BI-OBJECTIVE NON-STRICL SINGLE ALLOCATION HUB LOCATION PROBLEM: MATHEMATICAl PROGRAMMING MODEL AND A SOLUTION HEURISTIC

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Abstract: Hub location problem (HLP) is one of the most important problems in the areas of logistics and telecommunication network design. In this paper we address the bi-objective non-strict single allocation hub location problem (BONSSAHLP). The non-strict property of this problem which allows establishment of direct links between pairs of non-hub nodes makes it somehow different from the conventional hub location problem in which the linkage between any pair on non-hub nodes must be through one or two hub nodes. The first objective in BONSSAHLP is to minimize the total system-wide cost (including cost of locating hub facilities, cost of establishing linking arcs, and cost of transporting commodities) and the second objective is to minimize the maximum commodity routing distance between pairs of nodes in the network. A novel mathematical programming formulation is developed for the problem and since the problem belongs to the class of NP-hard problems (because it is a generalization of the conventional hub location problem which belongs to the class of NP-hard problems); an efficient heuristic based on tabu search (TS) is proposed to solve it. Numerical results indicate the efficiency of the proposed heuristic both in terms of solution quality and CPU time.

Keywords: Hub location; Logistics; Mathematical programming; Tabu search.

1. Introduction

Hubs are special facilities that serve as switching, transshipment and sorting points in many-to-many distribution systems. Instead of serving each origin-destination pair directly, hub facilities concentrate flows in order to take advantage of economies of scale [1]. Hub location problem has various applications in areas of transportation and telecommunication network design. The research on hub location was begun with the pioneering work of O’Kelly in 1986 [2] and the number of publications in this area has increased significantly over the past two decades. Campbell [3] presented formulations for four types of discrete hub location problems. Aykin [4] compared two hubbing policies which he named as strict and non-strict (direct connections are allowed). Sung and Jin [5] considered a hub network design problem where the non-stop service is allowed. The objective is to design a hub network under non-restrictive networking policy by determining all the required hub locations in the predetermined zones and also all the terminal-to-terminal routes such that the total network cost is minimized.

A typical non-strict single allocation hub location network is shown in Fig. 1. Here, hubs are shown as squares, thin lines are connections between non-hub and hub nodes, thick lines represent connections between hub nodes, and dashed lines show the direct connections between non-hub nodes (non-strict network). In single allocation hub location problems, every non-hub node (shown as circles in Fig. 1) must be connected to exactly one hub node. This is the case in many real applications such as logistic operations in supply chains where the entities in the chain prefer working one hub facility. Multi-objective models, among others, have received burgeoning attention over recent years. Koksalan and Banu [6] presented two bi-criteria uncapacitated multiple allocation $p$-hub location problems. In the first problem, the first objective is to minimize the total transportation costs while the second objective is to minimize the total traveling costs between hub points and origin-destination points. In their second problem, they addressed the delays occurring due to the congestion during service at the hubs and

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considered the trade-off between the first objective and a new objective function, which is to minimize the maximum delay at each hub. Costa et al. [7] proposed a different approach to the capacitated single allocation hub location problem. Instead of using capacity constraints to limit the amount of flow that can be received by the hubs, they introduced a second objective function to the model (besides the traditional cost minimizing function), that it minimizes the time to process the flow entering the hubs.

Fig. 1. A non-strict single allocation hub location network

This paper addresses the bi-objective non-strict single allocation hub location problem (BONSSAHLP) in which the following assumptions are made: 1) the hubs are interconnected and economies of scale is incorporated by a discount factor \(0 \leq \alpha \leq 1\) for using the inter-hub connections; 2) direct service is allowed according to non-strict hubbing policy and transportation cost associated with these arcs are increased by an increment factor \(\beta \geq 1\); 3) every non-hub node is connected to one and only one hub node (single allocation). The paper is organized as follows. In Section 2, the mathematical model of the problem is presented. A proposed solution heuristic based on tabu search is introduced in Section 3. The computational results are presented in Section 4 and finally the conclusions are given in Section 5.

