A note on tsunami amplitudes above submarine slides and slumps

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Abstract
Tsunami generated by submarine slumps and slides are investigated in the near-field, using simple source models, which consider the effects of source finiteness and directivity. Five simple two-dimensional kinematic models of submarine slumps and slides are described mathematically as combinations of spreading constant or sloping uplift functions. Tsunami waveforms for these models are computed using linearized shallow water theory for constant water depth and transform method of solution (Laplace in time and Fourier in space). Results for tsunami waveforms and tsunami peak amplitudes are presented for selected model parameters, for a time window of the order of the source duration.

The results show that, at the time when the source process is completed, for slides that spread rapidly \((c_R/c_T \approx 20)\), where \(c_R\) is the velocity of predominant spreading), the displacement of the free water surface above the source resembles the displacement of the ocean floor. As the velocity of spreading approaches the long wavelength tsunami velocity \((c_T = \sqrt{gh})\), the tsunami waveform has progressively larger amplitude, and higher frequency content, in the direction of slide spreading. These large amplitudes are caused by wave focusing. For velocities of spreading smaller than the tsunami long wavelength velocity, the tsunami amplitudes in the direction of source propagation become small, but the high frequency (short) waves continue to be present. The large amplification for \(c_R/c_T \sim 1\) is a near-field phenomenon, and at distances greater than several times the source dimension, the large amplitude and short wavelength pulse becomes dispersed.

A comparison of peak tsunami amplitudes for five models plotted versus \(L/h\) (where \(L\) is characteristic length of the slide and \(h\) is the water depth) shows that for similar slide dimensions the peak tsunami amplitude is essentially model independent. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction
In a previous paper [7], we studied the effects of source directivity and wave focusing on the amplitudes of tsunami generated by a slowly spreading uplift of the sea floor, using a simple kinematic source model. The results showed that amplification of up to an order of magnitude occurs in the direction of source propagation, when the velocity of the source is close to the long period tsunami velocity. This amplification occurs above the source, progressively, as the source evolves, by adding uplifted fluid to the fluid displaced previously by uplifts of preceding source segments. This amplification is larger for a wider source compared to the depth of water (the water has less opportunity to flow sideways), and is the largest for an infinitely wide source. It occurs as a short wavelength pulse, which has large amplitude only near the source. As the tsunami departs further and away from the source, the amplitude of this pulse decreases due to dispersion. In the far-field, the peak tsunami amplitude does not depend on the source velocity, but only on the volume of the displaced water by the source process.

In this paper, we describe tsunami generated by submarine slumps and slides using more detailed kinematic source models. The purpose of the study is to add to further understanding of the nature of waveforms of tsunami in the near-field. We present only the basic ideas and illustrate the possible range of tsunami amplitudes using most elementary forward source models. Our ‘source’ is a combination of rectangular or sloping blocks spreading laterally with assumed velocities. The vertical motions in our models may be thought of as approximations of the sea floor elevation associated with linear and non-linear movements of the body of a slump or a slide. The source parameters we choose are based on our study of the differences in the tsunami source characteristics for submarine slumps and earthquakes [8].

We describe three simple two-dimensional (2D) models for submarine slumps and slides. For models 1 and 3, we
consider two variants, A and B. Then, we present the equations of motion and solution by transform methods (Laplace in time and Fourier in space). Further details of the solutions and detailed results and discussion for each model are presented in Ref. [9]. In our numerical simulations, we consider only a time window of the order of the source duration, present results for selected model parameters, and limit the discussion only to wave amplitudes in the near-field. Finally, we summarize and compare the results for all the models, and present the conclusions.

2. Model

2.1. Tsunami source models for submarine slides and slumps

Figs. 1–3 show vertical cross-sections (through y = 0) of the mathematical models of submarine slides and slumps we consider in this study, at time \( t = t^* \), when the source process stops. We consider three models, of which models 1 and 3 have two variants, A and B, i.e. total of five models: 1.A, 1.B, 2, 3.A and 3.B.

All models are characterized by displacements spreading in one direction, without loss of generality coinciding with the x-axis, and tsunami propagating in the x–y plane. All slides are assumed to have constant width, \( W \). The spreading can be unilateral or bilateral, i.e. in the positive and negative x-direction. The vertical displacement, \( \zeta \), is negative (downwards) in zones of depletion and positive (upwards) in zones of accumulation.

Models 1.A (Fig. 1a,b), 3.A and 3.B (Fig. 3) represent mass movement triggered at \( x = 0 \) and spreading unilaterally in the positive x-direction (downhill) with velocities \( c_L \) and \( c_R \), respectively, for the zones of depletion and accumulation. Models 1.B and 2 (Figs. 1a,b and 2) represent mass movement starting at any point (including the foot of the slide) and spreading bilaterally. For these models, the zone of accumulation spreads with velocity \( c_R \) in the positive x-direction, and the zone of depletion spreads with velocity \( c_L \) if in the negative x-direction and with velocity \( c_C \) in the positive x-direction. For all examples presented in this paper, the balance of mass is assumed to be constant, i.e. the volume of the ‘accumulation’ zone is equal to the volume of the ‘depletion’ zone, except for model 2, when \( c_L \neq c_R \), and for model 3.B, which does not have a depletion zone.

