USC FBE APPLIED ECONOMICS WORKSHOP
presented by Alessandro Lizzeri
FRIDAY, Nov. 7, 2003
1:30 - 3:00 pm; **Room: HOH-601K**

The Incentive Effects of Interim Performance Evaluations

Alessandro Lizzeri, Margaret Meyer, and Nicola Persico*

Abstract

We study a dynamic moral hazard model where the agent does not fully observe his performance. We consider the incentive effects of providing feedback to the agent: revealing to the agent how well he is doing.

We show that, if the incentive scheme is exogenously given, there is a wide range of cases where the agent works harder if feedback is provided. However, the agent earns more money in this scenario.

We then characterize the optimal incentive schemes in the two scenarios and we show that the principal is better off if feedback is not provided; the expected cost of inducing any given level of expected effort is lower in the no-revelation scenario.

Incomplete Draft

NOTE: Dr. Lizzeri will be presenting a more recent version of this material. However, no new write-up is available at this time.

*Department of Economics, New York University, 269 Mercer Street, New York, NY 10003; e.mail: alessandro.lizzeri@nyu.edu; web: http://www.econ.nyu.edu/user/lizzeria/.

1 Introduction

How do organizations decide whether, and to what extent, to provide feedback to individuals on how their performance to date has been evaluated?

To examine this question, we study a dynamic principal-agent model in which the agent privately observes how hard he works but cannot fully observe the consequences of his efforts, i.e. his performance. In this setting the principal has a choice of whether to conduct an interim performance evaluation (IPE) and whether to reveal the outcome of this IPE to the agent.

To the extent that providing feedback on performance helps individuals do their jobs better, or plan their futures better, it is beneficial. But what effects does performance feedback have on incentives and motivation? What effects does it have on sorting within the hierarchy, i.e., ensuring that the more able individuals advance faster than the less able?

There are several organizational settings in which choices about interim performance evaluations are potentially important. In professional service firms such as law firms and consulting firms, junior professionals know that a substantial part of their reward takes the form of a potential promotion to partner. Long before the promotion decisions are made, some information about their prospects is typically revealed to these employees. Sometimes this information is revealed through a formal process of periodic evaluations by the partners; sometimes the information revelation process is more informal. In schools and universities, teachers and professors face choices about whether to assign grades based entirely on performance on final exams or whether, and to what extent, to use a more continuous form of assessment. A midterm exam is a particularly stark form of an IPE; if given a voice, most university students appear to favor having such an exam.

There are also other types of environments where it is possible to alter the amount of interim information individuals receive. In patent races, each firm's behavior will be affected by the interim information it receives on the progress made by rivals; how would a social planner concerned with innovation and welfare wish to influence how much firms learn about each other's progress? In auctions, the choice of rules can affect how much interim information contestants receive: an ascending auction provides more information to bidders about their rivals than does a sealed-bid auction. In multi-stage sports competitions, organizers could in principle influence how much players learned about others' performances. Finally, in electoral contests, candidates' choices about campaign spending and about platforms can be affected by the frequency and scope of pre-election polling.

Surprisingly, the economics literature has devoted very little attention to the effects of interim performance evaluations on behavior. There is extensive discussion of IPE's in the human resource management literature, but there is no consensus and little formal analysis.

The most obvious aspect of behavior that will be affected by IPE's is the motivation to exert effort following the interim evaluation. In addition, IPE's can affect incentives to exert effort prior to the IPE. Furthermore, the structure of the optimal incentive scheme will depend on whether or not IPE's are provided. Finally, when individuals have heterogeneous abilities, IPE's will affect how well incentive schemes *sort* agents according to ability, as well as how well effort choices are tailored to ability.

Our formal model consists of two periods. In each period, the agent exerts a privately-observed level of effort that determinines the probability of success. The agent does not observe whether the outcome is a success or a failure, but the principal does observe the outcome. The agent is assumed to be risk neutral but protected by limited liability. We contrast two informational scenarios. In the first, the 'no-revelation' scenario, when choosing second-period effort the agent does not know the first-period outcome. In the second, the 'revelation' scenario, the principal is assumed to provide a truthful interim performance evaluation, so the agent learns the first-period outcome before choosing second-period effort. (In order to focus on the effects of IPE's on the agent's behavior, we suppress consideration of incentive problems on the principal's part.)

We begin our analysis by contrasting the agent's behavior in the two scenarios when the incentive scheme, consisting of rewards conditional on the possible outcomes, is exogenously given and the same in the two scenarios. In this case we show that in some circumstances total expected effort is higher when the first-period outcome is revealed to the agent before the second period. However, the agent earns more money in the revelation scenario because he can tailor his effort to exploit differences in marginal compensation. Thus, the expected cost to the principal is higher in the revelation scenario. As a consequence, even if revelation raises the agent's expected effort, we cannot conclude that the principal prefers to provide an interim evaluation.

Furthermore, if the principal were to choose the incentive schemes optimally, i.e. to minimize the expected cost of inducing a given level of expected effort, he would choose different schemes in the two scenarios. When interim evaluations are not provided, the optimal incentive scheme rewards the agent if and only if he succeeds twice. By contrast, when interim evaluations are given, it is optimal to offer a strictly positive reward for a single success, while offering an even greater marginal reward for a second success.

Armed with these characterizations, we then assess the desirability of providing interim performance evaluations when incentive schemes can be designed optimally. In this setting, we show that it is better not to reveal any information, i.e. the expected cost of inducing any given level of expected effort is lower in the no-revelation scenario.

Related literature

There is a large literature on dynamic agency problems, though very little attention has been paid to the effect of interim performance evaluations. A number of papers (e.g. Fudenberg, Holmstrom, and Milgrom (1990) and Chiappori et al (1994)) investigate the conditions under which the optimal contract in a long-term agency relationship can be implemented through a sequence of short-term (spot) contracts. In our setting, whether or not IPE's are provided, a sequence of spot contracts would be strictly worse for the principal than the optimal long-term contract—this is a consequence of the assumption that the agent has limited liability.

Rogerson (1985) and Meyer (1992) study two-period moral hazard models (with one and two agents, respectively) in which agents, though not protected by limited liability, are risk averse. They assume throughout that the first-period outcome is observed by the agents before the second period. Both papers focus on how, in the optimal long-term contract, the second-period incentive scheme depends on the first-period outcome. Rogerson shows the desirability of having wage payments display "memory", and Meyer shows that the principal benefits by baising the second-period contest in favor of the first- period winner. Neither paper addresses the issue of whether the principal could induce efforts from the agents more cheaply by not providing interim performance evaluations.

Holmstrom and Milgrom (1987), along with the accounting literature on earnings management, study a dynamic agency setting in which the agent privately learns how he is performing and adjusts his subsequent efforts accordingly.[1] Holmstrom and Milgrom present conditions under which the optimal contract in such a setting is linear in cumulative performance. The assumption that the principal knows strictly *less* than the agent about the agent's performance, as well as about his effort, is very natural in some settings, such as when the principal is shareholders, the agent is a CEO, and performance is cash flows. In this paper, however, we are concerned with settings where the principal is *better* able than the agent to evaluate the (uncertain) consequences of the agent's efforts. There may be a significant subjective element in the evaluation of performance, as with clients' evaluations of professional services, or the agents may have little experience with the tasks they are performing, as when the agents are students learning new material and the principals are their teachers.

Malcomson (1984) and Kahn and Huberman (1988), among others, examine agency settings where principals are better informed than agents about agents' performance. They assume that it is intrinsically costless for principals to misreport agents' performance and study what incentive schemes are feasible in this case. In the professional serivces and educational contexts mentioned above, however, principals can, albeit at a cost, make performance evaluations verifiable. We assume that final performance is made verifiable, and our focus is on the effects of making interim performance verifiable as well.

There are a few papers which attempt to examine some of the consequences of interim performance evaluations. In Prendergast (1992), the firm privately observes workers' abilities after the first period and decides whether or not to use promotions to a fast track as a credible signal to high-ability workers. The cost of providing IPE's is that fast-track promotions occur inefficiently early, while the benefit is that workers are induced to tailor their training effort

[1] For recent overviews of the accounting literature on earnings management, see Arya, Glover, and Sunder (1998) and Lambert (2001).

to their level of ability. Gibbs (1991) presents some discussion of how interim evaluations affect subsequent efforts, when an agent's cumulative performance must surpass a threshold in order for him to receive a reward. Lazear (1999) asks a similar question in the context of rank- order tournaments.

Finally, our characterization of optimal incentive schemes with and without IPE's is related to work on the optimal design of prize structures in contests. Krishna and Morgan (1998) and Moldovanu and Sela (2001, 2002) examine when it is optimal to use a winner-take-all prize structure. We study how the desirability of rewarding only the very best cumulative performance is affected by whether or not an IPE is provided.

2 Model

There are two periods. In each period t, there are only two possible outcomes: success or failure. The outcome in period t is denoted by $X_t \in \{f, s\}$. The probability of a success in period t is equal to the effort e_t in that period: $P(X_t = s) = e_t$. Conditional on effort choices, outputs are independent across periods.

In each period the cost of effort e_t is denoted by $c(e_t)$, where c is increasing, three times differentiable, and convex, with $c(0) = c'(0) = 0$.

The agent is assumed to be risk neutral.

An incentive scheme for the agent is characterized by transfers conditional on all possible outcomes: $w(f, f), w(f, s), w(s, f), w(s, s)$. Given that the agent is risk neutral, the problem is uninteresting unless we assume that there is a limited liability constraint. We assume that $w(x, y) \geq 0$ for $x, y = f, s$.

