

TEACHING TO THE TOP AND SEARCHING FOR SUPERSTARS*

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October 2005

Abstract

In a two-period model that nests two forms of production, "routine" work where ability and effort are substitutes and "creative" work where they are complements, alternative ways of improving average ability have opposite implications for agents' career concerns. While teaching to the top (training complementary to ability) or identifying star performers increases agents' reputational concerns, teaching to the bottom has the opposite effect. The paper also makes more general comments relating to models of reputation.

*Versions of this paper have appeared under the title "Static Efficiency and Dynamic Incentives" and "Reputation for Excellence and Ineptitude". We thank Mariagiovanna Baccara, Jordi Blanes, Patrick Bolton, Luis Cabral, Guillermo Caruana, Pablo Casas, Antoine Faure-Grimaud, Leonardo Felli, Hugo Hopenhayn, Ian Jewitt, Clare Leaver, Alessandro Pavan, Larry Samuelson, Lucy White and audiences at ESSET 2004, IIOC 2004, Università Boconni, ESWC 2005 and a particularly helpful audience at UCLA for comments and helpful conversations relating to this work. The first author thanks Rutgers University and CEMFI for their hospitality.

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In the next decade and beyond, the ability to attract, develop, retain and deploy staff will be the single biggest determinant of a professional service firm's success.

Maister [1997] p.189

I do not approve of anything that tampers with natural ignorance. Ignorance is like a delicate exotic fruit; touch it and the bloom is gone. The whole theory of modern education is radically unsound. Fortunately in England, at any rate, education produces no effect whatsoever.¹

Lady Bracknell in Oscar Wilde [1895] *The Importance of Being Earnest*

1 Introduction

Popular press and academic literature have come to stress the importance of recruitment and development of staff in industries where human capital plays a critical role.² A central contribution of this paper is to highlight that in addition to affecting the quality of staff, training and recruitment policies also play a role in affecting the behaviour of employees through their career concern incentives.

In human capital intensive industries, including professional services such as the law, audit, consulting and architecture, career concern incentives are of paramount importance since firms often can not (or do not) use explicit outcome contingent contracts. As discussed in Fama [1980] and Holmstrom [1982/99], agents may exert effort in trying to persuade future employers that they are of high ability, that is, they may be motivated by career concerns. It is clear that their motivation will depend on the beliefs of potential future

¹One of the authors is at pains to point out that much has changed in English education in the last century.

²This literature includes Michaels et al. [2001], Maister [1997], Smart [1999], Hacker [2001] and no doubt many others.

employers; and recruitment and training policies will change these.³

In particular, we highlight that while many different training and recruitment policies might have the same effect on the average level of ability in the organization, they can have very different (and indeed exactly opposite) implications for incentives.

Specifically, in the model we present and discuss below, training policies that are targeted towards the top end of the ability distribution (or which will be more effective in raising the productivity of the most able) and recruitment policies that are geared towards finding the very best will lead to higher incentives for employees. Conversely, other policies which improve average ability by targeting training policies towards the lower end of the ability distribution or ensure that the least able are seldom recruited will reduce employees' incentives. Thus tampering with natural ignorance, as Lady Bracknell suggests, can indeed have deleterious effects.

We can separate between two channels through which training can have an effect. First, training the top increases the dispersion of type so observations are more informative. Secondly, training the top implies there is a greater pecuniary payoff to revealing yourself to be there. Searching for superstars has similar effects through both channels.

These results are shown to hold for different views of the productive process. Specifically, our model nests two models of production. One in which ability substitutes for effort (one might think in this case of ability as signifying knowledge of a routine task) and another in which ability and effort are complements (one might think of more able

³It is perhaps worth noting tangentially that as with any model of reputation, while beliefs are critical for incentives, these beliefs need not be an accurate representation of reality. In particular, critical for their incentives are employees' beliefs about the beliefs of the outside labour market, which will determine how observable outcomes are interpreted and thereby the wages that they might receive in the future. Most theoretical models, and the one below is no exception in this regard, suppose that in equilibrium, beliefs will accurately reflect the underlying state (or its distribution). In practice, however, it is easy to imagine that it is possible to fool some of the people some of the time, and that firms may generate greater effort from employees by stressing that their recruitment policies focus on hiring the very best, whether they do so or not.

agents in this case as more likely to have the inspiration which still requires hard work to succeed). The career concern literature and most of the reputation literature has viewed effort and ability as substitutes. However, more recent literature on reputational concerns surrounding ineptitude takes the opposite view.⁴ We discuss at some length how these different views of the production process relate to this recent literature on reputation as a concern to show that you are not inept.

1.1 Related Literature

Models of type-based reputation and career concerns bring together a number of elements, as highlighted in Figure 1.⁵ In models of reputation, typically agents know their own types and the rewards are type dependent (in particular for certain types certain actions are infeasible and can be thought of as having an infinite cost). By contrast, in career concern models, it is commonly assumed that agents do not know their own types and that the rewards depend only on posteriors and the actions taken. For reputation or career concerns models with more than two periods, the posteriors in one period form the priors for the following period.

The model in this paper will be a two period model and given the structure of the model, it will not matter whether agents are assumed to know their type or not. The type will determine the likelihood with which actions will turn out to be feasible but, given the realised feasible action set, all types will face identical decision problems.

