A DYNAMIC PROGRAMMING APPROACH FOR MINIMIZING THE NUMBER OF DRAWING STAGES AND HEAT TREATMENTS IN CYLINDRICAL SHELL MULTI-STAGE DEEP DRAWING

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Abstract
In this paper, the problem of minimizing the number of drawing stages and heat treatments needed for the multi-stage deep drawing of cylindrical shells is addressed. A conventional rule-based computer-aided process planning (CAPP) is utilized within a dynamic programming approach to generate a set of alternative optimal process plans. A finite element analysis is carried out to guide the selection of the appropriate process parameters and to verify the applicability of the selected process plans. A case study is presented to demonstrate the developed approach, and the results suggest that the combination of rule-based CAPP, dynamic programming and finite element validation could be a valuable, reliable and quick computer aided process planning approach to this complicated problem.

Keywords
Deep drawing; number of drawing stages; CAPP; dynamic programming; finite elements.

1. Introduction
In many deep drawing cases, the desired final shape of the product cannot be obtained in a single pass; for otherwise, undesirable deformation or failure will occur. In order to avoid that, redrawing of hollow shells is conducted along with heat treatment whenever necessary. In such a multi-stage deep drawing process, it becomes necessary to minimize the number of drawing stages and heat treatments which directly leads to the minimization of manufacturing cost and lead time. Traditionally, the determination of the number of drawing stages is conducted using rules-of-thumb and expert's judgment. As indicated by Huh and Kim (2001), this practice was commonly followed in dealing with the different optimization problems that arise in the deep drawing process until recently in the last decade when modern optimization techniques have been employed.

Wifi et al. (2007) provide a literature review on the different optimization techniques applied to the deep drawing process including the minimization of the number of drawing stages in the case of multi-stage deep drawing. It is shown that the research work that focused on the minimization of the number of drawing stages is quite few. Cao et al. (2001) developed an optimization approach based on suitable design rules and inverse finite element for constructing suitable die shapes for the first draw and subsequent drawing steps of an automotive part having an axisymmetric shape. That approach was compared with another one that is based on industrial experience. It was capable of reducing the number of drawing stages from 10 to 6. Sonis et al. (2003) developed an improved model for determining the limiting drawing ratio, which is the ratio between the starting diameter and the ending shell diameter, for the first and subsequent drawing stages. They demonstrated how this model can be used to reduce the number of drawing stages in two different case studies.

Other research work focused on the optimization of the different process parameters for a fixed number of drawing stages. Kim and Hong (2007) studied a multi-stage deep drawing process for molybdenum which
is characterized by high mechanical strength at high, as well as low temperatures. For a given case study, they used a basic rule-of-thumb for the process design which can achieve the desired final shell in 8 drawing stages. Due to failure that occurred at the seventh stage, they used a simulated annealing optimization approach concurrently with finite element simulation in order to find the safe working parameters. Ramírez et al. (2010) developed an evolutionary approach for optimizing the multistage forming process of aluminum cups. Through a case study, they showed that their proposed technique for optimization is capable of achieving a reduction of 12.60% in costs and 13.24% in total processing time.

The current paper provides the first attempt to minimize the number of drawing stages and heat treatments in the multi-stage deep drawing of cylindrical shells using conventional CAPP rules utilized within a dynamic programming (DP) approach. Different alternative optimal process plans can be generated from the DP approach. Finite element analysis is used to make a judgment on selecting a set of optimal process plans from among those generated by the DP approach, and to adjust some of the process parameters through checking the severity of deformation so that the success of the selected process plans is verified.

2. CAPP rules for multi-stage deep drawing of cylindrical shells
The first step in designing the multi-stage deep drawing process is to determine the initial blank diameter, denoted $d_0$, based on the final desired shell diameter, $D$, and other final shape parameters. The diameter of the resultant cylindrical shell after each stage of the multi-stage deep drawing process ($d_i, i > 0$) has to be selected such that no failure occurs. This can be done by keeping two constraints satisfied. The first constraint, as defined in Aida (1998) and Suchy (1998), is represented in the following simple form.

$$d_i / d_{i-1} \geq m_n \quad \text{for} \quad i > 0$$  \hspace{1cm} (1)

Where $m_n$ is defined as the drawing rate limit and $n$ is the number of drawing stages conducted since the last heat treatment is made. If $l$ is the index of the last drawing stage after which a heat treatment is conducted, then $n = i - l - 1$. It is assumed that $n = 0$ when $i = 1$. The value of $m_n$ is dependent on the type of material being processed. In Aida (1998), typical values of $m_n$ have been determined for common materials used in deep drawing for both the first and second stages after heat treatment (i.e. $m_0$ and $m_1$). For the third and subsequent stages, the drawing rate limit has been approximated by Tisza (1998) as $m_n = m_{n-1} + 0.02$ for $n > 1$.

