Generation of tsunamis by a slowly spreading uplift of the sea floor

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Abstract

Generation of tsunamis by slow submarine processes (faulting, slumps or slides) is investigated, in search for possible amplification mechanisms resulting from lateral spreading of the sea floor uplift. A linearized solution for constant water depth is derived by transform methods (Laplace in time and Fourier in space), for sea floor uplift represented by a sliding Heaviside step function (i.e. a simplified Haskell source model, with zero rise time). The model is used to study the tsunami amplitude amplification (wave amplitude normalized by the final sea floor uplift) as a function of the model parameters. The results show that, above the source, the amplification is larger for larger uplifted area and for smaller water depth, and is the largest in the direction of uplift spreading, for velocity of spreading comparable to the long period tsunami velocity. Near the source, this amplification could be one order of magnitude. This amplification mechanism seen in the near-field is a form of wave focusing, and is manifested by a high frequency pulse, with amplitude attenuating with distance due to dispersion and geometric spreading. In the far-field, the linear theory predicts maximum amplification equal to one, as predicted by point source models. An analogy between this form of focusing and resonance of a single-degree-of-freedom oscillator, and near-field radiation patterns are discussed. The magnitudes, seismic movements and source durations of selected earthquakes which generated tsunamis are cited in search of conditions which could lead to slow rupture and unusually large near-field amplification. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Tsunamis are long period sea waves, caused by a sudden uplift or depression of the sea floor. Among the natural phenomena that can cause a tsunami are submarine faulting, submarine slides and slumps triggered by earthquakes, submarine volcanic explosions and atmospheric conditions [1]. Their generation and propagation can be described by the linearized theory of long period gravity waves. Their period is of the order of 1000 s, and in the spectrum of oceanic waves they fall between swells (10 s) and tides (40,000–80,000 s). They are dispersive, and at long periods have phase velocity ~ \( \sqrt{gh} \), where \( h \) is the ocean depth and \( g \) is the acceleration due to gravity. This implies velocities of propagation 100–250 m/s for \( h \) between 1 and 6 km, and wavelengths 10–1000 km. In this paper, we use \( v_T = \sqrt{gh} \) as a measure of the tsunami velocity.

The mechanics of tsunami generation by a submarine earthquake faulting has been studied in the framework of a heavy incompressible fluid layer [2,3]. The typical velocities of faulting (and therefore of lateral spreading of the ocean floor uplift) are about 3 km/s, an order of magnitude larger than \( v_T \). Therefore, most tsunami earthquake source studies have neglected the details of wave propagation in the fluid during the source time, and it has been assumed that the initial conditions for tsunami propagation can be approximated by specifying the initial disturbance on the surface of the ocean. This disturbance is then assumed to have same amplitudes as the vertical displacement of the ocean bottom, corresponding to the permanent change of elevation caused by the faulting, sliding or slumping.

The tsunami theory, that is based on a model of a flat incompressible ocean layer, is convenient for exploratory studies of near-field motions in the fluid. It facilitates analytical and numerical studies of the fluid wave motion above finite sources, with irregular fault shapes and with arbitrary distribution of the ocean floor uplift, for variable spreading velocities and for various properties of the fluid layer. A disadvantage of this approach is that it requires a priori knowledge of the ocean floor displacements. The resulting amplitudes of the near-field waves are nonlinear functions of the governing parameters, and so the inverse analysis of maregrams for estimation of the fault slip become difficult and time consuming [4]. Another theory, based on representation of the earthquake source by a distribution of double-couple sources and on expansion in the eigenfunctions of an ocean covered spheroidal earth, is convenient to describe...
tsunamis in the far-field [4–6]. An important advantage of this approach is that the far-field tsunami amplitudes depend linearly on the parameters describing the point sources. This enables real time analysis of teleseismic body and surface waves and helps with further refinements in the tsunami warning system [7]. The disadvantage of this approach is that it cannot recover the details of the near-field wave motions. The above two representations are equivalent within the sub-space where their physical characteristics overlap. For example, the long period part of the former can be obtained asymptotically from the latter [8].

For most tsunamigenic earthquakes, the average tsunami runup (corrected for geometric spreading) correlates well with the surface wave magnitude $M_s$ (Fig. 1, top, [9]), but for some it may be 1–2 orders of magnitude larger than the average trend (e.g. the 1946 Aleutian, Fig. 1, top). These have been named ‘tsunami earthquakes’, and it has been suggested that they are caused by a long faulting process, so that $M_s$ underestimates the true value of the seismic moment [10]. Very long source times may result from multiplicity of the earthquake source [11,12], slow dislocation velocities in the accretionary wedge [9,13,14], submarine landslides triggered by an earthquake (1922 Atacama [1]; 1929 Grand Banks [15]), slumps (Hawaiian chain [16]; Loma-Prieta [17]; Hading Bay and Rangikoko [18]; Storegga [19,20]; Kalapana [21]), repeating submarine volcanic explosions (1883 Krakatau [22]) and changes in the atmospheric pressure [23].

Abe [24] studied 65 large tsunamigenic earthquakes which occurred between 1837 and 1974. About 10% of those have generated abnormally large runup relative to their surface wave magnitude, e.g. June 15, 1896, Sanriku; April 13, 1923, Kamchatka; April 1, 1946, Unimak Island, Alaska; November 20, 1960, Peru; October 20, 1963, Kurile Islands; and August 1, 1968, Philippines). The interpretations which have been proposed to explain these events require (1) scaling in terms of seismic moment $M_0$ or moment magnitude $M_W$ in place of $M_s (M_W = (\log_{10} M_0 – 16.1)/1.5$, for $M_0$ in dyne cm), and (2) very long source processes (i.e. very low dislocation velocities, resulting in small spreading velocities of the sea floor uplift). In spite of such ‘corrections’, the runup amplitudes for the 1946 Aleutian tsunami and several other larger tsunami events remain high. This is illustrated in Fig. 1 [9] showing tsunami runup at Hawaii versus $M_s$ (top) and $M_W$ (bottom), and tsunami-genic earthquakes between 1943 and 1987 (the runup has been equalized to a distance $\Delta = 90^\circ$). The solid circles show four tsunami earthquakes: 1946 Aleutian, 1960 Peru, 1963 Kurile and 1975 Kurile earthquakes. It is seen that these four have anomalously large runup relative to $M_s$. The solid line shows a fit to the data assuming that the runup is proportional to seismic moment. The dashed line follows the empirical relation of Abe [24] (redrawn from Pelayo and Wiens [9]). More recent examples of tsunami earthquakes are: Tonga, December 19, 1982; Nicaragua, September 2, 1992 [25]; and Java, June 2, 1994.

Searching for explanations for abnormally large tsunami amplitudes, we explore here amplification resulting from source directivity and wave focusing in the near-field. Using a simple kinematic source model (simplified Haskell model with zero rise time), we show that amplification of one order of magnitude may occur when the sea floor uplift spreads with velocity similar to the long wave tsunami velocity (i.e. during ‘slow’ earthquakes, slumps or slides). This amplification occurs above the source, progressively as the source evolves, by stacking the uplifted fluid over the fluid previously displaced by the uplift of the preceding source segments. The effect is larger for a wider source (the water has less opportunity to flow sideways), and is largest for an infinitely wide source. We suggest that this amplification may explain, in some cases, exceptionally large runup amplitudes in the near-field. This phenomenon can occur during ‘slow’ processes including earthquakes, sliding and slumping. Under favorable conditions in the near-field, it may lead to formation of a solitary wave.

