Defining equivalent stationary PSDF to account for nonstationarity of earthquake ground motion

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This paper presents three approaches to defining the stationary power spectrum density function (PSDF) of strong ground acceleration, for prediction of structural response corresponding to the strong-motion stationary part of the input excitation. The first approach defines the PSDF in terms of the Fourier amplitude spectrum and a stationary duration of ground acceleration. The PSDF obtained by this approach predicts accurately the response of structures with low to intermediate natural periods. In the second approach, we introduce the concept of stationary duration of response, which is defined as a function of the natural period and damping ratio of the oscillator. Using this approach, it is possible to get accurate estimates of response amplitudes for the broad range of natural periods. However, it is not convenient in practical applications to deal with several stationary durations for a given input excitation. Further, to evaluate these durations it is necessary to specify both the Fourier and the response spectra of ground accelerations; whereas the common engineering practice is to specify the response spectrum only. Therefore, the third approach suggests the use of the response ‘spectrum compatible’ PSDF. The paper presents several improvements in the general methodology used for this purpose. The improvements mainly relate to using more accurate peak factors and to using the transient nature of response. The spectrum compatible PSDFs, as evaluated in the present study, provide realistic specification of strong ground motion for stochastic seismic response analyses of structures. © 1998 Elsevier Science Limited.

Key words: stationary response, power spectrum density, earthquake excitation, stochastic seismic response, stochastic mechanics, random vibrations.

1 INTRODUCTION

For stochastic seismic response analyses of structures, the nonstationarity of ground acceleration is normally modeled by a stationary process multiplied by a deterministic envelope function.1–5 The corresponding evolutionary power spectrum density function (PSDF) is defined by the product of the PSDF of the associated stationary process and the square of the absolute value of the envelope function.6, 7 The response of structures to such nonstationary excitations, both in frequency6–14 and in time15–19 domains, is complicated and involves lengthy calculations.20 Perhaps because of this reason, even simple example results in literature are mostly limited only to the variance (mean-square value) of the response,8,11,14,15,19,21,22 and very few studies deal with the statistics of level crossings or peak factors.8,10,12,13,23 Further, in all the available studies, the stationary ground motion is represented by a band-limited white noise or the Kanai–Tajimi PSDF24, 25 and the envelope function is defined by some idealized regular shape which cannot be considered to be a close representation of any true ground acceleration. When we address questions of the utility of the results of such nonstationary response evaluation in earthquake resistant design of structures, the only information available for the purpose is the maximum response; the amplitude and time of its occurrence are greatly influenced by the choice of the
envelope function. Thus, if we can directly obtain the statistics of the largest response amplitudes to a given nonstationary input excitation, the conventional nonstationary response analysis can be considered redundant.

In contrast, several studies, based on the statistics of ordered peaks in a random time-history, are already available, and have shown (by comparing the statistical predictions with the time-history solutions) that it is possible to evaluate the amplitudes of all the significant response peaks with only a stationary analysis. The stationary PSDF of input motion in these studies was first defined from the Fourier amplitude spectrum and total duration of ground acceleration. Then, to correct for the nonstationarity of ground motion, the root-mean-square (RMS) value of the response was modified by matching the expected and most probable values of the modal responses with the response spectrum amplitudes. But such modifications were found not to be very straightforward when the effects of modal correlations, soil–structure interaction and simultaneous excitation by translational and rotational components of ground motion were taken into consideration. Therefore, in the present study, we have defined the PSDF of input excitation in such a way that the nonstationarity of response and the effect of various correlations and interactions are automatically accounted for without any modification in the RMS value of the response at a later stage.

