Abstract:
One of the most important things in pursuit of supply chain management is to prevent sub-optimization caused by decentralized decision making over the various entities. As a solution of the issue, supply chain coordination approach has frequently been used. However, a coordinated supply chain might fail to provide additional profit to one of the players. A supply chain contract can be to achieve the same profit of supply chain coordination and to improve the benefit of all entities involved. In this paper, we address the model of supply chain contract, which is a combination contract of a quantity discount contract based on a revenue sharing contract, which is used to coordinate under a multi-echelon supply chain facing a stochastic customer demand. It is shown that the proposed contract can achieve supply chain coordination and win-win situation.

Keywords: Supply chain management, Revenue sharing, Coordination, Contract, Win-Win

1. Introduction
A well-designed supply chain contracts are a useful tool to improve the performance of all entities included as well as prevent sub-optimization. A self-serving focus of the supply chain entities often results in sub-optimization with poor systemwide performance. A traditional solution to the issue is the use of supply chain coordination. To coordinate a supply chain, a centralized or decentralized decision-making approach can be adopted. The former option occurs when all decisions are made by a single entity in the supply chain, the latter when several independent entities make decision at the different supply chain stages. However, although a coordinated supply chain can achieve the best performance through optimizing the entire supply chain, win-win, which occurs if every supply chain entities make higher profit compared to the decentralized decision making situation, is not necessarily obtained. Furthermore, even if all the entities gain from their collaboration, the supply chain is necessarily optimized. That is, it is
to be noted that a coordinated supply chain does not imply win-win. When all supply chain entities is independent and autonomous organizations, clearly, from a practical point of view, win-win is probably more important than supply chain coordination. After all, the entity is only prompted to participate in joint actions if he will obtain a profit from the collaboration. As a useful means to achieve supply chain coordination and win-win, supply chain contracts is frequently adopted.

Most of literature on supply chain contracts focused on a situation with only two entities: a buyer and a supplier (Cachon, 2003). These models include revenue sharing (Cachon and Lariviere, 2000), quantity discounts (Burnetas et al., 2007), buy back (Emmons and Gilbert, 1998), quantity flexibility (Tsay, 1999).

The focus of this paper is on a combination contract of a Quantity Discounts (QD) contract based on a Revenue Sharing (RS) contract with demand uncertainty. Under a RS contract, the buyer shares some of its revenue with the supplier. A RS contract thus involves two parameters, namely the wholesale price per unit and a percentage of the buyer's revenue that goes to the supplier (Koulamas, 2006; Lee and Whang, 1999). Under a QD contract, a supplier who offers the buyer a quantity discount varies the price charged to the buyer retailer according to the quantity purchased by the retailer. The buyer obtains a discount for purchasing a larger quantity of the product from the supplier. A QD involve two parameters, with a fixed base price and an additional price which is a decreasing function in an order quantity (Burnetas et al., 2007). In result, a combination contract discussed in this paper can involve four parameters.

The literature on a multi-echelon supply chain contract has recently proposed two contract models called as pairwise contract and spanning contract. The former is to install contracts between all pairs of adjacent entities in a multi-echelon supply chain (Giannoccaro and Pontrandolfo, 2004), while the latter is that one supply chain entity takes the lead in negotiating a single contract with all other entities simultaneously (Van de rhee et al., 2010). In the previous research, the pairwise contract may be unlikely to occur in the whole multi-echelon supply chain. The main reason of this result is because it implicitly assumes that all contracts between the pairs of entities are installed simultaneously. However, spanning contract can overcome this drawback of pairwise contract because a single entity initiates a single contract with all other entities simultaneously. Van de rhee et al. (2010) have proposed the spanning RS contract where the most downstream entity initiates a single contract involving all upstream entities with considering a practical point of view.

This paper deals with a combination contract of a QD contract based on a RS contract, which is used to coordinate under a multi-echelon supply chain facing a stochastic customer demand. We call this type of RS contract the spanning RSQD contract. We focus on supply chain contracts addressing supply chain coordination, namely the spanning RSQD contract. Furthermore, particular attention is devoted to support the fine tuning of the contract parameters so as to achieve a win–win situation. The fundamental idea underlying the spanning RSQD contract is that if each entity decreases its wholesale price, their respective buyers have an incentive to decrease their prices also. As a consequence, the most downstream entity will motivate him to order more. This in turn increases availability to the final customer and therefore
positively influences the overall supply chain revenue. Clearly, the various entities might want to receive some compensation for their decreased prices, which in the spanning contract is given in terms of a share of the revenue of the most downstream entity.

