Government Policy with Time Inconsistent Voters

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Abstract

Behavioral economics presents a “paternalistic” rationale for benevolent government intervention. This paper presents a model of public debt where voters have self-control problems and attempt to commit by accumulating illiquid assets. We introduce politicians who may indulge/exploit voters’ behavioral biases. Three main insights emerge: (i) Individuals’ attempts to privately undo the consequences of debt accumulation feed the subsequent demand for government debt; (ii) There is large debt accumulation leading to wasted resources; (iii) Banning illiquid assets would improve first-period welfare. These results offer a new rationale for balanced budget rules in constitutions to restrain governments’ responses to voters’ self-control problems.

1 Introduction

An important and influential approach to government policy has grown out of the field of behavioral economics.1 A number of contributors to this area argue that some form of government policy interventions can be justified by “paternalistic attitudes” even in cases outside the realm of the textbook approach to public policy, i.e., even absent externalities, public goods, and asymmetric information.2 Within behavioral economics, a substantial body of

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1Camerer et al (2004) contains a number of “second generation” contributions to behavioral economics. See also Sunstein and Thaler (2009).
2See, for instance, Camerer et al. (2003) and Sunstein and Thaler (2009).
work discusses self-control problems and their consequences within a consumption-savings environment. Some argue that there are inefficiently low savings when individuals are left to their own devices (e.g., Camerer, Loewenstein, and Rabin 2003, Hurst 2003, and Madrian 2012). These insights have been used for justifying paternalistic interventions by governments aimed at helping individuals save, such as encouraging accumulation of illiquid assets, various forms of forced savings plans, as well as public pension systems (see Beshears et al. 2011, Camerer et al. 2003, Laibson 1998, Sunstein and Thaler 2003, and literature that followed). A related set of recent papers argues that individuals who suffer from self-control problems may accumulate excessive private debt, for instance, in the form of large credit card balances (see Ausubel and Shui 2005, Gottlieb 2008, and Heidhues and Koszegi 2010). These papers provide policy recommendations that attempt to rein in excessive debt by placing constraints on the actions of private intermediaries such as credit card companies (e.g., banning teaser rates, see Sunstein 2006).

This literature implicitly assumes a model of policy making that relies on a benevolent government. Of course, the assumption of a benevolent government is just a benchmark: a richer understanding of government intervention requires a more nuanced model of its decision making process. The political economy and public choice literature has investigated many of the more traditional realms of government intervention such as the provision of public goods, but there is little work related to how political incentives affect outcomes in environments with “behavioral” voters. Our goal in this paper is to understand how political incentives may interact with voters’ self-control problems in a consumption-savings environment, where government may accumulate public debt. In general, in environments where voters suffer from behavioral biases, one may worry that politicians seeking election may exploit or indulge voters’ behavioral distortions. In order to understand the impacts of government policy on capital accumulation, it is important to inspect how political incentives for debt are affected by voters’ self-control problems.

To study these issues, we embed politically determined government transfers in a stylized consumption-savings problem. We show that large (and distortionary) government debt accumulation occurs in equilibrium. Our model provides new justifications for restrictions on government debt accumulation. We also show that some of the policies that have been advocated in the prior literature (such as facilitating instruments of commitment like illiquid assets) may backfire in a world where government policy is endogenous.

We first study a simple three period model. We endow agents with ample commitment options by means of access to illiquid assets. Agents use illiquid assets to constrain their future selves’ consumption plans. The environment is designed to ensure that, absent government
intervention, agents can guarantee their commitment path of consumption.\footnote{An interpretation, of course, is that government policy has facilitated the availability of such illiquid assets.} We introduce government intervention by allowing office-seeking candidates to offer deficit-financed transfers to voters, subject to a maximal debt constraint. The logic of our model is the following. The first step is to recognize that when agents suffer from self control, government will accumulate debt to respond to the individuals’ desire to undo their commitments. The second step is to note that government debt gives sophisticated agents a motive to rebalance their portfolio to reestablish their commitment consumption sequence. But this feeds a subsequent demand for further debt accumulation. We show that for moderate debt constraints, in equilibrium, candidates choose the maximal debt, but voters are able to undo this by rebalancing their portfolios ex ante: a modified Ricardian equivalence result. When debt limits are high, however, a large government debt undermines individuals’ ability to commit.

This force is present as long as the marginal distortions from debt accumulation (e.g., distortionary taxes) are not too high relative to the present bias of the decisive voter. Equilibrium debt can then still be high, leading to high total distortions. We also show that this vicious cycle is not present in the context of an individual being offered liquid assets to undo her prior commitments (as, for instance, in Gottlieb 2008). In private credit arrangements, an individual understands that her first period choices affect her own choices in the second period. In contrast, this link between individual choices in the first period and collective choices in the second is absent in the case of government debt. Because debt is determined via collective choice in the future, individual portfolio decisions in prior periods have no impact on subsequent debt. In a three period economy, for instance, each individual voter has a private incentive to try and undo expected second period debt by an appropriate mix of liquid and illiquid assets, saving less for period 2 and more for period 3. But this individual optimization will, in the aggregate, generate demand for transfers in the second period, leading to a collective choice of even higher debt. Thus, portfolio decisions in period 1 produce collective demand for debt in the second period, even when debt is distortionary. The analysis therefore offers a new rationale for balanced budget rules in constitutions.

We also show that, when the population is not too heterogeneous, first period welfare of all agents is highest if none of the agents has access to illiquid assets and hence no one has any ability to commit to later consumption. This is because, in this scenario, no government debt is accumulated in equilibrium. Of course, for any fixed level of government debt, first-period selves of these agents are worse off because of their inability to commit. The inefficiency arises as a consequence of the feedback between the demand for illiquid assets in the first period and the demand for debt-financed transfers in the second period. This result provides a different interpretation of the policy recommendations from prior literature that suggest a beneficial
effect of policies facilitating savings commitments.

We finally extend the model to an arbitrary (finite) number of periods. We show that the equilibrium features a debt accumulation phase, where individuals consume exclusively out of government transfers, and a repayment phase where there are no additional transfers. We then show that when the number of periods approaches infinity, even when endowment per period remains fixed, the size of the debt becomes arbitrarily large.

Although we model the political process as the outcome of elections with office-motivated candidates, the key forces in the model will be present more generally in models where government is responsive to voters’ immediate desires. For instance, an alternative interpretation that leads to the same results in the case of homogenous agents is that the government is benevolent but lacks commitment and cannot control individual agents’ portfolio decisions.

2 Related Literature

Some authors (Benjamin and Laibson 2003, Caplan 2007, Glaeser 2006, Rizzo and Whitman 2009 a, b) have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glaeser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models. None of these papers considers time inconsistent agents. Bendor et al. (2011) present models based on aspiration-based learning to examine a wide variety of political phenomena. Hwang and Mollerstrom (2012) study political reforms with time inconsistent voters and show that gradualism emerges in equilibrium as a consequence of time inconsistency. They also show that election of a patient agenda setter can arise in equilibrium. Lizzeri and Yariv (2013) also studies an environment with voters suffering from self-control problems and discusses the desirability of various forms of collective action. The paper distinguishes interventions at a “commitment stage” from interventions at a “consumption stage,” They show that, if only commitment decisions are centralized, commitment investment is more moderate than if all decisions are centralized. Commitment investment is minimal when only consumption is centralized. First-period welfare is highest under either full centralization or laissez faire, depending on the distribution of the degree of present bias in the population. The model presented by Lizzeri and Yariv is not well suited for studying the interaction between public debt and private commitments that is the focus of the present paper.4

There are several papers that explore optimal disciplining of governments that are themselves prone to self-control problems (without explicitly linking their preferences to the elec-

4Ortoleva and Snowberg (2012) look at the potential effects of over-confidence on electoral outcomes.
toral process or the underlying preferences of the electorate). Amador, Werning, and Angeletos (2006) characterize the optimal budget set, balancing commitment and flexibility, for decision makers who are susceptible to temptation and face a consumption-savings problem in which independent taste shocks are experienced over time. Halac and Yared (2012) use this setting as a spring-board to study the optimal levels of discretion in policy making. They depart from Amador, Werning, and Angeletos (2006) by allowing for persistent taste shocks and interpret decision-makers as governments with a present bias towards public spending. Piguillem and Riboni (2013) also consider politicians who have a present bias for spending and bargain dynamically. They show that disagreement leads to more persistent policies and attenuation of the immediate desire of bargaining proposers to over-spend.

Our paper is also related to the literature on the political economy of government debt. Some of that literature explains debt as the outcome of a struggle between different groups in the population who want to gain more control over resources. The reason debt is accumulated is that the group that is in power today may not be in power tomorrow and debt is a way to take advantage of this temporary power. For instance, Cukierman and Meltzer (1989) and Song, Storesletten, and Zilibotti (2010) argue that debt is then a tool to redistribute resources across generations. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) argue that debt is a way to tie the hands of future governments which have different preferences from the current one. In Tabellini and Alesina (1990) voters choose the composition of public spending in an environment where the median voter theorem applies. If the median voter remains the same in both periods the equilibrium involves budget balance. If the median voter tomorrow has different preferences, the current median voter may choose to run a budget debt to take advantage of his temporary power and tie the hands of the future government. The equilibrium may also involve a budget surplus because there is an “insurance” component that links the two periods as well: a surplus tends to equalize the median voter’s utility in the two periods. Tabellini and Alesina give conditions such that debt will be incurred and show that increased polarization leads to larger debt levels. Battaglini and Coate (2008) present a dynamic model of taxation and debt, where a rich policy space is considered within a legislative bargaining environment. Velasco (1996) suggests a model where government resources are a “common property” out of which interest groups can finance their own consumption. Debt arises in his model as a consequence of a dynamic “common pools” problem. Lizzeri (1999) presents a model of debt as a tool of redistributive politics.5

In all these models voters are time consistent. Krusell, Kurusçu, and Smith (2002, 2010) examine government policy for agents who suffer self-control problems. Krusell, Kurusçu, and

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5Tabellini (1991) also illustrates how debt and social security differ as distributional instruments in an overlapping generations environment.
Smith (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (or unjust) world and ultimately affect redistributive politics.

3 Model

3.1 Economy

3.1.1 Preferences

We first consider a particularly simple three period model to highlight the basic idea in a particularly stark fashion.

There is a measure 1 of voters who live for three periods. To make things particularly simple, assume that in period 1 voters have a wealth $k$ from which to finance consumption over three periods. No endowment is available in the other two periods. As in Laibson (1997), preferences over consumption sequence $c_1, c_2, c_3$ are given by

\[
U_1 (c_1, c_2, c_3) = u(c_1) + \beta \delta u(c_2) + \beta^2 \delta^2 u(c_3),
\]

\[
U_2 (c_2, c_3) = u(c_2) + \beta \delta u(c_3),
\]

\[
U(c_3) = u(c_3),
\]

(1)

where $u$ is a continuous and strictly concave utility function. We also assume that the utility function is three times continuously differentiable. For now we assume that agents are identical. We consider the effects of heterogeneity in Section 6.1. For expositional simplicity, and since our main focus is on the impacts of time inconsistency, we assume that $\delta = 1$.

