A TWO-STAGE STOCHASTIC ASSEMBLY LINE BALANCING PROBLEM
WITH OUTSOURCING OPTION UNDER DEMAND UNCERTAINTY

Ufuk Kula 1, Rifat Gurcan Ozdemir 2

1 Sakarya University, Department of Industrial Engineering, Sakarya, Turkey
ukula@sakarya.edu.tr

2 Istanbul Kultur University, Department of Industrial Engineering, Istanbul, Turkey
rg.ozdemir@iku.edu.tr

Abstract

In recent years companies see outsourcing as an important strategy to level out their production and to deal with demand uncertainty. A common problem that manufacturers face is to determine which components of a product to outsource and which ones to assemble themselves. A component consists of a set of tasks may either be performed in house or be outsourced. When the demand is uncertain, an outsourcing option helps a manufacturer to hedge itself against demand exceeding the line capacity and at the same time to reduce its assembly line investment costs. In this paper, we consider a two-stage stochastic linear program to solve (i) a single product assembly line balancing problem and (ii) to determine the components to be assembled in the line and to be outsourced. In the first stage of the stochastic linear program, tasks are assigned to stations. In the second stage, after demand is realized, a recourse action is taken on the first stage decision and the sets of tasks, i.e. components to outsource are determined. We solve the two-stage problem by using sample average approximation (SAA) and perform a numerical study to provide insights on when outsourcing is most beneficial and on which components are to be outsourced.

Key words: Assembly line balancing, stochastic programming, outsourcing

1. INTRODUCTION

In mass production systems, an important problem is the assembly line problem. A large proportion of the studies related to assembly line systems deal with the determination of the set of tasks to be assigned to each work station under the cycle time and the precedence constraints, which is known as the simple assembly line balancing problem (SALBP). The assumptions of SALBP are very restrictive with respect to real-world assembly line systems (Becker and Scholl, 2003). Two versions of the well-known simple assembly line balancing problem i.e., SALBP-1 and SALBP-2 have intensively been studied in the case of deterministic demand. SALBP-1 is present when a new assembly line system has to be installed and the external demand can be well estimated since the cycle time and, hence, the production rate have to be specified as fixed parameters. SALBP-2 leads to the maximization of the production rate of an existing assembly line. Either SALBP-1 or SALBP-2 assumes that the demand is deterministic i.e., it is a well estimated value with no variation or possibility of shortage, hence, the shortage cost is negligible. However, the demand variation affects the decision on selecting of correct system parameters (desired cycle time or required number of stations). Therefore, in this study the external demand is assumed to be a random variable but follows a certain probability distribution.
Most of the research done on balancing problem deals with SALB in which no alternative equipment types are considered. Many exact algorithms have been developed for SALBP in the literature; the most popular ones are: FABLE by Johnson (1988), EUREKA by Hoffman (1992), and SALOME by Scholl and Klein (1997). For a very comprehensive survey on SALB solution methods see Baybars (1986), and Scholl and Becker (2003). Bukchin and Tzur (2000) consider equipment alternatives and minimize the total equipment costs for a given cycle time. Every station is provided with one equipment chosen from a set of equipment types. Each type has individual costs and an individual influence on the task times. So two problems arise: (1) A variable number of stations need to be installed and provided with equipment. (2) The tasks have to be assigned to the stations considering station related assignment restrictions because some tasks can only be performed with a subset of the equipment types. They present an exact and a heuristic algorithm for solving the problem. The first one is a branch-and-bound procedure which is based on the task-oriented construction scheme and uses the minimal lower bound (MLB) strategy. Bukchin and Rubinovitz (2002) show that the parallel station problem is a special case of the above mentioned selection problem (p stations in parallel correspond to an equipment which is p times as fast as the basic equipment in a single station). Therefore, the parallel station problem can be solved with the methods outlined above and can be combined with the equipment selection problem without changing the model.

