Abstract. Firms often specify break-up fees in their employment contracts where a worker must compensate the firm if he leaves to take up employment with a competitor. We highlight the role of such break-up fees in the presence of asymmetric information about the worker’s quality between his current employer and the outside labor market. Waldman (1984) argues that if the market attempts to learn the worker’s quality from the firm’s job assignment (or “promotion”) decision, it raises the wage of a promoted worker and leads to inefficiently few promotions. We argue that break-up fees can mitigate such inefficiencies by shielding the firm from the potential raiders. But in the presence of firm-specific matching, break-up fees thwart efficiency in turnover by muting the market’s incentive to bid for the worker. We characterize the optimal contract and show that the optimality of a break-up fee depends on the relative size of the worker’s expected productivity in the pre- and post-promotion jobs. It is optimal to specify a break-up fee if and only if the difference between the worker’s expected productivity levels in the two jobs is sufficiently large. Moreover, when the worker’s productivity critically depends on the firm-specific human capital, the use of break-up fee is less likely to be optimal.

1. Introduction

Firms often specify a break-up fee in their employment contracts in an attempt to dissuade their workers from leaving for the competing employers. Such break-up fees, also known as “golden handcuffs,” are a contractual obligation for the employee to pay back a compensation, or “damage fee”, to the firm should the employee choose to leave and join a competing firm in the industry.

A typical example of such break-up fees is deferred compensations that is commonly offered through retirement benefits and stock options. Often, such compensation is paid out at a pre-specified future date conditional on the continuing employment relationship between the firm and the worker. If the worker voluntarily leaves the employment relationship, he may forfeit his claim on a part of his compensation. For example, the employee’s retirement plan may not be vested or he may not be able to execute his stock options until he completes a certain length of tenure with the firm. Indeed, any back-loaded compensation plan where the employee forfeits her claim to a portion of her compensation should she decide to quit sooner than later can be conceived as a contract with break-up fees.

Several authors have highlighted the role of such back-loaded compensation in “locking in” the employees. Mehran and Yermack (1999) offer empirical evidence that stock options reduce CEO turnover (also see, Jackson and Lazear, 1991, and Scholes, 1991). Deferred compensation through pension plans has been found to be particularly effective in reducing turnover (Allen et al., 1993).
“Training contracts”—where a employee must reimburse her cost of training to the firm should she decide to leave—have also been found to reduce worker turnover (Manchester, 2009; Hoffman and Burks, 2013).

This paper seeks to highlight and analyze a novel trade-off associated with such break-up fees when there is asymmetric learning between the initial employer and the outside labor market about the workers’ productivity. We argue that in such an environment the use of employment contracts with break-up fees improves the efficiency in job assignment (or promotion) but hinders the efficiency in turnover. This trade-off emanates from the interplay of the following two economic effects.

First, when the worker’s productivity is gradually revealed, the initial employer is likely to be more informed (compared to the outside labor market) about its worker’s quality. A typical channel through which the outside labor market attempts to infer the worker’s quality is by observing the firm’s job assignment, or “promotion” decisions (Waldman, 1984). Promotions are more visible publicly than the actual quality of the worker, and the workers with higher quality are more likely to be promoted. Hence, the outside labor market may take promotion as a signal of high quality of a worker and make him an appropriately high wage offer in an attempt to raid him. Waldman (1984) argues that this effect makes promotion more expensive for the firm since the firm must increase the wage of the promoted worker accordingly in order to retain him. Consequently, too few workers are promoted compared to what is socially efficient.

A contract with break-up fee can alleviate this inefficiency by specifying an amount that the worker must pay back to the firm if he decides to leave once he is promoted. The break-up fee offsets the firm’s need to pay a steep wage to retain the promoted worker—the worker may continue to stay with his initial employer as his outside wage offer net of break-up fee may be dominated by his current wage offer. As a result, for the firm, the “cost” of promotion decreases and the firm gets a stronger incentive to promote a worker.

Second, if the productivity of a worker is governed by firm-specific matching, a break-up fee has its own cost. A high break-up fee may discourage an outside firm from bidding for the worker unless the matching gains from turnover are sufficiently large. Thus, contracts with break-up fees may lead to too few turnovers leading to a worker-firm matching inefficiency. Such matching inefficiencies, in turn, hurt the firm’s profit since the firm could extract the matching gains up-front from the worker.

Thus, a contract with break-up fee improves the efficiency in job assignment but reduces the efficiency in turnover, and the optimal contract balances this trade-off.

To capture this trade-off, we consider a simple two-period principal-agent model where the firm (principal) has two types of job, 1 and 2. In period one, the firm hires an agent with unknown ability level \(a\) and assigns him to job 1. Let the productivity of the worker in job 1 be \(v_1\). The initial contract specifies a wage and a break-up fee \(d\) payable to the firm should the worker decide to leave. In period two, the actual ability level of the worker is revealed to the firm, and the firm decides whether to promote the worker and assign him to job 2. In job 2, a worker with ability \(a\) produces \(v_2a\). The workers with higher level of ability are more productive in job 2 compared to job 1. Once the promotion decision is made, it is publicly observed and raiding firms—where the worker might be better matched—compete in wages to hire the worker. The initial employer can make a counteroffer upon observing the raiders’ offers. The worker chooses the employer who offers the highest wage net of break-up fee (if any such fee is stipulated in the initial contract).

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1Garmaise (2011) finds that non-compete clauses, too, help to reduce the turnover of top executives. Such a contract—where the worker the contractually prohibited to quit and join a rival firm within a specified time span—can be interpreted as an employment contract with a steep break-up fee. The worker can potentially make a buyout offer in order to be able to absolve herself from any legal binding while switching employers.

2DeVaro and Waldman (2011) offers some empirical support to this argument. Also see Baker et. al (1994a, 1994b) and McCue (1996) for empirical evidence that promotion is often associated with large wage increases.
Consider the role of break-up fee in the light of the above framework. Such a fee would create a wedge between what the market offers a promoted worker and what the firm needs to pay to retain him. Consequently, promotion becomes less expensive (for the firm) and the firm would have a higher incentive to promote a worker. Thus, the worker-job matching inefficiency (as highlighted by Waldman, 1984) is reduced.

But on the other hand, it affects the efficiency in turnover. This happens for two reasons: First, the raiding firms now correctly expect the ability of the promoted workers to be lower than before as the firm has lowered its threshold for promotion. As a result, the market reduces its bid and it becomes more likely that the firm would find it profitable to match such an offer. Therefore, the worker may stay back with the firm even when he is more productive with the raiders. Second, if the break-up fee is sufficiently high, the raiders may be foreclosed from the labor market. The raiders need to compensate the worker for the steep break-up fee and may find it unprofitable to do so unless the matching gains are sufficiently large. As a result, they may refrain from bidding altogether even when they are a better match for the worker.\(^3\) The trade-off between the efficiencies in job assignment and turnover shapes the optimal contract.

Our key finding is that the optimality of a break-up fee depends on the relative size of the worker’s expected productivity in the two jobs. More specifically, given \(\psi_2\), there exists a threshold \(\psi_1\), say \(\psi_1^*\), such that it is optimal to stipulate a break-up fee if and only if \(\psi_1 \geq \psi_1^*\). Moreover, in this case the inclusion of a break-up fee also increases the aggregate social surplus.

The intuition behind this finding is as follows: when \(\psi_1\) is small, the firm already has a strong incentive to promote most of the workers since they are much more productive in job 2 than in job 1. The workers who are inefficiently kept in job 1 are of low productivity and would have had little gains in productivity had they been assigned to job 2. Thus, in such a setting, the marginal gains from more efficient promotion that is brought about by stipulating a break-up fee is relatively small. However, such a break-up fee would hinder the efficient turnover of all promoted workers. And as most of the workers are promoted (all of whom should leave for the raiders when there are matching gains), the marginal loss due to inefficient turnover is relatively large. Thus, it is optimal not to stipulate such a fee. But when \(\psi_1\) is high, the firm would promote very few workers, only those with sufficiently high ability. Also, the marginal worker who misses the promotion would have been considerably more productive if he were promoted. Thus, the marginal gain from improved worker-job matching is high whereas the marginal loss from reduced turnover is low. Therefore, it becomes optimal to stipulate a break-up fee as it eases the inefficiency in promotion but costs little in terms of matching inefficiency it creates.

Finally, our findings also shed light on how the importance of firm-specific human capital influences the optimality of break-up fees. Note that the more critical is the role of firm-specific human capital in job 2, the less likely it is that the worker would be a better match with the outside labor market. We parameterize the distribution of the firm-specific matching gains and show that the more likely it is that the worker would be a better match with the firm, the less likely it is that the use of break-up fee would be optimal. The intuition behind this finding is simple: when firm-specific human capital is critical for production, the worker is less likely to be a better match outside, and hence, the threat of raids decreases. Consequently, promotion becomes less costly and worker-job matching efficiency improves. Thus, the role of break-up fee becomes less critical.

This observation offers a novel justification for compensation packages with “graduated vesting” where the vested part of the compensation gradually increases with the tenure of the worker. One may argue that the longer a worker stays with the firm the more firm-specific human capital she acquires. Thus, it becomes increasingly unlikely that the worker might be a better match with the outside labor market, and consequently, the use of break-up fee gradually becomes irrelevant.

\(^3\)This effect is similar in spirit with the role of long-term contracts in bilateral trading as discussed in Aghion and Bolton (1987).
Related literature: As discussed above, any deferred or “back loaded” compensation plan can be conceived as a contract with break-up fee (as the employee typically loses part of the compensation should he decide to quit). And it is has been long established that back loaded compensations play a significant role in various key aspects of an employment relationship, such as, human capital accumulation (Becker, 1964), effort incentive throughout the employment tenure (Lazear, 1979), and worker retention (Salop and Salop, 1976).

The key contribution of our paper is to highlight a novel trade-off between worker-job and worker-firm matching that may emanate from the use of such break-up fees. The environment where this trade-off appears has two salient features, both of which are well acknowledged in the existing literature: (i) Asymmetric information among employers leads to inefficient turnover (Greenwald, 1986; Lazear, 1986; Gibbons and Katz, 1991; also see Gibbons and Waldman, 1999, for a survey). (ii) The initial employer’s (publicly observable) decisions—e.g., promotions, outcome of a rank-order tournament, etc.,—may signal the outside labor market about a worker’s quality (Waldman, 1984, 1990; Bernhardt and Scoune, 1993; Zábojník and Bernhardt, 2001; Golan, 2005; Mukherjee, 2010, 2008a, 2008b; Ghosh and Waldman, 2010; Koch and Peyrache, 2011).

In the current literature on asymmetric information and learning in labor markets, our paper is most closely linked to Waldman (1984) (as discussed earlier). In a framework similar to Waldman (1984), Bernhardt and Scoones (1993) considers a more general model of promotion and turnover in the presence of firm-specific matching gains. The authors assume that the raiders can eliminate all information asymmetries if they invest in a costly information acquisition process. They argue that in order to dissuade the raiders from investing in information acquisition (as it increases the risk of losing the worker), the firm may promote the worker with a preemptively high wage. The wage signals a potentially good match between the worker and the firm and discourages the raiders to acquire information (as they anticipate a lower likelihood of successful raid). The assumption that the outside market can acquire the exact same information that the initial employer possesses is crucial for this finding. In our model such direct information acquisition is not feasible and the initial employer always enjoys some degree of information advantage. In many settings this is perhaps a more realistic assumption as the worker’s productivity is often a “soft” information that can only be learned through close observation of the worker performance over a considerable duration of time.

Another article that is closely related with our work is that of Burguet et al. (2002). Burguet et al. examine the role of the break-up clause when the firms compete for talented workers. They find that in the presence of complete information, the firms set high break-up fees to restrain the workers’ mobility in order to extract the maximum rent from a more efficient rival. Similar to the role of damage payments for breach of contract (see, Aghion and Bolton, 1987; Spier and Whinston, 1995), exclusive rights help the worker-firm coalition to capture a larger share of the surplus gained from efficient turnover. Burguet et al. study the link between the level of transparency about the worker’s ability and the use of exclusive employment contracts as a rent extraction mechanism. In contrast, we consider an environment where the firm’s decision on its job assignment reveals information to the market about the worker’s ability, and we focus on the interplay of two contrasting roles of a break-up fee: shielding a productive worker from the raiders and rent extraction from the outside labor market when there is turnover.

It is interesting to note that our main finding on the optimality of the break-up fees in employment contracts is somewhat contrary to the role of such fees in the product market. In the product market context, an influential article by Aghion and Bolton (1987) and the literature on exclusive contracts that followed (see, for example, Bernheim and Whinston, 1998; Rasmussen et al., 1991) argue that break-up fees are generally inefficient as they may foreclose the market for a more efficient entrant. While this effect is present in our model (as break-up fees reduce turnover efficiency), we also
highlight a countervailing effect. In our case, the worker-job matching efficiency is also important and we argue that the use of break-up fees can increase such efficiency.4

The rest of the paper is organized as follows. Section 2 elaborates on the baseline model that captures the key trade-off between the efficiencies in worker-job and worker-firm allocations. In light of this model, Section 3 explores the role of a break-up fee in a firm’s equilibrium job assignment policy. Section 4 elaborates on the inefficiencies in worker-job and worker-firm allocations that emerge in equilibrium and how they relate with one another. The optimal break-up fee is discussed in Section 5. Section 6 discusses some robustness issues related to our key findings. A final section draws a conclusion. All proofs are given in the Appendix.

2. The Model

We consider a two-period principal-agent model that formalizes the environment discussed in the introduction. The model is described below in terms of its five key components: players, technology, contracts and job assignment, raids and counteroffer, and payoffs.

Players. A firm (or “principal”), \( F \), hires a worker (or “agent”), \( A \), at the beginning of period one. The worker works for the firm in the first period of his life, but he may get raided in period two by the outside labor market where two identical raiding firms, \( R_1 \) and \( R_2 \), bid competitively for the worker.

Technology. The technology specification of the firm is similar in spirit to that in Waldman (1984). The firm (\( F \)) has two types of jobs: job 1 and job 2. Job 1 is the entry level job where the worker (\( A \)) is assigned in period one. The worker’s productivity in job 1 is assumed to be fixed at \( \psi_1 (> 0) \). However, in job 2, the worker’s productivity depends on his ability, or “type”, \( a \in [0, 1] \). The productivity of \( A \) in job 2 (with \( F \)) is solely driven by his ability \( a \) where he produces \( \psi_2 a \). For algebraic simplicity we will assume that \( \psi_2 \geq 2\psi_1 \). At the beginning of period one, the worker’s ability is unknown to all players (including the firm, the raider and the worker himself) but is known to follow a uniform distribution on \([0, 1]\).