The definition of the proposed PSDF is such that it corresponds to a stationary process with total energy equal to that in the actual nonstationary ground motion and a duration which depends upon the natural period and damping of the structure. From time-history solutions it can be shown that, for structural periods much shorter than the strong-motion part of an accelerogram, the response can be obtained accurately from the strong-motion part only. For very long-period response, almost the whole of an accelerogram can be treated to represent the strong motion. Because the strong-motion part of an accelerogram can be considered as a weakly stationary process, it is justified on physical grounds to model the ground motion, for excitation of long-period structures, by a stationary process of longer duration. Thus the desired PSDF can be defined from the Fourier spectrum of the complete accelerogram and a suitable duration such that the expected response amplitudes match the time-history results. Thus, we are required to know the complete spectrum of duration, along with the Fourier spectrum, to evaluate the response of different modes of an MDOF structure. It would, however, be convenient to define a fixed PSDF which ensures matching with the time-history solution for all the frequencies. Therefore, as an alternative approach, we consider an iterative method of obtaining the PSDF such that it is compatible with a given response spectrum. Because the response spectrum inherently accounts for the effect of nonstationarity of input excitation, the PSDF compatible with it automatically includes that effect, and there is no need to have a separate treatment for each frequency. It will be shown that definitions of the equivalent stationary PSDF by these two independent approaches, viz., by Fourier spectrum and several different durations, and the response spectrum compatible method, are mutually consistent. The use of the equivalent stationary PSDF is able to predict accurately the amplitudes of all the significant response peaks for a wide band of structural frequencies in a very simple and efficient way from the probability distribution functions of Gupta and Trifunac for different orders of peaks in a stationary random time-history. Thus the stationary response analysis using the proposed PSDF is expected to be of practical value for application, compared to the highly involved nonstationary response calculations.

2 STATIONARY DURATION OF GROUND ACCELERATION

The PSDF of a stationary stochastic process of duration $T_s$ can be defined from its Fourier amplitude spectrum $FS(\omega)$ as:

$$G(\omega) = \frac{1}{\pi T_s} |FS(\omega)|^2$$

(1)

However, to use this definition for getting the PSDF from the Fourier spectrum of an actual accelerogram, different values of stationary duration $T_s$ would have to be used to obtain an accurate estimate of the response of structures with different natural periods. But, in many studies, statistical computation of response spectra for all the frequencies has been attempted by using a fixed stationary duration $T_s$. Due to the lack of a formal definition of the stationary duration of ground acceleration which could provide accurate results over a broad frequency band, these studies have assigned the stationary duration, $T_s$, in different ad hoc ways. For the purpose of structural response analysis, Gupta has defined the stationary duration from the order statistics of acceleration peaks, which can be considered as a simple measure of duration providing accurate response amplitudes for a very broad frequency range. This stationary duration has been defined in such a way that it ensures conservation of total energy in the accelerogram and matching of the observed amplitudes of several highest acceleration peaks with the corresponding expected amplitudes. This method of evaluating the stationary duration of a real accelerogram can be described briefly as follows.

Let $\tilde{z}(t)$ be an accelerogram with total duration $T$. To use the probability distributions of Gupta and Trifunac to compute the theoretical expected amplitudes of its various peaks normalized by the root-mean-square acceleration, when the peaks are arranged in decreasing order of amplitude, we need to know the bandwidth parameter $\epsilon$ and the total number of peaks $N$. The parameter $\epsilon$ can be obtained from the ratio, $r$, of the negative peaks to the total number of peaks, $N$, in the accelerogram $\tilde{z}(t)$ using

$$\epsilon = \sqrt{1 - (1 - 2r)^2}.$$  

(2)
Let $E[\eta(1)], E[\eta(2)], E[\eta(3)], \ldots$ be the expected amplitudes of various orders of acceleration peaks normalized by the root-mean-square amplitude as computed by using the parameter $c$ obtained from eqn (2) and the actual total number of peaks, $N$, in the accelerogram. Also, if $a_{1}(\tau), a_{2}(\tau), a_{3}(\tau), \ldots$ are the observed peaks in the accelerogram and $a_{rms}$ the root-mean-square value of the acceleration, the $i$th observed normalized peak is given by $\eta_{i}(\tau) = a_{i}(\tau)/a_{rms}$. The total energy of the accelerogram $E(T)$ is proportional to $I(T) = \int_{0}^{T} \xi^{2}(\tau) d\tau$. If the equivalent stationary process with duration $T_s$ has the same total energy, the rms acceleration $a_{rms}$ is given by

$$a_{rms}^{2} = \frac{1}{T_s} \int_{0}^{T} \xi^{2}(\tau) d\tau = I(T)/T_s$$

(3)

