SOLVING VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS DELIVERY AND PICK-UP USING BACTERIAL FORAGING OPTIMIZATION ALGORITHM

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The vehicle routing problem with simultaneous delivery and pick-up (VRPSDP) is a type of the classical vehicle routing problem (VRP) where customers require simultaneous delivery and pick-up service. Deliveries are supplied from a single depot at the beginning of the vehicle’s service, while pick-up loads are taken to the same depot at the end of the service. The objective is to determine the optimal set of routes to satisfy both the delivery and pick-up demand of the customers. VRPSDP is an NP-hard combinatorial optimization problem; therefore exact methods are incapable of dealing with large scale VRPSDP instances. For this reason, in recent years, it is observed that studies have been focused on metaheuristic methods. In this study, a heuristic solution approach based on Bacterial Foraging Optimization Algorithm (BFOA) which is an algorithmic approach inspired by the foraging behavior of bacteria, has been proposed and its performance has been evaluated. In the scope of this study, VRPSDP is solved in order to minimize the total distance travelled and the results have been compared with the insertion based heuristic algorithm. The proposed algorithm is tested using a benchmark data set available from the literature. The computational result shows that the proposed method is generally better than the insertion based algorithm.

Keywords: vehicle routing problem with simultaneous delivery and pick-up, bacterial foraging optimization algorithm

1. Introduction

The VRP with simultaneous delivery and pick-up is a basic problem in reverse logistics and can be described as follows: customers require not only the delivery of goods but also the simultaneous pick up of goods from them (Ai and Kachitvichyanukul, 2009). A set of vehicles of limited capacity must visit a set of customers located on a transportation network. The goal is to minimize the overall length of the vehicle routes (Dell’Amico et al. 2006). VRPSDP is firstly proposed by Min (1989). Dethloff (2001) discussed the importance of VRPSDP in the reverse logistic operations. He proposed a mathematical formulation for the problem to minimize the total traveled distance subject to maximum capacity constraint of the vehicle. He also developed an insertion-based heuristic that use four different criteria to solve the problem. Crispim and Brandão (2005) are the first who presented a metaheuristic approach for VRPSDP. Their method is a hybrid of tabu search (TS) and variable neighborhood search (VNS). Several heuristic and metaheuristic methodologies have been proposed for VRPSDP; the most recent ones were published by Montané and Galvão (2006), Chen and Wu (2006), Bianchessi and Righini (2007), Gajpal and Abad (2009), Ai and Kachitvichyanukul (2009), Zachariadis et al. (2009). The detailed survey of VRPSDP studies can be found in Subramanian et al. (2010). In this paper, Bacterial Foraging Optimization Algorithm (BFOA) which is a novel optimization algorithm based on the social foraging behavior of E. coli bacteria is proposed to solve the VRPSDP. To the best knowledge of the authors, this is the first study which uses BFOA to solve VRPSDP. The test problems are used to compare the performance of the proposed BFOA and that of insertion based heuristic proposed by Dethloff (2001) using benchmark instances given by Dethloff (2001). The way Bacteria look for regions of high levels of nutrients can be seen as an optimization process. This idea was explored by Bremermann (1974). Passino (2002) proposed the bacterial foraging optimization algorithm (BFOA). He defined the biological features of bacterial foraging

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behavior and then used BFOA for distributed optimization and control. BFOA was successfully applied to several real-life problems such as optimal controller design (Kim ve Cho, 2005a), harmonic estimation (Mishra, 2005), machine learning (Kim and Cho, 2005b), transmission-loss reduction (Tripathy et al. 2006) and scheduling (Wu et al. 2007). BFOA was also hybridized with some other evolutionary computing techniques. Kim et al. (2007) proposed Genetic Algorithm (GA) – BFOA hybridization, Biswas et al. (2007a) proposed PSO-BFOA and Biswas et al. (2007b) differential evaluation – BFOA hybridization.

The remainder of the paper is structured as follows. Section 2 explains biological basis of BFOA. In Section 3 the proposed BFOA is given. Computational experimentations, comparisons and analyses are presented in Section 4. Finally, Section 5 contains the conclusion of the study and recommendations for future.