2. Mathematical programming formulation

The following notations are used to represent the parameters and decision variables in the proposed integer (zero-one) programming model.

**Parameters:**
- \(n\) Number of nodes
- \(d_{ij}\) Distance between nodes \(i\) and \(j\)
- \(w_{ij}\) Flow from node \(i\) to node \(j\)
- \(f_k\) Fixed cost associated with establishment of hub facility at node \(k\)
- \(f_{HH}\) Fixed cost associated with establishment of an arc between two hub nodes
- \(f_{HN}\) Fixed cost associated with establishment of an arc between a hub node and a non-hub node
- \(f_{NN}\) Fixed cost associated with establishment of an arc between two non-hub nodes
- \(\alpha\) Discount factor associated with the cost of transportation between two hub nodes
- \(\beta\) Increment factor associated with the cost of transportation between two non-hub nodes

**Decision variables:**
- \(x_{ij}\) \(=\) \begin{cases} 1 & \text{if the flow from non-hub node } i \text{ to non-hub node } j \text{ is transmitted via direct link} \\ 0 & \text{if not} \end{cases}
- \(z_{ik}\) \(=\) \begin{cases} 1 & \text{if node } i \text{ is assigned to hub } k \\ 0 & \text{if not} \end{cases}
- \(q_{ij}\) \(=\) \begin{cases} 1 & \text{if an arc is established between non-hub nodes } i \text{ and } j \\ 0 & \text{if not} \end{cases}
- \(y_{ijkm}\) \(=\) \begin{cases} 1 & \text{if the flow from node } i \text{ to node } j \text{ is transmitted through hubs } k \text{ and } m, \text{ respectively} \\ 0 & \text{if not} \end{cases}
The BONSSAHLP can now be formulated as follows:

\[
\begin{align*}
\min Z_1 &= \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{m=1}^{a} (d_{ik} + \alpha d_{km} + d_{mj}) w_{ik} y_{ikm} + \sum_{i=1}^{n} \sum_{j=1}^{n} (\beta d_{ij} w_{ij}) x_{ij} \right) \\
&+ \sum_{k=1}^{s} f_k z_{ik} + \sum_{k=1}^{s} \sum_{m=1}^{a} f_{km} y_{ikm} + \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} z_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} q_{ij} \\
\min Z_2 &= \max_{i,j,k,m} \left\{ (d_{ik} + \alpha d_{km} + d_{mj}) y_{ikm}, \ d_{ij} x_{ij} \right\} 
\end{align*}
\]

Subject To:

\[
\begin{align*}
x_{ij} + \sum_{k=1}^{s} \sum_{m=1}^{a} y_{ikm} &= 1 & \forall i, j \\
\sum_{k=1}^{s} z_{ik} &= 1 & \forall i \\
z_{ik} &\leq z_{ik} & \forall i, k \\
\sum_{m=1}^{a} y_{ikm} &\leq z_{ik} & \forall i, j, k \\
\sum_{k=1}^{s} y_{ikm} &\leq z_{jm} & \forall i, j, m \\
2x_{ij} &\leq 2 - z_{ik} - z_{kj} & \forall i, j \\
2q_{ij} &\geq x_{ij} + x_{ji} & \forall i < j \\
x_{ij}, y_{ikm}, z_{ik}, q_{ij} &\in \{0, 1\} & \forall i, j, k, m 
\end{align*}
\]

Objective function (1) is sum of the transportation costs and fixed hub and arc establishment costs. The objective function (2) seeks to minimize the maximum of the path length between every pair of nodes in the network. Constraint (3) states that flow from node \(i\) to node \(j\) can be transmitted either via hubs \(k\) and \(m\) or a direct link. Constraint (4) ensures the single allocation property. Constraint (5) implies that a node should be established as hub before any non-hub node is allocated to it. Constraints (6) and (7) state that in order to a customer use a hub as a transshipment point, that customer should be assigned to that hub beforehand. Constraint (8) assures that the two end nodes of a direct link (non-strict) cannot be hub nodes. Constraint (9) states that in order to a direct link be constructed, both the direct and reverse flow between the two end nodes of this arc should be directed through this link. Constraint (10) is the binary constraint of the decision variables. The model (1)-(10) is nonlinear, but it can simply be linearized by replacing the objective function \(Z_2\) with \(Z'_2\) and adding constraints (11) and (12) to the model as follows:

\[
\begin{align*}
\min Z_1 &= L_{\max} \\
\min Z'_2 &= L_{\max} \\
\end{align*}
\]