The paper is organized as follows. In Section 2, we introduce the basic supply chain model and discuss the spanning RSQD contract. In Section 3, necessary and sufficient conditions for the win-win solutions under the spanning RSQD contract are given. Finally, in Section 4 the study is concluded.

2. The multi-echelon supply chain model

The basic model setting introduced in this paper is similar to the one described in Van de rhee et al. (2010). In this model, the supply chain is a simple serial structure with a single entity at each of the \( n \geq 2 \) echelons. Denote Entity 1 as the most downstream and Entity \( n \) as the most upstream organization. Let Entity \( i \) buy a single product at purchasing cost \( W_{i+1} \) and operational costs \( c_i \) per unit and sell it to Entity \( i - 1 \) at \( W_i \), \( i = 2, ..., n \). For notational convenience, it is assumed that \( W_{n+1} = 0 \). Thus, Entity 1 faces a purchasing cost \( W_2 \) and operational costs \( c_1 \) per unit and sells it to the end-customers at \( p \) per unit and can decide on the order quantity \( Q \). Since the market price is higher than the total supply chain cost, it is assumed that \( p > \bar{c} = \sum_{i=1}^{n} c_i \), where the cost \( c_i \) can represent the operational costs and/or the shipping costs at the various entities. Furthermore, based on the spanning contract, it is assumed that all entities have full information on all data. For that reason, no further assumptions on the supply chain cost structure are made.

In a market-like setting, the system would work as follows. All entities face the newsvendor’s problem. Therefore, the product has a short life cycle and there is only a one-time order. Moreover, since the product has a short life cycle, there is no stock from previous periods, and any unused stock from previous periods cannot be used for future periods. Furthermore, it is assumed that each entity has the objective to maximize its own expected profit. The order quantity \( Q \) is passed on through the entire supply chain. And it is assumed that each entity has sufficient capacity to handle any realistic order size and any unmet demand is lost as reordering is not possible. Let \( D > 0 \) be end-customer demand during the selling season. Let \( F_D(D) \) be the distribution function of end-customer demand and \( f_D(D) \) its probability density function: \( F_D(D) \) is differentiable, strictly increasing and \( F_D(0) = 0 \).

The following sequence of events within this supply chain occurs. First Entity \( n \) determines its selling price \( W_n \), followed by Entity \( n - 1 \), all the way to Entity 2 that determines its price \( W_2 \). Given \( W_2, c_1 \) and the demand distribution \( f_D(D) \), Entity 1 submits an order quantity \( Q \) to Entity 2. The same order quantity is passed on from Entity 2 to Entity 3, all the way to Entity \( n \). Subsequently, the amount \( Q \) is shipped from Entity \( n \) all the way to Entity 1 before the selling season. Finally, the customer demand \( D \) occurs and Entity 1 sells the amount \( \min(Q, D) \) to the end-customer. Let \( F_D(Q) = \int_0^Q f_D(D) \, dD \), and \( G_D(Q) = \int_0^Q D \cdot f_D(D) \, dD \). The expected profit of Entity 1 is \( \pi_1 = (p - W_2 - c_1)Q - pQF_D(Q) + pG_D(Q) \). The
expected profit of Entity 2, ..., n is \( \pi_i = (W_i - W_{i+1} - c_i)Q \). And the expected profit of the total supply chain is \( \pi_{sc} = \sum_{i=1}^{n} \pi_i = (p - \bar{c})Q - pQF_D(Q) + pG_D(Q) \).

To achieve supply chain coordination, it is assumed that a centralized control involves the existence of a unique decision-maker in the supply chain. Therefore, all entities in supply chain work together as a single entity with the objective to maximize the supply chain profit. Clearly, the centralized supply chain (which is denoted by superscript \( c \)), is facing the well-known newsvendor problem with per-unit overstocking cost \( \bar{c} \) and the per-unit understocking cost \( p - \bar{c} \). The supply chain optimal order quantity \( Q^c \) is \( Q^c = (F_D)^{-1}\left(\frac{p-\bar{c}}{p}\right) \). The associated profit is \( \pi_{sc}^c = pG_D(Q^c) \).

