# Base Isolation by a Soft First Story with Inclined Columns

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**Abstract:** An idea of passive base isolation of buildings is explored, using inclined rubber base isolators or inclined ''soft'' first-story columns. Such a system behaves as a physical pendulum, ''pivoted'' above center of mass, and is more stable than the standard system. Another advantage of the inclination is that the inertia forces of the structure due to rotation about the pivot point cancel to some degree the inertia forces due to the base translation. This is expected to result in smaller relative deformations of the building and smaller internal forces. In this paper, the consequences of this concept are illustrated on a simple model (base isolated single-degree-of-freedom oscillator), but also considering the soil flexibility and the wave nature of strong earthquake shaking (these are associated with additional rotations of the system and also affect the system period). A frequency domain solution is presented for small deformations, rigid foundations embedded in a homogeneous elastic half-space, and horizontal in-plane and Rayleigh wave foundation driving motion.

## INTRODUCTION

Approaches for control of building response to strong earthquake shaking may be classified as active, passive, or hybrid. The active approaches require devices (actuators) that apply forces to counteract the inertia forces in the structure and rely on external supply of energy at the time of an earthquake. Due to cost and requirements for long-term reliability and maintenance, the active control methods are not likely to be used in practice for many structures. The passive approaches employ energy dissipating devices (dampers) or some mechanism of isolation of the building from the motion of the ground. The popular devices are rubber-based bearings, placed between the building and its foundation (Kelly 1986; Taylor et al. 1992). These are used mainly for buildings with <10 stories (to avoid the increasing contribution of the overturning moment and excessive isolator deformations). Another passive isolation system is the friction pendulum system (FPS) (Mokha et al. 1991; Tsopelas and Constantinou 1996). The aim of both of these systems is to reduce the structural response amplitudes by lengthening its ''fundamental'' period, that is, by ''shifting'' it toward the part of the spectrum where strong ground-motion amplitudes are expected to be smaller. [This, however, has to be done with care as the local site conditions may amplify the motions at longer periods (Filiatrault et al. 1990).] They also introduce additional damping into the system and ''decouple'' the structural response from the motion of the ground. [The latter is, however, not true for the higher modes of the system, if the rotational components of ground motion are considered (Wolf and Obernhuber 1991; Todorovska and Trifunac 1993).] There are published results on experiments in a controlled laboratory environment and on analytical and numerical studies of the performance of base isolators, but there is no sufficient evidence on their performance during strong ground shaking, in the near-field of strong earthquakes, and when the rotational (Lee and Trifunac 1985, 1987) and the differential (Trifunac 1997; Trifunac and Todorovska 1997) ground motions are significant.

Recent work on seismic base isolation and on control of structural response is usually based on simplified representation of the ground motion, and the effects of soil-structure interaction are usually ignored. Typically, the excitation is represented only by horizontal ground acceleration (Skinner et al. 1993; Mokha et al. 1991; Tsopelas and Constantinou 1996), and the rotational motions of the base, due to feedback forces from the soil and due to the wave nature of the earthquake motion, are usually ignored. This results in simplified governing equations that ignore potentially important degrees of freedom, present in actual structures. One consequence is that the analysis can then address only the apparent system frequency (which in the absence of soil-structure interaction coincides with the fixed-base fundamental frequency of the system). To suggest how such simplifications affect the model response, in this paper results are presented both for flexible and for rigid soil.

An illustration of one positive and one negative consequence of the wave nature of the excitation and of the soil flexibility is as follows. For excitation by surface Rayleigh waves, for example, the particle motion of the soil on the surface follows a retrograde elliptical orbit. Then the rocking component of the incident ground motion is out of phase with the relative building rotation (permitted by the compliance of the soil) and this reduces the relative response. This beneficial effect is lost and the rocking ground excitation adds to the rigid body rotation (caused by the flexible soil and by the horizontal inertial forces) during prograde elliptical particle motion [e.g., gravity waves (Lomnitz 1996)]. The foundation-structure model described in this paper derives beneficial properties from the fact that the rotation allowed by the inclined isolators (or by inclined columns), relative to the foundation, is 180° out of phase with respect to the relative response associated with constant horizontal acceleration.

Elegant and cost-effective reduction of the relative structural response should be achieved by design that is based on the dynamic characteristics of the complete soil-foundation-building system, with no reliance on external energy sources or complicated devices built in or mounted onto the building. The possibilities for such innovative design are many and should be explored, first by learning about the possible advantages via simple models. This paper presents one such elementary idea. The isolation mechanism is a variant of the old concept of soft first story (Martel 1929; Biot 1934; Kelly 1986), now with inclined rubber base isolators (columns). There are two main advantages of the inclination: (1) The system is more stable (beneficial effect of gravity); and (2) the structure translates and rotates in such a way that, at least for small deformations, the horizontal inertia forces due to rotation reduce, to some degree, the inertia forces due to the base translation. This paper presents a linear solution for a 2D model, in the frequency domain and considering the effects of soil-structure interaction and wave passage. The purpose of this paper is to illustrate the effectiveness of this concept on a simple structural model, but releasing the principal degrees of freedom of the foundation motion, associated with soil-structure interac-

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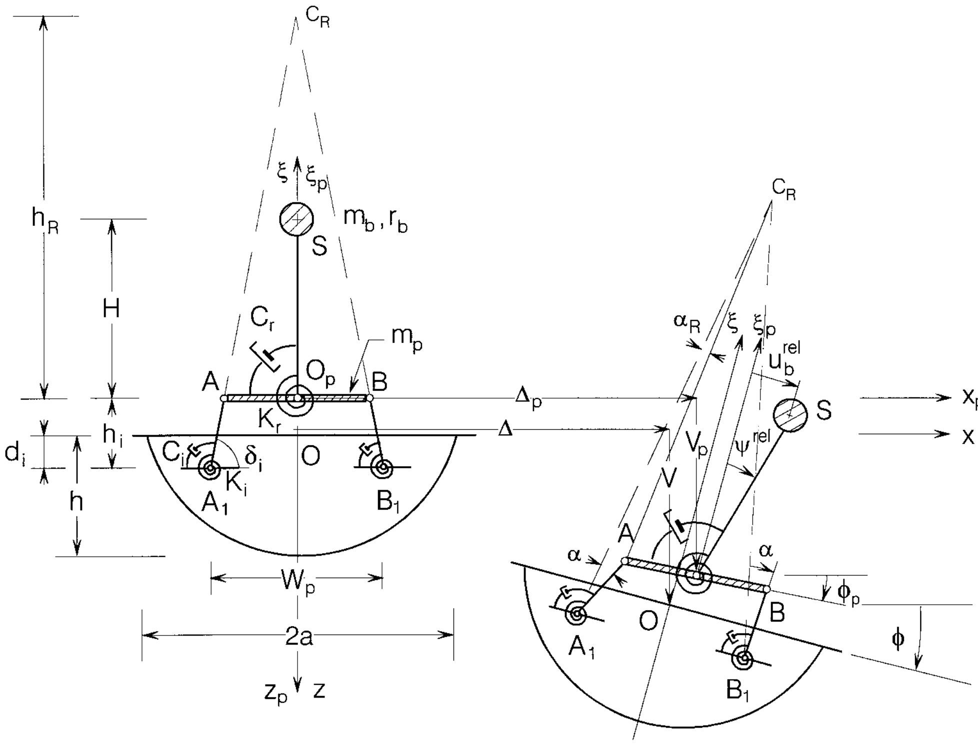


**FIG. 1. Model: Undeformed and Deformed Configurations**

tion and with the wave nature of the earthquake excitation. At this stage of the analysis, issues such as geometric nonlinearities, inelastic response, dynamic buckling of the inclined columns, dynamic instability of the model for coupled horizontal and vertical responses in the near-field of strong earthquakes, and various design issues are not considered.

