A note on the effects of nonuniform spreading velocity of submarine slumps and slides on the near-field tsunami amplitudes

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Abstract

The effects of variable speeds of spreading of submarine slides and slumps on near-field tsunami amplitudes are illustrated. It is shown that kinematic models of submarine slides and slumps must consider time variations in the spreading velocities, when these velocities are less than about 2cT, where cT = √gh is the long period tsunami velocity in ocean of constant depth h. For average spreading velocities greater than ~2cT, kinematic models with assumed constant spreading velocities provide good approximation for the tsunami amplitudes above the source. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Tsunami modeling; Tsunami with variable speed of spreading; Slides; Slumps

1. Introduction

Submarine slides and slumps can generate large tsunami, but usually result in more localized effects than the tsunami caused by earthquakes [9]. Nevertheless these localized waves can produce large run-up along the coast, especially in narrow bays and fjords [2,4–6].

Previous studies of tsunami generation have considered impulsive and spreading kinematic models of moving impermeable ocean bottom. For ‘fast’ movement (1–3 km/s), caused by most earthquakes, the interaction of water waves and of the tectonic motion may be neglected, and it can be assumed that the initial surface elevation of the water is the same as the displacement of the ocean bottom. It has also been assumed that the movement of the ocean bottom can be approximated by uplift or by depression spreading with constant velocity in one direction [7,8,10], and in two directions [11,12]. For tsunami generated by submarine slides and slumps, it becomes necessary to consider coupling of the slide motion and of the water waves, because the slide duration is ‘long’ (spreading velocities are ‘low’, e.g. 0.001–0.10 km/s; [9]), relative to the duration of typical earthquake sources.

Jiang and Le Blond [3] also show typical time dependent changes of the Froude number (Fr = cR/(gh)1/2, where cR is the frontal speed of the slide, h is the depth of ocean and g is acceleration due to gravity) during the slumping and slidding process. It first increases to reach a maximum as the slide accelerates, and then gradually decreases as the slide comes to rest.

The purpose of this paper is to illustrate the nature and the extent of variations in the tsunami waveforms caused by simple time variations of the frontal velocity of spreading, cR, for two-dimensional kinematic models of slides and slumps, and to compare the results with those for the slides spreading with constant velocity [10] cR. It will be shown how the changes of cR in time act to reduce focusing, when cR ∼ √gh. The problem is solved by transform method (Laplace in time and Fourier in space), with the forward and inverse Laplace transforms computed analytically, and the Fourier transforms computed by Fast Fourier Transform. Realistic simulation of water waves generated by submarine slumps and slides, considering all relevant parameters and physical properties, can be obtained only by numerical modeling. The usefulness of analytical and semi-analytical solutions for slides and slumps spreading with constant and with variable velocities lies in their ability to display the physical nature of the problem, directly in

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terms of the model parameters, and in providing a basis for independent verification of approximate numerical algorithms.

2. Mathematical model

We consider a kinematic tsunami source model which is a generalization of Model 1.B in Trifunac et al. [10], shown in Fig. 1. It represents a mass movement triggered at \( x = 0 \) and at time \( t = 0 \), and spreading bilaterally. The accumulation zone spreads downhill (in the positive \( x \)-direction) with frontal velocity \( c_R \), and the zone of depletion spreads downhill with velocity \( c_C \) and uphill with velocity \( c_L \) (this model is meaningful if \( c_R > c_C \)). The final uplift of the accumulation zone is \( \zeta_0 \) and the final depression of the depletion zone is \( \zeta_1 \), both assumed to be constant. It is assumed that the displacement of a point on the slide occurs instantaneously once the disturbance has reached that point. It is also assumed that the mass of the slide is conserved \( (A_1 = A_2 \) in Fig. 1) which implies

\[
\zeta_1 = \left( \frac{1 - \beta}{\gamma + \beta} \right) \zeta_0
\]

(1)

where \( \beta = c_C/c_R \) and \( \gamma = c_L/c_R \). The slide stops spreading at time \( t = t^* \). In the final configuration (Fig. 1), the length of the accumulation zone is \( L_R = c_R t^* \) when \( c_R \) is constant, used as characteristic length of the model. The width of the slide is \( W \).