It is well-known that agents with quasi-hyperbolic preferences suffer from time inconsistency, and therefore exhibit demand for commitment. We assume that agents are sophisticated: they are fully aware of their self-control problems.\textsuperscript{7} Let $c^*_1, c^*_2$, and $c^*_3$ denote the op-

\textsuperscript{6}It is easy to accommodate positive endowments in all periods. It is also possible, but more complicated, to allow for heterogeneity in wealth. What is important is that agents can make decisions that affect the future path of consumption. This can happen either because there is a substantial fraction of the population who desires to save in period 1. Alternatively we could assume that consumers are able to privately borrow against future endowments in period 1 and then wish to commit to a path of consumption.

\textsuperscript{7}The qualitative nature of our results would not change if agents only had partial awareness of their self-control problems, i.e., they are partially sophisticated ($\beta < \hat{\beta} < 1$ as in O’Donoughue and Rabin 1999). We can also allow a fraction of agents to be fully naive as long as this is not a majority.
timal consumption sequence with commitment in period 1. Namely, $c^*_1$, $c^*_2$, and $c^*_3$ maximize $U_1(c_1, c_2, c_3)$ subject to $c_1 + c_2 + c_3 \leq k$. Let $c^U_1$, $c^U_2$, and $c^U_3$ denote the optimal consumption sequence without commitment. For any savings $s_1$ in period 1, absent commitment, period 2 consumption $c_2(s_1)$ maximizes $U_2(c_2, c_3)$ subject to $c_2(s_1) + c_3 \leq s_1$. $c^U_1$ then maximizes $U_1(c_1, c_2(s_1), s_1 - c_2(s_1))$. Thus, $c^U_2 = c_2(k - c^U_1)$ and $c^U_3 = k - c^U_2 - c^U_1$.

To highlight the demand for commitment, consider any $\beta < 1$, and the commitment consumption sequence $c^*_1$, $c^*_2$, $c^*_3$. This sequence must satisfy $u'(c^*_2) = u'(c^*_3)$ which implies that $u'(c^*_2) > \beta u'(c^*_3)$. Thus, in period 2, the agent would like to transfer resources from the third period to the second to obtain a consumption that is strictly higher than $c^*_2$.

The following Lemma (proven in the Appendix) illustrates some basic effects of commitment on consumption that will be helpful for some of our subsequent analysis.

**Lemma 1** Commitment leads to lower second period consumption: $c^*_2 < c^U_2$.

### 3.1.2 Financial structure and commitment

We assume that in period 1 voters can choose to invest in liquid or illiquid assets. Assume all liquid and illiquid assets have the same exogenous rate of return of zero.\(^8\) Illiquid assets are two-period securities that cannot be sold in period 2. Liquid assets are one period securities. Absent government intervention in period 2, by appropriate choice of the mix of liquid and illiquid assets, a voter can commit to any desired consumption stream for periods 2 and 3. We denote savings in one period assets, in periods 1 and 2, by $s_{1,2}$ and $s_{2,3}$ respectively, and savings in illiquid assets by $s_{1,3}$.

The interplay between agents’ desire to commit in period 1 and government actions in the subsequent period is a key effect in our model. Allowing for imperfect commitment generates some interesting additional effects but the major forces are similar. We will return later to the consequences of allowing for differences in returns of liquid and illiquid assets, as well as of allowing the government to subsidize illiquid assets (for instance, as in the case of retirement plans).

### 3.2 Polity

We now introduce a government that takes actions in periods 2 and 3.\(^9\) We model government actions as arising out of electoral concerns.\(^10\) Specifically, there are two candidates running.

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\(^8\)We discuss a more general structure of returns in Section 6.2.

\(^9\)We consider the effects of first period elections in Section 6.3.

\(^10\)As mentioned above it can be shown that in the case of homogenous agents this is analogous to a social planner without commitment who cannot control agents portfolio decisions.
for office. Candidates are office motivated: they receive some positive benefit from electoral victory and hence choose electoral platforms to maximize the probability of winning.

It will soon be clear that candidates’ time preferences play no role, and that they need not be the same candidates in the two periods. There are simple majoritarian elections in periods 2 and 3. In period 2, each candidate offers a platform given by \((y, t)\) where \(y\) is a per-capita transfer and \(t\) is a lump-sum tax. Let \(d = y - t\) denote per-capita government debt in period 2. When taxes are non-distortionary, all that matters is debt. If taxes are distortionary, there is no reason to have positive contemporaneous taxes in this model. Thus, from now on, we assume that transfers are debt financed so we equate debt and transfers. We assume that, in each period, voters use strategies that are not weakly dominated. In particular, each agent votes for the candidate that offers the platform that they like better. As a tie-breaking rule, we assume that whenever individuals are indifferent between the two candidates, they vote for either with equal probability.\(^{11}\)

In what follows, we first consider a benchmark in which debt is non-distortionary and then move on to the case of distortionary debt. We assume debt is financed by foreign lenders at zero interest rate (tantamount to assuming a small open economy),\(^{12}\) to be repaid by third-period revenues raised by lump-sum taxes.\(^{13}\) We wish to study the effects of constitutionally imposed borrowing limits on the government; let \(\overline{d}\) denote the per capita value of this limit.

In period 1 an agent who predicts equilibrium per-capita debt levels of \(d\), chooses savings intended for period 2, denoted by \(s_{12}\) and for period 3, denoted by \(s_{13}\), to solve

\[
\max_{s_{12}, s_{13}} u(c_1) + \beta u(s_{12} + d - s_{23}) + \beta u(s_{13} + s_{23} - d).
\]

Note that, since there is a large number of voters, this agent takes as given the second period debt when making first-period choices.\(^{14}\)

\(^{11}\)This is akin to assuming that agents have lexicographic preferences that: a. respond to policy first, and upon indifference, to the identity of the candidate; and b. are uniformly distributed with respect to the preferred candidate.

\(^{12}\)The main forces present in our model would remain even if we considered interest rate determination in a closed economy. General equilibrium effects are subtle, however, when agents are quasi-hyperbolic (see Krussel et al. 2002).

\(^{13}\)For most of our analysis, period 3 elections are vacuous. We return to the effects of additional periods in Section 6.4.

\(^{14}\)The belief by agents that their individual savings behavior does not affect second-period debt is clearly correct for the case assumed here with a continuum of voters. With a finite electorate it would be possible to construct different equilibria but these would not be robust to adding some forms of noise in the second period (e.g., noise in second-period turnout). Roughly, what we require is that in the first period each agent believes that there is a negligible probability that he is the pivotal voter in the second period.
In period 2 a voter with preference parameter $\beta$ chooses savings $s_{23}$ to solve
\[
\max_{s_{23}} u(s_{12} + d - s_{23}) + \beta u(s_{13} + s_{23} - d).
\]
The resulting optimal consumption sequence is denoted $c_1(d), c_2(d), c_3(d)$. Suppose that candidate A chooses a debt $d_A$ and candidate B chooses debt $d_B$. Then the voter votes for A in period 2 whenever $u(c_2(d_A)) + \beta u(c_3(d_A)) > u(c_2(d_B)) + \beta u(c_3(d_B))$.

4 Equilibrium Debt and its Consequences

We now characterize equilibrium in the world with time-inconsistent agents for all possible constraints on debt accumulation. We first discuss the benchmark case of zero distortions from debt and taxes. This case is particularly simple. However, to highlight the richness of the environment we later move on to consider the case of distortionary debt (or taxes).

4.1 Equilibrium without Distortions

The following result characterizes equilibria for all possible debt limits. There is always an incentive for politicians to promise debt-financed transfers, but the consequences of such debt on agents’ equilibrium consumption depend on how tight debt limits are.

Proposition 1 (Incomplete Ricardian Equivalence)

1. If $\overline{d} \leq c_2^*$ then both candidates offer platforms with debt $\overline{d}$. Equilibrium consumption is $(c_1^*, c_2^*, c_3^*)$.

2. If $c_2^* < \overline{d} < c_2^U$ then both candidates offer platforms with debt $\overline{d}$. In equilibrium, second-period consumption is $c_2 = \overline{d}$.

3. If $\overline{d} \geq c_2^U$ then any $d$ such that $c_2^U \leq d \leq k$ is part of an equilibrium. Equilibrium consumption is $(c_1^U, c_2^U, c_3^U)$.

Proof. 1. Assume by way of contradiction that equilibrium debt is $d^* < \overline{d}$. If this is the case, a voter can implement the commitment sequence of consumption $c_1^*, c_2^*, c_3^*$ by choosing $s_{12} = c_2^* - d^*$, and $s_{13} = c_3^* + d^*$. This is feasible since $d^* < \overline{d} < c_2^*$. Hence, these are the optimal choices for the voter. But, by definition of $c_2^*, c_3^*, u'(c_2^*) > \beta u'(c_3^*)$, and therefore, in period 2 all voters would vote for a candidate who offered a slightly higher debt. Thus, the only debt that can be part of an equilibrium is $\overline{d}$. Given a debt of $\overline{d}$, in period 1, each voter chooses $s_{12} = c_2^* - \overline{d}, s_{13} = c_3^* + \overline{d}$. Given these saving choices, none of the voters would vote.
for a candidate that offered a lower debt in the second period, proving that debt and this sequence of consumption constitute a unique equilibrium.

2. Assume by way of contradiction that, in equilibrium, a debt \( d^* < \overline{d} \) is implemented. As in part (1), voters choose savings to restore commitment as much as possible. Assume that \( c^*_2 < d^* \) (otherwise, the proof of part (1) applies). Each agent maximizes

\[
 u(c_1) + \beta u(c_2) + \beta u(k - c_1 - c_2)
\]

\[ s.t. \quad c_2 \geq d^*. \]

The first order conditions yield

\[
 u'(c_1) = \beta u'(k - c_1 - d^*) > u'(c_2) = u'(d^*)
\]

because \( d^* > c^*_2 \) (recall that \( u'(c^*_2) = u'(c^*_3) \)). This means that the agent sets \( s_{12} = 0 \) since second-period consumption is already higher than desired by the first-period self. However, since \( d^* < c^*_2 \), \( u'(d) > \beta u'(c_3) \). Thus, in period 2 all voters would vote for higher debts contradicting the assumption that \( d \) is an equilibrium debt level. Finally, to conclude that a debt of \( d \) is indeed part of an equilibrium, observe that, given \( d \), by similar reasoning, the optimal saving choices of all voters would lead to \( u'(d) > \beta u'(c_3) \). Thus, no voter would vote for lower debts.

3. We first show that the claimed outcomes are part of an equilibrium. Given any candidate equilibrium debt \( k > d^* \geq c^*_2 \) that is expected by voters in period 1, an optimal policy of a voter in period 1 is a choice of \( s_{12} = 0 \) and \( s_{13} = c^*_3 - (d^* - c^*_2) \). In addition, given \( d^* \), in equilibrium, \( s_{23} = d^* - c^*_2 \) is to be saved in period 2 for period 3. Given this policy, by the definition of \( c^*_2, c^*_3 \), we have

\[
 u'(c^*_2) = \beta u'(c^*_3)
\]

giving no incentive to any period-2 self to change her savings plan away from \( s_{23} \). Suppose now that the period-1 self were to change (e.g., increase) \( s_{13} \). Then, the period-2 self would make an offsetting change (reduction) in \( s_{23} \) to restore period 2 optimality. Any change in \( s_{12} \) would similarly be offset (recall that since \( d^* \geq c^*_2 \), even if \( s_{12} = 0 \), the period-2 self can unilaterally choose \( c^*_2 \)). Thus, the period-1 self has no incentive to deviate.\(^{15}\)

Given these policies for the voters, consider a deviation to \( d < d^* \) in period 2. As long as the deviation is small (\( d \geq c^*_2 \)), all voters are indifferent (they can just make an offsetting reduction in \( s_{23} \) to restore the desired consumption sequence). If the deviation is large (\( d < c^*_2 \)), then

\(^{15}\)There are multiple ways for the period-1 self to implement the uncommitted sequence, involving increasing \( s_{12} \) and \( s_{23} \) by the same amounts with offsetting reductions to \( s_{13} \). All these are weakly dominated by the proposed sequence.
voters who can no longer make such offsetting reduction in \( s_{23} \). All voters would therefore vote against a candidate offering such a deviation. A deviation to \( d > d^* \) would leave all voters indifferent because they could make offsetting changes in \( s_{23} \).