## MODEL

The model is 2D. Fig. 1 shows a generic representation of the model. The building is represented by an equivalent single-degree-of-freedom (SDOF) oscillator, and the isolation mechanism is a pair of inclined base isolators or columns. Fig. 2 shows an illustration of the model variant with inclined base isolators. The equivalent oscillator consists of a lumped mass connected, by a massless rigid rod, to a rigid platform at point $O_p$. It has mass $m_b$, per unit length in the $y$-direction ($x - O - y$ is an inertia coordinate system), radius of gyration $r_b$, and height $H$. (For a wide building, the rotational inertia of the floors is then represented by $m_b r_b^2$.) The spiral spring stiffness $K_r$ and the dash-pot damping coefficient $C_r$ (both per unit length in the $y$-direction), are such that the natural frequency $\omega_N = \sqrt{K_r/(m_b H^2)}$ equals the fundamental fixed-base circular frequency of the building, and the damping ratio $\zeta$ is such that $2\omega_N\zeta = C_r/(m_b H^2)$. The relative oscillator response is measured by the angle $\psi^{\text{rel}}$, positive if clockwise from the normal to the platform.

The inclined base isolators or first story columns are represented by two massless bars at angle $\delta_i$ from the horizontal (Fig. 1). These are connected by pins to the platform at points $A$ and $B$, and by a pin, a spiral spring, and a dash-pot (with stiffness and damping coefficients $K_i$ and $C_i$, per unit length in the $y$-direction) to the foundation at points $A_1$ and $B_1$. The support points, $A_1$ and $B_1$, may be at depth $d_i$ from the top surface of the foundation, and the platform is at height $h_i$ from these points. The deformation of the inclined columns is measured by the angle $\alpha$ (positive clockwise), equal for both columns for small deformations. The platform has mass $m_p$, per unit length in the $y$-direction. In what follows, the equations of motion will be derived in reference to the generic model in Fig. 1. It is understood that these apply both to inclined isolators and to inclined columns, although explicitly only one or the other will be mentioned.

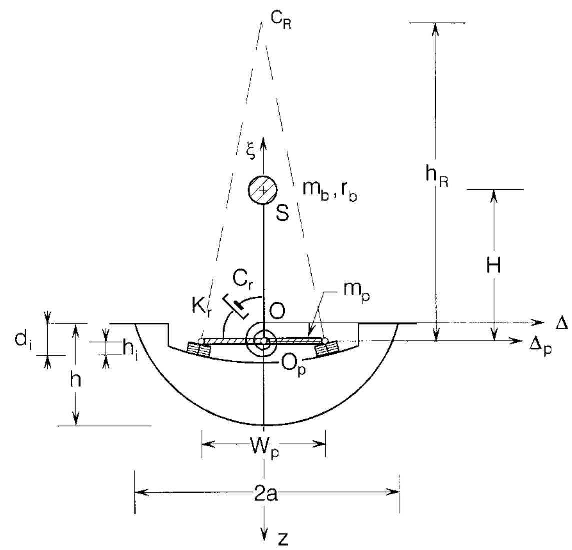


**FIG. 2. Variant of Model: SDOF Oscillator on Inclined Base Isolators**

The structure is supported by a rigid foundation embedded in an elastic half-space. In the examples in this paper, foundation impedance functions and foundation driving forces for semicircular foundation (with embedment depth $h$) and for homogeneous isotropic half-space (with shear-wave velocity $\beta$, shear modulus $\mu$, and Poisson's ratio $\nu$) are used. These were derived and used to study soil-structure interaction effects for shear-beam building models (Todorovska and Trifunac 1990; Todorovska 1993a,b), and the system response for equivalent SDOF oscillator building model (Todorovska 1992; Todorovska and Trifunac 1992). The equations of motion for the model, derived later in this paper, will be valid for an arbitrary shape rigid foundation and also for layered half-space (Luco and Wong 1987).

As the inclined columns deform, the platform rotates about an instantaneous center of rotation $C_R$. For small deformations, point $C_R$ does not change much with time, and is at the intersection of the two lines passing through the columns (Fig. 1). Let $h_R$ be the distance between point $C_R$ and the platform. When the columns rotate clockwise by angle $\alpha$, the platform rotates counterclockwise about point $C_R$ by angle $\alpha_R$. From geometry (Fig. 1), it follows that

$$\frac{\alpha_R}{\alpha} = \frac{h_i}{h_R} \tag{1}$$

Then, the distance traveled by point $O_p$, the arc $h_R\alpha_R$, is equal to $h_i\alpha$.

The absolute motion of points $O$ (on the foundation) and $O_p$ (on the platform) can be described, respectively, by horizontal translations $\Delta$ and $\Delta_p$ (positive in the positive $x$-direction), vertical translations $V$ and $V_p$ (positive in the positive $z$-direction), and rotations $\varphi$ and $\varphi_p$ (positive clockwise). Generalized displacement vectors $\mathbf{\Delta} = \{V, \Delta, \varphi a\}^T$ and $\mathbf{\Delta}_p = \{V_p, \Delta_p, \varphi_p a\}^T$ can be defined for the absolute motion of the foundation and the platform, where $a$ is the half-width of the foundation. From the geometry of the model in Fig. 1, it follows that

$$\Delta_p = \Delta + \frac{h_i - d_i}{a}\varphi a + \alpha_R h_R; \quad \varphi_p = \varphi - \alpha_R; \quad V_p = V \tag{2a–c}$$

Recalling that $\alpha_R h_R = \alpha h_i$, it follows that the generalized displacement vectors $\mathbf{\Delta}_p$ and $\mathbf{\Delta}$ are related by

$$\begin{Bmatrix} V_p \\ \Delta_p \\ \varphi_p a \end{Bmatrix} = [T_1] \begin{Bmatrix} V \\ \Delta \\ \varphi a \end{Bmatrix} + \alpha h_i \begin{Bmatrix} 0 \\ 1 \\ -a/h_R \end{Bmatrix} \tag{3}$$

where the transformation matrix $[T_1]$ is

$$[T_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (h_i - d_i)/a \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The coupling of the vertical accelerations with the horizontal and rocking accelerations will be neglected, as well as the moment due to the gravity forces ($P$-$\Delta$ effects).

## Motion of Equivalent SDOF Oscillator

For harmonic platform motion

$$\mathbf{\Delta}_p = \mathbf{\Delta}_{p,0} e^{-i\omega t} \tag{5}$$

where $\mathbf{\Delta}_{p,0} = \{V_{p,0}, \Delta_{p,0}, \varphi_{p,0} a\}^T$ = complex amplitude of $\mathbf{\Delta}_p$

$$\psi^{\text{rel}} = \psi_0^{\text{rel}} e^{-i\omega t} \tag{6}$$

and the complex amplitude $\psi_0^{\text{rel}}$ is

$$\psi_0^{\text{rel}} = \frac{\dfrac{m_b H^2}{I_{b,O_p}} \left(\dfrac{\omega}{\omega_N}\right)^2 \dfrac{\Delta_{O_p,0}}{H} + \left(\dfrac{\omega}{\omega_N}\right)^2 \varphi_{p,0}}{1 - 2i\zeta \dfrac{\omega}{\omega_N} - \left(\dfrac{\omega}{\omega_N}\right)^2} \tag{7}$$

In (7), $I_{b,O_p} = m_b H^2[1 + (r_b/H)^2]$ is the mass moment of inertia of the oscillator about point $O_p$.

Fig. 3 shows the dismembered suprastructure and the forces and moments of interaction between the equivalent SDOF oscillator, the platform, and the inclined first story columns. The inertia forces and moments are shown by dashed lines. The inertia moments are with respect to the center of the mass. The moment of inertia of the platform is $I_{p,O_p} = (1/12)m_p W_p^2$. The equivalent SDOF oscillator and the platform interact at point $O_p$ with horizontal and vertical forces, $f_{x,O_p}^{(b)}$ and $f_{z,O_p}^{(b)}$, and with moment $M_{O_p}^{(b)}$. The inclined columns interact with the platform at points $A$ and $B$, respectively, via forces $f_{1,t}$, $f_{1,n}$ and $f_{2,t}$, $f_{2,n}$.

