ME 525/ AE 525a HW#2

36. (a) Show that the set

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$

can possess a nontrivial solution only if $\lambda = 1$ or $\lambda = -3$.

37. The matrix

$$\begin{bmatrix} 0 & a & 1 & b \\ a & 0 & b & 1 \\ a & a & 2 & 2 \end{bmatrix}$$

is the augmented matrix of a system of linear algebraic equations $\mathbf{A}\mathbf{x} = \mathbf{c}$. Determine for what fixed values of a and b (if any) the system possesses the following
(a) A unique solution.
(b) A one-parameter solution.
(c) A two-parameter solution.
(d) No solution.

38. Determine the dimension of the vector space generated by each of the following sets of vectors:
(a) $\{1, 1, 0\}, \{1, 0, 1\}, \{0, 1, 1\}$
(b) $\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \{1, 1, 1\}$
(c) $\{1, 1, 1\}, \{1, 0, 1\}, \{1, 2, 1\}$.

39. Determine whether the vector $\{6, 1, -6, 2\}$ is in the vector space generated by the vectors $\{1, 1 - 1, 1\}, \{-1, 0, 1, 1\}, \{1, -1, -1, 0\}$.

40. (a) Determine the angle θ between the vectors

$$\mathbf{u} = \{1, 1, 1, 1\}; \mathbf{v} = \{\mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{1}\}$$

(b) Determine the Hermitian angle θ_H between the complex vectors

$$\mathbf{u} = \{0, i, 1\}; \mathbf{v} = \{i, 1 + i, 1\}$$

46. Show that the set of equations

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_1 + x_2 - x_3 &= 1 \\ 3x_1 + 3x_2 - 5x_3 &= 1 \end{aligned}$$

possesses a one-parameter family of solutions, and verify directly that the vector $\mathbf{c}$ whose elements compose the right-hand members is orthogonal to all vector solutions of the transposed homogeneous set of equations.