Dr. Burke
Dr. Gould
Dr. Johnson

Physics 151L

____

Final Examination
May 4, 2004

Name ______________________________

Signature ______________________________

Student ID ______________________________

First Letter
of Your
Last Name: [ ]

Problem	Score	Max	Grader
1	________	(20)	________
2	________	(25)	________
3	________	(25)	________
4	________	(25)	________
5	________	(20)	________
6	________	(20)	________
7	________	(25)	________
8	________	(40)	________
Total	________	(200)	________

Do Not Tear Off This Page

Possibly Useful Formulae

For constant acceleration in one dimension:

$$\mathrm{v} = at + \mathrm{v}_0$$

$$x = \tfrac{1}{2}at^2 + \mathrm{v}_0 t + x_0$$

$$\mathrm{v}^2 - \mathrm{v}_0^2 = 2a\Delta x$$

Momentum — linear and angular:

$$\vec{p} = m\vec{\mathrm{v}} \qquad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

Newton's Second Law:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$$

Circular Accelerations:

$$a_c = \frac{\mathrm{v}^2}{R} = \omega^2 R \qquad a_t = \frac{d\mathrm{v}}{dt} = \alpha R$$

Circular Accelerations:

$$a_r = \frac{\mathrm{v}^2}{R} = \omega^2 R$$

$$a_t = \frac{d\mathrm{v}}{dt}$$

Friction:

$$f_k = \mu_k N$$

$$f_s \le \mu_s N$$

Moment of Inertia:

$$I = \sum m_i r_i^2 = \int_{body} r^2 dm$$

$$I_{c.m.} = MR^2 \quad (ring)$$

$$I_{c.m.} = \frac{1}{2} MR^2 \quad (disc)$$

$$I_{c.m.} = \frac{1}{12} ML^2 \quad (thin\ rod)$$

$$I_{c.m.} = \frac{2}{5} MR^2 \quad (sphere)$$

Parallel axis theorem:

$$I = I_{c.m.} + Mh^2$$

Kinetic Energy — rotational and translational:

$$K = \frac{1}{2} m\mathrm{v}^2 + \frac{1}{2} I_{c.m.}\omega^2$$

Work:

$$W_{i\to f} = \int_i^f \vec{F} \cdot d\vec{s}$$

Potential Energy:

$$U(\vec{r}) - U(ref) = -\int_{ref}^{\vec{r}} \vec{F} \cdot d\vec{s}$$

$$U(y) = mgy \quad \text{(gravity, near Earth)}$$

$$U(r) = -\frac{Gm_1 m_2}{r} \quad \text{(gravity, generally)}$$

$$U(x) = \frac{1}{2} kx^2 \quad \text{(spring)}$$

Condition for rolling without slipping for a round object of radius R:

$$s = R\theta \qquad |\vec{\mathrm{v}}| = R|\vec{\omega}| \qquad |\vec{a}| = R|\vec{\alpha}|$$

Oscillations:

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \frac{\omega}{2\pi} = f = \frac{1}{T} \qquad x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

Temperature Scale T(K) = T(°C) + 273.15

First Law of Thermodynamics: $\Delta E = Q + W = Q - W_S$

W = Work done by environment

W_s = Work done by system

Ideal Gas Equations: $PV = nRT$ and $E = \left(\frac{3}{2}, \frac{5}{2}, or \frac{6}{2}\right) nRT$

Specific Heat: $C_V = \left(\frac{3}{2}, \frac{5}{2}, \text{ or } \frac{6}{2}\right) R$ Adiabatic Index: $\gamma = \frac{C_P}{C_V}$