With respect to Figure 1, the focus of this paper is on the role of prior beliefs in reputa-

⁴See, in particular, Tadelis [2002], Mailath and Samuelson [2002] and Bar-Isaac [2004a].

⁵The term “reputation” following the seminal work of Kreps and Wilson [1982] and Milgrom and Roberts [1982] has been used primarily for models in which agents have beliefs about the types of other agents. Others, for example Klein and Leffler [1981] have used the word “reputation” to discuss equilibria of infinitely repeated games; more recent contributions in this tradition include Rob and Sekiguchi [2004], Bar-Isaac [2005] and Cai and Obara [2004]. For a review of the literature on reputation, albeit one geared at particular applications, see Bar-Isaac [2004b].

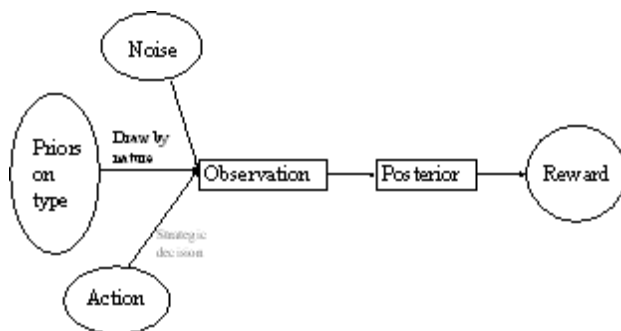


Figure 1: General representation of a reputation or career concerns model

tional concerns and factors that influence these prior beliefs (in particular recruitment and training).⁶ There is a wide literature which builds on the work of Holmstrom [1982/99] and examines other aspects of career concern models. Dewatripont, Jewitt and Tirole [1999] for example, provide a thorough analysis characterizing the impact of different information structures (mappings from ability and effort into observable outcomes). Others have focused on specific applications, whose primary effect is to alter such information structures, in particular through teamwork (Meyer [1994] and Jeon [1996]) and delegation of power (Ortega [2003], Blanes-i-Vidal [2002]). In a recent contribution Moav and Zeeman [2005] discuss how reputational concerns in a model with more than two periods and competent and inept agents might lead to more precise information reducing incentives, since late in the career the prior may be close to certain reducing incentives at that stage and through an unraveling argument throughout the game. Finally, with respect to Figure 1, while most of the literature does not closely examine the return function, Harstad [2004] derives interesting implications for organizational design by considering convex return functions, which depend on the degree of product market competition.⁷

⁶Somewhat related in highlighting the importance of widely-dispersed priors is Bar-Isaac (2004) which illustrates how partnerships and teamwork might foster such widely dispersed priors.

⁷See also Casas-Arce [2005] and Martinez [2005] who consider different micro-foundations for career concerns models with non-linear returns functions.

The relationship between recruitment policies, through its effect on firm reputation on individual reputation and behaviour has been discussed to some extent in Tirole [1996] and Bar-Isaac [2004b].

We postpone further discussion of the relationship between this paper and the literature on reputation, as it will be easier to discuss in the light of the full presentation of the model.

1.2 Preview of the Paper

In the rest of the paper, we first present a two-period model with a continuum of types. While the traditional career concerns literature, following Holmstrom [1982/99], views all agents as identical with regard to the strategies available to them and the reputation literature has focused on agents who are either entirely strategic or entirely non-strategic, we consider a continuum of agents who vary in their opportunities to be strategic.

We derive a simple expression which characterizes equilibrium effort. We go on to consider comparative statics of effort with respect to parameters and argue that these comparative statics exercises have natural applied interpretations as training to the top and searching for superstars. In particular, we find that different ways of raising average ability can have very different consequences for incentives and go on to discuss these results, both with regard to the applications of training and recruitment policies, and with regard to more methodological concerns.

It is worth highlighting that in the model we consider possible types of agents, where the type captures the agent's likelihood of getting an opportunity to engage in strategic behaviour. However, until all the parameters of the model are specified, it is unclear how the type relates to ability. To an extent, the general model acts as a nesting model between two different views of the productive process (in which effort and ability are complements or substitutes) and the choice of parameters leads to a switch between these two views.

For example one can think of the degenerate example where agents are either strategic or they are not (so there are only two types). The non-strategic agent is at the “top of the ability distribution” in the case where she always succeeds (that is she is an excellent or Stackelberg type) and so in this case one can think of effort and ability as substitutes. On the other hand if the non-strategic type always fails then she is at the “bottom of the ability distribution” and it is rather the strategic type who is the more able, and in this case ability and effort will act as complements. Thus in the model below the parameter t refers to “type” and the interpretation of high t or low t as more or less able depends on the values of parameters, as discussed in greater detail in Section 2.1.

2 Model

We introduce a two period model with a continuum of types of agents parameterized by $t \in [0, 1]$.⁸ Specifically in Period 1 a type t agent will have no strategic decision to make with probability t and in this case will succeed (for example by producing a high quality product) with probability μ and fails with probability $1 - \mu$. Otherwise (with probability $1 - t$) the agent must make an effort decision. In this latter case, when she chooses effort e , she succeeds with probability $\alpha + e$. Thus overall a t -type agent exerting effort e when given the opportunity to exert effort would succeed with probability $t\mu + (1 - t)(\alpha + e)$ and fail otherwise. Effort is costly and, specifically, exerting effort e costs the agent $\frac{e^2}{2\gamma}$, where $\gamma < 1 - \alpha$.