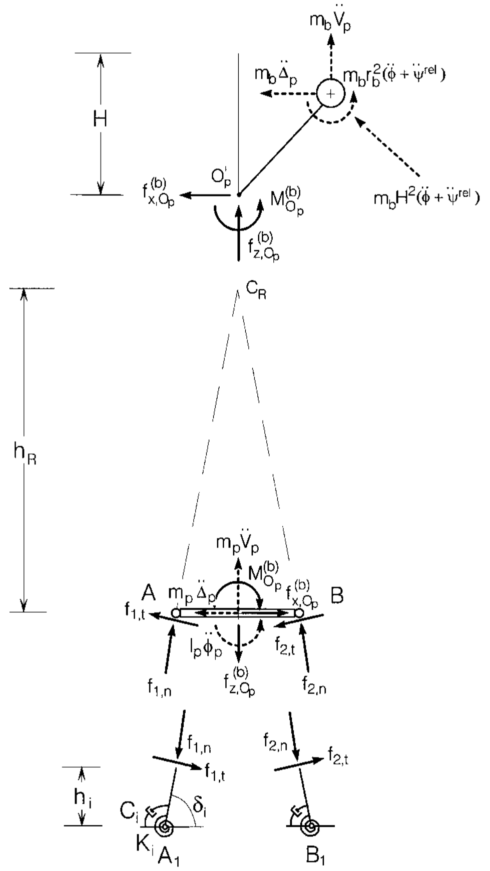


**FIG. 3. Model Dismembered, with Acting Forces and Moments**

Let us define generalized force vector $\mathbf{F}_{O_p}^{(b)} = \{f_{z,O_p}^{(b)}, f_{x,O_p}^{(b)}, M_{O_p}^{(b)}\}^T$. Dynamic equilibrium of the equivalent SDOF oscillator then implies

$$\mathbf{F}_{O_p}^{(b)} = m_b\omega^2[K_{O_p}^{(b)}]\mathbf{\Delta}_p \tag{8}$$

where $m_b\omega^2[K_{O_p}^{(b)}]$ = complex stiffness matrix for the forces of the building. The following matrix $[K_{O_p}^{(b)}]$ is dimensionless:

$$[K_{O_p}^{(b)}] = \begin{bmatrix} k_{11}^{(b)} & 0 & 0 \\ 0 & k_{22}^{(b)} & k_{23}^{(b)} \\ 0 & k_{32}^{(b)} & k_{33}^{(b)} \end{bmatrix} \tag{9}$$

and is identical to matrix $[K_{O_p}^{(b)}]$ in Todorovska and Trifunac (1992, 1993).

## Deflection of Columns of Soft First Story and Motion of Platform

For small deflections (Fig. 3)

$$f_{1,t} = f_{2,t} = \frac{K_i - i\omega C_i}{h_i/\sin\delta_i}\alpha \tag{10}$$

To express the deformation of the inclined columns, $\alpha h_i$, in terms of the displacement of point $O$, the condition of dynamic equilibrium of moments acting on the platform can be used (Fig. 3). Summing moments about point $C_R$ and normalizing by $a$ gives

$$\frac{M_{O_p}^{(b)}}{a} - f_{x,O_p}^{(b)}\frac{h_R}{a} + f_{1,t}\frac{\overline{AC}_R}{a} + f_{2,t}\frac{\overline{BC}_R}{a} - \frac{I_{p,O_p}}{a^2}\ddot{\varphi}_p a + m_p\ddot{\Delta}_p\frac{h_R}{a} = 0 \tag{11}$$

Substituting in (11) for $M_{O_p}^{(b)}$ and $f_{x,O_p}^{(b)}$ from (8), for $f_{1,t}$ and $f_{2,t}$ from (10), for the distances $\overline{AC_R} = \overline{BC_R} = h_R/\sin\,\delta_i$, and introducing frequency $\omega_{i,v}$ and damping ratio $\zeta_{i,v}$ such that

$$\omega_{i,v}^2 = \frac{2K_i/h_i^2}{(m_b + m_p)}; \quad 2i\omega_{i,v}\zeta_{i,v} = \frac{2C_i/h_i^2}{(m_b + m_p)} \qquad (12a,b)$$

gives

$$m_b\omega^2\left[\left(k_{32}^{(b)} - k_{22}^{(b)}\frac{h_R}{a} - \frac{m_p}{m_b}\frac{h_R}{a}\right)\Delta_p + \left(k_{33}^{(b)} - k_{23}^{(b)}\frac{h_R}{a} + \frac{I_{p,O_p}}{m_b a^2}\right)\varphi_p a\right]$$
$$+ m_b\omega^2\left(\frac{\omega_{i,v}}{\omega}\right)^2\left(1 - 2i\frac{\omega}{\omega_{i,v}}\zeta_{i,v}\right)\left(\frac{m_b + m_p}{m_b}\right)\frac{h_R}{a}\alpha h_i = 0 \qquad (13)$$

Substituting into (13) for $\Delta_p$ and $\varphi_p a$ from (3) and introducing

$$\bar{k}_{22} = \frac{m_b}{m_b + m_p}k_{22}^{(b)} + \frac{m_p}{m_b + m_p} - \frac{a}{h_R}\frac{m_b}{m_b + m_p}k_{22}^{(b)} \qquad (14a)$$

$$\bar{k}_{23} = \frac{m_b}{m_b + m_p}k_{23}^{(b)} - \frac{a}{h_R}\frac{I_{p,O_p}}{(m_b + m_p)a^2} - \frac{a}{h_R}\frac{m_b}{m_b + m_p}k_{33}^{(b)} \qquad (14b)$$

$$q_1 = \frac{\left(\dfrac{\omega}{\omega_i}\right)^2}{1 - 2i\left(\dfrac{\omega}{\omega_i}\right)^2\bar{k}_{22}} \qquad (14c)$$

gives

$$\alpha h_i = q_1\left[0 \;\; \bar{k}_{22} \;\; \bar{k}_{22}\frac{h_i - d_i}{a} + \bar{k}_{23}\right]\begin{Bmatrix} V \\ \Delta \\ \varphi a \end{Bmatrix} \qquad (15)$$

Eq. (15) gives $\alpha h_i$ directly in terms of the foundation motion. Then, from (3) and (15), the motion of point $O_p$ can be expressed directly in terms of the foundation motion and

$$\begin{Bmatrix} V_p \\ \Delta_p \\ \varphi_p a \end{Bmatrix} = [T]\begin{Bmatrix} V \\ \Delta \\ \varphi a \end{Bmatrix} \qquad (16)$$

where

$$[T] = [T_1] + [T_2] \qquad (17a)$$

$$[T_2] = q_1\begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{k}_{22} & \bar{k}_{22}\dfrac{h_i - d_i}{a} + \bar{k}_{23} \\ 0 & -\dfrac{a}{h_R}\bar{k}_{22} & -\dfrac{a}{h_R}\left(\bar{k}_{23} + \bar{k}_{22}\dfrac{h_i - d_i}{a}\right) \end{bmatrix} \qquad (17b)$$

## Solving for Motion of Point *O*

Having expressed the motion of point $O_p$, $\mathbf{\Delta}_p = \{V_p, \Delta_p, \varphi_p a\}^T$, in terms of the motion of point $O$, $\mathbf{\Delta} = \{V, \Delta, \varphi a\}^T$, and the forces and moment of interaction between the platform and the equivalent SDOF oscillator, $\mathbf{F}_{O_p}^{(b)}$, in terms of the motion of point $O_p$, the displacements of point $O$, $\mathbf{\Delta}$, can be evaluated by solving the dynamic equilibrium equations of the body in Fig. 4, consisting of the foundation, the columns, and the platform. In Fig. 4, $f_{x,O}^{(s)}$, $f_{z,O}^{(s)}$, and $M_O^{(s)}$ are the horizontal and vertical forces and the moment of interaction between the soil and the foundation (all acting at point $O$), respectively. Let us define generalized force vector $\mathbf{F}_O^{(s)} = \{f_{z,O}^{(s)}, f_{x,O}^{(s)}, M_O^{(s)}/a\}^T$. It consists of two parts (Todorovska and Trifunac 1990)

$$\mathbf{F}_O^{(s)} = \mathbf{F}_{O,0}^{(s)} + \mathbf{F}_{O,\Delta}^{(s)} \qquad (18)$$

