion curves (top) and the model TFs (bottom).
velocity on the band, which are representative of the beam response at the higher frequencies in the band.

4.3.3 Measured Velocities from Pulse Travel Time in Band-Pass Filtered IRFs

Next, we attempt to identify the pulse velocity in the subbands, each containing only one vibrational mode, by tracing the peak of the central lobe of the source pulse, as shown in Fig. 4.4a. The results are presented in Table 4.3 (right) and in Fig. 4.4b, in a similar fashion as for the low-pass filtered IRFs. It can be seen that the identified $c_{eq}^D$ in the bands follow the trend of the theoretical dispersion, and are close to the theoretical phase velocities (see section 4.2.2). Based on Eqn. (4.15) and subsequent discussion, the envelopes of the pulses for the second and third subband cannot be or can be barely resolved. Therefore, direct measurement of the group velocities in this building is not practical or possible. (If $c_{gr}^S \approx c_{SB}^T$, $\Delta f_s > 4.2$ Hz for TB model NS and 3.1 Hz for TB model EW).

4.3.4 Measured Velocities from Resonant Frequencies

Table 4.4 shows estimates of the phase velocity from the modal frequencies, $c_{res}(f_n)$ (see Section 4.2.3), also plotted in Fig. 4.4 as dots. It can be seen that the dots follow the trend of the theoretical phase velocity, but do not fall exactly on the curve for the reasons discussed in Section 4.2.3).
Table 4.4: Measured velocities, $c^{res}(f_i)$, of TB models NS (top) and EW (bottom) from the resonant frequencies, $f_i$, and comparison with the theoretical values, $c_1^{ph}(f_i)\,(m/s)$.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>$f_i(Hz)$</th>
<th>$c^{res}(f_i)$</th>
<th>$c_1^{ph}(f_i)$ (m/s)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NS TB Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.07</td>
<td>323</td>
<td>276</td>
<td>17.30</td>
</tr>
<tr>
<td>2</td>
<td>7.67</td>
<td>399</td>
<td>458</td>
<td>-12.88</td>
</tr>
<tr>
<td>3</td>
<td>15.95</td>
<td>498</td>
<td>554</td>
<td>-10.11</td>
</tr>
<tr>
<td><strong>EW TB Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.24</td>
<td>193</td>
<td>165</td>
<td>17.0</td>
</tr>
<tr>
<td>2</td>
<td>4.85</td>
<td>252</td>
<td>286</td>
<td>-11.9</td>
</tr>
<tr>
<td>3</td>
<td>10.24</td>
<td>320</td>
<td>359</td>
<td>-10.9</td>
</tr>
</tbody>
</table>

4.3.5 Modeling Error in Fitting Layered Shear-Beam in IRFs

In the next numerical experiment, a 3-layer shear-beam model is fitted in the response of the TB by matching the causal and acausal pulses in broader-band IRFs at several levels. The phase velocities were estimated directly from the pulse time shifts, and also by the waveform inversion algorithm, which fits the pulses as waveforms over selected time intervals (Rahmani and Todorovska, 2013). The corresponding estimates are indicated as $c_j^D$ and $c_j^{LSQ}$, where $j$ is the order of the layer measured from the top. Quality factor $Q = 50$ was assumed (equivalent to damping ratio $\zeta = 1/(2Q) = 2\%$). The results are shown in Table 4.5 ($\sigma_j$ in this table is the standard deviation of the fit in layer $j$). It is seen that the profile of the fitted model shows decreasing shear wave velocity towards the top, even though the structure that is identified has uniform properties. The distortion is the largest in the top layer, where it is a factor of 2. This distortion occurs because the
layered shear beam is compensating for the progressively larger deflection of the TB towards the top by the progressively lower shear wave velocity.

Table 4.5: Identified shear wave velocities, \(c_j^D\) and \(c_j^{LSQ}\), of an equivalent three-layer shear beam fitted in the response of TB model NS (top) and EW (bottom) by the direct and least squares fitting algorithms.

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Floors</th>
<th>(h_j) (m)</th>
<th>(\tau_j) (s)</th>
<th>(c_j^D) (m/s)</th>
<th>(c_j^{LSQ}) (m/s)</th>
<th>(\sigma_j) (m/s)</th>
<th>(\sigma_j/c_j^{LSQ}) (%)</th>
<th>(\frac{c_j^{LSQ}}{c_j^D})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Roof-7(^{th})</td>
<td>12.8</td>
<td>0.043</td>
<td>301</td>
<td>0.5</td>
<td>250</td>
<td>7.8</td>
<td>3.1</td>
</tr>
<tr>
<td>2</td>
<td>7(^{th})-4(^{th})</td>
<td>12.8</td>
<td>0.025</td>
<td>512</td>
<td>0.8</td>
<td>578</td>
<td>46.3</td>
<td>8.0</td>
</tr>
<tr>
<td>3</td>
<td>4(^{th})-GF</td>
<td>13.4</td>
<td>0.018</td>
<td>766</td>
<td>1.2</td>
<td>669</td>
<td>67.1</td>
<td>10.0</td>
</tr>
</tbody>
</table>

EW TB Model: 0-7.5 Hz, \(\zeta = 2\%\), \(c_T^{TB} = 480\) m/s

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Floors</th>
<th>(h_j) (m)</th>
<th>(\tau_j) (s)</th>
<th>(c_j^D) (m/s)</th>
<th>(c_j^{LSQ}) (m/s)</th>
<th>(\sigma_j) (m/s)</th>
<th>(\sigma_j/c_j^{LSQ}) (%)</th>
<th>(\frac{c_j^{LSQ}}{c_j^D})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Roof-7(^{th})</td>
<td>12.8</td>
<td>0.083</td>
<td>154</td>
<td>0.30</td>
<td>129</td>
<td>4.0</td>
<td>3.1</td>
</tr>
<tr>
<td>2</td>
<td>7(^{th})-4(^{th})</td>
<td>12.8</td>
<td>0.033</td>
<td>386</td>
<td>0.80</td>
<td>344</td>
<td>30.0</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>4(^{th})-GF</td>
<td>13.4</td>
<td>0.033</td>
<td>404</td>
<td>0.85</td>
<td>422</td>
<td>40.8</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Fig. 4.5 shows a comparison of the observed (TB) and fitted model (layered shear beam) IRFs at four levels (left), transfer-functions between roof and ground floor motions (center), and mode shapes of the first and second mode of vibration (right). The plot of the TFs shows that there is a mismatch of the frequencies of vibration, even though the pulse time shifts match closely. This demonstrates that, if the bending behavior of the structure that is identified is significant, the fitted layered shear beam cannot produce a good match of both IRFs and TFs. Then, the agreement of the TFs can be used as an indicator of the significance of bending for the structure that is identified, as suggested earlier in Rahmani and Todorovska (2013).

Despite the distortion of the identified velocity profile, the layering structure of a shear beam can still be used to monitor change in stiffness in different parts of the structure. However, further investigation is needed to find the possible effects of the distortion on the accuracy of the results.
Figure 4.4: Identification of the fixed-based TB models of Millikan library from pulse travel time in band-pass filtered IRFs. (a) Pulse propagation in three bands. (b) Comparison of the identified equivalent shear-beam velocity $c_{eq}^D$ (Table 4.3, right) and velocities measured from resonant frequencies with the theoretical dispersion curves.
Fig. 4.5: Assessment of modeling error in identified equivalent three-layer shear-beam velocity profile fitted in low-pass filtered IRFs of TB model NS (top) and TB model EW (bot) of Millikan library (Table 4.4). Comparison of fitted and observed impulse responses (left), transfer functions (center), and mode shapes (right).
4.4 Summary and Conclusions

The main findings of this study are as follows. (i) TB model depicts well the nature of dispersive wave propagation in Millikan Library at lower frequencies and may be an appropriate model for RC buildings with shear walls, in general. (ii) Both types of nonparametric estimates of phase velocity of the fixed-base TB models (the interval values from wave travel time and point values from natural frequencies of vibration) depict correctly the trends of the wave dispersion and are within the range/reasonably close to the true values. They differ from the true (theoretical) values because the former represent (biased) interval values, while the latter represent point values based on an approximation. As is the case with nonparametric estimation, in general, these techniques have the advantage that they do not require a prior assumption on the functional shape of the variation of wave velocity with frequency. However, as is the case with window type of analyses in general, the estimation from IRFs suffers from the tradeoff between resolution in frequency and resolution in time (consequence of the Heisenberg–Gabor uncertainty principle (Gabor, 1946)) and may not be always practical for buildings. (iii) For the particular shape of dispersion curves (phase velocity increasing with frequency but at a progressively smaller rate and eventually leveling off), the measured phase velocity over a broader band is biased toward the larger values on the band, which are representative of the higher frequencies in the band. This important finding helps explain, in the companion paper (Rahmani et al., 2015b), why the wave velocity estimated from broader band IRFs is not sensitive to the SSI effects while the fundamental frequency is sensitive (Todorovska, 2009b). (iv) Ignoring bending in interferometric identification of buildings from pulse travel time produces artificial softening in the identified velocity profile near the top. This may be important for application to SHM. It is concluded that the nonparametric interferometric
techniques considered in this paper present a good opportunity for exploratory analysis of wave dispersion in buildings. However, parametric identification, which may perform better in terms of accuracy, should also be explored, for example, for applications such as SHM, where the accuracy of the identification is important.

4.5 Acknowledgments

This work was in part supported by a grant from the U.S. National Science Foundation (CMMI-0800399). The authors also appreciate insightful discussions with F. Udwadia, T. Heaton, and M. Trifunac on the dynamic behavior of Millikan Library. The strong motion instrumentation network in this building is operated by the National Strong Motion Program of the U.S. Geological Survey. The strong motion data used can be downloaded from the Engineering Center for Strong Motion Data (www.strongmotioncenter.org).
Chapter 5

Structural System Identification Based on a Uniform Timoshenko Beam Model

This chapter is based on the article “Wave method for system identification and health monitoring of buildings – extension to fitting Timoshenko beam model” published in the Proceedings of the 10th National Conference in Earthquake Engineering (Ebrahimian and Todorovska, 2014b).

A significant new development of the wave method for structural system identification and health monitoring of buildings is presented. Previously, shear beam models were fitted, and changes in the identified equivalent shear wave velocities were monitored. These models are more appropriate for frame structures, which deform predominantly in shear. In this study, it is demonstrated, for the first time, that the method can be extended to fitting more complex models. As a first step, Timoshenko beam model is fitted, which accounts for bending deformation and rotary inertia, and is more appropriate for shear wall structures than the shear beam model. The model material parameters are estimated by minimizing the least squares (LSQ) error between the model and observed impulse response functions (IRF) over a time window, the IRFs representing the propagation of a pulse radiated from a virtual source. The Levenberg-Merquardt method for nonlinear LSQ estimation is employed. The initial values of the unknown parameters are
estimated from the first two apparent frequencies of vibration. However, the fit is performed over a frequency band that excludes the first mode of vibration, which is the one most affected by SSI, in order to minimize the effects of soil-structure interaction (SSI). A major complexity in the nature of wave propagation in Timoshenko beam, in comparison to shear beam, is the wave dispersion due to bending deformation. The identification is demonstrated on the NS and EW responses of Millikan library, a 9-story RC frame/shear wall structure in Pasadena, CA. It is concluded that the waveform inversion algorithm of the wave method for structural system identification of buildings can be extended to fitting dispersive models, such as Timoshenko beam model.

5.1 Introduction

Rapid damage assessment of instrumented buildings after a strong earthquake (or some other natural or manmade disaster) can be a valuable tool in emergency response. Timely evacuation of a weakened structure that may collapse during shaking from aftershocks would prevent loss of life and injuries. Likewise, not evacuating a structure that is safe would avoid needless service interruption of an important facility, such as e.g. a hospital, which is critically needed after the earthquake, or of a business center, for which interruption of service is very costly (Chang et al., 2003; Todorovska, 2009a). Moreover, structures identified to be safe can be used as shelters, when commute is disrupted and overcrowded streets obstruct emergency response (Hisada et al., 2012). However, to be practically useful, SHM systems need to satisfy simultaneously a number of requirements, which has been a challenge. E.g., they need to be sensitive enough to damage that may affect the structure serviceability, not sensitive to the environmental and
operating conditions and accurate enough to avoid false alarms, as well as robust to work on large amplitude data and real buildings (Chang et al., 2003).

Interferometric methods have been widely used for imaging the interior of bodies in many fields (Curtis et al., 2006). The representation of the response of a building as a superposition of bouncing waves, and the description of its properties by the velocity of wave propagation through the structure can be traced back to the early 1960s (Kanai and Yoshizawa, 1963). Seismic interferometry of buildings has received renewed interest recently (Snieder and Şafak, 2006), in particular for its use for structural health monitoring (SHM) (Todorovska and Trifunac, 2008a,b; Todorovska, 2009b,c; Trifunac et al., 2010; Todorovska and Rahmani, 2013; Rahmani and Todorovska, 2013, 2014, 2015). It has been shown that interferometric SHM based on representation of the building as shear beam is robust when applied to large amplitude earthquake data and real buildings (Todorovska and Trifunac, 2008a,b) and also not sensitive to the effects soil-structure interaction (Todorovska, 2009b,c), the latter being a major advantage over the SHM methods that are based on detecting changes in the fundamental frequency of vibration. In the previous work, wave propagation in buildings has been interpreted based on shear beam model. This paper presents a significant new development in interferometric SHM, which is extension to fitting models with dispersive wave propagation. As a first step, Timoshenko beam model is fitted (Timoshenko, 1921), which is more suitable than the shear beam model for buildings with significant deformation in flexure, such as buildings with shear walls (Ebrahimian and Todorovska, 2014c). The presented new development is a direct extension of the waveform inversion algorithm for system identification proposed by Rahmani and Todorovska (Rahmani and Todorovska, 2013), and consists of nonlinear least squares fit of impulse response functions in selected time windows. The objective of this paper is to
present the extension of the method and to demonstrate it on a case study. The case study used is Millikan library, a 9-story RC frame/shear wall structure in Pasadena, California (Snieder and Şafak, 2006; Rahmani and Todorovska, 2013).

5.2 Methodology

5.2.1 Impulse Response Functions and Parameter Estimation

Impulse Response Functions (IRF) are waveforms extracted from structural response as follows. Let \( \hat{u}(z,\omega) \) and \( \hat{u}_{\text{ref}}(z_{\text{ref}},\omega) \) be the Fourier Transforms of recorded motion at height \( z \) and at some reference height \( z_{\text{ref}} \) (any of the sensors can be chosen as reference). The transfer-function (TF) between the motions at levels \( z \) and \( z_{\text{ref}} \), \( \hat{h}(z,z_{\text{ref}};\omega) \), is

\[
\hat{h}(z,z_{\text{ref}};\omega) = \frac{\hat{u}(z;\omega)}{\hat{u}(z_{\text{ref}};\omega)}
\]

(5.1)

and the corresponding IRF, \( h(z,z_{\text{ref}};t) \), is its inverse Fourier transform

\[
h(z,z_{\text{ref}};t) = FT^{-1}\left\{\hat{h}(z,z_{\text{ref}};\omega)\right\}
\]

(5.2)

Both functions represent the system function, but in different domains. The IRF represents physically the response of the system at level \( z \) to unit input impulse (Dirac Delta function) at level \( z_{\text{ref}} \). Consequently, IRFs computed at different levels reveal how the pulse generated by the virtual source propagates through the structure and is modified during the propagation. We fit IRFs computed for virtual source at roof, which are simpler than those computed for virtual source.
at the base, and consist of acausal and causal pulses (Snieder and Şafak, 2006). Further, we compute the IRFs from regularized TFs computed as

\[
\hat{h}(z,z_{\text{ref}};\omega) \approx \frac{\hat{u}(z;\omega) \hat{u}^*(z_{\text{ref}};\omega)}{|\hat{u}(z_{\text{ref}};\omega)|^2 + \varepsilon}
\]  

(5.3)

where the regularization parameter \(\varepsilon\), is a small fraction of the average power spectral density of \(\hat{u}_{\text{ref}}(z_{\text{ref}},\omega)\) (10\% in the example in this paper) and \(*\) indicates complex conjugate. Band limited IRFs can be computed by filtering in the frequency domain. E.g., to compute IRFs in a band with central frequency \(\omega_c\) and half bandwidth \(\Delta\omega_s\)

\[
h(z,z_{\text{ref}},\omega_c,\Delta\omega_s; t) = FT^{-1}\{\hat{h}(z,z_{\text{ref}},\omega) \hat{S}(\omega_c,\Delta\omega_s;\omega)\}
\]

(5.4)

where the filter \(\hat{S}(\omega_c,\Delta\omega_s;\omega), \omega_c, \Delta\omega_s \geq 0\) is

\[
\hat{S}(\omega_c,\Delta\omega_s;\omega) = \begin{cases} 
1, & -\omega_c - \Delta\omega_s < \omega < -\omega_c + \Delta\omega_s \\
1, & \omega_c - \Delta\omega_s < \omega < \omega_c + \Delta\omega_s \\
0, & \text{otherwise}
\end{cases}
\]

\[
\overset{\text{FT}}{\Rightarrow} S(\omega_c,\Delta\omega_s; t) = \frac{2\Delta\omega_s}{\pi} \frac{\sin \Delta\omega_s t}{\Delta\omega_s t} \cos \omega_c t
\]

(5.5)

is the shifted box function. Other type of band-pass filters can also be used.

We consider fitting in a band that does not include the fundamental mode, to minimize the effects of SSI on the parameters of the fitted model, because the recorded motions reflect the response of the soil-structure system, which differs most from the fixed-base response in the neighborhood of the fundamental mode of vibration (Todorovska, 2009b). Extrapolation of the TF to lower frequencies
may give a proxy for the fundamental fixed-base frequency, like the proxy obtained by fitting uniform shear beam model (Todorovska, 2009c).

For the fit, the observed IRFs are computed from the recoded response, while, for the model, they are computed analytically, as shown in the next section. We fit the model by minimizing the least squares error between the observed and model IRFs over a selected time window. We employ the Levenberg-Merquardt method for nonlinear estimation, which is a fixed regressor, small residual method (Rahmani and Todorovska, 2013; Seber and Wild, 2003). It converged quickly but requires initial estimates that are close to the true values in order to converge. We show later in this paper how we choose the initial estimates.