We will contrast two scenarios on the information that is available to the agent when he chooses effort. In the no-revelation scenario, which we call the N-scenario, when choosing effort in the second period, the agent does not observe the first-period outcome. In this scenario, the agent's payoff is:

$$U^N(e_1, e_2) = w(s, s)e_1e_2 + w(s, f)e_1(1-e_2) + w(f, s)(1-e_1)e_2 + w(f, f)(1-e_1)(1-e_2) - c(e_1) - c(e_2)$$

In the revelation scenario, which we call the Y-scenario, the agent learns the first-period outcome before choosing second-period effort. Thus, in this scenario, the agent can choose a different effort in the second period depending on the first-period outcome. Letting $e_2(s)$ and $e_2(f)$ denote the second-period effort following success and failure, respectively, we can write the agent's payoff in this scenario as:

$$\begin{aligned} U^Y(e_1, e_2(s), e_2(f)) &= w(s, s)e_1e_2(s) + w(s, f)e_1(1 - e_2(s)) + w(f, s)(1 - e_1)e_2(f) \\ &\quad + w(f, f)(1 - e_1)(1 - e_2(f)) - c(e_1) - e_1c(e_2(s)) - (1 - e_1)c(e_2(f)) \end{aligned}$$

3 The Effect of IPE's When the Incentive Scheme is Fixed

3.1 Preliminaries

In this section, we discuss the effects of interim performance evaluations on effort choices when the incentive scheme is exogenously given and the same in both scenarios. For now, we assume that the cost of effort function is quadratic: $c(e) = ce^2/2$. We also assume that the rewards only depend on the number of successes, i.e., $w(f,s) = w(s,f) \equiv w(s)$, and that they are monotonically increasing in this number, so $w(f,f) \leq w(s) \leq w(s,s)$.[2]. Finally, to guarantee interior solutions we also assume that $w(s,s) < c$.[3]

3.2 The Effect of IPE's on Subsequent Efforts

We begin by analyzing the effect of IPE's on expected effort in the second period, taking as given an exogenously fixed probability of success p in the first period. In the no-revelation scenario the agent chooses second period effort e_2^N to maximize

$$p(w(s,s)e_2^N + w(s)(1 - e_2^N)) + (1-p)(w(s)e_2^N + w(f,f)(1 - e_2^N)) - \frac{1}{2}c(e_2^N)^2. \tag{1}$$

so the optimal value is

$$e_2^N(p) = \frac{p(w(s,s) - w(s)) + (1-p)(w(s) - w(f,f))}{c}. \tag{2}$$

Consider now the revelation scenario. If the first-period outcome is a success, the agent chooses $e_2(s)$ to maximize

$$w(s,s)e_2(s) + w(s)(1 - e_2(s)) - \frac{1}{2}c(e_2(s))^2, \tag{3}$$

so the optimal value is

$$e_2^Y(s) = \frac{w(s,s) - w(s)}{c}. \tag{4}$$

If the first-period outcome is a failure, the agent chooses $e_2(f)$ to maximize

$$w(s)e_2(f) + w(f,f)(1 - e_2(f)) - \frac{1}{2}c(e_2(f))^2, \tag{5}$$

so the optimal value is

$$e_2^Y(f) = \frac{w(s) - w(f,f)}{c} \tag{6}$$

We can now state the following benchmark result:

[2] In the next section, we relax the assumption that $w(f,s) = w(s,f)$, and we show that the optimal incentive schemes in the two scenarios are monotonic.

[3] See Section 3.4 for a discussion of the case where this assumption is relaxed.

Lemma 1 *Fix the first period probability of success at the same level p in both scenarios. Then expected effort in the second period is the same in the two scenarios.*

Proof: Expected second-period effort in the revelation scenario is $pe_2^Y(s) + (1-p)e_2^Y(f)$, which, using (4) and (6) equals $e_2^N(p)$ as given by (2). ∎

Lemma 1 says that, with exogenously given probability of first-period success and quadratic effort costs, interim performance evaluations have no effect on average second period effort. An IPE does, of course, induce the agent to tailor his second-period effort to reflect the marginal return, but since (by assumption) the IPE has no effect on the first-period probability of success, it has no effect on the *expected* marginal return to second-period effort. Since effort costs are quadratic, it then follows that an IPE leaves expected second-period effort unchanged.[4]

3.3 The Effect of IPE's on Prior and Subsequent Efforts

Denote by $u^Y(s)$ and $u^Y(f)$ and $u^N(s)$ and $u^N(f)$ the continuation utilities associated with success and failure in the revelation and no revelation scenarios, respectively. In either scenario i, the difference between the continuation utilities in the two states in the second period, $u^i(s) - u^i(f)$, represents the marginal benefit from increasing effort in the first period. The following result says that, if the probability of success in the first period is no larger than 1/2, this marginal benefit is larger in the revelation scenario than in the no revelation scenario. This is the key to the effort comparison across the two scenarios.

Lemma 2 *Given an exogenously fixed probability of success p in the first period, suppose that the agent chooses effort optimally in the second period. Then $u^Y(s) - u^Y(f) \geq u^N(s) - u^N(f)$ if and only if $p \leq \frac{1}{2}$. Equality holds only if $(w(s,s) - w(s)) = (w(s) - w(f,f))$ or $p = \frac{1}{2}$.*

Proof: To obtain $u^Y(s)$, substitute $e_2^Y(s)$ from (4) into (3). To obtain $u^Y(f)$ substitute $e_2^Y(f)$ from (6) into (5). Subtracting the second expression from the first gives:

$$u^Y(s) - u^Y(f) = (w(s) - w(f,f)) + \frac{1}{2c}((w(s,s) - w(s))^2 - (w(s) - w(f,f))^2) \tag{7}$$

To obtain the difference in utilities among the two states in the no-revelation scenario, substitute the optimal second-period effort (for a fixed p) into payoffs in the two states to obtain

$$\begin{aligned} u^N(s) - u^N(f) \quad = \quad & (w(s) - w(f,f)) + \frac{1}{c}\Big\{p\,[w(s,s) - w(s)]^2 - (1-p)\,[w(s) - w(f,f)]^2 \\ & +(1-2p)\,[w(s,s) - w(s)]\,[w(s) - w(f,f)]\} \end{aligned} \tag{8}$$

[4] With exogenously fixed probability of success in period 1, IPE's decrease (increase) expected second-period effort if the marginal cost of effort is convex (concave).

Define $x = w(s,s) - w(s)$ and $y = w(s) - w(f,f)$. Then expression (7) is greater than (8) if and only if

$$px^2 - (1-p)\,y^2 + (1-2p)\,xy < \frac{x^2}{2} - \frac{y^2}{2}.$$

We can rewrite this inequality as

$$(p - \frac{1}{2})x^2 + (p - \frac{1}{2})y^2 - 2(p - \frac{1}{2})xy < 0,$$

or

$$(p - \frac{1}{2})(x-y)^2 < 0.$$

Hence the result. ∎

The next proposition shows that first-period effort is higher in the revelation scenario.

Proposition 1 *Suppose that the agent is facing an incentive scheme defined by* $(w(s,s), w(s), w(f,f))$. *Then*

(i) First-period effort is higher if information is revealed: $e_1^Y \geq e_1^N$*; with strict inequaltiy if the incentive scheme is nonlinear, i.e.,* $w(s,s) - w(s) \neq w(s) - w(f,f)$.

(ii) $E(e_2^Y) > \ (<)\ e_2^N$ *if and only if* $w(s,s) - w(s) > \ (<)\ w(s) - w(f,f)$*, i.e., expected second-period effort is higher (lower) when information is revealed if the incentive scheme is convex (concave).*

Proof: **Part (i)**: In the revelation scenario, the agent's objective is to choose e_1 to maximize $e_1 u^Y(s) + (1-e_1)u^Y(f) - \frac{1}{2}c\,(e_1)^2$. The first-order condition is

$$u^Y(s) - u^Y(f) = ce_1^Y. \tag{9}$$

In the no-revelation scenario, given the optimal choice of e_2^N giving rise to $u^N(s)$ and $u^N(f)$, the agent chooses e_1 to maximize $e_1 u^N(s) + (1-e_1)u^N(f) - \frac{1}{2}ce_1^2$. The first-order condition is

$$u^N(s) - u^N(f) = ce_1^N. \tag{10}$$

Now observe that, since $c > w(s,s) - w(f,f)$, then

$$e_1^N = e_2^N = \frac{w(s) - w(f,f)}{c + w(s) - w(f,f) - (w(s,s) - w(s))} < \frac{1}{2}. \tag{11}$$

Suppose then that the first-period probability of success is fixed at the optimal effort in the N-scenario, e_1^N. Lemma 2 guarantees that given this probability of success in the first period, the left-hand side of equation (9) exceeds the left-hand side of equation (10). Thus, first-period effort must be higher in the Y-scenario.

Part (ii): In order to compare second-period efforts, observe that at the optimum, from equation (2),

$$e_2^N = \frac{e_1^N(w(s,s) - w(s)) + (1 - e_1^N)(w(s) - w(f,f))}{c}$$

and, from equations (4) and (6),

$$Ee_2^Y = \frac{e_1^Y(w(s,s) - w(s)) + (1 - e_1^Y)(w(s) - w(f,f))}{c}$$

Since, by part (i), $e_1^Y > e_1^N$, then we have that $Ee_2^Y > e_2^N$ if and only if $(w(s,s) - w(s)) > (w(s) - w(f,f))$ ∎

Proposition 1 says that first-period effort is higher under revelation. However, part (ii) of the Proposition says that second- period effort could be eiher higher or lower in the revelation scenario than in the no-revelation scenario depending on the convexity of the incentive scheme.

We now want to complete the effort comparison by comparing total expected efforts over the two periods in the two scenarios. In order to make the comparison, we need the following Lemma.