In Models 1.A and 1.B, the zones of accumulation and

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Fig. 1. (a) Tsunami source models 1.A (top) and 1.B (center), and a schematic cross-section of debris avalanches, flows or mud flows (bottom) those models represent. Model 1.A represents sliding downhill, while model 1.B can represent spreading of the source area uphill and downhill, at rates specified by \( c_L \), \( c_C \) and \( c_R \). It is assumed that \( A_1 = A_2 \). Distance \( L_a = c_R t^* \) is characteristic length. (b) Tsunami source models 1.A (top) and 1.B (center), and a schematic cross-section of a submarine slide (bottom) that those models represent.
depletion have uniform amplitudes, \( \xi_0 \) (accumulation) and \( \xi_1 \) (depletion), equal to the average amplitude over the area of these zones. The volumes of the uplifted and removed material are \( A_1 W \) and \( A_2 W \), where \( A_1 \) and \( A_2 \) are areas of the vertical cross-sections of these zones, as shown in Fig. 1.

For model 1.A, the final lengths of the zones of accumulation and depletion are, respectively, \( L_{\text{accum}} = t \left( c_R - c_L \right) \) and \( L_{\text{depl}} = t \left( c_L \right) \), their ratio is \( L_{\text{depl}}/L_{\text{accum}} = c_L/(c_R - c_L) \), \( A_1 = \xi_0 L_{\text{accum}} = \xi_0 t \left( c_R - c_L \right) \) and \( A_2 = \xi_1 L_{\text{depl}} = \xi_1 t \left( c_L \right) \). Conservation of mass then implies \( \xi_1/\xi_0 = (c_R - c_L)/c_L \). Characteristic length of this model is \( L_R = c_R t^* \).

For model 1.B, the lengths of the zones of accumulation and depletion are, respectively, \( L_{\text{accum}} = t \left( c_R + c_C \right) \) and \( L_{\text{depl}} = t \left( c_L + c_C \right) \), their ratio is \( L_{\text{depl}}/L_{\text{accum}} = (c_L + c_C)/(c_R + c_C) \), and conservation of mass implies \( \xi_1/\xi_0 = (c_R - c_C)/(c_L + c_C) \). Characteristic length of this model is also \( L_R = c_R t^* \).

In model 2, the amplitudes of the accumulation and depletion zones grow progressively in time and, at each moment of time are a linear function of \( x \), as shown in Fig. 2. At the leading edge of the accumulation zone, the amplitude is \( \zeta(x, y; t) = \zeta_0 (1 - e^{-at}) \), at an arbitrary \( x \) within this zone is \( \zeta(x, y; t) = \zeta_0 (x/c_R t)(1 - e^{-at}) \). The slope of these zones grows progressively in time, and their top surfaces rotate counterclockwise in time. The final lengths of the zones of accumulation and of depletion are, respectively, \( L_{\text{accum}} = t \left( c_R \right) \), and \( L_{\text{depl}} = t \left( c_L \right) \), their ratio is \( L_{\text{depl}}/L_{\text{accum}} = c_L/c_R \), \( A_1 = (1/2)c_R t \left( \zeta_0 (1 - e^{-at}) \right) \) and \( A_2 = (1/2)c_L t \left( \zeta_0 (1 - e^{-at}) \right) \). For this model, mass is conserved if \( c_R = c_L \). Characteristic length of this model is also \( L_R = c_R t^* \).

In model 3.A, the lengths of the accumulation and depletion zones grow until time \( t = L_0/c_R \), when they reach the final length \( L_0 \). For time \( t > L_0/c_R \), the accumulation zone slides as a ‘rigid block’ until time \( t = t^* \), while the depletion zone remains stationary. The length between the left edges of the depletion and accumulation zones is \( L_1 = c_R t^* \), and is used as characteristic length. In this model, mass is conserved. Model 3.B consists of an emerging sliding block of length \( L_0 \), with amplitude growing gradually in time as \( \zeta = \zeta_0 (1 - e^{-at}) \). The total length traveled by the block is \( L_1 = c_R t^* \), which is used as characteristic length.

The bottom parts of Figs. 1–3 show schematic representation of the physical processes modeled. In Fig. 1a, models 1.A and 1.B represent debris avalanches, debris flows or mud flows. In Fig. 1b, they represent submarine slide. Model 1.B could be thought of as approximating a slope failure, which is enlarged by the degradation of the head scarp, a process referred to as retrogressive failure. Model 1.A represents a slide with mass movement in the downslope direction. Model 1.B can be thought of as representing a retrogressive (upslope) landslide or slump. Model 2 (Fig. 2) represents a rotational slide, and models 3.A and 3.B (Fig. 3) represent a moving ‘block slide’ (this is a landslide.)
which may travel downhill significant distances, creating a scar and a displaced moving block).

The mathematical representations of these models and their representation as a superposition of simple propagating or non-propagating source models are presented in Appendix A.