5.2.2 Timoshenko Beam Model

The building model we consider is a uniform, cantilever Timoshenko beam (TB), stress free at the top and excited by horizontal motion at the base (Timoshenko, 1921). The beam has height $H$, and its cross-section is characterized by area $A$, second moment of inertia $I$ and shear factor $k_G$. Its material is characterized by mass density $\rho$, Young’s modulus $E$ and shear modulus $G$, which implies longitudinal and shear wave velocities in the material $c_L = \sqrt{E/\rho}$ and $c_S = \sqrt{G/\rho}$. Kelvin-Voight damping is assumed, with viscosity constant $\mu$ for both shear and bending deformation. The shear factor $k_G$ is correction for the fact that the shear stress is not uniform on the cross-sectional area $A$. It depends on the shape of cross-section, material properties and frequency (Cowper, 1966).
The horizontal displacement of the neutral axis, \( u(z, t) \), due to shear and bending deformation and considering the rotary inertia, satisfies the following differential equation

\[
c^2_L c_S^2 k_G (1 + \mu \frac{\partial}{\partial t})^2 \frac{\partial^4 u}{\partial z^4} - (c_L^2 + k_G c_S^2) \left(1 + \mu \frac{\partial}{\partial z^2} \frac{\partial^3 u}{\partial t^2}ight) + \frac{k_G c_S^2}{r_g^2} \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial t^4} = 0 \quad (5.6)
\]

where \( r_g = \sqrt{I/A} \) is radius of gyration. Substituting a trial propagating wave solution of the form \( e^{i(kz - \omega t)} \) leads to the dispersion relation of the beam

\[
c^2_L c_S^2 k_G (1 - i\omega \mu)^2 k^4 - (c_L^2 + k_G c_S^2)(1 - i\omega \mu) k^2 \omega^2 - \omega^2 \frac{k_G c_S^2}{r_g^2} (1 - i\omega \mu) + \omega^4 = 0 \quad (5.7)
\]

where \( \omega \) is the circular frequency and \( k \) is the wave number \( (k = \omega/c) \), where \( c \) is the phase velocity). The dispersion relation has two pairs of roots, each pair corresponding to a wave propagating in the positive and in the negative \( z \)-direction.

The four roots, in terms of dimensionless parameters are

\[
K = kr_g = \pm \frac{\Omega}{\sqrt{2}} \left[ \left( \frac{1}{\alpha} \right) \frac{1}{k_G} + R \right] \pm \sqrt{\left( \frac{1}{\alpha} \frac{1}{k_G} - R \right)^2 + \frac{4R}{\alpha \Omega^2}} \quad (5.8)
\]

where \( K = kr_g \) is dimensionless wave number, \( \Omega = \omega r_g / c_s \) is dimensionless frequency, \( R = G/E = c_s^2 / c_L^2 \) is the moduli ratio, \( \alpha = 1 - i\Omega M \), where \( M = \mu c_s / r_g \) is dimensionless damping. Dimensionless phase and group velocities corresponding to \( K_{1,2} \) can then be computed

\[
C_{1,2}^{ph} = \frac{C_{1,2}^{ph}}{c_s} = \frac{\Omega}{K_{1,2}} \quad (5.9)
\]
It can be seen from eqn (5.8) that, for an undamped beam, $K_1$ is always a real number and corresponds to a propagating wave, while $K_2$ corresponds to an evanescent wave up to some frequency and a propagating wave beyond that frequency. The critical frequency is $\Omega_{cr} = \sqrt{k_G}$. The solution in the frequency domain is then

$$U(z) = C_1 e^{ik_1z} + C_2 e^{-ik_1z} + C_3 e^{ik_2z} + C_4 e^{-ik_2z}$$

(5.11)

where $C_i$, $i = 1, \ldots, 4$, are constants. Known $U(z)$, the required transfer-functions and impulse response functions can readily be computed. Ebrahimian and Todorovska (2014c) present and analyze such functions for a range of the dimensionless parameters. Further, it can be shown that the ratios of the beam fixed-base frequencies depend only on the dimensionless height $H/r_g$, and moduli ratio $R$. This fact is used to find initial value of $R$ for the Levenberg-Merquardt algorithm. We find initial value of $R$ from the ratio of the observed first and second apparent frequencies of vibration, and further find initial value of the shear wave velocity from the frequency of either the second or the first mode.

### 5.2.3 Results and Analysis

Millikan Library is a densely instrumented 9-story reinforced concrete (RC) building in Pasadena, California (Snieder and Şafak, 2006; Rahmani and Todorovska, 2013). It has plan dimensions 21×23 m, and height 43.9 m above the ground and 48.2 meters above basement level (Fig. 5.1). Its lateral force resisting system is a combination of a moment resisting frame and shear walls. The shear walls providing resistance to NS motions are located at the east and west ends, while those
providing resistance to EW motions form a central core where the elevators are located (Fig. 5.1). Because of the added shear stiffness by these walls, the relative contribution of bending deformation to the total response is larger than what it would have been otherwise. The Yorba Linda, 2002 earthquake (M=4.8, R=40 km) was recorded by a dense network of accelerometers installed at every level of the building. In this paper, we use only data from sensors at the ground floor and at the roof.

![Figure 5.1: Millikan Library: a) photo (courtesy of M. Trifunac), b) vertical cross section (redrawn based on (Snieder and Şafak, 2006)), c) Timoshenko beam model, and d) typical floor plan (top) and basement plan (redrawn based on (Snieder and Şafak, 2006)).](image)

Fig. 5.2 shows IRFs of the EW response during this earthquake in the band 0-15 Hz, and in its subbands 0-7.5 Hz and 7.5-15 Hz. The dispersive nature of wave propagation in this building is apparent from the shape change of the propagating pulse, and from the faster velocity of propagation in the higher frequency subband (redrawn from Rahmani and Todorovska (2013)).
Millikan Library, Yorba Linda, 2002, observed IRFs for EW response in different frequency bands

-0.5 0 0.5
\[ t - s \]

0 - 7.5 Hz
7.5 - 15 Hz
0 - 15 Hz

Roof
9th
7th
6th
5th
4th
3rd
Gnd
2nd
8th

Roof
9th
7th
6th
5th
4th
3rd
Gnd
2nd
8th

Figure 5.2: Millikan Library: evidence of dispersion in the observed EW response during Yorba Linda, 2002 earthquake. Impulse responses are shown in different frequency bands: a) 0-15 Hz (solid line) and 0-7.5 Hz (dashed line); b) 0-7.5 Hz, and c) 7.5 – 15 Hz (Rahmani and Todorovska, 2013).

We specify the initial values of the model parameters for the LSQ fit as follows. The beam height \( H \) is taken to be distance from ground to the highest sensor (39 m), and \( r_g \) is approximated based on the plan dimensions, assuming uniform distribution of stiffness. Then \( H/r_g \) is computed and kept fixed. The frequency ratios of a TB depend only on \( R \) and \( H/r_g \). Then the initial value of \( R \) is obtained from the ratio of the first and second apparent frequencies \( f_{2,app}/f_{1,app} \) and \( H/r_g \). Then, the initial value of \( c_s \) is obtained by matching further the values of \( f_{1,app} \) or \( f_{2,app} \). A small initial value of the dimensionless damping \( M \) is assigned. These values are listed in 5.1.

The results of the fitting are shown in Tables 5.2 and 5.3 and Figure 5.2.3. Only the IRFs at the ground floor were fitted. The tables show results for fitting in two frequency bands, one band including and the other band excluding the first mode of vibration. The former is referred to as \textit{full} band and the latter, for which
Table 5.1: Building dimensions and initial values of the Timoshenko beam parameters

<table>
<thead>
<tr>
<th>TB Model</th>
<th>W (m)</th>
<th>H (m)</th>
<th>( r_g (m) )</th>
<th>Initial value of parameters to be fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>21</td>
<td>39</td>
<td>6.06</td>
<td>900 1.2 0.24</td>
</tr>
<tr>
<td>EW</td>
<td>23</td>
<td>39</td>
<td>6.64</td>
<td>650 1.7 0.003</td>
</tr>
</tbody>
</table>

The results are expected to be less affected by SSI, is referred to as high-pass band. The bands are 0-15 Hz and 5-15 Hz for the NS response, and 0-11 Hz and 3-11 Hz for the EW response. Table 5.2 shows the final values of \( c_s, R \) and \( M \).

Table 5.3 shows the observed apparent frequencies and the fitted fixed-base model frequencies of the first two modes of vibration, obtained from plots of the respective transfer-functions. The model first mode frequencies for fit in the high-pass band are obtained by extrapolating the fitted model beyond the band of the fit. It can be seen that the model frequencies for the two fits are close to each other, and are close to the apparent frequencies.

Table 5.2: Results of the LSQ fits in the full band and high pass band

<table>
<thead>
<tr>
<th>Initial values</th>
<th>LSQ fitted values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_s ) (m)</td>
<td>R</td>
</tr>
<tr>
<td>NS 0-10 Hz</td>
<td>900 1.2 0.24 23</td>
</tr>
<tr>
<td>NS 5-10 Hz</td>
<td>900 1.2 0.24 37</td>
</tr>
<tr>
<td>EW 0-11 Hz</td>
<td>650 1.7 0.003 52</td>
</tr>
<tr>
<td>EW 3-11 Hz</td>
<td>650 1.7 0.003 61</td>
</tr>
</tbody>
</table>

*Noramalized root mean squared error

Figure 5.2.3 shows results only for the high-pass band. Parts a) and b) compare the observed and model TFs between roof and ground level motions, and parts e)
Table 5.3: Comparison of natural frequencies of the fitted models with the observed apparent frequencies of vibration

<table>
<thead>
<tr>
<th>Data</th>
<th>$f_{1,app} (Hz)$</th>
<th>$f_{2,app} (Hz)$</th>
<th>$f_1 (Hz)$</th>
<th>$f_2 (Hz)$</th>
<th>$f_1 (Hz)$</th>
<th>$f_2 (Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>1.72</td>
<td>7.40</td>
<td>1.75</td>
<td>7.57</td>
<td>1.72</td>
<td>7.58</td>
</tr>
<tr>
<td>EW</td>
<td>1.12</td>
<td>4.88</td>
<td>1.16</td>
<td>4.81</td>
<td>1.22</td>
<td>4.85</td>
</tr>
</tbody>
</table>

and f) compare the corresponding IRFs at the ground floor, as well as at two other floors between ground and roof, where IRFs were not fitted. The frequency bands and the time windows of the fit are indicated by horizontal bars. It can be seen that, at all the floors, the model and observed IRFs are close. Parts c) and d) show the model phase and group velocities plotted vs. frequency. The equivalent uniform shear beam velocities, as identified in (Rahmani and Todorovska, 2013) by fitting IRFs at ground floor, are also shown as horizontal bars spanning the band of the fit.

The structural health may be monitored by detecting changes in the structural shear and flexural stiffness, as well by monitoring changes in the beam phase and group velocities, which reflect the combined effect of both flexural and shear stiffness. Change may be detected w.r.t. values fitted in the initial time window of smaller response, as in (Todorovska and Trifunac, 2008a,b). Demonstration of this method for health monitoring is out of the scope of this paper.

5.3 Conclusions

This study showed that the wave method for structural system identification of buildings can be extended to fitting building models in which wave propagation is dispersive, such as Timoshenko beam model. This model is more suitable than
Figure 5.3: a), b) Comparison of TB model transfer functions with observed data for Yorba Linda earthquake 2002 frequency window for band pass fitting is shown by red lines. c), d) Dispersion curves of fitted TB models in NS and EW directions. e), f) Comparison of model IRFs with observed data. Fitting interval at the ground is shown by red window below the IRF plot.
shear beam for structures with shear walls, e.g., for which flexural deformation is important. The Lavenberg-Merquardt algorithm was shown to converge in this case as well, for appropriately chosen initial conditions. The results are encouraging for possible future extension to fitting even more realistic model, such as e.g. finite element models.

### 5.4 Acknowledgments

The strong motion instrumentation network in this building is operated by the National Strong Motion Program of the U.S. Geological Survey. The strong motion data used can be downloaded from the Engineering Center for Strong Motion Data (www.strongmotioncenter.org).
Chapter 6

Structural System Identification
Based on a Nonuniform
Timoshenko Beam Model

This chapter is based on the article “Structural system identification of buildings by a wave method based on a nonuniform Timoshenko beam model” published in Journal of Engineering Mechanics (Ebrahimian and Todorovska, 2015).


A nonuniform Timoshenko beam model of a building, with piecewise constant properties along the height, is presented, along with an algorithm for structural system identification from earthquake records. The model accounts for shear and flexural deformation, rotatory inertia and variation of building properties with height. The model stiffness parameters are identified by matching, in the least squares sense, propagating pulses in impulse response functions. To minimize the effects of soil-structure interaction, the fit is performed on a band that excludes the fundamental mode of vibration. This algorithm is a new development in a wave method for structural health monitoring of buildings, intended for use in seismic alert systems to facilitate decision making on evacuation immediately after the earthquake, and for general condition monitoring. The model can also be used
with an earthquake early warning system for quick linear response prediction to
decide on safe shut down of sensitive equipment in advance of the strong shaking.
Identification and response prediction are demonstrated on the NS response of
Los Angeles 54-story Office Building, a tall steel frame building in downtown Los
Angeles.

6.1 Introduction

In this paper, we extend our recent analysis of dispersive wave propagation in a
uniform vertical cantilever model of a building to a Timoshenko beam (TB) model
with piecewise uniform properties along the height (Timoshenko, 1921; Ebrahimian
and Todorovska, 2014c). We refer to this variation as layering, and present solution
for its response in the frequency domain to base translation and rotation. The
solution is derived analytically by the propagator matrix approach (Gilbert and
Backus, 1966).

This model is more realistic than other simple models, such as lumped mass
and shear beam models, because it accounts for overall bending deformation of the
structure in addition to shear. Although high-rise buildings, as a whole, deform
more like shear beam, the bending deformation is not insignificant, in particular
for buildings with shear walls, and for taller buildings (see appendix B for details
about different beam theories). Shear-bending models are believed to be better
physical models even for moment frames buildings (Blume, 1968; Minami et al.,
2013; Miranda and Reyes, 2002). The layering structure allows modeling variation
of the building properties (plan dimensions, stiffness and mass density) with height.
This model differs from multifiber TB used to model RC walls, in which the fibers
are aligned in the longitudinal direction (Kotronis and Mazars, 2005).
The model is developed for use in system identification and health monitoring of high-rise buildings. We present an identification algorithm based on deconvolution interferometry, adapted to dispersive systems. The beam stiffness parameters are identified by least squares (LSQ) matching of travelling pulses. This algorithm accounts for wave dispersion due to bending, and is an improvement over the previous algorithms, which fit shear beam models (Todorovska and Rahmani, 2013; Rahmani and Todorovska, 2013, 2014, 2015).

The identification algorithm is demonstrated for a full-scale 54-story steel-frame building, located in downtown Los Angeles, using records of the Northridge, 1994 earthquake. The ability of the identified model to predict the linear response of the building during a future earthquake is also demonstrated using records of two subsequent earthquakes.

The identification algorithm is intended for use in seismic alert systems as a tool for rapid assessment of the structural health after a strong earthquake (or some other natural or man-made disasters), and for long term condition monitoring (Rahmani and Todorovska, 2015). Quick prediction of the linear response of the structure using this model may be useful with an earthquake early warning system to provide guidance on safe shut down of equipment in advance of the earthquake waves. Other uses are estimation of elastic displacement demand of existing structures under different levels of excitation, and calibration of numerical models (Gičev and Trifunac, 2009a).

TB models have been used for analysis of wave propagation in beams in non-destructive testing (Mead, 1985; Shen et al., 1994; Park, 2005). Such applications differ from the application to buildings in the ratio of the bending to shear stiffness. Structural elements deform primarily in bending, and TB is used to correct for shear, while a building as a whole deforms primarily in shear, and TB is used to
correct for bending. TB models have also been used to estimate modal properties of buildings from experimental data (e.g. Boutin et al. (2005); Michel et al. (2010); Cheng and Heaton (2015).

Coupled beam models have been used to approximate elastic deformation and acceleration demands in multistory buildings (Miranda and Reyes, 2002; Reinoso and Miranda, 2005; Taghavi and Miranda, 2005). Similarly, sandwich models have been used to estimate the natural frequencies of vibration (Kaviani et al., 2008). Dym and Williams (2007) and Aristizabal-Ochoa (2009) discuss the appropriateness of the coupled beam models (Euler-Bernoulli and shear beam connected in parallel) vs. TB models (Rayleigh beam and shear beam connected in series) for different types of structures. Further, Hans and Boutin (2008) analyzed a generic beam model and its special cases, and their physical relationship to different frame structures.

Minami et al. (2013), introduced a reduced order lumped mass model, with shear and rotational springs connected in series, and used it for system identification of super high-rise buildings in Japan shaken by the Tohoku, 2011 earthquake. Although conceptually similar, the layered Timoshenko cantilever beam model is more convenient for wave propagation studies because it provides analytic solution for the wave dispersion in the beam. To our knowledge, layered TB models of buildings have not been used previously. Preliminary results of this study have been reported in Ebrahimian and Todorovska (2014a).