Now consider the decentralized supply chain (which is denoted by superscript \( d \)), in which it is assumed that there is no coordination, i.e., all entities act independently and take decisions that maximize their respective profits. Although there are various approaches possible to analyze this situation, it will be assumed that optimization takes place in a Stackelberg sequence proposed by Van de rhee et al. (2010), where in the time-line Entity \( n \) is the first decision maker, and the model is solved through backwards induction. Thus, first he optimal order quantity \( Q^d \) of Entity 1 is determined (as a function of \( W^d_2 \)), then the optimal \( W^d_2 \) for Entity 2 is decided (as a function of \( W^d_3 \)), all the way to Entity \( n \) determining its optimal \( W^d_n \). In the decentralized scenario, the expected profit of Entity 1 is \( \pi_1^d = (p - W^d_2 - c_1)Q - pQF_D(Q) + pG_D(Q) \). Entity 1 faces a newsvendor problem with per-unit overstocking cost \( c_1 + W^d_2 \) and the per-unit understocking cost \( p - c_1 + W^d_2 \). The supply chain optimal order quantity is \( Q^d = (F_D)^{-1}\left(\frac{p-W^d_2-c_1}{p}\right) \). The expected profit of Entity 1 is \( \pi_1^d = pG_D(Q^d) \). The expected profit of Entity 2, ..., n is \( \pi_i^d = (W_i^d - W_{i+1}^d - c_i)Q^d \). And the expected profit of the total supply chain is \( \pi_{sc}^d = \sum_{i=1}^{n} \pi_i = (W^d_2 + c_1 - \bar{c})Q^d + pG_D(Q^d) \). Note that, unlike in the centralized supply chain, here the supply chain profit depends on the wholesale price \( W^d_2 \). According to Van de rhee et al. (2010)’ observations, for similar situation, the decentralized supply chain results in a suboptimal supply chain profit, i.e., \( \pi_{sc}^d > \pi_{sc}^c \).

The spanning RSQD contract (denoted by superscript \( sq \)) is a combinational contract of a quantity discount based on a revenue sharing. This contract is primarily governed by the wholesale prices \( W_i^{sq} \) and the percentages \( \varphi_i^{sq} \) of the revenues of Entity 1 that are shared with Entity \( i = 2, ..., n \) (with \( 0 \leq \varphi = \sum_{i=2}^{n} \varphi_i^{sq} \leq 1 \)). And all entities fix a base price \( \gamma_i^{sq} \) and charge an additional price \( \mu_i^{sq} \) which is a decreasing function in \( Q \). At its simplest form the price at which the supplier sells the product to the buyer is dependent upon the order quantity. The price of Entity 2, ..., n is \( W_i^{sq} = \gamma_i^{sq} + \mu_i^{sq} \). The expected profit of Entity 1 is \( \pi_1^{sq} = \left((1 - \varphi)p \right) - \gamma_2^{sq} - \frac{\mu_2^{sq}}{Q} - c_1 \) Q - (1 - \varphi)pQF_D(Q) + (1 - 1050)}.
The expected profit of Entity 2, \ldots, \textit{n} is
\[ \pi_i^{sq} = \left( \varphi_i^{sq} p + \gamma_i^{sq} + \frac{\mu_i^{sq}}{Q} - \frac{\mu_{i+1}^{sq}}{Q} - c_i \right) Q - q_i^{sq} p Q F_D(Q) + q_i^{sq} p G_D(Q). \]

\textbf{Theorem 1.} The spanning RSQD contract \( (\gamma_2^{sq}, \ldots, \gamma_n^{sq}; \mu_2^{sq}, \ldots, \mu_n^{sq}; \varphi_2^{sq}, \ldots, \varphi_n^{sq}) \) with
\[ \gamma_2^{sq} = (1 - \varphi) \bar{c} - c_1 \]
coordinates the supply chain.

\textbf{Proof.} For the spanning RSQD contract, Entity 1 would choose \( Q \) to optimize its profit as given by \( \pi_1^{sq} \).