2.2. Tsunami generation and propagation model

The model we consider (see Fig. 4) is a fluid layer of constant depth, $h$, bounded by the rigid ocean floor at $z = -h$ and by the free surface at $z = 0$, and excited by a ‘small’ uplift, $\zeta(x,y,t)$, at $z = -h$. The uplift of the free surface is $\eta(x,y,t)$. The motion of the fluid layer is such that the fluid velocity potential $\phi(x,y,z;t)$ satisfies the Laplace differential equation

$$\nabla^2 \phi(x,y,z;t) = 0$$

subject to the following three boundary conditions

$$\phi_x(x, y, z; t) - \eta(x, y, t) = 0 \quad \text{at} \quad z = 0$$

$$\phi_x(x, y, z; t) - g \eta(x, y, t) = 0 \quad \text{at} \quad z = 0$$

and

$$\phi_x(x, y, z; t) - \zeta(x, y, t) = 0 \quad \text{at} \quad z = -h$$

A linearized shallow water solution (i.e. for water depth much smaller than the tsunami wavelength) can be obtained by the Fourier–Laplace transform, defined as

$$f(\tilde{k};s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ikx}e^{ikt}\int_{0}^{\infty} e^{-st}f(x, y, t)dxdy$$

Transformation of the equation of motion and boundary condition, and the assumptions of linearity and shallow water lead to the following solution for the transform of $\eta$, $\tilde{\eta}(\tilde{k};s)$, in terms of the transform of $\zeta$, $\tilde{\zeta}(\tilde{k};s)$ [2,7]

$$\tilde{\eta}(\tilde{k};s) = \frac{s^2 \tilde{\zeta}(\tilde{k};s)}{s^2 + \omega^2 \cosh kh}$$

where

$$\omega^2 = gk \tanh kh$$

and $\omega$ is the circular frequency of the wave motion. Then, a solution for $\eta(x,y,t)$ is obtained as follows: (1) $\zeta(x,y,t)$ is transformed to obtain $\tilde{\zeta}(\tilde{k};s)$, (2) $\tilde{\eta}(\tilde{k};s)$ is computed from Eq. (4) after substitution for $\tilde{\zeta}(\tilde{k};s)$, and (3) $\eta(x,y,t)$ is computed by performing inverse transform. For the models we consider in this paper, the forward and inverse Laplace transforms are evaluated analytically using tables and the Fourier transforms are evaluated numerically using Fast Fourier Transform (FFT). Further details on the solutions for all five models are presented in Ref. [9].

The above constitutes ‘linearized’ solution, which is known as the ‘shallow water solution’. It works well if the water depth is much smaller than the length of water waves [2]. Inclusion of the neglected non-linear terms in the boundary conditions would have permitted a solution involving solitary waves [6]. Other simplifications adopted in the above solution involve the assumption that ocean floor is rigid and that $\zeta(x,y,t)$ may be assumed a priori. This assumption thus neglects the coupling of submarine slides and surface waves [5].

3. Results and discussion

Detailed results showing tsunami waveforms as functions of time and of the model parameters, and peak amplitudes versus selected model parameters have been presented in Ref. [9]. In this note, we only summarize the results for the five models.

3.1. Two-dimensional waveforms at $t = t^*$

Ref. [8] showed examples of tsunami waveforms generated by a simple propagating source for the case, when the velocity of spreading of submarine slides and slumps is comparable to the long period tsunami velocity $c_T = \sqrt{gh}$. Figs. 5–11 show these waveforms $\eta(x,y,t)$ for the five models presented in this paper, evaluated at time $t^*$ is the time when the source process stops.

Fig. 5 shows results for model 1.A for water depth $h = 2$ km (implying $c_T = 0.14$ km/s), total length of the regions of depletion and accumulation $L_R = 50$ km, width of these zones $W = 50$ km, and velocity ratios $c_R/c_T = 1$ and $c_L/c_R = 0.5$, where $c_R$ and $c_L$ are velocities of spreading of the zones of accumulation and depletion, both in the positive $x$-direction. It can be seen that the tsunami wave-form $\eta(x,y,t)$ has two large peaks of comparable amplitude, one pointing upwards and the other one downwards. Both of these peaks are due to the spreading of the zone of accumulation with velocity equal to the tsunami velocity.

