AE 525a/ME 525 Midterm Exam
Wednesday, June 9, 1999
5 - 7 PM

Professor: M. Dravinski   Name____________________________

Instructions. Closed books and notes. No calculators.

1. (25 pts.) Consider the following system of equations

$$\begin{aligned} ay + z &= b \\ ax + bz &= 1 \\ ax + ay + 2z &= 2 \end{aligned} \tag{1}$$

   (a) By investigating ranks of relevant matrices determine for which fixed values of a and b (if any) the system possesses unique solution, and

   (b) a one-parameter family solution.

2. (25 pts.) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \tag{2}$$

   (a) Determine the eigenvalues of $\mathbf{A}$.

   (b) Determine the three orthonormal eigenvectors of $\mathbf{A}$.

3. (25 pts.) Let the matrix $\mathbf{A}$ be given by Eq.(2) and let matrix $\mathbf{B}$ is defined by

$$\mathbf{B} = \mathbf{A}^3 - 3\mathbf{A} + \mathbf{I} \tag{3}$$

   (a) Determine the eigenvalues and eigenvectors of $\mathbf{B}$.

   (b) Investigate if $\mathbf{B}$ is a positive definite matrix and explain why. (Recall, $B = \mathbf{x}^T\mathbf{B}\mathbf{x} > 0$)

4. (25 pts.) Consider a least-squares fit of the form

$$b = x_0 + x_1 a_1 + x_2 a_2 \tag{4}$$

   (a) For the data $(b_i, a_{i1}, a_{i2}); i = 1, ..., m > 3$, derive the corresponding normal equations $\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b}$.

   (b) For a specific data

b_i	a_{i1}	a_{i2}
1	1	2
2	2	4
-1	3	6
-2	4	8

   solve the normal equations for the unknowns $x_i; i = 0, 1, 2$.