

A NOTE ON RESPONSE OF SHALLOW CIRCULAR VALLEYS TO RAYLEIGH WAVES: ANALYTICAL APPROACH

by

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ABSTRACT

The steady-state response of shallow alluvial valleys to plane monochromatic Rayleigh waves has been studied by a new analytical method. The shape of the valley is circular, with depth to half-width ratio ≤ 1 . The method is based on representation of the transmitted into the valley and the scattered waves in Fourier-Bessel series. Finite Fourier series have been used to approximate the free-field motion before the application of the continuity of stresses and displacements conditions. The results show that in general the valley amplifies the incident ground motion and that the amplification and the complexity of the displacement pattern depend on the frequency of the incident Rayleigh waves and on the shape of the valley.

INTRODUCTION

In earthquake engineering it is of importance for the design purposes to study the amplification patterns of strong ground motion in soft sediment valleys. Many studies of this subject have been carried out in the past twenty years, mostly employing various numerical techniques (e.g. Sánchez-Sesma and Esquivel, 1979; Moeen-Vaziri and Trifunac, 1988a, b; and many others). The numerical methods are applicable to inhomogeneities of arbitrary shape, but often met with difficulties in the numerical evaluations for higher frequencies of excitation. The exact analytical solutions are only few. Those of Trifunac (1971) and Wong and Trifunac (1974) for semi-circular and semi-elliptical valleys, and for incident plane SH-waves are commonly used for calibration of the numerical methods. Recently Todorovska and Lee (1989a, b, c) presented a new analytical method for circular cylindrical valleys and for incident plane SH, P and SV-waves, respectively. In their work for the first time this class of problems was treated analytically for incident in plane excitation. Their

method is a generalization of the method proposed by Cao and Lee (1988a, b), Lee and Cao (1988) and Todorovska and Lee (1989d), for circular canyons and plane SH, P and SV and Rayleigh waves. This method is based on representation of the scattered waves and the motion inside the valley in complete series of cylindrical wave functions and approximation of the half-space surface in the vicinity of the valley by a cylindrical surface with very large radius.

This paper is a continuation of the work of Todorovska and Lee (1989a, b, c). Their method has been extended here to circular valleys excited by monochromatic Rayleigh waves. The purpose of this paper is to check the applicability of the method for this type of excitation and to present some numerical results.

THE MODEL

The model which we consider is a two-dimensional, homogeneous, isotropic and linearly elastic alluvial valley imbedded into a homogeneous, isotropic and perfectly elastic half-space (Fig. 1). The shear wave velocity, shear modulus and the Poisson's ratio of the valley and of the half-space are β_v, μ_v , and ν_v , and β_s, μ_s and ν_s , respectively. Perfect bond is assumed between the two media. The shape of the valley is circular and its depth to half-width ratio can take values ≤ 1 . The radius of curvature of the valley is b and the center of curvature is at O_1 . In Fig. 1 d is the distance from O_1 to the half-space surface. The depth of the valley is $h = b - d$ and the half-width is $a = \sqrt{b^2 - h^2}$. The cylindrical surface with center at O_2 and large radius R approximates the half-space surface close to the valley (Todorovska and Lee, 1989a, b, c). The distance between the centers of curvature of the two cylinders, where also the rectangular coordinate systems $x_1 - O_1 - z_1$ and $x_2 - O_2 - z_2$ and the polar coordinate systems $r_1 - \theta_1$ and $r_2 - \theta_2$ originate, is D . The rectangular coordinate system $x - O - z$ is used to define the excitation.

The excitation is a monochromatic Rayleigh wave defined via its P- and S-wave potentials, ϕ and ψ , respectively, as

$$\phi = Ce^{-b_1 z} e^{i k (x - ct)} \quad (1a)$$

$$\psi = De^{-b_2 z} e^{i k (x - ct)} \quad (1b)$$

where C and D are complex constants, b_1 and b_2 , both real and positive, are the rates of the monothonic decay of the potentials with increasing depth, t is the time variable and $i = \sqrt{-1}$. The inhomogeneous wave is propogating in the positive x -direction with phase velocity c and with wave number $k = \omega/c$, where ω is the circular frequency. Those quantities are related as follows (Eringen and Suhubi, 1975):

$$b_1 = k\sqrt{1 - (c/\alpha_s)^2} \quad (2a)$$

$$b_2 = k\sqrt{1 - (c/\beta_s)^2} \quad (2b)$$

and

$$D = \frac{1}{2b_2 ik} (k^2 + b_2^2) C, \quad (2c)$$

where $\alpha_s = \sqrt{2(1-\nu_s)/(1-2\nu_s)}\beta_s$ is the P-wave velocity in the half-space. The eigenvalue of c is a solution of the Rayleigh equation and has values, e.g., $c = 0.9194\beta_s$ for $\nu = 1/4$ and $c = 0.9320$ for $\nu = 1/3$. The particle motion is elliptical (retrograde on the surface). In the $r_1 - \theta_1$ coordinate system the potentials ϕ and ψ can be written as

$$\phi = Ce^{b_1 z} e^{i k_\alpha r_1 \cos(\theta_1 - \theta_\alpha) - i \omega t} \quad (3a)$$

$$\psi = De^{b_2 z} e^{i k_\beta r_1 \cos(\theta_1 + \theta_\beta) - i \omega t} \quad (3b)$$

where $k_\alpha = \omega/\alpha$ and $k_\beta = \omega/\beta$ are the wave numbers of the P and the S-waves, respectively in the half-space and θ_α and θ_β are complex angles (Todorovska and Lee, 1989d) defined as

$$\theta_\alpha = \frac{\pi}{2} - i\phi_\alpha \quad (4a)$$

and

$$\theta_\beta = \frac{\pi}{2} - i\phi_\beta \quad (4b)$$

where

$$\phi_\alpha = \cosh^{-1}(\alpha_s/c) \quad (4c)$$

$$\phi_\beta = \cosh^{-1}(\beta_s/c) \quad (4d)$$

are real quantities.