Figs. 6–8 show results for model 1.B, for three sets of parameters. Common parameters for all three sets are: water depth $h = 2$ km (implying $c_T = 0.14$ km/s), total length of the regions moving in the positive $x$-direction (equal to part of the length of the region of depletion and the entire length of the region of accumulation) $L_R = 50$ km, width of these regions $W = 50$ km, and velocity ratios $c_L/c_R = 2$ and $c_R/c_T = 0.5$, respectively. These figures differ only in the value of the $c_R/c_T$ ratio, which equals 20, 1 and 0.5. For this
model, $c_R$ and $c_C$ are in the positive $x$-direction and $c_L$ is in the negative $x$-direction. In Fig. 6, all three velocities of spreading are much larger than $c_T$ ($c_R/c_T = 20$, $c_L/c_T = 40$ and $c_C/c_T = 20$) and $\eta(x,y;t)$ roughly follows the final displacement of the ocean floor. In Fig. 7, $c_R/c_T = 1$, $c_L/c_T = 2$ and $c_C/c_T = 0.5$. A large upwards peak of $\eta(x,y;t)$ is seen, and a smaller downward peak immediately following the large peak, both resulting from the motion of the zone of accumulation and due to the fact that $c_R/c_T = 1$. In Fig. 8, $c_R/c_T = 0.5$, $c_L/c_T = 1$ and $c_C/c_T = 0.025$. A large downward peak immediately followed by a smaller upward peak of $\eta(x,y;t)$ are seen traveling in the negative $x$-direction. Both of these peaks are due to the spreading of the zone of depletion in the negative $x$-direction with velocity equal to the tsunami velocity.

Fig. 9 shows results for model 2 for water depth $h = 2$ km (implying $c_T = 0.14$ km/s), length of the region of accumulation $L_R = 50$ km, width of this zone $W = 50$ km, and velocity ratios $c_R/c_T = 1$ and $c_L/c_T = 2$, where $c_R$ and $c_L$ are velocities of spreading of the zones of accumulation and depletion, the former in the positive $x$-direction and the latter in the negative $x$-direction. In this figure, two large peaks of tsunamis are seen, one traveling in the positive $x$-direction and the other one traveling in the negative $x$-direction, resulting, respectively, from the spreading of the zone of accumulation and depletion. The wave due to spreading of the zone of depletion has comparable amplitude to the one due to spreading of the zone of accumulation even though $c_L/c_T = 2.0$ while $c_R/c_T = 1$, because the volume of the depleted material is larger than the volume of the accumulated material.

Fig. 10 shows results for model 3.A for water depth $h = 0.25$ km (implying $c_T = 0.05$ km/s), length of the sliding block $L_0 = 20$ km, width of the sliding block $W = 20$ km, distance of the sliding block from the origin $L_R = 20$ km and velocity ratio $c_R/c_T = 1$, where $c_R$ is the velocity of the sliding block (in the positive $x$-direction). $L_R = L_0$ implies that the separation distance between the zone of depletion and the sliding block in its final position is zero. Two large pairs of peaks are seen, one in front of the block and due to sliding of the block forward, and the other one behind the block and due to spreading of the zone of depletion.

Fig. 11 shows results for model 3.B for water depth $h = 0.25$ km (implying $c_T = 0.05$ km/s), length of the sliding block $L_0 = 20$ km, width of the sliding block $W = 20$ km, distance traveled by the sliding block $L_R = 20$ km and velocity ratio $c_R/c_T = 1$, where $c_R$ is the velocity of the sliding block (in the positive $x$-direction). The waveform is very similar to the one for model 3.A. It also has two large pairs of peaks, one in front and the other one behind the block, although there is no depletion zone for this model. The peaks behind the block are due to the fact that, as the block slides and uplifts the ocean floor in front of it, at the same time the ocean floor behind it suddenly ‘moves’ down back to zero level. This motion also spreads forward with velocity $c_R$. 
Fig. 7. Top view of a tsunami waveform $\eta(x, y, t')$ simulated by source model 1.B for the same parameters as in Fig. 6 except that $c_R/c_T = 1$.

Fig. 8. Top view of a tsunami waveform $\eta(x, y, t')$ simulated by source model 1.B for the same parameters as in Fig. 6 except that $c_R/c_T = 0.5$, which implies $c_L/c_T = 1$.

Fig. 9. Top view of a tsunami waveform $\eta(x, y, t')$ simulated by source model 2 at time $t = t'$ the time when the sliding stops, in water of depth $h = 2$ km, for slide width $W = 50$ km, characteristic length $L_R = c_R t' = 50$ km, and velocities of spreading such that $c_R/c_T = 1$, and $c_L/c_R = 2$.

These plots show that, for given water depth, the peak tsunami amplitude depends on the ratios of spreading velocities ($c_R/c_T$ for unilateral spreading, and $c_R/c_T$ and $c_L/c_T$ for bilateral spreading), and also on the total accumulated/depleted volumes. The peak tsunami amplitude also depends on the depth in the sense that even a small area source generates a large amplitude tsunami if the water is very shallow (see, for example, models 3.A and 3.B in Figs. 10 and 11).

3.2. Dependence of wave forms on $c_R/c_T$—amplification caused by focusing

In the following, we summarize the results on waveforms along $y = 0$ and at $t = t'$ computed for different values of the spreading velocity. For model 1.B, the results are shown in Fig. 12, and for model 2 in Fig. 13.

The results show that, in all cases, the largest positive peak of the tsunami amplitude at time $t'$ occurs when $c_R/c_T > 1$, for predominant direction of sliding in the positive $x$-direction with velocity $c_R$. A comparable negative peak occurs when $c_L < 0$, i.e. for sliding initiated at the base of the slope and spreading ‘uphill’ with velocity $c_L > c_T$ (for example, see Fig. 12 for $c_R/c_T = 0.5$ and $c_L/c_R = 2$).