Interferometry has been applied to wave imaging in many disciplines, across a wide range of scales (Curtis et al., 2006). It has been applied to building response by many authors (e.g. Kanai and Yoshizawa (1963); Snieder and Şafak (2006); Kohler et al. (2007); Todorovska and Trifunac (2008b); Rahmani and Todorovska (2013); Nakata et al. (2013)).
6.2 Methodology

6.2.1 The Model

The building is modeled as a visco-elastic Timoshenko cantilever with piecewise continuous properties (Fig. 6.1a), stress free at the top and excited by horizontal motion at the base Timoshenko (1921). Each layer corresponds in general to a group of floors. It is assumed that within each layer the medium is homogeneous and isotropic, and that there is perfect bond between the layers. The layers are numbered from top to bottom, with their geometry characterized by height $h_j$, cross-sectional area $A_j$, moment of inertia $I_j$ and shear factor $k_G$, where $j = 1, \ldots, n$ is the layer index. In each layer, the material is characterized by mass density $\rho_j$, Young’s modulus $E_j$ and shear modulus $G_j$, which implies longitudinal and shear wave velocities in the material $c_{L,j} = \sqrt{E_j/\rho_j}$ and $c_{S,j} = \sqrt{G_j/\rho_j}$. Kelvin-Voigt model for the damping is assumed, with same viscosity constant $\mu_j$ for both shear and bending deformation. The elastic and damping properties of Kelvin-Voigt materials are modeled by an elastic spring and a viscous damper connected in parallel. The total stress then is the sum of the elastic stress and the viscous damping stress, which is proportional to the time derivative of the strain. The viscosity constant $\mu_j$ can be frequency dependent to accommodate differences in the damping for the different modes of vibration. The shear factor $k_G$ is a correction for the fact that the shear stress is not uniform on the cross-sectional area. It depends on the shape of the cross-section, material properties and frequency (Cowper, 1966; Mindlin and Deresiewicz, 1953).

Timoshenko beam theory accounts for both shear and flexural deformation, and also for rotatory inertia, and is essentially a combination of a shear and Rayleigh beam connected in series. It is a linear theory that assumes small deformations,
and that plane sections normal to the neutral axis remain plane but not normal. Fig. 6.1b shows a deformed element within a layer, where $\theta$ is rotation due to bending and $\gamma(z; t)$ is additional rotation, considered as shear deformation caused by uniform shear stress on the section. Although each cross section can rotate because of bending deformation, the neutral axis of the beam is assumed to move only horizontally, with displacement $u(z, t)$. Foundation rocking due to soil-structure interaction is ignored. Fig. 6.1c shows a free-body diagram of a beam element within a layer, where $M(z; t)$ and $V(z; t)$ denote bending moment and shear force. The state of the beam is described by the generalized vector $\{u(z; t), \theta(z; t), M(z; t), V(z; t)\}^T$. It is noted here that coupling of the bending moment with the axial force in the beam is neglected.

Figure 6.1: (a) Cantilevered layered Timoshenko beam model; (b) deflected element; (c) free body diagram of element.

### 6.2.2 Propagator for a Layered Timoshenko Beam

An analytical solution for the response of the beam to input horizontal and rocking base motion can be obtained in the frequency domain by the propagator matrix
approach (Gilbert and Backus, 1966) as follows. Assuming harmonic excitation, the elements of the state vector can be written as 
\[ u(z; t) = U(z)e^{-i\omega t}, \quad \theta(z; t) = \Theta(z)e^{-i\omega t}, \quad M(z; t) = \tilde{M}(z)e^{-i\omega t} \] and 
\[ V(z; t) = \tilde{V}(z)e^{-i\omega t}, \] where \( \omega \) is the circular frequency. Further, the two dynamic equilibrium equations of the beam element (Fig. 6.1c) and the constitutive relations for the beam can be written in matrix form as

\[
\frac{df}{dz} = B f(z)
\] (6.1)

where

\[ f(z) = \{U(z), \Theta(z), \tilde{M}(z), \tilde{V}(z)\}^T \] (6.2)

and

\[
B = \begin{bmatrix}
0 & 1 & 0 & \frac{1}{k_G GA(1-i\omega \mu)} \\
0 & 0 & \frac{1}{EI(1-i\omega \mu)} & 0 \\
0 & -\rho I \omega^2 & 0 & -1 \\
-\rho A \omega^2 & 0 & 0 & 0
\end{bmatrix}
\] (6.3)

For simplicity, the index of the layer, \( j \), has been omitted in eqn (6.3).

The propagator of the solution for \( f(z) \) from one point in a layer, \( z_0 \), to any another point, \( z \), in the same layer can be derived as follows (see appendix C for details). Because \( B \) is a continuous function of \( z \) within the layer, \( f(z) \) can be determined uniquely in terms of \( f(z_0) \) as

\[ f(z) = P(z, z_0) f(z_0) \] (6.4)

where \( P(z, z_0) \) is the propagator from \( z_0 \) to \( z \). The propagator can be constructed from the eigenvalues and eigenvectors of \( B \) as follows. Matrix \( B \) has four distinct eigenvalues \( \lambda_l, \quad l = 1, \ldots, 4 \) and consequently the corresponding eigenvectors are
linearly independent. Let \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \) and \( X \) be a matrix that has the eigenvectors as its columns. Then

\[
P(z, z_0) = e^{B(z-z_0)}
\]  

(6.5)

As it turns out, the eigenvalues of \( B \)

\[
\lambda_l = ik_l, \quad l = 1, \ldots, 4
\]  

(6.6)

where \( k_l = \omega/c_l \) are the allowable wave numbers and \( c_l \) are the corresponding phase velocities. This can be verified by substituting a propagating wave ansatz of the form \( e^{i(kz-\omega t)} \) in the equations of motion of a Timoshenko beam and comparing the corresponding characteristic equations. The allowable wave numbers, and phase and group velocities are discussed in the next section.

A solution from one point can be propagated in the neighboring layer by propagating it first to the layer boundary and then on the other side of the boundary based on the continuity condition

\[
f(z_j^-) = f(z_j^+)
\]  

(6.7)

Then, starting from the top, at \( z_0 = 0 \), at some point \( z_{j-1} < z \leq z_j \) in layer \( j \)

\[
f(z) = P^{(j)}(z, z_{j-1}) P^{(j-1)}(z_{j-1}, z_{j-2}) \cdots P^{(2)}(z_2, z_1) P^{(1)}(z_1, 0) \ f(0)
\]  

(6.8)

where \( P^{(j)} \) is the propagator in layer \( j \). What is left is to define completely \( f(0) \), from the partially known conditions at the top and base. At the top, it is known that shear force and bending moment are zero, while, at the bottom, it is assumed...
that horizontal displacement and rotation at the base have known values $U_g$ and $\Theta_g$

\begin{equation}
    f(z_0) = \{U(0), \Theta(0), 0, 0\}^T \quad (6.9)
\end{equation}

\begin{equation}
    f(z_n) = \{U_g, \Theta_g, M(z_n), V(z_n)\}^T \quad (6.10)
\end{equation}

Then, $U(0)$ and $\Theta(0)$ are found from the condition

\begin{equation}
    f(z_n) = \prod_{j=1}^{n} P^{(j)}(z_j, z_{j-1}) \ f(0) \quad (6.11)
\end{equation}

where $f(z_0)$ and $f(z_n)$ are as given in eqns (6.9) and (6.10). We note here that we further assume that the beam is clamped at the base, i.e. $\Theta_g = 0$.

### 6.2.3 Dispersion Relations

The allowable values of the wave number $k$ in each layer are

\begin{equation}
    k = \pm \frac{K_{1,2}}{r_g} = \pm \frac{\Omega}{\sqrt{2}} \sqrt{\left(\frac{1}{k_G} + R\right) \pm \sqrt{\left(\frac{1}{k_G} - R\right)^2 + \frac{4R}{\Omega^2}}} \quad (6.12)
\end{equation}

where $r_g = \sqrt{I/A}$ is the radius of gyration of the cross-section, $K = kr_g$ is dimensionless wave number, $\Omega = \omega r_g / c_S$ is dimensionless frequency, $R = G/E = c_S^2 / c_L^2$ is the moduli ratio and $M = \mu c_S / r_g$ is dimensionless damping parameter (Ebrahimian and Todorovska, 2014c). For an undamped beam, $K_1$ is real for all $\Omega$ and, therefore, corresponds to a propagating wave, while $K_2$ is real only for $\Omega > \Omega_{cr} = \sqrt{k_G}$ and corresponds to an evanescent wave otherwise. The corresponding phase and group velocities can be computed from their definitions, $c^{ph} = \frac{\omega}{k}$ and $c^{gr} = \frac{d\omega}{dk}$. The dependency on frequency indicates dispersive wave propagation in the beam segments. Analysis of dispersion in a uniform Timoshenko beam, for a
range of parameters representative of buildings, can be found in Ebrahimian and Todorovska (2014c).

6.2.4 Impulse Response Functions

The Impulse Response Function (IRF) of a linear system is the inverse Fourier transform of its Transfer-Function (TF). Both functions represent the system function, but in different domains. Let \( \hat{u}(z; \omega) \) and \( \hat{u}(z_{ref}; \omega) \) be the Fourier transforms of the motion of the building at height \( z \) and some reference height \( z_{ref} \). Then, the TF between the motions at levels \( z \) and \( z_{ref} \)

\[
\hat{h}(z, z_{ref}; \omega) = \frac{\hat{u}(z; \omega)}{\hat{u}(z_{ref}; \omega) \varepsilon (6.13)}
\]

and the corresponding IRF is

\[
h(z, z_{ref}; t) = FT^{-1}\left\{\hat{h}(z, z_{ref}; \omega)\right\} (6.14)
\]

where \( FT^{-1}\{\cdot\} \) indicates inverse Fourier transform. The IRF represents physically the response of the system at level \( z \) to a unit input impulse (Dirac Delta function) at level \( z_{ref} \). Consequently, IRFs computed at different levels of the structure reveal how a virtual pulse, applied at \( z = z_{ref} \), propagates through the structure. The IRFs represent Green’s functions for the system but for modified boundary conditions, i.e. such that the horizontal motion at the reference point is zero except when the virtual pulse is applied. We also note that the IRFs of recorded data were computed from regularized transfer functions

\[
\hat{h}(z, z_{ref}; \omega) \approx \frac{\hat{u}(z; \omega) \hat{u}(z_{ref}; \omega)}{\varepsilon} (6.15)
\]
where $\varepsilon$ is a regularization parameter (in this study, 5% of the average power spectral density of $\hat{u}(z_{\text{ref}}; \omega)$) and the bar indicates complex conjugate (Snieder and Şafak, 2006).

6.2.5 System Identification Algorithm

The stiffness related beam parameters are estimated by matching the model and observed IRFs, which are computed using Eqns (7.3) and (7.4), with $\hat{u}(z; \omega)$ and $\hat{u}(z_{\text{ref}}; \omega)$ being recorded motions or simulated motions by the model. The waveform inversion algorithm is used (Rahmani and Todorovska, 2014), which fits, in the least squares sense, the IRFs on a time window simultaneously at all levels where motion was recorded. In the case of Timoshenko beam, this physically means matching dispersed propagating pulses. Only the parameters related to stiffness are estimated ($G$ and $R = G/E$), while the other parameters are specified from geometry (e.g. $A$, $k_G$ and $I$) or are assumed (e.g. $\rho$ and $\mu$, which have been shown not to affect much the results). IRFs for virtual source at the top are matched.

The band for the fit $\omega \in (\omega_1, \omega_2)$ is chosen so that (1) the effects of the soil-structure interaction (SSI) on the estimated stiffness related parameters are minimized, and (2) Timoshenko beam is a valid model for the building. The former condition sets the limit for $\omega_1$, which is chosen so that the band excludes the fundamental mode of vibration, which is affected more by soil-structure interaction (SSI) than the higher modes (Todorovska, 2009b,c). It is noted that transfer-function $\hat{h}(z, z_{\text{ref}}; \omega)$ is affected by SSI, despite the normalization, because of the foundation rocking which is not eliminated by the normalization. The latter condition sets the limit for $\omega_2$. 
For the least squares fit, we employ the Levenberg-Marquardt method for non-linear estimation, which is a fixed regressor, small residual method (Levenberg, 1944; Marquardt, 1963; Rahmani and Todorovska, 2013). It converges quickly but requires initial estimates that are close to the true values in order to converge (examples are presented in the results section).

6.2.6 Response Prediction during Future Events

Assuming that the building is a linear time-invariant system, and the earthquake would produce motion $u_g(t)$ at level $z = z_n$ (base or ground level), the response at any level $z$ can be computed by convolution of $u_g(t)$ and the corresponding system function, i.e.

$$u(z; t) = u_g(t) * h(z, 0; t)$$  \hspace{1cm} (6.16)

where $*$ is the convolution operator. Extrapolated system function can be used to predict the response in a band that is broader than the band used for the identification. Further, empirical IRFs can be used to predict the response in the full recording band (typically 0-25 Hz), but that would restrict the prediction to the levels $z$ where motion was recorded previously.

6.3 Results and Analysis

6.3.1 Building Description

Los Angeles 54-story office building (Fig. 6.2a) is a steel-frame building in downtown Los Angeles, California designed in 1988 following the 1985 Los Angeles City Code and Title 24 of the California Administrative Code. The building was instrumented in 1991 by the California Strong Motion Instrumentation Program.
The instrumentation consists of a 20-channel digital accelerometer array distributed on 6 levels: basement (P4), ground, 20th, 36th, 46th, and Penthouse. As reported by the agency (www.strongmotioncenter.org), the building has 54 stories (210.2 m) above and 4 stories (14 m) below ground level. It has a rectangular plan with two rounded sides with dimensions decreasing toward the top in the EW direction as follows: 59.7 × 36.9 m ground to 35th floor, 53.6 × 36.9 m, 36th to 45th floor and 47.5 × 36.9 m 46th floor to penthouse (Fig. 6.2b, c). The lateral load resisting system is a moment resisting perimeter steel frame (framed tube) with 3 m column spacing. There are Vierendeel trusses and 1.22 m deep transfer girders at the 36th and 46th floors where vertical setbacks occur. The site geology is described as alluvium over sedimentary rocks.

Figure 6.2: Los Angeles 54-story office building (CSMIP 24629): (a) photo; (b) vertical cross section; (c) typical floor layouts and sensor locations on the plan (adapted from www.strongmotioncenter.org)
Additional information about the site geology is available based on site exploration by LeRoy Crandall Associates, conducted in 1981 and summarized in Naeim et al. (2008). According to this exploration, the soil profile consists of 20 m of sands with variable layers of silts and clays on siltstone and shale bedrock, which extends to the maximum exploration depth of 40 m. The shear wave velocity vs. depth is as follows: 300 m/s from the surface up to 9 m depth, 366 m/s for depth 9-20 m, 457 m/s for depth 20-30 m and 686 m/s for depth 30-37 m.

### 6.3.2 System Identification Based on Northridge, 1994 Earthquake Response

The building NS response is identified using records of the Northridge earthquake of January 17, 1994 (\(M_L = 6.4\)), which occurred 32 km to the North-West from the building. Fig. 6.3 shows the recorded accelerations at the West wall, which were used. The building showed no signs of damage after this earthquake (Naeim et al., 2006).

![Observed NS acceleration at the west wall during the Northridge, 1994 earthquake.](image)

Figure 6.3: Observed NS acceleration at the west wall during the Northridge, 1994 earthquake.
The building is identified based on an equivalent 4-layer cantilevered Timoshenko beam, representing the building between ground and penthouse levels. The layer boundaries at the levels where the strong motion instruments are located. The beam has rectangular cross section with same dimensions as the plan (Fig. 6.2b). The unknown parameters are the shear wave velocity $c_S$ and the moduli ratio $R$. The rest of the parameters are estimated from the building geometry or are assumed. Uniform mass density $\rho = 300 \text{ kg/m}^3$ is assumed, and a small value of the damping constant, $\mu = 0.005 \text{ s}$ (the results of the fitting are not very sensitive to the particular value of damping). The area moment of inertia $I$ was calculated assuming that the building mass and stiffness are uniformly distributed over the cross-section. The shear factor $k_G$ is taken to be $5/6$ which corresponds to rectangular cross section (Weaver et al., 1990). The band 0.3-1.7 Hz was chosen for the fit, which excludes the first system frequency of vibration and includes the 2\textsuperscript{nd} to 5\textsuperscript{th} NS modes of vibration. Table 6.1 shows the preset layer properties.

Table 6.1: Values of model parameters that are estimated from geometry or assumed

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Floors</th>
<th>h (m)</th>
<th>$\rho$ (kg/m(^3))</th>
<th>$\mu$ (s)</th>
<th>$W_{EW}$ (m)</th>
<th>$W_{NS}$ (m)</th>
<th>$A$ (m(^2))</th>
<th>$I_{EW}$ (m(^4))</th>
<th>$I_{NS}$ (m(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46th-Penth</td>
<td>27.9</td>
<td>300</td>
<td>0.005</td>
<td>47.5</td>
<td>36.9</td>
<td>1754</td>
<td>330,398</td>
<td>198,774</td>
</tr>
<tr>
<td>2</td>
<td>36th-46th</td>
<td>39.8</td>
<td>300</td>
<td>0.005</td>
<td>53.6</td>
<td>36.9</td>
<td>1976</td>
<td>474,463</td>
<td>224,258</td>
</tr>
<tr>
<td>3</td>
<td>20-36th</td>
<td>63.6</td>
<td>300</td>
<td>0.005</td>
<td>59.7</td>
<td>36.9</td>
<td>2203</td>
<td>655,288</td>
<td>249,742</td>
</tr>
<tr>
<td>4</td>
<td>Gnd-20</td>
<td>78.9</td>
<td>300</td>
<td>0.005</td>
<td>59.7</td>
<td>36.9</td>
<td>2203</td>
<td>655,288</td>
<td>249,742</td>
</tr>
</tbody>
</table>

The results of the identification are shown in Table 6.2. Two sets of initial estimates of $c_S$ and $R$ are shown, the final estimates of $c_S$ and $R$, and other derived parameters such as the static shear and bending stiffness, $GA$ and $EI$, and their ratio, and the critical frequency $f_{cr}$. The last two columns show the minimum wavelength in the data used for inversion $\lambda_{min} = c_{ph}^{th} (f_{max})/f_{max} \approx c_S/f_{max}$, where $f_{max}$ is the maximum frequency in the band of the fit, and the ratio $h/\lambda_{max}$ for each layer.
It can be seen from Table 6.2 that, in all but the top layer, \( f_{cr} > f_{max} = 1.7 \) Hz, implying existence of only one propagating wave in the band of the fit. In the top layer, \( f_{cr} = 1.54 \) Hz is below but very close to 1.7 Hz, and the contribution of the second propagating wave is small. It can also be seen that \( \lambda_{min} \sim 100 \) m/s (except for the top layer), and that \( h/\lambda_{max} \) is between 0.4 to 0.74. Similar analysis for the building as a whole, with \( c_S \approx 140 \) m/s (Rahmani and Todorovska, 2014), gives \( H/\lambda_{max} \approx 2.5 \), where \( H \) is the building height.