Consider now a candidate equilibrium debt \( d^* < c_2^U \). Such an expected debt would constrain period-2 consumption for the voters, leading to victory in period 2 for a candidate offering \( d > d^* \). ■

To gain intuition for this result, it is useful to first consider why zero debt is not an equilibrium. Suppose that individuals expect zero debt. Then, equilibrium outcomes would coincide with those in an economy with no government involvement, with agents committing to \((c_1^*, c_2^*, c_3^*)\). But then, in period 2, agents would find themselves constrained and would therefore vote for a candidate that offered positive debt. Next, consider increasing debt from step 1 (i.e., zero debt) and check if satisfying this initial demand for debt is sufficient to reach an equilibrium. Namely, let us set debt \( d_1 \) such that

\[
u'(c_2^* + d_1) = \beta u'(c_3^* - d_1).
\]

This is the level of debt that is the equilibrium of the period-2 election given that savings are determined by individuals expecting zero debt and committing to their desired sequence of consumption. This clearly is not an equilibrium either: if agents expect debt \( d_1 \), they react by reducing \( s_{12}(d_1) \) and increasing \( s_{13}(d_1) \) to restore the commitment allocation. We can proceed to find the (higher) equilibrium second-period debt that will be demanded by voters given the lower savings for the second period. It is easy to see that this leads to \( d_2 = 2d_1 \).

When will this process stop? If the debt limit is binding, the process continues until debt hits the debt limit (parts (1) and (2) of the proposition). If the debt limit is loose, the process continues until commitment is fully unraveled (case (3)) because \( s_{12} \) cannot go below zero.

In part (1) of this proposition, when the debt limit is low, voters can anticipate government debt and reduce savings intended for period 2 to restore the desired (commitment) sequence of consumption. The debt cap provides a form of commitment by the government not to succumb to individuals’ revised preferences in later periods.\(^\text{16}\) Because of this saving behavior in period 1, given any anticipated debt level in the feasible range, voters would like even higher debt in order to consume more in period 2 (they are endogenously liquidity constrained in period 2).

In contrast, in part (2), equilibrium debt is sufficiently high that agents are no longer able to restore their desired commitment consumption sequence in period 1. However, in period

\(^\text{16}\)Note that the stark nature of this result relies on the fact that agents foresee perfectly their susceptibility to temptations. Commitments would only be partially restored if agents were not fully sophisticated.
2 voters are still constrained so they vote for candidates who offer maximal debt. Clearly, in the scenario depicted in Proposition 1, debt is no longer neutral.

In the case of part (3), the debt cap \( \tilde{d} \) is large. In such cases, the government can no longer commit not to indulge agents’ period 2 preferences and consumption is distorted relative to the optimal commitment levels. This result shows that, even when there are no distortions, if constraints on government action are loose, then government policy is distortionary because it interferes with individuals’ ability to commit. Debt allows the government to undo the private commitments chosen by the voters in the prior period. Thus, the government acts as an enabler of the voters, substituting fiscal irresponsibility for private irresponsibility. Private commitments are not sufficient to induce consumption commitments: state commitment (such as tighter balanced budget constitutions) are essential. Agents’ period 1 selves are made better off by tighter limits that lower \( d \) and restore their abilities to commit.\(^{17}\)

We now comment on Ricardian equivalence with time inconsistent agents. Clearly there is no general (global) Ricardian equivalence since, in different regions of the debt limit, consumption is different. However, there is a “local” version of Ricardian equivalence for sufficiently low debt limits. Furthermore, there is no “contemporaneous” Ricardian equivalence in cases (1) and (2) of Proposition 1: if there is a surprise increase in the debt limit (and/or debt) in period 2, agents are unable to undo this by contemporaneous changes in their savings because they succumb to self-control problems. However, when debt limit is not too high (case (1)), voters can anticipate government debt and reduce savings intended for period 2 to restore the desired consumption sequence, which we view as a local Ricardian equivalence. In contrast, when the debt limit is loose there is local Ricardian equivalence even with contemporaneous small debt changes: since agents are no longer able to commit for sufficiently loose debt limits, in equilibrium, second period consumption is optimal for second-period selves and local changes in debt are fully undone by changes in second period savings.

These results may seem closely related to the inefficiency of competitive credit markets when consumers are time inconsistent: even if consumers can buy illiquid assets to attempt to commit to a future consumption path, intermediaries such as credit card companies have the incentive to enter the market, leading to an undoing of commitment.\(^{18}\) However, the force underlying these results is quite different, and can lead to more dramatic inefficiencies. In

\(^{17}\)Notice that we implicitly assume that individuals cannot commit themselves into debt (they can assure a minimal wealth of zero in period 2). Were they able to commit themselves to a personal debt of up to \( d_P \), the results of the proposition would carry through, with an appropriate shift of the debt limit by \( d_P \).

\(^{18}\)This point has been made by a number of authors. Gottlieb (2008) provides a detailed analysis of the effects of competition in markets with time inconsistent consumers.
order to see this we must move to a world with distortions. We discuss the comparison with private debt explicitly in Section 5.1.

4.2 Distortionary Debt

In the environment considered up to now, debt was not directly distortionary: the distortions originated only from the effect of debt on individuals’ private commitments.

We now consider the case in which government debt can be directly distortionary. There are a number of ways in which this can happen. For instance, debt could interfere with optimal smoothing of tax distortions, or because the small open economy assumption is violated, and debt has general equilibrium effects, or because the rate at which resources can be borrowed from abroad is high relative to citizens’ discount rate.

In this initial analysis we assume a simple distortion: for every dollar taken in the form of debt in period 2, there is a tax $\eta$ that is paid in period 3 (and is therefore destroyed wealth). Thus, a per-capita debt of $d$ taken in period 2 leads to $d(1 + \eta)$ that needs to be repaid in period 3. This is a reduced form way to capture distortions that could come from a variety of sources as mentioned above.\(^{19}\)

Given savings from period 1 of $s_{12}$ and $s_{13}$, in period 2 a voter would choose debt to maximize $u(s_{12} + d) + \beta u(s_{13} - d(1 + \eta))$. The first order condition is $u'(c_2) = \beta(1 + \eta)u'(c_3)$. In contrast, the analogous first order condition evaluated in period 1 is $u'(s_{12}) = u'(s_{13})$. It follows that for any individual with preference parameter $\beta(1 + \eta) < 1$, period-2 self still wants to transfer resources from the third to the second period at the commitment solution.

The definition of optimal consumption levels now involves a subtlety that was absent in the case of no distortions: debt now destroys wealth so feasible consumption depends on debt. Let $c^*_1(d), c^*_2(d),$ and $c^*_3(d)$ be the commitment sequence of consumption given debt $d$, namely, the solution to the following problem:

$$\max \{u(c_1) + \beta(u(c_2) + u(c_3))\}$$

$$s.t. \quad c_1 + c_2 + c_3 = k - \eta d$$

Analogously, let $c^U_1(d), c^U_2(d),$ and $c^U_3(d)$ be the corresponding quantities without commitment.

Notice that the Theorem of the Maximum guarantees that both the commitment and the no-commitment consumption sequences are continuous in $d$. They are also all decreasing functions of $d$.\(^{19}\)

\(^{19}\)Of course, there is no particular reason to expect these distortions to be proportional. This is assumed mainly for convenience. The qualitative analysis of this section does not depend on this assumption. We consider convex distortions in Section 6.4 and show that some of the main features remain unchanged.
As in the case of no distortions, the behavior of equilibrium debt and consumption is divided into three regions depending on the debt limit. In order to determine the limits of these regions we need to define two values of debt that we call $d^*$ and $d^{**}$.

Define $d^*$ as the solution of $c_2^* (d^*) = d^*$.\textsuperscript{20}

We now introduce an artificial constrained-maximization problem for a voter of preference parameter $\beta (1 + \eta) < 1$.

$$\begin{align*}
\max & \quad u(c_1) + \beta [u(c_2) + u(c_3)] \\
\text{s.t.} & \quad u'(c_2) = \beta (1 + \eta) u'(c_3), \\
& \quad c_1 + c_2 + c_3 = k - d\eta.
\end{align*}$$

\text{(2)}

The first constraint is a “relaxed” commitment constraint, where resources transferred between periods 3 and 2 are costly. This will be the relevant constraint in determining debt in the second period. The smaller the distortion $\eta$, the tighter this constraint. The second constraint reflects the loss of resources due to the distortion. Denote by $(c_1^d (d), c_2^d (d), c_3^d (d))$ the consumption sequence that solves the problem. We now define $d^{**}$ to be the solution of $d^{**} = c_2^d (d^{**})$.\textsuperscript{21} It is easy to show that $d^* < d^{**}$.

**Proposition 2 (Distortionary Equilibrium Debt)**

1. If $\beta (1 + \eta) > 1$ then in equilibrium there is no debt and consumption is given by $(c_1^*, c_2^*, c_3^*)$.

2. Assume that $\beta (1 + \eta) < 1$. If $d^* \geq d^*$, then equilibrium debt is given by $d^*$ and consumption is given by $(c_1^d (d), c_2^d (d), c_3^d (d))$. If $d^* < d^* \leq d^{**}$, then equilibrium debt is given by $d^*$ and period 2 consumption is given by $c_2 = d^*$. If $d^* > d^{**}$, then debt is given by $d^{**}$ and period 2 consumption is given by $c_2 = d^{**}$.

**Proof.** 1. We first show that there is an equilibrium with zero debt. Given an expected second-period debt of zero, in period 1 voters choose the mix of liquid and illiquid assets $s_{12} = c_2^*$ and $s_{13} = c_3^*$ that implements the commitment consumption sequence $(c_1^*, c_2^*, c_3^*)$. Given this mix of savings, $u'(c_2^*) = u'(c_3^*)$. Thus, if $\beta (1 + \eta) > 1$, $u'(c_2^*) < \beta (1 + \eta) u'(c_3^*)$ and voters have no incentive to vote for positive debt. Consider now any level of expected debt $d$. The mix of savings has to be such that $u'(s_{12} + d) \leq u'(s_{13} + s_{23} - d)$. But then $u'(s_{12} + d) < \beta (1 + \eta) u'(s_{13} + s_{23} - d)$, inducing voters to vote to reduce debt.

\textsuperscript{20}Notice that $c_2^d (0) \geq 0$, while $c_2^d (k/\eta) = 0 < k/\eta$, and so the Intermediate Value Theorem guarantees the existence of such a $d^*$.

