ANALYZING THE EFFECTS OF SCRAP, REWORK, RANDOM MACHINE BREAKDOWNS AND PARTIAL BACKORDERING ON THE ECONOMIC PRODUCTION QUANTITY MODEL USING TAGUCHI METHOD

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Abstract:
There are several production-inventory studies which consider economic production quantity (EPQ) models with scrap, rework and random machine failures in the literature. However, the effects of these critical factors on the total annual inventory cost haven’t been evaluated in an aggregated manner. In this paper, the major factors that affects the total annual inventory cost such as the characteristics of the machine break downs, defective item rates, rework rates, scrap rates and backordering rate are considered. The effects of these major factors are examined with the help of experimental design and Taguchi engineering. Experimental results obtained from the numerical example are evaluated and effects of related factors are examined through Taguchi’s signal/noise ratios, analysis of means graphs, and interaction graphs. Experimental results yield that except the scrap rate, all factors have cause a significant variation on the total annual production-inventory cost.

Keywords:
Economic production quantity, Machine breakdowns, Rework, Taguchi method.

1. Introduction

Economic production quantity (EPQ) model determines the optimal production volume while minimizing the total production-inventory costs. In other words, EPQ model is a mathematical inventory model that considers finite production rate and EPQ model assumes production rate is higher than the item’s demand rate, and machine has no breakdown (Widyadana & Wee, 2009).

Possibility of producing defective items is not taken into account in Classical EPQ models. Defective end items may be produced during the regular production up-time because of the several reasons such as deterioration of production processes, imperfect quality of the components and subassemblies that are purchased from the suppliers. Thus, several production-inventory models that consider the generation of the defective items and their rework processes at the end of the production cycle are proposed by many researchers in the literature. In addition to produce defective items, another important issue that is encountered in real life manufacturing systems is the possibility of the random machine failures or breakdowns. Since the effects of these machine breakdown occurrences in the anywhere of the replenishment period may be disruptive and costly, these can be considered as a critical factors.

Another critical factor is degree of partial backordering. Since partial backordering is more practical in a fair competitive market, the degree of partial backordering must be forecasted accurately by market researches (Taleizadeh et al., 2010). Shortages are considered as partial backordering and also taken into account in this study.

Chiu et al. (2007) investigated an EPQ model with scrap, rework, and stochastic machine breakdowns occurring in the production up-time while Taleizadeh et al. (2010) presented a multi-product EPQ model with random defective items, service level constraints, repair failure and partial backordering.
Groenevelt et al. (1992) studied two production control policies to deal with stochastic machine breakdowns. The first one assumes that the production of the interrupted lot is not resumed (called no resumption or NR policy) after a breakdown. While the second policy considers that the production of the interrupted lot will be immediately resumed (called abort/resume or AR policy) after the breakdown is fixed and if the current on-hand inventory is below a certain threshold level (Chiu et al., 2007).

Chiu (2003) considered a finite production model with random defective rate, scrap, the reworking of repairable defective items, and backlogging. The production run time problem with random machine breakdowns occurring in production up-time was examined by Chiu (2007) under AR policy and reworking of defective items produced. The robust planning for an EPQ model with random machine breakdown and failure in rework under AR control policy was studied in (Chiu, 2010). In this paper, AR policy is applied to the EPQ models with scrap, rework, partial backordering, random machine breakdowns taking place in the production up-time and in the reworking process which are used for experimental analysis.

Hayek & Salameh (2001) assumed that all of the defective items produced are repairable and derived an optimal operating policy for EPQ model under the effect of reworking of imperfect quality items. Chen et al., (2010) studied the optimal producer’s replenishment policy for an EPQ model with rework, scrap and backorder occurring in backorder replenishment period.

This paper extends the prior works by Taleizadeh et al., (2010) and studies the EPQ model with scrap, reworking of defective items, random machine breakdowns and partial backordering. The characteristics of the machine breakdowns are determined as follows; repair times for the general type of breakdowns, the different stages of the replenishment period where machine breakdowns may take place in and so forth. In the first part of this study, mathematical modeling, production-inventory cycles for both perfect quality and defective items and total annual cost functions which consist of production, set-up, repairing, reworking, holding, backordering and lost sale are derived for the EPQ models which are used in the experimental analysis phase. A numerical example are provided to illustrate the practical usage of the derived formulations. Finally, experimental results obtained from the numerical example are evaluated and effects of related factors are examined through Taguchi’s signal/noise ratios, analysis of means graphs, and interaction graphs. Experiments are carried out in MINITAB 14 according to Taguchi L27 orthogonal array. The aim of this paper is using Taguchi method for investigating the effects of major factors related to the machine breakdown characteristics along with the other factors such as percentage of defective items, scrap rate, and fraction of shortage that is backordered.

2. Assumptions and Formulation

The imperfect quality EPQ model considers a manufacturing process with a constant production rate $P$ and demand rate $D$, where $P > D$ (Taleizadeh et al., 2010). Consider that during the regular production up-time, $x$ portion of produced items is assumed to be defective and is generated randomly at a production rate $\lambda$. The production rate of defective items $\lambda$ could be expressed as the production rate times the defective rate: $\lambda = P \cdot x$. All items produced are screened and the inspection cost per item is included in the unit production cost $C_p$.