The potentials of the scattered waves, ϕ^R and ψ^R , and of the motion inside the valley, ϕ^T and ψ^T , have to satisfy the two-dimensional wave equations

$$\left(\frac{\partial^2}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial}{\partial r_1} + \frac{1}{r_1^2} \frac{\partial^2}{\partial \theta_1^2} \right) \phi^* = \frac{1}{\alpha_s^2} \frac{\partial^2 \phi^*}{\partial t^2} \quad (5a)$$

and

$$\left(\frac{\partial^2}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial}{\partial r_1} + \frac{1}{r_1^2} \frac{\partial^2}{\partial \theta_1^2} \right) \psi^* = \frac{1}{\beta_s^2} \frac{\partial^2 \psi^*}{\partial t^2} \quad (5b)$$

where ϕ^* and ψ^* stand for ϕ^R and ϕ^T , and for ψ^R and ψ^T , respectively, while α_s and β_s represent their corresponding velocities. The boundary conditions that the resultant motion has to satisfy are the zero-stress conditions

$$\begin{pmatrix} \tau_{rz}^{(s)} \\ \tau_{\theta z}^{(s)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ at } z = 0 \quad (6a)$$

$$\begin{pmatrix} \tau_{xx}^{(s)} \\ \tau_{xy}^{(s)} \\ \tau_{xz}^{(s)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ at } z = 0 \quad (6b)$$

and the continuity of stresses and displacements conditions

$$\begin{pmatrix} u_{r_1}^{(s)} \\ u_{\theta_1}^{(s)} \end{pmatrix} = \begin{pmatrix} u_{r_1}^{(v)} \\ u_{\theta_1}^{(v)} \end{pmatrix} \text{ at } r_1 = b \quad (7a)$$

and

$$\begin{pmatrix} \tau_{r_1 r_1}^{(s)} \\ \tau_{r_1 \theta_1}^{(s)} \end{pmatrix} = \begin{pmatrix} \tau_{r_1 r_1}^{(v)} \\ \tau_{r_1 \theta_1}^{(v)} \end{pmatrix} \text{ at } r_1 = b. \quad (7b)$$

where $\tau_{xx}^{(s)}$, $\tau_{xy}^{(s)}$, $\tau_{xz}^{(s)}$, $\tau_{r_1 r_1}^{(s)}$, $\tau_{r_1 \theta_1}^{(s)}$ and $\tau_{xx}^{(v)}$, $\tau_{xy}^{(v)}$, $\tau_{xz}^{(v)}$, $\tau_{r_1 r_1}^{(v)}$, $\tau_{r_1 \theta_1}^{(v)}$ are the components of the stress tensor in the half-space and in the valley and $u_{r_1}^{(s)}$ and $u_{\theta_1}^{(s)}$ and $u_{r_1}^{(v)}$ and $u_{\theta_1}^{(v)}$ are the displacement components in the respective media. Those displacement and stress components for the coordinate systems in Fig. 1 can be derived from the potentials as in Todorovska and Lee (1989d).

As in the preceding applications of this method (Todorovska and Lee, 1989b, c), the potentials of the scattered waves, ϕ^R and ψ^R , and of the transmitted waves into the valley, ϕ^T and ψ^T , will be represented as sums of two Fourier-Bessel series (one in the $r_1 - \theta_1$ coordinate system and the other in the $r_2 - \theta_2$ coordinate system), i.e.,

$$\phi^R = \phi_1^R + \phi_2^R \quad (8a)$$

$$\psi^R = \psi_1^R + \psi_2^R \quad (8b)$$

$$\phi^T = \phi_1^T + \phi_2^T \quad (9a)$$

and

$$\psi^T = \psi_1^T + \psi_2^T \quad (9b)$$

where

$$\phi_1^R = \phi_1^R(r_1, \theta_1, t) = \sum_n H_n^{(1)}(k_a^{(s)} r_1) \left(A_{1,n}^R \cos n \theta_1 + B_{1,n}^R \sin n \theta_1 \right) e^{-i \omega t} \quad (10a)$$

$$\psi_1^R = \psi_1^R(r_1, \theta_1, t) = \sum_n H_n^{(1)}(k_\beta^{(s)} r_1) \left(C_{1,n}^R \sin n \theta_1 + D_{1,n}^R \cos n \theta_1 \right) e^{-i \omega t} \quad (10b)$$

$$\phi_2^R = \phi_2^R(r_2, \theta_2, t) = \sum_n J_n(k_a^{(s)} r_2) \left(A_{2,n}^R \cos n \theta_2 + B_{2,n}^R \sin n \theta_2 \right) e^{-i \omega t} \quad (10c)$$

$$\psi_2^R = \psi_2^R(r_2, \theta_2, t) = \sum_n J_n(k_\beta^{(s)} r_2) \left(C_{2,n}^R \sin n \theta_2 + D_{2,n}^R \cos n \theta_2 \right) e^{-i \omega t} \quad (10d)$$

$$\phi_1^T = \phi_1^T(r_1, \theta_1, t) = \sum_n J_n(k_a^{(v)} r_1) \left(A_{1,n}^T \cos n \theta_1 + B_{1,n}^T \sin n \theta_1 \right) e^{-i \omega t} \quad (11a)$$

$$\psi_1^T = \psi_1^T(r_1, \theta_1, t) = \sum_n J_n(k_\beta^{(v)} r_1) \left(C_{1,n}^T \sin n \theta_1 + D_{1,n}^T \cos n \theta_1 \right) e^{-i \omega t} \quad (11b)$$

$$\phi_2^T = \phi_2^T(r_2, \theta_2, t) = \sum_n J_n(k_a^{(v)} r_2) \left(A_{2,n}^T \cos n \theta_2 + B_{2,n}^T \sin n \theta_2 \right) e^{-i \omega t} \quad (11c)$$

and

$$\psi_2^T = \psi_2^T(r_2, \theta_2, t) = \sum_n J_n(k_p^{(r)} r_2) \left(C_{2,n}^T \sin n\theta_2 + D_{2,n}^T \cos n\theta_2 \right) e^{-i\omega t} \quad (11d)$$

The series can be transformed from $r_1 - \theta_1$ into $r_2 - \theta_2$ coordinate system by means of the addition theorem (Abramowitz and Stegun, 1972) and the relationship between the coefficients of the different series can be established after the application of the zero-stress condition in the $r_2 - \theta_2$ coordinate system (Todorovska and Lee, 1989b). Finally the two independent sets of coefficients (one of the symmetric part of the displacements and the other of the anti-symmetric part) can be evaluated by the application of the continuity conditions at $r_1 = b$, equations (7a,b).