\textsuperscript{21}Again, the Intermediate Value Theorem assures that such $d^{**}$ always exists since $c_2^d (0) = c_2^d (0) \geq 0$, and $c_2^d (k/\eta) = 0 < k/\eta$, and the Theorem of the Maximum implies that $c_2^d (d)$ is continuous.
2. Consider now the case in which $\beta (1 + \eta) < 1$. Given any $\overline{d} < d^*$ and any expected $d \leq \overline{d}$, optimal savings in period 2 are given by $s_{23} = 0$ and $s_{12}, s_{13}$ are such that $u'(s_{12} + d) = u'(s_{13} - d)$. Thus, $u'(s_{12} + d) > \beta (1 + \eta) u'(s_{13} - d)$ and voters would vote to increase debt. Thus, in this scenario equilibrium debt must be $\overline{d}$ and consumption must be given by $(c_1^*(\overline{d}), c_2^*(\overline{d}), c_3^*(\overline{d}))$. If $d^* < \overline{d} \leq d^{**}$, then, by the same reasoning, equilibrium debt must be at least $d^*$. But then, by the definition of $d^*$, debt is higher than second-period commitment consumption, and optimal savings are at a corner: $s_{12} = s_{23} = 0$, implying that $c_2 = d$. Because $d < d^{**}$, we then have that $\beta (1 + \eta) u'(c_3) < u'(c_2) < u'(c_3)$. This implies that voters vote for higher debt unless $d = \overline{d}$. Finally, if $\overline{d} \geq d > d^{**}$, then by the definition of $d^{**}$, $u'(d) < \beta (1 + \eta) u'(c_3)$, so voters would vote to reduce debt. This proves that, for any $\overline{d} \geq d^{**}$ equilibrium debt is given by $d^{**}$. 

This result says that debt accumulation can result in very large distortions when voters suffer from self control problems. The intuition is fairly similar to the one that we described for the case of no distortions, and a similar iteration of steps can be illustrated for this case. Because debt is determined by voters’ collective choices, individual saving decisions in prior periods have no impact on debt: voters have an incentive to try to undo expected second-period debt by optimizing their mix of liquid-illiquid assets by saving less for period 2 and more for period 3. But, when the debt ceiling is not too low, this individual optimization will, in the aggregate, generate demand for transfers in the second period, leading to voting for a positive debt. Thus, savings decisions in period 1 generate their own demand for debt in the second period, even when debt is distortionary.

5 Institutions, Welfare, and Policy

We now evaluate how welfare in the equilibrium allocation presented in Proposition 2 compares with several alternative benchmarks/policies. We consider: 1. Private debt incurred via market intermediaries; 2. A social planner without commitment; 3. Banning of illiquid assets, thereby eliminating commitment possibilities; and 4. Tighter debt limits.

In order to evaluate these scenarios, it is useful to understand the welfare consequences of distortions. The following result provides a comparison of equilibrium welfare with and without distortions when debt limits are large (namely, $\overline{d} > d^{**}$).

**Proposition 3 (Welfare Effects of Distortions)** Whenever $\beta < \beta (1 + \eta) < 1$ the equilibrium with distortions determined by $\eta$ leads to lower first period welfare than the equilibrium corresponding to no distortions, when $\eta = 0$. If $\beta (1 + \eta) > 1$, then first period welfare is higher than that induced by any $\beta (1 + \eta) < 1$. 

15
The proof of this proposition is in the Appendix. As mentioned above, there are two contrasting effects of positive distortions. On the negative side, given that there is debt in equilibrium, the presence of distortions causes wealth destruction. On the positive side, distortions relax the commitment constraint in the artificial maximization that determines equilibrium debt. In fact, when $\eta$ is very high ($\eta > 1 - \beta$), distortions serve as a full commitment device since, in equilibrium, voters do not vote for positive debt in the second period. The proposition shows that the negative effect dominates.

Figure 1 illustrates the impact of distortions in the case of instantaneous log-utility, where we take the budget to be $k = 3$ and the population time preferences to be $\beta = 0.7$. The left panel of the figure illustrates the consumption patterns and wealth destroyed. Notice that consumption declines with $\eta$ in periods 1 and 2, but is increasing in period 3. This reflects the two effects discussed above that distortions have – on the one hand, they destroy wealth, and indeed, wealth destruction increases with $\eta$; On the other hand, they relax the constraints in period 2, which allows for more delayed consumption. The right panel of the figure illustrates the impact of distortions on welfare from the perspective of each self. Welfare for period-1 and period-2 selves declines with $\eta$, in line with the statement in the proposition. This indicates that the effect of wealth destruction outweighs the benefits of smoothing derived from greater distortions, and so overall greater distortions do not help individuals early in the process. However, since period 3 consumption is increasing, so does welfare in period 3.
5.1 Private Debt versus Public Debt

Consider now an environment in which there is no government but individuals can borrow on the private market from intermediaries such as credit card companies. The model is otherwise the same as in Section 3. For the purposes of comparison with our analysis of government debt, assume that credit card companies charge a proportional fee $\eta$ for every dollar borrowed in the second period. This could be due to markups in an imperfectly competitive credit market or to costs born by credit card companies. We do not claim that this is a rich and realistic model of credit card debt with self control. The point of this stark model is to draw an important contrast between private and public debt.

In order to properly compare with the public debt case we obtain the solution with private debt for any debt limit $\bar{d}$.

Given $s_{12}$ and $s_{13}$ and $\bar{d}$, in the second period the agent chooses $d$ to maximize $u(s_{12} + d) + \beta u(s_{13} - d(1 + \eta))$ subject to $d \leq \bar{d}$. Let $d(s_{12}, s_{13}, \bar{d})$ denote the solution to this problem.

In the first period, the agent solves the following problem:

$$\max_{s_{12}, s_{13}} u(c_1) + \beta \left[u(s_{12} + d(s_{12}, s_{13}, \bar{d})) + u(s_{13} - d(s_{12}, s_{13}, \bar{d})(1 + \eta))\right]$$

s.t. $c_1 + c_2 + c_3 = k - d(s_{12}, s_{13}, \bar{d}) \eta; \ d \leq \bar{d}$.

Notice that $U_1(c_1^*(d), c_2^*(d), c_3^*(d))$ is decreasing in $d$ and so there is a unique $d^C > 0$ for which $U_1(c_1^*(d^C), c_2^*(d^C), c_3^*(d^C)) = U_1(c_1^0(0), c_2^0(0), c_3^0(0))$. The debt $d^C$ is the debt level that renders the agent indifferent between borrowing $d^C$ but perfectly smoothing utility between periods 2 and 3, and not borrowing but accepting the constrained commitment allocation.

**Proposition 4 (Equilibrium with Credit Cards)** Assume $\beta (1 + \eta) < 1$.

1. If $\bar{d} \leq d^C$, then debt is $\bar{d}$, and consumption is $(c_1^*(\bar{d}), c_2^*(\bar{d}), c_3^*(\bar{d}))$.

2. If $\bar{d} > d^C$ then agents make portfolio decisions in period 1 that ensure no debt in the second period. The equilibrium consumption sequence is given by $(c_1^0(0), c_2^0(0), c_3^0(0))$. In equilibrium first period welfare is increasing in $\eta$.

**Proof.** Suppose $\hat{s}_{12}, \hat{s}_{13}$, and $d(\hat{s}_{12}, \hat{s}_{13}, \bar{d}) > 0$ constitute part of an equilibrium. If $d(\hat{s}_{12}, \hat{s}_{13}, \bar{d}) < \bar{d}$, then the first-order conditions of the agent must hold, and therefore we must have:

$$u'(\hat{s}_{12} + d(\hat{s}_{12}, \hat{s}_{13})) = \beta (1 + \eta) u'(\hat{s}_{13} - d(\hat{s}_{12}, \hat{s}_{13}, \bar{d})(1 + \eta)).$$

Consider a deviation to the following savings plan: $s_{12} = \hat{s}_{12} + d(\hat{s}_{12}, \hat{s}_{13}, \bar{d})$, $s_{13} = \hat{s}_{13} - d(\hat{s}_{12}, \hat{s}_{13}, \bar{d})(1 + \eta)$. Given these choices, the second period first-order conditions are satisfied with $d(s_{12}, s_{13}, \bar{d}) = 0$. But then, with this deviation the consumer will have saved $\eta d(\hat{s}_{12}, \hat{s}_{13}, \bar{d})$ which she can consume in period 1. This is a profitable deviation.
Assume now that the first-order conditions do not hold and therefore the debt limit is binding \((d(s_{12}, s_{13}, \bar{d}) = \bar{d})\). In this case, the agent must be consuming her commitment consumption sequence: otherwise, the agent could improve her first-period utility by reducing savings for the second period and increasing those for the third period without changing debt.

Thus, we have two possible equilibria: (1) \(d = \bar{d}\) and consumption \((c^*_1(\bar{d}), c^*_2(\bar{d}), c^*_3(\bar{d}))\) or (2) \(d = 0\) and consumption \((c^*_2(0), c^*_2(0), c^*_3(0))\) (indeed, \((c^*_2(0), c^*_2(0), c^*_3(0))\) clearly satisfies the second period first-order conditions. These two alternative plans yield utilities of \(U_1(c^*_1(\bar{d}), c^*_2(\bar{d}), c^*_3(\bar{d}))\) and \(U_1(c^*_2(0), c^*_2(0), c^*_3(0))\) respectively. The first of these is decreasing in \(\bar{d}\) whereas the second is independent of \(\bar{d}\). From the definition of \(\delta^C\), it follows that for \(\bar{d} \leq \delta^C\) the agent will choose to commit, while for \(\bar{d} > \delta^C\) the agent gives up commitment and chooses a debt of zero.

The logic of this result is the following. The availability of credit in the second period limits the commitment possibilities for time-inconsistent agents. However, sophisticated agents anticipate this issue and take appropriate steps to counteract this temptation. If the debt limit is particularly tight, then the consumer is willing to give up some resources in order to achieve commitment. However, when the debt limit is loose the cost is too large and therefore the consumer gives up trying to commit. Every consumption profile that is attainable via positive debt with credit cards is also attainable with an appropriate mix of liquid-illiquid assets. Thus, with positive distortions and loose debt limits it cannot be optimal to ever end up with positive credit card debt. Agents internalize the commitment constraint in period 2 and ‘give up on commitment’ just enough that they do not waste resources by dealing with credit card companies. Clearly, first period welfare is increasing in \(\eta\) because higher \(\eta\) relaxes the commitment constraint.

Agents’ sophistication is important for this characterization to hold. Indeed, were agents naive, believing their future selves will face no self-control problems, Heidhues and Koszegi (2010) illustrate a reverse pattern of welfare with respect to repayment penalties. In their setting, naive agents do not foresee taking any future debt and so the higher the repayment penalties, the greater the actual future cost agents end up paying. The commitment benefits of high distortions (high \(\eta\)) in our model are completely lost on naive agents.

Part 2 of this result contrasts Proposition 3. The key difference is that public debt is a result of collective action, so individuals have a private incentive to undo public debt.

We have discussed private debt and public debt separately, assuming that debt is either public or private but not both. One can easily examine a model with coexistence of private and public debt. In our model with linear distortions, the coexistence yields uninteresting results. Let \(\eta_G\) be the distortion associated with public debt and \(\eta_P\) the distortion (markup) associated with private debt. Then, if \(\eta_G < \eta_P\), it is possible to show that only public debt
matters. If $\eta_G > \eta_P$, then only private debt matters. It is not clear what assumption is more reasonable (e.g., interest rates on credit card debt often exceed 20%). This result hinges on the linearity of the distortions. It can in fact be shown that if distortions are convex, then private debt and public debt coexist. However, the model is much more complicated in this case.