Lemma 3 *Given an exogenously fixed probability of success p in the first period, suppose that the agent chooses effort optimally in the second period. In the revelation scenario, expected optimal effort in the second period changes less than one for one with p.*

Proof: We need to obtain expected second-period effort in the Y-scenario as a function of p. In order to do this, multiply by p the right-hand side of equation 4, multiply the right-hand side of equation 6 by $(1 - p)$ and add the resulting expressions to obtain

$$E\left(e_2^Y|p\right) = \frac{p(w(s,s) - w(s)) + (1 - p)(w(s) - w(f,f))}{c} \tag{12}$$

Thus,

$$\left|\frac{dE(e_2^Y|p)}{dp}\right| = \left|\frac{(w(s,s) - w(s)) - (w(s) - w(f,f))}{c}\right| < 1.$$

∎

The next proposition shows expected total effort is higher in the revelation scenario and shows that first period effort is higher than second period effort in the Y scenario.

Proposition 2 *Suppose that the agent is facing an incentive scheme defined by $(w(s,s), w(s), w(f,f))$. Then*

(i) Total expected effort is higher if information is revealed: $e_1^Y + E(e_2^Y) \geq e_1^N + e_2^N$;with strict inequality as long as $w(s,s) - w(s) \neq w(s) - w(f,f)$.

(ii) In the revelation scenario,, first period effort is higher than second period effort: $e_1^Y \geq E(e_2^Y)$; with strict inequality as long as $w(s,s) - w(s) \neq w(s) - w(f,f)$.

Proof: **Part (i):**

$$\begin{aligned} e_1^Y - e_1^N &\geq \left|E(e_2^Y|e_1^Y) - E(e_2^Y|e_1^N)\right| \\ &\geq E(e_2^Y|e_1^N) - E(e_2^Y|e_1^Y) \\ &= e_2^N - E(e_2^Y|e_1^Y). \end{aligned}$$

where the inequality in the first line comes from Lemma 3, and the third line comes from Lemma 1. Therefore, $e_1^Y + E(e_2^Y) > e_1^N + e_2^N$.

Part (ii): Observe first that $e_1^N = e_2^N$. Furthermore, by Lemma 1, if we evaluate the expectation of second-period effort in the revelation scenario according to the probability e_1^N, we obtain $Ee_2^Y = e_1^N$. Finally, by Lemma 1, Ee_2^Y increases less than one for one with increases in p. Thus, $e_1^Y > Ee_2^Y$. ∎

Corollary 1 *The expected cost of effort to the agent is higher in the Y scenario.*

Proof: From the above Proposition we have

$$e_1^N \leq \frac{e_1^Y + E(e_2^Y)}{2}.$$

Thus, using first monotonicity, and then concavity, of c, we can write

$$\begin{aligned} c\left(e_1^N\right) &\leq c\left(\frac{e_1^Y + E(e_2^Y)}{2}\right) \\ &\leq \frac{1}{2}c\left(e_1^Y\right) + \frac{1}{2}c\left(E(e_2^Y)\right) \\ &\leq \frac{1}{2}c\left(e_1^Y\right) + \frac{1}{2}E\left(c(e_2^Y)\right) \end{aligned}$$

whence

$$2c\left(e_1^N\right) \leq c\left(e_1^Y\right) + E\left(c(e_2^Y)\right)$$

∎

For future reference, observe that the optimum first period effort in the revelation scenario is

$$e_1^Y = \frac{(w(s) - w(f,f))2c + (w(s,s) - w(f,f))(w(s,s) + w(f,f) - 2w(s))}{2c^2} < \frac{1}{2}. \tag{13}$$

3.4 Discussion

Proposition 2 shows that there is a wide class of incentive schemes that lead the agent to exert more effort in the case where information is revealed to the agent. It would be tempting to conclude from this result that a principal who is interested in eliciting effort from the agent would choose to conduct interim performance evaluations and give all possible feedback to the agent. However, such a conclusion would be premature. In the revelation scenario, the expected wage bill is also higher. To see this, observe that the wage bill is equal to the expected utility of the agent plus the expected cost of effort. The expected utility of the agent is obviously higher in the revelation scenario and, by corollary 1 the total cost of effort is also higher in the revelation scenario. The inequalities are strict if the incentive scheme is nonlinear. Thus, it is not clear whether the principal would prefer to conduct interim performance evaluations. Furthermore, if the principal chooses the compensation scheme as well as the revelation policy, we have to consider the possibility that the optimal scheme in the no revelation scenario may be quite different from the optimal scheme in the revelation scenario.

One exception to this discussion is the case of a subprincipal (say a division manager) who has no control over the compensation scheme, but can choose whether to conduct interim performance evaluations. Our analysis comparing the incentive properties of exogenously fixed incentive schemes suggests that, if the division manager is rewarded on the basis of total output and not on the wage bill, he would choose to reveal information to employees.

We now comment on two assumptions that were made in the analysis of this section. The first is the assumption that $w(s,s) < c$. This assumption guaranteed interior solutions. If $w(s,s) > c$, then effort would be identical in the two scenarios: the agent would choose $e_1^N = e_2^N = e_1^Y = e_2^Y(s) = 1$. While the case of $w(s,s) = c$ seems unimportant, it turns out to be part of the optimal scheme in the case of no revelation for the case of quadratic costs. However, when analyzing exogenously given incentive schemes, it seems appropriate to ignore this case because when $w(s,s) = c$, effort is indeterminate: the agent is indifferent between all effort levels.

The second is the assumption of quadratic effort cost. If effort costs are not quadratic, then the analysis is more complex. As mentioned in footnote 3.2, for an exogenously fixed probability of success in the first period, expected effort in the second period is lower (higher) in the revelation scenario depending on whether the marginal cost of effort is convex (concave). However, we could find no general conditions guaranteeing a ranking of total expected efforts in the two scenarios.

The next section will not assume quadratic costs.

4 Optimal Incentive Schemes

We now allow the principal to choose the incentive scheme optimally in the two scenarios, we compare the properties of the incentive schemes in the two scenarios, and we look at which scenario is preferred by the principal. We dispense with the assumption of a quadratic cost function and the assumption that the incentive scheme must depend only on the number of successes. It is clear that in the optimal incentive scheme, $w(f, f) = 0$. Thus, from now on we will focus only on the remaining three values of the compensation scheme.

4.1 The Optimal Incentive Scheme in the No-Revelation Scenario

Let us express the problem of an agent in the following way

$$\begin{aligned} & \max_{e_1, e_2} U(e_1, e_2) \\ = \; & \max_{e_1, e_2} \alpha e_1 + \beta e_2 + \gamma e_1 e_2 - c(e_1) - c(e_2). \end{aligned} \tag{14}$$

The α, β, γ's are synthetic parameters which, in our problem must be a function of $w(s, f)$, $w(f, s)$, and $w(s, s)$. Specifically, $\alpha = w(s, f)$, $\beta = w(f, s)$ and $\gamma = w(s, s) - w(s, f) - w(f, s)$. The limited liability assumption implies that α and β are restricted to be nonnegative. We now discuss how the principal would choose a compensation scheme that depends on e_1, e_2, and $e_1 \cdot e_2$, two linear terms and a mixed term. It will become clear how such a scheme can be implemented with the instruments available to the principal. We assume that $c'(0) = 0, c'(1) = \infty$. These conditions guarantee that the optimal e_1 and e_2 are interior.

Let us first solve for the case in which the principal sets $\beta = \alpha$, meaning that the reward for just one success is independent of whether the success happened in period 1 or 2 (we will soon show that this contract is indeed optimal for the principal.) Provided that problem (14) is concave in e_1, e_2 (requiring $c''' \geq 0$), the agent will choose the same effort in both periods. Denote this effort by e. Then problem (14) becomes

$$\max_e 2\alpha e + \gamma e^2 - 2c(e).$$

The first-order condition for the agent is

$$2\left[\alpha + \gamma e - c'(e)\right] = 0.$$

Denote the solution by e^*. Integrating this expression over e between zero and e^* yields the surplus that the principal must allow the agent in order to implement e^* in the two periods. The per-period surplus enjoyed by the agent is depicted in Figure 1 as the area between the thin straight line originating at α, and the curve $c'(e)$. Adding the cost of exerting effort, i.e., the integral under the curve $c'(e)$, yields the expected per-period cost to the principal of implementing e^*.

Figure 1:

Any line going through e^* and with intercept greater than 0 corresponds to a contract that implements e^*. It is clear from Figure 1 that the contract that minimizes the per-period cost to the principal is the contract in which $\alpha = 0$. In this case the per-period cost is the area of the triangle $(0, c'(e^*), e^*)$. The expected total cost of implementing (e^*, e^*) is double the area of that triangle, i.e., exactly the area of the rectangle $(0, A, c'(e^*), e^*)$. Denoting by $R(e)$ the area of the rectangle with base of e and height of $c'(e)$,

$$R(e) \equiv \int_0^e c'(e)\, dy,$$

then the expected total cost to the principal of implementing e^*, e^* is simply $R(e^*)$.

Now let us verify that indeed the optimal contract indeed entails $\alpha = \beta$. Suppose not, and without loss of generality suppose that it were optimal to set $\alpha < \beta$. Then it must be that $e_1^* < e_2^*$. Write the agent's surplus as

$$U(e_1^*, e_2^*) = U(e_1^*, 0) + \int_0^{e_2^*} \frac{\partial U(e_1^*, y)}{\partial e_2} dy.$$

In light of the first-order condition and the fact that the compensation scheme is linear in e_2, we can rewrite the above equation as

$$U(e_1^*, e_2^*) = U(e_1^*, 0) + \int_0^{e_2^*} \left[c'(e_2^*) - c'(y)\right] dy.$$

Adding the agent's cost of effort yields the expense to the principal of implementing e_1^*, e_2^*. That equals

$$\begin{aligned} & \alpha e_1^* + \int_0^{e_2^*} c'\left(e_2^*\right) dy \\ = \quad & \alpha e_1^* + R\left(e_2^*\right). \end{aligned}$$

Notice that for a lesser expense of just $R\left(e_2^*\right)$ the principal can implement e_2^*, e_2^* (a strictly larger total effort) by setting $\alpha = \beta = 0$ and appropriately adjusting γ. This shows that it is suboptimal for the principal to set $\alpha < \beta$.

We collect these arguments in the following proposition.