Before the application of the continuity conditions the displacements and the stresses of the free-field motion (the incident Rayleigh wave), $u_{r_1}^{i=f}$, $u_{\theta_1}^{i=f}$, $\tau_{r_1\theta_1}^{i=f}$ and $\tau_{\theta_1 r_1}^{i=f}$, have to be transformed into a form compatible with the representation of the scattered and the transmitted waves. As in Todorovska and Lee (1989c) for incident SV waves beyond critical angle, in this paper the displacements and stress at $r_1 = b$ are approximated by the following finite Fourier series,

$$u_{r_1}^{i=f}(b, \theta_1, t) \approx \frac{1}{b} \sum_{n=0}^N \left(A_{0,n}^{u_r} \cos n\theta_1 + B_{0,n}^{u_r} \sin n\theta_1 \right) e^{-i\omega t} \quad (12a)$$

$$u_{\theta_1}^{i=f}(b, \theta_1, t) \approx \frac{1}{b} \sum_{n=0}^N \left(A_{0,n}^{u_\theta} \sin n\theta_1 + B_{0,n}^{u_\theta} \cos n\theta_1 \right) e^{-i\omega t} \quad (12b)$$

$$\tau_{r_1\theta_1}^{i=f}(b, \theta_1, t) \approx \frac{2\mu_f}{b^2} \sum_{n=0}^N \left(A_{0,n}^{\tau_r} \cos n\theta_1 + B_{0,n}^{\tau_r} \sin n\theta_1 \right) e^{-i\omega t} \quad (12c)$$

and

$$\tau_{\theta_1 r_1}^{i=f}(b, \theta_1, t) \approx \frac{2\mu_f}{b^2} \sum_{n=0}^N \left(A_{0,n}^{\tau_\theta} \cos n\theta_1 + B_{0,n}^{\tau_\theta} \sin n\theta_1 \right) e^{-i\omega t} \quad (12d)$$

The coefficients of those series can be calculated analytically as in Hamming (1962), after a modification, described and discussed in Todorovska and Lee (1989c,d).

Similarly as for the valleys excited by in-plane ground motion, (Todorovska and Lee, 1989b,c) the continuity conditions, equations (7a,b) imply the following two systems of equations, where the unknowns are the coefficients of ϕ_1^R and ψ_1^R :

$$\sum_{j=0}^n \left\{ \begin{bmatrix} R_{n,j}^+(k_a^{(r)} D) & 0 \\ 0 & R_{n,j}^-(k_p^{(r)} D) \end{bmatrix} - [Q(n)]^{-1} [P(n)] \delta_{n,j} \right\} \begin{pmatrix} A_{1,j}^R \\ C_{1,j}^R \end{pmatrix} = \begin{pmatrix} A_{0,n}^* \\ C_{0,n}^* \end{pmatrix} \quad n = 0, 1, 2, \dots \quad (13a)$$

and

$$\sum_{j=0}^n \left\{ \begin{bmatrix} R_{n,j}^-(k_a^{(r)} D) & 0 \\ 0 & R_{n,j}^+(k_p^{(r)} D) \end{bmatrix} - [Q(n)]^{-1} [P(n)] \delta_{n,j} \right\} \begin{pmatrix} B_{1,j}^R \\ D_{0,j}^R \end{pmatrix} = \begin{pmatrix} B_{0,n}^* \\ D_{0,n}^* \end{pmatrix} \quad n = 0, 1, 2, \dots \quad (13b)$$

where

$$R_{\pm i}^{\pm}(kD) = \frac{\epsilon_n}{2} \sum_{l=0}^n \frac{\epsilon_l}{2} \left[J_{n+l}(kD) \pm (-1)^l J_{n-l}(kD) \right] \left[\left[H_{l+i}(kD) \pm (-1)^l H_{l-j}(kD) \right] \right] \quad (14)$$

$$[P(n)] = \left[e^{(s,i)}(n,b) \right] - \frac{\mu_v}{\mu_s} \left[e^{(1,v)}(n,b) \right] \left[D^{(1,v)}(n,b) \right]^{-1} \left[D^{(s,i)}(n,b) \right] \quad (15a)$$

$$[Q(n)] = R[P(n)]. \quad (15b)$$

$$\begin{pmatrix} A_{0,n}^* \\ C_{0,n}^* \end{pmatrix} = [Q(n)]^{-1} \left\{ \begin{pmatrix} A_{0,n}^{\tau\tau} \\ A_{0,n}^{\tau\theta} \end{pmatrix} - \frac{\mu_v}{\mu_s} \left[e^{(1,v)}(n,b) \right] \left[D^{(1,v)}(n,b) \right]^{-1} \begin{pmatrix} A_{0,n}^{u\tau} \\ A_{0,n}^{u\theta} \end{pmatrix} \right\} \quad (16a)$$

$$\begin{pmatrix} B_{0,n}^* \\ D_{0,n}^* \end{pmatrix} = [Q(n)]^{-1} \left\{ \begin{pmatrix} B_{0,n}^{\tau\tau} \\ B_{0,n}^{\tau\theta} \end{pmatrix} - \frac{\mu_v}{\mu_s} \left[e^{(1,v)}(n,b) \right] \left[D^{(1,v)}(n,b) \right]^{-1} \begin{pmatrix} B_{0,n}^{u\tau} \\ B_{0,n}^{u\theta} \end{pmatrix} \right\} \quad (16b)$$

where R means real part and where $[e^{(l,i)}(n,b)]$ $l=1,2$ are 2×2 matrices with terms defined as in Todorovska and Lee (1989b,c). The above infinite systems of equations are first truncated and then solved numerically. Then displacements inside the valley and in the half-space can be calculated.

