ANALYSIS OF NON-STANDARD QUEUE SYSTEMS BY USING A HYBRID MODEL

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Abstract
Queuing theory can be used for analysis of various systems like production systems, transportation and stocking systems but applying queuing techniques and models owing to the existence of specific conditions in some systems is not easy. In this paper, a specific type of non-standard systems is studied. Queuing models in non-standard state occur when customers face limitations for getting service due to layout constraints. Basically, in such systems, delay in serving and exiting as a result of specific layout of servers is inevitable. The purpose of this paper is to present a hybrid model for non-standard queuing systems with layout constraints by using queuing theory concepts. This model will be able to compute major parameters like mean waiting time in queue for these systems and as well can be a base for evaluating similar non-standard queuing systems. Filling stations are one of these specific and non-standard queuing systems which can be analyzed by proposed model. Thus, filling stations as they are one of the most widely used queue systems in the country were selected to describe proposed model. Moreover, managers can evaluate and analyze their system by using this model and also achieve a better cognition of their system in order to improve it in the future.

Keywords: Queue Theory, Non-standard Systems, Hybrid Model, Filling Stations

1. Introduction
Waiting in a queue is unpleasant; however it is part of inevitable reality of life. Though, reducing waiting time usually requires extra investments [1]. The art of queue theory is to build a simple model first, and analyze its result with mathematical methods, then compare it to real life results and after that with addition of minor details makes the model compatible with the real life experience [2]. In the 1940s, queuing models were used to solve a variety of machine interference problems, i.e., how many repair persons are needed to be assigned to properly maintain a system, or how many telephone operators are required to handle traffic calls [3]. However, nowadays, manufacturing & services entities, use queue theories to optimize their decision making for reducing waiting time of their customers. This is not only to define resource level required but also to increase customer satisfaction. This subject is vital for survival of companies, specially in a competitive environment. Hence, research in describing queue theories in diverse environment is essential [4]. Important application areas of queuing models are production systems, transportation and stocking systems, communication systems and information processing systems.

Queuing models are particularly useful for the design of such system in terms of layout, capacities and control [5]. For example, Queue system layout affect waiting and serving time and plays a key role in the rate of productivity. Because of layout constraints, customers of many systems have not enough freedom to get
service or exit from system and this lead to increasing service and waiting time in system. In this paper modeling and analysis of these non-standard systems is presented. Thus, filling stations as they are one of the most widely used queuing systems in the country, were selected to describe proposed model.

2. Literature review

There have been researches about analysis of systems which have long waiting time. Hermida and et al [6], Wang and Tai [7], QinanWang [8], Maglaras and Van Mieghem [9] and Canonaco and Legato [10] have studied the problem of long waiting time in order to reduce customer dissatisfaction and server idleness and increase productivity by focusing on waiting time and presenting a new queuing model. Furthermore, filling stations have been studied beforehand. Shahmoradi computed the optimum number of servers in filling stations that both customer costs and service provider idleness meet minimum point [11]. Abedi and et al [12] focused on cognizance of gas providing systems in Iran and proposed a model for it based on queue theory. They introduced a framework for specific conditions by using Markov chain concepts. Teimoury and et al [13] presented a queue model for non-standard queue systems with specific state. This model is applicable to assembly and production lines.

3. Problem description

Queuing models in non-standard state occur when customers face limitations for getting service due to layout constraints. Filling stations are one of these specific non-standard queuing systems in Iran. The filling station Queuing system has a main waiting line that is formed out of filling station (due to space limitation in station). After a certain Period of time, vehicles get into one of sub-branches and it indicates that they cannot be served by other servers in other Branches [12]. Thus, each Branch can be considered as an independent queuing System and has Two Specific conditions. They are designed in a way that vehicles after finishing their fueling (except the front cars which are served by first server in each Branch), have not enough freedom to exit due to layout constraint and existence of other cars in front of them. This is called leaving limitation. Furthermore, Automobiles which are waiting in the queue, cannot be served, unless all the Automobiles in the branch have left the system. This is called Replacement limitation. These restrictions increase fueling mean time and Mean waiting time.