Proposition 3 *Assume $c''' > 0$. In the no-revelation scenario, the contract that implements a given total effort E at the lowest cost entails rewarding the agent only in the case of two successes. Thus, $w(s,f) = w(f,s) = 0$. The expected payment by the principal to the agent in the optimal contract implementing a total effort of E is $R\left(E/2\right) = c'\left(E/2\right) \cdot E/2$.*

To complete our characterization, we consider what happens if the principal wants to implement different efforts in the two periods, say because effort is more valuable to him in one period than]the other. The following proposition characterizes the optimal incentive scheme that implements $0 < e_1 < e_2$. The case $0 < e_2 < e_1$ has the complementary properties.

Proposition 4 *Assume $c''' > 0$. In the no-revelation scenario, the contract that implements an effort profile $0 < e_1 < e_2$ at the lowest cost entails setting $w(f,s) = 0$ and $w(s,s) > w(s,f) > 0$.*

4.2 The Optimal Incentive Scheme in the Revelation Scenario

In the revelation scenario, the agent is able to condition the choice of the second-period effort on the outcome of the first period effort. The principal, similarly, can give different second-period incentives depending on whether the first period effort resulted in success. We shall now describe some properties of the optimal incentive scheme in the revelation scenario.

Proposition 5 *Suppose the principal wants to implement an effort of $e_1 \in (0,1)$ in the first period and an expected effort of $E_2 > 0$ in the second period. Then the compensation scheme that minimizes the principal's expected cost induces the agent to exert a strictly positive effort after a faliure, but to work even harder after a success: $e_2(s) > e_2(f) > 0$.*

Proof: Given an incentive scheme described by $w(s,s), w\left(s,f\right)$ and $w(f,s)$, the agent's second-period effort choices satisfy:

$$w(s,s) - w(s,f) = c'(e_2(s)) \text{ and } w(f,s) = c'(e_2(f)),$$

so the utility of the agent conditional on a success can be written as

$$u(s) = w(s, f) + c'(e_2(s))e_2(s) - c(e_2(s)) \tag{15}$$

and conditional on failure

$$u(f) = c'(e_2(f))e_2(f) - c(e_2(f)) \tag{16}$$

The agent's first-period effort satisfies $u(s) - u(f) = c'(e_1)$. For any effort triple $(e_1, e_2(s), e_2(f))$ the principal chooses to induce, the necessary wage payments are determined by the equations above.

The principal's objective is to minimize expected wage payments subject to inducing a period-1 effort e_1 and period-2 expected effort $E_2 = e_1 e_2(s) + (1 - e_1)e_2(f)$. The principal's expected total cost is

$$TC = U + c(e_1) + E(c(e_2)),$$

where $U = e_1 u(s) + (1 - e_1)u(f) - c(e_1)$ and $E(c(e_2) = e_1 c(e_2(s)) + (1 - e_1)c(e_2(f))$. TC can be rewritten as

$$\begin{aligned} TC &= e_1 c'(e_1) + u(f) + E(c(e_2)) \\ &= e_1 c'(e_1) + c'(e_2(f))e_2(f) + e_1(c(e_2(s)) - c(e_2(f))) \end{aligned} \tag{17}$$

using the agent's first-order conditions above as well as expression (16). The principal chooses $e_2(s)$ and $e_2(f)$ to minimize TC subject to $e_1 e_2(s) + (1 - e_1)e_2(f) = E_2$. Using this constraint to substitute for $e_2(s)$ in TC and differentiating with respect to $e_2(f)$ yields the first-order condition

$$c''(e_2(f))e_2(f) + (1 - e_1)(c'(e_2(f)) - c'(e_2(s)) = 0 \tag{18}$$

This implies that $e_2(s) > e_2(f)$ since $c'' > 0$. Moreover, since at $e_2(f) = 0$ the left-hand side of equation (18) is strictly negative, it follows that $0 < e_2(f) < e_2(s)$. ∎

Proposition 5 shows that in the revelation scenario, the principal will choose to distort the effort of the agent in the second period away from what would be optimal in the absence of moral hazard. Specifically, the agent is induced to work harder after a success than after a failure despite the fact that this variation in effort per se raises the agent's ecpected effort costs. This distortion is optimal because it makes it cheaper for the principal to provide incentives for effort in the first period: The marginal reward to e_1 is $u(s) - u(f) = w(s, f) + [c'(e_2(s))e_2(s) - c(e_2(s))] - [c'(e_2(f))e_2(f) - c(e_2(f))]$, so for any e_1, the larger the gap between $e_2(s)$ and $e_2(f)$, the smaller the value of $w(s, f)$ required to make $u(s) - u(f) = c'(e_1)$.

An implication of Proposition 5 is that the probability of success is positively correlated across periods.

If the principal wants to implement a total expected effort of E and does not care directly about how the agent allocates his effort across periods, then we can show that, at the optimum, $0 < e_2(f) < e_1 < e_2(s)$.

Proposition 6 *Suppose the principal wants to implement a total expected effort of* $E = e_1 + e_1 e_2(s) + (1 - e_1)e_2(f)$. *Then the compensation scheme that minimizes the principal's expected cost induces the agent to choose efforts satisfying* $0 < e_2(f) < e_1 < e_2(s)$.

Proof: By Proposition 5, we already know that $0 < e_2(f) < e_2(s)$. We first show that $e_1 > e_2(f)$. The principal chooses $e_1, e_2(s)$, and $e_2(f)$ to minimize expression 17 subject to $e_1 + e_1 e_2(s) + (1 - e_1)e_2(f) = E$. Using this constraint to substitute for $e_2(s)$ in 17 and differentiating with respect to e_1 yields the following first-order condition

$$c'(e_1) + c''(e_1)e_1 - c'(e_2(s))(1 + e_2(s) - e_2(f)) + (c(e_2(s)) - c(e_2(f))) = 0. \tag{19}$$

The first-order condition with respect to $e_2(f)$ is the same as when e_1 is exogenously given, namely equation (18). Subtracting (18) from (19) yields

$$\begin{aligned} & c'(e_1) + c''(e_1)e_1 - (c'(e_2(f)) + c''(e_2(f))e_2(f))) \\ = \; & c'(e_2(s))(e_2(s) - e_2(f)) - (c(e_2(s)) - c(e_2(f))) + e_1(c'(e_2(s)) - c'(e_2(f))) > 0 \end{aligned}$$

because $e_2(s) > e_2(f)$ and c is strictly convex. Since $c'(e) + c''(e)e$ is strictly increasing if $c''' \geq 0$, it follows that $e_1 > e_2(f)$. Furthermore, it follows from $0 < e_2(s) - e_2(f) < 1$, $c''' \geq 0$, and equation (19) that

$$2c'(e_1) < c'(e_1) + c''(e_1)e_1 = c'(e_2(s))(1 + e_2(s) - e_2(f)) - (c(e_2(s)) - c(e_2(f))) < 2c'(e_2(s))$$

and therefore, since c is strictly convex, $e_1 < e_2(s)$. ∎

4.3 Comparing the Two Scenarios

The problem of the agent in the revelation scenario can be written as

$$\begin{aligned} & \max_{e_1, e_2^F, e_2^S} U\left(e_1, e_2^F, e_2^S\right) \\ = \; & \max_{e_1, e_2^F, e_2^S} \left[\delta e_1 - c\left(e_1\right)\right] + \left(1 - e_1\right)\left[\zeta e_2(f) - c\left(e_2(f)\right)\right] + e_1\left[\theta e_2(s) - c\left(e_2(s)\right)\right]. \end{aligned} \tag{20}$$

Our goal is to show that for any constellation of $e_1^*, e_2^*(f)$, and $e_2^*(s)$ giving rise to a total exepcted effort E, the same total expected effort can be implemented more cheaply in the no- revelation scenario.

Proposition 7 *Assume* $2c''(e) + ec'''(e) > 0$. *Achieving a total expected effort of* E *under revelation has an expected cost to the principal of more than* $R(E/2)$. *Thus, it is less costly to implement any given total expected if information is not revealed.*

Proof: **Case $e_1^* > E/2$.**

From the first order conditions with respect to e_1 we have

$$\frac{\partial U\left(e_1, e_2^*(f), e_2^*(s)\right)}{\partial e_1} = c'\left(e_1^*\right) - c'\left(e_1\right).$$

We can then express the agent's surplus as

$$\begin{aligned} U\left(e_1^*, e_2^{*F}, e_2^{*S}\right) &= U\left(0, e_2^*(f), e_2^*(s)\right) + \int_0^{e_1^*} \frac{\partial U\left(e_1, e_2^*(f), e_2^*(s)\right)}{\partial e_1} de_1 \\ &= U\left(0, e_2^*(f), e_2^*(s)\right) + \int_0^{e_1^*} \left[c'\left(e_1^*\right) - c'\left(e_1\right)\right] de_1. \end{aligned}$$

Adding to this expression the cost of effort, which is at least $c\left(e_1^*\right)$, yields the cost to the principal of implementing $\left(e_1^*, e_2^*(f), e_2^*(s)\right)$. The cost to the principal is therefore not smaller than

$$\begin{aligned} & U\left(0, e_2^*(f), e_2^*(s)\right) + R\left(e_1^*\right) \\ > \ & U\left(0, e_2^*(f), e_2^*(s)\right) + R\left(E/2\right). \end{aligned}$$

The inequality follows from the assumption that $e_1^* > E/2$. The term $U\left(0, e_2^*(f), e_2^*(s)\right)$ represents the expected surplus of an agent who has exerted no effort in the first period; it is nonnegative by individual rationality. This shows that a total expected effort of E is cheaper to achieve via no revelation.

Case $e_1^* \leq E/2$.