Job 1 is not available with the raiding firms, but they can employ the worker in job 2. However, the worker’s productivity with the outside labor market depends not only on his ability but also on the firm-specific matching factor, \( m \), where he produces \( \psi_2 a (1 + m) \). The matching factor \( m \) is unknown to all players at the beginning of the game and it is assumed to be distributed on \([-1, 1]\) according to a piece-wise uniform probability distribution function \( f(m) \) where

\[
  f(m) = \begin{cases} 
    \alpha & \text{if } m \leq 0 \\ 
    1 - \alpha & \text{if } m > 0 
  \end{cases}
\]

\[4\]It is also worth mentioning that the exclusive employment contracts have been studied extensively both by the legal scholars (Bishara, 2006; Gilson, 1999; Posner et. al, 2004; Rubin and Shedd, 1981) and by the labor economists (Burguet, et al., 2002; Franco and Mitchell, 2005; Kräkel and Sliwka, 2009). This literature is also closely related to the exclusive contracts literature in antitrust (see Posner, 1976; Aghion and Bolton, 1987; Bernheim and Whinston, 1998; Rasmusen et al., 1991) and has mostly focused on the role of such contracts in fostering investments in human capital and its implications on labor mobility.

\[5\]For expositional clarity, we are ruling out the possibility that the firm assigns its new hire directly to job 2. One may justify such a specification by assuming that job 1 offers some on-the-job training that is essential to perform in job 2. Also, we abstract away from the role of the worker’s effort in the production process as the moral hazard issues are not the central focus of our article.
and \( \alpha \in [1/2, 1) \).\(^6\)

Note that \( m \leq 0 \) implies that the worker is a better match with his initial employer than with the outside labor market, and the above specification implies that this event occurs with probability \( \alpha \). The parameter \( \alpha \) can be interpreted as the measure for the importance of firm-specific human capital in job 2. The more critical is the role of the firm-specific human capital in job 2, the less likely it is that the worker would be a better match with the outside labor market. At the extremes, if \( \alpha = 1 \) the worker is always more efficient with his initial employer indicating the scenario where firm-specific human capital is essential for performing job 2. In contrast, when \( \alpha = 1/2 \), a priori, the worker is as likely to be a better match with his initial employer as with the outside labor market, indicating that firm-specific human capital is completely irrelevant for the workers’ productivity in job 2.

The value of \( m \) is revealed in period two and we will elaborate on this shortly.

**Contracts and Job Assignment.** At the beginning of period one, \( F \) makes a take-it-or-leave-it offer \((w_1, d)\) to \( A \) where \( w_1 \) is the period-one wage and \( d \) is a break-up fee that \( A \) has to pay to \( F \) if \( A \) decides to leave for the raiders in period two. The offer is assumed to be publicly observable. Note that one can interpret \( d \) as a deferred compensation. Assuming no time discounting, the above contract specification is equivalent to the scenario where \( A \) receives \( w_1 - d \) upon accepting the employment and the firm is contractually bound to pay the worker at least \( d \) (as total compensation) in period two if the worker stays with the firm (i.e., the firm cannot set a negative period-two wage to offset the payment of \( d \)).

At the end of period one, the ability of the worker is observed by \( F \) (but not by the raiders) and \( F \) decides whether to assign (or “promote”) the worker to job 2.\(^7\) We assume that both the initial contract \((w_1, d)\) at the beginning of period one and the subsequent job assignment at the end of period one are publicly observed. Period-two wages are set by the spot market at the beginning of the period through an offer-counteroffer game as described below.\(^8\)

**Raiders and Counteroffer.** At the beginning of period two, the raiding firms \((R_1 \text{ and } R_2)\) observe \( F \)’s job assignment decision. For expositional clarity, we assume that it is never optimal for the raiders to bid for a worker who is not promoted.\(^9\) After the promotion decision is made, the (identical) raiders observe the matching factor \( m \) for a promoted worker and make simultaneous wage bids \( b_i \), \( i = 1, 2 \). We will maintain the convention that \( b_i = 0 \) when the raiders refrain from bidding. Observing the bids, \( F \) makes a counteroffer; let \( w_2^+ \) be the period-two wage that \( F \) offers

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\(^6\)We assume that job 1 is not available with the raider only to simplify the subsequent analysis. In fact, our analysis stays qualitatively the same if we assume that in period 2, the raiders also employ a worker in job 1 provided that in job 1 the worker is more productive with the firm than with the raiders due to firm-specific capital accumulation.

\(^7\)Note that this specification implies that there is always a vacancy in job 2. One can also consider a more general setting where a vacancy in job 2 arises with probability \( p \in (0, 1) \) and the job assignment is made only if there is an opening. The qualitative nature of our findings continue to hold in this general setting.

\(^8\)Here, we are implicitly assuming that long-term contracts are not feasible in the sense that the firms cannot commit to period-two wages at the beginning of period one. The infeasibility of long-term contracts and the spot market wage setting in period two are common assumptions in this literature (see, for example, Zabojnik and Bernhardt, 2001; DeVaro and Waldman, 2009) and we will revisit the role of long-term contracts later in Section 6. Note that we are also abstracting away from the possibility that the firm can announce an “initial” period-two wage to a promoted worker before the raiders make their offers. Such a wage, even if it may get revised in the offer-counteroffer stage, may serve as an additional signal of the worker’s underlying ability (see, Bernhardt and Scoones (1993) for a model on such signalling role of wage offers).

\(^9\)One can motivate this assumption as the equilibrium behavior of the raiders under a slight variation of the aforementioned technology: suppose that there exists \( \varepsilon > 0 \) sufficiently small such that a worker with ability \( a \in [0, \varepsilon] \) is only productive in job 1 and produces \(-K\) if assigned to job 2. Now, for \( K \) sufficiently large, it is never optimal for the raiders to hire a worker who remains in job 1.
to A in job i, i = 1, 2. The worker chooses the employer who offers the highest wage net of the break-up fee. In case of a tie, the worker stays with the initial employer.

A couple of remarks are in order. First, the assumption that m is revealed to the raiders after the promotion decision is made only as a modeling convenience. For the purpose of our analysis, a key assumption is that m is not known to the firm when it makes the promotion decision. This is a natural assumption in many environments where the initial employer may not have a complete information on the productivity of his worker in a competing firm (this information may be revealed only after the promoted worker generates offers from the potential raiders). One may also argue that the uncertainty about the match factor reflects uncertainty about job vacancy in a given firm. The firm may know the match factor for a given raider but whether the raiding firm has a job vacancy at a given point of time is private information to the raider. Only if the raider attempts to hire the worker, the information about the job opening becomes public.

Second, it can be argued that in the absence of any firm-specific matching gains, the possibility of counteroffer can remedy the inefficiencies in job assignment (see, e.g., Golan, 2005). However, as we will show below, if the matching gains can be positive (i.e., m > 0), the job assignment remains inefficient even when counteroffers are allowed (if break-up fees are not used). So, by allowing counteroffer we present a stronger result that shows the efficacy of the break-up fees even in an environment where they may appear to be redundant.

Payoffs. We assume that all players are risk neutral. Upon successfully hiring the worker, the firm’s payoff in period one is π1 = ψ1 − w1. But in period two, the payoff depends on the ability of the worker, whether the worker is promoted, and if promoted, whether the worker is retained. So, the firm’s payoff in period two from a worker with ability a is:

$$\pi_2(a) = \begin{cases} 
\psi_1 - w_2^{1} & \text{if A is not promoted} \\
\psi_2 a - w_2^{2} & \text{if A is promoted and retained by F} \\
d & \text{if A is promoted but subsequently raided}
\end{cases}$$

The firm’s aggregate payoff from hiring a worker with ability a is II = π1 + π2(a). Similarly, the worker’s payoff in period one is u1 = w1 but the period-two payoff, u2, depends on the promotion decision of the firm and the offer/counteroffer received upon promotion. That is,

$$u_2 = \begin{cases} 
\max \left\{ b_1 - d, b_2 - d, w_2^{2} \right\} & \text{if A is promoted} \\
w_2^{1} & \text{otherwise}
\end{cases}$$

and the worker’s aggregate payoff is U = u1 + u2. Finally, the raider’s payoff from a worker with ability a is:

$$\pi_{R_i} = \begin{cases} 
\psi_2 a (1 + m) - b_i & \text{if } R_i \text{ successfully raids the worker} \\
0 & \text{otherwise}
\end{cases}$$

We assume that both the worker and the firm have a reservation payoff of 0.

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10Our findings are robust to alternative modeling specifications as long as this key assumption is maintained. For example, one may assume that the raider knows m from the beginning of the game (the raiders may own certain complementary inputs that make the worker more productive) and it is revealed to the firm only through the raiders’ bids for a promoted worker.

11Golan’s argument, however, depends on the assumption that the worker’s productivity in job 1 is independent on his ability. If this assumption is relaxed, the inefficiency reappears; see DeVaro and Waldman (2011) and Waldman and Zac (2013) for details.
TIME LINE. The timing of the game is as follows.

- **Period 1.0.** $F$ publicly offers a contract $(w_1, d)$ to $A$. If accepted, the game proceeds but ends otherwise.
- **End of Period 1.** Period-one output is realized. Firm observes ability and decides on job assignment.
- **Period 2.0.** $R_1$ and $R_2$ observe job assignment, and matching factor $m$. $R_1$ and $R_2$ make simultaneous bids for the worker and offer $b_1$ and $b_2$ respectively.
- **Period 2.1.** After observing $b_1$ and $b_2$, $F$ decides whether to make a counteroffer and the period-two wages $w_2^1$ and $w_2^2$ are set.
- **Period 2.2.** $A$ chooses which employment contract to accept.
- **End of Period 2.** Period-two output is realized, wages are paid and the game ends.

**Strategies and equilibrium concept:** The strategy of $F$, $\sigma_F$, has three components: (i) at the beginning of period one, choose the initial contract offer $(w_1, d)$; (ii) at the end of period one, decide on job assignment $j \in \{\text{job 1, job 2}\}$ upon observing the worker’s ability, and (iii) at the beginning of period two, decide on the counteroffer $(w_2^i, i = 1, 2)$ upon observing the raiders’ offers. The worker’s strategy, $\sigma_A$, has two components: (i) accept or reject the firm’s initial contract, and (ii) choose period-two employer given the raiders’ offer and the firm’s counteroffer. Finally, the raiders’ strategy, $\sigma_{R_i}$, is to choose a wage bid $b_i$ given the matching factor and the firm’s job assignment decision.

We use perfect Bayesian Equilibrium (PBE) as a solution concept. Given the initial contract $(w_1, d)$ and the subsequent job assignment $j \in \{\text{job 1, job 2}\}$, let $\mu(a \mid (w_1, d), j)$ be the posterior belief of the raiders. We say that a profile of strategies $\sigma^* = \langle \sigma_F^*, \sigma_A^*, \sigma_{R_1}^*, \sigma_{R_2}^* \rangle$ along with the raiders’ belief $\mu^*$ constitute a PBE if (i) $\sigma^*$ is sequentially rational given $\mu^*$, (ii) on-equilibrium path $\mu^*$ is obtained through Bayes rule given the prior density of ability and the strategies of the firm and the worker, and (iii) off-equilibrium path $\mu^*$ satisfies the following restriction. If the firm deviates in period one and offers an initial contract $(w_1', d')$, the posterior belief of the raiders $\mu^*(a \mid (w_1', d'), j)$ must also be obtained through Bayes rule as defined as follows: Given an initial contract $(w_1', d') \in \mathbb{R}^2$ and the worker’s type $a \in [0, 1]$, denote $\sigma^*_F : \mathbb{R}^2 \times [0, 1] \to \{\text{job 1, job 2}\}$ as the component of the firm’s strategy $\sigma^*_F$ that defines the firm’s job assignment decision. We require

$$
\mu^*(a \mid (w_1', d'), j) = \frac{\Pr \big( j \mid a, (w_1', d'), \sigma^*_F \big) \Pr(a)}{\Pr \big( j \mid (w_1', d'), \sigma^*_F \big)}.
$$

Also, the worker’s belief on his ability remains unaffected by the firm’s initial offer.

In this context, it is important to note the following remarks: First, our restriction on the off-equilibrium belief invokes the “no signaling what you don’t know” condition imposed in Fudenberg and Tirole (2000) in their definition of PBE. As the initial contract $(w_1, d)$ is chosen by the firm before the worker’s type is revealed, it is natural to assume that the choice of contract cannot signal type, and hence, does not directly affect the raiders’ belief. That is, both both on- and off-equilibrium, the raiders’ belief on the worker’s ability is unaffected by the initial contract offer $(w_1, d)$.

Second, in the spirit of Fudenberg and Tirole’s condition that Bayes rule should be used to update belief “whenever possible,” we require that in every continuation game following any initial contract offer of $F$ the raiders update their beliefs using Bayes rule given their (common) prior belief and $F$’s job assignment behavior (under his strategy given the initial contract $(w_1, d)$). Thus, the initial contract affects belief only indirectly to the extent it influences the subsequent job assignment behavior of the firm. Note that, even if the initial contract has no information content, the raiders’ would have different beliefs on- and off-equilibrium as the firm’s subsequent behavior
would depend on its initial offer. Indeed, as we will show below, the continuation games following the initial contracts \((w_1, d)\) with different values of the break-up fee will be characterized by different promotion policies by the firm and different beliefs by the raiders.

Finally, our refinement implies that the equilibrium strategy profile and the belief induce a PBE in every continuation game following an initial offer \((w_1, d)\). Therefore, the optimal break-up fee is simply the one that induces the highest PBE payoff in the continuation game. In what follows, we analyze the optimal contracting problem accordingly.

3. Equilibrium Job Assignment Policy of the Firm

In order to derive the optimal contract for the firm, we first need to characterize the equilibrium in the continuation game for a given initial contract \((w_1, d)\). In this vain, we discuss below the firm’s equilibrium job assignment policy and analyze the offer-counteroffer subgame for an arbitrary value of \(d\) specified by the firm in period one.\(^{12}\) Our analysis also elucidates on the key trade-off between the efficiencies in the worker-job and the worker-firm matching.

But before we present the equilibrium analysis, it is instructive to consider the first best allocation of the worker as an efficiency benchmark.

3.1. First best allocation of the worker. The first-best allocation of the worker requires efficiency in both worker-job and worker-firm matching. Ex-post, when there is no uncertainty about ability and matching gains, the first-best allocation is straightforward. When the worker is a better match for the firm (i.e., \(m < 0\)), the worker stays with the firm and is promoted if and only if he is more productive in job 2 than in job 1, i.e., if and only if \(\psi_2 a \geq \psi_1\) or \(a \geq \psi_1/\psi_2\). In contrast, when the worker is a better match with the raiders (i.e., \(m > 0\)), the worker is promoted and leaves for the raiders if and only if \(\psi_2 (1 + m) a \geq \psi_1\) or \(a \geq \psi_1/\psi_2 (1 + m)\). Otherwise, the worker stays with the firm in job 1.

