Abstract:
Existing supply chain location-inventory models optimize over inventory policies and distribution center locations, but use a deterministic approach that does not take into consideration the uncertainty of demand. We present an extension to the deterministic multi-echelon joint location-inventory model that adds uncertainty by including a new variable that reflects the probability of different scenarios. We test the model using three different demand scenarios. To compare the results for the stochastic case with those of the deterministic case, the model is solved as a deterministic model under one scenario, and the cost of assuming a scenario other than the true scenario is calculated.

Keywords: Supply chains, location problem, integer programming, uncertainty modeling.

INTRODUCTION

Globalization and free trade policies have allowed raw materials and products to flow across the globe with drastically less delays and restrictions. In addition, the rise of the internet at the end of the nineties has expanded the reach of trade through e-commerce and online payments systems, resulting in low-cost and fast transactions. In the same time, the customers’ database has increased exponentially as companies have become unbounded by geography or location. Nowadays, a customer can order a product from a retailer located in another continent and receive it within a week. Therefore, companies and trading firms have faced a new challenge in designing an efficient supply chain that can reduce costs related to inventory storage and transportation.

In a competitive market, cost is one of the main indicators of a company’s health. Therefore, if the supply chain for a certain company adds more cost than value to the product, then the company will face an issue in expanding or even surviving in the market. On the other hand, a well designed supply chain can cut costs and boost the company’s growth. Walmart, the biggest retailer store in the USA, has experienced a phenomenal growth due to an efficient supply chain.

In addition to cost, an efficient supply chain will have a significant environmental effect by reducing carbon emissions and other green house gases (GHGs) through lowering inventory transportation frequency, subsequently a decrease in the gas consumption. Therefore, the supply chain efficiency has an environmental aspect that is becoming important as international agreements, such as Kyoto Protocol, are imposing restrictions on carbon emissions.

Demand is another issue that arises when supply chain management is discussed. Unplanned surges in demand can impose a pressure on the chain and create delays that usually result in higher costs and lower customers’ satisfaction. On the other hand, if demand turns up to be lower than expected, the company will face unnecessary costs of inventory stored at DCs. If the commodity is perishable, the company will face a higher cost of losing inventory with time. Therefore, demand adds uncertainty that can have a critical effect on the supply chain design.

Decisions in supply chain management are divided into strategic and operational decisions. Operational decisions deal with everyday processes, such as inventory policy, which deals with the amount of inventory ordered and stored at each warehouse along the supply chain. Furthermore, retailers’ assignment to DCs is another important operational variable that can reduce cost. Strategic decision includes long term (fixed) effects, such as distribution centers’ location and capacity.

This paper is a part of a continuous effort of providing a practical network design that can satisfy demand and reduce costs in the same time. Models previously have been approached differently. For instance, some models deal with inventory management while others deal with both inventory and DC locations.
Some models consider the supply chain as one stage with inventory flowing along it, whereas, some models consider the supply chain two stages, where the first stage is moving inventory between factories and DCs, and the second stage is shipping inventory from DCs to retailers. In terms of demand uncertainty, some models incorporated uncertainty using scenarios that reflect the probability of demand quantities. These models are referred to as stochastic or scenario-based models. All these models have different outputs and solution methods.

The model in this paper is an extension to model formulated by Diabat, Richard, and Codrington [7]. This extension adds uncertainty to the original deterministic model by including a new variable ($q$) that reflects the probability of different scenarios. This model optimizes the retailers' assignment to DCs and the amount of holding inventory at each DC. However, it does not optimize the number of open DCs and their locations. This decision is assumed to be made before the model can be solved.

The extension has been tested with three DCs and ten retailers and according to three different demand scenarios. The total cost and retailers’ assignment have been recorded. To test the efficiency of a scenario-based model, the model is solved as a deterministic model under one scenario. Then, another demand scenario is simulated to calculate the cost of the wrong demand expectation. Further discussion is provided in later Experimental Analysis section.

The remainder of the paper is organized as follows: Section II surveys the literature on stochastic supply chain management; Section III presents the mathematical formulation of the model; Section IV provides a numerical analysis of the model; and Section V concludes with future research directions.