Two sets of initial values for \( c_S \) and \( R \) were tried: (i) values obtained for a uniform Timoshenko beam model (Ebrahimian et al., 2014) that are assigned to all the layers, and (ii) values for \( c_S \) obtained by fitting a 4-layer shear beam (Rahmani and Todorovska, 2014) and some very small value of \( R \). Parameters for a uniform TB were obtained as follows. The frequency ratios of the beam depend only on \( R \) and an \( H/r_{g} \). First, \( H/r_{g} \) was approximated from geometry, then the initial value of \( R \) was obtained from the ratio of the first and second apparent frequencies \( f_{2,app}/f_{1,app} \) (obtained from the transfer-function between roof and base responses) and \( H/r_{g} \), and finally the initial vale of \( c_S \) was obtained from \( f_{1,app} \) or \( f_{2,app} \) (Ebrahimian and Todorovska, 2014b). For both sets, the Levenberg-Marquardt method converged to the same profiles for \( c_S \) and \( R \). The normalized root mean square (RMS) error (normalized with respect to the observed IRFs) is 25%, which is significantly smaller than the error for the initial values (72% and 92%).

In Fig. 6.4 the fitted model and the observed IRFs are compared over the band of the fit (0-1.7 Hz). The agreement is very good, confirming the low RMS error reported in Table 6.2. Similarly, in Fig. 6.5, the TFs between roof and ground level responses are compared over the band 0-3.5 Hz. It can be seen that the frequencies
Table 6.2: Identified stiffness parameters on the band 0.3-1.7 Hz and two sets of initial values used. The normalized root mean squared error (RMS) is also shown.

| Layer No. | Set 1: 4 layer SB | | Set 2: Uniform TB | | Nonlinear least squares fit: 4 layer TB model | |
|-----------|------------------|------------------|------------------|------------------|------------------|
|           | \(c_s\) (m/s) | \(R\) | \(c_s\) (m/s) | \(R\) | \(c_s\) (m/s) | \(R\) | \(GA\) (GN) | \(EI\) (TN.m²) | \(\frac{G A}{E A}\) | \(\lambda_{\min}\) \(\uparrow\) | \(h\) (m) | \(\lambda_{\min}\) \(\uparrow\) |
| 1         | 92.5 | 10^{-4} | 165 | 0.003 | 113 | 0.043 | 6.67 | 17.66 | 0.378 | 1.54 | 66.5 | 0.42 |
| 2         | 153.0 | 10^{-4} | 165 | 0.003 | 170 | 0.005 | 17.11 | 410.93 | 0.042 | 2.32 | 100.0 | 0.40 |
| 3         | 141.6 | 10^{-4} | 165 | 0.003 | 158 | 0.008 | 16.46 | 242.44 | 0.068 | 2.15 | 93.0 | 0.68 |
| 4         | 167.7 | 10^{-4} | 165 | 0.003 | 181 | 0.007 | 21.62 | 349.87 | 0.062 | 2.47 | 106.5 | 0.74 |
| RMS error (%) \(\uparrow\) | 72 | 92 | 25 |

\(\uparrow\) Normalized root mean squared error; \(\uparrow\) \(\lambda_{\min} = \frac{c_s}{f_{max}}\); \(R = \frac{G}{E}\)

of the 2\(^{nd}\) to 5\(^{th}\) modes of vibration, which fall in the fitting band (0.3-1.7 Hz) are matched closely.

The fitted model can be extrapolated to frequencies below the band of the fit. The frequency of the fundamental mode of vibration in the extended TF could serve as a proxy of the fundamental fixed-base frequency of the structure. Such proxy may be closer to the true value than a proxy obtained by fitting a uniform shear beam model as in Todorovska and Trifunac (2008b) and Todorovska (2009c). However, in general, it would differ from the true value because the model has been constructed from incomplete information about the structural response. For the case study in this paper, the extended TF happens to match closely \(f_{1,app}\), as it can be seen in Fig. 6.5.

Fig. 6.6 shows the \(GA\) and \(EI\) profiles, which represent the static bending and shear stiffness of the layers. It can be seen that the second layer from the top is stiffer than the layer below, and that is more so for the bending stiffness. Possible causes for this difference are discussed later in the Discussion section.
Figure 6.4: Comparison of impulse response functions of the fitted model with the observed ones computed from the Northridge earthquake response. A sketch of the model is shown on the left.
Figure 6.5: Comparison of the transfer-function (between penthouse and ground levels) of the fitted model with the observed one. The frequency window of the fit is also shown. In the background, the corresponding transfer-function of a 4-layer shear beam SB, fitted earlier on the same band (Rahmani and Todorovska, 2014), is shown by a dashed line.

### 6.3.3 Wave Dispersion in the Fitted Model

Fig. 6.7 shows the phase and group velocities of the first wave propagation mode, $c_{1}^{ph}$ and $c_{1}^{gr}$, which reflect the *dynamic* stiffness of the layers, in the band 0-2 Hz. The horizontal lines in the background represent the vertical wave velocities identified by fitting a 4-layer shear beam model on the band 0-1.7 Hz in (Rahmani and Todorovska, 2014). For reference, the frequencies of the first five modes of vibration are indicated by arrows.
Fig. 6.7 shows how $c_{ph1}^1$ and $c_{gr1}^1$ approach fast their high frequency asymptotes $c_s \sqrt{k_G}$= shear wave velocity in the beam. In the middle two layers, the curves are practically flat for frequencies including and beyond the third mode of vibration, suggesting small dispersion in that band. The variation with frequency is larger for the bottom and top layers, suggesting that their response is more affected by bending deformation than the response in the middle two layers.

In the second layer from the top, $c_{ph1}^1$ and $c_{gr1}^1$ are larger than in the layer below, in agreement with the results for fitted shear beam, shown by horizontal lines in Fig. 6.7 (Rahmani and Todorovska, 2014). It is seen that the latter estimates, obtained from IRFs on the band 0-1.7 Hz, are much closer to the higher frequency values of $c_{ph1}^1$ in the band. Except in the top layer, in which the ratio of shear to bending stiffness is the largest among all layers, they are very close to the high frequency asymptotes of $c_{ph1}^1$ and $c_{gr1}^1$. 

Figure 6.6: Bending stiffness (left) and shear stiffness (right) profiles of the fitted model.
Figure 6.7: Phase (thick lines) and group (thin lines) velocities of the fitted model, extended over a broader band. The band of the fit (0.3-1.7 Hz) is indicated on the top. The shear wave velocities of fitted 4-layer shear beam model are shown by horizontal lines (redrawn from Rahmani and Todorovska (2014)). The frequencies of the first five modes of vibration are indicated in the bottom for reference.

6.3.4 Predicted Response to Hector Mine, 1999 and La Habra, 2014 Earthquakes

Figs 6.8 and 6.9 show predicted linear response of the instrumented floors to two subsequent earthquakes, and comparison with the recorded motions for $f < 25$ Hz. The predictions were computed using Eqn (6.16), with IRFs of the fitted 4-layer TB (parts a) and c) extended to the band $f < 25$ Hz, and with empirical IRFs computed from the Northridge earthquake records also in the band $f < 25$
Hz (parts b) and d)). Both accelerations (parts a) and b)) and displacements (parts c) and d)) are compared. In Fig. 6.8, the excitation is Hector Mine, 1999 earthquake, a distant large earthquake with epicenter North-East from the building site (\(M_L=7.1, R=193 \text{ km}\)), and in Fig. 6.9 it is La Habra, 2014 earthquake, a local smaller magnitude event that occurred South-East from the building (\(M_L=5.1, R=34 \text{ km}\)). The former excited more the fundamental mode, while the latter excited more the higher modes.

The model has parameters as in Tables 6.1 and 6.2, except for the damping constant. To better match both the observed accelerations and the displacements, frequency dependent \(\mu\) was assumed: \(\mu = 0.03 \text{ s for } f \leq 0.25 \text{ Hz}\), which includes the first mode, \(\mu = 0.01 \text{ s for } 0.25 < f \leq 0.65 \text{ Hz}\), which includes the second mode, and \(\mu = 0.005 \text{ s for } f > 0.65 \text{ Hz}\). These values were chosen by trial and error to match approximately the transfer-function peaks for the Northridge, 1994 earthquake (Fig. 6.5).

Fig. 6.10 shows the observed IRFs used in the convolution (Eqn (6.16)), which have virtual source at ground floor, and how they are affected by low-pass filtering. IRFs are shown for \(f \leq 25 \text{ Hz}\) (the dark thin line) and for \(f \leq 1.7 \text{ Hz}\) (the light thick line). The upward propagation of the source pulse, its amplification upon its reflection from the roof, its further propagation downward and reflection back from the ground floor, and further bouncing between the top and bottom boundaries, can be seen clearly. It can be seen that the smoother IRF (for \(f \leq 1.7 \text{ Hz}\)) is close to the broader band IRF (for \(f \leq 25 \text{ Hz}\)) except for the slightly lower peaks of the initial upward propagating pulse.

Figs 6.8 and 6.9 show that both the empirical and model IRFs closely predict the acceleration and displacement responses during the subsequent events. There are several possible causes of the small differences: (i) permanent changes in the
Hector Mine Earthquake, October 16, 1999 ($M_L=7.1$, $R=193$ km)

Figure 6.8: Observed and predicted response during Hector Mine, 1999 earthquake. Parts a) and b) compare the accelerations, and parts c) and d) compare the displacements. The predicted response was computed by convolution with model (parts a) and c)) and empirical (parts b) and d) IRFs computed from records of Northridge, 1994 earthquake.
La Habra Earthquake, March 29, 2014 (M=5.1, R=33 km)

Figure 6.9: Observed and predicted response during La Habra, 2014 earthquake. Parts a) and b) compare acceleration, and parts c) and d) compare the displacements. The predicted response was computed by convolution with model IRFs computed from records of Northridge, 1994 earthquake, and the empirical IRFs computed from records of Northridge, 1994 earthquake.
Figure 6.10: Observed IRFs for virtual source at ground level, low pass filtered at 1.7 Hz and at 25 Hz. Both IRFs have been normalized to unit amplitude of the source pulse in the frequency domain.

building that have occurred between the earthquakes, (ii) natural variability of the building-soil system as function of the operating and environmental conditions (Rahmani and Todorovska, 2015; Trifunac and Ebrahimian, 2014); and (iii) differences in the rocking excitation, the response to which is implicitly included in the IRFs used, and which is in general different during different earthquakes.

6.3.5 Validity of Timoshenko Beam Model for the Case Study

The effects of bending on the building response are more prominent for structures with shear walls (see e.g. Fig 9a in Trifunac et al. (2001c)). Nevertheless, investigators have concluded that models with bending are better physical models even for
moment frame structures (Blume, 1968; Minami et al., 2013; Miranda and Reyes, 2002). In the following, such claim is demonstrated for the example building by analysis of the frequency ratios.

First, it is shown that decreasing plan dimension and decreasing rigidity towards the top not only cannot explain the differences between the observed frequency ratios and those for a uniform shear beam but produce the opposite effect. Table 6.3 shows the observed (apparent) modal frequencies and their ratios $f_{i,\text{app}}/f_{1,\text{app}}$. It also shows the (fixed-base) frequency ratios $f_i/f_1$ for a uniform shear beam, and three variants. The first variant has decreasing width $W$ of the cross section towards the top, by the same factor as for the case study (see Fig. 6.2). The second variant has decreasing shear wave velocity $c_S$ towards the top by the same factor as $W$ and the third variant has decreasing $W$ and $c_S$. Fig. 6.11 shows these frequency ratios normalized relative to uniform shear beam. It can be seen that the observed normalized ratios are all greater than 1, while those for the variants are all less than 1.

Next, we compare the transfer-functions of the fitted layered TB model and those of the fitted layered shear beam model (Rahmani and Todorovska (2014)) with the observed transfer-functions, all shown in Fig. 5. It can be seen that the layered TB model matches closely the observed modal frequencies, even for the fundamental mode, which is outside the band of the fit and is affected by soil-structure interaction. The fitted layered shear beam model does not match so well the first two modes. Soil-structure interaction affects most the fundamental mode, and much less the higher modes. Therefore, it cannot explain the mismatch of the second mode. The author believes that it did not affect much the fundamental frequency as well, because this is a flexible structure on relatively stiff soil (see the Building Description section). Bending deformation, however, produces softening,
which is progressively larger for the lower modes, and can explain the mismatch (see Fig. 6.7). The author believes that the lack of it in the shear beam model was the main reason of the disagreement, but other possible factors cannot be excluded.

Table 6.3: The observed apparent modal frequencies $f_{i,app}$ and their ratios $f_{i,app}/f_{1,app}$. R ratios $f_i/f_1$ are also shown for a uniform shear beam and three variants with decreasing properties with height: (i) decreasing width $W$ (based on decrease in plan dimension in Fig. 6.2), (ii) decreasing shear wave velocity $c_S$ (by the same factor as for $W$ in item (i)), and combination of (i) and (ii).

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Observed NS $f_{i,app}$ (Hz)</th>
<th>Uniform SB $f_i/f_1$</th>
<th>SB dec. $W$ $f_i/f_1$</th>
<th>SB dec. $c_S$ $f_i/f_1$</th>
<th>SB dec. $W$ &amp; $c_S$ $f_i/f_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>3.09</td>
<td>2.92</td>
<td>2.93</td>
<td>2.84</td>
</tr>
<tr>
<td>3</td>
<td>0.82</td>
<td>5.07</td>
<td>4.80</td>
<td>4.82</td>
<td>4.61</td>
</tr>
<tr>
<td>4</td>
<td>1.17</td>
<td>7.21</td>
<td>6.75</td>
<td>6.72</td>
<td>6.44</td>
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<td>5</td>
<td>1.51</td>
<td>9.32</td>
<td>8.62</td>
<td>8.58</td>
<td>8.14</td>
</tr>
<tr>
<td>6</td>
<td>1.83</td>
<td>11.28</td>
<td>10.49</td>
<td>10.53</td>
<td>10.03</td>
</tr>
</tbody>
</table>

6.3.6 Discussion

The fitted equivalent 4-layer Timoshenko cantilever models the response of the building as a whole in the lower frequency range, i.e. for wavelengths longer than 40% of the building height. The shortest wavelength $\lambda_{\text{min}}$ in the data used for the inversion is even longer compared to the dimensions of the layers (see Table 6.2). While it is sufficient to resolve the layers (Todorovska and Rahmani, 2013), it cannot resolve the individual floors and elements. Therefore, the identified values of $E$ and $G$ cannot correspond to any one of the individual structural members and the materials they are made of.
Figure 6.11: Normalized observed frequency ratios w.r.t. fixed-base uniform shear beam. Such ratios are also shown for three variants of decreasing properties towards roof: decreasing width $W$ (as per plan dimensions in Fig. 6.2), decreasing shear wave velocity $c_S$ (by the same factor) and decreasing $W$ and $c_S$.

As seen in Fig. 6.6, the static bending stiffness $EI$ is considerably larger in the second layer from the top, relative to the layer below, despite the smaller cross-sectional area; $GA$ is also larger but to a lesser degree. Their combined effect on $c_{ph}^i$ is a small increase. Such apparent "anomaly" appears in (Rahmani and Todorovska, 2014), who demonstrated that it was not due to measurement error. The second layer from the top is between the 36th and 46th floors, which is where the two vertical setbacks occur and where there are Vierendeel trusses and 1.22 m deep transfer girders. Such elements stiffen the floors where the setbacks occur (G. Brandow, personal communication). In the simplified 4-layer beam representation, their effect is felt most in the second layer, because it is bounded on both sides by the stiffened floors. As suggested by the results (Table 6.2, Fig. 6.6), they
also affect the balance of the shear and bending deformation in this layer, in favor of shear. It has been suggested that such effect can be achieved by increasing the floor stiffness and reducing joint rotation (Blume, 1968; Miranda and Reyes, 2002). Considering the simplicity of the model and incompleteness of information about the design of this building that was available for this study, more detailed explanation of the variations in the $EI$ and $GA$ profiles is beyond the resolution and scope of this paper.

The building was identified in a frequency band that excludes the fundamental mode of vibration, in order to minimize the effects of soil-structure interaction. The response of high-rise buildings is often dominated by the fundamental mode, especially for stiffer structures. This identification method, however, does not need the modes per se, but a band. The band, at least in principle, does not need to contain any of the modal frequencies; we have verified this on simulated response. What is important with real data is that sufficient energy of the structural response is contained in that band. The author have carried out successfully such inversion for the much stiffer Millikan library (9-story reinforced concrete building with shear walls/central core) using small amplitude earthquake response (Ebrahimian and Todorovska, 2014b). We are currently testing the method on a damaged structure and will report our findings in the near future.

6.4 Summary and Conclusions

A Timoshenko beam (TB) model of a high-rise building with piecewise constant properties was introduced, as well as a system identification algorithm based on this model. The model accounts for wave dispersion caused by flexural deformation, present in addition to shear, rotatory inertia and variation of the building
properties along the height. It automatically converges to shear beam model where bending deformation is not important and to Rayleigh beam where shear deformation is not important. The model stiffness parameters are identified by matching, in the least squares sense, with the Levenberg-Marquardt method, the model and observed band-pass filtered impulse response functions. The filter is such that it excludes the fundamental mode of vibration, which is most sensitive to the effects of soil-structure interaction. Therefore, by construction, the identification algorithm minimizes the sensitivity of the identified stiffness parameters to the effects of soil-structure interaction. The identification was applied to identification of the NS response of Los Angeles 54-story Office Building using acceleration records of the Northridge, 1994 earthquake and based on a 4-layer model. As initial values, results obtained by fitting a simpler model (uniform Timoshenko beam or a 4-layer shear beam) were used. The identified model was then used to predict the response to two subsequent earthquakes, one large and distant and the other one smaller and near.