RESULTS AND DISCUSSION

Fig. 2 through 8 illustrate the amplitudes and phases of the horizontal and the vertical components of the surface displacements when the excitation is a monochromatic Rayleigh wave with unit surface amplitude of horizontal displacement. The ratios of the material constants in all the cases are the following, $\beta_v/\beta_s = 1/2$ and $\rho_v/\rho_s = 2/3$ (ρ_v and ρ_s are the densities of the valley and the half-space, respectively). The Poissons ratio for both media is $\nu_v = \nu_s = \nu = 1/3$ and for this value the ratio between the amplitudes of the vertical and horizontal components of the free-field motion on the surface is 1.56. The frequency of the Rayleigh wave is expressed in terms of the dimensionless parameter η defined as the ratio of the width of the valley and the wave length of the S waves in the half-space, i.e. $\eta = 2a/\beta_s T$ where $T = 2\pi/\omega$ is the period of the Rayleigh wave. The phases have been drawn so that the phase at the center of the valley ($x=0, z=0$) is zero. In Fig. 2 through Fig. 6 the valley is shallow ($h/b=0.5$) and the frequency of the Rayleigh wave is $\eta=0.25, 0.5, 1, 2$ and 2.5 , respectively while in Fig. 7 and 8 the valley is semi-circular ($h/b=1$) and $\eta=0.5$ and 1 , respectively.

From such results it might be concluded that the valley amplifies the incident motion. This is not always the case. For exmple, for low frequency incident Rayleigh waves ($\eta=0.25$) in both deeper and shallower valleys the displacement amplitudes are smaller than the free-field amplitudes on the half-space surface. The depth of the valley influences the displacement patterns and amplitudes. For higher

frequencies of excitation larger variations and amplifications inside the valley are observed. For example, for $\eta=2$ the peak of the amplification of the horizontal displacement component on the surface of the shallow valley is approximately equal to 6.

The quality of the results by this method depend significantly on the number of terms in the truncated series (same as the number of terms in the finite Fourier series approximating the stresses and the displacements of the free-field motion along $r_1=b$). Best results are obtained when the depth of the valley and the number of terms are such that the finite Fourier series fit exactly the displacements and the stresses that they approximate at the edges of the valley, i.e. at $z=0$, where the change of these quantities with depth is most sudden.

CONCLUSIONS

Our analytical method has been extended in this paper to calculate the surface displacements of circular alluvial valleys for incident monochromatic Rayleigh waves. These valleys have sharp edges and shapes that can vary from semi-circular to very shallow. Fourier-Bessel series have been used to represent the waves scattered from and the waves transmitted into the valley. The displacements and stresses of the free-field motion along the bottom of the valley have been approximated by finite Fourier series before the application of the continuity conditions.

It can be concluded that the complexity of the motion on the surface of the valley depends on the wavelength of the incident Rayleigh wave. The amplification of the motion is in general larger for shorter waves. The displacement pattern also depends on the shape of the valley.

We found that the convergence of the proposed series representation is very fast. Thus the representation of the scattered and diffracted waves by irregular sedimentary basins, using this representation, should be faster than with direct use of cylindrical wave functions (Moeen-Vasiri and Trifunac, 1988a,b). Furthermore, for often flat and extended cross-sections of actual alluvial and sedimentary basins, the boundary conditions along the irregular layer-half-space interfaces will be more conveniently represented by a segment of a circular geometry than by the full half circle.

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FIGURE CAPTIONS

Figure 1 A shallow cylindrical valley imbedded in the homogeneous half-space. The surface of the half-space close to the valley is approximated by a cylindrical surface with very large radius R and with center of curvature at O_2 . The valley is a part of a circular cylinder centered at O_1 and with radius b .

Figure 2 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 0.25$. The assumed ratios of the material constants are $\beta_1/\beta_2 = 1/2$ and $\rho_1/\rho_2 = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley

and its radius of curvature is $h/b = 1/2$.

Figure 3 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 0.5$. The assumed ratios of the material constants are $\beta_s/\beta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1/2$.

Figure 4 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 1$. The assumed ratios of the material constants are $\beta_s/\eta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1/2$.

Figure 5 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 2$. The assumed ratios of the material constants are $\beta_s/\beta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1/2$.

Figure 6 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 2.5$. The assumed ratios of the material constants are $\beta_s/\beta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1/2$.

Figure 7 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 0.5$. The assumed ratios of the material constants are $\beta_s/\beta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1$.

Figure 8 The amplitudes and phases of the horizontal and vertical displacement components on the surface of the valley ($-1 \leq x/a \leq 1$) and of the half-space ($x/a < -1$ and $x/a > 1$) versus the distance x/a . The excitation is a Rayleigh wave with unit amplitude of the horizontal displacement component and dimensionless frequency $\eta = 1$. The assumed ratios of the material constants are $\beta_s/\beta_s = 1/2$ and $\rho_s/\rho_s = 2/3$ and the Poisson's ratio is $\nu = 1/3$. The ratio between the depth of the valley and its radius of curvature is $h/b = 1$.