LITERATURE REVIEW

Supply chain design has been an important topic of research in an effort to reduce cost related to inventory storage and transportation. Daskin and Owen [6] provide a qualitative discussion of different approaches to solve supply chain models. These approaches are static (or deterministic), dynamic, stochastic, and scenario planning. The study concludes that stochastic programming will experience advances that can provide better solutions and business decisions. Comparing different approaches, Zheng [23] discusses the difference between optimal solution under a deterministic EOQ policy and a stochastic model. His research indicates that at large quantities, the difference between deterministic and stochastic models is small and does not exceed one eighth.

The factors affecting a supply chain design can be divided into: strategic, tactical, and operational. The strategic factors included investment decisions, such as location and number of DCs. The tactical factors depend on short term and long term goals, such as inventory policies. Operational deals with the assignment or retailers to DCs and daily demand forecast. An early study conducted by Daskin, Coullard, and Shen [5] looks at the supply chain of blood banks in Chicago and results in a nonlinear integer programming that includes the supply chain of blood banks in Chicago and results in a nonlinear integer programming that includes the strategic and tactical decisions. [16] provides more information on the model and solution approach. You and Grossman [21] reformulate [16] model as mixed nonlinear integer programming. The reformulated model aims at removing the dependency on the assumption of identical variance-to-mean ratio because it reduces the applicability of the model in real situations. The inventory-location model is developed more to include demand uncertainty. Shu et al [17] explore [16] further incorporating uncertainty in demand and solving it using variable fixing.

Another approach is done by Amiri [3] who includes the number of plants in the model in addition to DCs. Goetschalckx, Vidal, and Dogan study the dynamics of domestic logistics systems, developing a model that determines the optimal mix considering production and inventory schedules. The model has seasonal and constant demand. Further research in this area is presented by MirHassani et al [13].

Tanonkou, Benyoucef, and Xie [19] explore the stochastic model further including a number of DCs and customers with random demand and supply lead time. The model is solved using Lagrangian relaxation-based approach. Stochastic modeling is also presented by Santos et al [15]. The model is based on Sample Average Approximation (SAA) with an accelerated Bender decomposition. The SAA method is based on solving a model for a number of random samples followed by deciding the confidence intervals for different solutions. The model has proved to decrease cost variability compared to deterministic models.

Miranda and Garrido [12] study the network design of a supply chain with stochastic demand and risk pooling (DNDRP). The model maximizes the reduction of total cost as the variability and holding costs increase because the number of warehouses and inventory costs decrease.

Mulvey et al [14] create a model that would help decision makers about uncertainty in the supply chain. The model was improved by Yu and Li [22], who aim at reducing the number of variables to make it more robust. The new proposed model included fewer variables, reducing CPU and time needed to solve it.

In [4], Cheung and Powell formulate a two stage model that minimized the cost of a stochastic demand. The first stage deals with moving inventory from the plant to the warehouses based on forecasted demand.
The second stage is moving the inventory from the warehouses to the customers when they send an order. Using an experimental case, the model indicates that having two warehouses per customer is more efficient than having one warehouse per customer as the standard policy. Another approach is done by Kamath and Pakkala [11]. A Bayesian model is formulated. The model covers long term planning. It is noted that previous information on demand trends could help significantly reduce the total cost of the supply chain because the cost drops as the variation of demand declines.

The stochastic model presented in this paper is an extension of a multi-echelon joint inventory-location model formulated by Diabat, Richard, and Codrington [7]. This model is based on the Economic Order Quantity policy (EOQ). Further description of this model can be found in the Model Description section.

Snyder, Deskin, and Teo [18] presented a similar stochastic extension to the inventory-location model by Deskin, Coullard, and Shen [5]. The original model is approached through the statistical distribution of demand unlike the EOQ policy approach. The extension adds the probability of different scenarios based on demand and cost. The extension is referred to as the stochastic location model with risk pooling (SLMRP). The SLMRP has proven to be cost effective compared to deterministic models by having low regret values. This is illustrated more in the Experimental Analysis section.

