Dr. Burke
Dr. Gould

Physics 151L

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Final Examination
December 15, 2003

Name ________________________________

Signature ________________________________

Student ID ________________________________

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First Letter of Your Last Name: [ ]

Problem	Score	Max	Grader
1		(20)	
2		(20)	
3		(25)	
4		(20)	
5		(20)	
6		(25)	
7		(20)	
8		(30)	
9		(20)	
Total		(200)	

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Possibly Useful Formulae

For constant acceleration in one dimension:

$$\mathrm{v} = at + \mathrm{v}_0$$

$$x = \tfrac{1}{2}at^2 + \mathrm{v}_0 t + x_0$$

$$\mathrm{v}^2 - \mathrm{v}_0^2 = 2a\Delta x$$

Momentum — linear and angular:

$$\vec{p} = m\vec{\mathrm{v}} \qquad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

Newton's Second Law:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

Circular Accelerations:

$$a_c = \frac{\mathrm{v}^2}{R} = \omega^2 R \qquad a_t = \frac{d\mathrm{v}}{dt} = \alpha R$$

Circular Accelerations:

$$a_r = \frac{\mathrm{v}^2}{R} = \omega^2 R$$

$$a_t = \frac{d\mathrm{v}}{dt}$$

Friction:

$$f_k = \mu_k N$$

$$f_s \le \mu_s N$$

Moment of Inertia:

$$I = \sum m_i r_i^2 = \int_{body} r^2 dm$$

$$I_{c.m.} = MR^2 \quad (ring)$$
$$I_{c.m.} = \frac{1}{2} MR^2 \quad (disc)$$
$$I_{c.m.} = \frac{1}{12} ML^2 \quad (thin\ rod)$$
$$I_{c.m.} = \frac{2}{5} MR^2 \quad (sphere)$$

Parallel axis theorem:

$$I = I_{c.m.} + Mh^2$$

Kinetic Energy — rotational and translational:

$$K = \frac{1}{2} m\mathrm{v}^2 + \frac{1}{2} I_{c.m.}\omega^2$$

Work:

$$W_{i \to f} = \int_i^f \vec{F} \cdot d\vec{s}$$

Potential Energy:

$$U(\vec{r}) - U(ref) = -\int_{ref}^{\vec{r}} \vec{F} \cdot d\vec{s}$$

$$U(y) = mgy \quad \text{(gravity, near Earth)}$$
$$U(r) = -\frac{Gm_1 m_2}{r} \quad \text{(gravity, generally)}$$
$$U(x) = \frac{1}{2} kx^2 \quad \text{(spring)}$$

Condition for rolling without slipping for a round object of radius R:

$$s = R\theta \qquad |\vec{\mathrm{v}}| = R|\vec{\omega}| \qquad |\vec{a}| = R|\vec{\alpha}|$$

Oscillations:

$$\frac{d^2 x}{dt^2} = -\omega^2 x \qquad \frac{\omega}{2\pi} = f = \frac{1}{T} \qquad x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

Temperature Scale T(K) = T(°C) + 273.15

First Law of Thermodynamics: $\Delta U = Q - W$

Ideal Gas Equations: $PV = nRT$ and $U = \left(\frac{3}{2}, \frac{5}{2}, or\, \frac{6}{2}\right) nRT$

Specific Heat: $C_V = \left(\frac{3}{2}, \frac{5}{2}, \text{ or } \frac{6}{2}\right) nR$ Adiabatic Index: $\gamma = \frac{C_P}{C_V}$

Elementary Thermodynamic Processes

Process	**Constants During Process**	**Change of Energy ΔU**	**Heat Given to System Q**	**Work Done by System dW = PdV**	**Change of Entropy dS = dQ/T**
1. Isochoric dV = 0	P/T	$C_V\,\Delta T$	$C_V\,\Delta T$	0	$C_V \ln\left(\frac{T_f}{T_i}\right)$
2. Isobaric dP = 0	V/T	$C_V\,\Delta T$	$C_P\,\Delta T$ $= (C_V + nR)\,\Delta T$	$P\,\Delta V$ $= nR\,\Delta T$	$C_P \ln\left(\frac{T_f}{T_i}\right)$
3. Isothermal dT = 0	PV	0	$nRT \ln\left(\frac{V_f}{V_i}\right)$	$nRT \ln\left(\frac{V_f}{V_i}\right)$	$nR \ln\left(\frac{V_f}{V_i}\right)$
4. Adiabatic Q = 0 (Isentropic)	PV^γ $TV^{\gamma-1}$ $T^\gamma P^{1-\gamma}$	$C_V\,\Delta T$	0	$\frac{C_V}{nR}\Delta(PV)$ $= -C_V\,\Delta T$	0