⁸Extending the model beyond two periods is not straightforward. First in a multi-period model if the agent does not know her type initially, she will learn it at a different rate to customers. Secondly if she does know her type (as in a reputation model) then it is no longer the case that when faced with a strategic decision all types face identical decisions as they anticipate different continuation value functions. In either case, a general characterization is far from trivial. Nevertheless, the intuitions provided in the simple two period model—that training and recruitment decisions will influence agents behaviour, mediated through the channels of “information” and “information” value effects—will still hold in a multi-period model and other more general settings.

Let $g(t)$ denote the distribution function for the types of agent and let T denote the average type (according to the *ex-ante* beliefs) $T = \int_0^1 tg(t)dt$ and let $V = \int_0^1 (t - T)^2 g(t)dt$ denote the variance of this prior distribution. This distribution function of types is common knowledge among the agent and customers.

Customers are risk neutral, value a success at 1 and a failure at 0 and they Bertrand compete for the agent's service in each period.⁹ Moreover, outcomes are observable but effort is not observable and contracts are incomplete, so that in effect an agent is paid in advance at a price which is simply the customers' common belief that the agent will produce a success.

There are two periods of trade, and outcomes are observed (and beliefs revised) in between the two periods.¹⁰ Specifically timing is as follows:

1. Period 1

- (a) customers Bertrand compete for the agent's service
- (b) the agent decides the level of effort if appropriate (that is if it is a task where effort will make a difference)
- (c) success/failure commonly observed
- (d) customers update beliefs according to Bayes rule

2. Period 2: customers Bertrand compete for the agent's service

Notice that in period 2, we could allow the agents the opportunity to exert effort but no agent would do so. Note that whether the agent knows her type or not would have

⁹The assumption that customers Bertrand compete for the product is not crucial, similar results would hold so long as the price paid was increasing in the customers' expected likelihood that the agent will be successful.

¹⁰One need not take the two periods of the model literally, rather the second period can be thought of as a reduced form payoff for a given reputation level, **albeit one that is linear in the reputation.**

no effect on this model since she has no ability to signal her type (we rule out long-term and outcome contingent contracts) and at the point where an agent has to make an effort decision then the problem is identical for all types. If agents had to make an effort decision and incur the costs of effort before she knew what kind of task she faced then we would obtain qualitatively similar results if the agent did not know her type, though the analysis of this problem when the agent did know her own type would be more complex.

We suppose that agents weigh the two periods equally and that agents maximize the sum of profits for the two periods and we solve for the effort exerted in the Perfect Bayesian equilibrium.¹¹

2.1 Interpreting the model

The model is intended to reflect that agents might be confronted with a variety of different tasks, whose nature is unobserved by customers. For example, customers hiring consulting firms find it difficult to determine the extent to which the project that they are assigning is a complex one or a simple one. Similarly, the difficulty of the project depends on the consultant's ability and experience. Depending on the value of μ , the model allows for somewhat different interpretations of the productive process and of the interpretation of high values of t as reflecting high or low ability.

Specifically, when $\mu = 1$ or μ is sufficiently high, high t reflects high ability; an agent with a high value of t finds it costless to succeed in a wide range of tasks, in this case ability and effort are substitutes and an agent would like customers to believe that she has a high value of t . One could think of this case as representing "routine" work, where more able workers know how to do more things and if they know how to perform the task that they

¹¹Allowing for discounting between periods or indeed allowing profits in the second period to be more valuable than in the first (consistent with an interpretation of the second period as a reduced form for the future) does not affect the qualitative results.

are assigned they will succeed with high probability.¹² If they do not have the requisite knowledge they could exert effort to acquire it, by reading the appropriate manuals for example. Examples include translation work, mechanics and routine work in low levels of management in an organizational hierarchy.

In contrast, when $\mu = 0$ or sufficiently low, then one can think of agents for whom $(1 - t)$ is higher as being more able, and effort and ability as complements—in this case even if the agent has some understanding of the task, exerting effort will still improve the outcome, however if she has no understanding of the task then she will surely fail. In this case one needs some ability in order to have any chance of success (one needs ability to have the flash of inspiration) but this in itself will not guarantee success, hard work is also required. This may be more appropriate for creative work, such as writing a Ph.D., writing advertisements or high level management which is not routine.

Notice, that in the case that $\mu = 0$, the agent would prefer customers to believe that she is a type with a low value of t and so when $\mu = 0$, one should think of agents with low values of t as more able.

In application, of course many jobs will include elements of both creative and routine work, however for different jobs or at different times in an individual's career (or within an organizational hierarchy) it will be more appropriate to think of work as primarily of one or other of these two production types.

In both these cases we can think of more able agents as having facility in some tasks but not in others. The difference in productivity for a task in which one has facility and in which one does not when exerting no effort (which is the case in period 2) is simply given by $|\mu - \alpha|$.