It is seen that for the slides that spread rapidly ($c_R/c_T \geq 20$), the displacement of the free surface resembles the
displacement of the ocean floor at time $t^*$. Then $\eta/\zeta_0 \approx 1$, which is the common assumption in numerical simulations that ignore the source finiteness. As $c_R/c_T$ decreases towards 1, the largest tsunami waveform has progressively larger peak amplitudes, and higher frequency content. For $c_R/c_T < 1$ the peak amplitudes decrease, but the high frequency (short) waves continue to be present. This is in excellent agreement with our description of slowly spreading uplift of ocean bottom [7], when $c_R \approx c_T$. It is a result of constructive interference, and as the process lasts longer, the peak amplitudes also increase. When the tsunami is faster than the uplift ($c_R/c_T < 1$), the initial wave ‘escapes’ ahead of the currently uplifted water and amplification does not occur. For $c_R/c_T = 0.5–0.7$, for example, the peak tsunami amplitude is smaller than $\zeta_0$.

### 3.3. Peak tsunami amplitudes as function of $L/h$

Next, we summarize the results presented in Ref. [9] and showing the peak positive tsunami amplitude $\eta_{\text{R, max}}/\zeta_0$ and the peak negative tsunami amplitude $\eta_{\text{L, min}}/\zeta_0$ as functions of $L/h$ (See figs. 2.A.7, 3.A.4 and 3.B.4 in Ref. [9]), for all models computed when $t = t^*$ and for $c_R/c_T = 1$.

These results show that the positive peak amplitude $\eta_{\text{R, max}}/\zeta_0$ is essentially model independent, and agrees well with our results for a simple spreading uplift of ocean bottom [7]. The results also show that, for $c_L/c_T \approx 1$, the negative peak wave amplitudes are approximately equal to the positive peak amplitudes ($\eta_{\text{L, min}}/\zeta_0 \approx \eta_{\text{R, max}}/\zeta_0$).

In Fig. 14, we compare $\eta_{\text{R, max}}/\zeta_0$ for all five models with those for the unilaterally spreading uplift of ocean floor, with constant amplitude $\zeta_0$: [7], for $W/L = 0.25, 0.5$ and $\geq 1$. It is seen that for $W/L \geq 0.5$ (and $c_R = c_T$), the peak amplitudes are governed by focusing, or piling up of water above the spreading uplift. In the vicinity of the largest positive peak, the waveforms $\eta(x, y; t^*)$ for all models considered here are remarkably similar. What is different are the periods of the waves near $\eta_{\text{R, max}}$. Those are determined by the relative participation of predominant wavelengths, which depend on the differences in the spatial and temporal dependence of the uplift of the ocean floor for the different source models.

### 3.4. Long period limit of tsunami amplitudes

The long wavelength limit $\lim_{\zeta_0 \to 0} \eta(k; t) = V$, total volume of the displaced ocean floor, following a slide (see, for example, Ref. [7]). In the case of earthquakes, $V \neq 0$, and the $\lim_{\zeta_0 \to 0} \eta(k; t)$ has a trend parallel to $M_d/\mu$ (= source area X average dislocation), but is one to two orders of magnitude smaller [3]. For the models of slides
and slumps described here, $V = 0$ for models 1.A, 1.B, and 3.A, by our definition of these models, $V \neq 0$ but is small for model 2, and $V \neq 0$ for model 3.B.

The total volume of the moving material during submarine avalanches, slides and slumps can be comparable to and larger than the volume displaced by shallow earthquakes. However, the differences in the nature of motion between earthquakes and slides and slumps will result in substantial values of $V$ for earthquakes and in $V \approx 0$ for most slides and slumps. In future, when instrumental recordings of tsunami waveforms $\eta(x, y; t)$ become more ubiquitous, it will be possible to compute $\tilde{\eta}(k; t)$ for $k \to 0$ and to use this result for discrimination of the physical nature of tsunami sources. At present, this is possible only via time consuming mapping of amplitudes and of the geographical extent of tsunami runup [4].

3.5. Evolution of tsunami waveforms in time

Trifunac et al. [9] studied tsunami waveforms $\eta(x, 0; t)$ along the axis of symmetry $y = 0$, as they evolve in time from $t = 0$ to several times the duration of the source process. Fig. 15a shows one such case for model 1.A. Fig. 15b shows how the large (short wavelength) peak caused by focusing is dispersed during $t > t^*$. 

3.6. Analogy between wave focusing and resonance of a SDOF oscillator

Todorovska and Trifunac [7] showed that when $c_R \sim \sqrt{gh}$, amplification of the peak amplitude $\eta_{\max}/\xi_0$ occurs, which when plotted versus $c_R/c_L$ resembles amplitude versus frequency behavior for a single degree of freedom system. This amplification increases with $L/h$, where $L$ depends on the model, and is the largest for $W/L \to \infty$. However, the largest amplitudes are essentially realized for $W/L \geq 1$. The ratio $h/(W L)$ could be thought of as representing ‘damping’. For the examples illustrated in this work, the ‘resonance peak’ occurs for $(h/W L) \approx 0.001$. No ‘resonance’ peak occurs for $(h/W L) \approx 0.001$, and thus there is no amplification by focusing. Trifunac et al. [9] showed such results for all five models, for $\eta_{R,\max}/\xi_0$ and for $\eta_{L,\max}/\xi_0$.