The main findings of this study are as follows. (1) The Levenberg-Marquardt method converged for both sets of initial values and produced quickly static shear and bending stiffness profiles, and frequency dependent phase and group velocity in the layers. (2) The results of the blind identification reveal stiffening in the layer between 36th and 46th floors, relative to the layer below, which may be due to the Vierendeel trusses and 1.22 m deep transfer girders at the 36th and 46th floors, where the vertical setbacks occur. (3) The fitted model is a better physical model for this building than a uniform or nonuniform shear beam model, even though the ratio of modal frequencies is close to the corresponding ratios for uniform shear beam. Finally, (4) the fitted model and observed impulse response functions predicted well the linear response of the building during subsequent.
It is concluded that the layered Timoshenko beam model and the least squares fitting algorithm may be suitable for implementation in seismic alert and warning systems, and for general condition monitoring. Inferences on loss of stiffness, possibly due to damage, would be made by monitoring changes in the vertical phase velocities of the fitted model. Such capability will be demonstrated in future publications. This model is valid for longer wavelengths, and can resolve blocks of floors. However, to detect more localized damage, more detailed models need to be fitted over a broader frequency band. The author leaves such task for the future.

6.5 Acknowledgments

The structural array in this building is operated by the California Strong Motion Instrumentation Program. The data were obtained from Engineering Center for Strong Motion Data (http://www.strongmotioncenter.org/). The authors are grateful to Gregg Brandow and Mihailo Trifunac for the insightful discussions on the design and response of tall buildings, and to the two anonymous reviewers, whose comments lead to improvements of this paper.
Chapter 7

Structural Health Monitoring of a 7-story Full-Scale Building Slice Tested on the UCSD-NEES Shake Table

Earthquake response records and ambient vibration test data, which differ both in amplitude and nature, are often used for structural health monitor of full-scale structures. It is of interest to compare their effectiveness for structural health monitoring. The shake table experiment of a full-scale slice of a 7-story reinforced concrete building, carried out on the UCSD-NEES shake table, presents such an opportunity. This study uses test data from four earthquakes, which progressively damaged it, and from ambient vibrations and white noise tests performed before and after each earthquake test. It analyzes the changes in two damage sensitive parameters, the fundamental frequency of vibration, $f_1$, and the compressional wave velocity, $c_L$, identified for each damage state and from the different types of test data. The latter was identified by least squares fit of a uniform Timoshenko beam model, with high shear stiffness, by matching low pass filtered impulse response functions. The results show that, for both $c_L$ and $f_1$, the identified values and the detected changes depend on the intensity of the excitation. The most
significant difference between $c_L$ and $f_1$ is in the detected changes from the earthquake test data. The change in $f_1$, as determined from the earthquake tests, are consistently larger than the changes in $c_L$ determined from the same data by at least 25%, while they are comparable for the weaker tests data. We interpret this larger difference to be due to nonlinear effects in the structure to which $f_1$ is more sensitive than $c_L$.

7.1 Introduction

While the ideal data for testing and calibrating structural health monitoring (SHM) methods are those recorded in real full-scale structures - in their built environment, such data are rare for higher levels of damage (Trifunac and Todorovska, 1999). Laboratory experiments of physical models provide an opportunity to fill in these gaps, albeit the findings may be specific to the particular conditions under which the experiment had been conducted. The control over these conditions may be even beneficial, enabling to study some aspects of the problem while minimizing other aspects that are difficult to separate otherwise. This study uses laboratory data for a slice of a full-scale 7-story reinforced concrete (RC) building shaken on the UCSD-NEES unidirectional shake table (USCD stands for University of California at San Diego and NEES stands for Network for Earthquake Engineering Simulations)(Panagiotou et al., 2011; Panagiotou and Restrepo, 2011). The test structure was shaken by four earthquakes of increasing intensity which progressively damaged it. Ambient vibrations and white noise excitation tests were also carried for each damage state, to be used for structural health monitoring studies. This study was motivated by the opportunity that these data present to further
test and calibrate the relatively recent wave method for structural health monitoring, in particular the algorithm that fits models with dispersive wave propagation (Ebrahimian and Todorovska, 2015).

The broader issues of interest in this study are to find out: (i) how the amplitudes and the nature of the excitation affect the system identification and detected changes of the wave damage sensitive parameter, and (ii) how the detected changes in the wave damage sensitive parameter compare with those in the fundamental frequency of vibration. The former issue is important for relating changes in the identified damage sensitive parameter to the level of structural damage, and for the transferability of results obtained from one type of excitation to other types of excitation. Of particular interest is this issue for ambient and earthquake data, which are commonly recorded in real structures. The study is extended also to the fundamental frequency of vibration \( f_1 \), which has been the most widely used damage sensitive parameter. The later issue is important because \( f_1 \) is believed to be the modal parameter most sensitive to damage and also most sensitive to other factors such as the boundary conditions and the environmental and operating conditions (Chang et al., 2003). Both of these issues have not been addressed before either for the wave method or in the previous studies of the same test data.

Additional technical issues addressed are testing the wave method: (iii) on a structure that deforms primarily in bending, and (iv) on relatively short segments of ambient vibration data. The former is of interest because, in such a case, the degree of dispersion of vertically propagating waves through the structure is greater than in the previously studied structures (e.g., moment frames and mixed moment/shear wall structures; (Ebrahimian and Todorovska, 2014b, 2015). The latter is of interest because ambient vibration testing presents an opportunity for cost effective SHM, as it does not require permanent instrumentation and waiting.
for an earthquake. In previous analyses of impulse response functions (IRF) of buildings, continuously recorded data over longer periods of time were used, which is less practical. For example, Prieto et al. (2010) recorded ambient noise in a 17-story steel building in Los Angeles for over a month, and analyzed stacked (averaged) IRFs, each obtained from 10 min ambient data and stacked over 1, 14, 30 and 50 days. In a similar analysis of ambient noise in an 8-story building in Japan recorded for two weeks, Nakata and Snieder (2014) computed IRFs from 30 s segments and averaged them over four day intervals. For the test structure studied in this paper, response to ambient noise was recorded only for 3 min. In the remaining part of this section, a brief review is presented of the wave method for SHM as well as of prior studies of the same test data, and the organization of this paper is outlined.

In the wave method for SHM, parameters related to wave propagation in the structure as a whole are identified and monitored (e.g. Safak (1999); Trifunac et al. (2003); Oyunchimeg and Kawakami (2003)). For this purpose, impulse response functions (IRF) have been proposed (Snieder and Şafak, 2006) and are increasingly being used. The velocity of vertically propagating waves was first estimated from the fundamental frequency of vibration, assuming the building behaves as a uniform shear beam, or from the time shift of pulses in the IRFs (Snieder and Şafak, 2006; Todorovska and Trifunac, 2008a,b; Kohler et al., 2007; Todorovska, 2009c; Todorovska and Rahmani, 2013) This was later extended to least squares fit of a simple beam model, such as layered shear beam, by matching pulses in the IRFs (Rahmani and Todorovska, 2013; Rahmani et al., 2015a). However, measuring time shift or fitting shear beam models is meaningful only for lightly dispersed wave propagation, or in a narrower frequency band (Ebrahimian et al., 2014). To avoid the shortcomings of the latter, it was proposed recently to fit simple beam
models that account for the dispersive wave propagation. E.g., Ebrahimian and Todorovska (2014b, 2015) fit uniform or layered Timoshenko beam models, which account for dispersion due to bending deformation. They demonstrated their algorithm on identification of two full-scale buildings from small amplitude records. In this paper, the algorithm is applied for the first time to a damaged structure.

The same test data have been used previously in several studies. The primary objective of the earthquake tests had been to validate a new displacement-based design methodology for reinforced concrete shear wall building structures (Panagiotou and Restrepo, 2011). In another study, Panagiotou et al. (2011) addressed issues relevant to construction optimization, and issues in the dynamic response, such as the interaction between walls, slabs, and gravity system. Further, Grange et al. (2009) addressed modeling issues in simulation of the nonlinear response of the specimen. They used a detailed finite element model, consisting of multifiber Timoshenko beam elements (with fibers in the longitudinal direction to account for different materials) and constitutive models, based on damage mechanics for concrete and plasticity for steel. The ambient and white noise tests had been carried out for use in structural system identification and health monitoring studies. Simoen et al. (2013) conducted a damage identification study by Bayesian finite element model updating, based on dividing it into substructures, and using the modal characteristics of the first three modes of vibration, extracted from the ambient and white noise test data. Moaveni et al. (2011) compared the identified modal properties of the test structure from the low amplitude test data (ambient and two levels of white noise) using six methods, and concluded that the results by the different methods are in good agreement. Further, Moaveni et al. (2010) used the modal parameters to quantify and locate the damage by updating a 3D linear elastic finite element model of the structure. They found that the damage
results for the different low amplitude test data did not exactly coincide, but were consistent in identifying the damage in the lower two stories. They interpreted this to be due to the significant differences in the identified modal parameters from the different test data, and the violation of the linearity assumption. The study in this paper differs from the previous SHM studies in the method used, in using the earthquake test data as well, and in analyzing also the variation of the detected changes in two damage sensitive parameters.

This paper is organized as follows. The methodology and data are first briefly summarized, presenting only the details most relevant for this study. Results of the wave method are presented for fitted uniform Timoshenko beam, as a first step, and therefore, the global health of the structure is monitored. Detecting the location of the damage is beyond the scope of this study. Results are presented also of the variation of the fundamental frequency of vibration. The instantaneous frequency is estimated from longer segments of the earthquake test data, which contain also weak (ambient) response before and after the strong (earthquake) portion. Interval estimates of the frequency are also obtained from the entire records of the weak motion tests and from the strong motion part of the records for the earthquake tests. The variations in the estimated values of both damage sensitive parameters are analyzed. Finally, the changes in both parameters are compared for the different type of tests and conclusions are drawn.

7.2 Methodology

7.2.1 Model

The building model is a uniform, cantilever Timoshenko beam (TB), stress free at the top and excited by horizontal motion at the base (Timoshenko, 1921). The
beam has height $H$, and its cross-section is characterized by area $A$, second moment of inertia $I$ and shear factor $k_G$. Its material is characterized by mass density $\rho$, Young’s modulus $E$ and shear modulus $G$, which implies longitudinal and shear wave velocities in the material $c_L = \sqrt{E/\rho}$ and $c_S = \sqrt{G/\rho}$. Kelvin-Voigt model for the damping is assumed, with same viscosity constant $\mu$ for both shear and bending deformation. The shear factor $k_G$ is a correction for the fact that the shear stress is not uniform on the cross-sectional area.

Timoshenko beam theory accounts for both shear and flexural deformation, and also for the rotatory inertia, and is essentially a combination of a shear and Rayleigh beam connected in series. The horizontal displacement of the neutral axis, $u(z,t)$, satisfies the following differential equation

$$
\begin{align*}
 c_L^2 c_S^2 k_G \left( 1 + \mu \frac{\partial}{\partial t} \right)^2 \frac{\partial^4 u}{\partial z^4} - (c_L^2 + k_G c_S^2) \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^4 u}{\partial z^2 \partial t^2} \\
 + \frac{k_G c_S^2}{r_g^2} \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial t^4} = 0
\end{align*}
$$

(7.1)

where $r_g = \sqrt{I/A}$ is the radius of gyration. The equation is solved in the frequency domain, to find the model system functions in the frequency and time domains. More details can be found in Ebrahimian and Todorovska (2014c). Introducing the moduli ratio $R = G/E = \sqrt{c_S^2/c_L^2}$, it can be shown that, as $R \to 0$, the Timoshenko beam equation approaches that of shear beam, and, as $R \to \infty$, it approaches that of a Rayleigh beam.

The allowable values of the wave number $k$ are

$$
 k = \pm \frac{K_{1,2}}{r_g} = \pm \frac{\Omega}{\sqrt{2}} r_g \sqrt{\left( \frac{1}{k_G} + R \right) \pm \sqrt{\left( \frac{1}{k_G} - R \right)^2 + 4R \frac{\Omega^2}{\Omega^2}}} 
$$

(7.2)
where \( K = kr_g \) is dimensionless wave number, \( \Omega = \omega r_g/c_S \) is dimensionless frequency and \( M = \mu c_S/r_g \) is dimensionless damping parameter (Ebrahimian and Todorovska, 2014c). For an undamped beam, \( K_1 \) is real for all \( \Omega \) and, therefore, corresponds to a propagating wave, while \( K_2 \) is real only for \( \Omega > \Omega_{cr} = \sqrt{k_G} \) and corresponds to an evanescent wave otherwise. The corresponding phase and group velocities can be computed as \( c^{ph} = \frac{\omega}{k} \) and \( c^{gr} = \frac{d\omega}{dk} \). The dependency on frequency indicates dispersive wave propagation. Analysis of dispersion in a uniform Timoshenko beam, for a range of parameters representative of buildings, can be found in Ebrahimian and Todorovska (2014c).

### 7.2.2 System functions in the time and in the frequency domain

In a linear system, the transfer-function (TF) and its inverse Fourier transform, the impulse response function (IRF), represent the system function in the frequency and time domains, respectively. Let \( \hat{u}(z; \omega) \) and \( \hat{u}(z_{ref}; \omega) \) be the Fourier transforms of the motion of the building at vertical coordinate \( z \) and some reference point \( z_{ref} \). Then, the TF between the motions at levels \( z \) and \( z_{ref} \) is

\[
\hat{h}(z, z_{ref}; \omega) = \frac{\hat{u}(z; \omega)}{\hat{u}(z_{ref}; \omega)}
\]  

(7.3)

and the corresponding IRF is

\[
h(z, z_{ref}; t) = FT^{-1}\left\{\hat{h}(z, z_{ref}; \omega)\right\}
\]  

(7.4)

where \( FT^{-1}\{\cdot\} \) indicates inverse Fourier transform. The IRF represents physically the response of the system at \( z \) to a unit input impulse at \( z_{ref} \). Consequently,
IRFs computed at different levels reveal how a virtual pulse, applied at $z = z_{ref}$, propagates through the structure. The IRFs of the recorded data were computed from regularized transfer functions

$$\hat{h}(z, z_{ref}; \omega) \approx \frac{\hat{u}(z; \omega) \overline{\hat{u}(z_{ref}; \omega)}}{|\hat{u}(z_{ref}; \omega)|^2 + \varepsilon}$$  \hspace{1cm} (7.5)$$

where $\varepsilon$ is a regularization parameter (in this study, 0.5% of the average power spectral density of $\hat{u}(z_{ref}; \omega)$) and the bar indicates complex conjugate (Snieder and Şafak, 2006).

### 7.2.3 System identification algorithm

The waveform inversion algorithm proposed by Rahmani and Todorovska (2014) is used. The algorithm involves fitting segments of the IRFs in the least square (LSQ) sense. Let $x$ be a vector of the unknown parameters, $\hat{h}_{obs}(z, z_{ref}, t_i)$ be the observed IRF at time $t_i$ and $\hat{h}_{mod}(z, z_{ref}, t_i, x)$ be the model IRF at the same point in time. Then, the LSQ estimate of $x$ is such that it minimizes the sum of the square of the error at all observation points, $z_j$

$$S(x) = \sum_{i,j} \left[ h_{obs}(z_j, z_{ref}, t_i) - h_{mod}(z_j, z_{ref}, t_i; x) \right]^2$$  \hspace{1cm} (7.6)$$

In this study, IRFs at the base for a reference point at roof are fitted. Band limited IRFs are fitted, because continuous beam models of structures are valid only for the lower frequency range of the response.

The Levenberg-Marquardt method for nonlinear LSQ estimation is used, which is a fixed regressor, small residual method, and is implemented in MATLAB optimization toolbox (Levenberg, 1944; Marquardt, 1963). It converges quickly but requires initial estimates that are close to the true values. We find such values by
computing the RMS error on a coarse grid and over a wide range of values of $x$, and choosing as an initial estimate $x$ for which the error is the minimum.

### 7.2.4 Instantaneous frequency estimation

The instantaneous frequency is estimated from the Gabor transform of the relative roof acceleration. The estimation follows closely Todorovska and Trifunac (2007); Todorovska (2001). The Gabor transform of a function $y(t)$ is

$$G_y(t, \omega) \triangleq \int_{-\infty}^{\infty} y(\tau) \hat{g}(t, \omega)(\tau) d\tau, \quad -\infty < t < \infty, \quad |\omega| > 0$$

(7.7)

where

$$g(t, \omega)(\tau) = g(\tau - t) \exp \left[ -i\omega(\tau - t) \right]$$

(7.8)

and

$$g(t) = \pi^{1/4} \exp \left[ -t^2/(2\sigma^2) \right]$$

(7.9)

The estimation is closely related to moving window Fourier analysis with a Gaussian window. The instantaneous frequency is obtained from the amplitude of the transform, as the projection of the ridge on the time-frequency plane. The value of the amplitude of the transform on the ridge is called the skeleton, $S(t)$, and represents a smooth estimate of the amplitude envelope of the signal. Parameter $\sigma$ controls the width of the Gaussian window and the uncertainty of the estimate. The meaning of the instantaneous frequency $f(t)$ is that, in the interval $t \pm \sigma_t$, the frequency is in the interval $f(t) \pm \sigma_f$. For the Gaussian window in Eqn (9), $\sigma_t = \sigma/\sqrt{2}$ and $\sigma_f = 1/(2\pi\sigma\sqrt{2})$. 

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7.3 Results and Analysis

7.3.1 Test structure and data

The test structure (Fig. 7.1a, b) is a portion of a 7-story RC mid-rise residential building. It was tested on the UCSD-NEES unidirectional shake table during the period from October 2005 to January 2006 (Panagiotou et al., 2011). In the transverse direction, lateral resistance is provided by the RC shear wall in the middle (web wall), and, in the longitudinal direction, it was provided by another shear wall on the east side of the building (flange wall). The walls are separated by a gap (Fig. 7.1c). The slabs are supported by gravity columns. A pre-cast column and bracing (horizontal trusses) are used to limit torsional behavior.