¹²Another interpretation for the case $\mu = 1$ is that high t agents get it “right first time”, but for all agents when they don't there is the opportunity of exerting effort to try to “fix mistakes”.

3 Equilibrium analysis

Trivially, when faced with an effort decision, all agents will make the same choice of effort (the benefits are identical for all agents and determined by equilibrium beliefs and the costs are identical for all agents, even though the frequency with which they have to make such decisions alters). Suppose that the equilibrium effort level is given by x .

Then a type t agent generates success in the first period with probability

$$\mu t + (1 - t)(\alpha + x) = (\mu - \alpha - x)t + (\alpha + x), \quad (1)$$

and generates a failure with probability

$$1 - \mu t - (1 - t)(\alpha + x) = 1 - \alpha - x - t(\mu - \alpha - x). \quad (2)$$

By Bayes rule, the probability density function given a success and given the belief that agents exert effort x in the first period can be written down as

$$s(t, x) = \frac{(\mu - \alpha - x)t + \alpha + x}{\int_0^1 [(\mu - \alpha - x)t + \alpha + x] g(t) dt} g(t) = \frac{(\mu - \alpha - x)t + \alpha + x}{(\mu - \alpha - x)T + (\alpha + x)} g(t), \quad (3)$$

and the probability density function given a failure in the first period is

$$f(t, x) = \frac{1 - (\mu - \alpha - x)t - (\alpha + x)}{\int_0^1 [1 - (\mu - \alpha - x)t - (\alpha + x)] g(t) dt} g(t) = \frac{1 - (\mu - \alpha - x)t - (\alpha + x)}{1 - (\mu - \alpha - x)T - (\alpha + x)} g(t). \quad (4)$$

In the second period, an agent of type t will exert no effort and so succeed with probability $\mu t + (1 - t)\alpha = t(\mu - \alpha) + \alpha$. In particular it follows, that if customers believed that

the types were distributed according to $h(\cdot)$ going into period 2 then they would be willing to pay the agent $\int_0^1 (t(\mu - \alpha) + \alpha)h(t)dt$.

It follows that the price that customers would pay following success and failure respectively are given by:

$$S(x) = \int_0^1 (t(\mu - \alpha) + \alpha)s(t, x)dt = (\mu - \alpha)E[t|S, x] + \alpha, \quad (5)$$

and

$$F(x) = \int_0^1 (t(\mu - \alpha) + \alpha)f(t, x)dt = (\mu - \alpha)E[t|F, x] + \alpha, \quad (6)$$

where

$$E[t|S, x] = \int_0^1 ts(t, x)dt = T + \frac{(\mu - \alpha - x)V}{(\mu - \alpha - x)T + (\alpha + x)} \quad (7)$$

and

$$E[t|F, x] = \int_0^1 tf(t, x)dt = T - \frac{(\mu - \alpha - x)V}{(1 - \alpha - x) - (\mu - \alpha - x)T} \quad (8)$$

Finally notice that an agent's problem is to choose e to maximize

$$(\alpha + e)S + (1 - \alpha - e)F - \frac{e^2}{2\gamma} \quad (9)$$

so the first order condition suggests that $\frac{e}{\gamma} = S(e) - F(e)$ and a rational expectations equilibrium is defined by the effort level x that satisfies:

$$\frac{x}{\gamma} = S(x) - F(x). \quad (10)$$

Lemma 1 (i) *The equilibrium effort e^* is lower than the efficient solution $e^{fb} = \gamma$. (ii) If $\mu > \alpha$ then $e^* < \mu - \alpha$.*

Proof. (i) Notice that $S(x) < 1$ for all x and $F(x) > 0$ for all x , so in particular $S(\gamma) - F(\gamma) < 1$, then the equilibrium effort level

$$\frac{e^*}{\gamma} = S(e^*) - F(e^*) < 1 \implies e^* < \gamma \quad (11)$$

(ii) For contradiction suppose that $\mu - \alpha - e^* < 0$, then consider

$$S(e^*) - F(e^*) = \frac{(\mu - \alpha)(\mu - \alpha - e^*)V}{[\mu T + (\alpha + e^*)(1 - T)][1 - \alpha - e^* - (\mu - \alpha - e^*)T]} \quad (12)$$

Note that $1 - \alpha - e^* > T(\mu - \alpha - e^*)$ since $T < 1$ and $1 > \mu$ so the denominator is positive and since $\mu > \alpha$ and $\mu - \alpha - e^* < 0$ it follows that $S(e^*) - F(e^*) < 0$ but then it is impossible that $S(e^*) - F(e^*) = \frac{e^*}{\gamma} > 0$, which is our desired contradiction. ■

Proposition 2 *Equilibrium effort is well-defined and unique.*

Proof. It suffices to show that there exists a unique solution for the equation (10), which is in the range $(0, \gamma)$.

Let $h(x) = -\frac{x}{\gamma} + S(x) - F(x)$. The equilibrium effort is then given by the solution of $h(e^*) = 0$ for $e^* \in (0, \gamma)$.

Note that $h(0) = S(0) - F(0) > 0$ and given Lemma 1, $h(0) > 0$ and $h(\gamma) < 0$. Moreover, $h(x)$ is continuous, and thus there exists at least one solution in the range $(0, \gamma)$.