However, as the firm makes its job assignment decision before observing the matching gains, one may consider the ex-ante efficient job allocation as a benchmark for evaluating the extent of allocative inefficiency in equilibrium. The ex-ante efficient promotion rule is the one that maximizes total production (i.e., aggregate surplus), assuming that turnover is efficient (following promotion).

Note that as the worker’s productivity in job 2 (i.e., \(\psi_2 a\)) is increasing in \(a\) while it is constant \((\psi_1)\) in job 1, the optimal promotion decision must follow a cut-off rule. Consider any arbitrary cut-off rule where a worker is assigned to job 2 if and only if his ability \(a \geq x\). Assuming efficient turnover following promotion, the ex-ante aggregate expected surplus under such a policy is:

\[
S(x) := \psi_1 x + \int_x^1 \psi_2 a \left[ \alpha \int_{-1}^0 dm + (1 - \alpha) \int_0^1 (1 + m) dm \right]da.
\]

The ex-ante efficient (or “first best”) promotion policy, \(a^{FB}\) (say), is the one that maximizes \(S\). That is,

\[
S'(a^{FB}) = 0, \text{ or } a^{FB} = \frac{2\psi_1}{(3 - \alpha) \psi_2}.
\]

Note that under the ex-ante efficient policy more workers are promoted to job 2 than what the firm would promote in the absence of any raiders (in that case, only the workers with ability \(a \geq \psi_1/\psi_2 > a^{FB}\) would be promoted). All workers with ability \(a \in [a^{FB}, \psi_1/\psi_2]\) are more productive in job 1 than in job 2 when working for their initial employer, but should be assigned to job 2 under ex-ante efficient promotion rule due to the potential matching gains from turnover.

\(^{12}\)Observe that the wage in period one, \(w_1\), has no direct impact on \(F’\)’s decision to promote the worker or raider’s decisions in period two. Thus, it can be ignore in the analysis of this section.
(recall that promotion to job 2 is necessary to realize the matching gains as a worker in job 1 is never raided).

We now consider the equilibrium job assignment and turnover and explore how the extent of inefficiency is affected by the break-up fee.

3.2. Equilibrium job assignment and turnover. Recall that the worker's productivity in job 2 is increasing in his ability and that in job 1 is constant. Now, as the worker's wage in period two is determined in the spot market, and the outside market does not observe the actual ability level of the worker, a worker's wage conditional on job assignment is independent of his ability. So, the firm's payoff from offering promotion is increasing in $a$ while denying promotion yields a constant payoff. Thus, as in the case of first-best allocation rule, the firm's promotion decision also follows a cut-off rule in equilibrium where the firm promotes a worker if and only if his ability is greater than a cut-off value $a^*$. 

In what follows, we consider the continuation game that follows from an arbitrary initial offer $(w_i, d)$ and solve the PBE of this game where the raiders' belief is updated through Bayes rule as define in (1). In particular, we are interested in solving for the equilibrium cut-off value $a^*$ as a function of the break-up fee $(d)$.

First, note that in period-two, if there is no market offer (i.e., $b_i = 0$ for all $i$) the firm offers a wage of 0. In other words, a worker who stays in job 1 as well as a promoted worker who does not receive any market offer earns $w_1^2 = w_2^2 = 0$ as the firm simply matches the worker's outside option.

Now consider the case where a promoted worker receives a market offer. In this case the firm's optimal counteroffer decision needs a more careful study. Let $b$ denote the highest bid that the worker receives (i.e., $b = \max\{b_1, b_2\}$). Throughout this article we refer to $b$ as the market bid for the promoted worker. If $b \leq d$ then the worker's outside option of 0 is better than his payoff from paying the break-up fee and joining the raider. Thus, the firm retains the worker by matching his outside option and offers $w_2^2 = 0$. But if $b > d$, the firm has two options. The firm can either retain the worker by making a counteroffer $w_2^2 = b - d$ and earn a profit of $\psi_2 a - (b - d)$, or it can let the worker go and earn $d$. So, the firm will make a counteroffer if and only if $\psi_2 a - (b - d) \geq d$, or, equivalently, $b \leq \psi_2 a$.

Note that the break-up fee reduces the retention wage of a promoted worker. Furthermore, if the market bids $b \leq d$ or $b \leq \psi_2 a^*$, it fails to raid the worker irrespective of his ability. But if $b > \max\{\psi_2 a^*, d\}$, the market successfully raids some types of the worker. More specifically, if $b < \max\{\psi_2, d\}$, the market raids all workers with ability $[a^*, \psi_2]$. That is, among the pool of promoted workers, only the relatively low ability workers leave the firm. But, if the market bids even higher, i.e., $b > \max\{\psi_2, d\}$, it raids all the workers who are promoted. So, to sum up, when the firm uses the promotion cut-off $a^*$, it retains every promoted worker with ability $a \geq \tilde{a}(b)$ where

$$\tilde{a}(b) = \begin{cases} 
  a^* & \text{if } b \leq \max\{\psi_2 a^*, d\} \\
  b/\psi_2 & \text{if } \max\{\psi_2 a^*, d\} < b \leq \psi_2 \\
  1 & \text{if } b > \max\{d, \psi_2\}
\end{cases}$$

Note that the function $\tilde{a}(b)$ captures the firm's optimal policy of worker retention.

Given the firm's optimal counteroffer decision, matching factor $m$, and the break-up fee $d$, raider $i$'s expected profit from bidding $b$ for a worker conditional on the worker's choice of period-two employer is $\tilde{\pi}_R(b, m, d; a^*) - b \Pr (a \in [a^*, \tilde{a}(b)])$, where

13For brevity of exposition, in what follows, we will refer to different “types” (or ability levels) of a worker simply as different “workers.”
When the break-up fee is too high then the market cannot pro…tably raid the worker. And if dominated strategies also do not survive the “market-Nash” re…nement of Waldman (1984). the raider is strictly better o¤ by not placing a bid that is higher than its valuation for the worker. Such equilibria “trembling hand perfect” — if there is a small probability that the worker may mistakenly accept the raiders’ bid, then the raider is strictly better o¤ by not placing a bid that is higher than its valuation for the worker. Such equilibria in dominated strategies also do not survive the “market-Nash” refinement of Waldman (1984).

\[
\hat{\pi}_R(b, m, d; a^*) := \mathbb{E}_a[\psi_2 a(1 + m) \mid a \in [a^*, \hat{a}(b)]]
\]

\[
= \begin{cases} 
0 & \text{if } b \leq \max\{\psi_2 a^*, d\} \\
\psi_2(\frac{a^*}{2} + \frac{b}{2\psi_2})(1 + m) & \text{if } \max\{\psi_2 a^*, d\} < b \leq \psi_2 \\
\psi_2(\frac{a^*+1}{2})(1 + m) & \text{if } b > \max\{d, \psi_2\}
\end{cases}
\]

Since the raiders compete in wages, they make zero expected profit in equilibrium and bid the entire expected value of the worker. That is, the raiders’ equilibrium wage bids must be \(b_1 = b_2 = b^*\) where \(b^*\) solves the equation \(\hat{\pi}_R(b, m, d; a^*) = \hat{a}(b)\) — \(b = 0\). The equilibrium bid \(b^*\) critically depends on the value of the break-up fee: if \(d < \psi_2\), then

\[
b^*(m, d; a^*) = \begin{cases} 
0 & \text{if } m \leq 0 \text{ or } a^* \leq \frac{d}{\psi_2 \left(\frac{1-m}{1+m}\right)} \\
\psi_2 a^* \frac{1+m}{1-m} & \text{if } m > 0 \text{ and } \frac{d}{\psi_2 \left(\frac{1-m}{1+m}\right)} < a^* \leq \frac{1-m}{1+m} \\
\psi_2(\frac{a^*+1}{2})(1 + m) & \text{otherwise}
\end{cases}
\]

and if \(d \geq \psi_2\), then

\[
b^*(m, d; a^*) = \begin{cases} 
0 & \text{if } m \leq 0 \text{ or } a^* \leq \frac{2d}{\psi_2(1+m)} - 1 \\
\psi_2(\frac{a^*+1}{2})(1 + m) & \text{otherwise}
\end{cases}
\]

Note that when the break-up fee is too high or the promotion cut-off is too low the raiders refrain from bidding even when the worker is a better match for them. The argument is straightforward. When the break-up fee is too high then the market cannot pro…tably raid the worker. And if \(a^*\) is too small, then promotion is not quite informative about the worker’s ability. So the market does not place any bid as it correctly anticipates attracting only a pool of suf…ciently low ability workers. But as \(a^*\) increases, promotion becomes a stronger signal of quality and the market finds it worthwhile to bid for the promoted workers.\(^\text{14}\)

Given the market’s bidding strategy, one can plug \(b^*\) in the cut-off function \(\hat{a}(b)\) and derive the firm’s retention threshold as follows: if \(d < \psi_2\), then

\[
\hat{a}(b^*(m, d; a^*)) = \begin{cases} 
a^* & \text{if } m \leq 0 \text{ or } a^* \leq \frac{d}{\psi_2 \left(\frac{1-m}{1+m}\right)} \\
a^* \frac{1+m}{1-m} & \text{if } m > 0 \text{ and } \frac{d}{\psi_2 \left(\frac{1-m}{1+m}\right)} < a^* \leq \frac{1-m}{1+m} \\
1 & \text{otherwise}
\end{cases}
\]

and if \(d \geq \psi_2\), then

\[
\hat{a}(b^*(m, d; a^*)) = \begin{cases} 
a^* & \text{if } m \leq 0 \text{ or } a^* \leq \frac{2d}{\psi_2(1+m)} - 1 \\
1 & \text{otherwise}
\end{cases}
\]

When \(b^* > d\), the firm retains a (promoted) worker if his ability \(a \geq \hat{a}(b^*(m, d; a^*))\) by matching the raiders’ bid net of the break-up fee (i.e., offers \(w_2^2 = b^* - d\)) but lets him leave otherwise (i.e., offers \(w_2^2 = 0\)). So, in any equilibrium, if a worker in job 2 receives a market offer \(b^* > d\), the offer matching policy of the firm is given as follows:

\(^\text{14}\)It is worth noting that the above characterization of the equilibrium bidding strategies implicitly assumes that the raiders do not play weakly dominated strategies. Otherwise, there may exist other equilibria where the raiders bid more than the expected value of the worker (to the raiders) if the firm is expected to retain the worker with certainty by making a counteroffer (this can happen if \(m < 0\)). One may rule out such equilibria as they are not “trembling hand perfect”—if there is a small probability that the worker may mistakenly accept the raiders’ bid, then the raider is strictly better o¤ by not placing a bid that is higher than its valuation for the worker. Such equilibria in dominated strategies also do not survive the “market-Nash” refinement of Waldman (1984).
\[
   w_2^2 = \begin{cases} 
   b^* - d & \text{if } a \geq \hat{a}(b^*(m, d; a^*)) \\
   0 & \text{otherwise}
   \end{cases}
\]

Now, we can also derive the firm’s profit from promoting the “marginal” worker, i.e., the worker with ability \(a^*\). This profit, \(\pi_p\) (say), depends on whether the firm will retain the worker or not, and, in case of retention, the wage it has to pay to the worker. From the analysis above, we obtain the following: if \(d < \psi_2\), then

\[
   \pi_p(a^*, m, d) = \begin{cases} 
   \psi_2 a^* & \text{if } m \leq 0 \text{ or } a^* \leq \frac{d}{\psi_2} \frac{1-m}{1+m}, \\
   d & \text{otherwise}
   \end{cases}
\]

and if \(d \geq \psi_2\), then

\[
   \pi_p(a^*, m, d) = \begin{cases} 
   \psi_2 a^* & \text{if } m \leq 0 \text{ or } a^* \leq \frac{2d}{\psi_2(1+m)} - 1, \\
   d & \text{otherwise}
   \end{cases}
\]

As we have argued before, when \(m \leq 0\) or if \(a^*\) is sufficiently small relative to \(d\), the raiders do not bid for the promoted workers. So, the firm retains every worker it promotes, including the marginal worker, and pays zero wage. In all other cases, the firm either lets all workers go or retains only the more able workers (among the promoted ones). Therefore, the marginal worker is never retained and the firm makes \(d\) on him.

As the productivity of the worker in job 1 is constant at \(\psi_1\), the cut-off ability level for promotion, \(a^*\), must solve \(\mathbb{E}_m \pi_p(a^*, m, d) = \psi_1\). The following proposition characterizes this solution.

**Proposition 1.** When the employment contract includes a break-up fee \((d)\), there exists a unique cut-off level \(a^* (d)\) such that the firm promotes a worker if and only if his ability \(a > a^* (d)\). Moreover, the cut-off \(a^* (d)\) decreases in \(d\) when \(d < \psi_2\), increases in \(d\) when \(\psi_2 < d < \psi_1 + \psi_2\), and is independent of \(d\) when \(d > \psi_2 + \psi_1\).

Proposition 1 indicates how the equilibrium promotion rule changes with the break-up fee specified in the contract: unless the specified break-up fee is significantly large, an increase in the break-up fee always induces the firm to promote more workers—for \(d < \psi_2\), the cut-off of ability, \(a^* (d)\), (above which the firm promotes the worker) is decreasing in \(d\). However, if the break-up fee is sufficiently large \((\psi_2 \leq d < \psi_2 + \psi_1)\), an increase in the fee may restrict promotion, and, at the extreme \((d > \psi_2 + \psi_1)\), break-up fee does not have any impact on the promotion rate (see Figure 1).\(^{15}\)

To see the intuition behind the equilibrium promotion policy, note that increasing \(d\) has two effects on the firm’s payoff from promoting a worker: an increase in \(d\) increases the compensation that the firm gets in case a promoted worker is raided, but it also increases the probability of retaining the promoted worker. The first effect always increases the firm’s expected profit from promotion. The second effect may increase or decrease expected profit depending on the attractiveness of retaining the worker relative to losing him to a raider. When \(d\) is not too large, retaining the worker is more attractive than losing him (since the break-up fee earned due to turnover is moderate). Hence, as \(d\) increases, both effects increase the firm’s profit from promoting a worker. This implies that the firm’s incentive to promote workers also increases with \(d\) (i.e., \(a^*\) decreases in \(d\)).