It is difficult to estimate, at present, how often this type of

![Fig. 1. Tsunami runup at Hawaii, equalized to a distance $\Delta$ of 90°, versus $M_s$ (top) and $M_W$ (bottom), for tsunamigenic earthquakes between 1943 and 1987. The tsunami earthquakes (solid circles) have anomalously large runup relative to $M_s$. The solid line is a fit to the data assuming that the runup is proportional to seismic moment. The dashed line follows the empirical relation of Abe [24] (redrawn from Pelayo and Wiens [9]).](image-url)
amplification may occur during actual faulting, sliding or slumping, because of the lack of detailed knowledge about the ground deformations in the source area of past tsunamis. Therefore, we present here only the basic ideas and illustrate the possible range of amplification factors by means of the most elementary models of the source region. Our ‘source’ is rectangular, with uniform final elevation, and the velocity of lateral spreading of the ocean floor uplift is constant. The elevation of the model may be thought of as approximating roughly the average of the sea floor elevation associated with linear and nonlinear movements of the accretionary prism during seismic and aseismic slip. This prism is forced to move by the failure of asperities along the deeper and more brittle segments of the inter-plate margin [13] or by submarine slides and slumps in the same region, during and following earthquakes [26].

In this paper, we first review the source duration of selected past submarine earthquakes, slumps and slides that generated tsunamis, to provide a physical justification for the small spreading velocities considered in this study. Then, we present results for selected model parameters, and a discussion of the mechanism of the amplification, radiation pattern in the near-field, and the relation to the well known far-field representation of the effects of source finiteness and directivity. In our numerical simulations, we consider only a time window of the order of the duration of faulting.

2. Duration of tsunami sources

The seismological source mechanism studies of tsunami-genic earthquakes are based on telesismic analyses of body and surface waves [4–6,8,10,27,28], which capture the long period ground motions (e.g. T > 10 s). For these and longer periods (long wavelengths), most earthquakes can be approximated by a point source. At the other end of the spectrum, high frequency recordings of strong motion in the near-field (0.1–25 Hz) can resolve fine time and space variations in the source time function and can be used to interpret the motions on the fault surface (e.g. the 1971 San Fernando earthquake [29]). At high frequencies, a long lasting rupture of a large earthquake often appears as a sequence of multiple events. For example, the 1940 Imperial Valley, California, earthquake was assigned $M_w = 7.1$ [30], but from strong motion records it could be interpreted as a sequence of four events, with the largest having local magnitude $M_s = 6.1$ [31]. The 1992 Landers, California, earthquake ($M_w = 7.3$), with total seismic moment $\sim 2.5 \times 10^{22}$ dyne cm, can be represented by three planar fault segments (Johnson Valley, Homestead Valley and Emerson Faults), with the central (second) segment contributing half of the total moment [32]. For the short period, near-field estimates of shaking near the Homestead fault, this motion would appear as an $M_s = 6.9$ earthquake.

The Nicaraguan tsunami earthquake of September 2, 1992 is a recent example of a multiple event sequence. This earthquake had $M_s = 7.2$, $M_w = 7.6$, $M_l = 7.6$ and total moment $\sim 2.5 \times 10^{22}$ dyne cm. According to Ide et al. [33] the rupture (strike = 302°, dip = 16°, slip = 88°) occurred bilaterally, consisted of three events and lasted about 100 s. The first event was 20 km southeast of the epicenter and occurred 11 s after origin time. The second event was located 80 km northwest and took place 36 s after origin time. The third event started 79 s after origin time and was 120 km southeast of the epicenter. From the focus, the rupture propagated with velocity $v = 2.2$ km/s towards northwest and with velocity $v = 1.5–1.8$ km/s to southeast. The overall effect had directivity towards southeast.

Imamura et al. [12] modeled the above source by a fault 200 km long and 100 km wide, with average dislocation $\bar{u} = 0.4$ m, and found that the computed tsunami amplitudes along the Nicaraguan coast are too small, by a factor between five and 10. Their computed runups magnified by a factor of 10 were larger than those observed in the northwest and smaller than those observed along the southeast coastline. Increasing the fault length to 250 km and decreasing the fault width to 40 km and $\mu$ to $1 \times 10^{10}$ N/m² gives $\bar{u} = 3$ m and maintains the seismic moment consistent with the seismic observations [25,34]. This also leads to correct overall runup amplitudes along the Nicaraguan coast, and is consistent with the interpretation of Ide et al. [33] that the overall average rupture velocity along the fault length, towards southeast, is $\nu = 1.5–1.8$ km/s. For ocean depth $h = 1–4$ km, $v_f = 0.1–0.2$ km/s and $\nu/v_f = 7.5–22$. As will be seen from the following, this $\nu/v_f$ ratio is too large to cause heaping (stacking) type of amplification, but could help reconcile the ratio of observed to computed runup amplitudes (<1 towards northwest and >1 towards southeast) as noted by Imamura et al. [12]. Consideration of a propagating rupture in Satake’s model [25,34] would have facilitated the use of a shorter fault length, also in better agreement with the observed distribution of aftershocks. It would have also eliminated the need to consider clockwise rotation of the longitudinal fault axis (strikes = 312° and 322°) in the work of Imamura et al. [12].

The generation of tsunamis by vertical displacements of the ocean floor depends on the characteristic size (length L and width W) of the displaced area and on the time, $\tau$, it takes to spread the motion over the entire source region. The ratio $L/\tau$ then defines the average spreading (and fault rupture) velocity, $\nu$ (assuming unilateral spreading of the fault slip along length, $L$). During ‘ordinary’ earthquakes ($1 < \nu < 10$ km/s), the dislocation propagates with velocities approaching the shear-wave velocity in the medium, and reductions in the overall average value of $\nu$ can result from multiplicity of the source and delays associated with breaking of barriers. The ‘slow’ earthquakes ($0.1 < \nu < 1$ km/s) may consist of one or several high velocity rupture events (thus producing the usual train of high frequency waves), with long delays between the successive events, accompanied by slip which can contribute large amplitude low-frequency excitation [35]. Examples of
Fig. 2. Earthquake moment $M_o$ and magnitude ($M_o$, $M_W$, and $M_L$) versus different measures of source duration. The examples of ‘duration’ shown are: (1) duration of T-waves ($\hat{s}$); (2) characteristic source duration, $\tau_c$ (○); (3) duration of three tsunami and one tsunamiogenic earthquakes (+); (4) average source duration, $\tau$ (dashed line); (5) rupture time of the dominant asperities, $T_{sd}$ (open large circles); (6) duration of faulting, $T_f$ (□); (7) duration of four strong motion earthquakes in Southern California (○); (8) the average trend of high frequency duration of strong motion (solid line); (9) approximate duration of the 1896 Sanriku, 1946 Aleutian, 1992 Nicaragua, and 1994 Java events (solid large circles); (10) the overall duration of two complex events: 1929 Grand Banks and 1960 Chile (horizontal bars). The shaded zones represent approximate boundaries between silent, slow and ordinary earthquakes. The solid line (bottom) boundaries of the shaded zones imply unilateral faulting, while top (left) boundaries imply symmetric bilateral faulting.

such ‘slow’ earthquakes are: (1) June 6, 1960, Chile earthquake which ruptured as a series of earthquakes for about an hour [11]; (2) February 21, 1978, Banda Sea earthquake [36]; and possibly (3) 1946 Aleutian, Unimak Island, earthquake (Beroza and Jordan [37], speculate that $v \sim 200$ m/s) which generated one of the largest tsunami runup amplitudes for a $M_o = 7.4$ event [38].