In order to demonstrate uniqueness, first take the derivative of $(S - F)$:

$$\frac{d(S - F)}{dx} = -(\mu - \alpha)V \left[\frac{\mu}{[(\mu - \alpha - x)T + (\alpha + x)]^2} + \frac{(1 - \mu)}{[(1 - \alpha - x) - (\mu - \alpha - x)T]^2} \right] \quad (13)$$

Then if $\mu > \alpha$, $\frac{d(S-F)}{dx} < 0$ and so $h(x)$ is monotonically decreasing in the range $(0, \gamma)$ and so the solution must be unique.

If $\mu < \alpha$ then $\frac{d(S-F)}{dx} > 0$ and so potentially, $h(x)$ could be increasing in some subset of $(0, \gamma)$ (note that since $h(0) > 0$ and $h(\gamma) < 0$ at must be decreasing in some of the range). However, we know that, when $\mu < \alpha$, $\frac{d^3h(x)}{dx^3} > 0$ since

$$\frac{d^3h(x)}{d^3x} = (\alpha - \mu)V \left[\frac{\mu 6(1-T)^2}{[(\mu - \alpha - x)T + \alpha + x]^4} + \frac{(1 - \mu)6(1-T)^2}{[1 - \alpha - x - (\mu - \alpha - x)T]^2} \right] > 0 \quad (14)$$

Suppose for a contradiction that $h(e^*) = 0$ has a number of solutions $0 < e_1 < . . . < e_N < \gamma$. Then first note that since $h(0) > 0$ and $h(\gamma) < 0$ then N must be an odd number. In particular therefore if there are multiple solutions to $h(e^*)$ in the range then there must be at least three. However $h(0) > 0$ and $0 < e_1 < e_2 < e_3$ with $h(e_1) = h(e_2) = h(e_3) = 0$ requires $\frac{dh(e_1)}{dx} < 0$, $\frac{dh(e_2)}{dx} > 0$ and $\frac{dh(e_3)}{dx} < 0$ which contradicts $\frac{d^3h(x)}{d^3x} > 0$. ■

Note in particular that since equilibrium effort is unique, comparative statics exercises are well defined and can be explored. Note, in particular, that as discussed at some length in Section 2.1, in the case where μ is high ($\mu > \alpha + \gamma$) or μ is low ($\mu < \alpha$), the model has natural interpretations as capturing “routine” and “creative” production in these two cases and is hard to interpret otherwise.

4 Comparative Static Results

The first result is a very intuitive one, if effort is less costly then the agent will exert more effort (when relevant) in equilibrium.

Proposition 3 *The equilibrium effort e^* is increasing in γ*

Proof. Note that by the arguments of the proof of Proposition 2, $h(x)$ is decreasing in the range $(0, \gamma)$.

Using the implicit function theorem $\frac{de^*}{da} = -\frac{\frac{\partial h(a)}{\partial a}}{\frac{\partial h(e)}{\partial e}}$ and since $\frac{\partial h(e)}{\partial e} < 0$, the sign of $\frac{de^*}{da}$ is simply the sign of $\frac{\partial h(x,a)}{\partial a}$ and so it is sufficient to consider that expression for $a = \gamma$.

Recall

$$h(x) = -\frac{x}{\gamma} + S(x) - F(x). \quad (15)$$

and so taking the derivative with respect to γ , we obtain

$$\frac{\partial h(x, \gamma)}{\partial \gamma} = \frac{x}{\gamma^2} > 0 \quad (16)$$

Therefore, we conclude that the optimal effort e^* is increasing in γ . ■

Next we turn to comparative statics with respect to V . The intuition here is clear, the greater the variance in the distribution of types, the more scope that the observation of a success or failure has to shift beliefs and associated reward. This is a familiar intuition (from Holmstrom [1982/99] for example); here we highlight that the result depends on V and no other characteristics of the distribution (for example we do not restrict that $g(\cdot)$ be normal).

Proposition 4 *The optimal effort e^* is increasing in V .*

Proof. As in the proof of Proposition 3, the sign of $\frac{de^*}{dV}$ is simply the sign of $\frac{\partial h(x,V)}{\partial V}$. So taking the derivative with respect to V :

$$\frac{\partial h}{\partial V} = \frac{\partial(S - F)}{\partial V} = \frac{(\mu - \alpha)(\mu - \alpha - x)}{[(\mu - \alpha - x)T + (\alpha + x)][(1 - \alpha - x) - (\mu - \alpha - x)T]} > 0. \quad (17)$$

■

Notice that while increasing V or γ has a clear monotonic effect on effort, it is reasonable to suppose that comparative statics with respect to other parameters might depend on which of the two interpretations alluded to in the description of the model in Section 2

applies, that is, whether an agent’s reputational concern is to try to convince customers that she is a “high t ” type or a “low t ” type.