A. Model Description

The presented model considers distribution centers that order inventory directly from a factory and ship orders to assigned retailers. It optimizes the number of DCs, assignment of retailers to DCs, and the holding inventory in order to minimize total cost. Holding inventory is the amount of inventory stored at each warehouse, and it usually has a variable cost depending on the size of the inventory.

This model is an extension to an existing model formulated by Diabat, Richard, and Codrington [7]. The original model is based on the Economic Order Quantity (EOQ) policy and investigates the strategic and tactical decisions previewed as number of DCs and inventory management. In specific, it includes the fixed cost of opening a DC, placing order costs, and shipment of inventory along the supply chain. The same model concept is investigated in [8], [9], and [10]. However, this model does not take into consideration the uncertainty of the supply chain and fluctuation of demand. Also, the model is only feasible for one product. The proposed extension will introduce new variables that will take into account scenarios of demand probabilities. However, it is important to note that the number of DCs is a strategic decision that will be made first regardless of the demand scenarios, whereas retailers assignment and inventory levels will be chosen according to the scenarios.

Four major cost components are considered in the objective function of the model. They are as follows:

i. DC fixed-location cost: the cost to establish and operate a distribution center;

ii. DC-retailer unit-shipping cost: the cost to ship one unit of a commodity from a DC to a retailer;

iii. Plant fixed-location cost: the cost associated with establishing and operating a plant; and

iv. Plant-DC unit-shipping cost: the cost to ship one unit of a commodity from a plant to a DC.

B. Notation

To formulate the problem, the following notation is used:

Sets

- \( I \) set of retailers, indexed by \( i \)
- \( J \) set of potential facility sites, indexed by \( j \)
- \( N \) set of scenarios, indexed by \( n \)

Parameters

Demand

- \( d_{in} \) daily demand by retailer \( i \in I, n \in N \)

Costs

- \( h_i \) holding cost per unit per day at retailer \( i \in I \)
- \( f_j \) fixed cost of opening site \( j \in J \)
- \( t_b \) number of days per planning period
- \( o_i \) fixed cost per order by retailer \( i \in I \)
- \( õ_j \) fixed cost per order by site \( j \in J \)
- \( s_{ijn} \) per unit shipping cost from site \( j \) to retailer \( i \in I, j \in J \)
- \( ñ_j \) per unit shipping cost from plant to site \( j \in J \)

Weights

- \( β_{tra} \) weighting factor for transportation
- \( β_{inv} \) weighting factor for inventory

Probabilities

- \( q_n \) probability that scenario \( n \) occurs, for \( n \in N \)

Others

- \( T_{ij} \) order period from site \( j \) by retailer \( i \in I \)
- \( Ŵ_j \) time between orders by site \( j \) from the plant, \( j \in J \)

Decision Variables
The formulation of the SLMRP problem can be stated as follows:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{n \in \mathcal{N}} q_n \left( \sum_{i \in \mathcal{I}} \left( \beta_{\text{inv}} \frac{u_{nk}}{T_{ij}} + \frac{1}{2} \beta_{\text{inv}} \left( h_i - \bar{h}_j \right) d_{in} t_b T_{ij} + \frac{1}{2} \beta_{\text{inv}} \bar{h}_j d_{in} t_b \max \{ T_{ij}, T_{ij} \} \right) Y_{ijn} + \\
& \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{\text{trn}} d_{in} t_b \left( s_j + z_{ijn} \right) Y_{ijn} \right) \\
\text{s.t.} & \quad \sum_{j \in \mathcal{J}} Y_{ijn} = 1, \quad \forall i \in \mathcal{I}, \quad \forall n \in \mathcal{N} \quad (1) \\
& \quad Y_{ijn} \leq X_j, \quad \forall i \in \mathcal{I}, \quad \forall n \in \mathcal{N} \quad (2) \\
& \quad Y_{ijn} \in \{0,1\}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \quad \forall n \in \mathcal{N} \quad (3) \\
& \quad X_j \in \{0,1\}, \quad \forall j \in \mathcal{J} \quad (4) \\
& \quad T_{ij} / T_{ij} = 2^{c_{ij}}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J} \quad (5) \\
& \quad C_{ij} \in \mathbb{Z}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J} \quad (6) \\
& \quad T_{ij} > 0, \quad \forall j \in \mathcal{J} \quad (7) \\
& \quad T_{ij} > 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J} \quad (8)
\end{align*}
\]