2. Bacterial Foraging Optimization Algorithm (BFOA)

*E. coli* bacteria exist in intestines of most animals on the earth. *E. coli* bacterium has a control system, which directs its behaviors in food foraging. Biological studies show that the foraging process includes four steps: (1) search for a possible food region, (2) decide to whether or not enter into the possible food region, (3) perform a careful search if it enters into a new region, (4) decide to either keep stay in the current region or emigrate into a new and more ideal region, after they consume some food in the current region. To mimic the aforementioned biological principles shown in the foraging behavior of *E. coli* bacteria, Passino (2002) presented BFOA for distributed optimization and control. An optimization period of BFOA consists of three events (Wu et al. 2007): chemotaxis event, reproduction event and elimination and dispersal event (Passino, 2002). These events are illustrated as follows:

2.1.Chemotaxis event: When all the flagella of bacteria sway counterclockwise, the *E. coli* bacteria move forward; and when all the flagella sway clockwise, the *E. coli* bacteria slow down and tumble in its place (Wu et al. 2007). And a result of these two behaviors bacteria take step called chemotactic step (Passino, 2002). In BFOA, the formula of direction change is given as follows:

$$
\theta(j+1,k,l) = \theta(j,k,l) + C(i)\phi(j)
$$

(1)

where \(\theta(j, k, l)\) stands for the current position of the \(i\)th individual; \(j, k \) and \(l\) indicate for the numbers of chemotaxis events, reproduction iterations and elimination and dispersal events, respectively; \(\phi(j)\) stands for the new advancing direction decided by flagella swaying and \(C(i)\) for the step length.

2.2.Reproduction event: To keep the population size constant, a portion of bacteria with superior foraging strategies are reproduced to take the places of removed ones. The reproduction event is fulfilled as follows: Let the population size is \(S\), the number of bacteria to be removed is defined as \(Sr = S/2\). Firstly, rank all of the individuals regarding to evaluations of their positions, and then remove out the last half (\(Sr\)) individuals and reproduce one copy for each of the residual half (\(Sr\)) ones to keep a constant population size (Wu et al. 2007).

2.3.Elimination and dispersal event: Some factors possibly cause gradual or sudden changes of the population. The population changes might include that all bacteria in the current region are killed or part of them move to a new region. The elimination and dispersal event is an evolution operation designed to imitate this biological process and triggered with probability \(P_{ed}\). If certain individual satisfies the dispersal condition, it should be deleted and then a new individual should be generated. This operation means that the individual moves to a new position. The procedure of solving optimization problem with bacterial foraging includes: (1) encoding the solution of the problem, (2) designing the evaluation function, (3) generating the initial population, (4) optimizing the objective function by the interaction of individuals(Wu et al. 2007). There are three loops of optimization in the algorithm. The outer loop is elimination and dispersal event, the middle loop is reproduction event and the inner loop is chemotaxis event. The inner loop, chemotaxis event, is the core of the three loops. It corresponds to the direction selection scheme which is the central step employed by a living creature to search food and in charge of the decisions that whether or not enter into a new region, how long does the individual stay in the current region, which direction should be selected in the next move. These decisions mean that the chemotaxis event has important influence to the algorithm convergence. The detailed survey of BFOA study can be found in Passino (2002).
3. The Proposed BFOA for VRPSDP

3.1. Encoding Method

As mentioned before, BFOA is a population-based search method that imitated the foraging behavior of the individuals in the swarm as a searching method. In the proposed BFOA, a swarm of S bacteria is served as searching agent for a specific problem solution. A bacterium’s position ($\theta$) simply is a sequence of n customer nodes and represents a solution of the problem. The ability of a bacterium to search for solution is represented by its direction vector $\phi(j)$ which drives bacterium movement. In the BFOA iteration step, every bacterium moves from one position to another position based on its direction. By moving from one position to another, a bacterium evaluates different solutions for the problem. An example of encoded and decoded solution are illustrated in Figure 1.

![Customer Sequence](image)

- **a) Encoded Solution**

| 3-1 | 2-1 | 1-1 | 4-1 | 9-1 | 10-2 | 8-2 | 5-2 | 7-2 | 6-2 |

- **b) The Information About Customer Assignments to Routes**

Route 1: 0-3-2-1-4-9-0  
Route 2: 0-10-8-5-7-6-0

- **c) Decoded Solution**

**Figure 1.** An Example of Encoded Solution and This Code’s Solution

Considering customer sequence given in Figure 1a, customers are started to be assigned to the first route. If capacity constraint is violated, a new route is opened and the customers not yet visited are assigned considering customer sequence. As all customers are assigned to the routes, the depot is added these routes and the traveled distance is calculated according to the Equation (2).

$$ J = \sum_{a,b} c_{ab} $$

$J$ is total traveled distance (objective function value), $n$ is number of customers, $c_{ab}$ is distance between nodes $a$ and $b$. As long as the solution improves in the algorithm, a memory table which has nodes is updated for the consecutive pairs of customers. In this table, the values of pairs of customers are incremented 1 and these values are used in elimination and dispersal loop to increase the probability of reaching the most appropriate solution. The detailed explanation of the steps is given in the next sub-sections:

3.2. Initial Solution: The initial route is constructed using the nearest neighborhood heuristic (NNH) (Gajpal and Abad, 2009). In this study, due to the number of bacteria is large, first customers can’t be selected randomly. NNH is used for each customer.

