NONLINEAR DAMAGE ACCUMULATION IN STOCHASTIC FATIGUE OF FRP LAMINATES

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Abstract

A number of nonlinear damage accumulation rules have been proposed to deal with the problem of fatigue damage accumulation in fiber reinforced plastics (FRPs). For FRP materials such nonlinear damage rules usually result from two major physical concepts, in which fatigue degradation is associated with a gradual reduction of stiffness or ultimate strength, respectively. Although such methods for composite materials have been investigated for a few decades, most comparisons with experimental results have considered only two-stress-level loadings; none has considered stochastic loadings. The objective of this investigation is to verify the applicability of a number of nonlinear damage accumulation theories in the context of composite FRP laminates subjected to simulated stochastic loadings. The experimental data needed for such an investigation was obtained on specimens made of cross-ply E-glass woven-roving fibers and vinyl ester resin. Variable-amplitude fatigue tests are carried out using simulated narrowband stochastic stress histories with two root-means-square (RMS) stresses on. The fatigue lives estimated using several different nonlinear damage accumulation rules are compared with those based on the linear damage accumulation rule and the mean values of the experimental lives.

Introduction

In recent years, the use of fiber-reinforced plastics (FRPs) in aerospace, automotive, marine, and civil infrastructure applications has been constantly increasing. In such applications, structures are constantly subjected to repetitive loadings, prompting questions about fatigue of the FRP materials. Due to the nature of the loadings encountered in those applications, a primary issue to be resolved in designing against fatigue is that of how damage accumulates under random (stochastic) loadings. Although variable-amplitude fatigue of composite materials has been studied before, most studies have considered two-stress-level (high-low or low-high) loadings or the so-called "block-spectrum" loadings, as applied to aerospace applications. Very little is known about how fatigue damage accumulates under more general (and more realistic) loading conditions like the wave-induced stochastic loadings encountered in marine structures.

The most simple approach to predicting fatigue life under any type of variable-amplitude loading is based on the linear [also known as the Palmgren-Miner (Miner, 1945)] damage accumulation rule. Simple analytical formulas, based on the Palmgren-Miner rule and random process theory, are available to calculate the expected fatigue life for stochastic loadings. Among these is the well-known Rayleigh approximation (Miles, 1954), for narrowband (*i.e.*, Rayleigh distribution of peaks) loadings.

At the same time, many investigators have suggested that the linear Palmgren-Miner rule may be inadequate to quantify the fatigue behavior of FRP composites because damage accumulation in these materials exhibits more load-sequence effects than it does in metals. Although some pure empirical concepts have been utilized, the bulk of nonlinear damage accumulation hypothesis proposed for FRPs can generally be classified into two main categories: “residual strength” or “residual stiffness.” Both approaches are phenomenological and associate fatigue damage with changes in the material properties.

The *residual strength* method is based on the assumption that the progressive fatigue damage gradually reduces the ultimate static strength of the material. A number of nonlinear damage accumulation rules resulting from this method have been developed by Broutman and Sahu (1972), Hashin (1985), Yang and Liu (1977), and, most recently, by Schaff and Davidson (1997a and 1997b). On its part, the *residual stiffness* method encompasses a number of approaches that seek to quantify fatigue life based on the reduction of stiffness that accompany the accumulation of fatigue damage. The main advantage of the method in comparison with the residual strength is that residual stiffness can be easily measured during testing. This approach has been utilized in the work of Hahn and Kim (1976), and Yang et al. (1990). A relatively simple to apply variation of the residual stiffness method is the “fatigue modulus” approach of Hwang, ***et al.*** (1995)

The great variation in matrix and fiber material systems and manufacturing techniques used today makes the selection and use of suitable damage accumulation method challenging. At the same time, given the many theoretical studies surrounding composite fatigue, there is very little known as to how these analytical predictions compare with experimental data in the cases of variable-amplitude loadings in general and stochastic loadings in particular. Among the few studies in this area, Read and Shenoi (1995) compared the predictions of a form of the residual strength approach, a form of the fatigue modulus approach, and the Palmgren-Miner rule with available experimental lives under two-stage loadings. It was observed that the Palmgren-Miner rule performed reasonably well in comparison with the more complex theories and the study could not reveal an advantage of any particular method. Sarkani ***et al.*** (1999) conducted a major investigation into the fatigue damage accumulation of FRP laminates and joints under simulated narrowband stochastic loading. This study, however, considered only the linear damage accumulation rule and the Rayleigh formula associated with narrowband loadings to predict the fatigue lives of the specimens. The main objective of the present study is to determine how various nonlinear damage accumulation rules compare with the linear damage accumulation rule in predicting the fatigue lives of FRP specimens under simulated stochastic loadings. Due to space limitation, only four residual strength-based rules are considered