Fig. 2 shows the seismic moment ($M_o$) and magnitude ($M_o$, $M_W$, and $M_L$) versus selected examples of different measures of the earthquake source duration. The open small circles represent the characteristic source time $\tau_c$ (source length divided by rupture velocity) for 11 slow earthquakes studied by Beroza and Jordan [37]. The crosses represent the durations of four events (three ‘tsunami’ and one ordinary) studied by Pelayo and Wiens [9]. The solid small circles represent 35 measured durations of T-waves, described by Okal and Talandier [39]. The solid large circles show the approximate duration of the 1896 Sanriku and 1946 Aleutian Islands earthquakes [10]. The bars show the range of possible overall durations of the 1929 Grand Banks earthquake, followed by a submarine slide, and the lower range of duration estimates of the anomalously long multiple shock sequence of the $M_o = 6.75$, June 6, 1960, event in Chile [11]. For relative comparison, the figure also shows the average trend of duration of high frequency radiation of strong ground motion in California (the short solid line; [40]), the durations of strong motions for four Southern California earthquakes (the diamonds; note the small magnitude, $M_L = 6.1$, and long duration of the source of the 1940 Imperial Valley event [41]), the average empirical relationship between $M_W$ and the source process time $\tau$ (the dashed line; [42]), the rupture time, $T_A$, of the dominant asperities (the open large circles), and the duration of faulting, $T_f$ (the squares [43]). Finally, the shaded zones represent approximate boundaries between ‘silent’ ($0.01 < v < 0.1$ km/s), ‘slow’ and ‘ordinary’ earthquakes, assuming $M = 3.94 + 1.94L$ for $v = 0.1, 1$ and 10 km/s [44]. The solid lines are for unilateral faulting, while top (left) ends of the grey zones are for symmetric bilateral faulting.

3. Model

The greatest hazard of tsunami runup is posed by shallow earthquakes in subduction zone areas and by submarine slumps and slides 100–150 km offshore, along the strip between the island arcs and the deep sea trenches. The velocity of tsunami propagation there depends on the water depth, while the configuration and the slope of the continental shelf affect the focusing and scattering along the propagation path. In mathematical modeling of tsunami sources, selecting representative fault mechanisms and evolution of slumps and slides is difficult because of the variety and complexity of the geometries of the frontal arcs and the fore arc basins, the small trench slope, the small rigidity of the sedimentary rocks in the accretionary prism and the variety of possible rupture and slide geometries and locations [13,14,26,45,46]. Two of the great tsunami earthquakes, the 1896 Sanriku and the 1946 Aleutian (Unimak Island), both occurred in regions with large normal fault earthquakes. Therefore, a simple linear representation of the ocean floor uplift, which can be computed for assumed dislocation amplitudes on a dipping thrust fault surface may be only one representation of a complex environment. For more accurate near-field numerical modeling, there is lack of information on the actual geometries and on the material properties of the medium. Therefore, in this paper, we consider only one simple aspect of the general problem by analyzing uniform uplift (depression) in its most elementary form. Our aim is to understand and describe first the basic aspects of the tsunami generation by a ‘slowly’ spreading uplift. We will address the effects of variations of the uplift in time and space and propagation of tsunamis away from the source in future work.
3.1. Equation of motion and boundary conditions

We consider the motion of a fluid domain, $D$, bounded by the rigid ocean floor, at $z = -h$, and the free surface at $z = 0$ (a side view is shown in Fig. 3), and excited by a sudden uplift of the bottom surface, $\zeta(x, y; t)$. Of particular interest is the resulting uplift of the free surface, $\eta(x, y; t)$. We assume irrotational fluid flow (i.e. $\nabla^2 \phi = 0$, $\vec{q}$-fluid velocity), inviscid fluid and negligible surface energy effects. The former implies existence of velocity potential $\phi(x, y, z; t)$ which fully describes the physical process. Finally it is assumed that tsunami generation and propagation are not strongly influenced by the elasticity of the solid earth, the compressibility of ocean, or the sphericity of the earth [28].

The potential $\phi(x, y, z; t)$ must satisfy the Laplace’s equation

$$\nabla^2 \phi = 0$$

(1)

and the following boundary conditions (kinematic and dynamic)

$$\phi_z = \eta_t + \phi_x \eta_x + \phi_y \eta_y \quad \text{on} \quad z = \eta(x, y; t)$$

(2a)

$$\phi_z = \zeta_t + \phi_x \zeta_x + \phi_y \zeta_y \quad \text{on} \quad z = -h + \zeta(x, y; t)$$

(2b)

and

$$\phi_t + \frac{1}{2} (\nabla \phi)^2 + g \eta = 0 \quad \text{on} \quad z = \eta(x, y; t)$$

(3)

where subscripts $t$, $x$ and $y$ indicate partial derivatives with respect to these variables.

3.2. Shallow water theory solution

A first order (linearized) solution of the otherwise difficult problem [47–49] is obtained if the nonlinear terms in the boundary conditions ($\phi_t \eta_s$, $\phi_x \zeta_x$, $\phi_y \eta_y$, $\phi_x \zeta_y$), and $(\Delta \phi)^2$ are neglected, and if the boundary conditions are applied on the undeformed instead of the deformed boundary surfaces ($z = -h$ and on $z = 0$ instead of $z = \zeta(x, y; t)$ and $z = \eta(x, y; t)$). This approximation is reasonable if the depth of the water, $h$, is much greater than the amplitudes of $\zeta$ and $\eta$, which is usually true for most tsunamis triggered by submarine earthquakes, slumps and slides [49]. This, and combining conditions (2a) and (3) in one equation yields

$$\phi_t(x, y, -h; t) = \zeta(x, y; t)$$

(4)

and

$$\phi_t(x, y, 0; t) + g \phi_x(x, y, 0; t) = 0.$$  

(5)

A solution of Eq. (1) satisfying boundary conditions (4) and (5) can be obtained by transform methods (Laplace in $t$ and Fourier in $x$ and $y$). The transforms of these equations are

$$\tilde{\phi}_{zz}(\tilde{k}, z; s) - k^2 \tilde{\phi}(\tilde{k}, z; s) = 0$$

(6)

$$\tilde{\phi}_s(\tilde{k}, -h; s) = s \tilde{\xi}(\tilde{k}; s)$$

(7)

Fig. 3. A model of tsunami wave generation. The fluid domain $D$ is bounded by the free surface on the top (initially at $z = 0$) and the ocean floor on the bottom (initially at $z = -h$). The displacement of the bottom surface is $\zeta(x, y; t)$ and of the free surface is $\eta(x, y; t)$.