Subject To:

\[
\begin{align*}
L_{\max} &\geq (d_{ik} + \alpha d_{km} + d_{mj}) y_{ikm} & \forall i, j, k, m \\
L_{\max} &\geq d_{ij} x_{ij} & \forall i, j
\end{align*}
\]

One of the most renowned solution approaches to solve bi-objective problems is to bound one of objective functions by a specified value and to optimize the other one. This process should be repeated using different values for the specified bound. In our model, the trade-off between two objective functions can be made by changing the value of \(L_{\max}\) and the Pareto optimal curve can be plotted from these
resulted points. In this paper, for the sake of brevity, the model is solved for only one value of $L_{\text{max}}$ ($L_{\text{max}} = 2500$).

3. A Solution Approach
One recent development in the solution of combinatorial problems is the introduction of meta-heuristics such as tabu search, genetic algorithms, simulated annealing and neural networks. All of these meta-heuristics show great promise in solving combinatorial problems such as the HLP. Among these, tabu search (TS) that is a local search procedure, uses memory structures to guide the movements from one feasible solution to another (usually to its best neighbor), even if this results in a deterioration of the objective function value allows the search to move out of the local optima and explore other regions of the solution space. (See Glover [8] and Glover et al. [9] for overviews of TS). We have proposed a heuristic based on TS approach to solve the BONSSAHLP.

3.1. Solution Representation
In this paper, we use one of the common solution representation methods in hub location literature. This representation uses two one-dimensional arrays of size $n$ (number of nodes). The first array, which is a zero-one string, is called hub location string in which the nodes corresponding to “one” elements are supposed to be hub nodes and the nodes corresponding to “zero” elements indicate non-hub nodes. The second array is called assignment string which shows the hub nodes to which every non-hub node is allocated. Fig. 2 shows the representation of the solution depicted in Fig. 1, which is a typical HLP with 13 nodes. As it is clear in Fig. 1, Four out of thirteen nodes are established as hub nodes (nodes 5, 9, 11, and 13).The corresponding elements in the first array in Fig. 2 take the value “one”.

![Fig. 2. Solution representation corresponding to the solution depicted in Fig. 1.](image)

3.2. Initial solution generation
The initial solution is constructed as follows. First, $p=[n/8]$ out of $n$ nodes are randomly selected as hub nodes (in which $[x]$ indicates the integer part of $x$) and then the remaining $(n-p)$ non-hub nodes are allocated to the hub nodes based on “nearest distance” criterion. This simple initial solution generation procedure is not only a quick way to produce a reasonable solution but it also produces diverse solutions which can help the algorithm not get trapped in local optima in different runs.

3.3. Neighborhood structures
We define and use four neighborhood structures to generate random neighbors in the proposed TS heuristic; “Close_Hub”, “Open_Hub”, “Hub_Exchange”, and “Reallocate” neighborhoods. In Close_Hub move, a randomly selected hub node becomes non-hub and its corresponding non-hub nodes are allocated to other hubs using the “nearest hub” policy. In Open_Hub move, a randomly selected non-hub node becomes hub and some of the non-hub nodes are allocated to the new hub using the “nearest hub” policy. In Hub_Exchange move, a randomly selected hub node becomes non-hub and a randomly selected non-hub node becomes hub. The non-hub nodes allocated to the former hub node now are allocated to the new hub node. The allocation of a randomly selected non-hub node to a hub node other than its current hub node is changed in Reallocate move.