By equating the expected amplitude, $E[\eta(1)]$, of the $i$th order peak to the observed amplitude, $\eta_{i}(\tau)$, the unknown stationary duration for the $i$th order acceleration peak can be expressed in terms of the known quantities as follows:

$$T_s = \left( \frac{E[\eta(1)]} {a_{i}(\tau)} \right)^{2} \frac{1}{I(T)}.$$  

(4)

By finding such stationary durations for a large number of acceleration peaks, the final stationary duration of ground acceleration is taken equal to a mean value which is maintained for several highest amplitude peaks.

Table 1 lists the stationary durations for the first 15 ordered peaks computed as above for several typical real accelerograms. In some cases, the durations corresponding to the few lowest orders (highest amplitude) of peaks increase rapidly with order of the peaks and are not found to be sustained for several higher order peaks. Such peaks cannot be considered to represent the strong-motion stationary parts of the accelerograms and have not been taken into account in defining the stationary durations. To find the stationary duration for each accelerogram, a sufficiently large number of lowest possible orders of peaks characterized by more or less a constant duration has been selected. Our choice of such bunches of peaks is indicated in Table 1 by enclosing them within square brackets. The final stationary duration of an accelerogram is taken equal to the mean of the durations corresponding to all the bracketed peaks, and these values are given in the last column of the table. By defining the PSDF of ground accelerations using these stationary durations and the Fourier amplitude spectrum we have computed the expected response spectra for a damping ratio of 5%. Comparison of these spectra with the corresponding time-history spectra for the first three accelerograms listed in Table 1 is shown in Fig. 1. From the results in Fig. 1 it is seen that the theoretical spectra (dotted curves) show good agreement with the time-history spectra (continuous curves) only for the short-period range ($t \leq 1.0$ s). This is because the strong-motion stationary part of an original accelerogram is generally dominated by high-frequency waves. By filtering out the high-frequency components and defining the stationary duration from the resulting accelerogram it is possible to get better matching for the long-period range, but then the matching for the low-period range will be spoiled. Thus it is not possible to have just one such duration for all the natural periods, and, in general, different durations for each natural period and damping ratio are required. We therefore define such frequency and damping dependent duration, and call it the stationary duration of response.

### 3 STATIONARY DURATION OF RESPONSE

Let $T_s(\omega_0, \zeta)$ be the stationary duration which would provide exact matching between the time-history response of an SDOF structure, with natural frequency $\omega_0$ and damping ratio $\zeta$, and the corresponding expected value of the first order (highest amplitude) response peak obtained from the PSDF defined in terms of the Fourier spectrum $FS(\omega)$ of ground acceleration as

$$G(\omega) = \frac{1}{\pi \tau_{s}(\omega_0, \zeta)} |FS(\omega)|^2$$

(5)