and

$$\tilde{\phi}_z(\tilde{k}, 0; s) + (s^2/g) \tilde{\phi}(\tilde{k}, 0; s) = 0$$

(8)

where the bar indicates the transform

$$\tilde{f}(\tilde{k}; s) = \int_{-\infty}^{\infty} e^{ik \tilde{x}} \int_{-\infty}^{\infty} e^{ik \tilde{y}} f(x, y; t) \, dx \, dy$$

(9a)

$\tilde{k}$ is a short notation for the pair $(k_1, k_2)$ and

$$k = \sqrt{k_1^2 + k_2^2}.$$  

(9b)

A general solution of Eq. (6) is

$$\tilde{\phi}(\tilde{k}, z; s) = A(\tilde{k}; s) \cosh k z + B(\tilde{k}; s) \sinh k z.$$  

(10)

Functions $A(\tilde{k}; s)$ and $B(\tilde{k}; s)$ are evaluated from the boundary conditions, Eqs. (7) and (8), yielding

$$\tilde{\eta}(\tilde{k}, z; s) = \frac{-gs \tilde{\zeta}(\tilde{k}; s)}{(s^2 + \omega^2) \cosh k h} \left( \cosh k z - \frac{s^2}{gk} \sinh k z \right)$$

(11)

where

$$\omega^2 = g k \tanh h.$$  

(12)

The uplift of the free surface, $\tilde{\eta}(\tilde{k}; s)$, is related to $\tilde{\phi}(\tilde{k}, z; s)$ via Eq. (3), which implies

$$\tilde{\eta}(\tilde{k}; s) = -(s/g) \tilde{\phi}(\tilde{k}, 0; s).$$

(13)

Eqs. (11) and (13) finally give $\tilde{\eta}(\tilde{k}; s)$ directly in terms $\tilde{\xi}(\tilde{k}; s)$

$$\tilde{\eta}(\tilde{k}; s) = \frac{s^2 \tilde{\xi}(\tilde{k}; s)}{s^2 + \omega^2 \cosh k h}.$$  

(14)

One can evaluate $\eta(x, y; t)$ for specified $\zeta(x, y; t)$ by computing its transform $\tilde{\zeta}(\tilde{k}; s)$, substituting it into Eq. (14) and inverting $\tilde{\eta}(\tilde{k}; s)$ to obtain $\eta(x, y; t)$. For simple functions $\tilde{\zeta}(x, y; t)$, $\tilde{\xi}(\tilde{k}; s)$ and the inverse Laplace transform of $\tilde{\eta}(\tilde{k}; s)$ can be evaluated analytically or by using tables. This gives $\tilde{\eta}(\tilde{k}; t) = \mathcal{L}^{-1}[\tilde{\eta}(\tilde{k}; s)]$, which is further converted to $\eta(x, y; t)$ by double inverse Fourier transform.

Eq. (12) describes the dispersion relationship for the tsunamis and implies phase velocity, $c = \omega/k$, and group
velocity, \(U = da/d\ell\), equal to
\[
c = \sqrt{\frac{g}{k}} \tanh kh
\]  
(15)

and
\[
U = \frac{1}{2}c \left[ 1 + \frac{2kh}{\sinh 2kh} \right].
\]  
(16)

As \(kh \rightarrow 0\), both \(c \rightarrow \sqrt{gh}\) and \(U \rightarrow \sqrt{gh}\). Therefore, \(v_T = \sqrt{gh}\) is the tsunami velocity for wavelengths long compared to the depth of water (e.g. \(\lambda = 2\pi/k > 100\) km for \(h = 5\) km). The shorter wavelength (higher frequency) components propagate with progressively smaller velocities.

The above ‘linearized’ solution is known as the ‘shallow water solution’. It does not account for time lags due to propagation of the disturbance vertically, and is a good approximation if the depth of the water, \(h\), is much smaller than the wavelength of the waves. The tsunamis have wavelengths of the order of 10–400 km, and the depth of the water where they are initiated ranges from 0.5 to 6 km, so that the ‘shallow water theory’ conditions are approximately satisfied. However, some of the examples presented in this paper will involve cases where these conditions begin to be violated, and consequently our results will represent only the corresponding linear approximation.

Keeping the term \((\nabla \phi)^2/2\) in Eq. (3) and the second order terms \(\phi_x \eta_y, \phi_y \eta_x, \phi_x \phi_y, \phi_y \phi_x\) in Eqs. (2a) and (2b) would permit the solution of the form known as a solitary wave [50]. This wave has a symmetric form, a single hump and propagates with uniform velocity over considerable distances without changing form. In this paper, we are not analyzing the propagation of solitary waves, only the conditions above the tsunami source which may lead to their formation.

3.3. Unilaterally propagating uplift of the ocean floor

We consider a constant uplift of the ocean floor, \(\zeta_0\), along the surface \(S\) defined by \(x \in [0, L] \times y \in [-W/2, W/2]\), which is realized during finite time interval \(t^*\), starting at \(x = 0\) and ending at \(x = L\), and occurring at \(x > 0\) at progressively later time. The disturbance propagates in the positive \(x\)-direction with finite velocity \(v\). In the \(y\)-direction, it propagates instantaneously. The source duration, \(t^*\), \(L\) and \(v\) are related by
\[
t^* = \frac{L}{v}.
\]  
(17)

To isolate the effects of the uplift propagation, we first assume that the final offset is constant throughout the uplifted domain, and once the disturbance reaches some \(x \leq L\), the uplift occurs at that \(x\) instantaneously. One simple faulting model for which the uplift of the sea floor could be approximated by such \(\zeta(x, y; t)\) in the vicinity of the trench axis, would be for a shallow dipping fault below the accretionary prism (with surface projection \(S = WL\); e.g. 40 × 250 km for the 1992 Nicaragua event), with rupture initiating along one edge (parallel to the \(y\)-axis; oriented up or down the dip, parallel to \(W\) and propagating horizontally towards the opposite edge of the fault with velocity \(v_T\) (along \(L\), parallel to the \(x\)-axis, in the direction of the fault strike). Here, for simplicity, we neglect the negative part of \(\zeta_0\) above and beyond the deep edge of the fault surface (it is easy to model this negative zone, for any \(\zeta(x, y; t)\) may also approximate the consequences of a submarine landslide or a slump spreading along a gently sloping ocean floor [15,17,18]. Then \(v\) is the apparent ‘rupture’ velocity in the \(x\)-direction, the source duration is \(t^*\), and the rise time is approximately equal to zero. It is also easy to incorporate different rise time functions (e.g. see Hammack [49]), but here, for simplicity, we assume instantaneous rise time. An analytical representation of such \(\zeta(x, y; t)\) is

\[
\zeta(x, y; t) = \zeta_0 H(t - x/v), \quad (x, y) \in S
\]  
(18)

where

\[
H(t - a) = \begin{cases} 
1, & t \geq a \\
0, & t < a 
\end{cases}
\]  
(19)

is the Heaviside unit step function. Fig. 4 shows (a) a plan view of the uplifted area, (b) a side view of the uplifted region and of the water domain, and (c) \(\zeta\) versus \(x\) and \(t\). Points 1–9 are at equal distance, \(R\), but at different azimuths, \(\theta\), from the origin (those will be referred to in the next section).