All repairable defective items are reworked at a rate of $P^d$ at the end of each production cycle or when the machine is repaired and restored. Among these defective items, $\theta$ portion is considered to be scrap and is discarded when regular production ends. The other portion $(1-\theta)$ can be reworked and repaired immediately after the regular process. Stock-outs are allowed and partial backordered, they are satisfied by the next replenishment. Further, according to the mean time between failures (MTBF) data, machine breakdown may take place randomly in the production up-time or in the reworking process. AR inventory control policy is adopted in this paper.
It is also assumed that due to tight preventive maintenance schedule, the probability of more than one machine breakdown occurrences in a production cycle is very small. However, if it does happen, safety stock will be used to satisfy the demand during machine repairing time. Therefore, multiple machine failures are assumed to have insignificant effect on the proposed model as stated in (Chen et al., 2010).

3. Mathematical Modeling

### Notations

- \( A \): set-up cost for each production run;
- \( M \): cost for repairing and restoring machine;
- \( C^R \): repairing cost for each defective product reworked, $/item;
- \( C^S \): disposal cost per scrap product, $/scrap item;
- \( C^H \): holding cost of per product per unit time, $/item/unit time;
- \( C^{H'} \): holding cost per reworked product per unit time;
- \( C^d \): back ordered cost per product per unit time, $/item/unit time;
- \( C^l \): lost sale cost per product per year;
- \( Q \): production lot size for each cycle;
- \( B \): maximum backorder level, in units for each cycle;
- \( t \): time required for repairing and restoring the machine;
- \( \alpha \): fraction of shortage that is backordered;
- \( N \): number of cycles per year;
- \( TC(t, B) \): total inventory costs per year;
- \( E(.) \): expected value.

#### 3.1. Case 1: A production-inventory cycle considering AR policy for perfect quality items with random machine failure taking place in reworking process

Breakdown of the machine may occur randomly in the reworking process and the AR control policy is assumed in this case. In this case, time before a machine breakdown taking place is larger than production up-time \( t^1 \).

![Figure 1. A production-inventory cycle considering AR policy for perfect quality items with random machine failure taking place in reworking process](image-url)

From Figure 1, one can obtain the level of on-hand inventory when the optimal production quantity completed \( H_1 \), the level of inventory when machine breakdown occurs in the reworking process \( H_2 \), the level of inventory when machine is repaired and restored \( H_3 \), maximum level of on-hand inventory when the remaining reworking process is accomplished \( H_4 \) as follows:
\[ t^1 = \frac{Q}{p} \]  
\[ H_1 = (P - \lambda - D).t^3 - \alpha B \]  
\[ H_2 = H_1 + (P^1 - D).(t - t^1) \]  
\[ H_3 = H_2 - t_r.D = (P - \lambda - D).t^3 - \alpha B + (P^1 - D).(t - t^1) - g.D \]  
\[ H_4 = H_3 + (P^1 - D).(t^2 - t + t^1 - t_r) \]  
\[ t^1 = \frac{H_1}{H_3} \]  
\[ t^2 = \frac{x.Q.(1 - \theta)}{P^1} + t_r = \frac{\lambda Q.(1 - \theta)}{P^1 + p} + \frac{\lambda t^1.(1 - \theta)}{p} + g \]  
\[ t^3 = \frac{H_4}{D} = (P - \lambda - D).t^3 - \frac{\alpha B}{D} + (P^1 - D)\left(\frac{t - t^1}{D} - \frac{g.D}{D} + \frac{(P^1 - D)}{D}\right) \]  
\[ t^4 = \frac{\alpha B}{D} \]  
\[ t^5 = \frac{H_5}{H_6} \]  
\[ T = t + t_r + (t^2 - t + t^1 - t_r) + t^3 + t^4 + t^5 \]  
\[ Q = \frac{D.T - (1 - \alpha)B}{(1 - \theta).x} \]  

Total production-inventory cost per year in the case of breakdown takes place (under AR policy) during reworking process can be explained as in Equation (13):

\[ TC_z(t, B) = N.C^P.P.t^1 + N.M + N.A + N.C^R.P.t^1.x.(1 - \theta) + N.C^S.P.t^1.x.\theta \]  
\[ + N.C^H.\left\{ \frac{H_5 + \lambda t^1}{2}.(t^1) + \frac{H_5 + H_6}{2}.(t - t^1) + \frac{H_6 + H_7}{2}.(t_r) + \frac{H_7 + H_8}{2}.(t^2 - t + t^1 - t_r) \right\} \]  
\[ + N.C^H.\left\{ \frac{p^1.(t - t^1)}{2}.(t - t^1) + \frac{p^1.(t^2 - t + t^1 - t_r)}{2}.(t^2 - t + t^1 - t_r) \right\} \]  
\[ + N.C^H.\left\{ \frac{\alpha.B}{2}.(t^4 + t^5) + N.C^L.\left(\frac{1 - \alpha}{2}.B\right).(t^4) \right\} \]  