First, we consider comparative statics with respect to α . Underlying the following result are two effects, first that it is more important to show oneself to be at the top of the ability distribution (or having facility in a greater range of tasks that is high t in the case when μ is high, low t in the case when μ is low) the greater the difference in the productivity of an agent in a task in which she has facility to her productivity in one which she does not regardless of her level of effort (that is the greater is $|\mu - \alpha - x|$ for all effort levels x). Secondly as $|\mu - \alpha - x|$ increases then an observation of success or failure becomes more informative. We distinguish explicitly between these two effects in the discussion in Section 5.3. In particular therefore when μ is high, one would expect that an increase in α should reduce effort, but when μ is low, it would increase equilibrium effort. Note however that in all cases, increasing α raises the average productivity of agents in period 1. Similar considerations apply with regard to comparative statics in μ . The Proposition below demonstrates that these intuitions are borne out.

Proposition 5 *Equilibrium effort is increasing in α but decreasing in μ when $\mu < \alpha$ but decreasing in α and increasing in μ when $\mu > \alpha$.*

Proof. As in the proof of Proposition 3, it is sufficient to consider $\frac{\partial h}{\partial \alpha}$, and $\frac{\partial h}{\partial \mu}$.

$$\begin{aligned} \frac{\partial h(x, \alpha)}{\partial \mu} = \frac{\partial(S-F)}{\partial \mu} = & \left[\frac{(\mu - \alpha - x)V}{(\mu - \alpha - x)T + \alpha + x} + \frac{(\mu - \alpha - x)V}{1 - \alpha - x - (\mu - \alpha - x)T} \right] \\ & + (\mu - \alpha) \left[\frac{(\alpha + x)V}{((\mu - \alpha - x)T + \alpha + x)^2} + \frac{(1 - \alpha - x)V}{(1 - \alpha - x - (\mu - \alpha - x)T)^2} \right] \end{aligned} \quad (18)$$

Notice that given that the denominators are always positive, if $\mu > \alpha + x$, which is true when $\mu > \alpha$ by Lemma 1(ii), then $\frac{\partial(S-F)}{\partial \mu} > 0$ since $(\mu - \alpha - x)$ and $(\mu - \alpha)$ are positive, similarly if $\mu < \alpha$, then $\frac{\partial(S-F)}{\partial \mu} < 0$.

Similarly

$$\begin{aligned} \frac{\partial h(x,\alpha)}{\partial \alpha} = \frac{\partial(S-F)}{\partial \alpha} = & - \left[\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T} \right] \\ & + (\mu - \alpha) \left[\frac{-V\mu}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{-V(1-\mu)}{(1-\alpha-x-(\mu-\alpha-x)T)^2} \right] \end{aligned} \quad (19)$$

Again the denominators are always positive and if $\mu > \alpha$, then $\frac{\partial(S-F)}{\partial \alpha} < 0$ since $(\mu - \alpha - x)$ and $(\mu - \alpha)$ are positive, similarly if $\mu < \alpha$, then $\frac{\partial(S-F)}{\partial \alpha} > 0$. ■

5 Discussion

In this section we highlight a number of aspects arising from the model and results presented above. First we return to the discussion in the introduction and illustrate how the model demonstrates that if training can be targeted towards having more of an effect on some types than others, then training that is complementary with ability would generate more effort. Next we turn to recruitment policies and highlight that among policies with the same effect on average ability, those which are more concerned with identifying the very best will lead to more effort from those hired than policies concerned about ruling out the very worst. We then turn to a couple of more methodological issues. First we adapt the model slightly in order to highlight that improving the ability of agents (through changing α or μ) affects reputational concerns and the equilibrium through two different channels: a “value effect” and an “information effect”. We conclude the section by illustrating two special cases of the model and through them relating this work to the literature on reputation and in particular the distinction between a reputational concern for excellence and ineptitude.

5.1 Targeted training and teaching to the top

Increasing α and increasing μ can both increase the ability of agents, and can readily be interpreted as the result of training directed towards different types of agents. However, as demonstrated in Proposition 5, these two means of increasing average ability have exactly opposite effects for equilibrium effort. In particular for low values of μ when an agent would like customers to believe that she is a “low t ” type then raising the ability of a “low t ” type relatively more than raising the ability of a “high t ” type (either by decreasing μ or by increasing α), heightens this reputational concern. As discussed below, it does so through two channels, by raising the pecuniary value of showing oneself to be a higher type and by making the outcome more informative about the agent’s type.

Similarly in the case where μ is high, then agents with high t are the most productive and a greater distinction between the most able and the least able (here by increasing μ or decreasing α) will heighten the reputational concern for agents seeking to convince consumers that they are the most able.

Thus in all cases raising the productivity of the most able agents (or reducing the productivity of the least able) increases the equilibrium effort.

5.2 Recruitment policies and searching for superstars

While training as described above affects the productivity of a given type and thereby affects the prior beliefs about an agent’s productivity, interviewing and recruitment policies directly affect the initial belief about the distribution of types $g(t)$. When employers seek recruitment policies which select better agents, there are various ways in which this can be achieved. Consider the case when μ is high (so that types with high values of t are the better agents); a recruitment policy that selects better agents will lead to a shift in the prior distribution from $g(t)$ with associated T and V , to a different prior distribution $g'(t)$ with

associated $T' > T$ and V' . Following Proposition 3, among all policies with the same effect on average ability (that generate the same T'), an employer would prefer to choose a policy that raised rather reduced the variance of the distribution. When superstars and disastrous potential recruits are rare, then it follows that employers concerned with employees' efforts would be better using recruitment policies that concentrated more on ensuring that any potential superstars were recruited than ruling out the worst of the applicants.