To obtain $T_s(\omega_0, \zeta)$ for a given accelerogram, we first consider the PSDF $\tilde{G}(\omega)$ with $T_s(\omega_0, \zeta)$ assumed as 1.0,

$$\tilde{G}(\omega) = \frac{1}{\pi} |FS(\omega)|^2$$

The PSDF of the displacement response of an oscillator with natural frequency $\omega_0$ and damping ratio $\zeta$ to this input PSDF can be written as

$$ED(\omega) = \tilde{G}(\omega) |H(\omega, T)|^2$$

(7)

where $H(\omega, T)$ is the transient transfer function of the oscillator at time $t$ equal to the total duration $T$ of the real ground acceleration, and is given by

$$H(\omega, T) = \frac{\omega_0}{\omega_0^2 - \omega^2 + 2i\zeta\omega_0}$$

(8)

with $H(\omega) = (\omega_0^2 - \omega^2 + 2i\zeta\omega_0)^{-1}$ as the steady-state transfer function of the oscillator and $\omega_0 = \omega_0 \sqrt{1 - \zeta^2}$ its damped natural frequency. By computing the zeroth, second and fourth order moments $m_0$, $m_2$ and $m_4$ of the PSDF $ED(\omega)$, the various statistical parameters of the response are given as

$$x_{rms} = \sqrt{m_0}$$

(9)

$$N = \frac{T}{\pi} \sqrt{\frac{m_4}{m_2}}$$

(10)

$$\epsilon = \frac{m_0 m_4 - m_2^2}{m_0 m_4}$$

(11)

Using the statistical parameters $N$ and $\epsilon$, we computed the
<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Comp.</th>
<th>Total duration (s)</th>
<th>Stationary duration of ground acceleration for order of peak equal to</th>
<th>Average stationary duration</th>
</tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>Imperial Valley EQ.</td>
<td>18 May 1940, $M = 6.7$, El Centro site</td>
<td>S00E</td>
<td>53.74</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>S90W</td>
<td>53.46</td>
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<td></td>
<td></td>
<td>VERT</td>
<td>53.78</td>
</tr>
<tr>
<td>2</td>
<td>Western Washington EQ.</td>
<td>13 April 1949, $M = 7.1$, Olympia–Washington Hyw. Tes-lab site</td>
<td>N04W</td>
<td>89.06</td>
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<td></td>
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<td></td>
<td>N86E</td>
<td>89.04</td>
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<td></td>
<td>VERT</td>
<td>88.88</td>
</tr>
<tr>
<td>3</td>
<td>Kern County, California EQ.</td>
<td>21 July 1952, $M = 7.7$, Taft Lincoln School Tunnel site</td>
<td>N21E</td>
<td>54.36</td>
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<td></td>
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<td>S69E</td>
<td>54.38</td>
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<td></td>
<td>VERT</td>
<td>54.26</td>
</tr>
<tr>
<td>4</td>
<td>Eureka earthquake, 21 December 1954, $M = 6.5$, Eureka Federal Bldg. site</td>
<td></td>
<td>N11W</td>
<td>77.96</td>
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<td></td>
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<td></td>
<td>N79E</td>
<td>79.56</td>
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<td></td>
<td></td>
<td></td>
<td>VERT</td>
<td>69.98</td>
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<tr>
<td>5</td>
<td>Parkfield, California EQ.</td>
<td>27 June 1966, $M = 5.6$, Cholame Shandon CA Array #5</td>
<td>N05W</td>
<td>43.92</td>
</tr>
<tr>
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<td></td>
<td>N85E</td>
<td>43.94</td>
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<tr>
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<td></td>
<td>VERT</td>
<td>53.86</td>
</tr>
<tr>
<td>6</td>
<td>San Fernando EQ.</td>
<td>9 February 1971, $M = 6.4$, Pacoima dam</td>
<td>S16E</td>
<td>41.80</td>
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<td>S74W</td>
<td>41.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>VERT</td>
<td>41.74</td>
</tr>
</tbody>
</table>
Defining equivalent stationary PSDF

expected value, $E[\eta_{11}]$, of the first order (highest amplitude) peak of the response normalized by the rms amplitude, $x_{rms}$, from the statistical distribution of Gupta and Trifunac. Thus the expected maximum response of the oscillator under excitation by the PSDF of eqn (6) is given by

$$\bar{SD}(\omega_0, \zeta) = E[\eta_{11}]x_{rms}$$

(12)

