Phase II Monitoring of MA (1) Linear Profiles

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Abstract

Recently, monitoring quality of a process or product that is characterized by a function or profile is becoming more popular. Many applications are shown for profile monitoring. Most existing control schemes, which have been suggested for profile monitoring in the literatures, consider the independence assumption of observation within profiles. However, in certain situation, this assumption can be violated. Violation of the independence assumption can affect the performance of most control charts strongly. There are some investigations to monitor auto correlated profiles in recent years. In this paper, we consider a simple linear profile where the observations within profiles are assumed to be correlated according to an MA(1) structure. We show the effect of correlation on the performance of monitoring process and use a transformation to eliminate the effect of correlation within profiles. A method is discussed to monitor the transformed profiles. The performance of the proposed method is evaluated using average run length (ARL) criterion. Results indicate satisfactory performance for the proposed method.

Keyword: Profile monitoring, Autocorrelation, Average Run Length, Phase II, Statistical process control

1. Introduction

In some statistical process control application, it is assumed that the quality of a process or product is characterized by a relationship between a response variable and one or more explanatory variables. This relationship is referred to as profile. A number of authors proposed different methods for profile monitoring. Further studies on both phase I and phase II monitoring of simple linear profiles have been investigated by some authors such as Kang and Albin (2000), Mahmoud et al. (2004 and 2007), Kim et al. (2003), Gupta et al. (2006), Zou et al. (2006)and Mahmoud et al.(2009). More advanced approaches have been addressed by many researches. Kazemzadeh et al. (2008 and 2009) have studied the phase I and phase II monitoring of polynomial profiles respectively. Mahmoud (2008) and Zou et al. (2007) investigated some methods for monitoring multiple linear profiles. Zou et al. (2008) used nonparametric regression methods in profile monitoring. Colosimo and Pacella (2007) applied principal component analysis for monitoring roundness profiles. Nonlinear profiles have been investigated by a number of authors such as Ding et al. (2006), Williams et al. (2007), Moguerza et al. (2007) and Vaghefi et al. (2009). Some methods based on the change point approach for monitoring the linear profiles are proposed by Zou et al. (2006) and Mahmud et al. (2007). Noorossana et al. (2010) and Eyvazian et al. (2010) proposed some methods for monitoring multivariate profiles. Noorossana et al. (2010) analyzed the effects of non-normality on the performance of simple linear profile.

In regression models, it is assumed the errors are independent and identically distributed (iid) according to a normal distribution but in some practical and economical applications this assumption can be violated. Violation of the independence assumption can affect the performance of most control charts strongly. Some methods for eliminate the effect of autocorrelation have studied by several authors such as Jensen et al. (2008 and 2009), Noorossana et al. (2008), Soleimani et al. (2009), Kazemzadeh et al.(2010). In this paper we use a transformation to eliminate the effect of correlation within linear profiles. The model of correlation within linear profiles is presented in the next section. Proposed transformation is discussed in the section 3. A method based
on the proposed approach is presented for monitoring of linear profile in section 4. Performance of the method is investigated in the last section.

2. MA(1) model in a simple linear profile

In this section we consider the case where the relationship between a response variable and a single explanatory variable is defined by a simple linear profile, where it is assumed that the error terms within each profile are correlated based on a given moving average model of order (1) or MA(1) model. MA(1) structure can occur in some industrial and economical processes. For instance, Box and Newbold (1971) suggested an integrated moving average model for a case study proposed by Coen et al (1969). Their example was a supposed lagged relationship between the financial times ordinary share index(y) and two inputs, \( x_1 \), the United Kingdom car production and \( x_2 \), the Financial Times commodity index with purpose of the use of lagged relationships for economic forecasting.

Here it is assumed when the process is under statistical control the \( t^{th} \) observation in the \( f^{th} \) sample can be modeled as

\[
y_{ij} = A_0 + A_1 x_i + \epsilon_{ij} \quad \text{and} \quad \epsilon_{ij} = \theta \epsilon_{i-1} + \eta_{ij}
\]

where \( \eta_{ij} \) are white noises and \( A_{ij} \sim N(0, \sigma^2) \) and \( \theta \) is the correlation coefficient. The variance–covariance matrix of error terms for a MA(1) model is given by

\[
\Sigma = \begin{bmatrix}
(1+\theta^2)\sigma^2 & -\theta\sigma^2 & 0 & 0 \\
-\theta\sigma^2 & (1+\theta^2)\sigma^2 & -\theta\sigma^2 & 0 \\
0 & -\theta\sigma^2 & (1+\theta^2)\sigma^2 & -\theta\sigma^2 \\
0 & 0 & -\theta\sigma^2 & (1+\theta^2)\sigma^2 
\end{bmatrix}
\]  

(2)

In this paper we consider the phase II monitoring of linear profile so we assume the \( A_0 \) and \( A_1 \) regression parameters and \( \theta \) in Eq.(1) are known.