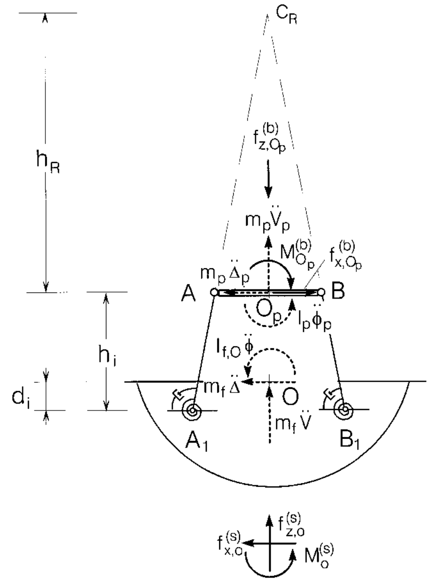


**FIG. 4. Free-Body Diagram of Composite Structure, Consisting of Foundation, Soft First Story, and Platform**

Vector $\mathbf{F}_{O,0}^{(s)}$ consists of the foundation driving forces and moment, and vector $\mathbf{F}_{O,\Delta}^{(s)}$ consists of the compliance forces and moment from the soil (i.e., forces and the moment with which the soil acts onto the foundation because it had moved by generalized displacement vector $\mathbf{\Delta}$). Vector $\mathbf{F}_{O,\Delta}^{(s)}$ can be written in terms of a complex stiffness matrix of the soil, $2\mu[K_O^{(s)}]$

$$\mathbf{F}_{O,\Delta}^{(s)} = 2\mu[K_O^{(s)}]\mathbf{\Delta} \qquad (19)$$

The $3 \times 3$ matrix $[K_O^{(s)}]$ is dimensionless and is related to the matrix $[Q]$ in Todorovska and Trifunac (1990) by

$$[K_O^{(s)}] = -[Q] \qquad (20)$$

For simicircular foundations, analytical expressions have been derived for the terms of matrix $[Q]$ (Todorovska and Trifunac 1990), as well as for $\mathbf{F}_{O,0}^{(s)}$ for incident plane P- and SV-waves and for surface Rayleigh waves.

The foundation driving forces can be expressed in terms of the foundation input motion $\mathbf{\Delta}^{\text{inp}}$, as

$$\mathbf{F}_{O,0}^{(s)} = 2\mu[K_O^{(s)}]\mathbf{\Delta}^{\text{inp}} \qquad (21)$$

If the wave passage effects are considered (kinematic interaction), then $\mathbf{\Delta}^{\text{inp}} = \{V^{\text{inp}}, \Delta^{\text{inp}}, \varphi^{\text{imp}}a\}^T$ is the response of a massless rigid foundation to the action of the incident waves, in the absence of any other external forces. When only the dynamic interaction is considered, $\mathbf{\Delta}^{\text{inp}}$ is a prescribed motion of the base equal to the free-field motion of point $O$ on the surface of the half-space. In the conventional analyses, $\mathbf{\Delta}^{\text{inp}}$ consists of horizontal translation only, which corresponds to vertically incident SV-waves and surface foundations.

Dynamic equilibrium for the structure in Fig. 4 (all moments are about point $O$) implies

$$m_f[M_f]\ddot{\mathbf{\Delta}} + m_p[T_1]^T[M_p]\ddot{\mathbf{\Delta}}_p = [T_1]^T\mathbf{F}_{O_p}^{(b)} - \mathbf{F}_O^{(s)} \qquad (22)$$

where

$$[M_f] = \text{diag}\{1, 1, I_{f,O_p}/a^2\}; \quad [M_p] = \text{diag}\{1, 1, I_{p,O_p}/a^2\} \qquad (23a,b)$$

are dimensionless mass matrices of the foundation and of the platform, and $[T_1]$ = transformation matrix given in (4). In (22), expressing $\ddot{\Delta}$ and $\ddot{\Delta}_p$ in terms of $\Delta$ and $\Delta_p$, and implementing (16), (18), (19), and (21), implies

$$[A]_{3\times3}\mathbf{\Delta} = \frac{1}{2\mu}\mathbf{F}_{O,0}^{(s)} \qquad (24)$$

where the 3 × 3 dimensionless block-diagonal matrix $[A]$ is

$$[A]_{3\times 3} = \frac{m_f\omega^2}{2\mu}[M_f] + \frac{m_b\omega^2}{2\mu}[T_1]^T\left[[K_{O_p}^{(b)}] + \frac{m_p}{m_b}[M_p]\right][T] - [K_O^{(s)}] \tag{25}$$

Then, (24) can be solved for $\mathbf{\Delta}$.

For semicircular foundations, analytical expressions for $\mathbf{F}_{O,0}^{(s)}$ and $[K_O^{(s)}]$ can be found in Todorovska and Trifunac (1990). Eq. (22) can also be used for 3D foundations, symmetric about the plane $x = 0$, and for incident in-plane motion, provided $\mathbf{F}_{O,0}^{(s)}$ and $[K_O^{(s)}]$ are available in numerical form, for example, at selected values of $\omega$.

In the limit when $h_R \to \infty$, $\delta_i \to 90°$ and the first-story columns (or base isolators) are vertical. An independently derived solution for this special case was presented by Todorovska and Trifunac (1993), both for an SDOF oscillator and for a shear-beam model (the condition of dynamic equilibrium of moments was written with respect to point $O$, whereas in this paper it was written with respect to point $C_R$). They analyzed the model response for incident plane P and SV waves, for Rayleigh waves, and for driving horizontal and rocking motions. Selected results for the SDOF model are also presented in Todorovska (1995b) and for the shear-beam model in Todorovska (1996). The equations in this paper become identical to those in Todorovska and Trifunac (1993) in the limit when $h_R \to \infty$. However, the solution for inclined columns cannot be derived from the solution in Todorovska and Trifunac (1993).

## SPECTRAL CHARACTERISTICS OF RESPONSE

### Natural Frequency $\omega_i$

It is convenient to express the stiffness and damping of the soft first story via the circular frequency $\omega_i$ and damping ratio $\zeta_i$ of free oscillations of the suprastructure if the building is rigid and only the soft first story deforms (i.e., for pendulum-like motion with pivot point at $C_R$). In fact, $\omega_{i,v}$ and $\zeta_{i,v}$, defined in (12), represent, respectively, circular frequency and damping ratio for the swing-like motion when the first-story columns are vertical, provided the story height $h_i$ and stiffness and damping constants, $K_i$ and $C_i$, remain the same (Todorovska and Trifunac 1993). For the model in Fig. 1

$$\omega_i^2 = \frac{\omega_{i,v}^2}{\frac{m_b}{m_b+m_p}\left[\left(1-\frac{H}{h_R}\right)^2 + \left(\frac{r_b}{h_R}\right)^2\right] + \frac{m_p}{m_b+m_p}\left[1+\frac{1}{12}\left(\frac{W_P}{h_R}\right)^2\right]} \tag{26}$$

and

$$\zeta_i^2 = \frac{\zeta_{i,v}^2}{\frac{m_b}{m_b+m_p}\left[\left(1-\frac{H}{h_R}\right)^2 + \left(\frac{r_b}{h_R}\right)^2\right] + \frac{m_p}{m_b+m_p}\left[1+\frac{1}{12}\left(\frac{W_P}{h_R}\right)^2\right]} \tag{27}$$

It is clear from (26) and (27) that $\omega_i \to \omega_{i,v}$ and $\zeta_i \to \zeta_{i,v}$ as $h_R \to \infty$. If the inertia of masses $m_b$ and $m_p$ due to gyration about the respective centroids is neglected [i.e., $r_b \to 0$ and $(1/12)(W_P/h_R)^2 \to 0$], for $h_R \geq H$ (which we will consider in this paper), $\omega_i \to \omega_{i,v}$. This indicates stiffening of the soft first story when the columns are inclined (provided $h_i$, $K_i$, and $C_i$ are kept the same). Even if those terms are not neglected, the above statement would still be true for sufficiently large $h_R$.