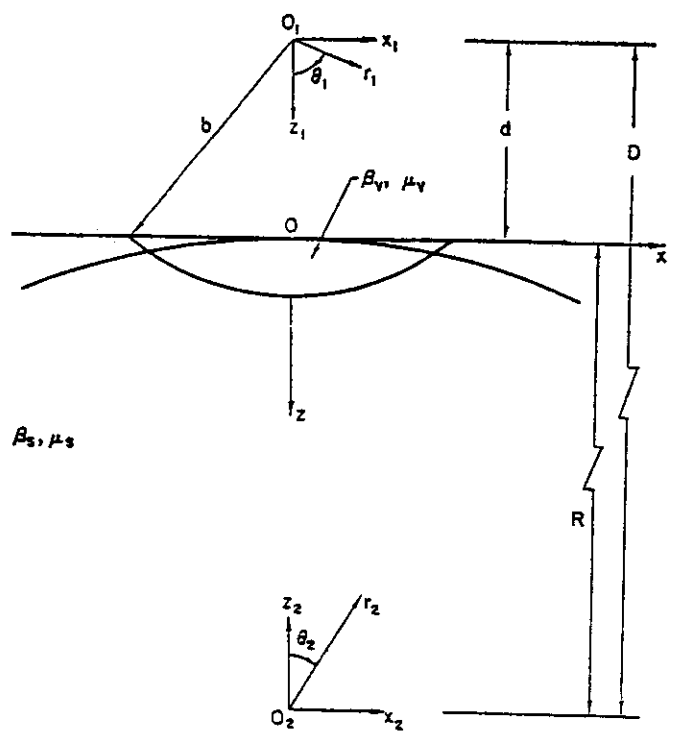


Fig. 1

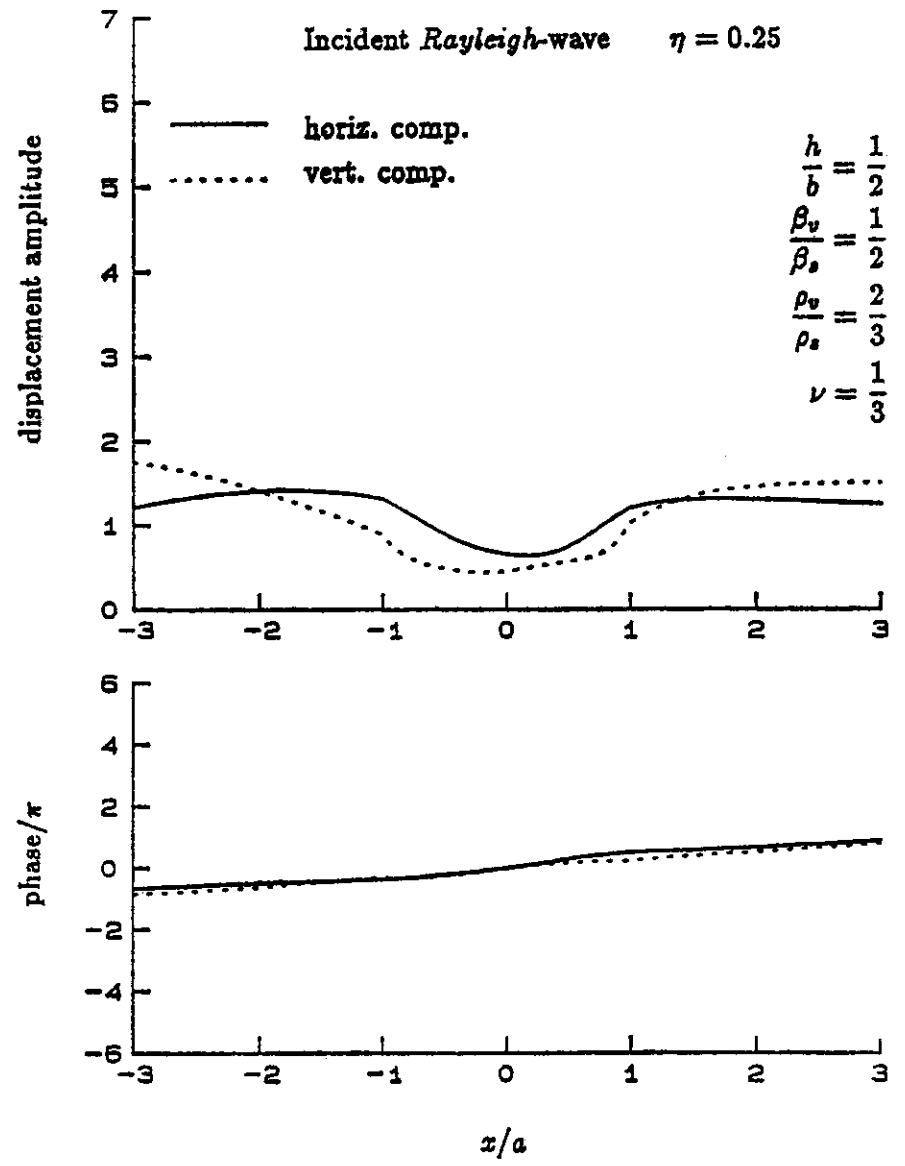


Fig. 2

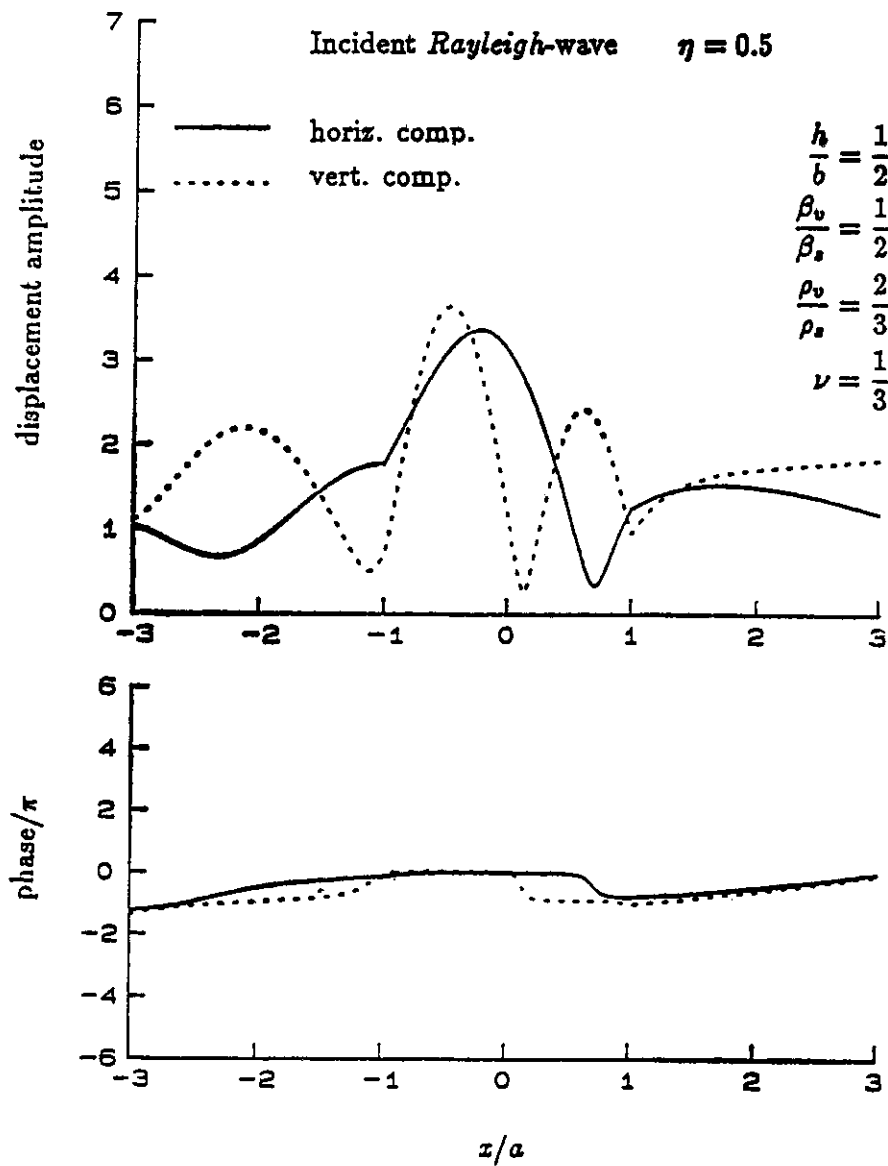


Fig.3

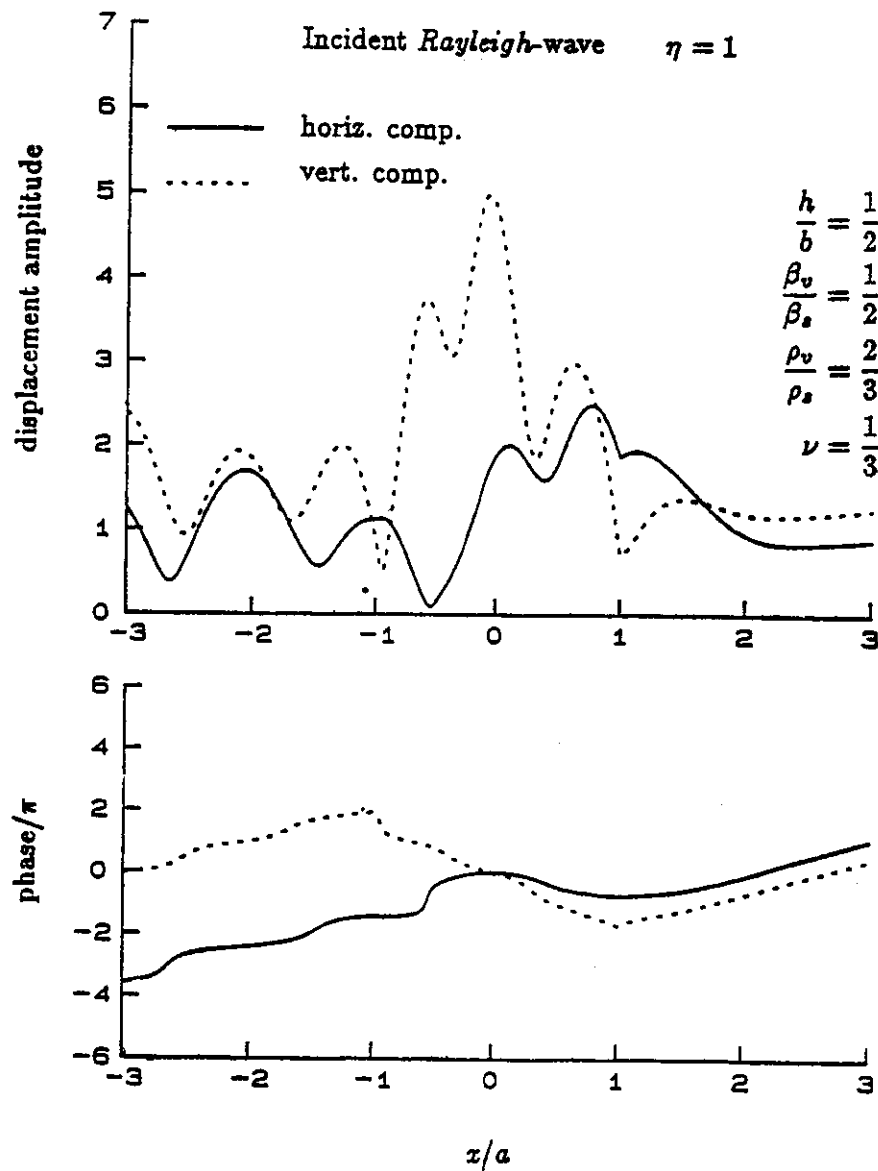