From this, the stationary duration $T_s(\omega_0, \zeta)$, which will ensure exact matching of the expected maximum response of the oscillator computed by using the PSDF of eqn (5) with the time-history solution $SD(\omega_0, \zeta)$, can be defined as

$$T_s(\omega_0, \zeta) = \left[ \frac{\bar{SD}(\omega_0, \zeta)}{SD(\omega_0, \zeta)} \right]^2$$

(13)

Examples of stationary durations of response computed as above for the Imperial Valley earthquake of 18 May 1940 and the Parkfield earthquake of 27 June 1966 are shown in Fig. 2(a) and (b) for damping ratios $\zeta = 0.02, 0.05$ and 0.20. For these durations, the total number of response peaks can approximately be obtained as

$$N_s(\omega_0, \zeta) = T_s(\omega_0, \zeta)\omega_0$$

(14)

This number of peaks is also shown plotted in Fig. 2(a) and (b). From the examples presented in Fig. 2, and from analysis of other accelerograms, it follows that, to satisfy the stationarity condition for long natural periods ($T_0 \approx 5.0$ s), the oscillator is required to experience about 5–10 cycles of response. For longer natural periods and smaller damping values, the equivalent stationary duration can be larger than the actual duration of the input ground motion. This is due to the fact that the PSDF of eqn (7) is not able to account exactly for the transient nature of the response. In the initial stage of excitation, before the oscillator attains the steady state, the oscillator responds to the ground motion with a phase delay. The probabilistic analysis based on the PSDF of eqn (7) assumes that the input and the oscillator are having the same phase, and hence it overestimates the root-mean-square value of the response for longer natural periods and smaller damping values. This overestimation of the response is ‘compensated’ by increase in the stationary duration of response beyond the actual duration of the input excitation. If the nonstationarity on account of the transient nature of the response (i.e., the response attains its steady-state value at some large time) were taken into account accurately, our stationary duration of response would not have been much greater than the physical duration of ground acceleration.

From the foregoing discussion, it is seen that to use the definition of eqn (1) to define the equivalent stationary PSDF from the Fourier spectrum, one additional requirement is to provide the spectra of stationary durations for different damping values. However, it would be more convenient in practical applications if we could unify the Fourier and stationary duration spectra to define a single PSDF for all the frequencies. This can, in fact, be achieved by defining the response spectrum compatible PSDF as described in the following.

4 RESPONSE SPECTRUM COMPATIBLE PSDF

Response spectra of ground acceleration correspond to the maximum response of an ensemble of SDOF oscillators with different natural frequencies and damping values, and hence have already accounted for the effects of nonstationarity. Thus the PSDF compatible with a given response spectrum would automatically include the effects of the nonstationarity of ground acceleration as well as the transient nature of the response for long natural periods. Further, as the design ground motion is normally specified in terms of a response spectrum, it is desirable to obtain the PSDF from a given response spectrum. Several formulations have been proposed by different investigators to obtain a spectrum compatible PSDF. All of these are based on some approximate statistical distribution of the maximum of a random function of time and some crude treatment for the transient nature of response. In the present study we describe a similar approach, using improved probability distribution functions for the statistics of response peaks and more accurate consideration for the transient nature of the response.

Let $SD(\omega_0, \zeta)$ be the amplitude at frequency $\omega_0$ of the given displacement response spectrum with damping ratio $\zeta$. 

Fig. 1. Comparison of time-history response spectra (solid curves) with the same derived from the stationary PSDF (dashed curves), for several recorded accelerograms.
Fig. 2. (a) Stationary durations of response versus natural period for damping ratios of 0.02, 0.05 and 0.20, to define the stationary PSDF for the S00E component of the Imperial Valley earthquake of 18 May 1940. The equivalent number of response peaks to attain the stationarity condition is also plotted in the figure. (b) Stationary durations of response versus natural period for damping ratios of 0.02, 0.05 and 0.20, to define the stationary PSDF for the N05W component of the Parkfield earthquake of 27 June 1966. The equivalent number of response peaks to attain the stationarity condition is also plotted in the figure.