An interesting case is the one when $H/h_R = 1$ (i.e., when the center of rotation $C_R$ coincides with at the center of mass of the SDOF oscillator). Again, to simplify the interpretation, we can neglect the inertia of the oscillator and the platform due to gyration about their center of mass. Then the mass of the oscillator does not contribute to the inertia during free oscillations of the system about point $C_R$. As only part of the system mass is engaged in the dynamic response, the natural frequency of the system is higher. As $H/h_R$ increases from zero toward unity, the system becomes stiffer and $\omega_i$ increases.

The expression for $\omega_i$ in (26) was derived assuming that the only restoring forces for the pendulum motion are the elastic forces in the springs of the soft first story. The tangent components of the gravity forces of the SDOF oscillator and of the supporting platform also act as restoring forces. For a simple mathematical pendulum with point mass and length $l$, the circular frequency $\omega_g$ is such that $\omega_g^2 = g/l$, where $g$ is the acceleration due to gravity. For our model, the corresponding circular frequency would be such that

$$\omega_g^2 = \frac{g}{h_R}\frac{1 - \frac{H}{h_R} + \frac{m_p}{m_b}}{\left[\left(1-\frac{H}{h_R}\right)^2 + \left(\frac{r_b}{h_R}\right)^2\right] + \frac{m_p}{m_b}\left[1+\frac{1}{12}\left(\frac{W_P}{h_R}\right)^2\right]} \tag{28}$$

The resulting circular frequency $\omega_{i,tot}$ if both the elastic forces and the gravity forces act as restoring forces, would be such that

$$\omega_{i,tot}^2 = \omega_i^2 + \omega_g^2 \tag{29}$$

If the inclination is very small (e.g., $H/h_R = 0.1$ or less), $\omega_g$ will be very small and $\omega_{i,tot} \approx \omega_i$.

### Fixed-Base Frequencies of the System

For vertical first-story columns, the system frequencies are as follows (Todorovska and Trifunac 1993). If the platform is massless ($m_p/m_b \to 0$) and the building is slender so that $r_b/H \to 0$, the model has only one natural frequency, $\omega_{eq}$, such that

$$\frac{1}{\omega_{eq}^2} = \frac{1}{\omega_N^2} + \frac{1}{\omega_i^2} \tag{30}$$

If the platform has mass ($m_p/m_b$) and the building is slender ($r_b/H \to 0$), the model has two natural frequencies, $\omega_1^*$ and $\omega_2^*$ ($\omega_1^* < \omega_2^*$) as follows:

$$\left(\frac{\omega_{1,2}^*}{\omega_N}\right)^2 = \frac{1}{2}\left[1+\left(\frac{\omega_i}{\omega_N}\right)^2\right]\left(1+\frac{m_b}{m_p}\right) \pm \sqrt{\frac{1}{4}\left[1+\left(\frac{\omega_i}{\omega_N}\right)^2\right]^2\left(1+\frac{m_b}{m_p}\right)^2 - \left(\frac{\omega_i}{\omega_N}\right)^2\left(1+\frac{m_b}{m_p}\right)} \tag{31}$$

If the platform is massless ($m_p/m_b \to 0$) but the building is not slender ($r_b/H \neq 0$), the transfer-function of $u_b^{\mathrm{rel}} = \psi^{\mathrm{rel}}H$ also has two peaks (Todorovska and Trifunac 1993). In the more general case, when $m_p/m_b \neq 0$ and $r_b/H \neq 0$, the transfer-function of $u_b^{\mathrm{rel}}$ has two peaks. The peak frequencies are such that $\omega_1^* < \omega_N < \omega_2^*$. Both peaks have lower amplitudes than the single peak in the case when the same building is on rigid isolators.

If the first-story columns are inclined so that $h_R/H = 1$, the platform is massless ($m_p/m_b = 0$), and the building is flexible and slender ($r_b/H \approx 0$), all the forces from the oscillator will be transmitted axially through the inclined columns. Since in this analysis the columns are assumed to be rigid in the axial direction, and there are no lateral forces acting on them, they will not deform ($\alpha = 0$) and will not decouple (isolate) the oscillator from the ground motion. The transfer-function of the relative response $u_b^{\mathrm{rel}}$, will then have peak at the natural frequency of the oscillator $\omega_N$. When $h_R/H \neq 1$, the columns will deform, and as $h_R/H$ progressively increases from 1 toward ∞, a smaller fraction of the forces from the oscillator will be transmitted axially through the columns. Then the soft first story will progressively act as an isolator, and the transfer-function of $u_b^{rel}$ will have a peak at the corresponding system frequency (this frequency will be higher than the one for $h_R/H = \infty$, because the system is stiffer). However, for $h_R/H < \infty$, it can be expected that the transfer-function will still have the peak at the natural frequency of the oscillator, which will progressively disappear as $h_R/H \rightarrow \infty$. This will be seen in the examples presented in the next section.

If the first-story columns are inclined, the platform has mass ($m_p/m_b \neq 0$), and the building is flexible, then there will be a third peak in the transfer-function of $u_b^{rel}$, corresponding to the second mode of the base isolated oscillator. The case $h_R/H = 1$ is again an interesting one. Then, if $r_b/H \rightarrow 0$, all the forces from the oscillator will be transmitted axially through the columns, and the motion of the oscillator and of the soft first story will be decoupled. The transfer-function of $u_b^{rel}$ will have one peak, at $\omega_N$. The transfer-function of $\alpha$ will have a peak at another frequency, $\omega_p$, which is the frequency $\omega_i$ from (30) evaluated for $m_b = 0$ (the deflections of the soft first story will be as if the building is not there).

## Flexible-Base Frequencies

When the foundation soil is ''soft'' and dynamic soil-structure takes place, the system ''softens'' and the system frequencies are modified. The system frequency for a conventional building (e.g., a building on rigid soft first story) is smaller than the fixed-base frequency $\omega_N$. For a building on soft first story with vertical columns, it is expected that the frequency corresponding to the first mode, $\bar{\omega}_1^*$, will be smaller than the corresponding fixed-base frequency, while the one corresponding to the second mode, $\bar{\omega}_2^*$ will not be much different. The change in frequency will be greater if the soil is ''softer'' relative to the building (i.e., if the shear-wave velocity of the soil $\beta$ is smaller and the fixed-base frequency of the building $\omega_N$ is larger).

# RESULTS AND ANALYSES

## Vertical First-Story Columns

To facilitate interpretation of results for the general case, first the results of Todorovska and Trifunac (1993) for vertical soft first-story columns (linear elastomeric bearings; Fig. 2) are briefly summarized. They analyzed the response for three prototype buildings (three-, six-, and 10-stories high). Their analysis showed that for a fixed-base model, more flexible isolators reduce more the amplitudes of the peaks of the transfer-function of the relative SDOF oscillator response. Due to radiation of energy back into the soil, as a result of dynamic soil-structure interaction, the amplitude of the peak in the transfer-function of conventional models is reduced. For the three- and six-story base isolated prototype models, the soil-structure interaction further reduced the amplitudes of those peaks, while for the 10-story model there was no significant additional beneficial effect from the soil-structure interaction. As a result of the wave passage and embedment, the foundation input motion also contains a rocking component. From among incident plane P and SV waves and Rayleigh waves considered, the amplitude of input rocking excitation, per unit amplitude of the horizontal component of the free-field motion, is largest for incident Rayleigh waves and incident SV-waves beyond critical angle. The amplitude of the input rocking depends on the ratio of the width of the foundation and the wavelength of S-waves in the soil. For example, for incident Rayleigh waves the input rocking is the largest when this ratio is 0.5. For very long incident waves, compared with the size of the foundation, the input rocking goes to zero. For the three prototype buildings they studied, the additional input rocking amplified the higher frequency peak in the relative response transfer-function. For incident SV-waves beyond critical angle and for Rayleigh waves, the higher frequency peak has larger amplitude than the lower frequency peak, while it is the opposite when the wave passage effects are ignored. Moreover, the transfer-function of $u_b^{rel}$ may have higher amplitude away from the two system frequencies, which is the case for the six- and 10-story buildings subjected to incident Rayleigh waves. Similar conclusions about the wave passage effects in base isolated buildings were reached by Wolf and Obernhuber (1981).