The second constraint is related to the strain severity condition. Due to the deformation of the sheet metal during first and subsequent stages, the strain severity of the material increases. To avoid failure, the current strain severity should not exceed the strain severity limit which is based on material properties. If at any drawing stage the strain severity has become close to that limit, heat treatment must be conducted to restore the formability of the processed material. However, heat treatment may also be conducted after any drawing stage at a point that is not necessarily close to the strain severity limit depending on the planner judgment which considers the remaining number of drawing stages.

At any stage $i$, the strain severity factor ($\varepsilon_i$) is calculated as $\varepsilon_i = [(d_{i-1} / d_i) + 1] / 2$ for $i > 0$. While, the cumulative strain severity ($E_i$) is given by $E_i = \prod_{j=i+1}^{l} \varepsilon_j$ for $i > 0$. Where, as indicated earlier, $l$ is the index of the last drawing stage after which heat treatment is conducted. The cumulative strain severity must not exceed the strain severity limit, denoted $E_{\text{max}}$, which is based on the material properties. Therefore, the second constraint on selecting the post-drawing shell diameter is expressed as follows.

$$E_i < E_{\text{max}}$$  \hspace{1cm} (2)
3. Dynamic programming coupled with finite element simulation

Dynamic programming (DP) is a mathematical technique that is based on the idea of separating a decision making problem into smaller sub-problems (Bellman, 1962). Each sub-problem represents a single stage whose solution is used to infer some properties that can be used to guide the solution of the other consecutive sub-problems (stages). A state definition for each stage needs to be developed such that any interdependence between stages is eliminated. In the studied problem, a state is defined using both the dimensions and the material properties of the shell after finishing the drawing stage. At an arbitrary stage $i$, the state is defined by the resultant shell diameter $d_i$ and the resultant material properties. The material properties for stage $i$ are summarized by a single factor based upon which the drawing rate limit and the cumulative strain severity are evaluated. This factor, referred to here as the age and denoted $t$, represents the number of drawing stages conducted since the last heat treatment is made. To summarize, the state at stage $i$, denoted $S_i$, is defined by the tuple $(d_i, a_i)$. The index of the initial state is set equal to zero where $S_0 = (d_0, 0)$, while there could be more than one terminal state for which the corresponding diameter equals $D$. The set of terminal states is defined as $\mathbb{T} = \{(D, a); a \geq 1\}.$

In order to move from state $S_i$ to state $S_j$, any arbitrary number of drawings and heat treatments may be conducted. We seek to conduct this transition with the minimum possible number of drawings and heat treatments such that constraints (1) and (2) are not violated. Starting from state $S_i$, we define $\Delta(S_i)$ as a mapping from that state to the set of all feasible shell diameters that can be reached in a single drawing step without violating constraints (1) and (2). Accordingly, the set of such feasible states is defined as $\mathcal{F}(S_i) = \{(d_j, a_i + 1); d_j \in \Delta(S_i)\}$. Furthermore, a heat treatment permits the transition from state $S_i = (d_i, a_i)$ to state $(d_i, 0)$. Accordingly, we define the set of all preceding states from which state $S_i$ can be reached as $\mathcal{P}(S_i) = \{S_j; S_j \in \mathcal{F}(S_i) \text{ and } a_i > 0 \} \cup \{(d_i, a); a_i > 0 \text{ and } a_i = 0\}.$

Let $N_{ij}$ denote the minimum number of drawing stages needed to move from state $S_i$ to state $S_j$, and $H_{ij}$ denote the minimum number of heat treatments. Since in this case we are considering two objectives, namely the minimization of the number of drawings and the minimization of the number of heat treatments, it is necessary to combine both objectives in a single one that can be used directly in the dynamic programming recursive search function. This integration is expressed in the form $O_{ij} = f(N_{ij}, H_{ij})$, where the function $f$ can take the form of a weighted summation of both objectives. In the trivial case when $D \in \Delta(S_0)$, the problem can be solved in just one drawing step. Otherwise, the following functional equation is used to evaluate the minimum number of drawing stages and heat treatments combination.