Figure 7.1: Seven-story test structure: (a) view of the structure on the shake table Panagiotou et al. (2013); (b) elevation view, sensor locations are shown by small squares; c) plan view of the structure adapted from Panagiotou and Restrepo (2011) and d) uniform Timoshenko beam model of the test structure.
The building slice was subjected to four earthquake records of increasing intensity (tests EQ1, EQ2, EQ3 and EQ4). The state of the specimen before EQ1 was applied is referred to as S0, and that after each earthquake motion was applied is referred to as S1, S2, S3 and S4. We refer to state S0 as that of the undamaged structure. During state S3, the bracing system between the slabs and the post tensioned column was stiffened and strengthened. Therefore, state S3 is subdivided into S3.1 (before modification of the braces) and S3.2 (after modification of the braces) (Moaveni et al., 2011).

The response was described as slightly nonlinear for EQ1, moderately nonlinear for EQ2, EQ3 and highly nonlinear for EQ4 (Moaveni et al., 2011). As reported in Panagiotou et al. (2011), limited yielding in the reinforcement and visible cracking up to the fourth floor occurred in the web wall after EQ1. the strain and inter story drift limits during EQ1 satisfied the values selected for immediate occupancy performance level as defined in Panagiotou and Restrepo (2011). During EQ2 and EQ3, moderate yielding occurred in the web wall longitudinal reinforcement. During EQ4 localized plasticity and spalling of the concrete cover was observed at the base of the web wall. The strain and inter story drift limits during EQ4 satisfied the life safety performance level as defined in Panagiotou and Restrepo (2011). The information about the earthquake motions and the corresponding response are summarized in Table 7.1.

Between the earthquake tests, ambient vibration (AV) of the test structure was recorded for 3 min. In addition, the structure was tested with two white noise base excitations, consisting of 8 min realizations of a banded white noise (0.25-25 Hz) process with RMS amplitudes of 0.03g and 0.05g (WN0.03g and WN0.05g tests) (Moaveni et al., 2011). The sampling frequency for all tests was 240 Hz. In this study, the data was low pass filtered at 40 Hz by an Ormsby filter (Ormsby, 1961).
Table 7.2 shows a summary of all the tests that are used in this study, in the order in which they have been performed.

Table 7.1: Earthquake records applied to the test structure from Panagiotou et al. (2011)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Date</th>
<th>Test</th>
<th>Seismic Event</th>
<th>Station</th>
<th>Comp.</th>
<th>PGA (g)</th>
<th>Roof drift (%)</th>
<th>Response description</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>11/21/2005</td>
<td>EQ1</td>
<td>1971 San Fernando Eq. (M_w=6.6)</td>
<td>Van Nuys</td>
<td>Long.</td>
<td>0.15</td>
<td>0.28</td>
<td>slightly nonlinear</td>
</tr>
<tr>
<td>43</td>
<td>11/21/2005</td>
<td>EQ2</td>
<td>1971 San Fernando Eq. (M_w=6.6)</td>
<td>Van Nuys</td>
<td>Trans.</td>
<td>0.27</td>
<td>0.75</td>
<td>moderately nonlinear</td>
</tr>
<tr>
<td>48</td>
<td>11/22/2005</td>
<td>EQ3</td>
<td>1994 Northridge Eq. (M_w=6.7)</td>
<td>Oxnard Blvd.</td>
<td>Long.</td>
<td>0.35</td>
<td>0.83</td>
<td>moderately nonlinear</td>
</tr>
<tr>
<td>62</td>
<td>01/14/2006</td>
<td>EQ4</td>
<td>1994 Northridge Eq. (M_w=6.7)</td>
<td>Sylmar</td>
<td></td>
<td>360°</td>
<td>0.91</td>
<td>highly nonlinear</td>
</tr>
</tbody>
</table>

†As reported by Panagiotou et al. (2011); ‡As reported by Moaveni et al. (2011)

Table 7.2: Dynamic tests on the slice of shear wall used in this study from Moaveni et al. (2010)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Date</th>
<th>Test Description*</th>
<th>Damage State</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>11/21/2005</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S0</td>
</tr>
<tr>
<td>40</td>
<td>11/21/2005</td>
<td>EQ1</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>11/21/2005</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S1</td>
</tr>
<tr>
<td>42</td>
<td>11/21/2005</td>
<td>8 min WN (0.05g)</td>
<td>S1</td>
</tr>
<tr>
<td>43</td>
<td>11/21/2005</td>
<td>EQ2</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>11/22/2005</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S2</td>
</tr>
<tr>
<td>47</td>
<td>11/22/2005</td>
<td>8 min WN (0.05g)</td>
<td>S2</td>
</tr>
<tr>
<td>48</td>
<td>11/22/2005</td>
<td>EQ3</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>11/22/2005</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S3.1**</td>
</tr>
<tr>
<td>50</td>
<td>11/22/2005</td>
<td>8 min WN (0.05g)</td>
<td>S3.1**</td>
</tr>
<tr>
<td>61</td>
<td>01/14/2006</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S3.2**</td>
</tr>
<tr>
<td>62</td>
<td>01/14/2006</td>
<td>EQ4</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>01/18/2006</td>
<td>8 min WN (0.03g)+3 min AV</td>
<td>S4</td>
</tr>
<tr>
<td>65</td>
<td>01/18/2006</td>
<td>8 min WN (0.05g)</td>
<td>S4</td>
</tr>
</tbody>
</table>

* WN: white noise base excitation test; AV: ambient vibration test
** During state S3, the bracing system between the slabs of the building and the post tensioned column was stiffened and strengthened. Therefore, state S3 is subdivided into S3.1 (before modification of the braces) and S3.2 (after modification of the braces)
Fig. 7.2 shows the acceleration time histories, their Fourier transform amplitudes and the transfer function between roof and ground floor motions for 30 s of the strongest motion of the EQ tests. Fig. 7.3 shows a similar plot for the AV, WN0.03g and WN0.005g tests of state S3.1. It can be seen that white noise input motion is banded between 0.25-25 Hz. The drop in amplitude of Fourier spectra at around 11.5 Hz is due to application of a notch filter in the control loop of the shake table (Moaveni et al., 2011).

![Graphs showing time history, Fourier amplitude spectra for earthquake input accelerations and transfer function of roof with respect to ground.](image)

Figure 7.2: Time history, Fourier amplitude spectra for earthquake input accelerations and transfer function of roof with respect to ground

### 7.3.2 System identification by the wave method

In this study, the transverse (EW) vibrations of the tests structure were used for the system identification, in particular acceleration records of channel H1, which is located on the web wall, to minimize the effects of torsion (Fig. 7.1c).
As mentioned in the Methodology section, the structure is identified by matching the model and observed impulse responses, representing physically the propagation of a virtual pulse vertically through the structure. The nature of such pulse propagation through the tests structure (Fig. 7.4a) is illustrated in Fig. 7.4b, c by impulse responses at different levels for virtual source at the base (part b)) and at the roof (part c)). The impulse responses were computed from recorded accelerations during the ambient vibration (AV) test of damage state S3.2. Fig. 7.4b shows the upward propagation of the pulse, its reflection from the top and further reflections from the base and top. The pulse, however, gets progressively more and more distorted along the path and its propagation is not possible to describe by a simple shift in time as was the case in frame structures (Rahmani and Todorovska, 2014). The pulse distortion can also be seen in Fig. 7.4c in the
acausal and causal propagation of the virtual pulse from the roof towards the base, which demonstrates the necessity of fitting dispersive models.

Figure 7.4: (a) Elevation of the test structure. (b), (c) IRFs at different levels in the test structure for virtual source at base and at roof, respectively. IRFs are calculated on the frequency band 0-40 Hz and correspond to ambient vibration test for damage state S3.2.

The building is modeled as an equivalent uniform cantilevered Timoshenko beam, representing the building between ground and 7th floor as shown in Fig. 7.1d. The beam has a rectangular cross section with dimensions as the floor slabs (3.66 m × 8.13 m; Fig. 7.1c), and has height $H=19.2$ m (Fig. 7.1b). Uniform mass density $\rho = 366$ kg/m$^3$ is obtained by dividing the total mass of 208.5 tons (Panagiotou and Restrepo, 2011) by the beam volume. The area moment of inertia $I$ was calculated assuming that the building mass and stiffness are uniformly distributed over the cross-section. The shear factor $k = 5/6$, which corresponds to a rectangular cross section (Weaver et al., 1990). The damping constant is chosen by trial and error to match approximately the TF peaks. It is assumed to be different for the frequency bands containing the first mode and second mode. The two remaining parameters,
longitudinal wave velocity $c_L$ and moduli ratio $R$, are to be determined by LSQ fit of the impulse responses at the ground floor, on specified time and frequency windows. Therefore, the structure is identified using only two of the records, at ground floor and roof.

To find reasonable initial values for $c_L$ and $R$, we calculate the RMS error on a coarse grid of $c_L$ and $c_S$ (the results of the fitting are not sensitive to the value of damping). Three examples of such error surfaces, for the AV and WN tests corresponding to damage state S2, are shown in Fig. 7.5. They show that the error is not sensitive to the value of $c_S$, for $c_S$ large enough on the grid (200 m/s to 4000 m/s), which suggests that the test structure deforms primarily in bending.

The simplest beam model for such behavior is a uniform Rayleigh beam.

A Rayleigh beam is emulated by a Timoshenko beam that is very stiff in shear, which in the limit approaches Rayleigh beam. The limit can be approached either by increasing $c_S$ or by increasing $R$, i.e. by setting to a large enough value either one of the two parameter, and fitting only $c_L$. Each of the two options has its own limitation. Assuming constant $c_S$ would not account for possible differences in $c_S$ for the different damage states, while assuming constant $R$ fixes the ratios of the frequencies of vibration (for a uniform Timoshenko beam, the frequency
ratios for given $H/r_g$ are only function of $R$). In this study, we assume $R = 5$ for all the damage states and find $c_L$ from the LSQ fit. Fig. 7.6 illustrates the error in the frequency ratio $f_2/f_1$ for the different damage states introduced by setting $R=5$. The bars show $f_2/f_1$ obtained from the ambient vibration data, as reported in Moaveni et al. (2011), while the thick horizontal line shows the value for a cantilever Timoshenko beam with $R = 5$. The other two horizontal lines represent the values for a shear beam and an Euler-Bernoulli beam, shown for reference. It can be seen that, although the actual frequency ratio is not constant, it is close to the value for the model for all the damage states.

![Bar chart showing frequency ratio $f_2/f_1$ for different damage states.](image)

Figure 7.6: $f_2/f_1$ for ambient vibration tests based on the values reported by Moaveni et al. (2011) in comparison with frequency ratios of shear beam, Euler-Bernoulli beam and Timoshenko beam with $R = 5$

Tables 7.3 to 7.6 show the results of the fit for the AV, WN0.03g, WN0.05g and EQ tests. The initial values of $c_L$ and the RMS (Root Mean Square) error, normalized by the RMS value of the observed IRF, are also reported in the tables. It can be seen than the identified value of $c_L$ decreases with progressive damage, as expected.

Fig. 7.7 shows an example of the goodness of fit by comparison of the observed and fitted model transfer function (between roof and ground level responses, Fig. 7.7a) and the impulse response function at ground level (Fig. 7.7b), for the AV
Table 7.3: Identified longitudinal $c_L$ on the band 0-15 Hz for ambient vibration (AV) tests. Initial values and the normalized RMS error are also shown. Damping parameter $\mu$ is found by trial and error to match peaks of transfer function.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>$\mu$ (s)</th>
<th>Initial values</th>
<th>Estimated values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>band 1 †</td>
<td>band 2 ‡</td>
<td>$c_L$ (m/s)</td>
</tr>
<tr>
<td>S0</td>
<td>0.005</td>
<td>0.002</td>
<td>1200</td>
</tr>
<tr>
<td>S1</td>
<td>0.005</td>
<td>0.003</td>
<td>1000</td>
</tr>
<tr>
<td>S2</td>
<td>0.008</td>
<td>0.002</td>
<td>1000</td>
</tr>
<tr>
<td>S3.1</td>
<td>0.009</td>
<td>0.003</td>
<td>800</td>
</tr>
<tr>
<td>S3.2</td>
<td>0.006</td>
<td>0.001</td>
<td>800</td>
</tr>
<tr>
<td>S4</td>
<td>0.015</td>
<td>0.005</td>
<td>600</td>
</tr>
</tbody>
</table>

† 0-5 Hz for S0 to S3.1 and 0-2.5 Hz for S4
‡ 5-15 Hz for S0 to S3.1 and 2.5-15 Hz for S4
* RMS error normalized with respect to RMS of observed IRF
Table 7.4: Identified longitudinal $c_L$ on the band 0-15 Hz for WNO.03g tests. Initial values and the normalized RMS error are also shown. Damping parameter $\mu$ is found by trial and error to match peaks of transfer function.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>$\mu$ (s)</th>
<th>Initial values</th>
<th>Estimated values</th>
<th>RMS error</th>
<th>RMS error</th>
<th>RMS error</th>
<th>RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>0.005</td>
<td>0.004</td>
<td>1200</td>
<td>45</td>
<td>1030</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>S1</td>
<td>0.006</td>
<td>0.004</td>
<td>1000</td>
<td>26</td>
<td>970</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>S2</td>
<td>0.008</td>
<td>0.007</td>
<td>800</td>
<td>33</td>
<td>821</td>
<td>32</td>
<td>20</td>
</tr>
<tr>
<td>S3.1</td>
<td>0.009</td>
<td>0.008</td>
<td>800</td>
<td>36</td>
<td>743</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>S3.2</td>
<td>0.012</td>
<td>0.006</td>
<td>800</td>
<td>33</td>
<td>712</td>
<td>20</td>
<td>31</td>
</tr>
<tr>
<td>S4</td>
<td>0.010</td>
<td>0.006</td>
<td>600</td>
<td>55</td>
<td>519</td>
<td>32</td>
<td>50</td>
</tr>
</tbody>
</table>

Note: 0-5 Hz for S0 to S3.1 and 5-15 Hz for S4; 0-2.5 Hz for S0 to S3.1 and 0-2.5 Hz for S4.
Table 7.5: Identified longitudinal $c_L$ on the band 0-15 Hz for WN0.05g tests. Initial values and the normalized RMS error are also shown. Damping parameter $\mu$ is found by trial and error to match peaks of transfer function.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>$\mu$ (s)</th>
<th>Initial values</th>
<th>Estimated values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>band 1 †</td>
<td>band 2 ‡</td>
<td>$c_L$ (m/s)</td>
</tr>
<tr>
<td>S1</td>
<td>0.007</td>
<td>0.005</td>
<td>1000</td>
</tr>
<tr>
<td>S2</td>
<td>0.008</td>
<td>0.007</td>
<td>800</td>
</tr>
<tr>
<td>S3.1</td>
<td>0.009</td>
<td>0.008</td>
<td>800</td>
</tr>
<tr>
<td>S4</td>
<td>0.012</td>
<td>0.006</td>
<td>600</td>
</tr>
</tbody>
</table>

† 0-5 Hz for S0 to S3.1 and 0-2.5 Hz for S4
‡ 5-15 Hz for S0 to S3.1 and 2.5-15 Hz for S4
* RMS error normalized with respect to RMS of observed IRF
Table 7.6: Identified longitudinal cl on the band 0-15 Hz for earthquake (EQ) tests. Initial values and estimated RMS error are also shown. Damper parameter µ is found by trial and error to match peaks of transfer function.

<table>
<thead>
<tr>
<th>Event</th>
<th>µ (s)</th>
<th>Initial values</th>
<th>Estimated values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RMS Error (%)</td>
<td>RMS Error (%)</td>
</tr>
<tr>
<td>EQ1</td>
<td>0.003</td>
<td>1000</td>
<td>0.003</td>
</tr>
<tr>
<td>EQ2</td>
<td>0.005</td>
<td>1000</td>
<td>0.003</td>
</tr>
<tr>
<td>EQ3</td>
<td>0.01</td>
<td>800</td>
<td>0.003</td>
</tr>
<tr>
<td>EQ4</td>
<td>0.02</td>
<td>600</td>
<td>0.003</td>
</tr>
</tbody>
</table>

RMS Error normalized wrt RMS of observed IRF.

<table>
<thead>
<tr>
<th>Event</th>
<th>Band 1</th>
<th>Band 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and error to match peaks of transfer function.
test data for state S3.2. The frequency window for the fit is 0-15 Hz, and the time window for the fit is shown by a red line in the figure. It can be seen that the qualitative agreement is very good. The fitted value is $c_L = 880 \text{ m/s}$ and the RMS error is 38%. Such plots look similar for all of the tests and are not presented.

The parameter of the beam, $c_L$, physically represents the velocity of propagation of compressional waves through the beam. The bending deformation propagates with phase velocity $c_{ph}$, which is frequency dependent (see Section 2.1) and asymptotically approaches $c_L$ at high frequency. Fig. 7.8 shows the phase velocity for the different damage states, computed analytically for the fitted beam models in the AV test data. It can be seen that $c_L$ increases rapidly with frequency, eventually approaching $c_L$ as $f \to \infty$, and decreases with increasing damage level.

Figure 7.7: Comparison of observed data with fitted model for ambient vibration test corresponding to damage state S3.2: (a) Transfer function; (b) IRF comparison (IRF has been calculated on 0-15 Hz and fitted on the time interval -3 s to 3 s, as shown with the bracketed line)
Figure 7.8: Phase velocity of the fitted model in the AV test data for the different damage states. The frequency window of the fit (0-15 Hz) is indicated.