The objective function measures the total cost according to specific scenarios. The term \( q_n \) measures the probabilities for different scenarios, where \( \sum_{n \in \mathcal{N}} q_n = 1 \). It is important to note that only the daily demand by retailers and the shipping cost between DCs and retailers will be affected by the scenarios, while other costs will remain constant. Furthermore, the \( Y \) variables change according to the scenario while \( X \) variables do not; as explained in the problem statement, this is because DCs numbers and locations are decided in the beginning regardless of the scenario while the assignment of the retailers to warehouses is scenario dependent.

The first term, in the brackets, of the objective function calculates the total inventory and ordering costs over the planning period. The expression \( t_{b} / T_{ij} \) indicates the number of orders by retailer \( i \) per planning period. Also, the expression, \( \frac{1}{2} \beta_{\text{inv}} \left( h_i - \bar{h}_j \right) d_{in} t_b T_{ij} + \frac{1}{2} \beta_{\text{inv}} \bar{h}_j d_{in} t_b \max \{ T_{ij}, T_{ij} \} \), indicates the inventory holding costs considering two cases, when the \( T_{ij} \geq T_{ij} \) and \( T_{ij} < T_{ij} \). In the first case, the holding cost will equal to the inventory the retailer has (the first term) and the holding cost of inventory that specified for the retailer at the distribution center. In the second case, the second part of the equation will cancel out with \( -\bar{h}_j \) leaving the holding cost at the retailer. The second term of the objective function donates the cost of opening a warehouse and operating it, basically the fixed cost of opening a warehouse and the cost of placing orders from it. The third term indicates the shipping costs from the plant to the warehouse and from the warehouse to the retailers. Constraint (1) ensures that only warehouse is assigned to each retailer. Constraint (2) prevents retailers from being assigned to unopened warehouses. Constraints (3) and (4) define the decision variables as binary. Constraints (5) and (6) are based on the power-of-two policy from EOQ. As mentioned previously, EOQ is a classical model to minimize cost in inventory management. The power of two policy states that the time ratio between the retailer orders and between warehouse orders is a power of two. Constraints (7) and (8) ensure that order times are not negative.

### Experimental Analysis

The analysis was based on three scenarios including 10 retailers and 4 DCs. The scenarios have a probability of 0.3, 0.5, and 0.2 respectively. The scenarios can be considered for one product with a three different demand probabilities or for three different products. The data used are taken from 49-node data set in Daskin (1995). Undefined data was estimated in regard to other variables. Table I shows the parameters’ values used in this analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>30</td>
</tr>
<tr>
<td>( \beta_{\text{inv}} )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \beta_{\text{trn}} )</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The model was solved using BARON MINLP on GAMS modeling language.

The solution is a total cost of $26,376 for all the scenarios. Each scenario has a significantly different retailer’s assignment. The demand under the three scenarios is indicated in Table II. The retailers’ assignment to DCs is shown in Figure I.

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To investigate the efficiency of the stochastic model, the model is run with one scenario (N1). The total cost in this case is $26,037. Then, the demand of scenario (N2) is used in the same model. In this case, the total cost exceeds the total cost of the stochastic model to reach $32,437. This total cost does not take into account the unsatisfied demand or extra inventory stored at the DCs. Assuming a penalty of $1 for each unit unsatisfied or for each unit left at the DC. Also, if we assume that inventory pooling is not possible. The modified total cost is $44,660. This is an increase by 69% of the stochastic model total cost. Therefore, the “regret” of not using stochastic model can be significant depending on the possible scenarios.

CONCLUSION

In this paper, we introduced an extension to a deterministic supply chain model. The new model was formulated as a MIP and solved using the BARON MINLP solver in GAMS. We conducted an experimental study on instances of small sizes, and found that taking into consideration the uncertainty of demand can have significant effects on total costs. The preliminary results show this extension is essential for modeling the supply chain. Further steps will include solving the model using genetic algorithm (GA) in order to optimize the problem considering its unique conditions.

REFERENCES