The Laplace–Fourier transform of \(\zeta(x, y; t)\) can be evaluated in the following order (the limits of integration are apparent from Fig. 4c)

\[
\tilde{\zeta}(\vec{k}; s) = \int_{-W/2}^{W/2} e^{ikx} dx \left[ \int_{0}^{t^*} e^{is\zeta_0 v \delta r} dr \right]
\]  
(20)

yielding

\[
\tilde{\zeta}(\vec{k}; s) = \zeta_0 \frac{2 \sin k_s W/2}{k_s^2} \frac{1}{s} \left[ 1 - e^{-is - ik_s v \delta r} \right]
\]  
(21)

Eqs. (14) and (20) then give

\[
\tilde{\eta}(\vec{k}; s) = \zeta_0 \left( \frac{2 \sin k_s W/2}{k_s^2} \right) \left( \frac{s}{s^2 + \omega^2} \right) \left[ 1 - e^{-is - ik_s v \delta r} \right].
\]  
(22)

The inverse Laplace transform of \(\tilde{\eta}(\vec{k}; s), \tilde{\eta}(\vec{k}; t)\), can be evaluated analytically as follows.

We recall that

\[
\frac{s}{\omega^2 + s^2} \Leftrightarrow \cos(\omega t)
\]  
(23)
Though continuous at \( t = \tau \), \( \eta(x, y; t) \) is evaluated as the double inverse Fourier transform of \( \tilde{\eta}(\tilde{k}; t) \)

\[
\eta(x, y; t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iky} \tilde{\eta}(\tilde{k}; t) dk \right] dk.
\]

This inversion is done efficiently by the Fast Fourier transform [51].

Inspection of Eqs. (24) and (27) indicates that \( \tilde{\eta}(\tilde{k}; t) \) is singular at \( k_2 = 0 \), \( k = 0 \) (i.e. \( k_1 = 0 \) and \( k_2 = 0 \)), and possibly at some \( k \neq 0 \) such that

\[
\omega^2 = (k_1v)^2.
\]

All of these are removable singularities, and the finite limits are as follows.

1. As \( k_2 \to 0 \), the singular term of \( \tilde{\eta}(\tilde{k}; t) \) has the limit

\[
\lim_{k_2 \to 0} \frac{2\sin k_2 W/2}{k_2} = W
\]

2. \( k \to 0 \), implies \( k_1 \to 0 \), \( k_2 \to 0 \) and \( \omega \to 0 \). Then, in addition to the term in item 1 above, \( I \) is also singular, but \( \tilde{\eta}(\tilde{k}; t) \) has limit

\[
\lim_{k \to 0} \tilde{\eta}(\tilde{k}; t) = \begin{cases} \frac{\xi_0 Wvt}{\tilde{k}^2} & t \leq t' \\ \frac{\xi_0 Wvt}{\tilde{k}^2} & t > t' \end{cases}
\]

This implies that the long wavelength limit of \( \eta \) does not depend on how the source is realized but only on the total effect, i.e. the amount of water that it has uplifted.

3. Condition \( \omega^2 = (k_1v)^2 \), leading to singularity of \( I \), can be rewritten as

\[
\tanh k h = \left( \frac{v}{\nu \gamma} \right)^2 \left[ 1 - \left( \frac{k_2}{k} \right)^2 \right] (kh).
\]

To evaluate the integrals in Eq. (26), we use

\[
\int e^{\alpha \tau} \cos \beta \tau d\tau = \frac{e^{\alpha \tau} (\alpha \cos \beta \tau + \beta \sin \beta \tau)}{\alpha^2 + \beta^2}
\]

with \( \beta = \omega \) and \( \alpha = ik_1v \). Finally, \( \eta(x, y; t) \) is evaluated as the double inverse Fourier transform of \( \tilde{\eta}(\tilde{k}; t) \)

\[
\eta(x, y; t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iky} \tilde{\eta}(\tilde{k}; t) dk \right] dk.
\]
velocities \( v/v \) just when the uplift process has been completed. The ratio of \( v/v \) of the ratio of displacement waveforms along the spreading velocity of the ocean floor uplift by comparing focusing are normalized by \( z \).

Results and discussion

seen that, for an uplift which is spreading rapidly, the displacements \( h \) £ 50 km, and the ratio of the velocity of the uplift and the tsunami moment when the spreading of the uplift of the ocean floor stops (i.e. at \( t = t^* = L/v \)). We first examine the significance of the finiteness of the 50 km² area of the ocean floor that is lifted, depth of \( h = 2 \) km, and the ratio of the velocity of the uplift and the tsunami wave is such that \( v/v_T = 0.5, 0.6, \ldots, 10, 20, 50. \)

where

\[
k_{1.0} = \pm \sqrt{k_0^2 - k_2^2} \tag{34}
\]

4. Results and discussion

4.1. Effects of \( v/v_T \) on waveforms—amplification due to focusing

We first examine the significance of the finiteness of the spreading velocity of the ocean floor uplift by comparing displacement waveforms along the \( x \)-axis for various values of the ratio \( v/v_T \). Fig. 5 shows such waveforms for a 50 x 50 km² area of the ocean floor that is lifted, depth of water \( h = 2 \) km \((v_T = 140 \text{ m/s})\) and at time \( t = t^* = L/v \), i.e. just when the uplift process has been completed. The ratio of velocities \( v/v_T = 0.5–50 \) (bottom to top). The displacements are normalized by \( \zeta_0 \) (the uniform ocean floor uplift). It is seen that, for an uplift which is spreading rapidly, the displacement of the free surface resembles the displacement of the ocean floor, and \( \eta/\zeta_0 \approx 1 \), as is commonly assumed in numerical simulations of tsunamis (e.g. in forward finite difference modeling). It is seen from this figure that for \( v/v_T > 20 \), i.e. \( v > 2.8 \text{ km/s} \) in \( h = 2 \) km deep waters, the common approximation is valid. A typical value of \( v \) for tsunamigenic earthquakes may be \( v \sim 1.5 \text{ km/s} \) [9], and this gives \( v/v_T = 10 \) for \( h = 2 \) km, or \( v/v_T = 15 \) for \( h = 1 \) km. As \( v/v_T \) decreases from 50 to one, the wavelength above the source and for \( x > L \) has progressively larger amplitudes (reaching about 3.5 in this example), and higher frequency content. When \( v/v_T < 1 \), the amplitudes decrease, but the higher frequency content continues to increase.

A physical explanation for this effect is that when the tsunami created by the previously uplifted source segments travels with velocity same (or similar) as the uplift, it interferes constructively with the newly uplifted water, and as the process evolves, the tsunami amplitude progressively increases. When the tsunami is faster than the uplift, the initial wave ‘escapes’ ahead of the currently uplifted water, and amplification does not occur. For \( v/v_T = 0.5 \), for example, the wave has smaller amplitude than the uplift of the bottom.