Experimental Investigation

The specimens used in the investigation were manufactured from a base FRP laminate consisting of an alternating 0/90° stacking sequence of thirty plies made of 0.6-mm thick E-glass woven-roving fabric and 510-A vinyl ester resin. This layup consisted of several panels measuring 610x914 mm using the Seemann Composite Resin Infusion Molding Process (SCRIMP). The specimens were approximately 95 mm wide and a hole of approximately 16-mm in diameter was drilled in the middle of each specimen to serve as a stress concentrator. The initial static strength of the specimens was determined at 168MPa. Constant-amplitude fatigue tests were conducted at five stress amplitude levels (137.9, 103.4, 69.0, 55.2, and 51.7 Mpa, respectively). The results were fit to a power-low S-N curve in the form of $\log(N) = \log(K) - m\log(S)$ in which S is the stress amplitude, N is the fatigue life, and K and m are the fatigue life coefficient and the fatigue strength exponent, respectively. The linear regression of the data yielded K=21.197 and m=8.504. Subsequent variable-amplitude tests were carried out under simulated stationary narrowband Gaussian loadings at two root-mean square (RMS) levels (41.4 and 31.0 Mpa) with three specimens per RMS level. The mean values of the experimental fatigue lives at those RMS levels were 81,100 and 633,900 cycles, respectively.

Linear Fatigue Damage Accumulation

Fatigue damage is most conveniently expressed in terms of a non-decreasing damage function D, which is postulated to be D=0 for material that has never experienced fatigue and D=1 for material experiencing a fatigue failure. Fatigue life under arbitrary loading can be evaluated using a function $D(n,S)$ yielding the damage after n cycles with constant stress amplitude S . In the case of linear damage accumulation, this function simplifies to $D=n/N$ in which N is the constant-amplitude fatigue life under this stress given by the S-N curve. Moreover, when the fatigue loadings represent a stationary narrowband Gaussian stochastic process, such as those used in this study, the fatigue life under the linear damage accumulation hypothesis can most simply be estimated by the so-called "Rayleigh approximation," which can be written as (Miles, 1954)

$$E[\Delta D] = \frac{1}{E[N]} = \frac{1}{K} 2^{m/2} S_{rms}^{m} \Gamma\left(1 + \frac{m}{2}\right) \tag{1}$$

where $E[\Delta D]$ is the expected (average) damage increment from a single cycle, $E[N]$ is the expected number of cycles to failure, S_{rms} is the RMS stress, and $\Gamma(\bullet)$ is the gamma function. The fatigue life predictions of Eq.(1) are given in Table 1 along with the average experimental lives.

RMS stress [MPa]	Rayleigh approximation [Cycles]	Experimental [Cycles]
41.4	41,900	84,100
31.0	483,600	633,900

Table 1. Fatigue life predictions based on the linear damage accumulation

Nonlinear Fatigue Damage Accumulation

When $D(n,S)$ is nonlinear, the approach followed is to compute the accumulated damage after each loading cycle S_i by using a recursive procedure in the form of $D(n_{i,eq}, S_i) = D_{i-1}$, $D_i = D(n_{i,eq} + 1, S_i)$. Here D_i and D_{i-1} denote the damage accumulated after the ith and (i-1)th cycle. The first of these two equations is used to computer an "equivalent" number of cycles, $n_{i,eq}$, such that the damage under $n_{i,eq}$ cycles with amplitude S_i is equal to D_{i-1} after which D_i is computed from the second equation for $n_{i,eq}$ +1 cycles. Four nonlinear damage accumulation models resulting from the application of the residual strength approach are considered below.