The uplift of the ocean floor, \( \zeta(x, y; t) \), for \( t \leq t^* \) and assuming \( c_R, c_C \) and \( c_L \) are constant, is

\[
\zeta(x, y; t) = \begin{cases} 
-\xi_1, & -c_C t \leq x < 0 \text{ and } -W/2 \leq y \leq W/2 \\
-\xi_1, & 0 \leq x < c_C t \text{ and } -W/2 \leq y \leq W/2 \\
\zeta_0, & c_C t \leq x \leq c_R t \text{ and } -W/2 \leq y \leq W/2 \\
0, & \text{otherwise}
\end{cases}
\]

(2)

and for \( t \geq t^* \) is same as for \( t = t^* \). This source process is equivalent to one that is a superposition of three simple spreading uplifts \( \xi^{(0)}(x, y; t) \), \( \xi^{(I)}(x, y; t) \) and \( \xi^{(II)}(x, y; t) \), as those in Todorovskia and Trifunac [7], subject to coordinate transformation. The first uplift has amplitude \( \zeta_0 \) and starts spreading at \( x = 0 \) in the positive \( x \)-direction with velocity \( c_R \), the second one has amplitude \( -(\zeta_0 + \xi_1) \) and starts spreading also at \( x = 0 \) in the positive \( x \)-direction, but with velocity \( c_C < c_R \), and the third one has amplitude \( -\zeta_1 \) and starts spreading also at \( x = 0 \), but in the negative \( x \)-direction and with velocity \( c_L \). This is expressed by

\[
\zeta(x, y; t) = \xi^{(0)}(x, y; t) + \xi^{(I)}(x, y; t) + \xi^{(II)}(x, y; t)
\]

(3)

\[
\xi^{(I)} = \zeta_0 H(t - x/c_R),
\]

\[0 \leq x \leq c_R t^* = L_R \text{ and } -W/2 \leq y \leq W/2\]

(4a)

\[
\xi^{(II)} = -(\zeta_0 + \xi_1) H(t - x/c_C),
\]

\[0 \leq x \leq c_C t^* \text{ and } -W/2 \leq y \leq W/2\]

(4b)

Fig. 2. Variable velocity of spreading \( c(t) \) and its piecewise constant approximation over \( N \) time intervals.

Fig. 3. Variable \( c(t) \) adopted for the examples in this paper (ramp up for \( 0 \leq t \leq t_1 \), constant for \( t_1 \leq t \leq t_2 \), and ramp down for \( t_2 \leq t \leq t^* \)).
Fig. 4. Test of convergence of $\eta/\zeta_0$ for $N = 15, 35$ and 39 segments.

Fig. 5. Normalized tsunami waveforms $\eta/\zeta_0$ along $y = 0$, at time $t = t^*$ (the time when the spreading of the slide stops), for $c_3/c_R = 0.3$ and $c_2/c_R = 0.1$ and for eight values of $c_R/c_T$ between 0.5 and 20. The ocean depth is $h = 2$ km, and the slide has width $W = 50$ km and characteristic length $L_R = 50$ km.
Fig. 6. Same as Fig. 5 but for $c_L/c_R = 0.3$ and $c_S/c_R = 0$.

Fig. 7. Same as Fig. 5 but for $c_L/c_R = 2$ and $c_S/c_R = 0.5$. 
Fig. 8. Left: an enlarged area from Fig. 5 for $c_t/c_T$, i.e. $\dot{c}/c_T = 0.7$, 0.8 and 0.9 showing peak amplitudes of the leading tsunami propagating in the positive $x$-direction. Right: comparison of the peak amplitudes for the Cases 1, 2 and 3 with those for a simple dislocation with constant velocity of spreading from Todorovska and Trifunac [7] with W/L > 1 and W/L > 0.5.