### 7.3.3 Identification of the fundamental frequency of vibration \( f_1 \)

The fundamental frequency \( f_1 \) is measured as follows. For the AV, WN0.03g and WN0.05g tests, it is measured from the peak frequencies in the transfer-function amplitude, obtained from the entire record. For the EQ tests, it is measured in the same way for 30 s of strongest motion (see Fig. 7.2). These estimates are referred to as interval estimates and are shown in Table 7.7. Those for the weak motion tests are in agreement with the values identified in Moaveni et al. (2011). To capture the time history of the changes during the EQ tests, the instantaneous frequency was estimated from the relative roof acceleration, as described in the Methodology section. For this purpose, the entire EQ test records available are used, which include segments of weak motion before and after the strongest motion.
Table 7.7: Frequency of the first mode read from transfer function plots

<table>
<thead>
<tr>
<th>Damage State</th>
<th>$f_1$ (Hz)</th>
<th>Amb</th>
<th>WN0.03g</th>
<th>WN0.05g</th>
<th>EQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>1.92</td>
<td>1.71</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>1.87</td>
<td>1.55</td>
<td>1.41</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>1.68</td>
<td>1.25</td>
<td>1.14</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>S3.1</td>
<td>1.50</td>
<td>1.12</td>
<td>1.05</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>S3.2</td>
<td>1.52</td>
<td>1.22</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>0.95</td>
<td>0.84</td>
<td>0.79</td>
<td>0.64</td>
<td></td>
</tr>
</tbody>
</table>

The instantaneous and the interval estimates of $f_1$ for all the tests are presented graphically in Fig. 7.9, in chronological order from left to right. The first row shows the time histories of the roof drift during the EQ tests, computed by double integration and filtering of the recorded accelerations (0.2-0.4 to 24-26 Hz). The second row shows the instantaneous frequency, $f_1(t)$, (the ragged lines) and the interval estimates from Table 7.7 (short horizontal lines), and the third row shows the skeleton of the transform, $S(t)$. The instantaneous frequency was estimated for three values of $\sigma$ of the Gabor window, corresponding to $\sigma_t = 1, 2$ and $4$ s, shown by lines of different thickness (see Section 3.3). The corresponding estimates of $\sigma_f$ are also stated in the legend. The instantaneous frequency curves are interrupted where the raw estimates are not reliable due to rapidly changing amplitude envelope of the signal (as illustrated by $S(t)$). The estimates from the AV, WN0.03g and WN0.05g tests are shown by dashes before or after the corresponding earthquake test. The longer dashes, which correspond to the EQ tests, span the corresponding time segment that was used to estimate them. It can be seen that they overlap with the instantaneous frequency curve on that segment, and represent the lowest estimate of the instantaneous frequency for that earthquake.
test. The AV estimates also are close to the instantaneous frequency estimate near the end of the previous and the beginning of the following EQ test.

Fig. 7.9 demonstrates the strong amplitude dependency of $f_1$. During the EQ tests, $f_1$ drops from its ambient level value, and recovers partially after the strong excitation is over. Therefore, comparison of different estimates of $f_1$ is meaningful only within the context of the levels of response that the structure experienced when the measurements were made. Possible causes for these nonlinear effects are, e.g., opening and closing of cracks in the concrete wall, rotation at the fixed end and yielding in the reinforcement, nonlinear effects in the construction joints and in the subgrade (the shake table foundation and soil part of the system) and other possible effects. Based on our limited investigation of the vertical records at the ground floor, we believe that the prevailing cause of these nonlinear effects was in the structure itself rather in the subgrade.

7.3.4 Global health monitoring from detected changes in $c_L$ and $f_1$

Next, the identified values of the two damage sensitive parameters, $c_L$ and $f_1$, and their change are compared. The values for the EQ tests are the interval ones, i.e. those estimated from 30 s segments of the strongest motion, for both $c_L$ and $f_1$.

In Fig. 7.10, the estimates of $f_1$ (part a)) and $c_L$ (part b)) are compared for the different states and tests (the values are from Tables 7.3 to 7.6 and 7.7). Similarly, in Fig. 7.11a the RMS values of the base motion for different test and states are compared. Note that the RMS value of the base motion for EQ1 is similar to that for WN0.05g tests. In Fig. 7.11b, the RMS error of the estimates of $c_L$ are compared. In Fig. 7.12, the detected changes in $f_1$ and $c_S$, relative to state S1, are shown.
Figure 7.9: Average roof drift and the instantaneous frequency, $f_1$, and skeleton, $S(t)$, obtained from the Gabor transform of the relative roof acceleration recorded during the earthquake tests.
Figure 7.10: Frequency of the first mode read from TFs for different tests at all damage states (left); Longitudinal wave speed found by fitting (right)

Figure 7.11: (a) RMS values of base motion for different tests and all damage states; (b) RMS error normalized with respect to observed RMS of IRF for different tests

It can be seen from Fig. 7.10 that the values of $f_1$ and $c_L$ differ for the same state, depending on the type of test, and have smaller values for the larger amplitude tests. The amplitude dependency is seen even among the weak motion tests.

Fig. 7.11b, shows that the RMS error of the estimate of $c_L$ is the smallest for the white noise tests and the largest for EQ tests, likely due to the violation of the assumption of linearity as well as the narrower band nature of the excitation. For
the AV tests, it is larger than that for the white noise tests. The reason for this may be the significantly shorter duration of the data segments, almost by a factor of three (3 min for the AV tests as opposed to 8 min for the white noise tests). The RMS error for the AV tests could be reduced by analyzing longer single segments or stacking (averaging) over different time windows.

Fig. 7.12 shows that changes with progressive level of damage are detected in both $f_1$ and $c_S$ and for all the tests. The level of the changes, however differs for the same state depending on the type of test data used. The changes are convincingly the largest in $f_1$ estimated from the EQ test data. They are the smallest for the AV tests, but only for the lower damage states (change from state S1 to S2). For the highest damage state, the changes detected form the AV tests are comparable to and even slightly larger than those from the other tests (except for the EQ tests and $f_1$, as mentioned earlier).

Finally, Fig. 7.13 shows the detected changes in $f_1$ and $c_L$ on separate plots for the different type of test data, for convenience in comparing the performance of the two damage sensitive parameters. It can be seen that, for the same test

Figure 7.12: Observed changes with respect to S1 for different tests and all damage states
data and damage state, the changes in both $f_1$ and $c_L$ are similar for the AV and WN tests, which means that both parameters are equally sensitive to damage for the lower level excitation tests. For the EQ test data, however, the changes in $f_1$ are significantly larger than the changes in $c_L$, by at least 25%. Considering the previously mentioned result, we believe that the changes in $f_1$ are larger because it is more sensitive to the nonlinear effects in the response to the EQ test excitation. For real structures in situ, $f_1$ would also be more sensitive to nonlinear effects in the soil-foundation response.

Figure 7.13: Comparison of observed changes with respect to S1 in $f_1$ and $c_L$ for different tests

### 7.4 Discussion and Conclusions

Shake table test data of a full-scale slice of a 7-story RC building were used to test and calibrate the wave method for structural system identification and health monitoring. Within the scope of this study was to monitor the global health
of the specimen. Therefore, a uniform beam model was fitted. Data were used from four earthquake (EQ) tests of increasing intensity, as well as from ambient vibration (AV) and white noise (WN) tests (with RMS values 0.03g and 0.05g), performed before the first and after each earthquake test. The specimen, which consisted of two perpendicular shear walls and deformed predominantly in bending, was modeled by a uniform Timoshenko beam (TB) with fixed larger value of the ratio of shear to bending stiffness (so that it corresponds to a Rayleigh beam). The model longitudinal wave velocity, $c_L$, was identified by matching, in the least squares sense, the model and observed impulse response functions obtained from two records only, at base and roof of the test structure. The fundamental frequency of vibration $f_1$ was also identified for comparison.

(1) One of the most important findings of this study is that stable estimates of $c_L$ could be identified from relatively short segments of ambient response data, of 3 min length in this case. Previously, much longer segments have been used and stacking to compute impulse response functions (e.g. Prieto et al. (2010); Nakata and Snieder (2014)). Such shorter duration tests for real buildings are practical and economical because they do not require permanent instrumentation, and can be carried out periodically using portable instruments, to monitor changes with time due to deterioration of the structure or changes in the operating and environmental conditions, and also after an extreme event, such an earthquake, to detect changes possibly due to structural damage.

(2) Another important finding is that $c_L$ was sensitive to the damage of the test structure, and changes proportional to the level of damage could be detected from all of the different type of test data. As estimated from the AV test data, the reduction in $c_L$ relative to the undamaged state, $S_0$, was about 1% after EQ1 (immediate occupancy), 6% after EQ2, 18% after EQ3 and 48% after EQ4 (life
(3) This study showed that the identified values of $c_L$ and $f_1$ are dependent on the amplitude of the response. They both drop during the EQ tests and recover partially afterwards during the AV tests. Such behavior of $f_1$ in full-scale structures has been known at least since the 1970s (Udwadia and Trifunac, 1974; Trifunac et al., 2001a,b). A recent example is a study of 1700 seismic events in a nine story RC shear wall building Boroschek et al. (2014). Amplitude dependency has been noted before also for the wave velocities (Todorovska, 2009c; Rahmani and Todorovska, 2015).

(4) This study also showed that the changes in both $c_L$ and $f_1$ depended on the type of test (Fig. 7.12). This is important in calibrating structural health monitoring methods, and when possible damage in the structure is inferred from observed parameter changes. For the lower damage levels, the larger amplitude tests generally revealed a larger change in $c_L$. For the highest damage level, however, the detected changes in $c_L$ were comparable, with the differences likely being mostly due to measurement error.

(5) The most dramatic dependence of the change on the level of the response was that of $f_1$ determined from the EQ test data (Fig. 7.12a), which was likely caused by recoverable nonlinear effects in the response of the tests structure during the EQ tests. In contrast, such an effect was not observed in $c_L$ (Fig. 7.12b), which suggests that $c_L$, as estimated from impulse response functions, is less sensitive to such effects than $f_1$.

According to Panagiotou et al. (2011), in this controlled experiment, the bending of the wall accounted for about 76-85% of the roof drift, while shear at the construction joints accounted for about 1-5% during the different earthquake tests.
Other two factors contributing to the roof drift that they measured are localized rotation of the fixed end (due to bar bond slip of the longitudinal reinforcing anchored in the foundation) which accounted for 1-7% of the roof drift, and effects of the subgrade, which accounted for 3-8% of the roof drift. Subgrade refers to the combined effects of the shake-table platen-hydraulic bearings foundation-soil test structure interaction (Luco et al., 2011). Because of the dominance of the contribution from bending of the wall, we interpret the observed changes in $c_L$ and $f_1$ and recoverable nonlinear effects to be largely related to the wall itself rather than to the soil.

It is concluded that the 7-story building slice test data provided a valuable insight into the effects of damage on the wave velocity in RC wall structures, and that the wave method with ambient test data presents an opportunity for practical use in full-scale structures that should be further explored and potentially developed for more complex structures.

7.5 Acknowledgments

The data were obtained from the George E. Brown Network for Earthquake Engineering Simulation (https://nees.org/). The authors are most grateful to Prof. Jose Restrepo from the University of California at San Diego for making the data available and to Prof. Babak Moaveni from Tufts University for his generous help in extracting the data.
Chapter 8

Summary and Conclusions

This study presented Timoshenko beam models to take into account dispersion due to flexural deformation for use in model-based system identification and health monitoring of buildings by the wave method. In this chapter, a summary of the main findings of this dissertation is presented. More detailed discussion of the results are provided at the end of each chapter. The general conclusions and suggestions for future work are also presented.

8.1 Summary of the Main Findings

In chapter 3, the building was modeled as a uniform viscoelastic cantilevered Timoshenko beam. Wave propagation in Timoshenko beam is dispersive meaning that phase velocity is frequency dependent. Timoshenko beam was used to understand how the presence of bending in the deformation of high rise buildings during seismic response may affect its identification from impulse responses in terms of velocities of wave propagation. A significant difference in the wave properties of a Timoshenko beam relative to shear beam, beyond the frequency dependency of the phase velocity is the existence of a second wave, which is evanescent for frequencies below critical frequency, \( \omega_{cr} = \sqrt{k_G c_S / r_g} \), and a propagating wave otherwise, adding complexity to the wave propagation. The first wave in the band \( \omega < \omega_{cr} \) is of greater interest for wave based structural system identification and health monitoring, because it requires simpler estimation (only one propagating wave). This
band is broader if the building is stiffer in shear and has smaller radius of gyration. The number of modes of vibration contained in this band further depends on the building axial stiffness and height, and is larger for larger $R = G/E$ and $H/r_g$. The dispersion is controlled by two independent parameters, the dimensionless frequency $\Omega = \omega r_g/c_S$ and $R$. The phase velocities deviate more from their high frequency asymptote (which is the velocity of shear waves in the beam $c_{TS}^B = \sqrt{k_G c_S}$ for $R < 1$) for smaller $\Omega$ and larger $R$. Consequently, the modal frequencies are affected more by dispersion for lower modal number, larger $R$, and larger $H/r_g$ (for which they start occurring at lower $\Omega$). The presented results for impulse response functions for a wider range of dimensionless parameters, typically corresponding to buildings, showed that, despite the dispersion, prominent pulses can be identified in the impulse responses in selected frequency bands, which could be used for SHM.

In chapter 4, two simple non-parametric methods were proposed to obtain approximately the dispersion characteristics of a building from earthquake records. The methods were demonstrated on a Timoshenko beam model of a building, for which analytical dispersion relation is known. Both nonparametric estimates of phase velocity (the interval values from wave travel time and point values obtained from the natural frequencies of vibration), depicted correctly the trends of the wave dispersion and were within the range/reasonably close to the true values. It was concluded that the two nonparametric interferometric techniques presented an opportunity for exploratory analysis of wave dispersion in buildings. It was also shown that Timoshenko beam model depicts well the nature of dispersive wave propagation in a 9-story RC building (Millikan Library) at lower frequencies and may be an appropriate model for RC buildings with shear walls, in general.
In chapter 5, a uniform Timoshenko beam model was used for system identification of a 9-story RC building (Millikan Library) by the wave method. A simple procedure was described how to approximate the geometric parameters of the beam model from the building geometry. The stiffness parameters were found by least square fit of the model impulse response functions to the observed ones. Since for the fitting initial values are needed such that they are reasonably close to actual values, a simple procedure was suggested to estimate them from the first two apparent frequencies of vibration.

In chapter 6, the uniform Timoshenko beam model was generalized to a layered one, i.e. with piecewise continuous properties along the height where each layer corresponds to a group of floors. The layering structure allowed modeling variation of the building properties (plan dimensions, stiffness and mass density) with height. A 4-layer TB model was used to identify a full-scale 54-story steel frame building from its earthquake response. The fitted model could successfully predict the linear response of the building during two subsequent earthquakes. The fitted Timoshenko beam model successfully detected increase in bending and shear stiffnesses in the part of the structure where the floors were stiffened by Vierendeel trusses and transfer girders. It was also observed that the model will approach shear beam (high values of bending stiffness) if shear deformation is more significant in the corresponding layer.

In chapter 7, a uniform Timoshenko beam model was used for global health monitoring of a full-scale 7-story slice of a shear wall building tested on the UCSD-NEES shake table. The test structure was progressively damaged by applying four earthquakes of increasing intensity. Ambient vibrations and white noise excitation tests were carried out for each damage state. As a general procedure to find reasonable initial values for the Levenberg-Marquardt algorithm, the mean
square error between model and observed impulse response functions was calculated on a coarse grid of beam stiffness parameters. The study showed that the wave method successfully detected the progressive level of damage for all tests. More importantly, it was shown that stable estimates of $c_L$ could be identified from relatively short segments of ambient noise response data, of 3 min length in this case. Short duration ambient vibration tests to identify the properties of real buildings and monitor the structural health are practical and economical because they do not require permanent instrumentation and can be carried out periodically using portable instruments.

8.2 General Conclusions and Recommendations for Future Work

In this study, the wave method for structural system identification and health monitoring of buildings was improved by introducing a non-uniform viscoelastic Timoshenko beam model to be fitted. This model takes into account shear and bending deformation and the effects of the rotatory inertia of the cross section. By choosing different parameters for the model, it can be used to identify buildings with different balance of shear and bending deformation, from pure shear beam at one end to Rayleigh beam at the other end. The usefulness of the model to identify building structures and its sensitivity to damage was demonstrated on three full scale examples. On the example of a 54-story steel moment frame building, the fitted 4-layer Timoshenko beam model successfully detected increase in bending and shear stiffnesses at the setbacks where the floor was stiffened by Vierendeel trusses and transfer girders. On the example of a 7-story slice of a shear wall building, changes in wave velocities calculated from dynamic tests of different
nature and intensity (ambient vibration, white noise and earthquake) successfully reflected the progressive damage. These tests were used to provide insight to relate the changes in model stiffness parameters to the level of damage.

In model-based system identification, one should always be aware of the bias that the model introduces and its limitations. The limitations of the non-uniform Timoshenko beam used in this study are as follows:

i The model is a linear elastic. However, it can be extended to treat nonlinear response of a building during earthquake by moving window analysis, by approximating the nonlinear system in each time window by an equivalent linear system.

ii The model is 1D and can take into account only variations of geometry and material properties of building along the height. Consequently, it can be used to model regular buildings but is unable to model 3D effects.

iii The model is a continuous. Consequently, it is suitable only for the lower frequency range (longer wavelengths).

In spite of these limitations, many buildings are sufficiently regular to be modeled as 1D structures. Future work should be focused on calibration of the method to relate changes in the damage sensitive parameters to the extent of damage in the structure for different structural systems. This is only possible through studying many buildings for which earthquake or ambient vibration records are available, complemented by data from full-scale laboratory tests. As more damaging earthquakes occur, an effort should be invested in detailed documentation of observed damage for the instrumented buildings. Having a database for different types of structures for a variety of amplitude motions, from ambient to small and large earthquakes, is crucial for calibration of any health monitoring methodology.
Based on the results of many case studies, one would be able to define a damage index that depends on the identified wave velocities and relate that to different structural performance levels. This will make the wave method ready to use in practice.
Reference List


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7-story building slice tested at UC-San Diego.” *Journal of Structural Engineering*, 137(6), 677–690.