## Inclined First-Story Columns

In the following, selected consequences of inclination of the first-story columns (or isolators) will be illustrated. It will also be commented on how to search for the ''optimal'' configuration that will be more effective in isolation of the building from strong ground motion. Detailed sensitivity studies and optimization over all the geometrical and material parameters are beyond the scope of this paper and will be presented elsewhere.

Results will be shown for the same three-story base-isolated prototype model of Todorovska and Trifunac (1993). (The results for the six-story model were not significantly different and will not be shown.) The model parameters are shown in Table 1. The stiffness of the soft first story is specified via the circular frequency $\omega_{i,v}$ defined by (12*a*). In the results that follow, this frequency and the height of the soft first story, $h_i$, will be kept constant while the angle of inclination changes (via the ratio $H/h_R$). In Table 1, $n$ is the number of stories, $H_{sb}$ and $W_{sb}$ are the height and width, respectively, of the building (e.g., shear-beam building model). The equivalent oscillator height $H$ and the radius of gyration $r_b$ are then evaluated from $H = H_{sb}/\sqrt{3}$ and $r_b = W_{sb}/\sqrt{12}$ (these are evaluated by assuming that the building deforms primarily in shear, requiring that the equivalent SDOF oscillator has the same mass moment of inertia about point $O_p$ as the shear beam). The dimensionless parameter $\eta = 2a/\beta T$ ($\beta$ is the shear-wave velocity in the soil and $T$ is the period of motion) is introduced. All frequencies are then expressed in terms of $\eta$, for given value of shear-wave velocity in the soil $\beta$. For example, $\eta_N$ is the natural frequency of the building, and in the table it corresponds to $\beta = 250$ m/s (e.g., a site in the Los Angeles basin). In the illustrations shown, the depth of embedment of the isolators, $d_i = 0$, and the effect of gravity is neglected. The foundation is semicircular and the Poisson ratio of the half-space is assumed to be $\nu = 0.333$. The frequencies $\omega_1^*$ and $\omega_2^*$, defined in (31) are such that $\omega_1^*/\omega_N = 0.102$ and $\omega_2^*/\omega_N = 0.507$.

**TABLE 1. Model Properties**

| $n$ (1) | $T_1$ (s) (2) | $H_{sb}$ (m) (3) | $W_{sb}$ (m) (4) | $H/a$ (5) | $r_b/a$ (6) | $m_b/m_f$ (7) | $m_f/m_s$ (8) | $\eta_N\vert_{\beta=250\text{ m/s}}$ (9) | $h_i/a$ (10) | $\omega_{i,v}/\omega_N$ (11) |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0.3 | 15 | 15 | 1.156 | 0.58 | 2.548 | 0.2 | 0.2 | 0.053 | 0.58 |

Note: It is assumed that $W_{sb} = 2a$, that $h_i = 0.4$ m, and that base isolators increase flexibility of building 4 times (in linear range). Assigned values are for $\rho_b/\rho_s = \rho_f/\rho_s = 0.2$, for semicircular foundation.

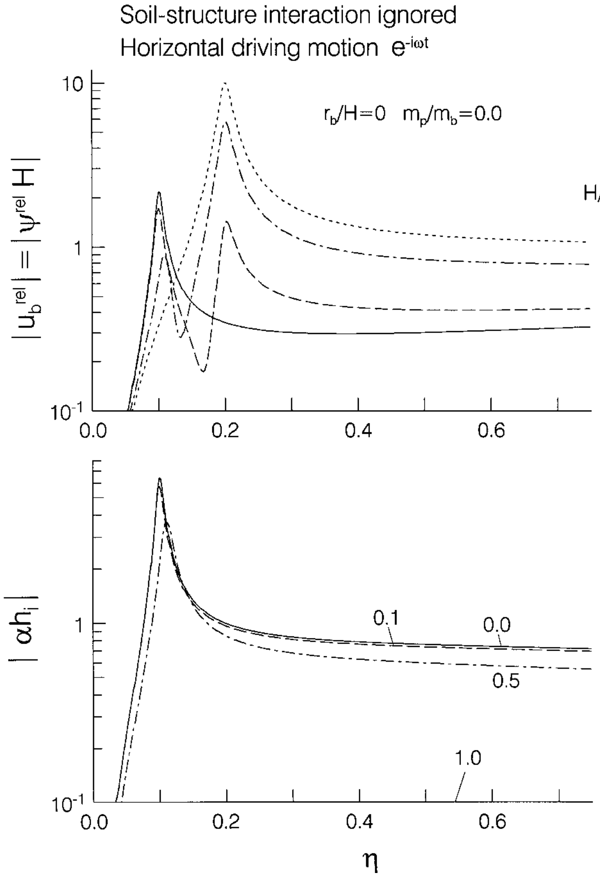


**FIG. 5. Oscillator and Isolators Relative Responses ($u_b^{\mathrm{rel}}$ and $\alpha_i h_i$) for Three-Story Model Building with Massless Platform ($m_p/m_b$ = 0). Soil-Structure Interaction Effects Are Neglected and Excitation Is Unit Amplitude Horizontal Driving Motion. Dimensionless Frequency η Corresponds to Shear-Wave Velocity in Soil β = 250 m/s**

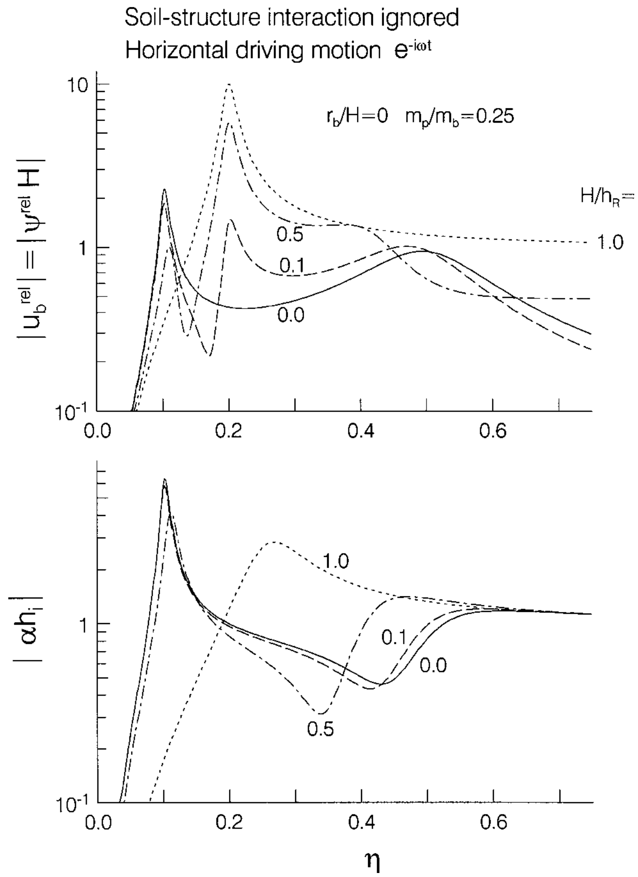


**FIG. 6. Oscillator and Isolators Relative Responses ($u_b^{\mathrm{rel}}$ and $\alpha_i h_i$) for Three-Story Model Building (Platform Has Mass Such that ($m_p/m_b$ = 0.25). Soil-Structure Interaction Effects Are Neglected and Excitation Is Unit Amplitude Horizontal Driving Motion. Dimensionless Frequency η Corresponds to Shear-Wave Velocity in Soil β = 250 m/s**