5.2 Social Planner without Commitment

In the environment we study, voters are time inconsistent while politicians simply pursue office in each period. As an alternative, consider a situation in which a time-inconsistent social planner, sharing the population preference parameter $\beta$, fully determines consumption allocations. Notice that this would correspond to the decision process emerging in a citizen-candidate version of our model. As for the case of private debt, it is easy to see that the allocation determined by such a social planner, at least for sufficiently high debt limits, is given by $(c^1_1(0), c^2_2(0), c^3_3(0))$, namely by the solution of the maximization problem given in (2). Therefore, first-period welfare is increasing in $\eta$ since higher distortions lessen the commitment constraint. This is clearly in stark contrast with the result in Proposition 3. As we pointed out before, however, a social planner who only controls aggregate intertemporal transfers and cannot control individual portfolio allocations would behave exactly as our politicians, leading to the same equilibrium as the political economy one. Thus, in our setting, there is an interesting non-monotonicity in the effect of government intervention: moderate government intervention in the form of debt-financed transfers leads to worse outcomes than either decentralized allocations or fully centralized allocations.

---

22 Notice that when private and public debt are both available, there can be a multiplicity of equilibria. Indeed, if no one takes on private debt at the outset, there is no demand for public debt later on (and any agent putting themselves into private debt at the beginning will not be able to repay). If everyone takes on private debt, there is a collective demand for public debt later on, which sustains the initial individual private demands for debt.

23 See Angeletos et al. (2001) for a model with coexistence of credit card debt and investment in illiquid assets.

24 Krusell et al. (2002) show that in an economy with capital accumulation there is an additional issue in contrasting a decentralized economy and a social planner without commitment. Specifically, while individuals take the returns to savings as given, the social planner takes into account the fact that, with decreasing returns, increased aggregate savings reduce returns to capital accumulation. This leads to even worse undersaving when a social planner is present.
5.3 Period 1 Financial Structure

We now consider the socially optimal mix of liquid and illiquid assets when government is fiscally irresponsible. A common argument in the behavioral literature is that in environments with time-inconsistent agents, an efficiency enhancing paternalistic policy is to subsidize or otherwise promote the existence of illiquid commitment assets.

Our results suggest that, in evaluating such policies, it is important to consider how this affects the political economy of debt.

When agents have no access to illiquid assets they have no commitment power. This can arise whenever, say, agents have access to personal credit cards (with a rate of return of 1) that allow them to undo any commitment plan they entered in earlier periods. Alternatively, whenever agents have access to illiquid assets and debt is non-distortionary and has no limit, agents are effectively tied to uncommitted consumption paths. The comparison with such environments is less straightforward since it presents a trade-off. On the one hand, debt allows for some level of commitment when illiquid assets are available. On the other hand, it entails a wealth loss. Specifically, in our model, an implication of Proposition 3 is that welfare is higher for all selves when illiquid assets are banned or taxed, rather than subsidized.

**Proposition 5 (Banning Illiquid Assets)** Assume that $\beta (1 + \eta) < 1$. Then, first period welfare is higher if illiquid assets are banned.

Of course, this result should be evaluated with caution since there may be many reasons why the personal benefits of commitment are not offset by subsequent increases of government debt. However, it provides a useful additional effect to be aware of when evaluating the appropriate asset mix. The result easily extends to allow for some heterogeneity in $\beta$. As long as the heterogeneity is not too large, all selves of all agents are made better off by eliminating illiquid assets. We discuss the effects of heterogeneity and a more general financial structure in more detail below.

6 Extensions

6.1 Heterogeneity

We now consider what happens when agents are heterogeneous in their present-bias parameter $\beta$. In analogy to our previous notation, we will denote by $c^*_t(\beta; d)$ and $c^*_t(\beta; d)$ the commitment solution for debt $d$ and the solution to the constrained problem (2) for each individual of preference parameter $\beta$. 
Figure 2: Consumption Patterns for a Given Debt Level

We start by assuming that second period consumption \( c_2^\beta (\beta; d) \) increases monotonically in \( \beta \). This holds when the utility function has sufficient curvature. We note that there are many preferences for which this does not hold. For instance, with log utility, consumption is not monotonic. However, even in such a case our initial discussion will be valid for a fairly wide class of distributions of the \( \beta \) parameter. We discuss the more general case below.

Let \( \beta^* \) be such that \( G(\frac{1}{1+\eta}) - (\beta^*) = 1/2 \). That is, half the population has preferences that are between \( \beta^* \) and \( \frac{1}{1+\eta} \). Figure 3 depicts the shape of commitment and no-commitment consumption levels in period 2 as a function of preferences for a particular debt level.

The agent of type \( \beta^* \) turns out to be the pivotal agent for determining debt in this environment. We can now define \( d^*(\beta^*) \) and \( d^{**}(\beta^*) \) as the solutions of \( d^* = c_2^\beta (\beta^*, d^*) \) and \( d^{**} = c_2^\beta (\beta^*, d^{**}) \).

**Proposition 6 (Equilibrium with Heterogeneous Voters)**

1. If \( \beta_M (1+\eta) > 1 \), then in equilibrium there is no debt, and consumption is given by \( c_1^\beta (\beta), c_2^\beta (\beta), c_3^\beta (\beta) \).

2. Assume that \( \beta_M (1+\eta) < 1 \). If \( \bar{d} \leq d^{**}(\beta^*) \), then equilibrium debt is given by \( \bar{d} \). If \( \bar{d} > d^{**}(\beta^*) \), then debt is given by \( d^{**}(\beta^*) \).

25Existence and uniqueness of these debt levels follow the same arguments used for the case of a homogenous electorate.
3. For any equilibrium debt level $d$, individual consumption for an agent of preference parameter $\beta$, period-2 consumption level in equilibrium is given by:

$$c_2(\beta; d) = \begin{cases} c_2^\theta(\beta; d) & \beta \leq \beta_L(d) \\ d & \beta_L(d) \leq \beta < \beta_H(d) \\ c_2^\theta(\beta; d) & \beta \geq \beta_H(d) \end{cases}$$

With respect to the distribution of preferences, notice that a shift in distribution changes the debt structure in the economy only when it modifies the preferences $\beta^*$ of the ‘pivotal agent’. As $\beta^*$ increases, $c_2^*(\beta^*; d)$ and $c_2^\theta(\beta^*; d)$ increase for all $d$, and therefore both $d^*$ and $d^{**}$ increase.

We say $G'$ is a median preserving spread of $G$ if both share the same median $\beta_M$ and for any $\beta < \beta_M$, $G'(\beta) \geq G(\beta)$, while for any $\beta > \beta_M$, $G'(\beta) \leq G(\beta)$. Intuitively, this implies that, under $G'$, more weight is put on more extreme values of $\beta$ (see Malamud and Trojani (2009) for applications to a variety of other economic phenomena).

The above discussion then implies the following corollary.

**Corollary 2 (Distributional Shifts)**

1. Assume $G(\frac{1}{1+\eta}) = G'(\frac{1}{1+\eta})$. If $G'$ First Order Stochastically Dominates $G$, and the corresponding medians $\beta_M, \beta_M' < \frac{1}{1+\eta}$, then equilibrium debt under $G'$ is (weakly) higher than that under $G$.

2. If $G'$ is a Median Preserving Spread of $G$, then equilibrium debt under $G'$ is (weakly) lower than that under $G$.

Part 1 of this corollary says that, as the population becomes more “virtuous” or less subject to self-control problems, equilibrium debt increases. This is potentially surprising but is a natural consequence of the logic of our model. There are two ways to glean intuition for this result. The more mechanical one is to recall that equilibrium debt is equal to second period consumption. As $\beta^*$ increases, so does the desired second period consumption of the pivotal agent $\beta^*$. Thus, equilibrium debt increases. Alternatively, notice that in our model debt arises because of the desire of the pivotal agent to constrain her future self, and the subsequent response of the political system undoing this commitment. The more virtuous the pivotal agent, the higher the level of debt that is required to prevent this agent from attempting to commit at an even higher level.
We now discuss the more general case in which second period consumption may not be increasing in $\beta$. For any $\eta$, denote by $d^p$ the debt level such that:

$$G \{ \beta \mid c^p_2(\beta; d^p) < d^p \} = \frac{1}{2}.$$ 

Proposition 6 can now be restated with $d^p$ playing the role of $d^{**}(\beta^*)$. If second period consumption is decreasing in $\beta$, then $d^p$ will correspond to $c^p_2(\beta_M; d^{**})$: the median voter will be pivotal. Otherwise, there may be multiple pivotal voters.

We now discuss how the welfare of different agent types is affected by the presence of illiquid assets. Our result in Proposition 5 showing that agents would be made better off in the first period if illiquid assets were penalized obviously extends to the case where the degree of heterogeneity is limited. Furthermore, if $c^p_2(\beta, d)$ is increasing in $\beta$, it is possible to show that, for any degree of heterogeneity, all agents with $\beta \leq \beta^*$ as well as those with sufficiently high $\beta$ are made worse off by the presence of illiquid assets: the former group because for these types, debt is higher than $c^p_2(\beta; d)$ and second period consumption is completely out of transfers, so the logic of Proposition 5 immediately holds for these agents; the latter group because these types do not have much of a self-control problem, so the presence of illiquid assets gains them little commitment but generates a destruction of resources through debt.

### 6.2 General Financial Structure

In our previous analysis we have assumed that liquid and illiquid assets have constant returns and that these returns are equal. In fact our analysis can readily be extended to allow for more general structure of returns. First of all, it can easily be seen that assuming that the returns are lower for the illiquid assets is equivalent to assuming a larger debt distortion $\eta$.

Secondly, we consider the case of decreasing returns in investment. Let $R_L(s)$ and $R_I(s)$ denote the returns on liquid and illiquid assets, respectively, as a function of the investment $s$. We assume that $R_L(0) \geq 1$, $R_I(0) \geq 1$, $R_L(k) < \beta$, and $R_I(k) < \beta$. Suppose that $R'_L(s)$ and $R'_I(s)$ are both negative. We also assume that there is no debt distortion, to focus on the novel distortion introduced by decreasing returns.

Given debt $d$, an individual chooses $s_{12}$ and $s_{13}$ to maximize

$$u(c_1) + \beta [u(R_L(s_{12})s_{12} + d) + u(R_I(s_{13})s_{13} - d)]$$

subject to

$$c_1 + s_{12} + s_{13} = k$$

Furthermore, electoral competition requires that, if there are no debt limits, debt $d$ in the second period is chosen to satisfy the first order condition of the political economy problem:

$$u'(R_L(s_{12})s_{12} + d) = \beta u'(R_I(s_{13})s_{13} - d).$$
There are two possibilities for the characterization of the equilibrium. We can have a corner where \( s_{12} = 0 \). This happens if decreasing returns are not very severe. If instead decreasing returns are sufficiently pronounced, then equilibrium is pinned down by the first period condition of Problem 3. Let \( \gamma_L(s) \) and \( \gamma_I(s) \) denote the marginal returns on liquid and illiquid assets respectively:

\[
\gamma_L(s) \equiv R_L(s) + R'_L(s) s, \quad \gamma_I(s) \equiv R_I(s) + R'_I(s) s
\]

The first order conditions corresponding to (3) are:

\[
\gamma_L(s_{12}) u'(R_L(s_{12}) s_{12} + d) = \gamma_I(s_{13}) u'(R_I(s_{13}) s_{13} - d).
\]

From (4) we obtain that, at an equilibrium level of debt, we must have:

\[
\frac{\gamma_I(s_{13})}{\gamma_L(s_{12})} = \beta. \tag{5}
\]

It is useful to consider the special case in which the returns to both assets are identical: \( R_L = R_I = R \) and \( \gamma_I = \gamma_L = \gamma \).