3.3. Chemotaxis loop: Bacteria begin to chemotaxis loop with their initial positions and initial objective function ($J$) values. The values of direction vector are chosen randomly from the discrete number sets {-1, 0, +1} and these numbers are assigned to each customer on the customer sequence. “-1” implies that the customer replaces with the customer located before step length ($C$). “0” implies this customer save its position and finally “+1” implies that the customer replaces with the customer located after step length ($C$). The customer sequence representing the bacterium is seperated five parts which have equal lengths and a part is chosen randomly. Defined bacterium direction is used for this part and bacterium’s position is modified. So the bacterium move along to the defined direction as ($C$). If solution improves the bacterium continues to move along the same direction and objective function ($J$) is computed according to this direction and the memory table is updated. This is maintained until the solution doesn’t improve or maximum number of swim steps ($N_s$) is reached. If one of these two criteria is met, one chemotactic step finishes. At the end of the chemotactic step bacterium’s objective
function \( J \) is the best value of this bacterium and all of the \( J \) values of the bacterium obtained in each chemotactic step are saved to be used at the reproduction step. Chemotactic step continues until the number of chemotactic steps \( (N_r) \) is reached. Each chemotactic step is repeated for each bacterium.

3.4. Reproduction loop: Three alternative reproduction methods are used for the proposed BFOA as follows: Copy, Reverse and Crossover. In Copy method, objective function values obtained in each chemotactic step are summed and called \( J_{\text{health}} \). Firstly, all of the bacteria are ranked regarding to evaluations of \( J_{\text{health}} \) and then the last half bacteria are removed out and one copy for each of the residual half ones are reproduced. In Reverse method, firstly the copy method is applied but instead of removing half of bacteria, each bacterium’s customer sequence is reversed and new bacterium is obtained. An example about reverse method is given in Figure 2.

In Crossover Method, firstly the copy method is executed then two bacterium is crossed according to the objective function values from the first half of the population using roulette cycle. The bacteria obtained end of the crossover are replaced with their copies at the second half of the population. This procedure continues as half number of bacteria. For the proposed BFOA, two point crossover known common in genetic algorithm is executed. In this situation, random two points are selected from two bacterium customer sequences which will be crossed and fields between these two points are exchanged. And so new bacteria are obtained.

3.5. Elimination and Dispersal loop

A random value is generated between (0-1) for each bacteria in this step and if this number is smaller than \( P_{\text{ed}} \) defined before, bacterium is eliminated. For eliminated bacterium, customer pairs with large memory values are integrated in memory table and new bacterium is generated. An example of memory table is given in Table 1.

Example memory table starts with depot (0) and ends with 10th customer. New customer sequences are generated integrating the largest customer pairs at each row considering the customers are assignable or not assignable. Firstly, the depot row (0) is started to generate the route. In this row, 0-6 cell has the largest value (24) in Table 1. So after the depot (0) customer 6 is placed at the first available position in the sequence. Then the next row is customer 6’s row. At this row 6-7 cell has the largest value (43) and customer 7 is placed at the next available position in the sequence. This procedure is maintained until all customers are placed to a position. The final customer sequence of the example is illustrated in Figure 3.

4. Computational Results

Parameter levels are determined and three reproduction methods are used to find the most appropriate parameter levels and the effect of alternative methods of reproduction for solving VRPSDP. In according to the ANOVA and Duncan’s test results, the most appropriate parameter levels and reproduction method used for experimental problems solutions are illustrated in Table 2.
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The results presented in Table 3 implies that RCRS results are better than RC, RS and TD. Against RCRS results, the proposed BFOA obtained better solutions for 24 problems of 40 test problems and is the more successful in SCA8 and CON8 data sets where vehicle capacity is smaller than the SCA3 and CON3 data sets. The improvement obtained for the SCA8 and CON8 is %95 while this rate is %25 for SCA3 and CON3. The better solutions against RCRS results are given in bold characters in Table 3.

5. Conclusion and Further Study
In this paper we proposed a new metaheuristic called BFOA to solve VRPSDP. The results are compared with RCRS introduced by Dethloff (2001) using Dethloff(2001)’s data sets. The proposed BFOA obtained generally better solutions. To the best of our knowledge, the BFOA has not been applied to the VRP and the different classes in the literature. In this manner this paper makes several significant contributions. First it presents a new metaheuristics method for VRPSDP and it describes a possible way for solving VRPSDP using BFOA. Second, the performance and efficiency of the algorithm can be increased by integrating BFOA with local search and other metaheuristics methods for solving VRPSDP. Third, the obtained results with BFOA can be applied to other hard combinatorial optimisation problems such as VRP with backhauls and mixed delivery and pick-up VRP and the performance of BFOA can be evaluated for these problems for future research.
References