Fig. 9. Normalized tsunami waveforms $\eta/\zeta_0$ along $y = 0$, at selected instants of time $t = t^*$ and for $c_t/c_R = 1$ and $c_p/c_T = 1$. The ocean depth is $h = 2$ km, and the slide has width $W = 50$ km and characteristic length $L_x = 50$ km. The solid lines show results for the constant velocity Model 1.B, and the dashed lines show results for Cases 1, 2 and 3 of the variable velocity model.
\[ \zeta^{(11)} = -\zeta_1 H(t - x'/c_L). \]
\[ x' = -x, \quad 0 \leq x' \leq c_L t^* \quad \text{and} \quad -W/2 \leq y \leq W/2 \]

The generalization of Model 1.B considered in this paper is that the velocities of spreading vary with time. In the numerical calculations, the true variation is discretized over \( N \) time intervals, with constant velocity approximation within each interval, as shown in Fig. 2 (\( c(t) \) here refers to velocity of spreading in general). In the examples shown in this paper, \( c(t) \), linearly increasing from zero to \( c \) for \( 0 \leq t \leq t_1 \), constant for \( t_1 \leq t \leq t_2 \) and linearly decreasing to zero for \( t_2 \leq t \leq t^* \), is assumed, as shown in Fig. 3. The average spreading velocity \( \overline{c} \) for this case is

\[ \overline{c} = \frac{1}{t^*} \int_0^{t^*} c(t) \text{d}t = \frac{1}{2} (1 + t_2 h^* - t_1 h^*) \]

The variability of the velocity of spreading can also be expressed as function of \( x \), where

\[ x(t) = \int_0^t c(\tau) \text{d}\tau \]

In our selection of the spreading velocities, we were guided by the time dependent changes of Froude number described by Jiang and Le Blond [3] in their numerical experiments on slide and slump motion. To keep the number of parameters small, only the time function for \( c_R \) is specified, and \( c_C \) and \( c_L \) are expressed as factors of \( c_R \) (hence having same type of time dependence).

A linearized shallow water solution (for water depth much smaller than the tsunami wavelength) for the tsunami generation and propagation is obtained by the Fourier–Laplace transform [1,7] defined by

\[ \tilde{f}(k; s) = \int_{-\infty}^{\infty} e^{iky} \left[ \int_{-\infty}^{\infty} e^{iks} f(x,y,t) \text{d}x \right] \text{d}y \]

(7)

In the transform space, the water elevation is

\[ \tilde{\eta}(k; s) = \frac{s^2 \tilde{\zeta}(k; s)}{s^2 + \omega^2} \frac{1}{\cosh kh} \]

(8)

where

\[ \omega^2 = gk \tanh kh \]

(9)

and \( \omega \) is circular frequency of the wave motion. For constant spreading velocities, the forward and inverse Laplace transforms can be computed analytically or from tables.
The Laplace–Fourier transform of $\zeta(x, y, t)$ in Eq. (3) is

$$\tilde{\zeta}(k; s) = \left[ \int_{-W/2}^{W/2} e^{ik_2y} dy \right] \left[ \int_{0}^{L_e} e^{ik_1x} \left( \int_{x/L_e(x)}^{\infty} \tilde{\zeta}_0 e^{-st} \, dx \right) \, dx \right] - \int_{0}^{L_e} e^{ik_1x} \left( \int_{x/L_e(x)}^{\infty} \tilde{\zeta}_1 e^{-st} \, dx \right) \, dx$$

$$- \int_{-L_e}^{0} e^{ik_1x} \left( \int_{x/L_e(x)}^{\infty} \tilde{\zeta}_1 e^{-st} \, dx \right) \, dx$$

where

$$L_\alpha = \int_{0}^{t} c_\alpha(\tau) \, d\tau, \quad z = R, C, L \tag{11}$$

For piecewise constant approximation of the velocities of spreading in $N$ intervals, the integrals in $x$ in Eq. (10) can be broken over $N$ segments, and

$$\tilde{\zeta}(k; s) = \left[ \int_{-W/2}^{W/2} e^{ik_2y} dy \right]$$

$$\times \sum_{n=1}^{N} \left[ \int_{x_{n-1}}^{x_n} e^{ik_1x} \left( \int_{x/L_e(x)}^{\infty} \tilde{\zeta}_0 e^{-st} \, dx \right) \, dx \right]$$