Pinto et al. (1983) discuss processing alternatives in a manual assembly line as an extension of SALB. Each processing alternative is related to a given set of tasks i.e., represents a limited equipment selection which may be added to the existing equipment in the station, and the decision is whether to use each such alternative in order to shorten the tasks duration, at a given cost. Since the line is manual, each task may be performed at each station. Their solution procedure consists of a branch and bound algorithm in which a SALB problem is solved in every node of the branch and bound tree, therefore this algorithm may be used only for a small number of possible processing alternatives. Malakooti (1991, 1994) and Malakooti and Kumar (1996) consider a multi-objective ALBP with capacity- and cost-oriented objectives and propose different solution approaches including generation of efficient alternatives, interactive approaches and goal programming. McMullen and Frazier (1998) and McMullen and Tarasewich (2003) develop an simulated annealing procedure and an ant algorithm for a GALBP with respect to parallel stations, stochastic task times, mixed-model production and alternative objectives. Pastor et al. (2002) consider mixed-model assembly line balancing problem with an additional objective that tries to increase the uniformity of tasks at the stations. Bukchin and Masin (2004) considered the problem of combining stations to larger units (aggregate stations) which are operated by teams of operators. For more detailed classifications and overviews on balancing issues we refer to, e.g., Baybars (1986), Erel and Sarin (1998), Scholl (1999), and Becker and Scholl (2003).

3. The problem formulation

The first stage of the problem is formulated as a binary- integer linear model. The following parameters and variables are defined in addition to the formerly given notations:

\[ i = \text{task index, } i = 1, \ldots, N \]
\[ j = \text{station index, } j = 1, \ldots, M \]
\[ k = \text{component index, } k = 1, \ldots, K \]
\[ t_i = \text{task time of task } i \]
\[ c = \text{the pre-determined cycle time along the line that can achieve the pre-specified service level.} \]
\[ o_k = \text{total fixed and variable costs of outsourcing component } k. \]
\[ w = \text{total in-house production cost associated with the time performed to accomplish a product.}\]
\( f \) = fixed cost of establishing a station in the line.
\( E_k \) = set of tasks required to be performed to make the component \( k \).
\( SIP_i \) = set of immediate predecessors of task \( i \).
\( SIP_k \) = set of immediate predecessors of component \( k \).

And the following binary integer variables are defined for every station \( j \) and for every task \( i \):

\[
X_{ij} = \begin{cases} 1, & \text{if task } i \text{ is assigned to station } j. \\ 0, & \text{otherwise} \end{cases}
\]

\[
Y_{kj} = \begin{cases} 1, & \text{if component } k \text{ is supplied to station } j. \\ 0, & \text{otherwise} \end{cases}
\]

\[
R_j = \begin{cases} 1, & \text{if station } j \text{ is open} \\ 0, & \text{otherwise} \end{cases}
\]

\[
\text{Min } Z = \sum_{j=1}^{M} \sum_{k=1}^{K} o_k Y_{kj} + w \sum_{j=1}^{M} \sum_{i=1}^{N} t_i X_{ij} + f \sum_{j=1}^{M} R_j \tag{1}
\]

Subject to:

\[
\sum_{j=1}^{M} X_{ij} = 1, \quad \forall i \in \left\{ \bigcup_{k=1}^{K} E_k \right\} \tag{2}
\]

\[
\sum_{j=1}^{M} X_{ij} = 1 - \sum_{j=1}^{M} Y_{kj}, \quad \forall k \text{ and } i \in E_k \tag{3}
\]

\[
\sum_{j=1}^{M} Y_{kj} \leq 1, \quad \forall k \tag{4}
\]

\[
\sum_{j=1}^{M} j \cdot X_{ij} \leq \sum_{j=1}^{M} j \cdot X_{ij}, \quad \text{for } h \in SIP_i \tag{5}
\]

\[
\sum_{j=1}^{M} j \cdot X_{ij} \leq \sum_{j=1}^{M} j \cdot Y_{kj}, \quad \text{for } i \in SIP_k \tag{6}
\]

\[
\sum_{j=1}^{M} j \cdot Y_{kj} \leq \sum_{j=1}^{M} j \cdot X_{ij}, \quad \text{for } k \in SIP_i \tag{7}
\]

\[
\sum_{j=1}^{M} j \cdot Y_{kj} \leq \sum_{j=1}^{M} j \cdot Y_{kj}, \quad \text{for } g \in SIP_k \tag{8}
\]

\[
\sum_{i=1}^{N} t_i X_{ij} + \sum_{k=1}^{K} t_k Y_{kj} \leq c \cdot R_j, \quad \forall j \tag{9}
\]

\[
X_{ij}, Y_{kj}, R_j = 0,1, \quad \forall i, j, k \tag{10}
\]