Fig.1. schematic Plan of fuel station

It is obvious that owing to the layout constraints, we cannot use available queuing Models to analyze this system. So, considering that the Performance of each Row (branch) is independent and similar to each other, Firstly the existent queuing system will be modeled based on K servers in a branch and then it will be developed to the whole system.

3.1. System assumptions and limitations

To model queuing system, we will define parameters, variables and assumptions as below:

- Customers arrival is based on Poisson Process with $\lambda$ parameter.
- K servers are serving as a series in special conditions
- $x_1, x_2, ..., x_k$, are Service Time of first (front car in each Branch), second, ..., kth customer Respectively and has exponential distribution with parameter $\mu$
- The selected Branch by customer, cannot be changed again.
- If K servers were idle, customer would be referred to the first Server and if the first server was busy, customer would be referred to the second server and this Procedure will be continued up to server K.
- Any of K customers (except front customers) in each Branch will not be able to leave the
system unless all customers who are in front of them, have left the system (leaving limitation).
- If Second Server or third server or ... or server number K was busy and first server or second server or ... or server number (K-1) was idle, the next customer in queue would not be able to Refer to the Idle server owing to the existence of server K (replacement limitation).

3-2 Contrasts between M/M/C model and filling station and similar queuing systems

In this section two reasons for non-conformity of these type of systems with M/M/C model in order to clear the uncertainties which have existed due to the similarity of these systems to M/M/C model, is presented.

A) Independence of service time: in M/M/C model fluctuation of service time will have direct effect on system measurement parameters while such case is not observed in filling stations or similar service systems. For example consider a branch which has two servers. Assume that the front car finish it's fueling and exit. In this case the automobile in the queue, cannot be served immediately (replacement limitation). Therefore, faster service of server one does not reduce the waiting time in the line because the next car in the queue must wait until the second vehicle finish its fueling and this contradicts with M/M/C model.

B) delay in Leaving and replacement: in the standard M/M/C model, customer leave without delay and when a customer leave system, the next customer is replaced immediately whereas in these non-standard systems, in the first state, compulsive waiting owing to layout constraints exist and in the second state, replacement is done when all K customer have left the system not anytime a car finish its fueling. This apparent contradiction by itself can be a reason to reject the claim of being M/M/C model for such systems.

4. System recognition

In this section, real queuing model, real service and arrival rate in each branch is presented.

4.1. The actual Model of non-standard filling stations description in each Branch and the Real serving Rate ($\mu'$) computation

As it was noted, in this system, in each Branch, on the one hand the cars for getting service, are forced to wait in the line until the car number K leave the system (Replacement limitation). On the other hand, the car number K will not be able to leave the system unless all the cars which are a head of it, have left the system. From the view point of customer who is waiting in the line, This state of affairs is like this status that all K Customers leave the system altogether at the same time and immediately after finishing fueling of the car which has the most fueling time because the cars which left the system earlier and were in front of the car which has the greatest service time, haven't helped waited customers in the line to be served sooner and haven't Reduced their waiting time.

Considering this, customer arrival will be individually and leaving system will be simultaneous and as a group. According to this, the Mentioned conditions can be considered equivalent to the system which has one sever that serve K customer simultaneously in each Branch. In other words owing to the expressed limitations in this type of system, existence of K server that each of them in each Branch can serve one customer exactly will be equivalent to existence of one server which can serve K customer at once.