In contrast, when \(d\) is large, the break-up fee is sufficiently lucrative and letting the worker leave is more profitable than retaining the worker. In this case, the second effect lowers the firm’s

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\(^{15}\)The closed form expression of \(a^* (d)\), is given in the proof of Proposition 1.
profit (it restricts turnover, and hence, the firm fails to collect the break-up fee) and may dominate the first effect. When that happens, the firm’s profit from promoting a worker decreases with \( d \), meaning that its incentives to promote a worker also decreases in \( d \) (i.e., \( a^* \) increases in \( d \)). Finally, for \( d \) significantly large (i.e., \( d > \psi_1 + \psi_2 \)) neither effect is relevant, as the market never attempts to raid the promoted workers. In this case the break-up fee has no effect on the firm’s profit from promotion and, as a consequence, the incentives to promote the worker remain unchanged with \( d \).

\[
\begin{align*}
\text{Figure 1. The optimal cut-off for promotion as a function of the break-up fee } (d) \\
\end{align*}
\]

We conclude this section with the following important observations. First, note that in absence of any break-up fee, job assignment remains inefficient—too few workers would be promoted to job 2 compared to the first-best level (i.e., \( a^*(0) = \psi_1/\alpha \psi_2 > a^{FB} \)). This inefficiency is similar in spirit to that discussed in Waldman (1984) and we will further elaborate on this in the next section. It is, however, worth noting here that we obtain such inefficient job assignment in spite of allowing for counteroffers by the firm. This observation is in contrast to Golan (2005) who argue that in the Waldman’s setting, if the initial employer is allowed make counteroffers, the resulting winner’s curse problem sufficiently depresses the raiders’ bid and ensures efficient job assignment. Our result indicates that even if counteroffers are allowed in the Waldman’s model, job assignment remains inefficient in the presence of firm-specific matching gains. In the same vein, DeVaro and Waldman (2011) and Waldman and Zax (2013) argue that in Waldman’s model, counteroffer fails to restore efficiency when the worker’s ability affects his productivity not only in job 2 but also in job 1. Collectively, these observations indicate that the inefficiency in job assignment is indeed a robust implication of the signaling role of job assignment in the presence of asymmetric information about workers’ type.

Second, it is interesting to note the impact of firm-specific human capital on the firm’s promotion policy. One can verify that for a given \( d \), the cutoff \( a^*(d) \) decreases in \( \alpha \) if \( d < \psi_1 \), increases in \( \alpha \) if \( \psi_1 < d < \psi_1 + \psi_2 \), and is independent of \( \alpha \) if \( d \geq \psi_1 + \psi_2 \) (see proof of Proposition 1). So, \( d \) and \( \alpha \) have similar qualitative impact on the promotion cut-off as both factors protect the firm from potential raiders.

Third, it is also important to note that we can combine the optimal promotion policy of the firm and the bidding strategy of the raiders to further characterize the raiders’ bidding behavior as a function of the break-up fee. In particular, we can ask: given a value of the break-up fee, when is it
optimal for the raiders to bid for the worker? The following lemma suggests that when $d$ is large, raiders may refrain from bidding even when there are positive matching gains.

**Lemma 1.** If $d \leq \psi_1$, the raiders bid for the promoted worker whenever $m > 0$. Otherwise, the raiders may refrain from bidding even if matching gains are positive. In particular, if $\psi_1 < d < \psi_1 + \psi_2$, the raiders bid for the promoted worker whenever $m > \tilde{m}(d) \in (0, 1)$ where $\tilde{m}$ is an increasing function of $d$. Finally, if $d \geq \psi_1 + \psi_2$, the raider never bids for the worker irrespective of the value of $m$.

Lemma 1 indicates that the use of break-up fees may lead to a “foreclosure” effect; i.e., the raiders may not be able to compete to hire the worker even when the worker would be more productive with the raiding firms. The foreclosure effect emanates from the fact that when the break-up fee is sufficiently high, the winner’s curse effect becomes too severe and the raiders refrain from bidding unless the matching gains are also sufficiently large. At the extreme, when the break-up fees are significantly large, the raiders never bid for the worker.

Finally, it is worth mentioning that from an empirical perspective, the case where $d > \psi_1$ is somewhat less relevant as such a contract would imply that the break-up fee exceeds the (per period) value of the worker in the firm. It is a rather uncommon practice to use such a contract for executive compensation and if the firm must use a very steep break-up fee, it is perhaps more common to use a contract with a non-compete clause. Nevertheless, in what follows, we analyze the optimal contract for the entire parameter space. While the findings for some parameter settings are empirically more relevant than the others, our results provide a complete characterization of the equilibrium and highlights the precise conditions under which the key economic effects emerge in equilibrium.

Having characterized the promotion rule for a given break-up fee, we can now address the question of the optimal break-up fee. But before we do so it is instructive to discuss the allocative inefficiencies arising from a given promotion policy. The optimal break-up fee is simply the one that induces a promotion policy that minimizes these inefficiencies.

### 4. The Nature of Allocative Inefficiencies

In this section we elaborate on the nature of allocative inefficiencies that arise with an arbitrary promotion policy (given the offer-counteroffer game that follows the promotion decision). We do so with the help of Figure 2 below that plots the range of matching gains ($m$) and the worker’s ability ($a$). The following discussion elucidates on the key economic effects that shape the firm’s optimal contract and facilitates the characterization of the optimal break-up fee.

Consider an arbitrary promotion policy where the firm assigns a worker to job 2 if and only if his ability $a \geq a_0 > \psi_1/\psi_2$ (see panel (i)). There are three potential sources of inefficiencies: first, for $m < 0$, there is a worker-job matching inefficiency—all workers with $a \in [\psi_1/\psi_2, a_0]$ should have been assigned to job 2 but were kept in job 1 instead (shown by area $A$). Second, when $m > 0$, for the the set of workers with $a \in [\psi_1/\psi_2 (1 + m), a_0]$ there is both worker-firm and worker-job inefficiencies (shown by area $B$)—it is socially efficient for all of these workers to work in job 2 at the raiding firm but they remain in job 1 at the initial employer. Finally, even among the promoted workers there is a set of workers who are inefficiently matched with their initial employer. Given the raiders’ bidding strategy, the raiders make a bid for all workers in job 2 when $m > 0$ but

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16However, an important exception is professional sports where for some of the key athletes break-up fees often far exceed their annual compensations. For example, in European soccer leagues, the break-up fees (usually known as “transfer fees”) have escalated significantly over the last two decades, and so much so that the European Commission has recently called for an overhaul of such fee structure to maintain the competitiveness of the leagues (see “Football transfers need overhaul to keep game competitive, says EC study,” by Jamie Jackson, *The Guardian*, February 7, 2013).
the firm matches the offer if the worker’s ability is sufficiently high, i.e., if \( a \geq \hat{a} (b^*(m, d; a_0)) \), which is equivalent to \( m \leq (a - a_0) / (a + a_0) \) (using equations (6) and (7)). Thus, among the promoted workers there is an inefficient worker-firm matching when \( m \leq (a - a_0) / (a + a_0) \) (shown by area \( C \)). Note that this effect is similar to the “winner’s curse” problem in common value auctions—the raiding firms lower their bids as a successful raid may carry a negative signal about the worker’s ability, namely, the initial employer did not find the worker productive enough to warrant a matching wage offer. Thus, the initial employer finds it profitable to match the raiders’ offer even though the worker would have been more productive at the raiding firm.

![Panel (i)](image1)

![Panel (ii)](image2)

**Figure 2.** The allocative inefficiencies associated with a given promotion policy

Accounting for these three sources of inefficiencies, the aggregate expected surplus (in period-two) under the above promotion policy can be written as:

\[
\hat{S}(a_0) := \psi_1 \Pr[\text{no promotion}] + \mathbb{E}[\psi_2 a \mid \text{promotion, no turnover}] \Pr[\text{promotion, no turnover}] + \mathbb{E}[\psi_2 (1 + m) a \mid \text{promotion, turnover}] \Pr[\text{promotion, turnover}]
\]

\[
= \psi_1 a_0 + \int_{a_0}^1 \psi_2 a \left[ \alpha \int_{-1}^{0} dm + (1 - \alpha) \int_{0}^{a - a_0 / (a + a_0)} dm \right] da
\]

\[
+ \int_{a_0}^1 \psi_2 a \left[ (1 - \alpha) \int_{a - a_0 / (a + a_0)}^{1} (1 + m) dm \right] da.
\]
The optimal promotion policy *given* the information asymmetry in the offer-counteroffer game is the one that maximizes $\tilde{S}$.

Now, to see the marginal effects of promotion threshold ($a_0$) on the expected surplus, suppose that the promotion threshold is lowered from $a_0$ to $a_1$ (see panel (ii)). This change leads to a more efficient worker-job and worker-firm matching (areas $A'$ and $B'$). But the improved worker-job matching comes at a cost of worse worker-firm matching that results from an aggravated winner’s curse problem. As the ability threshold for promotion lowers, the expected productivity of the promoted worker decreases. And so does the equilibrium bid. Thus, the firm will retain a higher share of the workers: now a worker of ability $a$ is successfully raided only if $m > (a - a_1) / (a + a_1) > (a - a_0) / (a + a_0)$ (the increased worker-firm matching inefficiency is shown by area $C'$).

The promotion policy that maximizes $\tilde{S}$ must balance the trade-off between improved worker-job matching and worsened worker-firm matching. Let us denote the policy that maximizes $\tilde{S}$ as the “second best” promotion policy, or $a^{SB}$ (in contrast with the “first best” policy discussed earlier in Section 3.1 where turnover following job assignment is always assumed to be efficient).

The allocative inefficiencies discussed above illustrate the costs and benefits of using break-up fees. Note that in absence of any break-up fee, even the second-best promotion policy may not be attained as $a^* (0) \neq a^{SB}$. Also note that as the raiders make zero profit due to competition and the firm can ensure zero rents for the worker by sufficiently lowering his first-period wage, the firm appropriates the entire surplus that is generated by the coalition of the firm, worker and the raiders. Consequently, the problem of choosing the optimal break-up fee can be conceived as the problem of choosing $d$ such that equilibrium promotion rule $a^* (d)$ coincides with the second-best optimal policy, $a^{SB}$.

However, as highlighted in Lemma 1, if a sufficiently high break-up fee is needed to implement the second-best promotion, it may create an additional source of inefficiency through market foreclosure: the raiders bid for the worker only if the realized matching gain is sufficiently large (i.e., if $m \geq \tilde{m} (d) > 0$). Thus, if the firm needs to specify a $d$ that is larger than $\psi_1$ in order to implement the second-best promotion policy, implementing such a policy is no longer optimal. In this case the firm must also account for the loss of surplus due to market foreclosure, and the promotion policy associated with the optimal contract falls short of even the second-best level.

In what follows, we elaborate on the optimal break-up fee in the light of the above discussion. It turns out that the optimality of such fees critically hinges on the worker’s relative productivity in jobs 1 and 2.

5. Characterization of the Optimal Break-up Fee

Consider the firm’s optimal contracting problem at the beginning of period one (when the worker’s ability is unknown to all parties). Recall that the firm’s payoff in period one is simply $\pi_1 = \psi_1 - w_1$ (the worker is assigned in job 1 at a wage of $w_1$). However, the derivation of the firm’s expected payoff in period two, $E \pi_2$, is a bit more involved. Similar to the aggregate social surplus $\tilde{S}$ (equation (10)), $E \pi_2$ also depends on likelihood of promotion and turnover. But the expression for $E \pi_2$ differs from that of $\tilde{S}$ for two reasons: (i) if there is turnover (that is, there is a market offer and the firm decides not to make a counteroffer), the firm earns only the break-up fee ($d$) and (ii) if there is no turnover, the firm’s payoff depends on whether there is market offer or not. If there is no market offer, the firm makes $\psi_2 a$ on the worker (since wage stays at 0). But if there is market offer then the firm pays $b - d$ and earns a profit of $\psi_2 a - (b - d)$. So, drawing parallel to equation (10), one obtains:

\[
E \pi_2 = \psi_1 \times \text{Pr}[\text{no promotion}] + E[\psi_2 a \mid \text{promotion, no offer}] \times \text{Pr}[\text{promotion, no offer}] \\
+ d \times \text{Pr}[\text{promotion, offer, no counteroffer}] \\
+ E[\psi_2 a - (b - d) \mid \text{promotion, offer, counteroffer}] \times \text{Pr}[\text{promotion, offer, counteroffer}].
\] (11)
Now, in equilibrium, the raiders’ bid \( b^* (m, d; a^*) \) is given by equations (4) and (5). Moreover, the raiders’ bidding function in conjunction with the firm’s equilibrium job assignment policy (i.e., the cut-off from promotion \( a^* = a^* (d) \) as discussed in Proposition 1) and the offer-matching policy (i.e., the retention cut-off \( \hat{a} = \hat{a} (b^* (m, d; a^*)) \) as given in equations (6) and (7)) determines the (joint) probability of a worker being promoted, receiving a wage offer from the raiders, and receiving a counteroffer from the firm. Thus, the firm’s optimal contracting problem boils down to maximizing its aggregate expected profit \( \pi_1 + \mathbb{E} \pi_2 \) by choosing period-one wage \( w_1 \) and the break-up fee \( d \) subject to the raiders’ bidding function, the firm’s job assignment and counteroffer policies, and the worker’s individual rationality constraint \( IR \). That is, the firm solves:

\[
\max_{w_1, d} \quad \Pi = \pi_1 + \mathbb{E} \pi_2
\]

subject to bidding strategy \( b^* (\cdot) \), retention policy \( \hat{a} (\cdot) \), promotion policy \( a^* (d) \), and

\[
w_1 + \mathbb{E} [b - d | \text{promotion, offer}] \times \Pr [\text{promotion, offer}] \geq 0. \tag{IR}
\]

Because the worker’s \( (IR) \) constraint always binds in equilibrium (else the firm can lower \( w_1 \) and increase its profit), one can plug the \( (IR) \) constraint in the firm’s objective function to eliminate \( w_1 \). Let the resulting profit function be \( \hat{\Pi} (d) \). Hence, the firm’s optimal contracting problem \( \mathcal{P} \) boils down to an unconstrained maximization problem of finding \( d \) that maximizes \( \hat{\Pi} (d) \). The following lemma offers a useful characterization the function \( \hat{\Pi} \).

**Lemma 2.** The firm’s expected profit function \( \hat{\Pi} \) is continuous in \( d \) and given by the following functional form:

\[
\hat{\Pi} (d) = \left\{ \begin{array}{ll}
\psi_1 + \hat{S} (a^* (d)) & \text{if } 0 \leq d < \psi_1 \\
\psi_1 + \hat{S} (a^* (d)) - \phi (d) & \text{if } \psi_1 \leq d < \psi_1 + \psi_2 \\
(\psi_1 + \psi_2)^2 / 2\psi_2 & \text{otherwise}
\end{array} \right.
\]

where \( \phi \) is a positive valued function of \( d \).

(Recall that \( \hat{S} (a^*) \) is the aggregate expected surplus (in period-two) given the promotion cut-off \( a^* \), as given in equation (10).)