Elementary Thermodynamic Processes

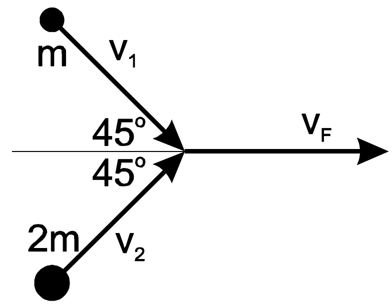
Process	**Constants During Process**	**Change of Energy ΔE**	**Heat Given to System Q**	**Work Done by System $dW_s = PdV$**	**Change of Entropy dS = dQ/T**
1. Isochoric dV = 0	P/T	$nC_V\,\Delta T$	$nC_V\,\Delta T$	0	$nC_V \ln\left(\frac{T_f}{T_i}\right)$
2. Isobaric dP = 0	V/T	$nC_V\,\Delta T$	$nC_P\,\Delta T$ $= n(C_V + R)\,\Delta T$	$P\,\Delta V$ $= nR\,\Delta T$	$nC_P \ln\left(\frac{T_f}{T_i}\right)$
3. Isothermal dT = 0	PV	0	$nRT \ln\left(\frac{V_f}{V_i}\right)$	$nRT \ln\left(\frac{V_f}{V_i}\right)$	$nR \ln\left(\frac{V_f}{V_i}\right)$
4. Adiabatic Q = 0 (Isentropic)	PV^γ $TV^{\gamma-1}$ $T^\gamma P^{1-\gamma}$	$nC_V\,\Delta T$	0	$\frac{C_V}{R}\Delta(PV)$ $= -nC_V \Delta T$	0

Efficiency of a Heat Engine:

$$\eta = \frac{W_S}{Q_H} = 1 - \frac{|Q_C|}{Q_H} \quad \rightarrow \quad 1 - \frac{T_C}{T_H} \quad (Carnot)$$

Heat Transfer:

$$I = \frac{\Delta Q}{\Delta x} = kA\frac{\Delta T}{\Delta x} \qquad I = e\sigma AT^4$$

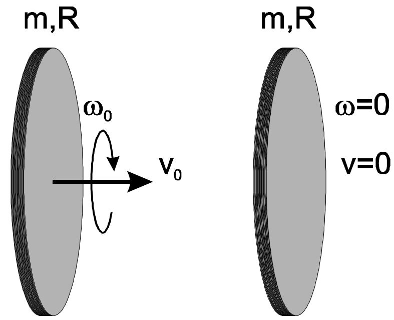


1. (20 pts) 2-D Collision. Two pieces of putty, of mass *m* and 2*m* are traveling at speeds v_1 and v_2 perpendicularly to each other when they collide and stick together, then moving with *known* speed v_F and at an angle of 45° with respect to each of the incoming directions. Your job here is to determine what v_1 and v_2 are in terms of *m*, v_F, and the angle between them.

(a) (10 pts) Is momentum conserved? If so, then also write down here the equations expressing its conservation which you will later use to solve for v_1 and v_2.

(b) (10 pts) Solve for v_1 and v_2 in terms of the other given quantities.

2. (25 pts) Collision with Angular Momentum. Two identical solid disks collide and stick together. The first travels initially with speed v_0 and is rotating about its axis with angular speed ω_0 (in the clockwise direction as seen by the second disk), while the second is initially not moving or rotating. The disks are perfectly aligned so that the centers of the two disks touch each other after the collision. All answers should be in terms of m, R, v_0, ω_0, or a subset of these quantities.

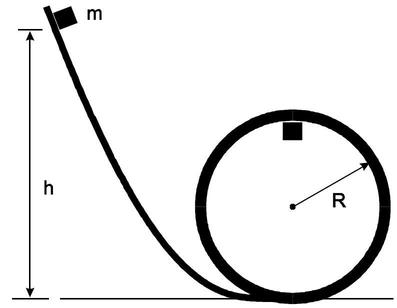


(a) (3 pts) What is the direction and magnitude of the linear momentum of the system of the two disks after the collision?

(b) (4 pts) What is the direction and magnitude of the angular momentum of the system of the two disks after the collision?

(c) (6 pts) What is the energy of the system of the two disks *before* the collision?

(d) (12 pts) What is the energy of the system of the two disks *after* the collision?



3. (25 pts) Loop-the-Loop. A roller coaster car of mass *m* initially sits motionless on a frictionless track a vertical distance *h* above the bottom of the roller coaster's loop. The loop has radius *R*.