Write the agent's surplus as

$$\begin{aligned} & U\left(e_1^*, e_2^*(f), e_2^*(s)\right) \\ = \ & U\left(e_1^*, 0, 0\right) + \int_0^{e_2^*(f)} \frac{\partial U\left(e_1^*, e_2(f), 0\right)}{\partial e_2^F} de_2(f) + \int_0^{e_2^*(s)} \frac{\partial U\left(e_1^*, e_2^*(f), e_2(s)\right)}{\partial e_2^S} de_2(s) \end{aligned}$$

Make use of the first order conditions to rewrite the agent's surplus as

$$U\left(e_1^*, 0, 0\right) + \left(1 - e_1^*\right) \int_0^{e_2^*(f)} \left[c'\left(e_2^*(f)\right) - c'\left(e_2(f)\right)\right] de_2(f) + e_1^* \int_0^{e_2^*(s)} \left[c'\left(e_2^*(s)\right) - c'\left(e_2(s)\right)\right] de_2(s).$$

Adding the cost of effort to this expression yields the cost to the principal of implementing $\left(e_1^*, e_2^*(f), e_2^*(s)\right)$, which is

$$\delta e_1^* + \left(1 - e_1^*\right) R\left(e_2^*(f)\right) + e_1^* R\left(e_2^*(s)\right).$$

The function $R\left(e\right)$ is strictly convex when $2c''\left(e\right) + ec'''\left(e\right) > 0$. This means that the average area of the two rectangles in Figure 2 is larger than the area of the rectangle with average base. Then the cost to the principal of implementing $\left(e_1^*, e_2^*(f), e_2^*(s)\right)$ is strictly greater than

$$\delta e_1^* + R\left(\left(1 - e_1^*\right) e_2^*(f) + e_1^* e_2^*(s)\right).$$

Figure 2:

Since by assumption $e_1^* \leq E/2$, it must be $(1-e_1^*)\, e_2^*(f) + e_1^* e_2^*(s) \geq E/2$. Therefore, the previous expression is greater than

$$\delta e_1^* + R\,(E/2)\,.$$

The term $R\,(E/2)$ represents the cost of implementing E in the no-revelation scenario. Since $\delta \geq 0$, that cost is smaller than the cost to the principal of implementing $(e_1^*, e_2^*(f), e_2^*(s))$ under revelation. ∎

Note that the proof of Proposition 7 can be readily adapted to prove the following:

Proposition 8 *Suppose the principal wants to implement an effort of e_1 in the first period and an expected effort of E_2 in the second period. This can be done at lower cost when information is **not** revealed to the agent.*

4.4 Example where revelation is optimal

Consider the following cost function:

$$c(e) = \begin{array}{cc} \frac{e^2}{2} & e \leq \widehat{e} \\ e\widehat{e} - \frac{(\widehat{e})^2}{2} & e > \widehat{e} \end{array} \tag{21}$$

with corresponding marginal cost:

$$c'(e) = \left\{ \begin{array}{cc} e & e < \widehat{e} \\ \widehat{e} & e > \widehat{e} \end{array} \right\}$$

We will show that, for this cost function, the principal's total cost of implementing a total effort over two periods of at least $2\widehat{e}$ is lower if the principal reveals information to the agent. This is driven by the fact that the first order approach fails, and the first order conditions of the agent are not sufficient to guarantee that incentive constraints are satisfied. Once global incentive constraints are taken into account, the power of the incentive scheme under no revelation becomes very weak, and this gives the edge to revelation.

Claim: Assume that the agent's cost function is given by equation (21), and that the principal wants to implement a total effort over two periods of at least $2\widehat{e}$. Then, in the optimal incentive scheme in the no-revelation scenario, $w(s,s) - w(s,f) - w(f,s) = 0$.

This claim can easily be verified in the case in which the effort to be implemented is e^* in both periods, i.e. the principal wants the agent to implement the same effort in both periods. Indeed, assume that $w(s,s) - w(s,f) - w(f,s) > 0$, then consider a choice of effort of e for the agent in both periods. The second order conditions for this common effort are: $w(s,s) - w(s,f) - w(f,s) - c''(e^*) < 0$. but, if $e^* > \widehat{e}$, $c''(e^*) = 0$, and if $w(s,s) - w(s,f) - w(f,s) > 0$, the first order conditions describe a local minimum. The case in which efforts to be implemented are different in the two periods the logic is somewhat more complex but quite similar nevertheless. Since the costs to the principal can easily be shown to be decreasing in $w(s,s) - w(s,f) - w(f,s)$, this proves the above claim.

Given this claim, it is easy to show that the principal is better off in the revelation scenario. Because of the claim, in the no revelation scenario we have $w(s,s) - w(s,f) = w(f,s)$. This wage profile implements exactly the same effort in the revelation scenario since the marginal benefit of effort to the agent is the same in both states of the second period. Thus, this effort profile is implementable at the same costs in the two scenarios. However, we know from Proposition 5 that, in the revelation scenario, the principal benefits from choosing different efforts in the two states. Thus, the principal does better under revelation.

It turns out that even in the no revelation scenario, once the first order approach fails, it can be optimal to implement different efforts in the two periods. To see this, note that to implement $0 < e_1 < \widehat{e}$, $1 > e_2 \geq \widehat{e}$, we must have: $w(f,s) = \widehat{e}$, $w(s,f) = e_1$. This leads to a total cost to the principal of $TC = e_2\widehat{e} + e_1^2$ where $e_1 + e_2 = 2\widehat{e}$. Thus, the optimal profile is obtained by $e_1 = \frac{\widehat{e}}{2}, e_2 = \frac{3\widehat{e}}{2}$

Thus, total cost in the N-scenario are $TC^N = \frac{3\widehat{e}^2}{2} + \frac{\widehat{e}^2}{4} = \frac{7}{4}\widehat{e}^2$

In the revelation scenario, consider the following effort profile: $e_1 = \widehat{e}, e_2^S = 1, e_2^F = 0$. To obtain $e_2^S = 1$, we must set $w(s,s) - w(s,f) = c'(1) = \widehat{e}$. This leads to a utility for the agent conditional on success of $u^S = w(s,s) - \left(\widehat{e} - \frac{(\widehat{e})^2}{2}\right) = w(s,f) + \frac{(\widehat{e})^2}{2}$

Furthermore, to obtain $e_1 = \widehat{e}$, we must have $u^S - u^F = \widehat{e}$ or $w(s,s) = 2\widehat{e} - \frac{(\widehat{e})^2}{2}, w(s,f) + \frac{(\widehat{e})^2}{2} = \widehat{e}$. Thus, $w(s,f) = \widehat{e} - \frac{(\widehat{e})^2}{2}, w(s,s) = 2\widehat{e} - \frac{(\widehat{e})^2}{2}$

This leads to $TC^Y = \widehat{e}w(s,s) = 2\widehat{e}^2 - \frac{(\widehat{e})^3}{2}$. It can easily be verified that $TC^Y < TC^N$ if $\frac{1}{4}\widehat{e}^2 - \frac{1}{2}\widehat{e}^3 < 0$ i.e., $\frac{1}{2}\widehat{e}^2(\frac{1}{2} - \widehat{e}) < 0$ i.e.,$\widehat{e} > \frac{1}{2}$.

4.5 Ability

We now add to the model a component of ability. Agents can be more or less able. Ability translates into higher value of effort to the principal: effort exerted by a more able agent is more valuable to the principal. Formally, the value to the principal of effort e from agent of ability a is given by the function $v(e,a)$ which is increasing in a. In this formulation, a good agent is not more likely than a bad agent to succeed, nor is his effort cheaper.

After the first period effort has been exerted, the principal draws a signal that is informative about the agent's ability. This signal is separate from and independent of the signal about effort, which as before is thought of as success or failure.

As before, we allow the contract to be conditional on the agent's revealed ability. The question for the principal is whether, in this new scenario, there should be interim evaluations. If so, what kind of evaluation should this be, i.e., should it reveal to the agent how well his effort turned out and/or reveal the signal about his ability?

We show that, while an the interim evaluation is generally preferable to no interim evaluation, the subject of the evaluation should be the ability of the candidate, not how well his or her effort turned out. Formally, we show that revealing how the effort turned out is dominated by not revealing.

To this end, we first study the case in which ability is revealed but effort is not. We examine the case in which the agent is rewarded only for two successes.

Proposition 9 *Suppose the principal give an interim evaluation that only reveals the agent's ability a but not whether the agent succeeded or failed in the first period. Suppose further that the agent is rewarded only for two successes. Then the cost of implementing any implementable plan $e_1^*, \mathbf{e}_2^*$ is $R(e_1^*) = E_a[R(e_2^*(a))]$.*

Remark 1 *By the equality, and since R is convex, we obtain $e_1^* \geq E_a[e_2^*(a)]$. Thus, the agent's expected effort (though not necessarily its value to the principal) is greater in the first period.*

Proof of the Proposition

Denote with $\gamma(a)$ the reward for two successes that implements $e_1^*, e_2^*(a)$. Notice that this reward depends on the ability of the agent as revealed by the signal a. Given this system of rewards, the agent's utility from taking plan $e_1, e_2(a)$ is

$$U(e_1, \mathbf{e}_2) = E_a [e_1 \cdot e_2(a) \cdot \gamma(a) - c(e_2(a))] - c(e_1)$$

Write

$$\begin{aligned} U(e_1^*, \mathbf{e}_2^*) &= U(0, \mathbf{e}_2^*) + \int_0^{e_1^*} \frac{\partial U(e_1, \mathbf{e}_2^*)}{\partial e_1} de_1 \\ &= E_a [-c(e_2^*(a))] + \int_0^{e_1^*} \left[c'(e_1^*) - c'(e_1)\right] de_1. \end{aligned}$$

Adding the expected cost of effort yields the cost to the principal of implementing the action plan, which equals $R(e_1^*)$. Conversely, write

$$\begin{aligned} U(e_1^*, \mathbf{e}_2^*) &= U(e_1^*, \mathbf{0}) + E_a \left\{ \int_0^{e_2^*(a)} \left[c'(e_2^*(a)) - c'(e_2(a))\right] de_2(a) \right\} \\ &= -c(e_1^*) + E_a \left\{ \int_0^{e_2^*(a)} \left[c'(e_2^*(a)) - c'(e_2(a))\right] de_2(a) \right\}. \end{aligned}$$

Adding the expected cost of effort yields the cost to the principal of implementing the action plan, which equals $E_a [R(e_2^*(a))]$.