Our objective is to minimize the sum of the variable and fixed outsourcing cost per period and total in-house production cost per period, and total installation cost for stations. \( w \), here is the unit operation
cost, and \( o_{ik} \) is the total variable and fixed cost of outsourcing in the period, \( f_i \), fixed cost of installing a station. The equation (1) shows the objective function of our formulation which involves three parts as total costs incurred due to outsourcing components, total in-house production costs and total fixed costs for installation of all stations in the line. Constraint (2) ensures that each task which is not part of any component should be assigned to a station. Constraint set (3) ensures that each task of an unassigned component should be assigned to a station. Constraint set (4) ensures that each component can be supplied to only one station. Constraint sets (5 – 8) show the precedence relations among tasks, between tasks and components and among components, respectively. Constraint set (9) ensures that station content can not exceed the predetermined cycle time. Finally, constraint set (10) defines the decision variables.

Our formulation assumes that each component has a unique set of tasks (i.e. there is no common task between any pair of components).

If we assume that a certain task that should be performed to assemble a certain outsourced component on to the product still remains in the task set, then the following constraint should be added to the formulation:

\[ X_{i_k, j} \geq Y_{kj} \quad , \forall k \text{ and } j \]  

(11)

Where, \( i_k \) is the task required to be performed to assemble the component \( k \) on to the product.

Then, constraint set (11) can be re arranged based on the modification formulated in constraint set (12) as follows:

\[ \sum_{i=1}^{N} t_i X_{ij} \leq c \cdot R_j \quad , \forall j \]  

(12)

Here, we will illustrate the case modeled with constraint sets (2) – (8) and (12), the objective function remains the same as in equation (1). For this case, we relaxed the assumption 6, whereas unlimited supplier capacity is supposed as additional assumption.

4. Numerical Example

In this section we will illustrate our proposed model developed for the first stage over an example case problem. The problem is gathered from a TV-set manufacturer, and the components of the single model TV-set produced in the line are defined with the help of the design engineers of the company. Figure 1 presents the all assembly tasks that should be performed to make a TV-set, and also predecessors and times of each task are given in the same figure. Tasks 4 through 8 and 10 through 12 are performed to make components Hi-Fi, Chasis, and Tube.
Figure 1. Times and predecessors of all assembly tasks.

The engineering team defined three components (Hi-Fi, Chasis and Tube) that can be outsourced separate from regular assembly process. Each component requires different sets of tasks to be produced and all production data are provided in figure 2.

![Table](https://example.com/table.png)

**Figure 2. Components’ assembly process data.**

To make a TV-set, 22 tasks need to be performed, however, first 15 tasks are separated from the rest due to the fact that task 16 is a special process in assembly of a TV-set and this task divide whole process in to two parts. Thus, we just focus on the first 15 tasks in determining a line balance solution for the implementation of the first stage of our approach. The model presented in the previous section of the paper is coded in GAMS 23.1 and solved using CPLEX 12.0 for obtaining a solution to the problem of line balancing and outsourcing decision. The solution to the first stage of the problem for given components and tasks is presented in the following figure.
As seen in figure 3, four stations are open and maximum load attained from among all stations is realized to be 43 seconds. All three components are decided to be outsourced and components 1 and 2 are supplied to the first station and component 3 is supplied to the third station of the line. Objective function value for this solution is obtained as $192,000.

Conclusion

This paper has considered a two-stage stochastic linear program to solve the problem of line balancing and outsourcing decision. In the first stage of the stochastic linear program, tasks are assigned to stations using a linear programming model for given sets of components and tasks under certain cycle time constraint. In the second stage, after demand is realized, a recourse action is taken on the first stage decision, i.e. components to outsource are determined and fed to the first stage to solve the linear line balancing problem for the second iteration. The two-stage problem is solved by using sample average approximation (SAA). A numerical study has been provided for the first stage of the proposed approach and the second stage will be incorporated in to the study in presentation. The early results show that the idea of consideration of outsourcing decision with line balancing problem is highly significant and provide a new perspective to the decision maker in assembly process.

References