(a) (8 pts) When the roller coaster car goes around and around the loop, what is the normal force exerted by the track on the car *at the bottom of the track*? Give your answer in terms of *m*, *g*, *h*, and *R*, or a subset of these quantities. Show how you reached this conclusion.

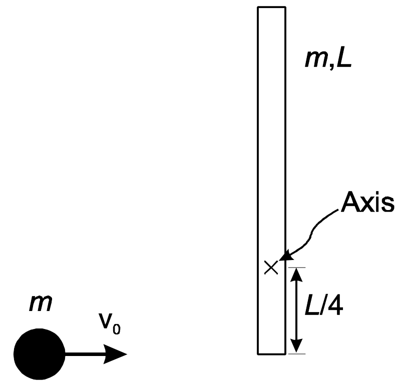
.

(b) (6 pts) When the roller coaster car goes around the loop, what is the speed of the car *at the top of the track*? Give your answer in terms of *m*, *g*, *h*, and *R*, or a subset of these quantities. Show how you reached this conclusion.

.

(c) (11 pts) What is the smallest that *h* can be in order that the roller coaster car not fall off the track at the very top, plunging its occupants into... oh, wait – the lawyers say not to worry about what happens *after* it falls off. Give your answer in terms of *m*, *g*, and *R*, or a subset of these quantities. Show how you reached this conclusion.

4. (25 pts) Putty and Rod. A uniform rod of mass *m* and length *L* is initially motionless. A piece of putty of the same mass *m* traveling at speed v_0 perpendicular to the rod strikes the rod at its very end, as shown in the figure to the right, and sticks to it. At the instant of the collision the center of mass of the system is located at the position marked "×" in the figure. The location of the "×" axis is fixed in space, though the rod and putty freely move without restraint.



(a) (5 pts) What is the angular momentum of the system of rod and putty about the axis "×" marked in the figure *before* the collision?

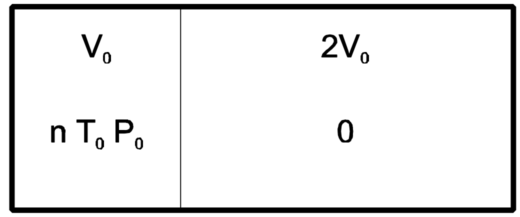
(b) (5 pts) What is the angular momentum of the system of rod and putty about the axis "×" marked in the figure *after* the collision?

(c) (5 pts) After the collision, what is the moment of inertia of the system of rod and putty about the system's center of mass?

(d) (5 pts) After the collision, what is the angular frequency of rotation of the system of rod and putty about its center of mass?

(e) (5 pts) After the collision, what is the total mechanical energy of the system, including both translational and rotational kinetic energy?

5. (20 pts) Free Expansion. In a chamber of volume V_0 we confine n moles of a diatomic gas with temperature T_0 and pressure P_0. Suddenly, one wall of the chamber vanishes and the gas freely rushes to fill the now-larger chamber of total volume $3V_0$. The gas eventually settles down to a new equilibrium state in this larger volume.



(a) (4 pts) How much heat is added to the gas during the free expansion?

(b) (4 pts) How much work does the gas do during the free expansion?

(c) (4 pts) What does Conservation of Energy tell you that the temperature of the gas is when it settles down after the free expansion? Show how you found the temperature.

(d) (4 pts) What is the entropy change of the gas due to the free expansion?

(e) (4 pts) After everything settles down the gas is then recompressed adiabatically back to its initial volume. What is the entropy change of the gas during this recompression?

6. (20 pts) Calorimetry. Maria and Arnold are hosting a delegation of legislators. She brings 1 liter of chilled white wine at a temperature of 2°C, while he brings 2 liters of the same type of wine straight from the store at 20°C. They decide to make the best of it and mix the two together. (Assume that the container in which they mix the wine does not change its temperature during the process.) The specific heat of wine is C = 4 kJ/kg-K, and its density is 1 kg/liter.