Lemma 2 suggests that the firm’s profit as a function of the break-up fee \( d \) has the following characteristics: when \( d \) is small (i.e., \( d < \psi_1 \)), the effect of the break-up fee on the firm’s profit can be completely characterized by the break-up fee’s impact on the equilibrium promotion rule, \( a^* (d) \). In this case, break-up fee affects the firm’s profit only by accentuating the winners’ curse effect. In contrast, for \( d \) sufficiently large (\( d > \psi_1 + \psi_2 \)), the break-up fee has no impact on the profit since the market is completely foreclosed and there is no turnover. But for all intermediate values of \( d \), the market is partially foreclosed and the firm’s profit reflects both the winner’s curse effect and the market foreclosure effect. The latter effect is captured by the function \( \phi (d) \). Also note that in absence of market foreclosure, the firm’s profit is simply equal to the aggregate expected surplus generated across the two periods by the coalition of the firm, worker and the raiders, given the firm’s promotion policy \( \psi_1 \) in period one and \( \hat{S} (a^*) \) in period two.

Given the characterization of the firm’s profit function, the first question we ask is the following: under what circumstances is it optimal for the firm to specify a break-up fee? The proposition below addresses this question.
Proposition 2. Given $\alpha$ and $\psi_2$, there exists a value $\psi_1 (> 0)$ such that the firm’s optimal contract specifies a strictly positive break-up fee if and only if $\psi_1 > \psi_1$. Moreover, under this condition the use of a break-up fee is socially optimal as it increases the aggregate surplus.

The above proposition implies a clear testable prediction that the optimality of a break-up fee is driven by the relative productivity of the worker in the two jobs: it is never optimal to use the break-up fee in the employment contract if the worker’s productivity in job 1 (i.e., $\psi_1$) is too low compared to his expected productivity in job 2 (as reflected by $\psi_2$). Otherwise, it is always optimal to specify a break-up fee in the employment contract. This finding can also be interpreted as one that links the underlying production technologies of the two jobs with the nature of the employment contract: break-up fees are more likely to be used if the production technologies in the pre- and post-promotion jobs are considerably similar (e.g., they involve similar sets of tasks, and hence, expected productivity difference of the worker in the two jobs is not too large).

The intuition behind this finding is as follows. As discussed above, the use of a break-up fee improves worker-job matching but worsens worker-firm matching. When $\psi_1$ is small, the marginal gain from the former effect is lower than the marginal loss from the latter effect. To see this, note that for low $\psi_1$, the equilibrium promotion rule $a^*$ is also low even in the absence of any break-up fee—as the worker is hardly productive in job 1, the firm has a strong incentive to promote a worker provided that his ability is not too low. Indeed, one can verify that $a^*(0) = \psi_1 / \alpha \psi_2$, which is increasing in $\psi_1$ and decreasing in $\psi_2$ (see the proof of Proposition 1). As most workers are promoted (when $\psi_1$ is small), the marginal worker who “misses” promotion has a relatively low ability and assigning him to job 2 (as efficiency in worker-job allocation dictates) has only a small impact on productivity. Thus, while the introduction of a break-up fee does improve worker-job matching, its impact on aggregate surplus is small. In contrast, its impact resulting from a less efficient worker-firm matching is still large. As almost all workers are promoted, the introduction of a break-up fee reduces the likelihood of turnover for many workers. Hence, when $\psi_1$ is small, the marginal positive effect from a break-up fee (in terms of efficient promotion) is more than offset by the marginal negative effect (in terms of reduced turnover) and it is optimal not to use such a fee in the employment contract.

But when $\psi_1$ is high, the opposite happens—the marginal gain from worker-job matching dominates the marginal loss from inefficient worker-firm matching. When $\psi_1$ is large, very few workers are promoted in equilibrium if a break-up fee is not used (recall that $a^*(0)$ is increasing in $\psi_1$). Thus, the marginal worker who misses promotion has high ability and the gain in productivity from (efficiently) promoting him is relatively large. In contrast, the loss for reduced turnover from introducing a break-up fee is minimal. This is due to the fact that very few workers are promoted in the first place and those are the only workers whose turnover is affected by the existence of a break-up in the labor contract. Hence, when $\psi_1$ is large, the firm can increase its profit by stipulating a break-up fee that ensures more efficient promotion.

Now, consider the optimality of using break-up fees from the social welfare perspective. Since the firm extracts the entire surplus generated by the worker, if the inclusion of a break-up fee is profit-enhancing for the firm, it is also socially optimal—it increases the aggregate social surplus generated by the coalition of the firm, worker and the outside labor market.

As we have discussed in the previous section, even when it is feasible for the firm to implement $a^{SB}$, it may not always be optimal to do so. If the associated break-up fee leads to market foreclosure (i.e., if $d > \psi_1$) the optimal break-up fee must also account for this additional source of inefficiency.

\[\text{Unfortunately, empirical findings on this issue are rather scant as task variations across jobs in the organizational hierarchies may be difficult to measure. An empirical test of our prediction can potentially follow the approach suggested in DeVaro et al. (2012). In an analysis of discrimination in labor markets, they construct a measure of task variability from information on factors such as knowledge required, supervision received, guidelines, etc., that describe the nature of a given job within the organizational hierarchy of a given firm.}\]
But when do we observe market foreclosure in equilibrium? The next proposition addresses this question.

**Proposition 3.** Given \( \alpha \) and \( \psi_2 \), there exists a value \( \bar{\psi}_1 (> \psi_1) \) such that for \( \psi_1 < \bar{\psi}_1 \leq \bar{\psi}_1 \), the optimal break-up fee \( d^* \in (0, \psi_1] \), \( a^*(d^*) = a^{SB} \), and the firm’s profit \( \Pi^* = \psi_1 + \bar{S}(a^{SB}) \). But for \( \psi_1 > \bar{\psi}_1 \), \( d^* \in (\psi_1, \psi_2) \) and \( \Pi^* < \psi_1 + \bar{S}(a^{SB}) \).

Proposition 3 suggests that as long as \( \psi_1 \) is not too large (i.e., \( \psi_1 < \bar{\psi}_1 \)), the optimal break-up fee never forecloses the market (as \( d^* < \psi_1 \)). So, only the winner’s curse effect remains, and as discussed earlier, whenever feasible (i.e., when \( \psi_1 > \bar{\psi}_1 \)), the optimal \( d \) is the one that implements \( a^{SB} \). But when \( \psi_1 \) is high (> \( \bar{\psi}_1 \)) there is direct foreclosure of the market since \( d^* > \psi_1 \). In this case, the associated profit of the firm falls short of the second-best due to the additional inefficiency (i.e., market foreclosure) that the break-up fee creates.

Recall that the break-up fee affects efficiency in two margins: worker-job matching and worker-firm matching. Propositions 2 and 3 directly speak to the issue of worker-job matching and highlight how the difference in the workers’ productivity across jobs affects the optimal break-up fee. The larger is the expected gain in productivity from promotion, the less likely it is that the firm would specify a break-up fee in its employment contract.

In parallel to this observation, one can analyze the optimality of break-up fee as the efficiency in turnover, i.e., worker-firm matching, becomes more critical. Note that \( \alpha \) captures how critical the efficiency in turnover is for the maximization of the aggregate surplus. The larger is the value of \( \alpha \), the less likely it is that the worker is more productive with the raiders, and hence, the loss of surplus resulting from any inefficiency in turnover is small. The following proposition states that as \( \alpha \) increases, the firm is less likely to use break-up fees.

**Proposition 4.** The threshold \( \bar{\psi}_1 \), i.e., the value of \( \psi_1 \) below which it is never optimal to specify a break-up fee, is increasing in \( \alpha \).

To see the intuition, note that the higher is \( \alpha \) the less likely it is that the worker would be better matched with the raiders. Thus, the raiders are less likely to bid for the worker and as the threat of successful raids reduces, promotion becomes less costly. Consequently, the firm has a stronger incentive to promote the worker even in the absence of any break-up fee. As the extent of inefficiency in the worker-job matching decreases, the role of break-up fee becomes less critical. Since \( \alpha \) can be interpreted as the importance of firm-specific human capital in job 2, Proposition 4 indicates that break-up fees and the essentiality of the firm-specific human capital can be conceived as alternative mechanisms that may shield the firm from the threat of raids.

The above findings allude to the fact that in equilibrium, higher values of break-up fee may be associated with higher values of \( \psi_1 \) but lower values of \( \alpha \). While an analytical derivation of such a comparative statics result appears to be algebraically intractable, Figure 3 presents a numerical solution of the optimal break-up fee as a function of \( \psi_1 \) and \( \alpha \). In conformity with the argument presented above, we find \( d^* \) to be increasing in \( \psi_1 \). The impact of \( \alpha \) is, however, a bit subtle. For \( d \leq \psi_1 \), \( d^* \) decreases in \( \alpha \), but it is not monotone over the parameter range where \( d^* > \psi_1 \). The ambiguity arises from the fact that when \( d^* > \psi_1 \), the market foreclosure effect emerges in equilibrium and it weakens with higher \( \alpha \) as turnover becomes less likely to be efficient at the first place. Consequently, it reduces the marginal cost of raising the break-up fee and creates a countervailing effect that tends to increase \( d^* \). But as we have mentioned previously, the parameter space where \( d^* > \psi_1 \) is empirically less relevant as employment contracts with break-up fees larger than the worker’s productivity is not very common in practice.
An important implications of the above finding is that it offers a novel justification for back-loaded compensation with “graduated vesting” where the percentage of vesting increases with the employee’s tenure within the firm.\textsuperscript{18} One may argue that the longer a worker stays with a firm, the larger is his stock of firm-specific human capital. And hence, it becomes increasingly unlikely that the worker would be a better match with the raiders. Thus, gradually, the use of break-up fee becomes less relevant.

6. Discussion and extensions

The analysis above highlights the trade-off between worker-job and worker-firm matching efficiencies that originates with the use of break-up fees in employment contracts. In this section, we explore the robustness of our key findings to a set of alternative modeling assumptions and explore some possible generalizations of our contracting environment.

6.1. Additional signals on worker’s type. In many environments with asymmetric learning about the worker’s type, job assignment is not the only signal available to the outside labor market. Note that the availability of any additional signal about worker’s quality dilutes the signaling role of job assignment. As a result, the worker’s job assignment is likely to have a smaller influence on raider’s bid easing the degree of inefficiency in the worker-job matching. For example, DeVaro and Waldman (2012) shows that there is less distortion in promotion decision for workers with higher schooling levels. As higher schooling level may already serve as publicly observed signal of higher productivity, promotion is likely to serve a smaller role as a signal of productivity. A similar issue is also highlighted by Milgrom and Oster (1987) in their “Invisibility hypothesis.” If the worker’s performance is publicly visible, it is more difficult for the initial employer to earn rent on the worker as the competitive bidding by the outside labor market would raise the worker’s wage. Thus, for workers whose performance is not publicly visible, the firm has an strong incentive to conceal their productivity from the outside labor market and promotes inefficiently few workers to position of prominence.

\textsuperscript{18}Love (2007) reports that according to Bureau of Labor Statistics’ 2000 National Compensation Survey, close to 50% of all 401(k) plans held by all private industry workers had graduated vesting.
In this context, a testable implication of our model is that the break-up fee should be smaller (or less likely to be used) the weaker is the signaling role of promotions—if there is little distortion in worker-job matching to begin with, use of break-up fee yields little marginal benefit but it still has a relatively large cost in terms of the distortion in worker-firm matching that it induces.\footnote{We are thankful to an anonymous referee for suggesting this implication.}

However, one may argue that casual empiricism often appears to run contrary to this prediction. For example, professional athletes (such as soccer players in European clubs or basketball players in the US) often have contracts with a large break-up or “transfer” fee even though their performance is clearly visible to all prospective employers. But it is critical to note that in many such settings, break-up fee not only shields the worker from potential raiders—a role that we do highlight in our model—but also works as a channel of surplus extraction from potential raiders should they succeed to raid the worker—a role that we do not consider in our model. As the worker does not face any liquidity constraint in our model, the firm can extract the rent (that the worker expects from his future employer) simply by lowering the worker’s first-period wage. In contrast, if the worker is liquidity constrained, one may not be able to disassociate the allocational role of the break-up fee from its rent-extraction role. And in such contexts, a more visible worker may indeed face a higher break-up fee (see, e.g., Burguet et al., 2002). The more information the market gets about the worker the less severe is the adverse selection problem associated with the turnover. As turnover becomes more efficient, gains from trade increases and a larger fee becomes necessary to extract the resulting surplus.

6.2. Break-up fee based on ability. While the above subsection highlights the role of additional signals that arise exogenously, one may also consider environments where such additional signals on ability arise endogenously. A potential source of such signal is the break-up fee that is tied to the ability of the worker.

In our baseline model we have assumed that the break-up fee is set at the beginning of the game. An immediate implication of this assumption is that the break-up fee is independent of the worker’s ability level. This assumption is realistic in many settings; e.g., in case of deferred compensation the vesting rule is usually specified at the beginning of the employment relationship. But it is not inconceivable that in some settings that the firm can choose the break-up fee after observing the workers’ type. That is, the firm may simultaneously decide on the promotion of the worker and on the break-up fee in case the worker is promoted. How would the optimal contract change in such a setting?

While a complete characterization of the equilibrium appears intractable, two salient observations can be made: first, in equilibrium, the break-up fee may vary with ability, and hence, the optimal contract also serves as a signal (in addition to signal implied by job-assignment) on the worker’s quality.\footnote{This aspect of the equilibrium is reminiscent of Bernhardt and Scoones (1993) where the promotion may be associated with a preemptively high wage to signal the worker’s value and dissuade potential raiders.} Second, the break-up fee is used regardless of the difference of the workers’ productivity between the two jobs. The latter observation is somewhat nontrivial and the argument is as follows.

Note that in our baseline model, the issue of allocational efficiencies and surplus extraction can be decoupled: surplus extraction is done using the period 1 wage \( w_1 \) and \( d \) is chosen so as to implement the promotion policy that maximizes the aggregate surplus. When the difference in an worker’s productivity between jobs is high, the gain in worker-job allocation from using \( d \) does not compensate the loss in worker-firm allocation and the firm optimally sets \( d = 0 \). But if \( d \) is specified along with the promotion decision at the end of period 1, the choice of \( d \) also affects surplus extraction, i.e., it protects the firms’ profit in case it decides to promote the worker. As long as there is a chance that a promoted worker would receive an offer from the raiders, it will be optimal to set a break up fee—with a break-up fee it is always cheaper to retain a worker and the firm obtains a compensation in case the worker leaves. This argument holds even if the difference...
in the worker’s productivity between jobs is large. So, in this case, \( d \) is used more as a tool to appropriate surplus than as a tool to achieve allocational efficiency. Of course, even in this case, the use of \( d \) still has the trade-off we highlight earlier: it leads to more efficient promotion but compromises turnover efficiencies. But this trade-off never precludes the use of break-up fees in the optimal contract.