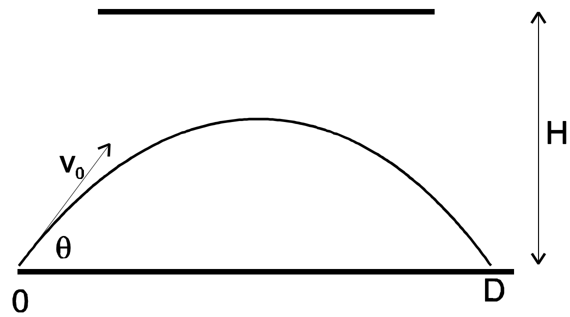
Efficiency of a Heat Engine:

$$\varepsilon = \frac{W}{Q_H} = 1 - \frac{|Q_C|}{Q_H} \quad \rightarrow \quad 1 - \frac{T_C}{T_H} \quad (Carnot)$$

Heat Transfer:

$$I = \frac{\Delta Q}{\Delta x} = kA\frac{\Delta T}{\Delta x} \qquad I = e\sigma A T^4$$

1. (20 pts) Trajectory. An object of mass *m* is projected from the ground at an angle θ with respect to the horizontal. Your goal is to make the projectile strike a target on the ground a distance *D* away.

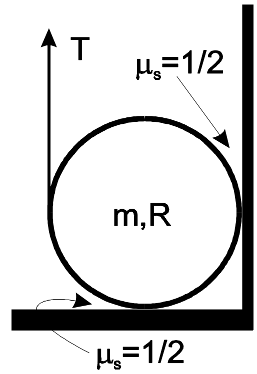


(a) (6pts) In terms of *D*, *g*, and θ, or a subset of these quantities, with what speed v_0 must you launch the projectile?

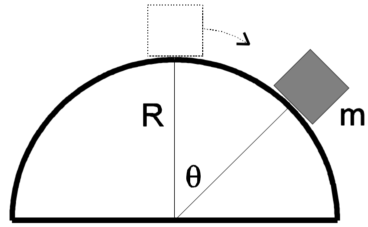
(b) (10 pts) Okay, now we have to admit that we didn't tell you everything up front. It turns out that this experiment is being done inside a long room with a ceiling a distance *H* above the floor. If you launch the projectile at too large an angle with the speed required to reach the target, the projectile will hit the ceiling and fall short of the target. In terms of only *D*, *g*, and *H*, or a subset of these quantities (*note*: you cannot use v_0 in the answer!), what is the maximum angle θ at which you can launch the projectile and still reach the target?

(c) (4 pts) If you did part (b) correctly, your answer does *not* involve *g*. (Hint: if it does, you should go back and find your mistake now!) Without doing any calculations, given that the only allowed quantities in your answer were *D*, *g*, and *H*, why should you have known that *g* could not appear in that answer?

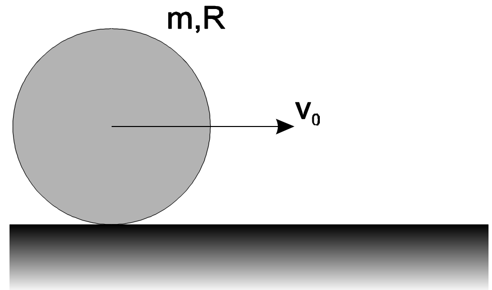
2. (20 pts) Statics. A cylinder of mass *m* sits at the corner between the floor and wall with a string wrapped around it. The string is then pulled upwards as shown in the figure. The coefficient of static friction between the cylinder and both floor and wall is exactly μ_s=½. Find the maximum tension *T* with which the string can be pulled without causing the cylinder to slip against the wall and floor. (*Hint*: When the cylinder is on the verge of slipping, both friction forces are at their respective maximum values.) Show how you solved the problem. An answer without that solution will not be credited. We recommend you start with a free body diagram.