3. A remedial measure

In this section, we use a transformation to eliminate the effect of correlation within linear profiles when the variance–covariance matrix of the errors is known, Shumway(2006). Suppose the regression model of the \( f^{th} \) sample profile collected over time is

\[
y_j = X A + e_j
\]

(3)

where \( y_j \) is \( n \times 1 \) response vector, \( X \) is \( n \times 2 \) independent variables matrix, \( A \) is \( 2 \times 1 \) regression parameters vector and \( e_j \) is \( n \times 1 \) vector of errors. If the variance–covariance matrix of errors (\( \Sigma \)) is known, it is possible to find a transformation matrix (H), such that \( H \Sigma H^T = \sigma^2 I \) where I is the \( n \times n \) identity matrix, then the Eq.(3) can be rewritten as

\[
H y_j = X A + H e_j
\]

(4)

where \( X = [1, x_j] \), \( A = [A_0, A_1]^T \) and therefore

\[
y'_j = X A + U'_j
\]

(5)

where \( y'_j = H y_j \), \( F = H X \) and \( U'_j \) is a white noise vector with variance–covariance matrix of \( U'_j \). Then by applying the least square method to estimate the vector of \( A_y = [A_{0y}, A_{1y}]^T \)
To find the transformation matrix \( (H) \) we can use Cholesky Decomposition method. Then we have a linear profile with uncorrelated errors that we can use the existing methods in the literature for monitoring uncorrelated linear profiles in phase II.

4. Monitoring procedure – The EWMA_3 method

In this part to show the application of the proposed method, we chose one of the recommended procedures to monitoring the linear profiles. This method is based on the approach proposed by Kim et al (2003). Some authors such as Kim et al (2003) and Soleimani et al. (2009) showed the EWMA_3 method is one of most efficient methods in comparison with other methods such as \( T^2 \) or EWMA-R control charts for linear profiles monitoring.

We coded the z-values to change the average to the zero. In this situation, the estimators of the intercept and slope are independent. Hence, we can apply separate control charts to monitor the intercept and slop. The new model is given by

\[

t^j = E^j + \beta_1 y^j + \epsilon^j
\]

where \( \epsilon^j = \xi^j - \bar{\xi} \), \( E^j = \hat{\beta}_0 + \hat{\beta}_1 y^j \), \( \beta_1 = \hat{\beta}_1 \).

Similar to Kim et al (2003), for monitoring the intercept \( (E^j) \), we use the estimates of intercept, \( \hat{\beta}_0 \), to compute the EWMA statistics as

\[
EWMA_t(E^j) = \theta \hat{E}_t + (1 - \theta)EWMA_{t-1}(E^j - 1)
\]

and

\[
\hat{E}_t = \hat{E}_t - \hat{\beta}_0 \]

where \( \theta (0 < \theta < 1) \) is a smoothing constant and \( EWMA_0(E^j) = E^j \). The lower and upper control limits for monitoring the statistics in Eq. (8) are

\[
LCL = E^j - L_T \sigma \quad \text{and} \quad UCL = E^j + L_T \sigma \sqrt{\frac{\theta}{1 - \theta}}
\]

respectively, where \( L_T \) is chosen to achieve a specified in control ARL.

For monitoring the slope, we use the estimate of the slope, \( \hat{\beta}_1 \), to compute the EWMA statistic as follow

\[
EWMA_t(\beta_1) = \theta \hat{\beta}_1 + (1 - \theta)EWMA_{t-1}(\beta_1 - 1)
\]

and

\[
\hat{\beta}_1 = \hat{\beta}_1 - \hat{\beta}_1 \]

where \( EWMA_0(\beta_1) = \beta_1 \).

The lower and upper control limits for monitoring the statistics in Eq. (10) are

\[
LCL = \beta_1 - L_T \sigma \sqrt{\frac{\theta}{1 - \theta}} \quad \text{and} \quad UCL = \beta_1 + L_T \sigma \sqrt{\frac{\theta}{1 - \theta}}
\]

respectively, where \( L_T \) is chosen to give a specified in control ARL.