6.3. On the role of competition among the raiders. Our model assumes that the outside labor market is perfectly competitive. But how would the level of competition in the outside labor market affect the optimal break-up fee? For example, if the raider is a monopsonist, how would the optimal break-up fee vary? Even though a complete analysis of this case would be somewhat unwieldy as it involves a re-derivation of all the key results discussed earlier, the answer to this question is quite intuitive: A monopsonist raider would bid less aggressively than the competitive labor market leading to a decrease in the equilibrium break-up fee.

To see the intuition, consider how the bidding behavior of the raider would change under a monopsony raider. Note that given a promotion policy \( a^* \) and break up fee \( d \), the raider’s bid \( b^{**} \) is

\[
b^{**}(m; d; a^*) = \arg \max_b \mathbb{E} [\psi_2 a (1 + m) - b \mid a \in [a^*, \hat{a}(b)]] \Pr (a \in [a^*, \hat{a}(b)]) ,
\]

where \( \hat{a}(b) \) is the retention policy of the firm as given in equation (3) above. Note that even when \( d = 0 \), the solution to the raider’s bidding problem is as follows:

\[
b^{**}(m, 0; a^*) = \begin{cases} 
0 & \text{if } m \leq 0 \\
\frac{1}{1-m} \psi_2 a^* & \text{if } m > 0 \text{ and } a^* \leq 1 - m \\
\psi_2 & \text{otherwise}
\end{cases}
\]

Now, recall the equilibrium bid by a competitive labor market as given by equation (4):

\[
b^{*}(m, 0; a^*) = \begin{cases} 
0 & \text{if } m \leq 0 \\
\psi_2 a^* \frac{1+m}{1-m} & \text{if } m > 0 \text{ and } a^* \leq \frac{1-m}{1+m} \\
\psi_2 (\frac{2}{(a^*+1)}(1 + m)) & \text{otherwise}
\end{cases}
\]

Comparing the two equations, it is straightforward to note that when break-up fee is not used, the bids as a function of \( a^* \) are lower under monopsony irrespective of the value of \( m \). As promotion becomes less costly to the firm, the firm would promote more workers for the same \( d \) and the reduced bids would also imply more inefficient retentions more frequent. These two effects decreases the marginal benefit for \( d \) but increases its marginal cost. Hence, the optimal \( d \) decreases.

6.4. Renegotiation of break-up fee. Observe that the market foreclosure effect stems from the fact that when the break-up fee is set at a sufficiently high level, the raiders need to raise their bid significantly in order to successfully hire the worker. And unless the matching gains are substantially large, it is not worthwhile for the raiders to do so. But note that if it is efficient for the worker to leave for the raiders, it would be profitable for the firm to let the worker go provided the firm can extract the matching gains generated through turnover. One way to do so is to renegotiate the initial contract if the (promoted) worker receives a better offer from the market. In what follows, we explore the role of renegotiation in our model and argue that with renegotiation break-up fees never foreclose the market.\(^{21}\)

Suppose that the firm and worker can renegotiate the amount of the break-up fee if the worker receives an external offer. All other aspects of the model are kept unchanged and we assume that

\(^{21}\)However, turnover continues to be inefficient due to the information asymmetries in the offer-counteroffer game.
the firm continues to have the entire bargaining power even at the renegotiation stage. Note that the possibility of renegotiation makes a difference in our initial analysis only in the case where \( b < d \). In our initial model, if \( b < d \), the worker necessarily stays with the firm. But with renegotiation, the firm would lower \( d \) and let the worker leave if it is optimal for the firm-worker coalition to do so. This happens whenever \( b > a \psi_1 \), i.e., the bid exceeds the worker’s value with the firm. At the renegotiation stage the firm sets \( d = b \) to extract the matching gain and the worker leaves for the raiding firm. So, irrespective of the value of \( d \), the worker stays with the firm if and only if \( \psi_2 a > b \), or \( a \geq \tilde{a}(b) := b/\psi_2 \). Note that with renegotiation, the market’s bid need not exceed the break-up fee for the raid to be successful. Whenever the market offers \( b > a \psi_2 \), it will successfully raid workers with ability \( a \in [a^*, b/\psi_2] \), \( a^* \) being the ability threshold for promotion.

Given the above observation, the subsequent derivation of the optimal contract parallels our analysis of the initial model. As before, competition ensures that the market’s equilibrium bid, \( b^* \), is equal to its expected payoff from bidding, i.e., \( b^* = \mathbb{E}[\psi_2 a(1 + m) \mid a \in [a^*, b/\psi_2]] \), or,

\[
\hat{b}(m, a^*) = \begin{cases} 
0 & \text{if } m \leq 0 \\
\psi_2 a^{*\frac{1+m}{1-m}} & \text{if } m > 0 \text{ and } a^* \leq \frac{1-m}{1+m} \\
\psi_2(1 + m)\frac{1}{2}(a^* + 1) & \text{otherwise}
\end{cases}
\]

Two issues are important to note: first, \( \hat{b} \) does not depend on \( d \) when renegotiation is allowed. Indeed, the bidding behavior is the same as in our initial model when \( d = 0 \) (see equation (13)). This finding is intuitive as the market’s bid no longer has to exceed \( d \) to raid the worker. Second, the market is never foreclosed in equilibrium—the raiders always bid for the worker as long as there are matching gains. The latter observation is an immediate implication of the former; as the bid need not have to exceed the break-up fee for a successful raid, it is always optimal for the raiders to place a bid if there are matching gains. This finding is reminiscent of the result discussed in Spier and Whinston (1995) who argue that in a model of bilateral trade with potential entrants, any break-up fee specified by the seller does not foreclose the market for a more efficient entrant if the buyer and the seller can renegotiate the break-up fee up on entry.

Now, as break-up fees never foreclose the market, our earlier discussion on the optimal break-up fee suggests that the firm chooses \( d \) such that \( a^*(d) = a^{SB} \), whenever such a value of \( d \) is feasible. And whenever such a value of \( d \) is feasible in our initial model, it is also feasible even when renegotiation is allowed. To see this, note that given \( b^* \), we can compute the firm’s profit from promoting the marginal worker (with ability \( a^* \)) as:

\[
\pi_p(a^*, m, d; a^*) = \begin{cases} 
\psi_2 a^* \min\{b^*(m, a^*), d\} & \text{if } m \leq 0 \\
\psi_2 a^* \min\{b^*(m, a^*), d\} & \text{if } m > 0 
\end{cases}
\]

Observe that \( \pi_p \) is always larger than its “no-renegotiation” counterpart.\(^{22}\) This is because with renegotiation, when \( \psi_2 a^* < \hat{b} < d \), the worker leaves the firm and the firm collects \( \hat{b} \) whereas without renegotiation, the worker stays back and the firm earns only \( \psi_2 a^* \). As the expected profit from promotion is higher with renegotiation, the firm has a stronger incentive to promote a worker, i.e., with renegotiation the equilibrium promotion threshold \( a^*(d) \) is always lower than that without renegotiation. But we have already argued that in the absence of renegotiation, it is feasible to set \( a^*(d) = a^{SB} \). Since \( a^*(0) \) is the same with or without renegotiation (trivially, renegotiation does not play any role when no break-up fee is specified) and for any \( d > 0 \), \( a^* \) is lowered when renegotiation is allowed, it must be still feasible to set \( a^*(d) = a^{SB} \).

\(^{22}\)The derivation of \( \pi_p^m \) is straightforward. If \( m < 0 \) the firm makes \( a^* \psi_2 \) on him and when \( m > 0 \), the firm makes \( \min\{b, d\} \) while the worker always leaves for the raider (if the market offers \( b > d \), the firm collects \( d \) and if \( b < d \), renegotiation implies that the firm sets \( d = b \)).
So, one may conclude that in the presence of renegotiation, a better matched raider is never foreclosed from the market. However, both worker-job and worker-firm matching continue to remain inefficient (i.e., in equilibrium $a^* (d) \neq a^{FB}$) due to the winner’s curse problem at the offer-counteroffer stage.\(^{23}\)

### 6.5. Contracts with severance payments.

The key role of the break-up fee that we highlight here is that it shields the promoted worker from the outside labor market, and, as a result, improves the worker-job matching efficiencies. But the break-up fee need not be the only contracting device that achieves this goal. The same can be achieved with, for example, severance payments. The firm may commit to make these lump-sum payments to the worker (depending on his job assignment) when the employment relation terminates in period two. However, the payments are made irrespective of whether the worker stays with the firm in period two (and leaves at the end of period) or leaves at the beginning of the period to join the raider’s firm.

Let $s_1$ and $s_2$ be the severance payments in job 1 and 2 respectively. We now rule out break-up fees in our model but keep its other aspects unchanged. Note that as the severance payments are made regardless of whether the worker stays or not, these payments do not affect the worker’s decision on whether to switch employers. So the worker’s choice of period two employer depends solely on the wage proposed by the firm in period two and the wage offer made by the raiders. The severance payments also do not affect the firm’s counteroffer.\(^{24}\) So, in order to derive the equilibrium promotion policy, we can continue to use our initial analysis and set $d = 0$.

This observation has two important implications: (i) when $d = 0$ there is no market foreclosure—the raiders always make a bid whenever there are matching gains (see equation (13)). (ii) The equilibrium promotion rule depends on the difference of the severance payments across the two jobs, $\Delta s := (s_2 - s_1)$. To see this, note that the firm’s profits associated with promoting and not promoting the marginal worker are given by $\pi_p = \alpha a^* \psi_2 - s_2$ and $\pi_{np} = \psi_1 - s_1$, respectively. And the equilibrium promotion rule solves $\pi_{np} = \pi_p$, which implies that

$$a^* = \frac{\psi_1}{\alpha \psi_2} + \frac{\Delta s}{\alpha \psi_2}.$$  

So, by choosing $\Delta s$ the firm can implement any promotion rule ($a^*$) in equilibrium. As the market foreclosure effects are absent, it is always optimal to choose $\Delta s$ such that $a^* = a^{SB}$.

In this context, it is important to note the following. First, the optimal contract with severance payment is a (weakly) more efficient than the optimal contract with break-up fees as it never forecloses the market and always guarantees the second-best. But note that similar to the case of renegotiation, both worker-job and worker-firm matching remain inefficient due to the winner’s curse problem in the offer-counteroffer stage. Second, in equilibrium $\Delta s < 0$; that is, the firm commits to a larger severance pay in job 1 compared to job 2. As a result, at the beginning of period two, the firm creates a stronger incentive for itself to promote the worker. Finally, even though the use of the severance payments appear to be more efficient than the use of break-up fees, it has its own issues. The contract with severance payment is profitable provided that the firm can ex-ante recover such payments by lowering the period-one wage of the worker. As these payments are made to all workers irrespective of their ability and job assignments, it would require

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\(^{23}\)Spier and Whinston also note that even with renegotiation, the market foreclosure effect reappears if the seller needs to make relationship specific investments and the entrant has some market power. In the context of our model, this finding suggests that if the initial employer invests in its worker for firm-specific human capital accumulation and if the raider can make take-it-or-leave-it offer, then contract renegotiation need not rule out the possibility of market foreclosure. A complete analysis of this issue is beyond the scope of this article and remains an interesting topic for future research.

\(^{24}\)Indeed, if we assume that the severance payments are paid immediately after the promotion decision, it is clear that they will not affect subsequent behavior of the firm and worker.
the firm to significantly lower the worker’s period-one wage to extract all rents. So, if the worker has liquidity constraints, such a low period-one wage may not be feasible and the optimal contract may still fall short of achieving even the second-best promotion policy.25

7. Conclusion

Break-up fee and more generally, deferred compensation, is a contracting tool that firms frequently use to retain their key employees. Such contracts dissuade potential raiders by making successful raids more expensive—the raiders need to compensate the employee for the break-up fee if they were to induce him to switch employers. This article highlights a novel trade-off associated with the use of such break-up fees: In the presence of asymmetric learning about the worker’s productivity where job assignment becomes a signal of quality (a la Waldman, 1984), the use of break-up fees improves efficiencies in worker-job matching but undermines the efficiencies in worker-firm matching.

Our key finding is that the optimality of the break-up fee depends on the relative size of the worker’s expected productivity across jobs. If there is substantial (expected) productivity gains from promotion, then it is never optimal to specify any break-up fee in the employment contract. Our analysis also suggests that the more critical is the role of firm-specific human capital in the production process, the smaller is the optimal break-up fee likely to be. Moreover, the use of break-up fee increases the aggregate social welfare by trading off gains in job assignment efficiencies with the loss from inefficient turnover.

There are several other economic effects that are interesting and relevant in our environment albeit beyond the scope of our model. One may assume that to be productive in the “post-promotion” job, it is necessary that the worker (and/or the firm) invests in human capital. How would the presence of break-up fees affect the incentives for investment? The answer to this question depends on whether the human capital is general or firm-specific and who undertakes the investments.26 It would also be interesting to consider the case where the market can screen the promoted workers (see Ricart i Costa (1988) for a related model on managerial job assignment). Here, the firm’s promotion policy continues to play an important role as it can affect that information rent that the worker earns from the market (which, in turn, can be extracted by the initial employer). Finally, if there is a moral hazard problem in the production process, the use of break-up fee may create an additional cost: it mutes work incentives by dampening the raiders’ bid, and therefore, lowering the prospect to future wage increments (see, Kräkel and Sliwka (2009) for a similar discussion).

The issues raised above offer useful directions for future research and may offer additional insights into the firm’s job assignment policies. However, the key trade-off between the worker-job and the worker-firm matching that we highlight in this article continues to play a critical role in all these setting and we expect our findings to be informative in analyzing such complex environments.

Appendix

This appendix contains the proofs omitted in the text.