$$O_{0j} = \min_{S_i \in \mathcal{P}(S_j)} \{O_{0i} + O_{ij}\}$$

(3)

Since the shell diameter is a continuous variable, there is infinite number of states in the search space. Fortunately, it is not necessary to investigate all states in the search space since constraints (1) and (2) define discrete values for the shell diameter beyond which no feasible states can be reached. Therefore, it is sufficient to discretize the search space by dividing the range from $d_0$ to $D$ into smaller sub-ranges. For simplification, these sub-ranges are selected to be equally-sized with a size denoted $\delta$. Accordingly, it is required only to evaluate the values of $O_{ij}$ at diameters that equal $d_0 + \delta$, $d_0 + 2\delta$, $d_0 + 3\delta$ and so on. The selection of the sub-ranges size $\delta$ is crucial for the success of the application of dynamic programming to the studied problem. If it is selected to be too wide, the optimal solution might be missed and the obtained solution will be sub-optimal. If it is selected to be very small, unnecessary extra computational time will be needed without really improving the quality of the optimal solution obtained.

By solving the recursive equations (3), all feasible, optimal paths from state $S_0$ to any of the terminal states in set $\mathbb{T}$ can be easily determined. Each of these paths defines a process plan in which the post-drawing shell diameter after each stage along with the requirement of conducting heat treatment is
defined while having the minimum objective value. For a complete multi-stage deep drawing process plan, there are other important parameters that must be defined. These parameters include the die profile radius \((r_d)\), the punch nose radius \((r_p)\), the clearance between the punch and the die and the working range for the holder pressure \((P_h)\). The literature contains empirical recommendations for each of the previously mentioned parameters which depend mainly on the selected post-drawing shell diameters. These ranges are summarized in Abdel-Magied et al. (2003) and used in the current study.

The output of the dynamic programming approach is given in the form of a set of feasible process plans that have the same minimum objective value. The proposed approach in this paper does not stop at this point. A validation of the applicability of the selected process plans is conducted through a finite element simulation using ABAQUS-Explicit software. In that simulation, the deformation severity and the resulting strains and thickness variations are investigated. The forming limit diagram (FLD) is adopted as a basic reference to part failure in the process and an indicator for the need for heat treatment. If the process fails at any stage, some process parameters are manually adjusted based on the possible causes of failure, and the simulation is repeated until the success of the process is achieved.

4. Case study

The developed approach is applied to a case study from the literature in order to demonstrate its effectiveness. In this case study, a stainless steel 304 sheet with a thickness of 2.3 mm is drawn from a blank diameter of 451 mm down to a shell with a diameter of 127 mm. The main material properties that are needed for applying dynamic programming are given as \(E_{\text{max}}=1.9\), \(m_0=0.5\) and \(m_1=0.8\). Smith (1990) reported that this process is conducted in practice in seven stages. Abdel-Magied et al. (2003) were able to reduce this number to four stages with one heat treatment using a rule-based approach.

The DP calculations, presented in the previous section, are programmed using c++. Equal weights are used for \(\mathcal{N}_{ij}\) and \(\mathcal{H}_{ij}\). The selected increment \(\delta\) is set equal to 2, which is sufficient to obtain the same optimal objective value compared to lower values, while resulting in sufficiently large number of alternative optimal plans. Each of these plans can reach the required final shell dimensions in three drawing stages and one heat treatment, which represents a reduction of one drawing stage compared to Abdel-Magied et al. (2003). The running time for the DP approach is found to be less than two seconds on a dual core 1.86GHz processor with 2GB RAM.

The multistage drawing process is simulated in one job where all the dies are built in tandem i.e. one after the other as shown in Fig. 1. The details of the simulation process are given by Wifi et al. (2010). The present analysis considers the use of the forming limit diagrams (FLD) as described in Goodwin (1968) to predict the material’s failure during the drawing process. The forming limit diagram for stainless steel 304 used in the present analysis is developed by Tourki et al. (2005).