![Fig.2.Real and equivalent system](image)

With the above assumption, queuing model of these type of systems in each branch in every service operation is exactly conform to M/M/1 model with first kind bulk service and maximum service capacity K [14]. In this model each time, group of customers are served and it is completely similar to filling station system with layout constraints. Furthermore, it is first kind because server does not wait until reach K customers and it starts serving anytime a
customer arrives. The important and key point is that serving point owing to the mentioned limitations is not $\mu$ anymore because waiting customers in the line must wait as long as the most service time among $K$ customers who are being served not as long as $t_{ave}$. Therefore service time will be equal to $\text{Max}(x_1, x_2, \ldots, x_K)[15]$. Consequently, mean service time from viewpoint of waiting customers is:

$$E(\text{Max}(x_1, x_2, \ldots, x_K)), \ x_i \sim \text{exp}(\mu), \ i = 1, 2, \ldots, K$$

Thus, in this case we will have an M/M/1 system with first kind bulk service and real serving rate($\mu'$) as below:

$$\mu' = \frac{1}{E(\text{Max}(x_1, x_2, \ldots, x_K))} \quad (1)$$

4.2. Arrival Rate($\lambda'$) computation for each branch by Jackson networks approach

Jackson queuing network assumptions for the noted special system are expressed as follows [16]:

- Customer arrival to branch number "i" is based on Poisson Process with Rate $\lambda_i'$
- The queue capacity of every branch is not limited
- $p_{ij} = 0$ (The Probability of leaving station i and getting into station j). Consequently, $p_{io} = 1$ (The Probability of leaving system after receiving Service from station i)
- If $P_i$ is Probability of selecting branch number "i", The actual arrival rate of branch number "i" will be computed as follow($\lambda$=The whole arrival rate of the entire system):

$$\lambda_i' = \lambda p_i \quad (2)$$

Given that in the long term period, the Probability of selecting branch "i" will tend to an equal volume, the obtained result of Jackson and M/M/1 with the first kind bulk service can be extended from one branch to the entire system, by conversion of whole arrival rate to modified arrival rate for each branch, according to the following Relation:

$$\lambda_1' = \lambda_2' = \ldots = \lambda_i' = \lambda' = \frac{\lambda}{i} \quad (3)$$

5. Modeling filling stations queuing system

In this section major parameters will be calculated. For this purpose like previous, the first formulation in each branch will be done and then it will be developed to the whole system.

5.1. major parameters calculation and formulation for each independent branch

In fact, here the filling station can be considered as a queue system which includes “i” independent queue system (i branches) that each has k servers. Consequently, regarding the fact that practically every branch performance is independent and identical to each other, firstly, the way of the major parameters calculation is expressed by using Markov chains concepts [17] and ultimately, the final model in the form of a queue system which has i rows and k servers in each row, will be extracted. For this reason, in this part, system state will be equal to one row state.

Here, for formulating the problem in the form of Markov chain, $n$ is assumed as the system state and is considered as a situation where $n$ customers are in the system. So, entering a new customer with $\lambda'$ arrival rate adds system status by one and leaving by server with service rate $\mu'$ that reduces the system state. In this case, system state depends on the current system conditions. If the number of customers in the system is equal to k or more, the number of customers who leave system will be k but if the system state is less than k person, server will serve all the people in the system and the system state will reach to zero. According to this, transition rate diagram and equilibrium equation set are as below:
And finally we will have:

\[
\left\{ \begin{array}{l}
\lambda \pi_0 = \mu (\pi_k + \pi_{k-1} + \cdots + \pi_1) \\
(\lambda + \mu) \pi_n = \mu \pi_{n+k} + \lambda \pi_{n-1} : n \geq 1
\end{array} \right.
\]

(4)

In order to solve the above equilibrium equation set, the second equation is written as follows [14]:

\[
[\mu x^{k+1} - (\lambda + \mu) x + \lambda] \pi_n = 0 : n \geq 0
\]

(5)

And we know that if \( (x_1, x_2, \ldots, x_n) \) are the roots of the above characteristic equation, differential equation desired answers are as below [14]:

\[
\pi_n = \sum_{i=1}^{k+1} c_i x_i^n : n \geq 0
\]