Proof of Proposition 1. Using equations (8) and (9) we can calculate \( E_m \pi_p(a^*, m, d) \) as follows. When \( d < \psi_2 \),

\[
E_m \pi_p(a^*, m, d) = a^* \psi_2 \Pr [m \leq \max \{0, (d - a^* \psi_2)/(d + a^* \psi_2)\}] \\
+ d \Pr [m > \max \{0, (d - a^* \psi_2)/(d + a^* \psi_2)\}].
\]

\[25\] Liquidity constraints can be less binding under contracts with break-up fee as the worker may have lower rents in period two (hence, period-one wage need not have to be lowered as much to ensure complete rent extraction).

\[26\] Golan (2005) addresses these issues in a related environment but does not consider break-up fees or matching gains with the outside labor market. Also see Bernhardt and Scoones (1998) for a related discussion on the incentives to invest on human capital.
The exact values of the probabilities above depend on whether $d - a^*\psi_2$ is positive or not. By considering the two cases, we obtain that

$$
\mathbb{E}_m\pi_p(a^*, m, d) = \begin{cases} 
\frac{a^*\psi_2[d(3 - 2\alpha) + \psi_2 a^*(2\alpha - 1)]}{(d + a^*\psi_2)} & \text{if } a^* \leq d/\psi_2 \\
\frac{a^*\psi_2\alpha + d(1 - \alpha)}{(d - a^*\psi_2)} & \text{if } a^* > d/\psi_2.
\end{cases}
$$

Consider now the case where $d > \psi_2$. In this case, $\mathbb{E}_m\pi_p(a^*, m, d) = a^*\psi_2 \Pr[m \leq 2d/(\psi_2(1 + a^*)) - 1] + d \times \Pr[m > 2d/(\psi_2(1 + a^*)) - 1]$. When $d \geq \psi_2(1 + a^*)$, $m \leq 2d/(\psi_2(1 + a^*)) - 1$ for all possible realizations of $m$. Hence, when $d > \psi_2$,

$$
\mathbb{E}_m\pi_p(a^*, m, d) = \begin{cases} 
a^*\psi_2 & \text{if } a^* \leq d/\psi_2 - 1 \\
a^*\psi_2 \left[2\alpha - 1 + \frac{2(1 - \alpha)\psi_2}{\psi_2(1 + a^*)} \right] + 2d(1 - \alpha) \left[1 - \frac{d}{\psi_2(1 + a^*)}\right] & \text{if } a^* > d/\psi_2 - 1.
\end{cases}
$$

Using the above characterization of $\mathbb{E}_m\pi_p(a^*, m, d)$, it is routine to solve for the equation $\mathbb{E}_m\pi_p(a^*, m, d) = \psi_1$ as follows:

$$
(14) \quad a^*(d) = \begin{cases} 
\frac{(\psi_1 - d(1 - \alpha))}{(\alpha\psi_2)} & \text{if } 0 \leq d < \psi_1 \\
\frac{\psi_1 d}{(2d\psi_2 - \psi_1\psi_2)} & \text{if } \psi_1 \leq d < \psi_2 \text{ and } \alpha = \frac{1}{2} \\
\frac{(\psi_1 - d(3 - 2\alpha) + \sqrt{A_0})}{(2\psi_2(2\alpha - 1))} & \text{if } \psi_1 \leq d < \psi_2 \text{ and } \alpha \neq \frac{1}{2} \\
\frac{(\psi_1 - d + \frac{d^2}{\psi_2})}{(2d - \psi_1)} & \text{if } \psi_2 \leq d < \psi_2 + \psi_1 \text{ and } \alpha = \frac{1}{2} \\
\frac{(\psi_1 - d + \frac{d^2}{\psi_2})}{(2d - \psi_1)} & \text{if } \psi_2 \leq d < \psi_2 + \psi_1 \text{ and } \alpha \neq \frac{1}{2} \\
\psi_1/\psi_2 & \text{if } d > \psi_2 + \psi_1,
\end{cases}
$$

where $A_0 = (d(3 - 2\alpha) - \psi_1)^2 + 4(2\alpha - 1)d\psi_1$, $A_1 = 4d(1 - \alpha) - \psi_1 + \psi_2(2\alpha - 1)$ and $A_2 = A_1^2 - 4(2\alpha - 1)(2d(1 - \alpha)(\psi_2 - d) - \psi_1\psi_2)$. The properties of $a^*(d)$ highlighted in the proposition follow from direct inspection of (14).

**Proof of Lemma 1.** Suppose first that $d < \psi_2$. We know from (4) that $b^*(m, d; a^*) = 0$ if and only if $m \leq 0$ or $a^* \leq d(1 - m)/(\psi_2(1 + m))$. The second inequality is equivalent to $m \leq (d - a^*\psi_2)/(d + a^*\psi_2)$. Thus, $b^*(m, d; a^*) = 0$ if and only if $m \leq \max\{0, (d - a^*\psi_2)/(d + a^*\psi_2)\}$.

From Proposition 1, it follows that in equilibrium $a^*$ is continuous and decreasing in $d$ when $d < \psi_2$, which implies that $(d - a^*(d)\psi_2)/(d + a^*(d)\psi_2)$ is continuous and increasing in $d$ when $d < \psi_2$. This, together with the fact that

$$
\lim_{d \to \psi_1} \frac{d - a^*(d)\psi_2}{d + a^*(d)\psi_2} = 0,
$$

(where $a^*(d)$ is as given in (14)) implies that $(d - a^*(d)\psi_2)/(d + a^*(d)\psi_2) \leq 0$ when $d \leq \psi_1$ and $(d - a^*(d)\psi_2)/(d + a^*(d)\psi_2) > 0$ when $\psi_1 < d < \psi_2$. Hence, if $d \leq \psi_1$, then $b^*(m, d; a^*(d)) = 0$ if and only if $m \leq 0$. If $\psi_1 < d < \psi_2$, $b^*(m, d; a^*(d)) = 0$ if and only if $m \leq (d - a^*(d)\psi_2)/(d + a^*(d)\psi_2) = \tilde{m}(d)$, which is strictly greater than 0 and lower than 1 and, as mentioned above, increasing in $d$.

Suppose now that $d \geq \psi_2$. We follow the same steps as above. From (5) we know that $b^*(m, d; a^*) = 0$ if and only if $m \leq \max\{0, 2d/(\psi_2(1 + a^*)) - 1\}$. Clearly, when $d \geq \psi_2$, $2d/(\psi_2(1 + a^*)) - 1 > 0$. Hence, $b^*(m, d; a^*) = 0$ if and only if $m \leq 2d/(\psi_2(1 + a^*)) - 1 \equiv \tilde{m}(d)$, which can be shown to be increasing in $d$ when $d \geq \psi_2$. Since $\lim_{d \to (\psi_1 + \psi_2)} \tilde{m}(d) = 1$, then $\tilde{m}(d) < 1$ for all $d < \psi_1 + \psi_2$. Next, observe that

$$
\lim_{d \to \psi_1} \frac{d - a^*(d)\psi_2}{d + a^*(d)\psi_2} = \frac{1 - a^*(\psi_2)}{1 + a^*(\psi_2)} = \lim_{d \to \psi_2} \frac{2d}{\psi_2(1 + a^*(d)) - 1},
$$

meaning that $\tilde{m}(d)$ is continuous at $d = \psi_2$. Hence, $\tilde{m}(d) \in (0, 1)$ and is monotone increasing when $\psi_1 < d < \psi_2 + \psi_1$. Finally, observe that when $d \geq \psi_1 + \psi_2$, in equilibrium $a^* = \psi_1/\psi_2$ and $2d/(\psi_2(1 + a^*)) - 1 \geq 1$. In this case there is total market foreclosure.

**Proof of Lemma 2.** In what follows, we will prove a more general version of this lemma that would be useful later in proving subsequent results (we do not present this version in text for
where $H : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}$ and $J : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}$. Furthermore, (i) $H(a^*(d), d) > 0$ for all $d \in (\psi_1, \psi_2)$, (ii) $J(a^*(d), d) > 0$ for all $d \in [\psi_2, \psi_2 + \psi_1)$, and (iii) for any $d_1 \in [\psi_1, \psi_2)$ and $d_2 \in [\psi_2, \psi_2 + \psi_1)$ such that $a^*(d_1) = a^*(d_2)$, $J(x, d_2) > H(x, d_1)$.

Under the optimal contract, $w_1$ is such that the agent’s individual rationality constraint is binding. Moreover, in any equilibrium, the raiders’ bid the entire expected production of the workers they successfully raid. Hence, both the worker’s expected utility and the raiders’ expected profit are zero, which implies that $\Pi(d)$ is always equal to the aggregate expected surplus. The aggregate expected surplus depends on the firm’s promotion policy, the raiders’ equilibrium decision to bid for a promoted worker and the firms’ decision to make a counter-offer and retain the worker. The remainder of the proof consists of the following for steps.

**Step 1:** $\Pi$ when $d < \psi_1$. It follows from Lemma 1 that when $d < \psi_1$ the raiders bid for a promoted worker if and only if $m > 0$. Hence, there is no market foreclosure. This implies that the expected aggregate period-two surplus is $\tilde{S}(a^*(d))$ and $\Pi(d) = \psi_1 + \tilde{S}(a^*(d))$.

**Step 2:** $\Pi$ when $\psi_1 \leq d < \psi_2$. When $\psi_1 \leq d < \psi_2$ the raiders bid for a promoted worker if and only $m > (d - a^*(d)\psi_2)/(d + a^*(d)\psi_2) \geq 0$ (see proof of Lemma 1). There is partial foreclosure, since a promoted worker is retained by $F$ whenever $m \leq (d - a^*(d)\psi_2)/(d + a^*(d)\psi_2)$. The allocative difference between this case and that underlying the second best aggregate surplus is that when the realization of $a$ and $m$ is such that $a^*(d) \leq a \leq d/\psi_2$ and $(a - a^*(d))/(a + a^*(d)) < m \leq (d - a^*(d)\psi_2)/(d + a^*(d)\psi_2)$ the worker is retained by $F$ instead of joining a raider firm where he is more efficient by a factor of $m$. Hence, in this case, $\Pi(d) = \psi_1 + \tilde{S}(a^*(d)) - H(a^*(d), d)$, where

$$H(x, y) := \int_\mathbb{R} \int_\mathbb{R} \omega x m(1 - \omega) d\omega d\alpha.$$

Clearly, $H(x, y) > 0$ for all $x \in [0, \frac{\psi_1}{\psi_2})$ and $y > \psi_1$.

**Step 3:** $\Pi$ when $\psi_2 \leq d < \psi_1 + \psi_2$. When $\psi_2 \leq d < \psi_1 + \psi_2$ the raiders bid for a promoted worker if and only if $m > 2d/[\psi_2(1 + a^*(d))] - 1$ (see proof of Lemma 1). There is partial foreclosure, since a promoted worker is retained by $F$ whenever $m \leq 2d/[\psi_2(1 + a^*(d))] - 1$. The allocative difference between this case and that underlying the second best aggregate surplus is that when the realization of $a$ and $m$ is such that $a^*(d) \leq a \leq 1$ and $(a - a^*(d))/(a + a^*(d)) < m \leq 2d/[\psi_2(1 + a^*(d))] - 1$ the worker is retained by $F$ instead of joining a raider firm where he is more efficient by a factor of $m$. Hence, in this case, $\Pi(d) = \psi_1 + \tilde{S}(a^*(d)) - J(a^*(d), d)$, where

$$J(x, y) := \int_\mathbb{R} \int_\mathbb{R} \omega x m(1 - \omega) d\omega d\alpha.$$

Observe that $2y/([\psi_2(1 + x)] - 1 > (a - x)/(a + x)$ for all $a \in [x, 1)$, when $y > \psi_2$. Hence, $J(x, d) > 0$ for all $x \in [0, 1)$ and $y > \psi_2$. 

expositional clarity. We will show that the firm’s expected profit function $\Pi$ is continuous in $d$ and given by the following functional form:

$$\Pi(d) = \begin{cases} 
\psi_1 + \tilde{S}(a^*(d)) & \text{if } 0 \leq d < \psi_1 \\
\psi_1 + \tilde{S}(a^*(d)) - H(a^*(d), d) & \text{if } \psi_1 \leq d < \psi_2 \\
\psi_1 + \tilde{S}(a^*(d)) - J(a^*(d), d) & \text{if } \psi_2 \leq d < \psi_2 + \psi_1 \\
(\psi_1 + \psi_2)^2/2\psi_2 & \text{otherwise}
\end{cases}$$


Step 4: \( \mathcal{H} \) when \( d \geq \psi_1 + \psi_2 \). It follows from Lemma 1 that when \( d \geq \psi_1 + \psi_2 \) the raiders never bid for a promoted worker irrespective of the value of \( m \). Hence, there is full market foreclosure since all promoted workers are retained by \( F \). The allocative difference between this case and that underlying the second best aggregate surplus is that when the realization of \( a \) and \( m \) is such that \( a^*(d) \leq a \leq 1 \) and \( (a - a^*(d))/(a + a^*(d)) < m \leq 1 \) the worker is retained by \( F \) instead of joining a raider firm where he is more efficient by a factor of \( m \). Hence, in this case, \( \mathcal{H}(d) = \psi_1 + \hat{S}(a^*(d)) - L(a^*(d), d) \), where

\[
L(x, y) := \int_0^1 \int_0^1 a \psi_2 m(1 - \alpha) d\alpha dx.
\]

Clearly, \( \mathcal{H}(d) = \psi_1 + \psi_1 a^*(d) + \int_a^{a^*(d)} \left[ f^0_1(1 - \alpha) \psi_2 d\alpha m(1 - \alpha) \right] d\alpha = \psi_1 + \psi_1 a^*(d) - \frac{1}{2} \psi_2 (a^*(d) - 1) (a^*(d) + 1) \). Since \( a^*(d) = \psi_1/\psi_2 \) when \( d \geq \psi_1 + \psi_2 \), we obtain that \( \mathcal{H}(d) = \left( \psi_1 + \psi_2 \right)^2 / 2 \psi_2 \).

Step 5: Comparing \( H(x, y) \) with \( J(x, y) \). To compare \( H(x, y) \) with \( J(x, y) \) one needs to compare the upper limits of integration. Clearly, \( \frac{x}{\psi_2} \leq 1 \) for all \( y < \psi_2 \). Furthermore, \( (y_1 - x \psi_2)/(y_1 + x \psi_2) < 2 \psi_2 / \psi_2 (1 + x) - 1 \) for all \( y_1 < \psi_2 \), \( y_2 \geq \psi_2 \), and \( x > 0 \). Since both in \( H(x, y) \) and in \( J(x, y) \) the integrand is the same and is positive, we obtain that \( H(x, y_1) > J(x, y_2) \) for all \( y_1 \in [\psi_1, \psi_2) \), \( y_2 \geq \psi_2 \), and \( x \in [0, 1] \).

Proof of Proposition 2. The proof is given in the following steps.