Four different plans have been selected from the output of dynamic programming, where each has three drawing stages and one heat treatment. Their post-drawing shell diameters and the associated process parameters are shown in Table 1. These parameters are further refined via careful finite element modeling and simulation trials. Attention was given to the die profile radius. This parameter is crucial in affecting the severity of deformation and should be carefully selected to get successful cup produced by multistage deep drawing.
Table 1: Optimum plans for a case study

<table>
<thead>
<tr>
<th>Stage No.</th>
<th>Initial Diameter</th>
<th>Final Diameter</th>
<th>Drawing Rate</th>
<th>$E_i$</th>
<th>$r_d$ (mm)</th>
<th>$r_p$ (mm)</th>
<th>$P_h$ (MPa)</th>
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<tbody>
<tr>
<td>Plan 1</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>451</td>
<td>291</td>
<td>0.645</td>
<td>1.275</td>
<td>12.0</td>
<td>36.0</td>
<td>1.99-49.03</td>
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<tr>
<td>2</td>
<td>291</td>
<td>239</td>
<td>0.821</td>
<td>1.414</td>
<td>6.0</td>
<td>18.0</td>
<td>1.59-167.7</td>
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<td>3</td>
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<td>127</td>
<td>0.531</td>
<td>1.441</td>
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<td>20.0</td>
<td>2.73-77.48</td>
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<td></td>
<td></td>
<td></td>
<td>Heat treatment</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Plan 2</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>451</td>
<td>240</td>
<td>0.532</td>
<td>1.44</td>
<td>12.0</td>
<td>18.9</td>
<td>2.72-37.92</td>
</tr>
<tr>
<td>2</td>
<td>240</td>
<td>154</td>
<td>0.642</td>
<td>1.279</td>
<td>8.0</td>
<td>14.9</td>
<td>2.0-99.22</td>
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<tr>
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<td>154</td>
<td>127</td>
<td>0.825</td>
<td>1.415</td>
<td>5.0</td>
<td>20.0</td>
<td>1.592-488.7</td>
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<td></td>
<td>Heat treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plan 3</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>451</td>
<td>240</td>
<td>0.532</td>
<td>1.44</td>
<td>12.0</td>
<td>18.9</td>
<td>2.722-37.92</td>
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<tr>
<td>2</td>
<td>240</td>
<td>197</td>
<td>0.821</td>
<td>1.597</td>
<td>5.0</td>
<td>11.9</td>
<td>1.593-210.26</td>
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<tr>
<td>3</td>
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<td>1.276</td>
<td>7.0</td>
<td>20.0</td>
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<tr>
<td>Plan 4</td>
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<tr>
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<td>18.9</td>
<td>2.39-41.0</td>
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<tr>
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<td>212</td>
<td>0.822</td>
<td>1.523</td>
<td>6.0</td>
<td>12.9</td>
<td>1.59-202.187</td>
</tr>
<tr>
<td>3</td>
<td>212</td>
<td>127</td>
<td>0.599</td>
<td>1.335</td>
<td>8.0</td>
<td>20.0</td>
<td>2.21-103.3</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Heat treatment</td>
<td></td>
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</table>

The four different plans were simulated according the technique described earlier. In all simulations, the die profile radius for Stages 2 and 3 were used as given by the conventional CAPP rules given in Abdel-Magied et al. (2003). In all cases the simulations did not proceed during both stages. Increasing the die profile radius relaxes the drawing process and allows larger thickness to be achieved at the end of the drawing stage. Only plans 2 and 3 were successful and completed to the end of the simulation. Plans 1 and 4 failed due tearing in the cup wall in stage 2 and 3 respectively after increasing the die profile radii in all cases.

Fig. 2 shows the variations in thickness and FLD failure criterion respectively at the end of Stage 3 for Plans 2 (a) and Plan 3 (b); and FLD damage factor Plan 2 (c) and Plan 3 (d).
fact that the heat treatment process was performed before Stage 3 in Plan 3. These simulations will help the designer to choose between two cases, one with higher final thickness and the other with a lower thickness but the material has higher residual strength.

5. Conclusion
In this paper, a new approach for CAPP of the multi-stage deep drawing of cylindrical shells is presented. In this approach, dynamic programming is used to optimize the selection of drawing stages and heat treatment decisions. Finite element simulation is used to guide the selection of the optimal process plans generated, and to adjust the process parameters so that the success of the selected process plans is verified. It is shown that the developed approach is a fast and reliable approach. In a future study, further investigation is needed to provide an automated way of optimizing the process parameters of a selected optimal process plan instead of using expert's judgment.

References