The desired spectrum compatible PSDF has to be such that the expected value of the maximum displacement response of an oscillator with natural frequency \( \omega_n \) and damping ratio \( \zeta \) matches closely the given response spectrum amplitude. If \( G(\omega) \) is the required PSDF compatible with the given response spectrum, then the PSDF of the displacement response of an SDOF oscillator with natural frequency \( \omega_n \) and damping value \( \zeta \) can be written as

\[
ED_n(\omega) = G(\omega) |\tilde{H}(\omega, T)|^2
\]

where \( \tilde{H}(\omega, T) \) is the transient transfer function of the oscillator at time \( t \) equal to the total duration \( T \) of the ground motion, as given in eqn (8). To get an initial approximation to \( G(\omega) \), we replace \( \tilde{H}(\omega, T) \) by the steady-state value \( H(\omega) \) and further approximate \( |\tilde{H}(\omega)|^2 \) by a rectangle of height \( 1/(4\zeta^2 \omega_n^2) \) and width \( \pi \omega_n \) centered at \( \omega_n \). Also, we approximate the PSDF over a narrow frequency band from \( \omega_n - \frac{1}{2} \pi \omega_n \) to \( \omega_n + \frac{1}{2} \pi \omega_n \) by a constant value \( G_0(\omega) \). With these assumptions, the zeroth, second and fourth order moments \( m_0, m_2, m_4 \) of \( ED_n(\omega) \) can be obtained as

\[
m_0 = \frac{\pi G_0(\omega_n)}{4 \zeta^2 \omega_n^2}
\]

\[
m_2 = \frac{G_0(\omega_n)}{4 \zeta^2 \omega_n^2} \left( \pi \zeta + \frac{1}{12} \pi^3 \frac{1}{\zeta^3} \right) \omega_n^4
\]

\[
m_4 = \frac{G_0(\omega_n)}{4 \zeta^2 \omega_n^2} \left( \pi \zeta + \frac{1}{2} \pi^3 \frac{1}{\zeta^3} + \frac{1}{80} \pi^5 \frac{1}{\zeta^5} \right) \omega_n^8
\]

The statistical parameters \( x_{\text{rms}}, N \) and \( \epsilon \) can thus be obtained from eqns (9)–(11) as

\[
x_{\text{rms}} = \sqrt{\frac{\pi G_0(\omega_n)}{4 \zeta^2 \omega_n^2}}
\]

\[
N = \frac{T \omega_n}{\pi} \left( \pi \zeta + \frac{1}{2} \pi^3 \frac{1}{\zeta^3} + \frac{1}{80} \pi^5 \frac{1}{\zeta^5} \right)^{\frac{1}{2}}
\]

\[
\epsilon = \left[ 1 + \frac{1}{8} \pi^2 \frac{\zeta^2}{\omega_n^2} + \frac{1}{16} \pi^4 \frac{\zeta^4}{\omega_n^4} \right]^{\frac{1}{2}}
\]

From eqns (20) and (21) it is seen that parameters \( N \) and \( \epsilon \) can be obtained from a knowledge of only the damping and natural frequency of the oscillator, and the total duration \( T \) of the ground motion. The value of \( N / \left( \frac{T \omega_n}{\pi} \right) \) is found to be
very close to 1.0 for all the damping ratios up to 0.20 and that of $\epsilon$ is found to increase almost linearly from 0.0 to 0.33 as $\xi$ increases from 0.0 to 0.20. From a knowledge of these parameters, one can compute the expected amplitude, $\bar{X}_{max}$, of the highest (first order) peak of the oscillator response normalized by the rms amplitude $x_{rms}$, using the probabilistic expressions developed by Gupta and Trifunac. The expected value of the maximum response of the SDOF oscillator, which by definition is the expected response spectral amplitude $\bar{S}D(\omega, \xi)$, can be written as

$$\bar{S}D(\omega, \xi) = \bar{X}_{max} \cdot x_{rms} = \bar{X}_{max} \sqrt{\frac{\pi G_0(\omega_0)}{4\xi \omega_0^2}}$$  \hspace{1cm} (22)$$