(6)

And since \( \sum_{n=0}^{\infty} \pi_n = 1 \), every \( x_i \) must be less than one or \( c_i = 0 \). It was proved that mentioned equation has just one root \( (x_0) \) whereas \( x_0 \in (0,1) \). Hence:

\[
\pi_n = c x_0^n : n \geq 0, 0 < x_0 < 1
\]

(7)

\[
\sum_{n=0}^{\infty} \pi_n = 1 \Rightarrow C \sum_{n=0}^{\infty} x_0^n = 1 \Rightarrow C \frac{1}{1-x_0} = 1
\]

\[
\Rightarrow C = 1 - x_0 : n \geq 0
\]

On the other hand:

\[
\pi_0 = c x_0^0 = c = 1 - x_0
\]

(8)

And finally we will have:

\[
\pi_n = (1-x_0)x_0^n : n \geq 0, 0 < x_0 < 1
\]

(9)

Ultimately \( L' \) and \( W' \) (Expected number of customers and mean Waiting time in the system for each branch), can be calculated by the following relations:

\[
L' = \frac{x_0}{1-x_0} \quad (10)
\]

\[
w' = \frac{x_0}{\lambda (1-x_0)} \quad (11)
\]

by considering little’s relations:

\[
W_q = w' - \frac{1}{\mu} \quad (12)
\]

\[
L_q = \lambda W_q' \quad (13)
\]

5.2. Developing proposed model and parameters calculation for the entire system by assuming i branches

Measurement parameters like \( L_q, w_q, L', w' \) were calculated for each branch but because of the similarity of conditions in “i” branches, computation of these parameters which will be shown as \( W, L_q, W_q, L \) for the entire system, are as follow(mean waiting time for individual customer in the system and in the queue for branch \( i \) is described respectively \( W_i, W_q' \)):

\[
W = \frac{\lambda_1}{\lambda} W_1 + \frac{\lambda_2}{\lambda} W_2 + \cdots + \frac{\lambda_k}{\lambda} W_i \quad (14)
\]

\[
W_q = \frac{\lambda_1}{\lambda} W_{q_1} + \frac{\lambda_2}{\lambda} W_{q_2} + \cdots + \frac{\lambda_k}{\lambda} W_{q_i} \quad (15)
\]

But according to the result of previous section \( \lambda_i = \lambda = \frac{\lambda}{i} \) and performance similarity of all branches, \( W_1 = W_2 = \cdots = W_i = W, W_{q_1} = W_{q_2} = \cdots = W_{q_i} = W_q' \). As a result:

\[
W_q = w_q', \quad W = w' \quad (16)
\]

In the other words, waiting time in the queue and in the system for each branch is equal to waiting time in queue and system for the entire system and is calculated by the relations which were acquired before.

\( L \) And \( L_q \) calculations as integrative criteria are a little different. Since all branches are independent and identical to each other, if \( L' \) is the number of people in one row, it also will be the number of customers in other branches. It
means $L'_1 = L'_2 = \cdots = L'_i = L'$. These conditions are exactly the same for the total number of individuals in the entire system queue. Therefore, according to the Jackson networks theorem:

$$L_q = L'_q \times i \quad (17)$$
$$L = L' \times i \quad (18)$$

6- Conclusion

In this paper non-standard queuing systems with layout constraints was analyzed by using queuing theory concepts and presenting a hybrid model which can compute major parameters like mean waiting time in the queue and system for this kind of systems. Moreover, this study can be a base for evaluating similar non-standard queuing systems. Therefore, managers could analyze and evaluate their system by using this model and also could compare the results with former conditions, after improving their system and achieve a better cognition of their system. In addition, customers can make optimum decision for entry by obtaining information from this model. Finally to describe the proposed model, filling stations in Iran has been studied because they are one of the largest and most widely used queue systems in the country.

References