In the case in which there is no government intervention this simplifies the solution because the first order conditions of the individual agent’s problem must be solved by \( s_{12} = s_{13} \) and hence

\[
\frac{\gamma(s_{13})}{\gamma(s_{12})} = 1.
\]

Equation (5) then implies that in the political economic equilibrium individuals face a lower return at the margin as a consequence of investing too much in the illiquid asset. Therefore, there is a waste of resources.

We now consider the effects of taxing the returns on illiquid asset. Let \( \phi \) be the unit tax. Then, for given \( s_{12}, s_{13} \), the political economy equilibrium gives the first order conditions

\[
u'(R(s_{12}) s_{12} + d) = \beta u'((R(s_{13}) - \phi) s_{13} - d).
\]

Given \( d \), each agent chooses \( s_{12} \) and \( s_{13} \) to maximize

\[
u(k - s_{12} - s_{13} + \Phi) + \beta [u(R(s_{12}) s_{12} + d) + u((R(s_{13}) - \phi) s_{13} - d)],
\]

where \( \Phi \) is a lump sum reimbursement of revenues from tax. This leads to the following first order conditions:

\[
\gamma(s_{12}) u'(R(s_{12}) s_{12} + d) = (\gamma(s_{13}) - \phi) u'((R(s_{13}) - t) s_{13} - d)
\]

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and so, substituting from the condition for the political equilibrium, we obtain:
\[
\frac{\gamma(s_{13}) - \phi}{\gamma(s_{12})} = \beta.
\]
These taxes lead to lower investment distortions by reducing investment in illiquid assets, which in turn leads to a reduction of the debt in the second period. Thus, in this equilibrium, when we introduce taxes on illiquid assets we still have \( u'(c_2) = \beta u'(c_3) \) so the self-control problem is unavoidable. However, when \( \phi \) is high, this intertemporal condition is satisfied by lower levels of \( s_{13} \) and lower levels of debt. Thus, there are values of \( \phi \) that lead to greater efficiency by reducing excessive investment in illiquid assets (and reducing debt).

6.3 Elections in period 1

The model studied so far allowed for government actions and elections in periods 2 and 3. We now extend the model to consider elections in period 1 as well. The objective of this extension is to evaluate whether collective action in period 1 could effectively satisfy the demand for commitment agents display in period 1 or at least limit the distortions associated with debt accumulation in period 2. It turns out, however, that the equilibrium consumption sequence and the total amount of resources destroyed is completely unaffected by period 1 elections. The only thing that changes is that there is a multiplicity of equilibria determining the timing of distortions.

The economic environment is the same as the one assumed in previous sections. There are two candidates running for office, both in period 1 and in period 2. The candidates are office motivated. The policy space is extended to allow candidates to offer a transfer \( y_1 \) and a lump-sum tax \( t_1 \) in period 1, as well as a transfer \( y_2 \) and tax \( t_2 \) in period 2 (elections in period 3 are redundant as before). Debt financing is allowed. Tax collection in any period carries distortions of a unit loss \( \eta > 0 \) for every unit collected. For this robustness check, we focus on the case of high debt limit, \( \bar{d} \geq d^{**} \).

By taxing themselves in period 1 and investing the proceeds in the liquid asset agents can effectively commit resources for consumption in period 2 and hence reduce debt accumulation. On the other hand, if the proceeds of taxes carried to period 2 are smaller than \( d^{**} \), in per-capita terms, a strict majority of agents in period 2 will support a positive debt level so as to increase consumption in period 2. Let \( t = t_1 - y_1 \) denote per-capita taxes in period 1 and \( d = y_2 - t_2 \) denote debt in period 2. It turns out that even though by taxing themselves in period 1 agents can indeed limit debt accumulation in period 2, this strategy simply shifts some of the repayment of debt from period 3 to period 1, but does not alter total distortions and has no ultimate effects on consumption profiles.
Proposition 7. In the economy with elections in every period and with high debt limit, \( d \geq d^{**} \), the set of equilibria is characterized by pairs of period 1 taxes and period 2 debt of the form \((t, d)\), such that \( t \in [0, d^{**}] \) and \( d = d^{**} - t \); total agents’ distortions and consumption profiles are unchanged relative to the case in which elections take place only in period 2.\(^{26}\)

It is easy to show that debt limits would be the only way to reduce distortions even in the model with period 1 elections. The reason is that any limit on period 1 surpluses would just shift the financing to higher debt in the second period. It is also easy to show that in the case of convex distortions the indeterminacy would disappear and period 1 elections would have no effect whatsoever.

Of course, the debt limit may be endogenously determined via voting. Suppose, for instance, that in period 1 agents vote on the debt limit that would affect the debt imposed in period 2 as in the model studied thus far. In such a setting, all agents would favor low debt limits in period 1. In fact, since illiquid assets allow agents to commit without experiencing the loss of wealth that results from distortional debt, equilibrium would entail a debt limit fixed at zero. Of course, if agents could vote again on the debt limit in period 2 (prior to determining the debt level itself, as in the model studied thus far), they would collectively choose a positive debt limit and consumption would be distorted (relative to the commitment paths). This suggests the importance of timing in constitutional reform. Since most amendments take a substantial amount of time to pass, changes in debt limits are likely to occur a significant time prior to the ‘temptation’ of consumption. Even if multiple elections occurred over such amendments, it would be difficult to achieve a super-majority to agree over time on an increase on the debt limit itself (as pointed above, early in the process, one would expect voters to reject debt limit increases).

6.4 Arbitrary Number of Periods and Convex Distortions

We now study an economy that lasts for an arbitrary number of period \( T \). We later consider the limit case as \( T \to \infty \). For simplicity, in this section we assume that agents’ preferences also satisfy certain Inada conditions, namely we assume that \( \lim_{c \to 0} u'(c) = \infty \).

As in the analysis of the previous sections, we assume that agents can choose to invest in liquid or illiquid assets and that these assets have equal zero returns. An illiquid asset with maturity \( m \), acquired in period \( t \), pays off in period \( t + m \) and cannot be sold before then. To isolate the effects of the interaction of time-inconsistency and fiscal policy on debt

\(^{26}\) Only if the distortion on taxes at \( t = 1 \) were smaller than the distortion on taxes at \( t = 3 \) election in period 1 would help reducing debt in period 2 and hence distortions in period 3.
accumulation as in the previous sections, we make the strong assumption that in any period $t = 1, \ldots, T - 2$ illiquid assets are available with any maturity $m$ between 2 and $T - t$. A liquid asset has maturity 1. We assume that liquid assets are available in any period $t = 1, \ldots, T - 1$. Absent government intervention, by appropriate choice of the mix of liquid and illiquid assets with different maturities, an agent can commit to any desired consumption stream.

Elections occur in any period $t \geq 2$. Period $T$ elections are vacuous and period 1 elections can be shown to be irrelevant (when distortions are convex). Let $D_t$ denote accumulated debt at $t$, while $d_t$ denotes the deficit at time $t$.

Consider first the economy with linear distortions $\eta$ such that $\beta(1 + \eta) < 1$. It is easy to extend the analysis of Section 4.2 to construct an equilibrium in which debt is accumulated until period $T - 1$ and is repaid completely only in the last period, time $T$. Furthermore, at each time $2 \leq t \leq T - 1$ agents consume exclusively off of debt; whereas at time $T$, agents consume off of time 1 savings:

$$c_t = d_t, \text{ for any } 2 \leq t < T; \quad c_T = s_{1T} - (1 + \eta)D_{T-1}.$$ 

With linear distortions agents have no incentive to smooth debt repayment and the repayment is thus concentrated at time $T$.\footnote{Note that in our formulation distortions are incurred only when debt is repaid, independently of how far in the future it is in fact repaid (recall returns are zero). This is intended to represent an environment in which debt is repaid by means of distortionary taxation.} Debt then explodes as the number of periods increases. It is clear, however, that the linearity of distortions plays a fundamental role in this construction. We show next that even when distortions are strictly convex, so that there is a motive to smooth repayments over time, debt accumulation can be large when voters are time inconsistent and the political system does not impose debt limits.

To clarify notation, though somewhat redundantly, we make explicit the distinction between deficit and repayment and let $d_t \geq 0$ and $q_t \geq 0$ denote, respectively deficits and repayments at time $t$. Recall that $D_t$ denotes debt accumulated up to (and including) time $t$: $D_t = \sum_{t'=2}^{t} d_{t'} - q_t$. We assume that tax distortions $\eta(q)$ are smooth, non-negative, strictly increasing and strictly convex in $q$:

$\eta(q)$ is a twice continuously differentiable function which satisfies:

$$\eta(q) > 0, \eta'(q) > 0, \eta''(q) > 0, \text{ for } q > 0; \text{ and } \eta(0) = \eta'(0) = 0$$

(6)

The total cost of repayment $q$, defined as $A(q) = q(1 + \eta(q))$, is then also increasing and convex, strictly for any $q > 0$, with $A(0) = 0$ and $A'(0) = 1$.

As in the economy studied in the previous sections, at equilibrium deficits and repayments are the outcome of the electoral process, while investments in the liquid and illiquid assets.
available in financial markets are derived from individual choices, taking as given election outcome.

At time $t = 1$ agents invest in liquid and illiquid assets, determining a sequence of savings in illiquid assets $s_{1t}$ for any time $t > 1$. Agents can in principle rebalance their asset portfolio at any time $t > 1$. But since a full set of illiquid assets are available in financial markets, agents can effectively implement commitment strategies and hence the option to rebalance investment portfolios in the future has no effect on the equilibrium consumption sequence nor on the equilibrium of the electoral process.\(^\text{28}\) Therefore, for any given deficit and repayment sequence $\{d_t, q_t\}_{t=2}^{T}$, the investment problem of any agent at time $t = 1$ involves the choice of the sequence of period 1 savings in illiquid assets $\{s_{1t} \geq 0\}_{t=1}^{T}$ corresponding to the maximization problem:

$$\max_u u(s_{11}) + \beta \sum_{t=2}^{T} u(s_{1t} + d_t - A(q_t))$$
$$\text{s.t. } \sum_{t=1}^{T} s_{1t} = k$$

The political economy problem at any election at time $t \geq 2$ involves two candidates running for office choosing electoral platforms to maximize the probability of elections. As in Section 3, however, the strategic interaction between the candidates is reduced at equilibrium to the solution of a single choice problem at any election time $t$. In the economy with an arbitrary but finite number of periods $T$, by backward induction, this problem can be reduced to the choice of maps $d_t(D_{t-1})$, $q_t(D_{t-1}) \geq 0$, for given time 1 transfers $\{s_{1\tau}\}_{\tau=t}^{T}$ and given expected future maps $d_\tau(D_{\tau-1}), q_\tau(D_{\tau-1})$ for all $t + 1 \leq \tau \leq T$ to

$$\max_u u(s_{1t} + d_t(D_{t-1}) - A(q_t(D_{t-1}))) + \beta \sum_{\tau=t+1}^{T} u(s_{1\tau} + d_\tau(D_{\tau-1}) - A(q_\tau(D_{\tau-1})))$$
$$\text{s.t. } \sum_{t=2}^{T} d_t - q_t = 0.$$  \(^{(8)}\)

Of course, deficits and repayments will not both be positive at the same time $t$: $d_t \cdot q_t = 0$.