3.7. Radiation pattern

Radiation pattern of tsunami wave amplitudes in the near-field can be evaluated starting with data on the inundation heights. In the past, this has been combined with inverse refraction diagrams to estimate the distribution of wave amplitudes in the source region [3]. In long-period teleseismic seismological studies, radiation pattern has been used to infer the velocity and direction of a spreading dislocation [1].
patterns for our five slide and slump models, because they are not general and not detailed enough to help visualize radiation patterns from actual submarine slides and slumps. An example of a radiation pattern from a rectangular spreading uplift of the ocean floor can be found in Ref. [7]. In Fig. 16, we illustrate variation of wave amplitudes and predominant periods for model 1.A and for variation of azimuths of observation station, within $0 \leq \theta \leq 180^\circ$.

3.8. Bores

Todorovska and Trifunac [7] noted that very little has been written about the near-field mechanisms, which lead to creation of bores during tsunamiigenic earthquakes, slides or slumps. The example calculations in this work show bore-like behavior of $\eta(x, 0; t)$ for many combinations of the source parameters (see also Ref. [9]).

4. Summary and conclusions

The results show that, for given constant water depth, the peak tsunami amplitude depends on the ratios of spreading velocities ($c_0/c_T$ for unilateral spreading, and $c_0/c_T$ and $c_1/c_T$ for bilateral spreading), and also on the volumes of accumulated/depleted material. The peak tsunami amplitude also depends on the water depth in the sense that even a small
area source can generate a large amplitude tsunami if the water is shallow.

Largest amplitudes are observed, when $c_R \sim c_T = \sqrt{gh}$ the long wavelength tsunami velocity. A plot of the peak amplitude $\eta_{\text{max}}/\zeta_0$ versus $c_T/c_R$ has a peak at $c_T/c_R = 1$ and resembles amplitude versus frequency behavior for a single degree of freedom oscillator. This amplification increases with $L/h$, where $L$ depends on the model, and is the largest for $W/L \to \infty$. The ratio $h/(WL)$ could be thought of as representing damping. For the examples considered in this work, the ‘resonance peak’ occurs for $(h/WL) \approx 0.001$. No ‘resonance’ peak occurs for $(h/WL) \approx 0.001$.

The results show that, in all cases, the largest positive peak of the tsunami amplitude at time $t^*$ occurs when $c_R/c_T \sim 1$, for predominant direction of sliding in the positive $x$-direction with velocity $c_R$. A comparable negative peak occurs when $c_L \neq 0$, i.e. for sliding initiated at the base of the slope and spreading ‘uphill’ with velocity $c_L \sim c_T$.

For the slides that spread rapidly ($c_R/c_T \approx 20$), the displacement of the free surface resembles the displacement of the ocean floor at time $t^*$. Then $\eta/\zeta_0 \approx 1$, which is the common assumption in numerical simulations that ignore the source finiteness. As $c_R/c_T$ decreases towards 1, the largest tsunami waveform has progressively larger peak amplitudes, and higher frequency content. For $c_R/c_T < 1$ the peak amplitudes decrease, but the high frequency (short) waves continue to be present. This is in excellent agreement with our description of slowly spreading uplift of ocean bottom [7], when $c_R \approx c_T$. It is a result of constructive interference, and as the process lasts longer, the peak amplitudes also increase. When the tsunami is faster than the uplift ($c_R/c_T < 1$), the initial wave ‘escapes’ ahead of the currently uplifted water and amplification does not occur. For $c_R/c_T = 0.5-0.7$, for example, the peak tsunami amplitude is smaller than $\zeta_0$.

A comparison of the results for the positive peak amplitude $\eta_{\text{max}}/\zeta_0$ show that it is essentially model independent, and agrees well with our results for a simple spreading constant uplift of ocean floor model [7]. The results also show that, for $c_L/c_T \sim 1$, the negative peak wave amplitudes are approximately equal to the positive peak amplitudes ($\eta_{\text{max}}/\zeta_0 \approx \eta_{\text{max}}/\zeta_0$).

The observed amplification, when $c_R \sim c_T = \sqrt{gh}$ is a near-field effect. This amplification occurs in a short wavelength pulse, which gets dispersed as the tsunami travels further away from the source.

For all the models in this work, it has been assumed that the spreading of submarine slides or slums is in one direction only, here chosen to coincide with positive and/or negative $x$-direction. The methods we use in this work can be generalized to study also the tsunami generated by slides and slums spreading in two directions.

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Appendix A

This appendix presents the mathematical representations of the five source models studied in this paper.