4.2. Evolution in time

Next, we illustrate the nature of the tsunami build up and propagation by observing the wave at selected instants of time, during and after the uplift process (Figs. 6 and 7). The final uplifted area of the ocean floor is \( 50 \times 50 \) km², the water depth is \( h = 2 \) km \((v_T = 140 \text{ m/s})\), and we choose the velocity of spreading of the uplift to be equal to \( v_T \). We express the time in terms of the duration of the uplift process, \( t^* = L/v \). Fig. 6a shows the normalized waveforms, \( \eta/\zeta_0 \), along the axis of the symmetry, \( y = 0 \), at times \( t = 0.1t^*, 0.2t^*, \ldots, t^* \). It is seen how the amplitude of the wave builds up as progressively more water is lifted below the leading wave. In this example, the peak of \( \eta/\zeta_0 \) is about 3.5 (at \( x = L \)) as the wave leaves the source region. For \( x < 0 \), \( \eta/\zeta_0 \) is small (<0.5). Fig. 6b shows \( \eta(x, 0; t) \), again for \( t = 0.2, 0.4, 0.6, 0.8 \) and 1.0, but now plotted in the moving coordinate system, as it would appear to a boat crew traveling in the positive \( x \) direction with velocity \( v = v_T \). The boat is progressively lifted by the heaping (stacking) amplification mechanism (below it), the water column continuously receives the new volume of water equal to \( \zeta_0 \delta W/\delta t \). Fig. 7 follows the evolution of \( \eta/\zeta_0 \) for times between \( t = t^* \) and \( t = 4t^* \). It is seen that the maximum wave amplitude decreases with time, due to the geometric spreading and also due to the dispersion. At \( t = 4t^* \), the wave front is at about \( x = 200 \) km, and \( \eta/\zeta_0 \) drops to about 1.25.

Fig. 8 shows the normalized wave amplitude \( \eta/\zeta_0 \) for the same ocean floor uplift as in Figs. 6 and 7, at time \( t = t^* = L/v \), in the region \(-100 < x < 100 \) km and \( 0 \leq y \leq 75 \) km (the wave forms for \( y < 0 \) are mirror images of those for
4.3. Analogy between wave focusing and resonance of a single-degree-of-freedom oscillator

It was seen from Figs. 5–7 that the amplification $\eta/\zeta_0$ is the largest when the velocity of the uplift is close to the velocity of the water waves (i.e. $v = v_T = \sqrt{gh}$). This amplification also depends on the length, $L$, over which the uplift has propagated (see Fig. 6), on the depth of water, $h$, and on the width of the uplift, $W$ (it is proportional to $L$ and $W$ and inversely proportional to $h$). It may also be expressed as a function of the ratios $L/h$ and $W/L$. For example, it is larger when $W/L$ is larger, and is the largest when $W/L \rightarrow \infty$. It is also larger when $W/h$ is larger. For small $W$ and large $h$, it is easier for the elevated water to flow $y > 0$ because of the source symmetry). The strong directivity is evident.

Fig. 6. (a). Normalized displacement of the free surface $\eta(x,0,t)/\zeta_0$ at selected times $t = t'$, where $t' = L/v$ is the time when the uplift of the ocean floor is completed. The model is characterized by $h = 2$ km (implying $v_T = 140$ m/s), $W = L = 50$ km and $v/v_T = 1$. It is seen how the amplitude of the tsunami wave builds up as progressively more water is lifted by the ocean floor. (b). Normalized amplitudes of free surface displacement $\eta(x,0,t)/\zeta_0$, for $t'/L = 0.2, 0.4, 0.6, 0.8$ and $1.0$, for $h = 2$ km, $W = L = 50$ km $v/v_T = 1$, plotted in the moving $x$ coordinate system, so that $x = 0$ is above the tip of the uplift at $t'/L$.

Fig. 7. Same as Fig. 6, but for times $t$ between $t'$ and $4t'$. The dispersion of the large pulse leaving the source region ($x = 50$ km), propagating in the positive $x$-direction is evident.
sideways (in the $y$-direction), reducing the efficiency of this amplification mechanism.

We next analyze the normalized peak amplitude $A_{\text{max}}/\zeta_0$ (where $A_{\text{max}}$ is the peak of $\eta$) as a function of the above mentioned variables, assuming that $\eta$ is the largest when $v/\nu_1 = 1$ and when $t = t' = L/v$. All of our numerical estimates indicate that this is a good approximation for the purposes of this analysis. We note however that the train of waves contributing to and following the largest amplitude pulse is dispersed and consists of progressively higher frequencies (shorter wavelengths), which propagate with velocities smaller than $\sqrt{gh}$. We calculated $A_{\text{max}}/\zeta_0$ for $h = 2$ km and for various values of $L$ and $W$. Table 1 shows $A_{\text{max}}/\zeta_0$ for various values of the ratios $W/L$ and $L/h$, and Fig. 9 shows $A_{\text{max}}/\zeta_0$ plotted versus $L/h$ for $W/L = 0.25$, $W/L = 0.5$ and $W/L > 1$. It is seen that for $L/h$ between 0 and 500, $A_{\text{max}}/\zeta_0$ is between 0 and 27.

The sharp peak(s) of $\eta$ (Figs. 8 and 9) will have significant amplitude in the source region and at ‘small’ and ‘intermediate’ distances from the source. At large distances, they will be ‘smoothed’ and reduced by dispersion, and the

Table 1

<table>
<thead>
<tr>
<th>$L/h$</th>
<th>$W/L$</th>
<th>0.25</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
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<td>–</td>
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<td>1.19</td>
<td>1.21</td>
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<tr>
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<td>1.33</td>
<td>1.78</td>
<td>1.95</td>
<td>1.96</td>
<td></td>
</tr>
<tr>
<td>25</td>
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<td>2.96</td>
<td>3.58</td>
<td>3.67</td>
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<tr>
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<td>5.85</td>
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<tr>
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<td>9.37</td>
<td>9.37</td>
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<tr>
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<td>17.30</td>
<td>17.30</td>
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</tr>
<tr>
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<td>27.50</td>
<td>27.55</td>
<td>27.55</td>
<td>27.55</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. A map view of the displacements of the free surface $\eta(x, y, t')$, in the region $y \geq 0$ and $-100 < x < 100$ km, at time then the uplift spreading has just stopped, for $W = L = 50$ km, $h = 2$ km, $v_1/\nu_1 = 1$ and $\zeta_0 = 1$.

Fig. 9. Amplification of the tsunami wave amplitude (the peak wave amplitude, $A_{\text{max}}$, normalized by the amplitude of the ocean floor uplift, $\zeta_0$) versus the dimensionless parameter $L/h$, for $v_1/\nu_1 = 1$, $W/L \geq 0.25$ and $h = 2$ km.
For typical ocean depths during past tsunami earthquakes, we gathered data for selected events recorded in Japan during 1931 and 1968 [52,53], and plotted a histogram of the distribution of events over depth. We show this histogram in the bottom right part of Fig. 10, with \( h \) plotted on a vertical log scale. For this data, \( h \) ranges between 0.5 and 6 km, with average around 2 km. Next, in the bottom part of Fig. 10 we show a plot of \( h \) versus \( v_T/v \), such that the \( h \) axis is aligned with the histogram and the \( v_T/v \) axis is aligned with the plot on the top. The shaded region in this plot shows the domain corresponding to slowly spreading uplift (\( v < 0.1 \) to 0.25 km/s) and the range of depths shown in the histogram (\( h = 0.5 \) to 6 km). For this range of depths, \( v_T/v \approx 1 \) is realized if \( v \approx 0.1 \) to 0.25 km/s. This implies that large amplification of \( h/\zeta_0 \) could be caused by earthquakes producing uplifts of the ocean floor, or by submarine slides and slumps which spread with velocities 100–250 m/s.