3. (25 pts) Igloo. An eskimo of mass *m* sits precisely at the top of a frictionless hemispherical igloo of radius *R* when he starts to slip. As he slides further down the igloo he speeds up, eventually (but before hitting the ground) losing contact with the igloo. At what angle with respect to the vertical, θ, does he lose contact (*i.e.* flies off)? Show how you solved the problem. An answer without that solution will not be credited.



4. (20 pts) Billiard Ball. A billiard ball of mass *m* and radius *R* is initially moving with speed v_0, but is not rotating. Thanks to friction between the billiard ball and table top (coefficient of kinetic friction μ_s) the ball slows down, but at the same time starts rotating. Eventually the ball rolls without slipping. In the following parts give your answer in terms of the known quantities *m*, v_0, *R*, *g*, and μ_k, or a subset of them. Show how you solved the problem. An answer without that solution will not be credited.

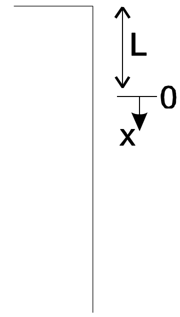


(a) (6pts) Draw the free body diagram for the billiard ball during the time that it is translating (and possibly rotating) but has not yet reached the condition of rolling without slipping. Make sure to draw coordinate systems as appropriate, as they are also included in the grading.

(b) (6 pts) Write down the components of Newton's Second Law in both the translational (F=ma) and rotational ($\tau = I\alpha$) forms along all relevant coordinates.

(c) (8 pts) How much time elapses between the instant when the ball was translating without rotating, and when it rolls without slipping?

5. (20 pts) Bungee Cord. A person of mass *m* attached to a bungee cord, of unstretched length *L* and spring constant *k*, jumps off of a cliff. After free-falling the distance *L* the bungee cord starts to stretch and eventually stops the person from hitting the ground far below. Use the coordinate system shown in the figure to the right where the origin is located at a distance *L* below the top of the cliff and the downward direction is taken to be positive. In the following give your answers in terms of the known quantities *m, g, L, k*, or a subset of these quantities. Show how you solved the problem. An answer without that solution will not be credited.

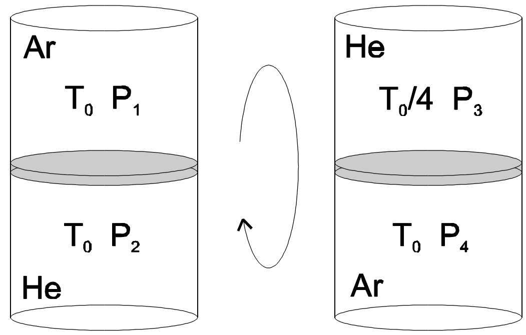


(a) (10 pts) What is the maximum speed of the person during the fall?

(b) (5 pts) After the person bounces up and down on the bungee cord many times, enough energy is dissipated so that he/she comes to rest at some equilibrium position. Where is this?

(c) (5 pts) Why is the location you found in part (b) the same as the location in part (a)?

6. (25 pts) Inverted Gas Volumes. A vertical closed cylinder of cross-sectional area A is divided into two equal volumes by an insulating freely-movable piston of mass m. There is Argon in the upper chamber at temperature T_0, and Helium in the lower chamber also at temperature T_0. The cylinder is then inverted, putting the Helium on top. Holding the Argon temperature at T_0 it is found that reducing the temperature of the Helium to $T_0/4$ causes the volumes of the two gasses in this inverted configuration to again be equal. None of the pressures are given, so to refer to them we will simply call the pressures of the two gasses in the two configurations P_1 through P_4 as shown in the figure.



(a) (5 pts) What is the pressure P_2 in terms of P_1, m, g, and A, or a subset of these quantities?

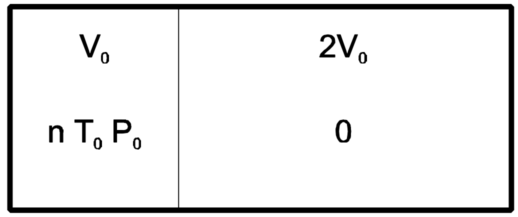
(b) (5 pts) What is the pressure P_3 in terms of P_2, m, g, and A, or a subset of these quantities?

(c) (5 pts) What is the pressure P_4 in terms of P_3, m, g, and A, or a subset of these quantities?

(d) (5 pts) What is the pressure P_4 in terms of P_1, m, g, and A, or a subset of these quantities?