Let $R(n,S)$ denote the residual strength of such specimen after it is subjected to n cycles of constant amplitude S. A number of proposed $R(n,S)$ functions can be expressed as the particular cases of the solution of the following general differential equation:

$$\frac{dR(n)}{dn} = -B\frac{Cn^{C-1}}{AR(n)^{A-1}} \tag{2}$$

in which A, B, and C are model (material) parameters. Integrating the equation and taking into account the initial condition $R(0)=R_0$ results in

$$R(n,S)^A = R_0^A + (R_0^A - S^A)(n/N)^C \tag{3}$$

Taking into account the failure condition $R(N,S)=S$, a damage function can be defined as

$$D(n,S) = [R_0 - R(n,S)]/(R_0 - S) \tag{4}$$

Note that the use of Eqs.(3) and (4) in the general nonlinear damage accumulation procedure outlined above results in continuous damage but discontinuous residual strength from one loading cycle to another. The continuity of the residual strength function can be assured if instead of Eq.(3) one uses the following function (Hashin, 1985)

$$R(n,S)^A = R_{0i}^A + (R_0^A - S^A)(n/N)^C \tag{5}$$

in which R_{0i} is the residual strength prior to the application of the constant amplitude loading with amplitude S. Note, however, that now the damage function (4) becoming discontinuous across the loading cycles. It should be noted that the two approaches defined by Eqs. (3) and (5) lead to different fatigue life predictions. Particular cases of the general approach are considered below.

1) A=C=1. Using those values in Eqs. (4) and (5) yields the model first introduced by Broutman and Sahu (1972) . Residual strength and damage can be computed as

$$R_k = R_{k-1} + (R_0 - |S_k|)\frac{1}{2N_k} \quad D_k = D_{k-1}\frac{R_0 - |S_{k-1}|}{R_0 - |S_k|} + \frac{1}{2N_k} \tag{6}$$

in which D_k, D_{k-1}, R_k, and R_{k-1} denote the accumulated damage and residual strength after the application of k and k-1 segments of the loading sequence, respectively. Note that one segment is equal to half a cycle (n=1/2). Note also that using Eq.(3) with such values of A and C yields the linear damage accumulation. The fatigue life predictions resulting from Eqs.(6) are shown in Table 2. As can be seen, both predictions are lower than the fatigue lives predicted by the Palmgren-Miner rule.

RMS stress [MPa]	Predicted Fatigue Lives [Cycles]	Experimental [Cycles]
41.4	28,100	84,100
31.0	378,100	633,900

Table 2. Fatigue lives predicted by the Broutman and Sahu residual strength model

2) A=1. Using Eqs. (3) or (5), damage can be computed respectively as:

$$D_k = \left(D_{k-1}^{\frac{1}{C}} + (1/2N_k)\right)^C \quad D_k = D_{k-1}\frac{R_0 - |S_{k-1}|}{R_0 - |S_k|} + \left(\frac{1}{2N_k}\right)^C \tag{7}$$

The first of these equations is equivalent to the model used by Schaff and Davidson (1997). In this study the parameter C is assumed to depend on S by the following expression

$$C(S) = 1 + \nu(S_{\max} - |S|)/S_{\max} \tag{8}$$

where S_{max} is the maximum of all $|S_i|$ in the loading sequence and ν is a stress-independent model parameter. The fatigue life predictions based on Eq. (7a) for ν values varying between −1 and +1 are shown in Table 3. As can be seen by those results, the Schaff and Davidson model fails to improve the predictions in comparison with the Palmgren-Miner rule. It also seems that the Palmgren-Miner rule results are an upper bound of the predictions of this approach. The predictions obtained using the modified Schafff and Davidson (Eq.7b) are presented in Table 4. It can be seen that this equation results in predictions that are closer to the experimental results. The values of ν, for which those "optimal" predictions are obtained, are not the same for the two RMS levels considered, prompting the conclusion that a stress dependent ν would be a better choice.

ν	Predicted Fatigue lives, [Cycles]	
	RMS=41.4 MPa	RMS=31.0 MPa
1.0	42,400	476,700
0.9	42,400	478,000
0.5	42,400	481,300
0.2	42,400	483,000
0.1	42,400	483,100
0 *	42,400	483,100
-0.1	42,400	483,100
-0.2	42,400	482,500
-0.5	42,000	478,100
-0.9	40,000	462,100
-1.0	9860	107,471
Exper.	84,100	633,900

*Equal to the linear damage model

Table 3. Fatigue lives predicted by the Schaff and Davidson residual strength model