Fig.4

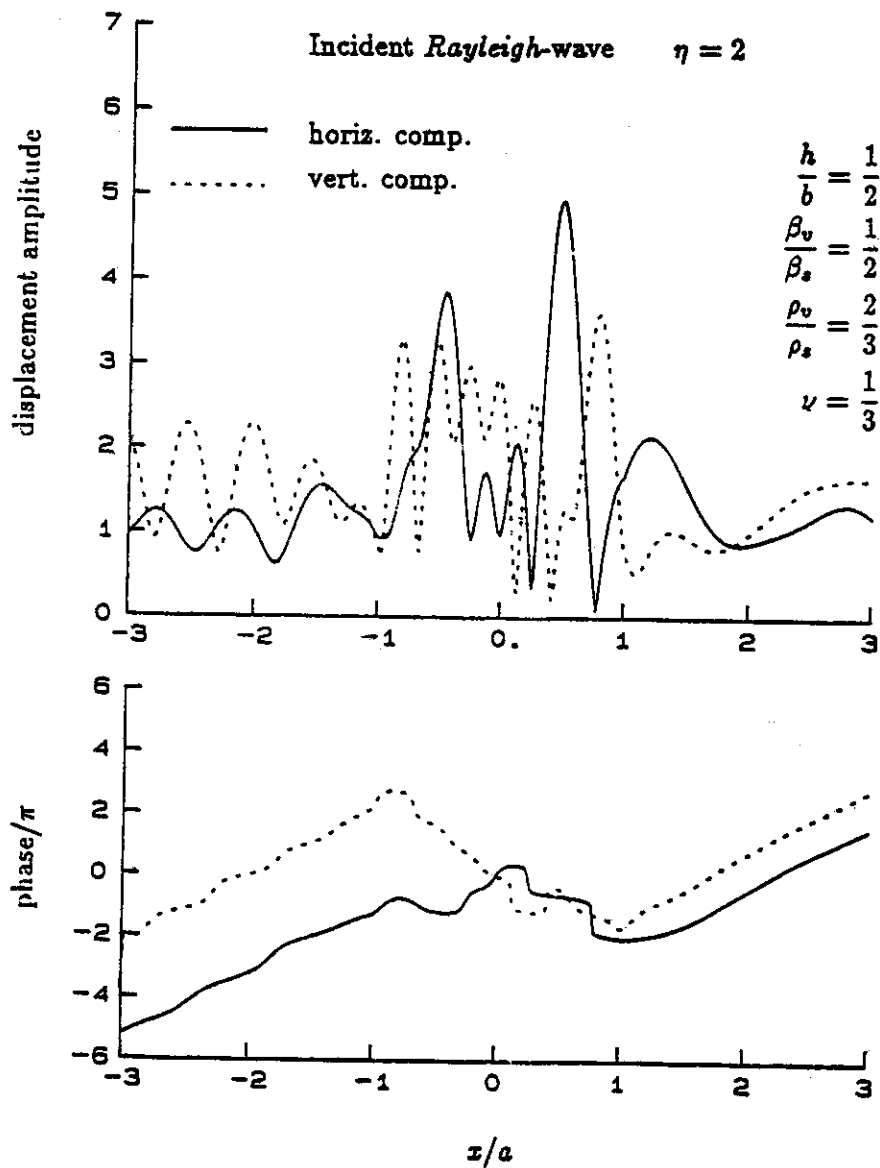


Fig.5

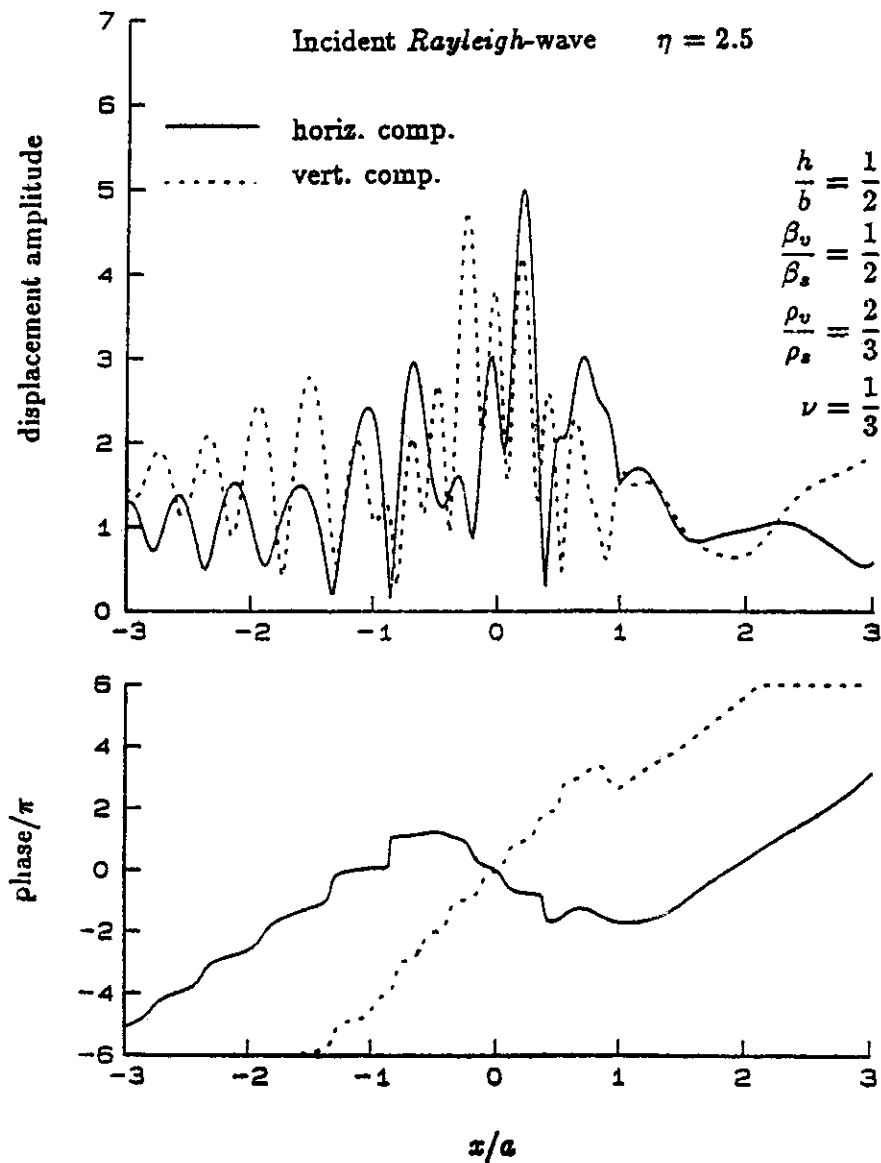


Fig.6

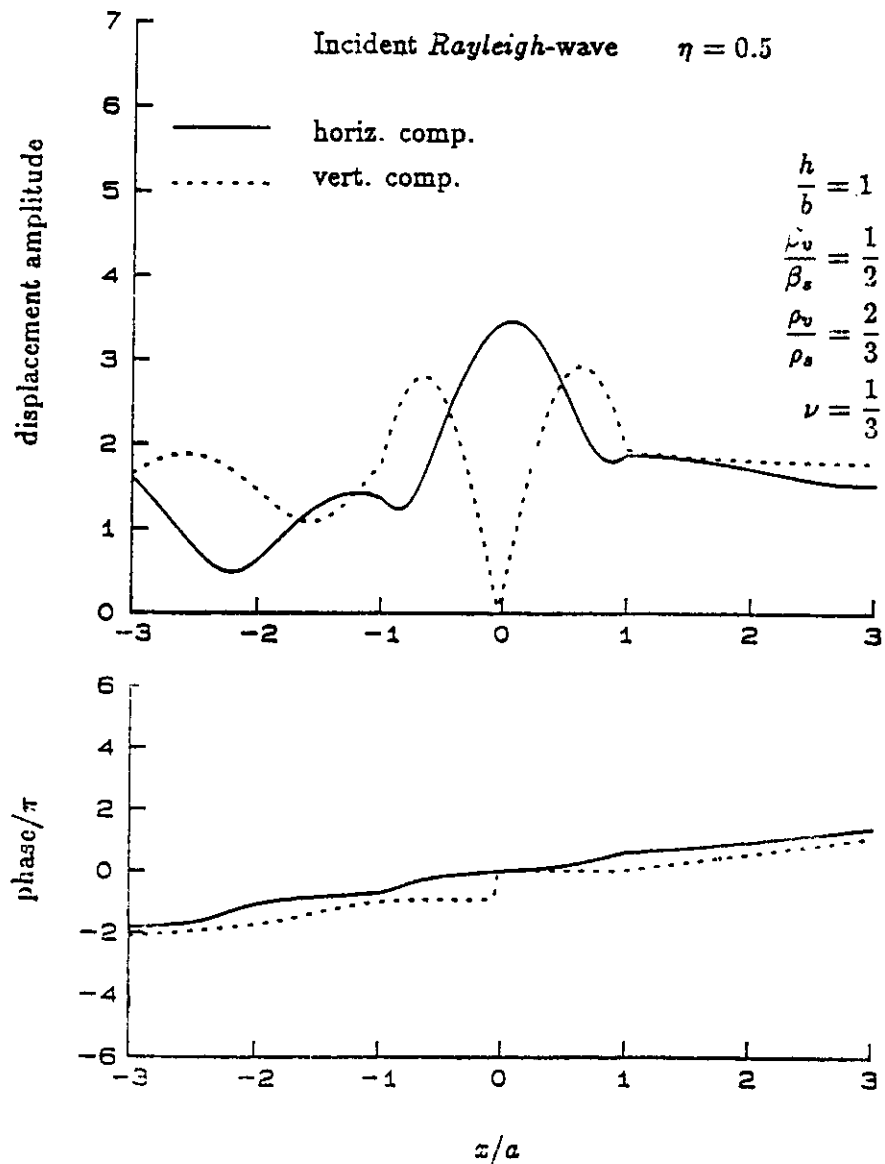