The results are illustrated in Figs. 5–8. In each figure, the complex amplitude of the transfer function of the relative oscillator response $u_b^{\mathrm{rel}} = \psi^{\mathrm{rel}} H$ and of the isolators response $\alpha h_i$ are shown. The ground excitation is horizontal input displacement of the foundation (with unit amplitude) or incident Rayleigh wave (with unit amplitude of the horizontal displacement on the ground surface). The different types of lines in each figure correspond to different inclinations of the isolators. The inclination is defined in terms of the ratio $H/h_R$. Value $H/h_R$ = 0 corresponds to vertical isolators, and $H/h_R$ = 1 corresponds to the case when the instantaneous center of rotation $C_R$ is at the center of mass of the oscillator. Results are shown for $H/h_R$ = 0, 0.1, 0.5, and 1.0. The corresponding angles of inclination of the isolators are then $\alpha_0$ = 90, 85.06, 66.61, and 49.14°.

The simplest case to study is when the platform has negligible mass compared with the building ($m_p/m_b$ = 0), when the floor inertia is neglected ($r_b/H$ = 0), when the soil-structure interaction is neglected (rigid soil), and when the input motion is horizontal translation only. Fig. 5 shows results for the model under those conditions. It is seen that the transfer functions of $u_b^{\mathrm{rel}}$ and $\alpha h_i$ have one peak when $H/h_R$ = 0, at dimensionless frequency $\eta \approx 0.1$. When $H/h_R$ = 1, the transfer functions of $u_b^{\mathrm{rel}}$ has again only one peak, but now at the fixed-base frequency $\eta \approx 0.2$, and $\alpha h_i = 0$ for all frequencies. The isolators do not deform in shear for this configuration because there are only axial forces in the base isolators. The effect is as if the isolators were rigid, and therefore there is no change in the natural frequency of the model. For intermediate values ($0 < H/h_R < 1$), there are two peaks, one at the natural frequency, $\eta_N = 0.2$, and the other one at $\eta \approx 0.1$. The latter moves toward larger values of η as the isolators become more inclined (i.e., the system stiffens). The relative amplitude of the two peaks varies with $H/h_R$. While the amplitude of the first peak decreases as the isolators become more inclined, it is the opposite for the peak at the fixed-base frequency. For sufficiently large $H/h_R$, the second peak exceeds the amplitude of the first peak. The rotation of the platform, which occurs when the isolators are inclined, reduces only the amplitude of the first peak.

Fig. 6 shows results for the same prototype building model, but now the platform has mass ($m_p/m_b$ = 0.25). The ground and the loading conditions are the same as in Fig. 5, and $r_b/H$ = 0. It is seen that, for vertical isolators ($H/h_R$ = 0), the transfer-function of $u_b^{\mathrm{rel}}$ has two peaks, and when $H/h_R$ = 1 only one peak. In this case, the relative response of the isolators is not zero, and has one peak (at $\eta_p$ = 0.26) as predicted in the discussion on the spectral characteristics of the response. For the intermediate values of $H/h_R$, $u_b^{\mathrm{rel}}$ has three peaks; the second peak is at the fixed-base frequency of the model $\eta_N$. For this case, when the isolators become more inclined, the amplitude of the first peak becomes smaller. The amplitude of the second peak, however, becomes larger. There will be some optimal configuration, that compromises between these two competing effects. This optimal configuration also will depend on the frequency content of the excitation.

Figs. 7 and 8 illustrate results for a model with platform mass (same as in Fig. 6) but for flexible foundation soil (β = 250 m/s). In Fig. 7, only dynamic soil-structure interaction is accounted for (the input motion is horizontal driving motion), whereas in Fig. 8 the wave passage effects (kinematic interaction) are also considered. The excitation is a Rayleigh wave

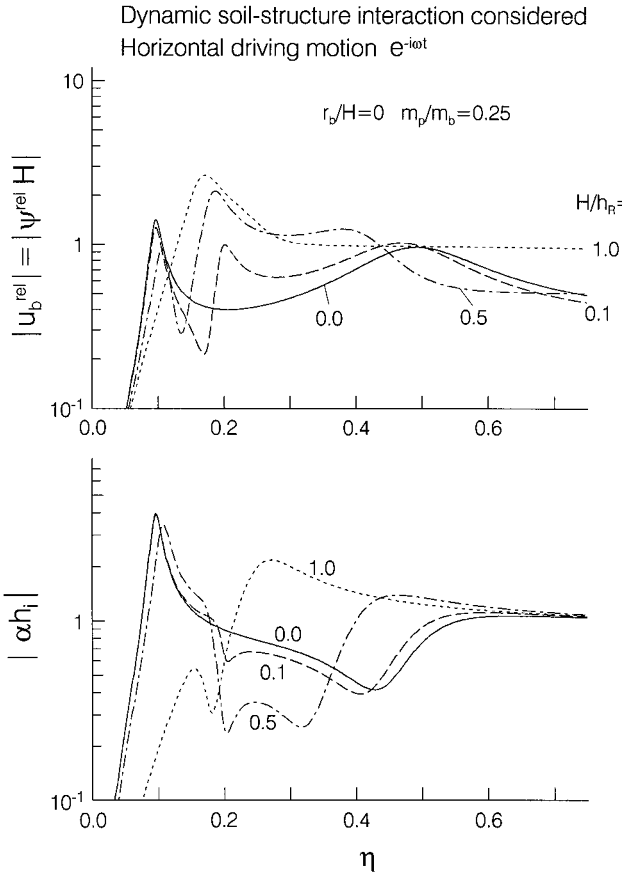


**FIG. 7. Oscillator and Isolators Relative Responses ($u_b^{rel}$ and $\alpha_i h_i$) for Three-Story Model Building (Platform Has Mass Such that ($m_p/m_b$ = 0.25). Dynamic Soil-Structure Interaction Effects Are Considered, and Excitation Is Unit Amplitude Horizontal Driving Motion. Shear-Wave Velocity in Soil Is β = 250 m/s**

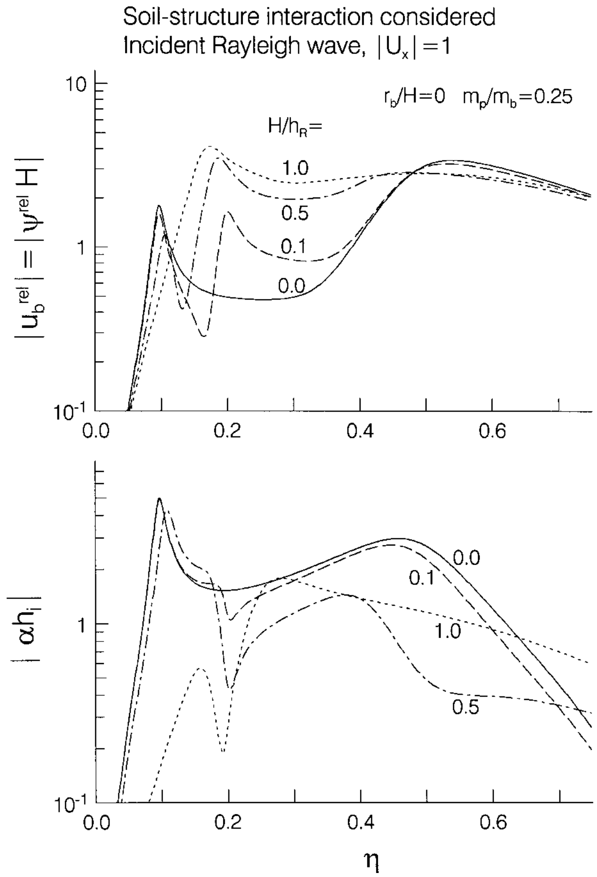


**FIG. 8. Oscillator and Isolators Relative Responses ($u_b^{rel}$ and $\alpha_i h_i$) for Three-Story Model Building (Platform Has Mass Such that ($m_p/m_b$ = 0.25). Dynamic Soil-Structure Interaction and Wave Passage Effects Are Considered, and Excitation Is Rayleigh Wave with Unit Horizontal Amplitude. Shear-Wave Velocity in Soil Is β = 250 m/s**

with unit horizontal amplitude at the free surface. At the free surface, the particle motion during passage of a Rayleigh wave is a retrograde ellipse, and for $\nu$ = 0.33 the amplitude of the vertical motion is 1.56.