Equating this to the given spectral amplitude $S_0(\omega, \xi)$, the initial approximation to the PSDF can be obtained as

$$G_0(\omega_0) = \frac{4\xi \omega_0^2}{\pi} \left[ \frac{\bar{S}D(\omega_0, \xi)}{\bar{X}_{max}} \right]^2$$  \hspace{1cm} (23)$$

Iterations can be performed to improve upon this initial approximation of the PSDF. By using this in eqn (15) and computing the moments $m_0, m_2, m_4$ of the PSDF $ED_d(\omega)$ by numerical integration, we get the improved expected spectral amplitude $\bar{S}D(\omega_0, \xi)$ from probability theory, as mentioned before. From this, the improved PSDF for the next iteration is defined by

$$G_1(\omega) = G_0(\omega) \left[ \frac{SD(\omega, \xi)}{SD(\omega_0, \xi)} \right]^2$$  \hspace{1cm} (24)$$

The foregoing procedure has been found to converge rapidly, and very good agreement can be achieved between the given and the theoretical response spectra after only two to three iterations. Fig. 3(a) and (b) show examples of the PSDFs thus derived from the 5% damped response spectra of Imperial Valley (El Centro Site, S00E, 1940) and Parkfield (Cholame Shandon, California Array Station #5, N85E, 1966) earthquakes. The comparison between the given and the theoretical spectra are also shown in these figures. The expected spectra are found to provide excellent matching with the given time-history spectra. Thus the response spectrum compatible PSDFs are able to account for the effects of the nonstationarity of ground motion and the transient nature of the response. The use of such equivalent stationary PSDFs is able to predict accurately the response of multi-degree-of-freedom structures.

Fig. 3. (a) Response spectrum compatible PSDF and comparison between the theoretical and time-history response spectra with damping ratio of 0.05 for the S90W component of the Imperial Valley earthquake of 18 May 1940, recorded at the El Centro site. (b) Response spectrum compatible PSDF and comparison between the theoretical and time-history response spectra with damping ratio of 0.05 for the N05W component of the Parkfield earthquake of 27 June 1966, recorded at the Cholame Shandon, California Array, Station #5.
obtained from the empirical scaling relations due to Lee\(^{66,67}\)
and extended for both high and low periods using the methods developed by Trifunac\(^{40-53}\) from simple source functions and physical processes involved in the attenuation of high frequency ground motion. For our further study we have used such extended empirical spectra for magnitudes \(M = 4.0, 6.0\) and 8.0 and for a confidence level of 0.5, epicentral distance \(R = 0\), focal depth \(H = 0\) and focal soil and regional geological conditions, both as ‘rock’ type.\(^{68}\) The result for \(M = 6.0\) is shown in Fig. 4. The quality of results for \(M = 4.0\) and 8.0 is very similar, and indicates that our iterative procedure is able to provide excellent matching almost over the entire period range, except for very low periods where the theoretical spectra lie slightly above the specified spectra.