For monitoring the standard deviation, first we calculate MSE for the \( j^{th} \) sample as follow

\[
\hat{\sigma}^2 = (S' \Sigma^{-1} S)^{-1} S^T \sigma
\]

\[
= (W \Sigma^{-1} W)^{-1} W^T \Sigma^{-1} W
\]

Because

\[
\sigma^2 \Sigma^{-1} = H^T H
\]
Then the value of \( \text{MSB} \) is used to calculate the EWMA statistics. Therefore

\[
\text{EWMA}_2(t) = \max \{ \theta (\text{MSB} - 1) + (1 - \theta) \text{EWMA}_2(t-1), 0 \}
\]

(13)

where the \( \text{EWMA}_2(0) = 0 \). The upper control limit is

\[
\text{UCL} = l \sqrt{\frac{\text{var}(\text{MSB})}{\theta} - \phi}
\]

(14)

and \( l > 0 \) is chosen to give a specified in-control ARL.

5. Simulation studies

To show the performance of proposed method, the underlying simple linear profile which is used in this paper is defined as

\[
y(t) = 3 + 2x + e(t) \quad \text{and} \quad e(t) = \alpha(t) - \rho \alpha(t-1)
\]

(15)

where \( \alpha(t) \sim N(0, \sigma^2) \) and the \( x \)-values are equal to 2, 4, 6, 8.

The first, we show the effect of correlation with structure of moving average, MA(1) on the performance of EWMA_3 control charts. Tables 1-3 summarize the simulation results.

**Table 1.** The effect of MA(1) correlation on the ARL performance under intercept shifts from \( \delta \) to \( \delta + \Delta \) with \( \rho = 0.1 \) and \( \rho = 0.9 \) in EWMA_3 method

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi=0 )</td>
<td>200</td>
<td>59.1</td>
<td>16.2</td>
<td>7.9</td>
<td>5.1</td>
<td>3.8</td>
<td>3.1</td>
<td>2.6</td>
<td>2.3</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>( \phi=.1 )</td>
<td>170</td>
<td>65.1</td>
<td>17.9</td>
<td>5.1</td>
<td>3.8</td>
<td>3</td>
<td>2.5</td>
<td>2.2</td>
<td>2</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>( \phi=.9 )</td>
<td>7.3</td>
<td>7.1</td>
<td>6.7</td>
<td>5.3</td>
<td>4.1</td>
<td>3.4</td>
<td>2.8</td>
<td>2.4</td>
<td>2.1</td>
<td>2</td>
<td>1.9</td>
</tr>
</tbody>
</table>

**Table 2.** The effect of MA(1) correlation on the ARL performance under slope shifts from \( \beta \) to \( \beta + \Delta \) with \( \rho = 0.1 \) and \( \rho = 0.9 \) in EWMA_3 method

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>0</th>
<th>0.025</th>
<th>0.05</th>
<th>0.075</th>
<th>0.1</th>
<th>0.125</th>
<th>0.15</th>
<th>0.175</th>
<th>0.2</th>
<th>0.225</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi=0 )</td>
<td>200</td>
<td>101.6</td>
<td>36.5</td>
<td>17.0</td>
<td>10.3</td>
<td>7.2</td>
<td>5.5</td>
<td>4.5</td>
<td>3.8</td>
<td>3.3</td>
<td>2.9</td>
</tr>
<tr>
<td>( \phi=.1 )</td>
<td>170</td>
<td>156</td>
<td>111.7</td>
<td>76.3</td>
<td>49.2</td>
<td>33.2</td>
<td>23.3</td>
<td>17.4</td>
<td>13.2</td>
<td>10.6</td>
<td>8.9</td>
</tr>
<tr>
<td>( \phi=.9 )</td>
<td>7.3</td>
<td>7.2</td>
<td>7</td>
<td>6.9</td>
<td>6.8</td>
<td>6.6</td>
<td>6.5</td>
<td>6.3</td>
<td>6</td>
<td>5.7</td>
<td>5.3</td>
</tr>
</tbody>
</table>

**Table 3.** The effect of MA(1) correlation on the ARL performance under standard deviations shifts from \( \sigma \) to \( \sigma + \Delta \) with \( \rho = 0.1 \) and \( \rho = 0.9 \) in EWMA_3 method