We are now ready to characterize equilibrium debt accumulation and repayment. To better illustrate the structure of equilibrium it is convenient to re-consider first the case in which $T = 3$ and construct the equilibrium for the case of convex distortions. The $T = 3$ economy is special in that debt accumulation necessarily occurs in period 2 and repayment is concentrated in the last period $T = 3$. The first order condition of the political economy problem (the commitment constraint from our analysis of linear distortions) is given by:

$$u'(d_2 + s_{12}) = \beta A'(q_3)u'(s_{13} - A(q_3)).$$

In contrast with the model with linear distortions, however, the equilibrium level of debt need not be determined by a corner solution for consumption. At an interior solution, positive

\(^{28}\)Note however, that the same consumption pattern could in principle be obtained with different transfer sequences, if portfolio rebalancing after period 1 were allowed.
savings $s_{12}$ and $s_{13}$ are chosen to smooth consumption so that $u'(d_2 + s_{12}) = u'(s_{13} - A(q_3))$ and hence

$$\beta A'(q_3) = 1$$

and $c_2 = c_3$. When, on the other hand, a corner solution obtains with $s_{12} = 0$ ($s_{13}$ is always positive by Inada conditions), $u'(d_2) < u'(s_{13} - A(q_3))$ and $c_2 = d_2 > c_3$. In this case, the political economy conditions imply that $\beta A'(q) < 1$. Corner solutions obtain when the $A'(q)$ does not grow sufficiently quickly, given the size of the debt, to guarantee that $\beta A'(q) = 1$. Alternatively, these corners arise when the size of the economy, measured by the total endowment $k$, is not large enough to ensure that debt and $q$ are sufficiently large to guarantee that $\beta A'(q) = 1$. This is also the case in general, for $T > 3$: when $k$ is relatively small with respect to $q$, the economy behaves effectively like the one with linear distortions and $\beta(1 + \eta) < 1$: it displays corners of debt accumulation until $t = T - 1$, with repayment concentrated in the last period, at $T$.\footnote{This is formally shown as a by-product of the proof of Proposition 8 in the Online Appendix.}

From now on we restrict ourselves to the more interesting case in which, fixing the function $A(q)$, the total endowment $k$ is sufficiently “large.” In this case, when $T > 3$, we show that the dynamics of fiscal policy is characterized by two distinct phases: debt is accumulated first and then repaid. It still turns out that in the debt accumulation phase agents are at a corner in the sense that they consume exclusively off of government spending, as in the economy with linear distortions. However, debt repayment is smoothed over time and the equilibrium is interior during the repayment phase.

**Proposition 8** In the economy with $T > 3$ the equilibrium consumption sequence has the following properties: there exists a $\tilde{t} \geq 2$ such that: for $t \leq \tilde{t}$ the government accumulates debt; for $t > \tilde{t}$ the government gradually repays the debt. Furthermore:

1. In the repayment phase, for $t > \tilde{t}$, the equilibrium is interior and $c_t = s_{1t} - A(q_t)$.

2. Up to the last period of the debt accumulation phase, for $2 \leq t \leq \tilde{t} - 1$, agents consume exclusively off of deficit-financed spending: $s_{1t} = 0, c_t = d_t$;

\footnote{This statement is empty if $\tilde{t} = 2$. However, for $T$ and/or $k$ sufficiently large we must have $\tilde{t} > 2$.} in contrast, in the last period accumulation period ($t = \tilde{t}$) savings and debt are both positive $c_{\tilde{t}} = s_{1\tilde{t}} + d_{\tilde{t}}$, with $s_{1\tilde{t}} > 0$.

The proof of this proposition is in the Online Appendix.

It may be surprising that, even in the case of convex distortions, in the accumulation phase agents consume exclusively off of deficit (the equilibrium deficit is determined by a
corner condition for savings). The intuition for this result is the following: the marginal condition that characterizes the voting equilibrium at \( \tilde{t} \) essentially determines the maximal level of debt \( D_{\tilde{t}} \). Other things being equal, this condition trades off the marginal cost of future distortions due to an increase in debt and the marginal benefit of an increase in consumption at \( \tilde{t} \). At every time \( t < \tilde{t} \), therefore, an increase in debt has a positive marginal effect on current consumption without affecting the level of debt at \( \tilde{t} \) (since consumption in period \( \tilde{t} \) falls by the same amount), and hence without affecting the future cost of distortions at the margin. The smoothing of distortions therefore only plays a role in the repayment phase.

The following corollary characterizes the equilibrium in more detail.

**Corollary 9** *In the economy with \( T > 3 \), in equilibrium, the sequences of deficits, repayments, and consumption have the following properties:*

1. In the repayment phase, for \( t > \tilde{t} \), the sequence of repayments \( q_t > 0 \) is strictly increasing over time and consumption \( c_t \) is constant in \( t \).
2. Up to the last period of the debt accumulation phase, for \( 2 \leq t \leq \tilde{t} - 1 \), consumption \( c_t \) and the deficit \( d_t \) are decreasing over time; in contrast, consumption in period \( \tilde{t} \) is equalized to subsequent consumption \( c_{\tilde{t}} = c_t \) for any \( t > \tilde{t} \).

The proof of this Corollary is in the Online Appendix, but some details are necessary to understand the characterization of the sequences of deficits, repayments, and consumption it provides.

We show in the Appendix that along the repayment phase, the first order conditions reduce to:

\[
A'(q_t) = \frac{\beta}{T - t} \left[ \sum_{\tau=t+1}^{T} A'(q_{\tau}) \right] \tag{12}
\]

Equation (12) implies that distortions are indeed smoothed at the margin: it requires in fact that at any time \( t \) in the repayment phase, the marginal distortion \( A'(q_t) \) be equal to the average future marginal distortion, \( \frac{1}{T-t} \left[ \sum_{\tau=t+1}^{T} A'(q_{\tau}) \right] \) discounted by \( \beta \). It is the discounting by \( \beta > 0 \) which induces an increasing sequence of marginal distortions \( A'(q_t) \) and hence of repayments \( q_t \).

Furthermore, in equilibrium, it has to be the case that, at any time \( t \) in the repayment phase, the marginal distortion is \( > 1 \). Otherwise the agent would vote for accumulating debt in period \( t \): an increase in consumption at \( t \) would have a larger effect at the margin than the induced marginal cost of future distortions. This is guaranteed by the condition that \( k \) be large enough. Finally, as \( q_t > 0 \) in the repayment phase, \( s_{1t} \) must also be, by Inada conditions.
The solution of the problem of the agent at time 1, Problem (7), is then interior: agents at time 1 will use transfers to equalize consumption and hence $c_t$ will be constant. This is the case for the entire repayment phase.

The last period of the debt accumulation phase is characterized by the fact that at the margin an increase in consumption has the same effect as the induced cost of future distortions. Indeed, we show in the Appendix that the first order condition of Problem (8) at $\tilde{t}$ reduces to:

$$\frac{\beta}{T - \tilde{t}} \left[ \sum_{\tau = \tilde{t} + 1}^{T} A'(q_\tau) \right] = 1. \quad (13)$$

Furthermore, we show that the solution of Problem (7) is interior at $\tilde{t}$. This implies that $c_{\tilde{t}} = c_{\tilde{t} + 1}$; and, by our characterization of the repayment phase in this corollary, $c_{\tilde{t} + 1} = c_{\tau}$, for any $\tau > \tilde{t} + 1$. Note that this is a $T$ period version of the equation that characterizes the interior equilibrium repayment in the $T = 3$ economy, equation (9).

On the contrary, in the debt accumulation phase, up to period $\tilde{t} - 1$, the agent consumes off of deficit spending; that is, transfers from time 1 are zero. As a consequence, consumption is not equalized across periods. In fact, it is declining over time due to the agents’ self-control problem ($\beta < 1$).

We discuss now the consequences of extending the horizon $T$ of this economy. We investigate whether the maximal debt $D_{\tilde{t}}$ grows without bound as $T$ goes to infinity. To this end we construct a sequence of replica economies by allowing the aggregate endowment of the economy to grow at the same rate as $T$, so that the endowment per period remains constant along the sequence, and consumption does not become infinitesimal nor unboundedly large in every period. More precisely, the replica economies are characterized by aggregate endowment $\rho k$ and $\rho T$ periods, for some $\rho > 1$ (such that $\rho T$ is an integer). This construction guarantees that the characterization obtained in Proposition 8 and Corollary 9 hold for all replicas, along the sequence for which $\rho \to \infty$.

Let $\tilde{t}(\rho)$ denote the last accumulation period at the equilibrium of the replica economy corresponding to $\rho$; $\tilde{t}(1)$ is then the last accumulation period in the original economy with endowment $k$ and $T$ periods.

**Corollary 10** Along the sequence of replica economies, the maximal level of debt $D_{\tilde{t}(\rho)}$ increases with $\rho$ and $D_{\tilde{t}(\rho)} \to \infty$ as $\rho \to \infty$.\(^{31}\)

The proof of the Corollary is in the Online Appendix. The intuition is as follows. Consider a $k$ and a $T$ such that the characterization in Proposition 8 holds. Now double both $k$ and $T$

\(^{31}\)It should be noted that $\tilde{t}(\rho)$ also grows without bounds along the sequence of replica economies; see the Online Appendix.
(that is, consider the replica economy corresponding to \( \rho = 2 \)). At equilibrium all transfers must go to support consumption in period 1, in the last accumulation period, and in the repayment phase. If the repayment phase stayed the same in terms of its length and of the size of repayments, consumption along the repayment phase would be larger: the aggregate endowment available to transfer over the same number of periods would have essentially doubled. In this case, however, the political process represented by the solution to Problem (8) would require smoothing and hence higher deficits and for a longer number of periods implying a higher debt. Indeed, at equilibrium, the repayment phase is longer and the maximal debt it supports is higher. More generally, along the sequence of replica economies in which the aggregate endowment grows at the same rate as the number of periods, the maximal level of debt increases. But the sequence of maximal debt cannot have an upper bound as this would imply that the length of the repayment phase is finite in the limit and consumption along this phase would grow unboundedly, violating the first order conditions of the accumulation phase.

Another possible intuition for the result is that spreading the repayment of any finite amount of debt over a large number of future periods induces smaller and smaller marginal distortions that converge to 0. As a consequence, debt accumulation must also grow without bounds as the number of periods in the repayment phase.

In conclusion, when voters are time inconsistent, while convex distortions induce debt repayments to be smoothed over time, debt accumulation can nonetheless be very large to the point that debt grows without bound as the number of periods increases. This is the case, of course, unless debt limits are imposed. In other words, debt limits are necessary to limit the inefficient distortions which the economy must incur to repay large accumulated debts at equilibrium.