Proposition 10 *Suppose the principal give an interim evaluation that only reveals the agent's ability a but not whether the agent succeeded or failed in the first period. Then it is optimal for the principal to reward the agent only for two successes.*

Proof of the Proposition

Consider any action plan $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$ that can be implemented by rewarding more outcomes than just two successes, and denote the agent's utility by

$$U(\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*) = \widetilde{\alpha}\widetilde{e}_1^* - c(\widetilde{e}_1^*) + E_a \left[\widetilde{\beta}(a) \cdot \widetilde{e}_2^*(a) - c(\widetilde{e}_2^*(a))\right] + E_a [\widetilde{\gamma}(a) \cdot \widetilde{e}_1^* \cdot \widetilde{e}_2^*(a)].$$

Performing the usual transformations yields

$$U(\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*) = \widetilde{\alpha}\widetilde{e}_1^* + E_a [R(\widetilde{e}_2^*(a))] = E_a \left[\widetilde{\beta}(a) \cdot \widetilde{e}_2^*(a)\right] + R(\widetilde{e}_1^*).$$

Now we construct $e_1^*, \mathbf{e}_2^*$, an action plan that is implementable by rewarding only two successes and has greater or equal expected value as $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$. Start from a reward scheme that only rewards two successes and in which the vector $\gamma(a)$ is chosen so that the resulting vector of second period efforts $\widetilde{\mathbf{e}}_2^*$. Now, look at the resulting first period effort level. Two scenarios are possible. Either the first period effort is larger or equal than $\widetilde{e}_1^*$, in which case we denote the resulting action plan by $e_1^*, \mathbf{e}_2^*$. The action plan $e_1^*, \mathbf{e}_2^*$ gives the agent greater expected value than $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$ and, by Proposition 9, costs $E_a\left[R\left(e_2^*\left(a\right)\right)\right] = E_a\left[R\left(\widetilde{e}_2^*\left(a\right)\right)\right]$. This cost is no greater than what it costs to implement $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$, which proves our claim that it is optimal for the principal to reward the agent only for two successes.

In the second scenario, the first period effort associated with the reward scheme γ is smaller than $\widetilde{e}_1^*$. This means that the expected value to the principal under scheme γ is smaller than the expected value of $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$. Then, increase the vector γ along all its components, and keep doing this until the resulting expected value of the effort taken by the agent equals the expected value of $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$. Denote the resulting effort levels by $e_1^*, \mathbf{e}_2^*$. Notice that since by construction $e_2^*\left(a\right) > \widetilde{e}_2^*\left(a\right)$ for all a, therefore $e_1^* < \widetilde{e}_1^*$. But then the expected cost of implementing $e_1^*, \mathbf{e}_2^*$ which, by Proposition 9, equals $R\left(e_1^*\right)$, is smaller than $R\left(\widetilde{e}_1^*\right)$ and thus smaller than the cost of implementing $\widetilde{e}_1^*, \widetilde{\mathbf{e}}_2^*$.

Now we want to show that, assuming that the principal reveals the agent's ability in the interim evaluation, then a given expected value of effort can be more cheaply implemented by not disclosing in the interim evaluation whether the first period effort resulted in success or failure.

Proposition 11 *Given any action plan $e_1^{*R}, \mathbf{e}_2^*(f), \mathbf{e}_2^*(s)$ that is implementable with revelation of success, there is plan $e_1^*, \mathbf{e}_2^*$ that is implementable under no revelation and gives the principal an expected value at least as large.*

Proof of Proposition

Write

$$\begin{aligned} U\left(e_1^R, \mathbf{e}_2(f), \mathbf{e}_2(s)\right) &= \delta e_1^R - c\left(e_1\right) + \left(1 - e_1^R\right) E_a\left[e_2\left(f,a\right)\zeta\left(a\right) - c\left(e_2\left(f,a\right)\right)\right] \\ &\quad + e_1^R E_a\left[e_2\left(s,a\right)\theta\left(a\right) - c\left(e_2\left(s,a\right)\right)\right]. \end{aligned}$$

Denote $\overline{\mathbf{e}}_2^* = \left(1 - e_1^{*R}\right)\mathbf{e}_2^*(f) + e_1^{*R}\mathbf{e}_2^*(s)$ as the ability-indexed vector of expected efforts in period 2 in the case of revelation. Pick the vector $\gamma\left(a\right)$ so that the second period effort vector in the case of no revelation equals $\overline{\mathbf{e}}_2^*$. Then, look at the corresponding first period effort in the case of no revelation, e_1. If the resulting value of effort to the principal is greater with no revelation than with revelation, then the resulting allocation in the no revelation case is our candidate plan $e_1^*, \mathbf{e}_2^*$. We call this configuration Case A.

If, instead, the the resulting value of effort to the principal is smaller with no revelation than with revelation (which must mean that $e_1 < e_1^{*R}$), then increase all the components of the vector $\gamma\left(a\right)$ until the resulting value of effort to the principal is with no revelation equals that with revelation. The resulting vector of effort in the no-revelation case is our candidate plan $e_1^*, \mathbf{e}_2^*$. Note that by construction in this case we have $e_1^* < e_1^{*R}$. We call this configuration Case B.

Case A.

Write

$$\begin{aligned} U\left(e_1^{*R}, \mathbf{e}_2^{*F}, \mathbf{e}_2^{*S}\right) &= U\left(e_1^{*R}, \mathbf{0}, \mathbf{0}\right) + \left(1 - e_1^{*R}\right) E_a \left[\int_0^{e_2^*(f,a)} \left[c'\left(e_2^*(f,a)\right) - c'\left(e_2(f,a)\right)\right] de_2(f,a)\right] \\ &\quad + e_1^{*R} E_a \left[\int_0^{e_2^{*S}(a)} \left[c'\left(e_2^*(s,a)\right) - c'\left(e_2(s,a)\right)\right] de_2(s,a)\right] \\ &= \delta e_1^{*R} - c\left(e_1^{*R}\right) + \left(1 - e_1^{*R}\right) E_a \left[\int_0^{e_2^*(f,a)} \left[c'\left(e_2^*(f,a)\right) - c'\left(e_2(f,a)\right)\right] de_2(f,a)\right] \\ &\quad + e_1^{*R} E_a \left[\int_0^{e_2^*(s,a)} \left[c'\left(e_2^*(s,a)\right) - c'\left(e_2(s,a)\right)\right] de_2(s,a)\right]. \end{aligned}$$

Adding the cost of effort yields the cost to the principal of implementing $e_1^{*R}, \mathbf{e}_2^*(f), \mathbf{e}_2^*(s)$, which is

$$\delta e_1^{*R} + \left(1 - e_1^{*R}\right) E_a \left[R\left(e_2^*(f,a)\right)\right] + e_1^{*R} E_a \left[R\left(e_2^*(s,a)\right)\right].$$

Because R is convex, this is greater than

$$\begin{aligned} &\delta e_1^{*R} + E_a \left[R\left(\left(1 - e_1^{*R}\right) e_2^*(f,a) + e_1^{*R} e_2^*(s,a)\right)\right] \\ = \; &\delta e_1^{*R} + E_a \left[R\left(\overline{e}_2^*(a)\right)\right]. \end{aligned}$$

This is not smaller than $E_a\left[R\left(\overline{e}_2^*(a)\right)\right]$, the cost of implementing the plan $e_1^*, \mathbf{e}_2^*$ under no revelation. Since that plan gives the principal an expected value of effort at least as large as that in the revelation case, we have proved our claim.

Case B.

Write

$$\begin{aligned} U\left(e_1^{*R}, \mathbf{e}_2^*(f), \mathbf{e}_2^*(s)\right) &= U\left(0, \mathbf{e}_2^*(f), \mathbf{e}_2^*(s)\right) + \int_0^{e_1^{*R}} \left[c'\left(e_1^{*R}\right) - c'\left(e_1^R\right)\right] de_1^R \\ &= E_a\left[e_2^*(f,a)\,\zeta(a) - c\left(e_2^*(f,a)\right)\right] + \int_0^{e_1^{*R}} \left[c'\left(e_1^{*R}\right) - c'\left(e_1^R\right)\right] de_1^R. \end{aligned}$$

The first term is nonnegative by individual rationality. Adding the cost of effort, which is at least $c\left(e_1^{*R}\right)$, yields the cost to the principal of implementing $e_1^{*R}, \mathbf{e}_2^*(f), \mathbf{e}_2^*(s)$, which is therefore not smaller than

$$\int_0^{e_1^{*R}} \left[c'\left(e_1^{*R}\right)\right] de_1^R = R\left(e_1^{*R}\right).$$

Since by construction we have $e_1^* < e_1^{*R}$, this quantity is strictly greater than $R\left(e_1^*\right)$, which is the cost to the principal of implementing the plan $e_1^*, \mathbf{e}_2^*$ under no revelation. Since that plan gives the principal an expected value of effort at least as large as that in the revelation case, we have proved our claim.