Appendix A

Phase Velocity, Group Velocity and Dispersion Relation

Consider a harmonic wave of the form $f(x - c^{ph}t) = ae^{i(kt - wt)}$ where $\omega$ is frequency and $k$ is the wave number. For a moving observer which is at location $x(t)$ at time $t$, waveform is $ae^{i(kx(t) - wt)}$. If the wave is to appear stationary to the moving observer, its phase, $(kx(t) - wt)$, must be constant with time as shown in eqn. A.1. $c^{ph}$ is called phase velocity because at this velocity argument of the harmonic which can be thought as a constant phase angle. In other words, a fixed phase of the harmonic travels with the velocity $c^{ph}$.

$$\frac{d}{dt}(kx(t) - \omega t) = 0 \rightarrow k\dot{x}(t) - \omega = 0 \rightarrow c^{ph} = \frac{\omega}{k} = \dot{x}(t)$$ \hspace{1cm} (A.1)

If there is more than one frequency, harmonic waves with close frequencies within a band form a wave packet. As shown in A.1, while different harmonics propagate with their phase velocity $c^{ph}$, wave energy is transported by the group velocity $c^{gr}$ of the packet, which is also the velocity of propagation of its amplitude envelope. Equation for group velocity is shown in eqn. A.2 (Hagedorn and DasGupta, 2007; Saad, 2009).

$$c^{gr} = \frac{d\omega}{dk} = c^{ph} + k\frac{d}{dk}(c^{ph})$$ \hspace{1cm} (A.2)
Figure A.1: Phase and group velocity

In a non-dispersive medium, the phase velocity is equal to the group velocity but in a dispersive medium phase velocity may be greater than, equal to or less than the group velocity (Saad, 2009). Relationship between wave number and frequency in a dispersive medium is called the dispersion relation. Phase velocity and group velocity of the medium can be found from the dispersion. On the plot of wave number versus frequency, at each point phase velocity is slope of the line connecting the point to the origin and group velocity is the slope of the tangent line. For a non-dispersive medium wave number has a linear relationship with frequency so dispersion curve is a line and phase velocity is equal to group velocity.
Appendix B

Different Beam Theories

Governing equations of different beam theories are presented in this section.

B.1 Shear beam

Motion of shear beam is defined by a simple wave equation as shown in eqn. B.1

\[ \frac{\partial^2 u_s}{\partial z^2} - \left( \frac{\rho}{k_G G} \right) \frac{\partial^2 u_s}{\partial t^2} = 0 \]  

where \( \rho \) is the mass density, \( k_G \) is the shear factor and \( G \) is the shear modulus. By trying a solution of the form \( e^{i(kx - \omega t)} \), it can be easily shown that \( k = \omega c_S \) where \( c_S = \sqrt{\frac{\rho}{k_G G}} \) is the shear wave velocity and \( c^{ph} = c^{gr} = c_S \).

B.2 Euler-Bernoulli beam

Euler-Bernoulli beam equation is as follows

\[ EI \frac{\partial^4 u_{EB}}{\partial z^4} + \rho A \frac{\partial^2 u_{EB}}{\partial t^2} = 0 \]  

(B.2)

Dividing eqn. B.2 by \( \rho I \) results in

\[ c_L^2 \frac{\partial^4 u_{EB}}{\partial z^4} + \frac{1}{r_g^2} \frac{\partial^2 u_{EB}}{\partial t^2} = 0 \]  

(B.3)
where \( c_L = \sqrt{\frac{E}{\rho}} \) is the longitudinal wave velocity and \( r_g = \sqrt{\frac{T}{A}} \) is the gyration radius.

Trying a traveling wave solution of the form \( e^{i(kx-\omega t)} \), dispersion relation of Euler-Bernoulli beam is as follows

\[
c_L^2 k^4 - \frac{\omega^2}{r_g^2} = 0 \quad \text{(B.4)}
\]

Based on \( \pi \) theorem if an equation involves \( n \) physical variables and \( m \) is number of fundamental dimensions required to describe them, then there will be \( m \) primary variables and the remaining \((n-m)\) variables can be expressed as \((n-m)\) independent dimensionless quantities of \( \pi \) groups \( \pi_1, \pi_2, \pi_3, \ldots, \pi_{n-m} \) (Bridgman, 1931)

\[
q_1 = f(q_1, q_2, \ldots, q_n) \rightarrow \pi_1 = \varphi(\pi_2, \pi_3, \ldots, \pi_{n-m}) \quad \text{(B.5)}
\]

Based on Table B.1 \( n = 4 \) and \( m = 2 \) so there are \( n-m = 2 \) dimensionless quantities. Here, we introduce dimensionless wave number as \( K = kr_g \) and dimensionless frequency as \( \Omega_2 = \frac{\omega r_g}{c_L} \). Using dimensionless parameters, eqn. B.4 can be written as follows

\[
K^4 - \Omega_2^2 = 0 \rightarrow \Omega_2 = K^2, \quad K = \sqrt{\Omega_2} \quad \text{(B.6)}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_L )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>( r_g )</td>
<td>( L )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td>( k )</td>
<td>( LT^{-1} )</td>
</tr>
</tbody>
</table>
Now, dimensionless phase and group velocity can be found as follows

\[ C^{ph} = \frac{c^{ph}}{c_L} = \frac{\Omega_2}{K} = K = \sqrt{\Omega_2} \quad (B.7) \]

\[ C^{gr} = \frac{c^{gr}}{c_L} = \frac{d\Omega_2}{dK} = 2K = 2\sqrt{\Omega_2} = 2C^{ph} \quad (B.8) \]

It can be seen that as frequency \( \rightarrow \infty \) both phase and group velocity \( \rightarrow \infty \). Infinite wave velocity is not physically meaningful, so for high frequencies Euler-Bernoulli beam is not a realistic model. It can also be seen that for all frequencies, group velocity is twice as large as phase velocity \( C^{gr} = 2C^{ph} \).

### B.3 Rayleigh beam

Rayleigh beam equation is as follows.

\[ EI \frac{\partial^4 u_{RB}}{\partial z^4} + \rho A \frac{\partial^2 u_{RB}}{\partial t^2} - \rho I \frac{\partial^4 u_{RB}}{\partial t^2 \partial z^2} = 0 \quad (B.9) \]

Dividing eqn. B.9 by \( \rho I \) results in

\[ c_L^2 \frac{\partial^4 u_{RB}}{\partial z^4} + \frac{1}{r_g^2} \frac{\partial^2 u_{RB}}{\partial t^2} - \frac{\partial^4 u_{RB}}{\partial t^2 \partial z^2} = 0 \quad (B.10) \]

where \( c_L = \sqrt{\frac{E}{\rho}} \) is the longitudinal wave velocity and \( r_g = \sqrt{\frac{T}{A}} \) is the gyration radius.

Trying a traveling wave solution of the form \( e^{i(kx - \omega t)} \), dispersion relation of Rayleigh beam is as follows

\[ c_L^2 k^4 - \frac{\omega^2}{r_g^2} - k^2 \omega^2 = 0 \quad (B.11) \]
Similar to the case of Euler-Bernoulli beam (Table B.1), number of physical variables \((n)\) is 4 and number of fundamental dimensions \((m)\) is 2, so based on \(\pi\) theorem there are \((n - m) = 2\) dimensionless quantities (Bridgman, 1931). Again, we introduce dimensionless wave number as \(K = kr_g\) and dimensionless frequency as \(\Omega_2 = \frac{\omega_2}{c_L}\) to rewrite dispersion relation (eqn. B.11) as follows

\[
K^4 - \Omega_2^4 - K^2 \Omega_2^2 = 0 \rightarrow \Omega_2 = \sqrt{\frac{K^4}{1 + K^2}} , \quad K = \frac{\Omega_2}{\sqrt{2}} \sqrt{1 \pm \sqrt{1 + \frac{4}{\Omega_2^2}}} \quad (B.12)
\]

Now, dimensionless phase and group velocity can be found as follows

\[
C_{ph}^{ph} = \frac{c_{ph}}{c_L} = \frac{\Omega_2}{K} = \frac{\sqrt{2}}{\sqrt{1 \pm \sqrt{1 + \frac{4}{\Omega_2^2}}}} \quad (B.13)
\]

\[
C_{gr}^{gr} = \frac{c_{gr}}{c_L} = \frac{d\Omega_2}{dK} = \frac{(-K^4 + 2K^3 + 2K)}{(1 + K^2)^{\frac{3}{2}}} \quad (B.14)
\]

### B.4 Timoshenko beam

Dispersion characteristics of Timoshenko beam (eqn. B.15) are presented in detail in chapter 3. Here, limiting cases of Timoshenko beam are presented.

\[
EI \frac{\partial^4 u}{\partial z^4} - \rho I \left(1 + \frac{E}{k_G G}\right) \frac{\partial^4 u}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 u}{\partial t^2} + \rho^2 I \frac{\partial^4 u}{\partial t^4} = 0 \quad (B.15)
\]

As \(G \rightarrow \infty\), Timoshenko beam goes toward Rayleigh beam

\[
EI \frac{\partial^4 u}{\partial z^4} - \rho I \frac{\partial^4 u}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 u}{\partial t^2} = 0 \quad (B.16)
\]

Divide eqn. B.15 by \(EI\)
\[
\frac{\partial^4 u}{\partial z^4} - \rho \left( \frac{1}{E} + \frac{1}{k_G G} \right) \frac{\partial^4 u}{\partial z^2 \partial t^2} + \frac{\rho A}{E I} \frac{\partial^2 u}{\partial t^2} + \left( \frac{\rho^2}{k_G G} \right) \left( \frac{1}{E} \right) \frac{\partial^4 u}{\partial t^4} = 0 \quad (B.17)
\]

Now, as \( E \to \infty \), Timoshenko beam goes toward shear beam

\[
\frac{\partial^4 u}{\partial z^4} - \frac{\rho}{k_G G} \frac{\partial^4 u}{\partial z^2 \partial t^2} = 0 \quad (B.18)
\]

### B.5 Timoshenko beam without rotatory inertia

Neglecting rotatory inertia of the cross section in Timoshenko beam will results in

\[
EI \frac{\partial^4 u}{\partial z^4} - \rho \frac{E}{k_G G} \frac{\partial^4 u}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 u}{\partial t^2} + \frac{\rho^2 I}{k_G G} \frac{\partial^4 u}{\partial t^4} = 0 \quad (B.19)
\]

As \( G \to \infty \), eqn. B.19 goes toward Euler-Bernoulli beam

\[
EI \frac{\partial^4 u}{\partial z^4} + \rho A \frac{\partial^2 u}{\partial t^2} = 0 \quad (B.20)
\]

Divide eqn. B.19 by \( EI \)

\[
\frac{\partial^4 u}{\partial z^4} - \rho \frac{1}{k_G G} \frac{\partial^4 u}{\partial z^2 \partial t^2} + \frac{\rho A}{E I} \frac{\partial^2 u}{\partial t^2} + \left( \frac{\rho^2}{k_G G} \right) \left( \frac{1}{E} \right) \frac{\partial^4 u}{\partial t^4} = 0 \quad (B.21)
\]

Now, as \( E \to \infty \), eqn. B.19 goes toward shear beam

\[
\frac{\partial^4 u}{\partial z^4} - \frac{\rho}{k_G G} \frac{\partial^4 u}{\partial z^2 \partial t^2} = 0 \quad (B.22)
\]
Appendix C

Propagator Matrix for the Viscoelastic Timoshenko Beam

Characteristic equation of matrix B (defined in chapter 6) is as follows.

\[
\text{det}(A - \lambda I) = 0 \rightarrow \lambda^4 + \left(\frac{\rho}{E} + \frac{\rho}{k_G G}\right)\left(1 - i\omega\mu\right)\lambda^2\omega^2 - \left(\frac{\rho A}{EI(1 - i\omega\mu)}\right)\omega^2 + \left(\frac{\rho^2}{k_G GE(1 - i\omega\mu)^2}\right)\omega^4 = 0 \quad (C.1)
\]

Simplifying eqn (C.1) using eqns (C.2), (C.3), (C.4) leads to equation (C.5) where \(c_L\) is longitudinal wave velocity, \(c_S\) is shear wave velocity and \(r_g\) is radius of gyration.

\[
c_L = \sqrt{\frac{E}{\rho}} \quad (C.2)
\]

\[
c_S = \sqrt{\frac{G}{\rho}} \quad (C.3)
\]

\[
r_g = \sqrt{\frac{I}{A}} \quad (C.4)
\]

\[
\lambda^4 + \left(\frac{1}{c_L^2} + \frac{1}{k_G c_S^2}\right)(\frac{1}{1 - i\omega\mu})\lambda^2\omega^2 - \left(\frac{1}{c_L^2 r_g^2(1 - i\omega\mu)}\right)\omega^2 + \left(\frac{1}{c_L^2 k_G c_S^2(1 - i\omega\mu)^2}\right)\omega^4 = 0 \quad (C.5)
\]
Take dimensionless wavenumber, frequency, damping parameter and modulus ratio as defined by the following equations (same as chapter 3). It can be shown that eqn (C.5) has 4 distinct eigenvalues in the form shown in eqn (C.11). By substituting $\lambda = ik$ in eqn (C.5), it can also be shown that eqn (C.5) is equivalent to dispersion relation of Timoshenko beam (See chapter 3 for dispersion relation).

$$K = k_r g$$  \hspace{1cm} (C.6)

$$\Omega = \frac{\omega r_g}{c_s}$$  \hspace{1cm} (C.7)

$$M = \frac{\omega c_s r_g}{h}$$  \hspace{1cm} (C.8)

$$R = \frac{G}{E} = \frac{c_s^2}{c_L^2}$$  \hspace{1cm} (C.9)

$$\alpha = 1 - i\omega \mu$$  \hspace{1cm} (C.10)

$$\lambda = \pm \frac{i}{r_g} \frac{\Omega}{\sqrt{2}} \sqrt{\left( \frac{1}{\alpha} \right) \left( \frac{1}{k_G} + R \right) \pm \sqrt{\left( \frac{1}{\alpha^2} \right) \left( \frac{1}{k_G} - R \right)^2 + \frac{4R}{\alpha \Omega^2}}}$$  \hspace{1cm} (C.11)

Since matrix B has four distinct eigenvalues, they will lead to four linearly independent eigenvectors. Eigenvector corresponding to each eigenvalue can be easily calculated from $Bv = \lambda v$ as shown in (C.12). To find eigenvectors, one can assume $v_4 = 1$ and use three of four eqns (C.12) to solve for other components of the vector $(v_1, v_2, v_3)$. Since $\det(A - \lambda I) = 0$ is already satisfied, it doesn’t matter which three equations are chosen and the results will be multiples of each other. Here, fourth, first and second equation from (C.12) are used. Introducing $m = \rho A$ and using equations (C.6)-(C.10) and $\lambda = ik$, eigenvectors can be written in terms of dimensionless parameters as shown in (C.13)-(C.17).
\[ Bv = \lambda v \rightarrow \begin{bmatrix}
0 & 1 & 0 & \frac{1}{k_G G A (1 - i \omega \mu)} \\
0 & 0 & \frac{1}{E I (1 - i \omega \mu)} & 0 \\
0 & -\rho I \omega^2 & 0 & -1 \\
-\rho A \omega^2 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4
\end{bmatrix} = \lambda \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4
\end{bmatrix}
\]

\[ \begin{align*}
v_1 + \frac{1}{k_G G A} v_4 &= \lambda v_1 \\
\frac{1}{E I} v_3 &= \lambda v_2 \\
-\rho I \omega^2 v_2 - v_4 &= \lambda v_3 \\
-\rho A \omega^2 v_1 &= \lambda v_4
\end{align*} \quad (C.12) \]

\[ v_4 = 1 \quad (C.13) \]

\[ v_1 = \frac{-\lambda v_4}{\rho A \omega^2} = \frac{-\lambda}{\rho A \omega^2} = \frac{-i K r_g}{m c_s^2 \Omega^2} \quad (C.14) \]

\[ v_2 = \lambda v_1 - \frac{1}{k_G G A} = \frac{-\lambda^2}{\rho A \omega^2} - \frac{1}{k_G G A} = \frac{1}{m c_s^2} \left( \frac{K^2}{\Omega^2} - \frac{1}{k_G \alpha} \right) \quad (C.15) \]

\[ v_3 = EI \alpha \lambda v_2 = EI \alpha \lambda \left( \frac{-\lambda^2}{\rho A \omega^2} - \frac{1}{k_G G A (1 - i \omega \mu)} \right) \]

\[ = EI \alpha \left( \frac{i K}{r_g} \right) \left( \frac{1}{G A} \right) \left( \frac{K^2}{\Omega^2} - \frac{1}{k_G \alpha} \right) = \left( \frac{i K r_g}{R} \right) \left( \frac{K^2 \alpha}{\Omega^2} - \frac{1}{k_G} \right) \quad (C.16) \]
\[ v = \begin{cases} 
\frac{-iKr_{g}}{mc_{S}^{2}\Omega} \\
\frac{1}{mc_{S}^{2}} \left( \frac{K^{2}}{\Omega^{2}} - \frac{1}{kG\alpha} \right) \\
\frac{iKr_{g}}{R} \left( \frac{K^{2}\alpha}{\Omega^{2}} - \frac{1}{kG} \right) \\
1 
\end{cases} \]  
(C.17)

Matrix B has 4 distinct eigenvalues \((\lambda_1, -\lambda_1, \lambda_2, -\lambda_2)\) so the corresponding eigenvectors are linearly independent and can be used to form one fundamental matrix for the system of O.D.E.s based on equation \(e^B = Xe^\Lambda X^{-1}\), where \(X\) is a matrix that has eigenvectors as its columns and \(\Lambda\) is a diagonal matrix that has eigenvalues on its main diagonal. Fundamental matrix can be used to propagate the solution from one point in a layer to any other point in the same layer, so in geophysics community it is called the propagator matrix. Propagator matrix wrt \(z_0\) is shown in (C.18).

\[ P(z, z_0) = e^{B(z-z_0)} = Xe^{\Lambda(z-z_0)}X^{-1} \]

\[ = X \begin{bmatrix} 
\lambda_1(z-z_0) & 0 & 0 & 0 \\
0 & e^{-\lambda_1(z-z_0)} & 0 & 0 \\
0 & 0 & \lambda_2(z-z_0) & 0 \\
0 & 0 & 0 & e^{-\lambda_2(z-z_0)} 
\end{bmatrix} X^{-1} \]  
(C.18)