Substitution in Eq. (3) of the relationship between $\zeta_1$ and $\zeta_0$ from Eq. (1) and integration gives

$$\tilde{\zeta}(k; s) = \tilde{\zeta}_0 \frac{2 \sin k_2 W/2}{k_2} \left[ K_R - \frac{1 + \gamma}{\gamma + \beta} \frac{1}{\gamma + \beta} K_C - \frac{1 - \beta}{\gamma + \beta} K_L \right] \tag{14}$$

where

$$K_z = \sum_{n=1}^{N} \frac{e^{-(s-ik_1c_{z,n})d_{z,n}} - e^{-(s-ik_1c_{z,n}d_{z,n})}}{s(s - ik_1c_{z,n})} c_{z,n}, \quad z = R, C \tag{15a}$$
and

\[ K_L = \sum_{n=1}^{N} \frac{[e^{-(s+ik_1c_{L,n})x} - e^{-(s+ik_1c_{L,n})x}]c_{L,n}}{s(s + ik_1c_{L,n})} \]  

(15b)

Substitution of \( \tilde{\zeta}(k; s) \) in Eq. (8) gives

\[ \tilde{\eta}(k; s) = \frac{\sin^2 k_2 W/2}{s^2 + \omega^2} \frac{2\sin k_2 W/2}{k_2} \frac{1}{\cosh kh} \frac{2\sin k_2 W/2}{k_2} \]

\[ \times \left[ K_R - \frac{1 + \gamma}{\gamma + \beta} K_C - \frac{1 - \beta}{\gamma + \beta} K_L \right] \]

(16)

The inverse Laplace transform of \( \tilde{\eta}(k; s) \), \( \eta(k; t) \), can be evaluated analytically, as follows

\[ \eta(k; t) = \frac{1}{\cosh kh} \frac{2\sin k_2 W/2}{k_2} \left\{ \frac{s^2}{s^2 + \omega^2} \right\} L^{-1} \left\{ \frac{s^2}{s^2 + \omega^2} \right\} \]

\[ \times \left[ K_R - \frac{1 + \gamma}{\gamma + \beta} K_C - \frac{1 - \beta}{\gamma + \beta} K_L \right] \]

(17)

with

\[ L^{-1} \left\{ \frac{s^2}{s^2 + \omega^2} K_L \right\} \]

\[ = \sum_{n=1}^{N} \frac{i c_{L,n}}{\omega^2 - k_1^2 c_{L,n}^2} \left\{ e^{ik_1 x_{L,n}} e^{k_1 (x_{L,n} - x_{L,n-1})} \right\} \]

\[ - k_1 c_{L,n} \cos \omega \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) - i \omega \sin \omega \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) \]

\[ \times H \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) - e^{ik_1 x_{L,n}} k_1 c_{L,n} e^{k_1 (x_{L,n} - x_{L,n-1})} \]

\[ - k_1 c_{L,n} \cos \omega \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) \]

\[ - i \omega \sin \omega \left( t - \frac{x_{L,n}}{c_{L,n}} \right) \]

\[ H \left( t - \frac{x_{L,n}}{c_{L,n}} \right) \]

\[ Z = \mathbb{R}, \mathbb{C} \]

(18a)

and

\[ L^{-1} \left\{ \frac{s^2}{s^2 + \omega^2} K_L \right\} \]

\[ = \sum_{n=1}^{N} \frac{i c_{L,n}}{\omega^2 - k_1^2 c_{L,n}^2} \left\{ e^{ik_1 x_{L,n}} e^{k_1 (x_{L,n} - x_{L,n-1})} \right\} \]

\[ + \omega \sin \omega \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) - \exp \left\{ i k_1 c_{L,n} \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) \right\} \]

\[ \times H \left( t - \frac{x_{L,n-1}}{c_{L,n}} \right) + e^{-ik_1 x_{L,n}} i k_1 c_{L,n} \cos \omega \left( t - \frac{x_{L,n}}{c_{L,n}} \right) \]
\[ h = 2 \text{ km} \quad L_R = 50 \text{ km} \quad c_T = 0.14 \text{ km/s} \quad c_T/c_R = 1 \quad t = t^* \]
\[ W = 50 \text{ km} \quad c_L/c_p = 0.3 \quad c_U/c_R = 0 \]

\[ \eta/\zeta_0 \]

