

ON THE ROOTS OF $f(z) = J_0(z) - iJ_1(z)$ *

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Abstract. The function $f(z) = J_0(z) - iJ_1(z)$ is examined to determine its behavior in the complex plane. It is shown that $f(z)$ has no zeroes in the upper half plane.

Introduction. The function

$$f(z) = J_0(z) - iJ_1(z) \quad (1)$$

arises frequently in problems of shallow-water waves propagating over constant depth and then encountering a sloping beach. In this context, it was first encountered by Keller and Keller [1] in the matching of a linear inner solution valid over the sloping beach to a linear outer solution valid over the constant depth region.

Linear theory is known to describe the evolution of long gravity waves reasonably well away from the shoreline. Carrier [2] and Synolakis [3] have used different solutions of the linear theory to specify a boundary condition for the Carrier and Greenspan transformation [4], and they have obtained analytical results for the climb of relatively arbitrary long waves up sloping beaches. These results have been shown to be in good agreement with experimental data [3].

The process of determining the evolution of a long wave over a sloping beach using either linear theory or a combination of linear and nonlinear theory involves calculating integrals of the form

$$\Psi(\beta, \sigma, \theta) = \oint \frac{\Phi(z)}{z} \frac{J_0(\sigma z) e^{-i\theta z}}{J_0(\beta z) - iJ_1(\beta z)} dz. \quad (2)$$

$\Phi(z)$ is the frequency distribution of the incoming wave, β is a parameter which depends on the beach angle, and σ and θ are functions of the space and time coordinates. To evaluate this integral with standard methods it is necessary to determine the behavior of $f(z) = J_0(z) - iJ_1(z)$. In this paper, I will show that $f(z)$ has no zeroes in the upper half plane.

Solution. Consider a contour \mathcal{C} in the upper half plane consisting of the real axis segment $(-r, r)$ and the semicircular arc Γ defined by $z = |r|e^{i\theta}$, $0 < \theta < \pi$. \mathcal{D} is the domain bounded by \mathcal{C} . Let N be the number of zeroes, and let P be the number of

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