ν	Predicted Fatigue lives,	
	RMS=41.4 MPa	RMS=31.0 MPa
1.0	143,100	2,428,000
0.9	133,100	2,193,000
0.5	78,100	1,278,000
0.2	48,100	683,100
0.1	38,100	518,100
0 *	28,100	378,100
Exper.	84,100	633,900

*Equal to the Broutman and Sahu model

Table 4. Fatigue lives predicted by the modified Schaff and Davidson residual strength model

3) C=1. Using C=1 and Eq.(3) results in Hashin's approach (Hashin, 1985). The corresponding equation is

$$D_k = \frac{R_0 - \left\{ \left[R_0 - D_{k-1}(R_0 - |S_k|) \right]^A - (R_0^A - |S_k|^A)\frac{1}{2N_k} \right\}^{\frac{1}{A}}}{R_0 - |S_k|} \qquad (9)$$

Following Hashin's approach, the parameter A is treated it as a constant independent of S. Table 5 shows the predicted fatigue lives using Eq. (9) for different values of A. This model does not result in improved predictions in comparison with the Palmgren-Miner rule.

A	Predicted Fatigue lives [Cycles]	
	RMS=41.4 MPa	RMS=31.0 MPa
0.1	42,400	483,000
1.0*	42,400	483,100
2.0	42,400	481,700
5.0	40,900	474,300
10.0	39,600	481,700

*Equal to the linear damage model

Table 5. Fatigue lives predicted by the Hashin's residual strength model

Conclusions

The fatigue life predictions of four nonlinear damage accumulation theories based on the residual strength approach are investigated in the context of composite FRP laminates subjected to simulated stochastic loadings. The predictions of the nonlinear damage accumulation models are compared with the linear damage accumulation rule and with experimental data obtained from specimens of cross-ply E-glass woven-roving laminates. Almost all nonlinear damage accumulation models considered predicted fatigue lives that were generally lower than those predicted by the linear damage accumulation rule. In only one case, the Schaff and Davidson residual strength model modified to provide continuous change of residual strength across the variable-amplitude loading, the predictions could be made closer to the experimental results than those yielded by the Palmgren-Miner.

References

Broutman J., and Sahu S. (1972), "A New Theory to Predict Cumulative Fatigue Damage in Fiberglass Reinforced Plastics." *Composite Materials: Testing and Design*, ASTM STP 497, pp. 170-188.

Hahn, H.T., and Kim, R.Y. (1976), "Fatigue Behavior of Composite Laminate." *J. Composite Materials*, Vol.10, p.156-180.

Hashin, Z. (1985). "Cumulative Damage Theory for Composite Materials: Residual Life and Residual Strength Method. " *Composites Science and Technology*, 23, pp. 1-19.

Hwang, W., Lee, C. S., Park, H. C., and K. C. Han. (1995). "Single and Multi-Stress Level Fatigue Life Prediction of Glass/Epoxy Composites, " *Journal of Advanced Materials* , 20, pp. 3-9.

Miles, J.W., (1954). "On Structural Fatigue Under Random Loading," *J. Aero. Soc.*, 21, pp. 753-762.

Miner, M.A., (1945), "Cumulative Damage in Fatigue, " ASME, *J. Applied Mechanics*, 12(3), pp159-164.

Read, R. J. C. L., and Shenoi, R. A. (1995). "A Review of Fatigue Damage Modeling in the Context of Marine FRP Laminates. " *Marine Structures*, 8, pp. 257-278.

Sarkani, S., Michaelov, G., Kihl, D.P., and Beach,J.E., (1999). "Stochastic Fatigue Damage Accumulation of FRP laminates and joints, " *J. Structural Engineering*, Vol. 125(12), pp.1423-1431.

Schaff, J.R., and Davidson, B.D. (1997a). "Life Prediction Methodology for Composite Structures. Part I –Constant Amplitude and Two-Stress Level Fatigue." *J. Composite Materials*, Vol.31, pp. 128-157.

Schaff, J.R., and Davidson, B.D. (1997b). "Life Prediction Methodology for Composite Structures. Part II –Spectrum Fatigue." *J. Composite Materials*, Vol.31, pp. 158-181.

Yang, J.N., and Liu, M.D. (1976). "Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates." *J. Composite Materials*, Vol. 11, p. 176-203.

Yang, J. N., Jones, D. L., Yang, S. H., and Meskini, A. (1990). "A Stiffness Degradation Model for Graphite Epoxy Laminates. " *J. Composite Materials*, 24, pp. 753-769.