As Entity 1 faces a newsvendor problem with per-unit overstocking cost \( \gamma_2^{sq} + c_1 \) and the per-unit understocking cost \( (1 - \varphi) p - \gamma_2^{sq} - c_1 \), the optimal order quantity is
\[ Q^{sq} = (F_D)^{-1} \left( \frac{(1-\varphi)p-\gamma_2^{sq}-c_1}{(1-\varphi)p} \right). \]

Under condition (1) it follows that \( Q^{sq} = Q^c \). Using the optimal order size, the expected profit of Entity 1 is
\[ \pi_1^{sq} = -\mu_2^{sq} + (1 - \varphi) p G_D(Q^{sq}) = -\mu_2^{sq} + (1 - \varphi) \pi_1^{sc}. \]
Furthermore, using condition (1), the profits for Entity \( i = 2, \ldots, n \) are
\[ \pi_i^{sq} = \left( \varphi_i^{sq} \bar{c} + \gamma_i^{sq} + \frac{\mu_i^{sq}}{Q^{sq}} - \gamma_{i+1}^{sq} - \frac{\mu_{i+1}^{sq}}{Q^{sq}} - c_i \right) Q^{sq} + \varphi_i^{sq} \pi_i^{sc}. \]
And, again condition (1), the total supply chain profit is \( \pi_{sc}^{sq} = \sum_{i=1}^{n} \pi_i^{sq} = \pi_i^{sc} + \varphi \bar{c} Q^{sq} + (1 - \varphi) \bar{c} - c_1) Q^{sq} - \sum_{i=2}^{n} c_i Q^{sq} = \pi_i^{sc} \). Therefore, the maximal supply chain profit is achieved.

\[ 3. \text{ Win-Win Solutions} \]

Note that in order to achieve supply chain optimization under the RSQD contract, only a condition is needed on how \( \gamma_2^{sq} \) is related to \( \varphi_i^{sq} \) (\( i = 2, \ldots, n \)). Furthermore, note that there is still considerable freedom to choose \( \gamma_2^{sq}, \varphi_2^{sq}, \ldots, \varphi_n^{sq} \). These observations are used to derive win-win solutions.

Clearly, under the assumption that condition (1) is fulfilled, which means that the spanning RSQD contract coordinates the supply chain, the necessary conditions for achieving a win-win outcome are
\[ \pi_1^{sq} = -\mu_2^{sq} + (1 - \varphi) \pi_1^{sc} > \pi_1^{d}, \]
for \( i = 2 \), since \( \gamma_2^{sq} \) is determined by condition (1)
\[ \pi_i^{sq} = \left( \varphi_i^{sq} / (1 - \gamma_2^{sq}) - c_1 + \frac{\mu_2^{sq}}{Q^{sq}} - \gamma_3^{sq} - \frac{\mu_3^{sq}}{Q^{sq}} - c_2 \right) Q^{sq} + \varphi_2^{sq} \pi_i^{sc} > \pi_i^{d}, \]
and for \( i = 3, \ldots, n \).
\[
\pi_i^q = \left( q_i^{sq} c + y_i^{sq} + \frac{\mu_i^{sq}}{Q^{sq}} - y_{i+1}^{sq} - \frac{\mu_{i+1}^{sq}}{Q^{sq}} - c_1 \right) Q^{sq} + \varphi_i^{sq} \pi_{Sc}^{i} > \pi_i^d. \tag{4}
\]

Let \( \Omega \) denote the set of all RSQD contract \((y_2^{sq}, \ldots, y_n^{sq}; \mu_2^{sq}, \ldots, \mu_n^{sq}; \varphi_2^{sq}, \ldots, \varphi_n^{sq})\) that satisfy the above conditions (2)-(4). Clearly, all RSQD contracts in \( \Omega \) achieve both coordination and a win-win outcome.

4. Conclusion

This paper has addressed supply chain contract model, which is a combination contract of a quantity discount contract based on a revenue sharing contract. In particular, a combination contract model has been proposed to coordinate a multi-echelon supply chain. It has been shown that a combination contract lets all entities select order quantities that are optimal for the whole supply chain. Therefore, the model guarantees supply chain coordination. Furthermore, knowing that a win–win condition is really achievable could be a good incentive to the collaboration required in the contract design.

References