Fig. 7 shows that the amplitudes of all three peaks decrease due to the dynamic soil-structure interaction. Comparison of Figs. 7 and 8 shows that the relative response amplitudes can be noticeably larger for the incident Rayleigh wave, although the input horizontal motion has the same amplitude. This is due to the rocking component of the ground excitation of the incident Rayleigh wave. Analysis of foundation input motion for embedded foundations (Todorovska 1993; Todorovska and Trifunac 1993) shows that the rocking component of the foundation input motion for Rayleigh waves increases with frequency, reaching a maximum at about $\eta$ = 0.5, then decreases reaching a minimum at about $\eta$ = 1, again increases, and so on. In Fig. 8, the additional (rocking) excitation amplifies the higher frequency peaks. It is seen that the third peak (corresponding to the second mode of the base-isolated SDOF oscillator) is higher than the first peak (corresponding to the first mode of the base-isolated SDOF oscillator) for all values of $H/h_R$ considered.

It is apparent from the examples in Figs. 5–8 that, for the parameters considered, the inclination is close to ''optimum'' for small values of $H/h_R$ (e.g., $0 < H/h_R < 0.25$)—then all the peaks have comparable amplitudes. For response in the time domain, the frequency content of the excitation should also be considered in determining the optimum configuration. As the shape of the Fourier spectrum of an earthquake may vary significantly with the local site conditions, earthquake magnitude, and distance, no general rules can be established a priori. However, for given building characteristics, site conditions and building location with respect to the surrounding earthquake faults, an optimal configuration could be determined (e.g., for the most damaging earthquake). An optimal configuration could also be determined for all possible earthquakes during the life of the building, via probabilistic seismic hazard analysis (Todorovska 1994, 1995a), for example, subject to constraint(s) that the probability $P[u_b^{rel} > x]$ is minimum (with respect to some norm). Another constraint could be that, for a given probability of exceedance, the value of $u_b^{rel}$ that will be exceeded during the exposure (e.g., expected service time of the structure) is minimized. Examples of such analyses are beyond the scope of this paper.

## Implementation of Inclined Base Isolators

In the above examples, the stiffness of the soft first story was specified by the circular frequency $\omega_{i,v}$ that is related to the stiffness of the spiral spring $K_i$, vertical height $h_i$, and the sum of the masses of the platform and of the SDOF oscillator [see (12*a*)]. Therefore, $\omega_{i,v}$ defines the stiffness of the base isolators $K_i$. In the model in Fig. 1, the base isolators were represented by rigid rods and spiral springs, which is equivalent to base isolators infinitely stiff in the vertical direction. The stiffness of the equivalent spiral spring can be evaluated from the stiffness of the base isolators for lateral deformations. To implement base isolators at an angle, they have to be attached to inclined surfaces at the base and to a rigid ''platform'' at their top, also with an inclined surface, so that when the system is at rest, only axial static forces are transmitted through the isolators.

## DISCUSSION AND CONCLUSIONS

An analysis was presented of the response of a simple building model isolated by inclined rubber based isolators (or a soft first story with inclined columns). It was anticipated that, due to the inclination, in addition to stabilizing the system, it would also be possible to reduce the relative building response. The purpose of the paper was to illustrate in the simplest possible terms the effect of the inclination, and some physical phenomena associated with its response, resulting from the soil flexibility and the wave nature of earthquake ground motion. The model is 2D. The building is represented by an equivalent SDOF oscillator. The axial deformation of the base isolators (columns of the soft story) is ignored. The effects of dynamic and kinematic soil-structure interaction are considered via placing the structure on a rigid foundation embedded in an elastic homogeneous half-space. Solution for small (linear) deformations was presented in the frequency domain. The effects of gravity, coupling of the vertical motions with the horizontal translations and the rotations, buckling of the columns, inelastic material response, and dynamic instability were neglected at this preliminary stage of analysis.

The deformation of the soft first story with inclined columns or base isolators is such that it forces the building in a pendulum-like motion with the pivot point coinciding with the instantaneous center of rotation of the soft first story, $C_R$. For a classical soft first story (vertical columns), $C_R$ is at infinity. When $C_R$ is at the center of mass of the oscillator, the deformations of the oscillator and of the first story columns are decoupled. Then, the soft first story can transmit only axial forces from the building, the building and the first story vibrate independently, and the isolation mechanism is not effective.

The system has, in general, three frequencies, two of which are associated with the two modes of the conventionally base isolated SDOF oscillator (by vertical rubber base isolators), and the third (intermediate) is at the natural frequency of the oscillator. In general, the transfer-function of the relative oscillator response has three peaks, at the above mentioned frequencies. For vertical base isolators, the middle peak disappears, whereas when the instantaneous center of rotation is at the center of mass of the oscillator, the other two peaks disappear. If the isolators are more inclined, the amplitudes of the lower and higher frequency peaks in the transfer-function of the relative oscillator response are smaller, but the middle peak is larger.

This analysis shows that the rocking component of the foundation input motion can amplify the high frequency amplitudes of the response transfer function, so that the relative response amplitudes corresponding to the second mode of the base isolated SDOF system are higher than the first peak, corresponding to the first mode of the base isolated system. Thus, both soil-structure interaction and realistic description of actual ground motion, based on the principles of wave propagation, should be included in future analyses of base isolated structures.

If the excitation has a broad band of frequencies, an optimum value of the inclination angle is such that $0.05 < H/h_R < 0.1$ (for the three-story prototype model analyzed in this paper, $H/h_R = 0.1$ corresponds to angle $\delta_i = 85°$). The inclination stiffens the system. However, for such a small inclination, the change of the frequency of the first mode is small.

In view of the pendulum-like rigid body motion of the SDOF oscillator, comparison with the FPS comes to mind (Mokha et al. 1991). One fundamental difference between the two is in the nature of the restoring forces. For the FPS case, at each bearing, it is a fraction of the weight supported, whereas for the system in this paper it is mainly the elastic force in the isolators (the action of gravity, however, would contribute toward stability of the system). Although the period of this ''rigid body motion mode'' does not depend on the mass for the FPS case, but only on the radius of curvature (pendulum length), for the model in this paper the period is not significantly affected by the gravity, due to the small inclination and consequently large radius of curvature. Another difference is that, for the FPS case, the FPS bearings move as individual penduli with their own pivot point. For synchronous horizontal base motion and symmetric spatial arrangement of identical bearings (usually considered in studies of the FPS system), the motion of the building itself is only translation. For the model in this paper, however, there is only one pivot point and the building itself moves as a pendulum (this is so even for synchronous horizontal base motion and no soil-structure interaction).

For earthquake excitation, for the model presented in this paper, the optimum inclination and its effectiveness will depend on the frequency content of the excitation and can be evaluated for a scenario earthquake or via probabilistic seismic hazard methodology, subject to specified constraints (e.g., minimization of the relative displacement response of the building). Examples of such an analysis are beyond the scope of this paper.

The model described in this paper may represent a good approximation of the actual response for excitations with wavelengths longer than the width of the foundation (Fig. 1). For shorter wavelengths, the simplifying assumption that the foundation is rigid results in excessive scattering and filtering of the incident wave motion. In the real soil-structure system, the situation is further complicated by relative deformation of the foundation, which will result in differential motion of the column supports (isolators). We leave analyses of these effects for future study.

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