Example 1 ***The Value of Interim Evaluations***. *Suppose that ability a can be 0 or 2, with equal probability. Suppose further that $v(e,a) = e \cdot a$. In the absence of interim evaluations about ability (we have shown already that interim evaluations about effort are suboptimal), whatever incentive scheme the principal offers that is a function of a will be perceived as its expected value by the agent. Thus, the agent's effort will not depend on information about his ability and the situation is like that in Proposition 3. So, the optimal plan for the principal is to implement the same effort in both periods, call it e^*, e^*. The expected value of this effort in the first period is e^*, in the second period is $\frac{1}{2} \cdot 2e^* + \frac{1}{2} \cdot 0 = e^*$, so the expected value in total is $2e^*$ and that is achieved at cost $R(e^*)$.*

Suppose now that the principal implements the following effort scheme with revelation of ability (but not of effort). The principal will pay the able agent γ in case of two successes, and zero otherwise. The unable agent receives zero in any case. It is clear that an agent who learns that he is unable will exert no effort in the second period. So, letting $\widehat{e}_2$ denote the agent's effort in the second period, the agent solves

$$\max_{\widehat{e}_1,\widehat{e}_2} \widehat{e}_1 \frac{1}{2} \left(\widehat{e}_2 \gamma - c\left(\widehat{e}_2\right)\right) - c\left(\widehat{e}_1\right)$$

The resulting value of effort is $\widehat{e}_1 + \widehat{e}_2$. Pick γ so that $\widehat{e}_1 + \widehat{e}_2$ equals $2e^*$. The cost of implementing $\widehat{e}_1 + \widehat{e}_2$ is, by Proposition 9, $R(\widehat{e}_1)$. So, if we are able to show that $\widehat{e}_1 \leq \widehat{e}_2$ then it follows that $\widehat{e}_1 \leq e^*$ and so it is cheaper to implement $\widehat{e}_1 + \widehat{e}_2$ with revelation of ability rather than e^*, e^* without revelation. To verify that $\widehat{e}_1 \leq \widehat{e}_2$ inspect the first order conditions that determine $\widehat{e}_1$ and $\widehat{e}_2$,

$$\begin{aligned} \frac{1}{2}\left[\widehat{e}_2 \gamma - c\left(\widehat{e}_2\right)\right] &= c'\left(\widehat{e}_1\right) \\ \gamma &= c'\left(\widehat{e}_2\right). \end{aligned}$$

Since $\widehat{e}_2 \leq 1$ then $\frac{1}{2}\left[\widehat{e}_2 \gamma - c\left(\widehat{e}_2\right)\right] < \gamma$, which implies that $\widehat{e}_1 < \widehat{e}_2$.

5 Multi-Outcome Model (Incomplete)

We now generalize the analysis to the case in which there are more than two outcomes in each period. Assume that in each period there are N outcomes denoted by $i = 1, ..., N$ outcomes. Denote

$$p_i(e) \equiv \Pr[\text{outcome } i \text{ is realized given effort } e].$$

We assume that there is an i_0 with the property that $p_i'(e) \leq 0$ iff $i \leq i_0$ and $p_i'(e) \geq 0$ iff $i > i_0$. In other words, low-indexed outcomes are such that an increase in effort reduces the probability of those outcomes, whereas high-indexed outcomes are such that effort increases the probability of those outcomes.

5.1 The one-period problem via first order approach

An incentive scheme in the one period problem is a vector of payments $\mu = [\mu_1, ..., \mu_N]$ that specifies a payment for each outcome realization. Given an incentive scheme μ and a probability vector p, denote the agent's surplus from taking action e as

$$S\left(e, \sum_i \mu_i p_i\right) = \left(\sum_i \mu_i p_i(e)\right) - c(e).$$

Given the incentive scheme, the agent solves

$$\max_e S\left(e, \sum_i \mu_i p_i\right).$$

At an interior optimum e^* the first order conditions must be satisfied, i.e.

$$\sum_i \mu_i p_i'(e^*) = c'(e^*). \tag{22}$$

Assume that the principal wishes to implement e^*. Then, the principal's problem is to minimize the cost of getting the agent to choose e^*. Doing so involves minimizing the agent's surplus conditional on the surplus being nonnegative and on the agent being willing to choose e^*.

The principal, chooses μ to solve

$$\begin{aligned} & \min_\mu S\left(e^*, \sum_i \mu_i p_i\right) \\ \text{s.t.} \quad & S\left(e^*, \sum_i \mu_i p_i\right) \geq 0 \\ & \sum_i \mu_i p_i'(e^*) = c'(e^*) \\ & \mu_i \geq 0. \end{aligned}$$

The first constraint is the standard individual rationality constraint. The second constraint is the agent's first order condition. We assume that this constraint is enough to ensure that the agent's incentive constraint is satisfied; we thus follow the first order approach (Rogerson 1985). We will later remove this assumption which turns out to be very stringent in the dynamic model. The final constraint is the limited liability constraint.

Define

$$\begin{aligned}\widetilde{\mu}_i &= \mu_i \frac{p_i'(e^*)}{c'(e^*)},\\ \widetilde{p}_i(e;e^*) &= p_i(e)\frac{c'(e^*)}{p_i'(e^*)}.\end{aligned}$$

Note that $\widetilde{\mu}_i \widetilde{p}_i = \mu_i p_i$. Then the principal's problem can be rewritten as that of choosing the $\widetilde{\mu}$ that solves

$$\min_{\widetilde{\mu}} S\left(e^*, \sum_i \widetilde{\mu}_i \widetilde{p}_i\right) \tag{23}$$

$$\text{s.t.} \quad S\left(e^*, \sum_i \widetilde{\mu}_i \widetilde{p}_i\right) \geq 0 \tag{24}$$

$$\sum_i \widetilde{\mu}_i = 1 \tag{25}$$

$$\widetilde{\mu}_i \leq 0 \text{ iff } i \leq i_{0.} \tag{26}$$

The next proposition characterizes the optimal incentive scheme.

Proposition 12 *Assume the first order approach is valid.*

(i) Assume first that the agent's individual rationality constraint is not binding at the optimum. Then, the optimal incentive scheme entails paying the agent a positive wage only when outcome $k^+ \equiv \arg\min_{i>i_0}\{S(e^*, \widetilde{p}_i)\}$ *is realized. In this case, the cost of implementing action* e^* *is* $\widetilde{p}_{k^+}(e^*;e^*) = p_{k^+}(e^*)\frac{c'(e^*)}{p'_{k^+}(e^*)}$.

(ii) If there is full extraction at the optimum, then the optimal scheme may entail rewarding the agent upon the realization of more than one outcome.

Proof: Let L denote the lagrangean for problem (23) subject to the constraint (24), and let $\lambda \leq 0$ denote the multiplier of the constraint. The principal's problem reduces to choosing the vector $\widetilde{\mu}$ that satisfies the constraints (25), (26) and minimizes

$$\begin{aligned}L(\widetilde{\mu}) &= (1+\lambda) S\left(e^*, \sum_i \widetilde{\mu}_i \widetilde{p}_i\right)\\ &= (1+\lambda)\sum_i \widetilde{\mu}_i S(e^*, \widetilde{p}_i),\end{aligned}$$

where the last equality obtains because $\sum \widetilde{\mu}_i = 1$. Denote

$$k^{-} = \arg\max_{i \leq i_o} \{S(e^*, \widetilde{p}_i)\}; k^{+} = \arg\min_{i > i_o} \{S(e^*, \widetilde{p}_i)\}.$$

Suppose first that $\lambda = 0$. Since the lagrangean is linear in $\widetilde{\mu}$, the minimizer $\widetilde{\mu}^*$ will be such that

$$\widetilde{\mu}_i^* = 0 \text{ for } i \neq k^{-}, k^{+},$$

and, in light of constraint (26),

$$\widetilde{\mu}_{k^+}^* = 1 - \widetilde{\mu}_{k^-}^*.$$

So the lagrangean reads

$$L(\widetilde{\mu}) = \widetilde{\mu}_{k^-}^* S(e^*, \widetilde{p}_{k^-}) + (1 - \widetilde{\mu}_{k^-}^*) S(e^*, \widetilde{p}_{k^+}).$$

Therefore, if $S(e^*, \widetilde{p}_{k^-}) - S(e^*, \widetilde{p}_{k^+}) < 0$ the optimal solution is $\widetilde{\mu}_{k^-}^* = 0$ and the value of the lagrangean is $S(e^*, \widetilde{p}_{k^+})$. This vector is the solution provided that $S(e^*, \widetilde{p}_{k^+}) \geq 0$, so that the constraint (24) is not violated and then $\lambda = 0$. In this case the agent's surplus equals $S(e^*, \widetilde{p}_k)$. If, instead, $S(e^*, \widetilde{p}_{k^+}) < 0$, then the solution cannot put weight 1 on $\widetilde{p}_k$ since otherwise the constraint (24) would be violated. Given the linearity of the Lagrangean, this requires that λ equal -1. But then the lagrangean equals zero, i.e., the agent's surplus is zero in the optimal contract.If $S(e^*, \widetilde{p}_{k^-}) - S(e^*, \widetilde{p}_{k^+}) > 0$ then it is optimal to choose $\widetilde{\mu}_{k^-}^*$ arbitrarily close to $-\infty$. But then constraint (24) would be violated. This means that λ must equal -1. But then the lagrangean equals zero, i.e., the agent's surplus is zero in the optimal contract. ∎

With additional assumptions on the probability vector p it is possible further to characterize the optimal scheme.

Assumption (Monotone Likelihood Ratio). For every $e < e'$, $i < j$, we have

$$\frac{p_i(e)}{p_j(e)} \geq \frac{p_i(e')}{p_j(e')}.$$

Under this assumption we can provide a sharper characterization of the incentive scheme.

Corollary 2 *Assume the first order approach is valid. When $p(e)$ has the MLR property then $k^+ = N$. Unless full extraction obtains, the agent is rewarded only when the highest outcome is realized.*

Proof: When $p(e)$ has the MLR property then $\widetilde{p}_i(e^*; e^*) > \widetilde{p}_j(e^*; e^*)$ when $j > i > i_0$. To verify this, observe first that if $p(e)$ has the MLR property then $\widetilde{p}(e; e^*)$ also does for outcomes $j > i > i_0$. This means that for every $e, j > i > i_0$ implies

$$\frac{\partial}{\partial e} \log\left(\frac{\widetilde{p}_i(e; e^*)}{\widetilde{p}_j(e; e^*)}\right) \leq 0,$$

which can be written as

$$\frac{\widetilde{p}_i(e;e^*)}{\widetilde{p}_j(e;e^*)} \geq \frac{\widetilde{p}'_i(e;e^*)}{\widetilde{p}'_j(e;e^*)}.$$

The right hand side equals 1 at $e = e^*$, which proves our claim.