A.1. Model 1A (Fig. 1a,b)

The uplift of the ocean floor for $t \leq t^*$ is
\[ \zeta(x, y; t) = \begin{cases} -\zeta_1, & 0 \leq x < c_L t \text{ and } -W/2 \leq y \leq W/2 \\ \zeta_0, & c_L t \leq x \leq c_R t \text{ and } -W/2 \leq y \leq W/2 \\ 0, & \text{otherwise} \end{cases} \]  

(A1)

and for \( t \geq t^* \) is

\[ \zeta(x, y; t) = \begin{cases} -\zeta_1, & 0 \leq x < c_L t^* \text{ and } -W/2 \leq y \leq W/2 \\ \zeta_0, & c_L t^* \leq x \leq c_R t^* \text{ and } -W/2 \leq y \leq W/2 \\ 0, & \text{otherwise} \end{cases} \]  

(A2)

The characteristic length of the model is \( L_R = c_R t^* \).

The representation for this model is equivalent to a superposition of three simple spreading uplifts, \( \zeta^{(i)} \) and \( \zeta^{(ii)} \), both starting to spread at \( x = 0 \) and with velocity of spreading in the positive \( x \)-direction [7]:

\[ \zeta(x, y; t) = \zeta^{(i)}(x, y; t) + \zeta^{(ii)}(x, y; t) \]  

(A3a)

\[ \zeta^{(i)} = \zeta_0 H(t - x/c_R), \]

\[ 0 \leq x \leq c_R t^* = L_R \text{ and } -W/2 \leq y \leq W/2 \]  

(A3b)

\[ \zeta^{(ii)} = -(\zeta_0 + \zeta_1) H(t - x/c_L), \]

\[ 0 \leq x \leq c_L t^* \text{ and } -W/2 \leq y \leq W/2 \]  

(A3c)

Then the Laplace–Fourier transform of \( \zeta(x, y; t) \) is

\[ \tilde{\zeta}(\tilde{k}, \tilde{s}) = \left[ \int_{-W/2}^{W/2} e^{i\tilde{k}x} d\tilde{x} \right] \left[ \int_0^{c_L t^*} e^{i\tilde{s} \tau} \left( \int_{-c_L \tilde{s}}^{\infty} \zeta_0 e^{-st} d\tau \right) d\tilde{x} \right] - \left[ \int_{-W/2}^{W/2} e^{i\tilde{k}x} d\tilde{x} \right] \left[ \int_0^{c_R t^*} e^{i\tilde{s} \tau} \left( \int_{-c_R \tilde{s}}^{\infty} \zeta_1 e^{-st} d\tau \right) d\tilde{x} \right] \]  

(A4)

A.2. Model 1.B (Fig. 1a,b)

The uplift of the ocean floor for \( t \leq t^* \) is

\[ \zeta(x, y; t) = \begin{cases} -\zeta_1, & -c_L t \leq x < 0 \text{ and } -W/2 \leq y \leq W/2 \\ -\zeta_1, & 0 \leq x < c_C t \text{ and } -W/2 \leq y \leq W/2 \\ \zeta_0, & c_C t \leq x \leq c_R t \text{ and } -W/2 \leq y \leq W/2 \\ 0, & \text{otherwise} \end{cases} \]  

(A5)

The uplift of the ocean floor for \( t \geq t^* \) is

\[ \zeta(x, y; t) = \begin{cases} -\zeta_0 \frac{x}{L_R} (1 - e^{-at}), & -c_L t \leq x \leq 0 \text{ and } -W/2 \leq y \leq W/2 \\ -\zeta_0 \frac{x}{L_R} (1 - e^{-at}), & 0 \leq x \leq c_R t \text{ and } -W/2 \leq y \leq W/2 \\ 0, & \text{otherwise} \end{cases} \]  

(A9)
and for \( t \geq t^* \) is

\[
\zeta(x, y; t) = \begin{cases} 
-\frac{\zeta_0}{L_R} \frac{x}{(1 - e^{-a r^*})}, & -c_R t^* \leq x \leq 0 \text{ and } -W/2 \leq y \leq W/2 \\
\frac{\zeta_0}{L_R} \frac{x}{(1 - e^{-a r^*})}, & 0 \leq x \leq c_R t^* \text{ and } -W/2 \leq y \leq W/2 \\
0, & \text{otherwise}
\end{cases}
\] (A10)

The characteristic length of the model is \( L_R = c_R t^* \).

The Laplace–Fourier transform of \( \zeta(x, y; t) \) is

\[
\tilde{\zeta}(\tilde{k}; s) = \left[ \int_{-W/2}^{W/2} e^{i k y} dy \right] \times \left[ \int_0^{c_R t^*} e^{i k x} \left( \int_{-\infty}^{\infty} \frac{\zeta_0}{L_R} \frac{x}{(1 - e^{-a r^*})} e^{-s t} dx \right) dr \right]
\] (A11)

A.4. Model 3.A (Fig. 3)

The mathematical representations of the sea floor uplift at these three instants of time is:

- \( t < L_0/c_R \) :
  \[
  \zeta(x, y; t) = \begin{cases} 
  -\zeta_0, & 0 \leq x \leq c_R t \text{ and } -W/2 \leq y \leq W/2 \\
  \zeta_0, & L_0 \leq x \leq L_0 + c_R t \text{ and } -W/2 \leq y \leq W/2 \\
  0, & \text{otherwise}
  \end{cases}
  \] (A12)

- \( L_0/c_R < t < t^* \) :
  \[
  \zeta(x, y; t) = \begin{cases} 
  -\zeta_0, & 0 \leq x \leq L_0 \text{ and } -W/2 \leq y \leq W/2 \\
  \zeta_0, & c_R t \leq x \leq L_0 + c_R t \text{ and } -W/2 \leq y \leq W/2 \\
  0, & \text{otherwise}
  \end{cases}
  \] (A13)

- \( t \geq t^* \) :
  \[
  \zeta(x, y; t) = \begin{cases} 
  -\zeta_0, & 0 \leq x \leq L_0 \text{ and } -W/2 \leq y \leq W/2 \\
  \zeta_0, & c_R t^* \leq x \leq L_0 + c_R t^* \text{ and } -W/2 \leq y \leq W/2 \\
  0, & \text{otherwise}
  \end{cases}
  \] (A14)

The motion of this model can also be represented as a superposition of the solutions for two simple spreading uplifts subject to co-ordinate transformation. Both of these two simple sources, \( \xi^{(I)} \) and \( \xi^{(II)} \), have velocity of spreading \( c_R \) in the positive \( x \)-direction, but one starting to spread at \( x = L_0 \) (with amplitude \( \zeta_0 \)), and the other one at \( x = 0 \) (with amplitude \( -\zeta_0 \)).

\[
\zeta(x, y; t) = \xi^{(I)}(x, y; t) + \xi^{(II)}(x, y; t)
\] (A15a)

\[
\xi^{(I)} = -\zeta_0 H(t - x/c_R),
\] (A15b)

\[
\xi^{(II)} = -\zeta_0 H(t - x/c_R),
\] (A15c)

A.5. Model 3.B (Fig. 3)

The mathematical representations of the sea floor uplift for this model for time \( t \leq t^* \) is

\[
\xi(x, y; t) = \begin{cases} 
\zeta_0(1 - e^{-a r^*}), & c_R t \leq x \leq L_0 + c_R t \text{ and } -W/2 \leq y \leq W/2 \\
0, & \text{otherwise}
\end{cases}
\] (A17)

and for \( t > t^* \) is

\[
\xi(x, y; t) = \begin{cases} 
\zeta_0(1 - e^{-a r^*}), & c_R t^* \leq x \leq L_0 + c_R t^* \text{ and } -W/2 \leq y \leq W/2 \\
0, & \text{otherwise}
\end{cases}
\] (A18)
where $\alpha$ is a constant used to describe the ‘duration’ of the uplift.

The uplift for this model can be represented as a superposition of three simple source models, one with positive amplitude, starting to spread at $x = L_0$ and spreading with velocity $c_R$ in the positive x-direction, the second one with negative amplitude, starting to spread at $x = 0$ and spreading with velocity $c_R$ in the positive x-direction, and the third one with positive amplitude, which does not spread, and extends between $x = 0$ and $x = L_0$. The amplitude of the uplift of all the three simple sources changes in time as $\zeta_0(1 - e^{-\alpha t})$. The mathematical description of these motions is

$$\zeta(x, y; t) = \zeta^{(i)}(x, y; t) + \zeta^{(ii)}(x, y; t) + \zeta^{(iii)}(x, y; t) \quad (A19a)$$

$$\zeta^{(i)} = \zeta_0(1 - e^{-\alpha t})H(t - x/c_R), \quad x' = x - L_0, 0 \leq x' \leq c_R t^* \text{ and } -W/2 \leq y \leq W/2$$

$$\zeta^{(ii)} = -\zeta_0(1 - e^{-\alpha t})H(t - x/c_R), \quad 0 \leq x \leq c_R t^* \text{ and } -W/2 \leq y \leq W/2$$

$$\zeta^{(iii)} = \zeta_0(1 - e^{-\alpha t}), \quad 0 \leq x \leq L_0 \text{ and } -W/2 \leq y \leq W/2 \quad (A19d)$$

Then the Laplace–Fourier transform of $\zeta_R(x, y; t)$ is

$$\tilde{\zeta}(k; s) = \zeta_0 \left[ \int_{-W/2}^{W/2} e^{iky} dy \right]$$

$$\times \left[ \int_{c_R t^*}^{L_0 + c_R t^*} e^{ikx} \left( \int_{x - L_0/c_R}^{\infty} (1 - e^{-\alpha t})e^{-st} dr \right)dx \right.$$

$$- \int_{0}^{c_R t^*} e^{ikx} \left( \int_{x/c_R}^{\infty} (1 - e^{-\alpha t})e^{-st} dr \right)dx$$

$$+ \int_{c_R t^*}^{L_0} e^{ikx} \left( \int_{0}^{\infty} (1 - e^{-\alpha t})e^{-st} dr \right)dx \] \right]. \quad (A20)$$

References