For example, suppose that with no recruitment policy, types are distributed according to the degenerate distribution $g(0) = \frac{1}{10}$, $g(\frac{1}{2}) = \frac{4}{5}$ and $g(1) = \frac{1}{10}$ so that $T = \frac{1}{2}$ and $V = \frac{1}{20}$. Now consider, two recruitment policies which raise the average ability, one does so by reducing the probability of recruiting disasters. Specifically Policy A leads to the distribution $g_A(0) = \frac{1}{20}$, $g_A(\frac{1}{2}) = \frac{17}{20}$, and $g_A(1) = \frac{1}{10}$ so that $T_A = \frac{21}{40}$ and $V_A = \frac{1}{20}(\frac{21}{40})^2 + \frac{17}{20}(\frac{1}{40})^2 + \frac{1}{10}(\frac{19}{40})^2 = \frac{59}{1600}$. Policy B increases the probability of identifying superstars and leads to the distribution $g_B(0) = \frac{1}{10}$, $g_B(\frac{1}{2}) = \frac{3}{4}$, and $g_B(1) = \frac{3}{20}$, then $T_B = \frac{21}{40}$ and $V_B = \frac{1}{10}(\frac{21}{40})^2 + \frac{3}{4}(\frac{1}{40})^2 + \frac{3}{20}(\frac{19}{40})^2 = \frac{99}{1600}$. Since $V_B > V_A$ it follows by Proposition 3 that, while both policies raise average ability equally, the latter policy would lead to greater equilibrium effort compared to the first and so would be preferred.

5.3 The information and value effects

By adapting the model slightly to suppose that in period 2 a type t agent succeeds with probability $t\mu' + (1-t)\alpha'$, we can distinguish between two channels through which changes in ability as discussed in Proposition 5 and Section 5.1 affect incentives. Specifically, in this modified model $S - F = (\mu' - \alpha') \left[\frac{(\mu - \alpha - x)V}{(\mu - \alpha - x)T + \alpha + x} + \frac{(\mu - \alpha - x)V}{1 - \alpha - x - (\mu - \alpha - x)T} \right]$ and similar qualitative results apply.

Proposition 6 *Equilibrium effort is increasing in α and α' but decreasing in μ and μ' when $\mu, \mu' < \alpha$ but decreasing in α and α' and increasing in μ and μ' when $\mu, \mu' > \alpha$.*

Proof. Similar to the proof of Proposition 5, it is sufficient to consider $\frac{\partial(S-F)}{\partial\alpha}$, $\frac{\partial(S-F)}{\partial\alpha'}$, $\frac{\partial(S-F)}{\partial\mu}$, and $\frac{\partial(S-F)}{\partial\mu'}$.

First

$$\frac{\partial(S-F)}{\partial\mu'} = \frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} + \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T} \quad (20)$$

which is negative when $\mu < \alpha$ but positive when $\mu > \alpha$. Similarly,

$$\frac{\partial(S-F)}{\partial\alpha'} = -\frac{(\mu-\alpha-x)V}{(\mu-\alpha-x)T+\alpha+x} - \frac{(\mu-\alpha-x)V}{1-\alpha-x-(\mu-\alpha-x)T} \quad (21)$$

which is positive when $\mu < \alpha$ but negative when $\mu > \alpha$.

Next

$$\frac{\partial(S-F)}{\partial\mu} = (\mu' - \alpha') \left[\frac{(\alpha+x)V}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{(1-\alpha-x)V}{(1-\alpha-x-(\mu-\alpha-x)T)^2} \right], \text{ and} \quad (22)$$

$$\frac{\partial(S-F)}{\partial\alpha} = (\mu' - \alpha') \left[\frac{-VT}{((\mu-\alpha-x)T+\alpha+x)^2} + \frac{-V(1-\mu)}{(1-\alpha-x-(\mu-\alpha-x)T)^2} \right]. \quad (23)$$

Similarly to the proof of Proposition 5, the derivatives have the signs claimed in the relevant parameter ranges. ■

Thus Proposition 6 demonstrates that the overall effect of changing the abilities of types through changes to μ and α described in Proposition 5 can be decomposed into two distinct mechanisms.

First consider the comparative statics with respect to the second period productivities μ' , and α' . Fixing some effort level, then the beliefs about the type of an agent following either a success or a failure do not change as μ' or α' changes. However, since the belief that the agent is excellent following a success is higher than it is following a failure, raising (lowering) μ' in the case where $\mu' > \alpha$ (where $\mu' < \alpha$) increases S —the agent's wage following a success—by more than it increases F , the wage following failure. Since incentives

are stronger the greater the difference between S and F , an increase in μ' therefore raises incentives. A similar argument applies for α' . Notice that changing μ' , and α' does not affect the inferences that customers draw from the outcomes (in equilibrium when they correctly anticipate x) but they affect the value to the agent of being thought of as a particular type. We therefore term this channel for influencing an agent's incentives a "value effect".