Fig.7

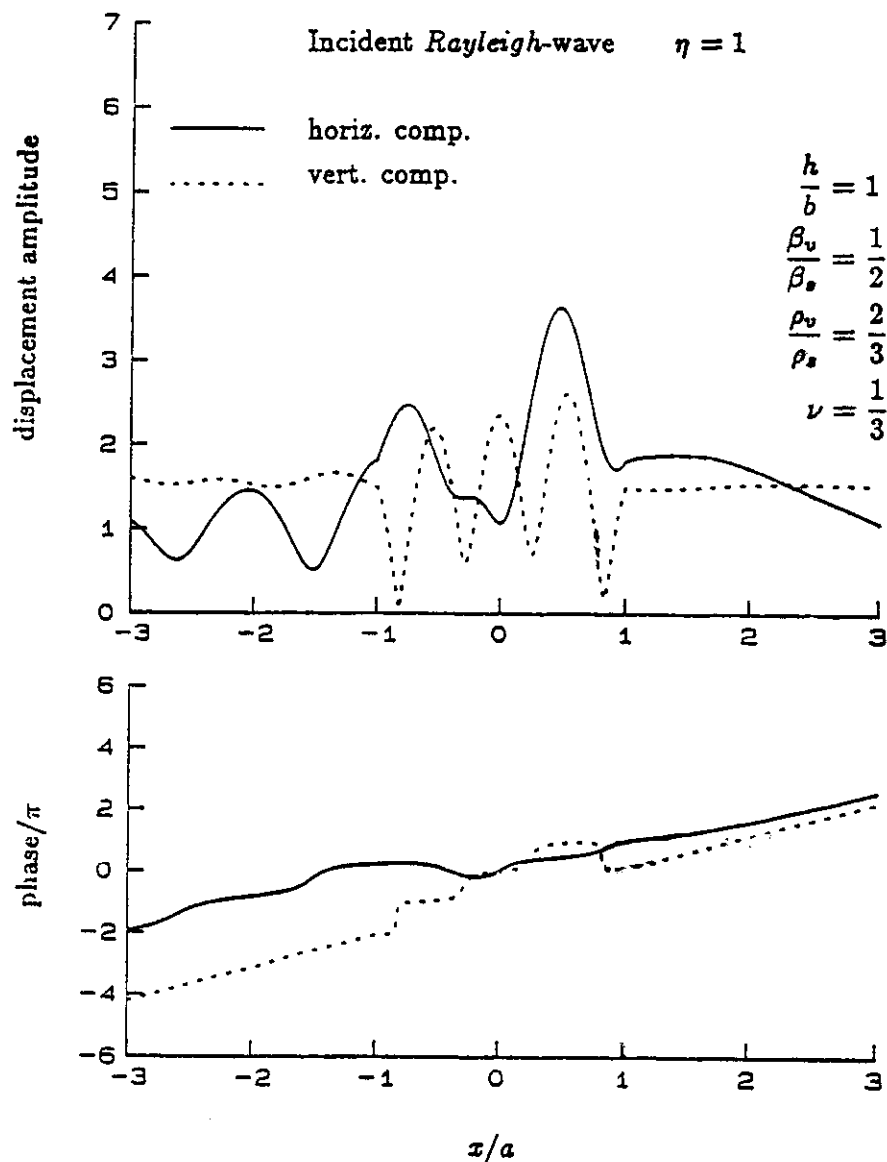


Fig.8

关于圆形浅谷对Rayleigh波的反应的注记——分析方法

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提 要

本文采用一个新的分析方法研究了冲积浅谷对平面单频Rayleigh波的稳态反应。谷地的形状是圆的，其深度对半宽的比 ≤ 1 。将传入谷地以及散射的波表为Fourier-Bessel级数，本文的方法就是建立在这个基础之上。在应用应力和位移的连续条件之前，用有限的Fourier级数来近似自由场运动。研究结果表明，谷地一般放大了入射的地面运动，同时，位移的放大和复杂性决定于入射Rayleigh波的频率以及谷地的形状。