**Model 1.B**

<table>
<thead>
<tr>
<th>( y )-km</th>
<th>( -100 )</th>
<th>( 0 )</th>
<th>( 100 )</th>
<th>( x )-km</th>
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<th>( y )-km</th>
<th>( -100 )</th>
<th>( 0 )</th>
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<th>( x )-km</th>
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</table>

**Case 1**

**Case 2**

**Case 3**

\[ \eta(x, y; t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ik_2} \eta(k; t) dk_1 dk_2 \]

Fig. 13. Comparison of tsunami waveforms \( \eta/\zeta_0 \) for the constant velocity Model 1.B [10] and for Cases 1, 2 and 3 of the variable velocity model, all computed at time \( t = t^* \), and for \( h = 2 \text{ km}, L_R = W = 50 \text{ km}, c_T/c_R = 1, c_L/c_p = 0.3 \) and \( c_U/c_R = 0 \).

Finally inverse Fourier transform is performed on \( \tilde{\eta}(k; t) \) by Fast Fourier Transform to obtain \( \eta(x, y; t) \).
\[
\begin{align*}
\eta/\zeta_0 \\
\text{Model 1.B} \\
\eta/\zeta_0 \\
\text{Case 1} \\
\eta/\zeta_0 \\
\text{Case 2} \\
\eta/\zeta_0 \\
\text{Case 3}
\end{align*}
\]

Fig. 14. Same as Fig. 13, but for \( c_l/c_R = 2 \) and \( c_c/c_R = 0.5 \).

The long wavelength limit of \( \tilde{\eta}(\tilde{k}; t) \) is

\[
\lim_{\tilde{k} \to 0} \tilde{\eta}(\tilde{k}; t) = 0 \tag{20}
\]

The other limits are almost the same as for the Model B [10].

3. Numerical results

We illustrate results for three cases of trapezoidal \( c_R(t) \), as shown in Fig. 3, defined by parameters \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \), which are equal to the duration of each segment normalized by the total duration of the
\( h = 2 \text{ km} \quad L_R = 50 \text{ km} \quad c_T = 0.14 \text{ km/s} \quad c_T/c_R = 2 \quad t = t^* \)
\( W = 50 \text{ km} \quad c_L/c_R = 2 \quad c_C/c_R = 0.5 \)

Fig. 15. Same as Fig. 13, but for \( c_K/c_T = 0.5 \), \( c_L/c_R = 2 \) and \( c_C/c_R = 0.5 \).
slide
\[ \alpha_1 = t_f t^* \quad \alpha_2 = (t_2 - t_1) t^* \quad \alpha_3 = (t^* - t_2) t^* \]

The three cases are:

- **Case1:** \( \alpha_1 = 0.6, \quad \alpha_2 = 0.35, \quad \alpha_3 = 0.05 \)
- **Case2:** \( \alpha_1 = 0.5, \quad \alpha_2 = 0.45, \quad \alpha_3 = 0.05 \)
- **Case3:** \( \alpha_1 = 0.4, \quad \alpha_2 = 0.55, \quad \alpha_3 = 0.05 \)

Fig. 4 shows the effects of the discretization on the accuracy of the normalized wave amplitudes \( \eta/\xi_0 \). The different curves correspond to \( N = 15, 35 \) and 39 intervals. It is seen that the results for \( N = 35 \) and 39 are essentially the same, and therefore we adopt \( N = 39 \) for all subsequent examples in this paper.

Figs. 5–12 show variations of \( \eta/\xi_0 \) with the model parameters. They show \( \eta/\xi_0 \) versus \( x \) and along \( y = 0 \), for ocean depth \( h = 2 \) km (\( c_T = 0.14 \) km/s), \( W = 50 \) km and \( L_R = 50 \) km. The solid lines correspond to Model 1.B in Trifunac et al. [10], with constant velocities of spreading, equal to the average computed by Eq. (5), and the dashed lines correspond to Cases 1, 2 and 3.