(e) (5 pts) Solve for the pressure P_1 in terms of m, g, and A only (*i.e.* without any other pressures in your final expression).

7. (20 pts) Free Expansion. In a chamber of volume V_0 we confine n moles of a diatomic gas with temperature T_0 and pressure P_0. Suddenly, one wall of the chamber vanishes and the gas freely rushes to fill the now-larger chamber of total volume $3V_0$. The gas eventually settles down to a new equilibrium state in this larger volume.



(a) (4 pts) How much heat is added to the gas during the free expansion?

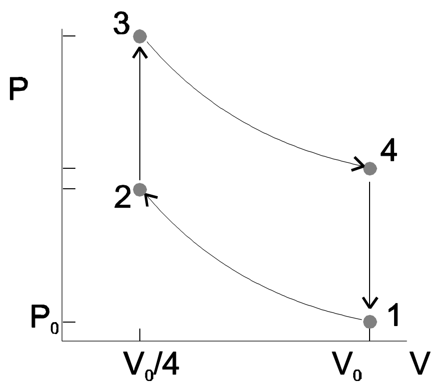
(b) (4 pts) How much work does the gas do during the free expansion?

(c) (4 pts) What is the temperature of the gas when it settles down after the free expansion?

(d) (4 pts) What is the entropy change of the gas due to the free expansion?

(e) (4 pts) After everything settles down the gas is then recompressed adiabatically back to its initial volume. What is the entropy change of the gas during this recompression?

8. (30 pts) Otto Cycle. *n* moles of an ideal gas with adiabatic index γ=3/2 is placed initially in state 1 characterized by a known pressure P_0, volume V_0, and temperature T_0, when it is carried through the Otto cycle shown in the figure to the right. (Note: this diagram is not necessarily to scale.)



1→2 Adiabatic compression to volume $V_0/4$.
2→3 Isochoric heating with $Q=3nRT_0$.
3→4 Adiabatic expansion back to volume V_0
4→1 Isochoric cooling back to the initial state.

(a) (12 pts) Calculate the *P*, *V*, *T* coordinates of each of the states in terms of P_0, V_0, and T_0 respectively, and place the answer into this table, which has already been partially filled with the information given above. Show your work, where needed, below the table.

State	Pressure	Volume	Temperature
1	P_0	V_0	T_0
2		$V_0/4$	
3		$V_0/4$	
4		V_0	

(continued on next page)

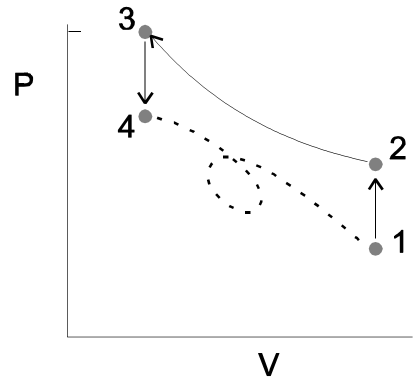
Otto Cycle Problem (continued):

(b) (12 pts) For each leg of this cycle, calculate the heat *added to* the system, Q, the work done by the system, W, and the change in the system's internal energy ΔU.

Process	**Q**	**W**	**ΔU**
1→2			
2→3			
3→4			
4→1			

(c) (6 pts) What is the efficiency of this cycle?

9. (20 pts) Irreversible Refrigerator. An ideal gas is taken through a series of thermodynamic processes as shown in the figure, finishing with an irreversible free expansion.



1→2 Isochoric heating
2→3 Isothermal compression
3→4 Isochoric cooling
4→1 Free expansion back to the initial state.

(a) (12 pts) In each cell of the following table, indicate with a "+", "0", or "-" sign whether the quantity at the top of the column increases, stays the same, or decreases during the process on the left.

Process	**ΔU**	**Q**	**W**	**ΔS**
1→2				
2→3				
3→4				
4→1				

(b) (8 pts). In terms of the heat added and work done along each leg ($Q_{1\rightarrow2}$, $Q_{2\rightarrow3}$, $Q_{3\rightarrow4}$, $Q_{4\rightarrow1}$, $W_{1\rightarrow2}$, $W_{2\rightarrow3}$, $W_{3\rightarrow4}$, $W_{4\rightarrow1}$, or a subset of these eight quantities) what is the coefficient of performance of this refrigerator (*i.e.* its efficiency)?