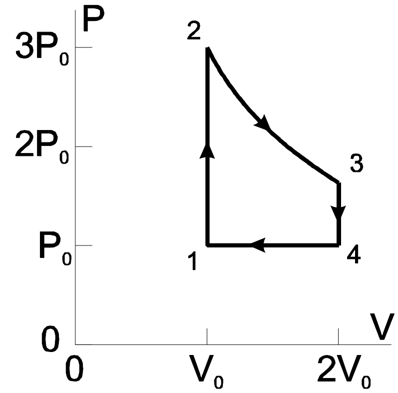
(a) (5 pts) Using conservation of energy, determine the final temperature of the mixture.

(b) (5 pts) What is the amount of heat transferred from Arnold's wine into Maria's ?

(c) (5 pts) How much heat needs to be extracted from the mixture in order to cool it down to 0°C?

(d) (5 pts) In order to effect this final chilling, the Chief of Staff brings some ice at exactly 0°C and wants to add it to the modestly cool wine in order to cool it further. The latent heat of fusion of ice is 336 kJ/kg. How much ice should be added so that the wine is cooled all the way to 0°C and does not leave any tell-tale ice chips floating in the container?

7. (25 pts) Four Legged Cycle. *n* moles of an ideal gas with adiabatic index γ=3/2 is placed initially in state 1 characterized by a known pressure P_0, volume V_0, and temperature T_0, and then carried through the cycle shown in the figure to the right.



1➞2 Isochoric heating to pressure $3P_0$.
2➞3 Isothermal expansion to volume $2V_0$.
3➞4 Isochoric cooling back to the initial pressure.
4➞1 Isobaric compression back to the initial state.

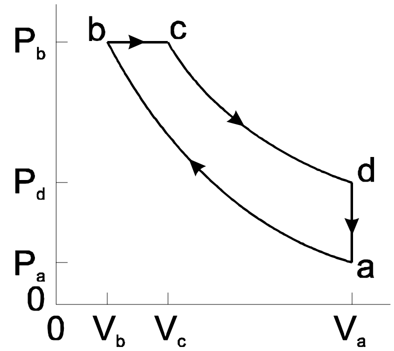
(a) (8 pts) Calculate the *P*, *V*, *T* coordinates of each of the states in terms of P_0, V_0, and T_0 respectively, and place the answer into this table, which has already been partially filled with the information given above.

State	Pressure	Volume	Temperature
1	P_0	V_0	T_0
2	$3P_0$	V_0	
3		$2V_0$	
4	P_0	$2V_0$	

(b) (6 pts) Would the device described by the cycle in this problem be more appropriately described as an engine or as a refrigerator, or does it make a difference? Explain your answer.

(c) (11 pts) Compute the total work done by the gas on the environment, W_S, when it goes around a complete cycle. Express your answer in terms of *n*, *R*, and T_0 only. Since you don't have a calculator handy, you can leave your answer in terms of square roots and logarithms as needed.

8. (40 pts) Diesel Cycle. This thermodynamic cycle is composed, in order, of an adiabatic compression, an isobaric expansion, an adiabatic expansion, and an isochoric cooling. Consider *n* moles of an ideal gas with an adiabatic index of γ.



(a) (22 pts) In the table below, calculate all of the energy flows indicated, leaving them in terms of the temperature of each state, T_a, T_b, T_c, T_d, and *not* the pressures or volumes. If you need space to calculate anything, use the back of this page only.

Process	**ΔE**	**Q**	**W_S**	**ΔS**
a→b adiabatic				
b→c isobaric				
c→d adiabatic				
d→a isochoric				
Total		**Don't add these two columns**		

(b) (10 pts) In terms of the same temperatures as above, what is the efficiency of this cycle?

We would now like to express the efficiency of this cycle now solely in terms of the volumes V_a, V_b, and V_c. (Note that V_d=V_a so only three symbols are needed, not four.) Here is one step along the way toward that goal. You can use the back of this page if you need more space.

(c) (8 pts) Express T_d in terms of T_c and whatever volumes you find necessary.