3.4. The Proposed TS procedure
The proposed TS heuristic here have two stages (Locate_TS and Allocate_TS) which can be briefly described as follows: heuristic starts with an initial solution and at each iteration of Locate_TS, 100 hub
neighbor solutions are generated (probability of Close_Hub, Open_Hub, and Hub.Exchange moves are 0.2, 0.6, and 0.2, respectively). Then for each of these solutions, non-hub nodes are allocated to nearest hub. In the following, Allocate_TS heuristic run on reallocation move to improve allocation part of solution. The proposed TS heuristic has two parameters Max_Iter and Tabu_Tenure. Max_Iter is the maximum number of iterations performed by TS and Tabu_Tenure is the famous tabu tenure of TS. The values of these two parameters for Locate_TS are 8 and 4 while the corresponding values for Allocate_TS are 10 and 3. For each solution, the feasibility (according to the coverage radius) is checked. If the solution is infeasible, specified penalty cost is added to the objective function.

4. Computational Results

To demonstrate the effectiveness of the proposed TS heuristic, different instances of the well-known CAB data set which is generated from the Civil Aeronautics Board Survey of 1970 passenger data in the United States are first solved using the GAMS integrated development environment with CPLEX 12.2 solver. Then the same instances are solved using the proposed TS heuristic which is coded in MATLAB R2010b and run on a PC with 2.53 GHz of CPU and 4 GB of RAM. Problem instances are created as different combinations of two parameters α and β, each assuming several values (4 values for α and 6 values for β). To compare the results obtained through the assumption of non-strict hubbing policy with those of strict policy, we have set an additional level for β to be a sufficiently large number (M) in order to prohibit the establishment of direct arcs between non-hub nodes. So, we have created 24 (4×6) problem instances from CAB data set and used in our computational study. The selected values for other parameters used in our study are listed in Table 1.

<table>
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<tr>
<th>Table 1. Selected values for the additional parameters for the CAB data set</th>
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<td>n</td>
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Numerical results obtained from solving different BONSSAHLP instances for only one value of \(L_{max}\) (\(L_{max} = 2500\)) are shown in Tables 2. For each test problem instance, ten runs of the proposed TS heuristic are executed in order to test the consistency of the procedure i.e. the number of the algorithm runs in which optimal solution is resulted (successful runs). The first two columns characterize the problem instances. The next three columns (columns 3, 4, and 5) present the optimal objective function value, CPU time required for the CPLEX to solve the problem, and nodes acts as hubs in the optimal solution, respectively. Finally, the last three columns, which present the results obtained through solving the problem instances with the proposed TS heuristic, are as follows:

- Avg.CPU – average running time of heuristic spent (in seconds) through 10 runs;
- Min.Gap – minimum percentage gap of TS best solution from the optimal solution (out of 10 runs);
- S.Run – number of times the optimal solution is reached (out of 10 runs);

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<th>Table 2. Solution results obtained for the CAB data set using GAMS and proposed TS based heuristic</th>
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<td>Problem</td>
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It is clear from Table 2 that using non-strict hubbing policy, the total cost of the system can be reduced significantly in most of the cases comparing to strict policy. These results approve the effectiveness of the more flexible hubbing policies and also the importance of the developed model. It should also be noted that the proposed heuristic significantly reduces the time needed to solve the problem comparing to CPLEX software. As it is reported, the TS heuristic obtains the optimal solution for all instances in more than half of the times and this indicates the robustness of the proposed solution method.

5. Conclusion
In this paper, the new problem of bi-objective non-strict single allocation hub location problem (BONSSAHLP) is considered. A novel integer programming formulation is presented to model the problem. We developed an efficient tabu search heuristic approach to solve the problem and the validity of the proposed solution heuristic is shown by applying it on a set of test problems with different parameter settings. The results indicate the efficiency of the proposed heuristic in terms of CPU time required to obtain the optimal solution comparing to the CPLEX software. Also, it is shown that the total cost of the system may be reduced considerably when direct transportations between non-hub nodes (non-strict policy) are permissible in comparison to the case of strict hubbing policy. A promising avenue for further research is to impose limitations on the capacity of the arcs or hubs in the network.

6. References