Figs. 5–7 show how \( \eta/\xi_0 \) changes with respect to \( c_R/c_T \), when \( c_R = c_T \). They show results at \( t = t^* \) for chosen \( c_T/c_R \), and \( c_T/c_R \) and for eight values of \( c_T/c_T \) between 0.5 and 20 (\( c_L/c_R = 0.3 \) and \( c_C/c_R = 0.1 \) in Fig. 5, \( c_L/c_R = 0.3 \) and \( c_C/c_R = 0.1 \) in Fig. 6, and \( c_L/c_R = 2 \) and \( c_C/c_R = 0.5 \) in Fig. 7). It is seen that the largest near-field amplitudes of \( \eta/\xi_0 \) occur for \( c_R/c_T \sim 1 \) and \( c_L/c_R \sim 1 \).

Fig. 8 (left) shows the leading tsunami peaks, propagating in the positive x-direction, at time \( t = t^* \) and for \( \partial/c_T = 0.7, 0.8 \) and 1.0. The numerical values of the normalized peak amplitudes, \( \eta_{\text{max}}/\xi_0 \), are also shown. The right half of Fig. 8 shows the peak values plotted versus \( L/h \), where \( h \) is the depth of ocean and \( L \) represents that part of \( L_R \) (Fig. 1) during which \( c_R \) is constant and equal to \( c \) (Figs. 3 and 4). The peak amplitudes corresponding to Model 1.B are plotted at \( L/h = 50/2 \), for \( c_C = 0, 0.1 c_T \) and 0.5 \( c_T \). It is seen that as \( L \rightarrow L_R \), \( \eta_{\text{max}}/\xi_0 \) approaches the estimates for \( L/h > 1 \) from Todorovska and Trifunac [7]. For Cases 1, 2 and 3, the peak values are plotted as open circles in three groups, corresponding to \( c_T/c_T \) (i.e. \( \partial/c_T \) = 0.7, 0.8 and 1.0). Adjacent to each point is the value of the constant velocity \( c \) (during time interval \( t_2 - t_1 \)) in terms of \( c_T \). It is seen that for each case \( \eta_{\text{max}}/\xi_0 \) increases as \( \partial/c_T \) of the peaks approach the estimates of \( \eta_{\text{max}}/\xi_0 \) for \( L/h > 1 \) in Model 1.B.

Figs. 9–12 show how \( \eta/\xi_0 \) evolves in time. Waveforms for selected \( c_L/c_R, c_C/c_R \) and \( c_R/c_T \) are shown in time windows \( 0.1 \leq t^*/T \leq 1 \) and \( 1 \leq t^*/T \leq 5 \). In all four figures, \( c_L/c_T = 1, i.e. \partial/c_T = 1 \).
$c_L/c_R = 1$, $c_C/c_R = 0$ and $tt^*$ is, respectively, in the intervals $0.1 \leq tt^* \leq 1$ and $1 \leq tt^* \leq 5$. In Figs. 11 and 12, $c_L/c_R = 0.3$, $c_C/c_R = 0.3$ and $tt^*$ is again in the intervals $0.1 \leq tt^* \leq 1$ and $1 \leq tt^* \leq 5$. It is seen that for most slides with downhill motion (which in most cases would correspond to the direction away from coastline), the slides that originate near the foot of the slide and then spread uphill will produce the largest run-up.

Figs. 13–15 show top views of the tsunami waveforms at time $t = t^*$, for Model 1.B and for the three cases of variable velocity model. Fig. 13 corresponds to Fig. 6 with $c_R/c_T = 1$, while Figs. 14 and 15 correspond to Fig. 7 for $c_R/c_T = 1$ and $c_R/c_T = 0.5$, respectively.