4.4. Radiation pattern

The source areas of many tsunamis have been estimated roughly by inverse refraction diagrams based on the arrival...
time of the tsunami front at the tide recording stations. The initial wave heights (at the source) have been approximated by applying refraction and scaling coefficients to the inundation heights [52–54]. This has led to association of the observed directivity with the ellipticity ratio of the source zone. Let \( a \) and \( b \) designate the major and minor elliptical axes of the source area, and let \( H_a \) and \( H_b \) be the amplitudes, and \( T_a \) and \( T_b \) be the periods of tsunamis in these principal directions. The data suggest that \( H_b = H_a \), \( a = b \); but \( T_b = T_a \), \( b = a \): Fig. 11, redrawn from Hatori [54] illustrates \( H \) and \( T \) for the 1933 Sanriku and the 1952 Kamchatka events. In this figure, \( \theta \) is measured counterclockwise from the major axis of the source ellipse (as in Fig. 4a). The trend indicates coincidence of large \( H \) with small \( T \) and vice versa. The above data was later used to develop empirical relationships between the linear dimension (\( L \)), area (\( S \)) and ellipticity (\( b/a \)) of tsunami sources, versus earthquake magnitude [53]. In all this work it was assumed that the waves are created by instantaneously elevating the elliptical area of the source.

Next, we show radiation patterns (for amplitudes and periods) for our model, at distance \( R \approx 100 \) km from the origin. The purpose is to illustrate what the effects of source directivity might be on the radiation pattern in the near-field during ‘slow’ earthquakes, slides or slumps. The peak amplitudes and periods were measured from plots of \( \eta/\zeta_0 \) versus time at various points around the source. One such plot, for \( \nu/\nu_T = 1.2, h = 2 \) km and \( L = W = 50 \) km, is shown in Fig. 12. It shows time history of \( \eta/\zeta_0 \) at nine points, all at distance \( R = 100 \) km from the origin and at azimuths, \( \theta \), equally spaced between 0 and 180° (\( \theta = 0, 22.5, 45, \ldots, 157.5 \) and 180°; see Fig. 4a). The ‘periods’ were computed by doubling the ‘half period’ estimates determined by measuring the time between the maxima (solid circles) and minima (open circles) of \( \eta/\zeta_0 \). Fig. 13 shows the actual radiation patterns in the \( r-\theta \) coordinate system, for \( L = W = 50 \) km and \( h = 2 \) km, and for \( \nu/\nu_T = 1.0, 1.2, 1.5, 2, 5 \) and 20. The scale on the right corresponds to the peak amplitudes of \( \eta/\zeta_0, A_{\text{max}}/\zeta_0 \), and the one on the left corresponds to the ‘periods’. For sources such that \( L \neq W \), and for different depths of the ocean, \( h \), this pattern will change accordingly, but the general trends will remain similar. In the direction of source propagation (\( \theta = 0^\circ \)), the peak amplitudes will be larger when \( \nu/\nu_T > 5 \), while the ‘periods’ will be shorter when \( \nu/\nu_T \leq 2 \).

The above indicates that the directivity trends, usually ascribed to ellipticity of idealized instantaneous sources [54], will contain significant contributions from the source directivity, when \( \nu/\nu_T \) is smaller than 2–5. Comparison of the results in Figs. 11 and 13 indicates that both ellipticity of idealized instantaneous sources and directivity of ‘slowly’ spreading sources imply that along the azimuth for which the peak amplitude is the largest the ‘period’ is the smallest.

4.5. Source finiteness and directivity

In the following, we comment on the near-field effects of source finiteness and directivity and on their representation in the far-field.
The effects of the finiteness of simple sources on tsunami amplitudes were studied by Ben-Menahem and Rosenman [27]. They described these effects in the far-field (at distance $R$ such that $L^2 \ll \lambda R$, where $\lambda$ is the wavelength) via the finiteness factor

$$\text{fin}(k) = \frac{\sin X \sin Y}{X \cdot Y}$$

(35)
in the direction of rupture propagation and vice versa in the opposite direction; [55]). The near-field problem, however, is more complicated.

Our model implies that, as $k \to 0$ and after the source process has been completed ($t \geq t^*$), $\eta \to WL_c$, which is equal to the volume of water uplifted by the source and also to the source potency (see Eq. (31) and recall that $v_f = L$). This means that, at large distances, the amplitude of the long period waves (in deep sea water) will depend on the total amount of water raised, regardless of how the source was realized. Unless conditions exist in the source region for creation of solitary waves, the large tsunami amplitudes resulting from wave focusing (‘heaping’ amplification) will be diminished due to dispersion.

Next, we simplify our model by assuming $W \to \infty$ (1D tsunami propagation) and analyze $\eta$ in the wave-number-time domain. For this model

$$\tilde{\eta}(k; t) = \frac{-i\xi_0}{\cosh k h} \frac{1}{\left(\frac{c}{v}\right)^z} \left[ \cos at + i \frac{c}{v} \sin at e^{-ikL} \right].$$

$$t \leq t^* = L/v.$$  

(38)

The above expression is singular for some $k$ such that $c(k) = v$, which can happen only if $v < v_T = c(0)$ (subsonic case). If $v = v_T$, $k_0 = 0$, and $k_0$ is progressively larger for $v$ progressively smaller than $v_T$. Although the singularity at $k = k_0$ is removable, somewhat larger amplitudes of $\tilde{\eta}(k, t)$ are to be expected in the neighborhood of $k_0$. We illustrate $|\tilde{\eta}(k; t)|$ for $v/v_T = 1$, and for one supersonic and one subsonic case ($v/v_T = 50$ and 0.7) in Fig. 14. All of these are for water depth $h = 2$ km and for time $t = t^*$ (moment when the uplift spreading is completed). It is seen that the amplitude of $|\tilde{\eta}(k; t)|$ is modulated by a function which depends on $v/v_T$, and that there is a hump near $k = k_0$. The effect of the hump on the tsunami is the strongest when $k_0$ is small because then the amplified tsunami is less dispersed.