Step 1: There exists \( \psi \) such that the optimal damage fee is strictly positive if \( \psi_1 > \psi \). Using Lemma 2 we obtain that \( \hat{\mathcal{H}}(0) = \hat{S}'((a^*(0)) a^*(0)) \). We know that \( a^*(0) = -(1 - \alpha)/(\alpha \psi_2) < 0 \). Moreover, computing the integrals in \( \hat{S} \), we obtain that

\[
\hat{S}(x) = \psi_1 x + \frac{1}{2} \left( 4 \alpha x - 6 x - 7 x^2 + 6 x^2 \alpha - 1 \right) \frac{(x - 1) \psi_2}{x + 1} + 4 \psi_2 x^2 \left( \ln 2 \frac{x}{x + 1} \right) (1 - \alpha) .
\]

Using (15), we obtain that

\[
\hat{S}'((a^*(0)) = \psi_1 (\psi_1/(\alpha \psi_2)) = \frac{(\alpha - 1) 7 \psi_1^2 + 6 \alpha \psi_1^2 \psi_2 - 2 \alpha^3 \psi_2^2 - 5 \alpha^2 \psi_1^2 \psi_2}{(\psi_1 + \alpha \psi_2)^2} - \frac{(\alpha - 1) 8 \psi_1 \ln \frac{2 \psi_1}{\psi_1 + \alpha \psi_2},}
\]

which, in turn, can be used to obtain that \( \lim_{\psi_1 \to 0} \hat{S}'((a^*(0)) = 2 (1 - \alpha) \psi_2 > 0 \) and that

\[
\lim_{\psi_1 \to \psi_2/2} \hat{S}'((a^*(0)) = \frac{1}{2} (1 - \alpha) \frac{20 \alpha^2 - 12 \alpha + 16 \alpha^3 - 7}{(2 \alpha + 1)^2} + 8 \ln \frac{2 \alpha}{2 \alpha + 1} \psi_2 < 0
\]

for all \( \alpha \in [1/2, 1) \). (Observe that \( \lim_{\psi_1 \to \psi_2/2} \hat{S}'((a^*(0)) = 0 \) for \( \alpha = 1 \).) Next, note that for all \( \alpha \in [1/2, 1) \), \( \hat{S}'((a^*(0)) \) is continuous and decreasing \( \psi_1 \) in \( (0, \psi_2/2) \). Thus, for all \( \alpha \in [1/2, 1) \) there exists \( \psi_1 \in (0, \psi_2/2) \) such that \( \hat{S}'((a^*(0) < 0 \) if and only if \( \psi_1 > \psi_1 \). Hence, for \( \psi_1 > \psi_1 \), \( \hat{H}(0) > 0 \), meaning that the optimal damage fee is strictly positive.

Step 2: The optimal damage fee is strictly positive only if \( \psi_1 > \psi_1 \). To show this, we show the equivalent statement that the optimal damage fee is zero if \( \psi_1 \leq \psi_1 \). Lemma 2, together with the fact that for all \( d_1 \in (\psi_2, \psi_2 + \psi_1) \) there exists \( d_2 \in (\psi_1, \psi_2) \) such that \( a^*(d_1) = a^*(d_2) \) (see the version of Lemma 2 given in this Appendix), implies that \( d > \psi_2 \) is never optimal. Hence, it remains to show that \( 0 < d \leq \psi_2 \) is not optimal either. If \( \psi_1 \leq \psi_1 \), then \( \hat{S}'((a^*(0)) \geq 0 \) (see the analysis in Step 1). Moreover, observe that \( \hat{S}''(x) = \psi_4(9 - 10 \alpha - 2 \alpha x - x - 13 x^2 + 10 x^2 \alpha + 6 x^3 \alpha - 7 x^3 - 8 (x + 1)^3 (\ln 2 \frac{x}{x + 1}) (\alpha + 1)) / (x + 1)^3 < 0 \) for all \( x \in [0, 1] \), meaning that \( \hat{S}(x) \) is concave. Hence, when \( \psi_1 \leq \psi_1 \), \( \hat{S}'(x) > 0 \) for all \( x < a^*(0) \). Since \( \hat{H}(d) = \psi_1 + \hat{S}(a^*(d)) \) and \( a^*(d) < a^*(0) \) for all
and note that

\[ H(d) = \psi_1 + \tilde{S}(a^*(d)) - H(a^*(d), d) \]

where \( H(a^*(d), d) \leq 0 \) (see the proof of Lemma 2). Hence, \( \tilde{H}(d) \leq \psi_1 + \tilde{S}(a^*(d)) \). Moreover, concavity of \( \tilde{S}(x) \), and the fact that \( a^*(d) < a^*(0) \) and \( a^*(d) < 0 \) when \( \psi_1 \leq d < \psi_2 \), implies that \( \psi_1 + \tilde{S}(a^*(d)) \) is decreasing in \( d \) when \( \psi_1 \leq d < \psi_2 \). Thus, \( \tilde{H}(d) < \psi_1 + \tilde{S}(a^*(d)) \leq \psi_1 + \tilde{S}(a^*(0)) = \tilde{H}(0) \) if \( \psi_1 \leq d < \psi_2 \). Finally, because \( \tilde{H} \) is continuous at \( d = \psi_2 \), then \( \tilde{H}(\psi_2) = \lim_{d \to \psi_2} \tilde{H}(d) < \tilde{H}(0) \). This observation, together with Step 1 above, establishes that the optimal damage fee is strictly positive if and only if \( \psi_1 > \psi_1 \).

**Step 3:** Aggregate surplus increases with inclusion of damage fee when \( \psi_1 > \psi_1 \). This follows from the fact that the firm’s profit is identical to the expected aggregate surplus. ■

**Proof of Proposition 3.** In what follows, we use \( f^i(x^-) \) and \( f^i(x^+) \) to denote, respectively, the left and the right derivative of a function \( f \) at point \( x \). The proof is given in the following steps.

**Step 1:** \( \tilde{H} \) is differentiable at \( d = \psi_1 \) and \( \tilde{H}'(\psi_1) = \tilde{S}'(a^*(\psi_1)) \times (-1 - \alpha)/(\alpha \psi_2) \). Since \( \tilde{S} \) is differentiable at \( a^*(\psi_1) \), \( \tilde{H}(d) = \psi_1 + \tilde{S}(a^*(d))a^*(d) \) when \( d < \psi_1 \), and \( \tilde{H} \) is continuous at \( \psi_1 \), then

\[
\tilde{H}'(\psi_1) = \tilde{S}'(a^*(\psi_1))a^*(\psi_1) = \tilde{S}'(a^*(\psi_1)) \times (-1 - \alpha)/(\alpha \psi_2).
\]

When \( d \geq \psi_1 \), \( \tilde{H}(d) = \psi_1 + \tilde{S}(a^*(d)) - H(a^*(d), d) \), where \( H \) is as defined in the proof of Lemma (2). Hence,

\[
\tilde{H}'(\psi_1) = \tilde{S}'(a^*(\psi_1))a^*(\psi_1) - H_x'(a^*(\psi_1), \psi_1^+)a^*(\psi_1) - H_y'(a^*(\psi_1), \psi_1^+).
\]

Differentiating \( H \) with respect to \( x \) and to \( y \), we obtain, respectively,

\[
H_x' = \int_{0}^{\psi_2} x \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm
\]

\[
H_y' = \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm + \int_{\psi_2}^{\psi_1} \psi_2 m(1 - \alpha)dm
\]

Using, (17) and (18), we obtain that

\[
H_x'(a^*(\psi_1), \psi_1^+) = H_y'(a^*(\psi_1), \psi_1^+) = 0.\]

Since \( a^*(\psi_1^+) = -(1 - \alpha)/(\alpha \psi_2) = a^*(\psi_1^-) \), we obtain that

\[
\tilde{H}'(\psi_1^-) = \tilde{S}'(a^*(\psi_1)) \times (-1 - \alpha)/(\alpha \psi_2) = \tilde{H}'(\psi_1^-),
\]

which implies that \( \tilde{H} \) is differentiable at \( d = \psi_1 \) and \( \tilde{H}(\psi_1) = \tilde{S}'(a^*(\psi_1)) \times (-1 - \alpha)/(\alpha \psi_2) \).

**Step 2:** There exists \( \overline{\psi}_1 > \psi_1 \) such that \( \tilde{S}'(a^*(\psi_1)) > 0 \) if \( \psi_1 < \overline{\psi}_1 \), \( \tilde{S}'(a^*(\psi_1)) < 0 \) if \( \psi_1 > \overline{\psi}_1 \), and \( \tilde{S}'(a^*(\psi_1)) = 0 \) if \( \psi_1 = \overline{\psi}_1 \). Concavity of \( \tilde{S} \) and the fact that \( a^*(\psi_1) < a^*(0) \) implies that \( \tilde{S}'(a^*(\psi_1)) > \tilde{S}'(a^*(0)) \). Since \( \tilde{S}'(a^*(0)) \geq 0 \) when \( \psi_1 = \overline{\psi}_1 \) (see proof of Proposition 2), it follows that \( \tilde{S}'(a^*(\psi_1)) > 0 \) when \( \psi_1 = \overline{\psi}_1 \). We next analyze \( \tilde{S}'(a^*(\psi_1)) \) when \( \psi_1 \to \psi_2/2 \). Using (15), we obtain that

\[
\tilde{S}'(a^*(\psi_1)) = \tilde{S}'(\psi_1/\psi_2) = 2(1 - \alpha)\left(\frac{\psi_3^2 - 3\psi_1^2 + 3\psi_1\psi_2^2 - 2\psi_1^2\psi_2 + 4\psi_1\ln \frac{2\psi_1\psi_2}{\psi_1 + \psi_2}}{(\psi_1 + \psi_2)^2}\right).
\]

Using this result, we obtain that \( \lim_{\psi_1 \to \psi_2/2} \tilde{S}'(a^*(\psi_1)) = \frac{1}{3} (36 \ln \frac{2}{3} + 13) \psi_2 (1 - \alpha) < 0 \). Next, note that \( \tilde{S}'(a^*(\psi_1)) \) is continuous and decreasing in \( \psi_1 \) in \( [0, \psi_2/2] \). Hence, there exists \( \overline{\psi}_1 \in \)
Step 3: For \( \psi_1 < \psi_1 \leq \bar{\psi}_1 \), the optimal break-up fee \( d^* \in (0, \psi_1] \) and \( a^* (d^*) = a^{SB} \). We know from Proposition 2 that \( d^* > 0 \) when \( \psi_1 > \psi_1 \). We next show that if \( \psi_1 < \psi_1 \), then \( d^* \notin (\psi_1, \psi_2) \). Let \( \psi_1 < \bar{\psi}_1 \) and \( \psi_1 < d < \psi_2 \). Since \( \psi_1 < \bar{\psi}_1 \), then \( \hat{S}'(a^*(\psi_1)) \geq 0 \) (see Step 2). Concavity of \( \hat{S}(x) \) together with the fact that \( a^*(d) < a^*(\psi_1) \) implies that \( \hat{S}'(a^*(d)) > 0 \). Since \( a^*(d) < 0 \), then \( \hat{S}'(a^*(d)) \times a^*(d) < 0 \), meaning that \( \psi_1 + \hat{S}(a^*(d)) \) decreases with \( d \). This, together with the fact that \( \hat{\Pi}(d) = \psi_1 + \hat{S}(a^*(d)) - H(a^*(d), d) \) where \( H(a^*(d), d) < 0 \), implies that \( \hat{\Pi}(d) < \psi_1 + \hat{S}(a^*(d)) < \psi_1 + \hat{S}(a^*(\psi_1)) = \hat{\Pi}(\psi_1) \). Hence \( d^* \notin (\psi_1, \psi_2) \). Since \( d^* < \psi_2 \) by Lemma 2, this means that \( d^* \in (0, \psi_1] \). We next show that \( a^*(d^*) = a^{SB} \). When \( \psi_1 < \psi_1 \leq \bar{\psi}_1 \), \( \hat{S}(a^*(0)) < 0 \) and \( \hat{S}(a^*(\psi_1)) \geq 0 \). Because \( \hat{S} \) is concave, \( a^{SB} \in [a^*(\psi_1), a^*(0)) \). Finally, since \( \hat{\Pi}(d) = \psi_1 + \hat{S}(a^*(d)) \) when \( 0 \leq d < \psi_1 \), then \( a^* = a^{SB} \) necessarily.

Step 4: For \( \psi_1 > \bar{\psi}_1 \), \( d^* \in (\psi_1, \psi_2) \) and \( \Pi^* < \psi_1 + \hat{S}(a^{SB}) \). We first show that \( d^* > \psi_1 \). Concavity of \( \hat{S} \) together with the fact that \( a^*(d) \geq a^*(\psi_1) \) for all \( d \leq \psi_1 \) implies that \( \hat{S}'(a^*(d)) < \hat{S}'(a^*(\psi_1)) \). Hence \( \hat{S}'(a^*(d)) < 0 \) for \( d \leq \psi_1 \). Furthermore, \( a^*(d) \) decreases with \( d < \psi_1 \). Hence, \( \hat{\Pi}'(d) = \hat{S}'(a^*(d))a^*(d) > 0 \) for \( d \leq \psi_1 \). Thus, \( d^* > \psi_1 \). Since \( d^* < \psi_2 \) by Lemma 2, then \( d^* \in (\psi_1, \psi_2) \). Finally, observe that \( \Pi^* = \psi_1 + \hat{S}(a^*(d^*)) - H(a^*(d^*), d^*) < \psi_1 + \hat{S}(a^*(d^*)) \leq \bar{\psi}_1 + \hat{S}(a^{SB}) \), where the second inequality follows from the fact that \( a^{SB} \) is a maximizer of \( \psi_1 + \hat{S}(x) \).

Proof of Proposition 4. The value \( \hat{\psi}_1 \) is characterized by the condition: \( \hat{\Pi}'(0) = 0 \) when \( \psi_1 = \hat{\psi}_1 \). Since \( \hat{\Pi}'(0) \) increases with \( \psi_1 \) when \( \psi_1 \in (0, \psi_2/2] \) (see step 1 of the proof of Proposition 2), it suffices to show that \( \hat{\Pi}'(0) \) evaluated at \( \psi_1 = \hat{\psi}_1 \) decreases with \( \alpha \). Using Lemma 2 we obtain that \( \hat{\Pi}'(0) = \hat{S}'(a^*(0))a^*(0) \). We know that \( a^*(0) = -\alpha/(\alpha \psi_2) < 0 \). Using (16), we obtain

\[
\hat{\Pi}'(0) = \frac{(1 - \alpha)^2}{\alpha^2 \psi_2} \left( \frac{7 \psi_1^3 + 6 \alpha \psi_2^2 - 2 \alpha^2 \psi_1^2 / \psi_2^2 - 5 \alpha^2 \psi_1 \psi_2^2}{(\psi_1 + \alpha \psi_2)^2} - 8 \psi_1 \ln \frac{2 \psi_1}{\psi_1 + \alpha \psi_2} \right).
\]

Differentiating \( \hat{\Pi}'(0) \) with respect to \( \alpha \) and evaluating it at \( \psi_1 = \hat{\psi}_1 \) (i.e., use the fact that \( \hat{\Pi}'(0) = 0 \) at \( \psi_1 = \hat{\psi}_1 \)), we obtain

\[
2 (1 - \alpha) \left( \frac{\psi_2^3 \alpha^3 (1 - \alpha) (\psi_1 - \alpha \psi_2)}{\psi_2 (\alpha \psi_1 + \alpha^2 \psi_2)^3} \right) < 0.
\]

Hence the proof.

References