Fig. 16 shows $\eta_{R,max}/L_0$ (top) and $\eta_{L,min}/L_0$ (bottom) versus $c_T/c_R$ for rectangular slides with areas $100 \times 100$, $50 \times 50$ and $10 \times 10$ km$^2$, for $c_L/c_R = 0.3$, $c_C/c_R = 0.1$ and for the ocean depth $h = 2$ km. Here $\eta_{R,max}/L_0$ is the largest positive wave amplitude associated with the accumulation zone, and $\eta_{L,min}/L_0$ is the largest negative wave amplitude, associated with the depletion zone and propagating in negative $x$-direction. It is seen that for Model 1.B the peak of $\eta_{R,max}/L_0$ is near $c_T/c_R = 1$, and the amplitudes of $\eta_{R,max}/L_0$ resemble amplification in the vibration of a single-degree-of-freedom oscillator [7], with $h/WL$ acting as fraction of critical damping. For Cases 1, 2 and 3, the effective length of sliding $L_E$, during which the spreading velocity is constant and equal to $c$ (Fig. 3), is shorter than $L_R$ ($L_R = 12.96$, $15.52$ and $17.74$ km, respectively, for Cases 1, 2 and 3, when $L_R = 50$ km, and $L_E = 25.92$, $31.04$ and $35.48$ km, when $L_R = 100$ km). Because $c = 1.48\tilde{c}$, $1.38\tilde{c}$ and $1.29\tilde{c}$ for Cases 1, 2 and 3, respectively, and because the results shown in Fig. 16 are plotted versus $c_T/c_R$, i.e. $c_T/c\tilde{c}$, the peaks $\eta_{R,max}/L_0$ occur at $c_T/c_R = 1.50$, $1.37$ and $1.30$ for the $50 \times 50$ km$^2$ source, and at $c_T/c_R = 1.50$, $1.43$ and $1.32$ for the $100 \times 100$ km$^2$ source (Fig. 16 top) rather than at $c_T/c_R = 1$, as for Model 1.B.

Fig. 17 shows selected peak amplitudes of $\eta_{L}/L_0$ in the near-field versus $L/h$. These peak amplitudes were evaluated at $t = t^*$ and correspond to different $c_R/c_T$ ratios. The four sets of peak amplitudes correspond to Model 1.B and to Cases 1, 2 and 3. The $c_T/c_R$ ratios where the peaks occur (Fig. 16) are shown adjacent to each point. Open circles show that the $\eta_{L,min}/L_0$ peaks are larger than the trend for constant velocity of spreading and $W/L > 1$ [7], apparently because of the parts of increasing and decreasing ramps in $c = c(t)$ (Fig. 3), which tend to extend the ‘equivalent’ $L_E$ beyond the values computed only for the segment when $c(t) = c$, during time interval $t_2 - t_1$. All $\eta_{L,min}/L_0$ peaks are smaller than the corresponding $\eta_{R,max}/L_0$ for Model 1.B.

It can be seen from Figs. 5–17 that the differences in the waveforms between the results for Model 1.B and for Cases 1, 2 and 3 are small for $c_R/c_T > 2$. For $2 \leq c_R/c_T \leq 1$, and in particular for $c_R/c_T \sim 1$, peak amplitudes of $\eta_{L}/L_0$ are reduced significantly, because the time dependent $c_R$ results in shorter length $L_E$ of sliding with constant velocity and thus smaller build up of amplitudes by focusing.

4. Discussion and conclusions

We illustrated the effects of variable sliding velocity of submarine slides and slumps on the near-field amplitudes of the resulting tsunami. The simple nature of the assumed time dependence of these changes was motivated by the reported changes in Froude number, in numerical simulations of slump motions, in the studies of Jiang and Le Blond [3].

We found that the overall nature of near-field tsunami amplitudes, involving directivity, focusing and strong dependence of waveforms on the overall average speed of slumping and sliding remains unchanged, with respect to our previous studies, which approximated the motions of submarine slides by kinematic models with constant velocities of spreading [10,11]. The details of the wave motions, however, can change appreciably, when the spreading velocity $c_R$ varies with time, especially when the average spreading velocity $\bar{c}$ is near the long period tsunami velocity $c_T = \sqrt{gh}$.

For the accumulation and depletion zones of submarine slides and slumps spreading only downhill and away from the coast, essentially all the tsunami energy is directed away from the coast. Large wave amplitudes which propagate towards the coast occur mainly during retrogressive evolution of slides and slumps (that is when $c_L \neq 0$).

For velocities of spreading $c_R$ and $c_L$ such that $c_R/c_T > 2$ and $c_L/c_T > 2$, the effects of variable velocities of sliding are small for the model studied in this paper. These effects become significant for $c_R/c_T < 2$ and/or for $c_L/c_T < 2$.

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