Other mechanisms may limit debt accumulation and hence distortions. Reducing the frequency of elections so that there is voting every \( n > 1 \) periods may lead to greater political commitment. A formal analysis of the effect of such a restrictions turns out to be complex. One important modeling choice is how the government is expected to behave in non-election periods. If the government chooses policies in non-election periods by attempting to satisfy popular opinion (governing by opinion polls), then the outcome would be equivalent to the one characterized in this Section. However, if the government can commit in election periods to its behavior in non-election periods, then this would presumably enhance political commitment and reduce debt accumulation, thereby producing beneficial effects on the equilibrium outcome.
7 Concluding Remarks

We introduced a political process determining fiscal policy when voters are time inconsistent. Several messages arise from our analysis. First, absent distortions, as long as debt limits are low enough, the availability of illiquid assets makes debt irrelevant for ultimate consumption levels since agents can adjust foreseen debt income by an appropriate ex-ante allocation of liquid and illiquid assets. In particular, there is a Ricardian equivalence of sort. When debt limits are high, agents’ ability to commit is impaired. That is, electorally accountable politicians ultimately choose policies that interfere with individuals’ ex-ante desire to commit. When debt is distortionary some of these effects are accentuated since debt entails an effective loss of wealth. In fact, we show that there can be a substantial loss in welfare relative to the case in a world without any ability to commit and without debt. The paper highlights the importance of analyzing the political process when contemplating enlarging the menus of policies directed at enhancing the welfare of ‘behavioral’ electorates. Indeed, when focusing on time inconsistency, the underlying message of our paper is that governments may not be very effective in satisfying the demand for commitment.

In assessing the relevance of these results, it is important to note that agents’ behavior does not require particularly complicated behavior. Agents need to forecast the time path of transfers and debts to understand their expected consumption and tax liabilities so that they can optimize their savings and portfolio behavior. Equilibrium imposes restrictions on this path of debts but the agents do not need a particularly sophisticated understanding of the economy. In particular, no more knowledge is required than in a standard neoclassical model.

8 Appendix

Proof of Lemma 1

Suppose by way of contradiction that \( c^*_2 > c^U_2 \). Since \( c^*_2 = c^*_3 \) and \( c^U_2 > c^U_3 \), this implies that \( c^*_3 > c^U_3 \). Thus, by the resource constraint, \( c^*_3 < c^U_3 \). Denote by \( e_2 = e_1 - c^U_1 \). For any wealth \( e_2 \) left at period 2, denote by \( c^U_2(e_2) \) and \( c^U_3(e_2) \) the uncommitted solution. Then, optimization requires that:

\[
 u' \left( c^U_1 \right) = \beta \left[ u' \left( c^U_2(e_2) \right) \frac{\partial c^U_2(e_2)}{\partial e_2} + u' \left( c^U_3(e_2) \right) \frac{\partial c^U_3(e_2)}{\partial e_2} \right].
\]

Since \( c^U_3(e_2) = e_2 - c^U_2(e_2) \), we then have:

\[
 u' \left( c^U_1 \right) = \beta \left[ u' \left( c^U_2(e_2) \right) \frac{\partial c^U_2(e_2)}{\partial e_2} + u' \left( c^U_3(e_2) \right) \left( 1 - \frac{\partial c^U_2(e_2)}{\partial e_2} \right) \right].
\]
From the constraint of period 2 self,

\[ u'(c_2^U (e_2)) = \beta u'(e_2 - c_2^U (e_2)). \]

Using the Implicit Function Theorem,

\[ \frac{\partial c_2^U (e_2)}{\partial e_2} = \frac{\beta u''(e_2 - c_2^U (e_2))}{u''(c_2^U (e_2)) + \beta u''(e_2 - c_2^U (e_2))} < 1. \]

Since \( c_2^U (e_2) \geq c_3^U (e_2) \), we get that \( u'(c_1^U) > \beta u'(c_2^U) \). Optimization with commitment implies that \( u'(c_1^*) = \beta u'(c_2^*) \). But these conditions cannot hold simultaneously if \( c_1^* < c_1^U \) and \( c_2^* > c_2^U \).

\[ \square \]

**Proof of Proposition 3**

Consider the following maximization problem:

\[
\begin{align*}
\max & \quad u(c_1) + \beta [u(c_2) + u(c_3)] \\
\text{s.t.} & \quad u'(c_2) = \beta (1 + \eta) u'(c_3) \\
& \quad c_1 + c_2 + c_3 = k - \eta c_2.
\end{align*}
\]

(10)

This is an artificial problem corresponding to an agent who chooses the debt level and her consumption plan in tandem but consuming \( c_2 \) destroys resources just as debt does. In particular, this problem generates a higher overall utility (from period 1’s perspective) than that experienced by an agent who consumes \( c_1^*(d^{**}), c_2^*(d^{**}), c_3^*(d^{**}) \) because such an agent takes the equilibrium level of debt as given and cannot alter it unilaterally. The latter generates the equilibrium level of welfare for distortions \( \eta \). Furthermore, the two coincide when \( \eta = 0 \).

We now show that the maximized objective of problem (10) is decreasing in \( \eta \). Indeed, suppose \( \eta_1 > \eta_2 \). Denote the solution of (10) for distortions \( \eta_1 \) by \( (c_1, c_2, c_3) \). We now approximate a policy under distortions \( \eta_2 \) small enough that it satisfies the constraints and generates a strictly higher value for the objective.

For \( \eta_2 \) close enough to \( \eta_1 \), there exists \( \varepsilon > 0, \varepsilon < c_3 \) such that

\[ u'(c_2) = \beta (1 + \eta_2) u'(c_3 - \varepsilon). \]

Therefore,

\[ u'(c_2) = \beta (1 + \eta_2) [u'(c_3) - \varepsilon u''(c_3) + O(\varepsilon^2)]. \]

Since \((c_1, c_2, c_3)\) is a solution to the problem with distortions \( \eta_1 \), \( u'(c_2) = \beta (1 + \eta_1) u'(c_3) \). It follows that:
Consider then the policy \((c_1 + \varepsilon + (\eta_1 - \eta_2) c_2, c_2, c_3 - \varepsilon)\) when the distortions are \(\eta_2\). Notice that, by construction, this policy satisfies the two constraints in problem (10). The difference between the generated objective and the maximal value of the objective under distortions \(\eta_1\) is then:

\[
\Delta = [u(c_1 + \varepsilon + (\eta_1 - \eta_2) c_2) - u(c_1)] + \beta [u(c_3 - \varepsilon) - u(c_3)].
\]

Using a first order approximation,

\[
\Delta = (\varepsilon + (\eta_1 - \eta_2) c_2) u'(c_1) - \beta \varepsilon u'(c_3) = \\
= (\eta_1 - \eta_2) c_2 u'(c_1) + \frac{(\eta_2 - \eta_1) u'(c_2) u'(c_1)}{\beta (1 + \eta_2) u''(c_3)} - \frac{(\eta_2 - \eta_1) u'(c_2) u'(c_3)}{(1 + \eta_2) u''(c_3)} + O(\varepsilon^2)
\]

Notice that the solution to problem (10) with distortions \(\eta_1\) must satisfy \(u'(c_1) = \beta [u'(c_2) + u'(c_3)]\)

and so:

\[
\Delta = \frac{(\eta_1 - \eta_2)}{(1 + \eta_2)} u'(c_2) \left[ \frac{u'(c_1) c_2}{u'(c_2)} - \frac{u'(c_2)}{u''(c_3)} \right] + O(\varepsilon^2),
\]

which from concavity of the instantaneous utility \(u\), is positive whenever \(\eta_1\) and \(\eta_2\) are close enough. In particular, the optimal solution for problem (10) with distortions \(\eta_2\) must generate a strictly higher level of the objective function than the solution with distortions \(\eta_1\). It follows that welfare in our distortion economy is lower under any \(\eta > 0\) relative to the case of \(\eta = 0\).

Last, notice that when \(\beta (1 + \eta) < 1\), all agents achieve their commitment solution absent debt, an consequently the maximal period 1 utility under the budget constraint. From Proposition 2, this is no longer the case when \(\beta (1 + \eta) > 1\) and so period 1 utility is lower for distortions exceeding \(1 - \beta\).

**Proof of Proposition 6**

Recall that this proposition covers the case in which for all \(d\), \(c^*_d(\beta; d)\) and \(c^{U}_d(\beta; d)\) are increasing in \(\beta\).

Assume first that \(\bar{d} \leq d^*\). Suppose equilibrium debt is \(d < \bar{d}\). Monotonicity implies that \(c^*_d(\beta; d) \geq c^*_d(\beta^*; d) \geq d\) for all \(\beta \geq \beta^*\). Furthermore, by definition of \(d^*\) and continuity of \(c^*_d(\beta; d)\), for sufficiently small \(\varepsilon > 0\), \(c^*_d(\beta; d) \geq d\), for all \(\beta \geq \beta^* - \varepsilon\). It follows that all agents with preference parameter \(\beta \in [\beta^* - \varepsilon, 1 - \eta]\) best respond by investing in illiquid assets leaving them with period 2 wealth of \(c^*_2(\beta; d) - d\). However, in period 2, these agents would vote for a
slightly higher debt level. There would therefore be a strict majority in favor of higher debt. In particular, the only candidate for equilibrium debt is $d$. When debt is expected to be $d$, for sufficiently small $\varepsilon > 0$, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond in period 1 by investing in illiquid assets so that $\max\{0, c_2^U(\beta; d) - d\}$ is left for the period 2 self. Thus a majority of agents would vote against any reduction of debt in period 2. It follows that $\overline{d}$ constitutes the equilibrium debt level and the commitment consumption stream is implemented for the agent with preferences $\beta^*$.

Consider now the case $d^* < \overline{d} \leq d^{**}$. As above, for any $d < d^*$, when voters use best responses, there would be a strict majority support for an increase in debt in period 2. Suppose, then, that in equilibrium the debt is $d$, $d^* \leq d < \overline{d}$. For all $\beta \geq \beta^*$, monotonicity implies that $c_2^U(\beta; d) \geq c_2^U(\beta^*; d) \geq d$. Furthermore, by definition of $d^{**}$ and continuity of $c_2^U(\beta; d)$, for sufficiently small $\varepsilon > 0$, $c_2^U(\beta; d) \geq d$, for all $\beta \geq \beta^* - \varepsilon$. Therefore, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond by investing in illiquid assets so that $\max\{0, c_2^U(\beta; d) - d\}$ is left for the period 2 self. In particular, all agents with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$, constituting a strict majority, would support a higher debt. It follows that $\overline{d}$ is the only candidate for equilibrium debt. Repeating the argument for the case of $\overline{d} \leq d^*$, a majority would oppose reduction in debt, showing that equilibrium debt is given by $\overline{d}$, coinciding with period 2 consumption for $\beta^*$.

Finally, suppose that $\overline{d} > d^{**}$. As above, for any $d < d^{**}$, a strict majority support for an increase in debt in period 2. Assume now that $d > d^{**}$. As above, from monotonicity and continuity of $c_2^U(\beta; d)$, and the definition of $d^{**}$, there exists $\varepsilon > 0$ such that for all $\beta \leq \beta^* + \varepsilon$, $c_2^U(\beta; d) < d$. Since distortions imply that, in period 2, agents never desire a debt level that exceeds their intended consumption, it follows that all agents with preference parameters $[0, \beta^* + \varepsilon]$, a strict majority, desire a lower debt level.

References