3.2 Case 2: EPQ model with stochastic machine failure occurred during the production up-time using the AR policy

Breakdown of the machine may occur randomly in the regular production up-time and the abort/resume (AR) control policy is assumed in this case. In this case, time before a machine breakdown taking place \( t \) is smaller than production up-time \( t^1 \). Total production-inventory cost per year can be calculated as in Equation (14) as similar in the previous case.

\[ TC_z(t, B) = N.C^P.P.t^1 + N.M + N.A + N.C^R.P.t^1.x.(1 - \theta) + N.C^S.P.t^1.x.\theta \]  
\[ + N.C^H.\left\{ \frac{H_5 + \lambda.E(t)}{2}.(t) + \frac{H_5 + H_6}{2}.(t_r) + \frac{H_6 + H_7}{2}.(t^1 - t) + \frac{\lambda.E(t) + \lambda.t^1}{2}.(t^1 - t) \right\} \]  
\[ + N.C^H.\left\{ \frac{p^1.(t - t^1)}{2}.(t - t^1) + \frac{p^1.(t^2 - t + t^1 - t_r)}{2}.(t^2 - t + t^1 - t_r) \right\} \]  
\[ + N.C^H.\left\{ \frac{\alpha.B}{2}.(t^4 + t^5) + N.C^L.\left(\frac{1 - \alpha}{2}.B\right).(t^4) \right\} \]
4. Numerical Example and Analysis

Assume in a manufacturing company, the demand rate of a product is 40,000 units per year, and this product has a production rate of 200 units per day. The unit production cost is $30, the holding cost is $10 per unit per year, the backorder cost is $5 per unit per product and the set-up cost is $400 per production run. There are 250 working days in a year. Additional data used in the non-ideal situations such as machine breakdowns, production of defective items and lost sale are given as follows; reworking rate is 180 units per day, C_r=$3 reworking cost for each defective product, C_s=$1.8 disposal cost per scrap product, C_l=$20 lost sale cost for each product per year, h_1=$13 holding cost for each reworked product g=0.012 years, constant machine repairing time, M=$600 cost for repairing and restoring machine for each breakdown. For analyzing the effects of critical factors and parameter changes on the optimal total annual cost value, Taguchi method was performed on the numerical example as an alternative method to the sensitivity analysis. The parameter (factor) levels were determined as given in Table 1.

Table 1. Factor levels used in the Taguchi design

<table>
<thead>
<tr>
<th>Breakdown occurrence</th>
<th>% of defective product</th>
<th>% of partial backordering</th>
<th>Repairing time (years)</th>
<th>Scrap rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>none</td>
<td>%0</td>
<td>0.0027</td>
<td>%0</td>
</tr>
<tr>
<td>Level 2</td>
<td>in production uptime</td>
<td>%8</td>
<td>0.0081</td>
<td>%10</td>
</tr>
<tr>
<td>Level 3</td>
<td>in reworking process</td>
<td>%10</td>
<td>0.015</td>
<td>%20</td>
</tr>
</tbody>
</table>

Because of the total degrees of freedom is equal to 10, we can use the L27 orthogonal design matrix which has least experiment number for 5-factors, 3-levels Taguchi design. Total annual cost was determined as response variable for the Taguchi technique. For the evaluation of experiment results and effects of related factors, Taguchi’s signal/noise ratios, analysis of means graphs and interaction graphs were used and the experiments were carried out in MINITAB 14 according to Taguchi L27 scheme. According to Figure 2a and 2b, since the place of breakdown occurrence, defective product rate, degree of partial backordering and repairing time cause significant variability on the total cost, these factors are important for the performance criterion. But scrap rate has no significant effect on the total cost.

It is obviously seen that, the main factor effects with analysis of means graph and the S/N ratios supported the same optimal factor levels. As a result, main factor levels breakdown occurrence-1(none), percentage of defective product-2 (%8), degree of partial backordering-2 (%50), repairing time-3 (0.015) and scrap rate-1,2 or 3 are observed as the factor levels decreasing the total annual production-inventory cost. Another important result is that total cost will increase after the %50 backordering rate. According to Figure 3, except the scrap rate there are strong or weak interactions between the factors. Since the
interaction lines do not intersect for the scrap rate, there is no interaction or weak interaction between scrap rate and other factors.

Figure 3. Interaction graphs for the total annual production-inventory cost.

5. Conclusion

EPQ models with scrap, rework, random machine breakdown occurring in the different stages of the replenishment period and partial backordering was proposed using the AR inventory control policy in this paper. According to the results of Taguchi method, optimal factor combinations were determined as follows: breakdown occurrence-1 (none), percentage of defective product-2 (%8), degree of partial backordering-2 (%50), repairing time-3 (0.015) and scrap rate-1, 2 or 3. In addition, it is specified that too high and low backordering rates will increase the total annual production-inventory cost.

For the future works, a production-inventory model considering AR policy for deteriorating items with random machine failure occurring in the production up-time, reworking process or backorder replenishing time may be investigated by the researchers. Also, new mathematical formulations can be derived for multiple machine breakdowns.

References