As mentioned before, the stationary PSDF for each frequency, \(\omega_0\), of the SDOF oscillator can also be defined using the Fourier spectrum and an appropriate stationary duration of response, \(T_s(\omega_0, \zeta)\). Conversely, if we know the stationary duration of the response and the stationary PSDF, we can obtain the Fourier spectrum of ground acceleration from eqn (5) as

\[
F_S(\omega) = \sqrt{\pi T_s(\omega_0, \zeta)} G(\omega) 
\]

(25)

By assigning the durations equal to 10.0, 20.0 and 35.0 s to the three magnitudes \(M = 4.0, 6.0\) and 8.0\(^{66,67}\) considered in the above to generate the extended spectra, we have evaluated the stationary durations of response and plotted those along with the equivalent number of response peaks in Fig. 5. Using these stationary durations and the response spectrum compatible PSDFs, we have obtained the Fourier amplitude spectra from eqn (25). These spectra are plotted along with the actual Fourier spectra in Fig. 6. For better comparison, both the Fourier spectra have been normalized to a maximum value of 1.0. The results in Fig. 6 show that the shapes of the Fourier spectra derived from the PSDF match very closely those of the original Fourier spectra. The two corner frequencies in the original extended spectra are also preserved in the theoretical Fourier spectra, thus establishing that the process of obtaining the PSDF from a response spectrum is able to preserve all the main characteristics of the actual ground motion.

**5 DISCUSSION AND CONCLUSIONS**

The commonly used approach for nonstationary seismic response analyses of structures involves considerable computational effort, which does not seem worthwhile in view of little practical utility of such studies. The final outcome of a nonstationary study of which some use can be made in design applications is the highest value of the response and the duration of time over which such large response amplitudes are maintained. But both of these depend on the choice of the deterministic envelope function and the PSDF of the
associated stationary process and thus are not unique. In the present study we have suggested a method of defining the PSDF of input ground acceleration in such a way that, using it, one can obtain accurate information on all the significant response peaks corresponding to the strong-motion part of the input excitation from a simple and convenient stationary analysis only. Such PSDFs have been termed equivalent stationary PSDFs, because they represent only that part of a nonstationary process which can be treated as weakly stationary.

The PSDF of a stationary process can be defined conveniently from a knowledge of its Fourier amplitude spectrum and the total duration, using eqn (1). This definition has also been used by several investigators to study the strong-motion part of earthquake ground motion, which can be considered as weakly stationary. Due to the abundance of high-frequency waves in the strong motion, this approach is able to predict accurately the response of high-frequency structures only, and due to nonstationarity of ground motion it generally overestimates the response of long-period (low-frequency) structures. To improve upon this drawback, we have introduced the stationary duration of response, which is defined as a function of the natural period and damping value of an SDOF oscillator. These durations can be obtained from a knowledge of the Fourier and response spectra of the ground acceleration. To use this approach for an MDOF structure, one has to define a separate PSDF for each mode of vibration. This does not add much additional calculation because all these PSDFs have the same shape, but it does not seem convenient to deal with several stationary durations. The response spectrum compatible PSDF provides a more elegant formulation, because a single PSDF is able to predict the response for all the frequencies.

To derive the PSDF from a given response spectrum in the present study we have used an improved probability distribution for the maximum value of a random time-history and have treated the transient nature of response more accurately. Thus, this can be considered an improvement over the existing formulations for the purpose. The spectrum compatible PSDF is able to predict accurately the response of MDOF structures without any need to make corrections for the nonstationarity effects later on. Because the ground motion for design of structures is most commonly specified in terms of a response spectrum, the transition from response spectrum to PSDF is expected to be of considerable practical utility.

From a number of results on the PSDFs of real accelerograms, it is seen that, similarly to the Fourier and response spectra, the amplitudes at different frequencies and the shape of the PSDF varies with earthquake magnitude, source-to-site distance and the geological and site soil conditions. The widely used Kanai–Tajimi PSDF is not able to reflect all these effects in a consistent manner. It would therefore be useful to develop frequency-dependent scaling relations for the PSDF using the PSDFs of recorded accelerograms, similarly to those for Fourier and response spectra which could be used directly to get the design PSDF with a desired confidence level for given earthquake specifications and local site conditions. Such PSDFs will be able to account implicitly for the random effects due to various uncertainties associated with source, path and site characteristics as well as those due to the stochastic nature of the earthquake ground motion.

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