We now turn to the comparative statics with respect to the first period productivities through changes in μ , and α . If the beliefs about the type of the agent are fixed, then increasing μ , and α has no effect whatsoever on the value of the agent in Period 1. Changing μ and α however can affect the inferences that customers draw from an observation of success or failure, we therefore term such changes as having an "information effect". In particular, intuition can be drawn from the observation that for a fixed level of effort, increasing (reducing) μ in the case where $\mu > \alpha$ (where $\mu < \alpha$) increases the probability that "better types" generate success and decreases the probability that they generate failure. Therefore, conditional on observing a success, customers believe that the agent is more likely to be at the top of the ability distribution and so S is higher, while conditional on observing a failure, customers believe that the agent is less likely to be at the top of the distribution and so F is lower. In particular, therefore, $(S - F)$ increases. Similar arguments apply with regard to changes in α .

5.4 Reputation for excellence or ineptitude

This note relates to a wider literature on reputation. Much of the economic literature on reputation has focused on a reputation for excellence (trying to show that you are a type who always does well, or where reputation is about "who you'd like to be").¹³ In these

¹³Following Kreps and Wilson [1982] and Milgrom and Roberts [1982] and later Fudenberg and Levine [1989], the formal economic literature on reputation has been used primarily to discuss beliefs about the

models, the “most able” are non strategic and implicitly, they are assumed to be somewhat unusual or scarce, so for example one might expect general “untargeted” training to have little effect on them. More recent literature (Mailath and Samuelson [2001], Tadelis [2002], Bar-Isaac [2004a]) and common intuition suggests that often reputational concerns might also relate to avoiding a reputation for ineptitude (trying to show that you are not a type who always does badly or where reputation is about “who you’re not”) where the top of the distribution is the strategic type, and the bottom of the distribution is an inept types whom one might expect to be little affected by training. This distinction between these two approaches to reputation has been forcibly made recently by Mailath and Samuelson [1998] who highlight, in particular, that the latter view of reputation leads to increasing certainty about the agent’s type over time and so reputational incentives disappear over time unless type uncertainty is continually introduced.¹⁴

In practice, it is far from obvious whether it is more appropriate to think of agents as particularly concerned that others should think them to be excellent or that they should not think them to be inept. However as we illustrate, modelling reputational concerns in these two ways can lead to opposite conclusions.

This paper highlights an important distinction between the two approaches in a simple type of the agent. Previous literature (Klein and Leffler [1981] for example) and a great deal of intuition has also used the term in a somewhat looser fashion to consider sustaining certain actions in infinitely repeated games. As highlighted in Fudenberg and Levine [1989] this corresponds closely to the notion of reputation where reputation is a concern to show that you’re a “Stackelberg” type—that is a type whose behavior a strategic agent would like to promise to commit to—similar to what we term later in this note a reputation for excellence.

¹⁴Further in Mailath and Samuelson [1998], the model is constructed in such a way that there is unravelling so that if there are no reputational incentives at some point, there are no such incentives throughout.

The more general point on reputational incentives disappearing over time without some kind of replenishment of type uncertainty applies more widely. Indeed, Cripps, Mailath and Samuelson [2004] show this to be the case unless actions are perfectly observable, even in the case when reputation is about excellence and a competent agent can perfectly mimic an excellent agent (though incentives may disappear only in the very long run).

Bar-Isaac [2004a] suggests an endogenous mechanism to maintain type uncertainty by allowing agents to choose to work in teams.

two-period model. Specifically, following the intuition of the paragraphs above, making the strategic agent more efficient diminishes reputational concerns (reducing effort) when reputation is about excellence but increases reputational concerns when reputation is about ineptitude.

To see this more clearly consider setting $\mu = 1$ and the degenerate distribution $g(0) = 1 - p$ and $g(1) = p$. This corresponds to a fairly typical model where type $t = 0$ corresponds to the strategic type whose reputational concern is to try to convince customers that she is the “excellent” or Stackelberg type. Following Proposition 3, in this case improving the strategic agent by raising α would reduce effort.

In contrast suppose that a strategic agent’s reputational concern is to avoid a reputation for ineptitude. This corresponds to the model where $\mu = 0$ with $g(0) = 1 - p$ and $g(1) = p$ and in this case improving the strategic agent by raising α would increase equilibrium effort.

It is worth noting that in those papers that have considered something akin to the inept type we consider in this note (in particular Diamond [1989], Mailath and Samuelson [1998], Tadelis [2002] and Bar-Isaac [2004a]), there are no reputational incentives from trying to avoid a reputation for ineptitude or gain a reputation for competence *per se*.¹⁵ Essentially this is because in the notation of this paper they have taken $\alpha = 0$.

6 Summary

At heart this note highlights the simple observation that the distribution of prior beliefs is a crucial determinant of reputational incentives. There are numerous considerations which affect such prior beliefs, including for example contemporaries and social peers.

¹⁵In those papers, it is only useful to be thought of as a competent type if you can somehow also commit to exerting effort. Diamond also allows for an inept type that always fails, but has a strategic type for whom $\alpha = 0$, Cabral and Hortacsu [2004] is an exception in supposing that $\alpha \neq 0$.

Further, there are many policies that firms undertake (in particular, we have focused on training and recruitment policies) which affect the shape of the distribution of these priors. Different policies which affect the mean ability in the same way can have exactly opposite implications for reputational concerns.

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