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
<th>2.2</th>
<th>2.4</th>
<th>2.6</th>
<th>2.8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi=0 )</td>
<td>200</td>
<td>33.5</td>
<td>12.7</td>
<td>7.2</td>
<td>5.1</td>
<td>3.9</td>
<td>3.2</td>
<td>2.8</td>
<td>2.5</td>
<td>2.3</td>
<td>2.1</td>
</tr>
<tr>
<td>( \phi=.1 )</td>
<td>170</td>
<td>28</td>
<td>11.3</td>
<td>6.6</td>
<td>4.8</td>
<td>3.8</td>
<td>3.1</td>
<td>2.7</td>
<td>2.4</td>
<td>2.2</td>
<td>2</td>
</tr>
<tr>
<td>( \phi=.9 )</td>
<td>7.3</td>
<td>4.7</td>
<td>3.6</td>
<td>2.8</td>
<td>2.5</td>
<td>2.3</td>
<td>2</td>
<td>1.9</td>
<td>1.8</td>
<td>1.6</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Tables 1-3 show the MA(1) correlation structure within profile leads to increase the false alarm rates. The results show that when the value of correlation coefficient gets larger, the in-control ARL for the EWMA_3 control charts decreases.

Here to show the application of the recommended transformation, we set the overall in control ARL into 200. The ARL values are evaluated by 10000 simulation runs under different shift in the intercept, slopes and standard deviation. We considered two correlation coefficients $\rho=0.1$ and $\rho=0.9$ (both weak and strong correlation). In the EWMA_3 method, for weak correlation coefficient ($\rho=0.1$) we set the values of $L_a, L_t$ and $L_1$ equal to 3.01, 2.86 and 1.82 respectively and for strong correlation coefficient ($\rho=0.9$), we set the values of $L_a, L_t$ and $L_1$ equal to 1.54, 3.07 and 1.37 respectively in order to obtain the ARL for each control chart approximately 584 and overall in-control ARL of roughly 200. The results of simulation studies are illustrated in Table 4-6.

Table 4. The ARL performance under intercept shifts from $\lambda_i$ to $\lambda_i + \lambda_t$ with $\rho=0.1$ and $\rho=0.9$ in EWMA_3 method, after transformation

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi=.1$</td>
<td>52.3</td>
<td>14.1</td>
<td>6.9</td>
<td>4.6</td>
<td>3.4</td>
<td>2.8</td>
<td>2.3</td>
<td>2.0</td>
<td>1.8</td>
<td>1.6</td>
</tr>
<tr>
<td>$\phi=.9$</td>
<td>27.5</td>
<td>7.5</td>
<td>4.0</td>
<td>2.7</td>
<td>2.0</td>
<td>1.6</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 5. The ARL performance under slope shifts from $\beta_i$ to $\beta_i + \beta_t$ with $\rho=0.1$ and $\rho=0.9$ in EWMA_3 method, after transformation

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0.025</th>
<th>0.05</th>
<th>0.075</th>
<th>0.1</th>
<th>0.125</th>
<th>0.15</th>
<th>0.175</th>
<th>0.2</th>
<th>0.225</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi=.1$</td>
<td>93.7</td>
<td>31.7</td>
<td>14.6</td>
<td>8.8</td>
<td>6.2</td>
<td>4.8</td>
<td>3.9</td>
<td>3.3</td>
<td>2.9</td>
<td>2.5</td>
</tr>
<tr>
<td>$\phi=.9$</td>
<td>60.0</td>
<td>16.8</td>
<td>8.0</td>
<td>5.2</td>
<td>3.8</td>
<td>3.0</td>
<td>2.5</td>
<td>2.2</td>
<td>2.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 6. The ARL performance under standard deviations shifts from $\sigma$ to $\delta \sigma$ with $\rho=0.1$ and $\rho=0.9$ in EWMA_3 method, after transformation

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
<th>2.2</th>
<th>2.4</th>
<th>2.6</th>
<th>2.8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi=.1$</td>
<td>39.1</td>
<td>14.2</td>
<td>7.1</td>
<td>4.4</td>
<td>3.1</td>
<td>2.5</td>
<td>2.1</td>
<td>1.8</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi=.9$</td>
<td>44.6</td>
<td>16.8</td>
<td>9.1</td>
<td>5.8</td>
<td>4.2</td>
<td>3.3</td>
<td>2.7</td>
<td>2.3</td>
<td>2.1</td>
<td>1.9</td>
</tr>
</tbody>
</table>

As we can see the transformation method can improve the ARL performance in most different values of shift in intercept, slope and standard deviation.

5. Conclusions

In this paper, we showed the effect of correlation with structure of moving average, MA(1) on the performance of one of the most famous methods in the linear profiles monitoring procedures or EWMA_3 control chart. Then we proposed a transformation method to eliminate the effect of correlation. We showed the performance of our recommend method with simulation study and ARL criteria. Results indicated the acceptable performance for the suggested remedial measure.

References