A physical interpretation for Eq. (38) for a special case ($t = t'$ and $v > v_T$) can be obtained as follows. At time $t = t'$, Eq. (38) gives

$$\tilde{\eta}(k; t') = \frac{\xi_0}{ik \cosh kh} \frac{1}{\left(\frac{c}{v}\right)^z} \left[ \cos \left( kL \frac{c}{v} \right) \right]$$

$$+ i \frac{c}{v} \sin \left( kL \frac{c}{v} \right) e^{-ikL}.  \tag{39}$$

It can be verified by straightforward integration that $\tilde{\eta}(k, t')$ is the Fourier transform of an approximation of the waveform at $t = t'$ by two box functions, except for the factor $1/\cosh kh (= 1$ for small $k$). Such an approximation is illustrated in Fig. 15. (The waveform in this figure is actually for a square uplifted area and for $L = W = 50$ km, $h = 2$ km and $v/v_T = 2$. Examples of

where

$$X = \frac{1}{2} kL \left( \frac{c}{v} - \cos \theta \right)$$

$$Y = \frac{1}{2} kW \sin \theta$$

and where $\theta$ is as shown in Fig. 4. The factor $\text{fin}(k)$ has maximum value 1, for $k \to 0$. If the fault width is much smaller than the length ($W \ll L$), then $\sin Y / Y = 1$ and $\text{fin}(k) = \sin X / X$. Then, for $v \geq c$, $\text{fin}(k)$ is maximum for $\theta \approx \pm 90^\circ$, i.e. normal to source. For $v \approx c$, $\text{fin}(k)$ is maximum for $\theta \approx 0^\circ$, i.e. in direction of rupture propagation. In both cases $\text{fin}(k)$ does not exceed unity.

The results illustrated in this paper show that above and near the source the amplification due to source directivity and finiteness can be greater than unity when $v \approx c$. The effects predicted by the finiteness function, $\text{fin}(k)$, are again present (larger amplitudes and more high frequencies...
waveforms for other values of $v/\nu_T$ can be seen in Fig. 5. The waveforms for an infinitely wide fault are very similar.) The approximation, $\eta_b(x; t')$, by two box functions is

$$\eta_b(x; t') = \begin{cases} \frac{1}{2} \xi_0 \frac{1}{v} + 1, & -L \frac{c}{v} \leq t < -L \frac{c}{v} \\ \xi_0 \frac{1}{v}, & L \frac{c}{v} \leq t \leq L. \end{cases} \tag{40}$$

Both amplitudes are such that the total volume of water uplifted (along unit width) is $\xi_0 L$. The larger amplitude box is a result of the wave focusing (‘stacking’ amplification).

The transforms of the exact waveform and of its application are related by

$$\tilde{\eta}(k; t') = \frac{1}{\cosh \kappa h} \tilde{\eta}_b(k; t'). \tag{41}$$

We use this fact to express conveniently $\tilde{\eta}(k; t')$ in terms of the finiteness function $\sin \tau = \sin \pi \theta / L$. Following the definition of Fourier transform and using coordinate transformation $x' = x - (L/2)(c/v + 1)$ gives

$$\tilde{\eta}_b(k; t') = \xi_0 L \frac{1}{v} \left[ \frac{1}{2} \frac{c}{v} \frac{\sin X_1}{X_1} + e^{i(kL/2)(c/v + 1)} \frac{\sin X_2}{X_2} \right] \tag{42}$$

where

$$X_1 = k L \frac{c}{v} \tag{43a}$$

$$X_2 = 2 k L \left( \frac{c}{v} - 1 \right). \tag{43b}$$

It is seen that $X_2$ is identical to $X$ for the far-field line source representation, evaluated for $\theta = 0$ (see Eq. (36)). We could have expressed $\tilde{\eta}_b(k; t')$ in terms of these finiteness functions by rearranging terms and by simple transformations, but the steps would not have been obvious. Eq. (42) implies that for a ‘fast’ rupture ($c/v \to 0$), the contribution to $\tilde{\eta}_b(k; t')$ from the box to the left ($|k| < L c/v$ in Fig. 15) would be small, and

$$\lim_{c/v \to 0} \tilde{\eta}_b(k; t') = \xi_0 L e^{i kL/2} \frac{\sin kL/2}{kL/2 \nu_T}. \tag{44}$$

We showed that for $v > \nu_T$ and at $t = t'$, the spatial Fourier transform of $\eta(x; t)$ contains the source finiteness function, same as the line source representation in far-field studies. We used the sketch in Fig. 15 and assumed $v > \nu_T$ in representing $\tilde{\eta}(k; t')$ in terms of the finiteness functions, $\sin X_1/X_1$ and $\sin X_2/X_2$; to facilitate the logic, we could have arrived at the same result by purely formal manipulation of Eq. (38). Therefore, the same representation holds for any $v/\nu_T$. The procedure can also be repeated for any $t < t'$.

For two-dimensional tsunami propagation, a similar expression would be obtained for the part of $\tilde{\eta}(k; t')$ which depends on $k_1$. The part that depends on $k_2$ is actually $\sin Y$ with $Y = kW/2$, i.e. the same as in Eq. (37) for $\theta = 0$.

### 4.6. Bores

Finally, we comment on the formation of bores by our simple model. Very little has been written about the near-field mechanisms which lead to creation of bores during tsunamigenic earthquakes, slides or slumps [56,57]. A bore is formed when the moving water is deeper than the still water. If a bore is turbulent, the breaking zone of water can be several times longer than the water depth. If the change in water level is much smaller than the water depth, an undular bore can form, consisting of a train of waves with wavelengths several times longer than the water depth. Consequently, the formation and propagation of such bores may be studied via shallow water theory.

In Fig. 5, it is seen that an undular bore is created and propagates in the opposite direction of the uplift (negative $x$-direction). Its amplitude decreases as $v$ decreases. For $v/\nu_T$ smaller than about 0.7, another undular bore is formed, but now in front of the moving uplift, propagating in the positive $x$-direction.

The eyewitness reports on bore-like motions associated with documented tsunami events apply only to conditions along coasts and rivers. Those reports are difficult to interpret because of complexities of the interaction of the incident waves with the coastline. Some of these bores may have been created during the process of runup, not necessarily originating in the tsunami source region [58–60]. From the observational viewpoint, it is interesting to see that the slowly spreading motions of the ocean floor can create conditions for generation of bores in the near-field.

### 5. Conclusions

The analysis of the model presented in this paper shows that the common assumption in modeling of tsunamis in the source region, that the ocean floor is uplifted simultaneously...
in the entire source area, is not valid for slowly spreading uplift of the sea floor \((0.1 < v < 1 \text{ km/s})\). The effects of the spreading of the ocean floor deformation (faulting, submarine slides or slumps) on the amplitudes and periods of the generated tsunamis are largest when the spreading velocity of uplift and the tsunami velocity are comparable. Then, the amplitudes of the tsunami propagating in the direction of the spreading uplift can be amplified, by as much as one order of magnitude. The mechanism for this amplification is wave focusing. Comparison with data of past tsunamis suggests that the properties of this model (amplitudes, periods and directivity) are in agreement with the trends seen in the data. The presented analysis suggests that some abnormally large tsunamis could be explained in part by a slowly spreading uplift of the sea floor.

The leading amplitudes of the wave created above the ‘source’ are monotonic. For the examples we considered, the tail of this wave shows oscillatory nature and tends to be associated with shorter wavelengths. It is expected, however, that a family of \(g(x, y, t)\) with amplitudes increasing towards the center of the uplift can lead to smooth decay of these tail amplitudes. This may create conditions for a large soliton to leave the source area of slow tsunamigenic uplift of the sea floor. We leave analysis of this interesting possibility for future work.

The model described in this paper represents only very simple spreading sources. It can be generalized to model more realistic ocean floor displacements. Coupling of such models with detailed numerical models for inverse wave refraction will provide valuable tools for inverse modeling of water waves in the source area, to study the movement of the ocean floor, and the causative earthquake sources or submarine slides and slumps.

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References

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