The fact that $\widetilde{p}_i(e^*;e^*) > \widetilde{p}_j(e^*;e^*)$ implies that $S(e^*, \widetilde{p}_i) > S(e^*, \widetilde{p}_j)$ and, therefore, that $k^+ = N$. ∎

5.2 The 2-period problem via first order approach (The Free Lunch property)

We now move to the dynamic agency environment and we use we extend the properties of the incentive scheme that we obtained in the previous subsection to derive an important feature of the dynamic problem that will be useful for our comparison with the revelation scenario.

Denote by h, j the outcome in which h is realized in the first period and j in the second period. For example, if the outcome can be success (s) or failure (f), the four possible two-period outcomes are: ss, sf, fs, ff. The principal rewards any two-period outcome realization h, j with payment μ_{hj}. Therefore, an incentive scheme is a nonnegative vector $\mu_{11}, ..., \mu_{NN}$. The agent's surplus is

$$S\left(e_1, e_2, \sum_{hj} \mu_{hj} p_h p_j\right) = \left(\sum_{h=1}^{N}\sum_{j=1}^{N} \mu_{hj} p_h(e_1) p_j(e_2)\right) - c(e_1) - c(e_2).$$

Let us start from the problem in which the principal wants to implement the same effort e^* in both periods. We show that it is possible to implement this outcome at an expected cost $\widetilde{p}_{k^+}(e^*)$, the same cost of implementing e^* in one period only!

Proposition 13 *(The free lunch property) Assume the first order approach is valid.*

(i) The cost to the principal of implementing effort e^* in both periods is $\widetilde{p}_{k^+}(e^*)$ this is the same as the cost of implementing e^* in the one period problem.

(ii) In the 2-period problem any principal with preferences that are strictly monotonic in e_1^*, e_2^* will wish to implement the same effort in both periods.

(iii) The optimal incentive scheme to implement e^* entails paying the agent only when the outcome $k^+ = \arg\min_{i > i_0^2} \left\{S\left(e^*, \widetilde{p}_i^2\right)\right\}$ is realized in both periods, and nothing otherwise, unless there is full extraction (i.e. the agent retains no surplus). If there is full extraction then the optimal scheme may entail rewarding the agent upon the realization of more than one outcome.

Proof: Consider the incentive scheme which pays the agent only when outcome k^+ is realized twice. We can write the agent's problem as

$$\max_{e_1, e_2} \left(\mu_{k^+k^+} p_{k^+}(e_1) p_{k^+}(e_2)\right) - c(e_1) - c(e_2).$$

The first order conditions read

$$\mu_{k^+k^+} p'_{k^+}(e_h)\, p_{k^+}(e_j) = c'(e_h),$$

so to implement e^* it must be

$$\mu_{k^+k^+} = \frac{c'(e^*)}{p'_{k^+}(e^*)\, p_{k^+}(e^*)}.$$

The cost to the principal of implementing e^*, e^* is

$$\mu_{k^+k^+} p_{k^+}(e^*)\, p_{k^+}(e^*) = \widetilde{p}_{k^+}(e^*; e^*).$$

This proves part (i).

Now let us show that this contract is the optimal contract for any principal with preferences that are strictly monotonic in e_1^*, e_2^*. We will show that, given any incentive scheme that implements e_1^*, e_2^* with $e_1^* < e_2^*$, the principal can implement e_2^*, e_2^* at a lower cost. To see this, let us write down the cost of impementing e_1^*, e_2^* as

$$\sum_{h,j} \mu_{hj} p_h(e_1^*)\, p_j(e_2^*).$$

Since it is optimal for the agent to take e_2^* in the second period, the first order conditions must be satisfied

$$\sum_{h,j} \mu_{hj} p_h(e_1^*)\, p'_j(e_2^*) = c'(e_2^*).$$

Define

$$\widetilde{\mu}_{hj} = \mu_{hj} \frac{p'_j(e_2^*)}{c'(e_2^*)}.$$

Then the first order conditions read

$$\sum_{h,j} \widetilde{\mu}_{hj} p_h(e_1^*) = 1,$$

and the cost of impementing e_1^*, e_2^* can be written as

$$\sum_{h,j} \widetilde{\mu}_{hj} p_h(e_1^*)\, \widetilde{p}_j(e_2^*; e_2^*) \geq \sum_{h,j} \widetilde{\mu}_{hj} p_h(e_1^*)\, \widetilde{p}_{k^+}(e_2^*; e_2^*),$$

where the inequality holds because $\widetilde{p}_j(e_2^*; e_2^*) \geq \widetilde{p}_{k^+}(e_2^*; e_2^*) \geq 0$ when $\widetilde{p}_j$ and $\widetilde{\mu}_{hj}$ are positive, while when $\widetilde{p}_j$ and $\widetilde{\mu}_{hj}$ are negative then $\widetilde{\mu}_{hj}\widetilde{p}_j \geq 0 \geq \widetilde{\mu}_{hj}\widetilde{p}_{k^+}$. In light of the first order conditions, the right hand side equals $\widetilde{p}_{k^+}(e_2^*; e_2^*)$: the cost of implementing e_2^*, e_2^*, and therefore the inequality shows that the cost of impementing e_1^*, e_2^* is greater than the cost of implementing e_2^*, e_2^*.

We have ignored the issue of the second order conditions relative to e_h and e_j. But since the second order conditions are satisfied with respect to a joint deviation in e_h and e_j, that is, since

$$\mu_{k^+k^+}\left[p_{k^+}(e)\right]^2 - 2c(e)$$

is maximized at e^*, it follows that

$$\mu_{k^+k^+}p_{k^+}(e^*)\,p_{k^+}(e) - c(e) - c(e^*),$$

is maximized at e^*, i.e., the second order conditions are satisfied with respect to deviations in e_h and e_j. ∎

It is easy to see that the logic of the proof of this proposition extends to any number of periods. As long as the first order approach remains valid, the cost to the principal of implementing effort e^* for T periods is the same as the cost of implementing e^* in a single period. This of course means that the first order approach cannot remain valid indefinitely, because at some point, this would violate the individual rationality constraint of the agent. We will see later the implications of violations of the first order approach.

5.3 Comparison of revelation policies in the two period model

With revelation the agent can condition his second period effort on the first period realization h. We denote the second period effort by e_{2h}, and the vector that collects all the e_{2h} is denoted by $\mathbf{e}_2$. The agent's surplus with revelation is

$$\sum_{h=1}^{N} p_h(e_1)\left(\sum_{j=1}^{N}\mu_{hj}p_j(e_{2h}) - c(e_{2h})\right) - c(e_1). \tag{27}$$

Suppose the principal maximizes

$$U(e_1) + \delta\mathbb{E}U(\mathbf{e}_2) - m,$$

where the expectation is taken using the probability generated by $p_h(e_1)$, and m represents the transfer to the agent.

Theorem 1 *Assume the first order approach is valid. Then the principal prefers not to reveal information.*

Proof: We will show that given any pair $e_1^*, \mathbf{e}_2^*$ that is implementable with revelation, the principal can at a lower cost implement either e_1^*, e_1^* or $\mathbf{e}_2^*, \mathbf{e}_2^*$ by not revealing ($\mathbf{e}_2^*, \mathbf{e}_2^*$ denotes a lottery which implements e_{2h}^*, e_{2h}^* with probability $p_h(e_1^*)$). Let us start by showing that

e_1^*, e_1^* can be implemented more cheaply than $e_1^*, \mathbf{e}_2^*$. The first order conditions with respect to e_1 from problem (27) read

$$\sum_h \frac{p_h'\left(e_1^*\right)}{c'\left(e_1^*\right)}\left[\sum_j \mu_{hj} p_j\left(e_{2h}^*\right) - c\left(e_{2h}^*\right)\right] = 1.$$

The cost of implementing $e_1^*, \mathbf{e}_2^*$ is

$$\begin{aligned}
& \sum_h p_h\left(e_1^*\right) \sum_j \mu_{hj} p_j\left(e_{2h}^*\right) \\
\geq\ & \sum_h p_h\left(e_1^*\right)\left[\sum_j \mu_{hj} p_j\left(e_{2h}^*\right) - c\left(e_{2h}^*\right)\right] \\
=\ & \sum_h \widetilde{p}_h\left(e_1^*; e_1^*\right) \frac{p_h'\left(e_1^*\right)}{c'\left(e_1^*\right)}\left[\sum_j \mu_{hj} p_j\left(e_{2h}^*\right) - c\left(e_{2h}^*\right)\right] \\
\geq\ & \sum_h \widetilde{p}_{k^+}\left(e_1^*; e_1^*\right) \frac{p_h'\left(e_1^*\right)}{c'\left(e_1^*\right)}\left[\sum_j \mu_{hj} p_j\left(e_{2h}^*\right) - c\left(e_{2h}^*\right)\right] \\
=\ & \widetilde{p}_{k^+}\left(e_1^*; e_1^*\right).
\end{aligned}$$

The last equality makes use of the first order conditions. To verify that the second inequality holds, consider first that the term in brackets is nonnegative because it represents the agent's expected surplus in period 2 after h is realized in period 1. Examine then the two cases separately depending on the sign of $\widetilde{p}_h$. When $\widetilde{p}_h > 0$ then $p_h' > 0$ and since $\widetilde{p}_{k^+}\left(e_1^*; e_1^*\right) \leq \widetilde{p}_h\left(e_1^*; e_1^*\right)$, the inequality is verified. When $\widetilde{p}_h < 0$ then $p_h' < 0$ and therefore substituting $\widetilde{p}_{k^+} > 0$ for $\widetilde{p}_h$ gives rise to a negative term where previously there was a positive one, again consistent with the inequality. This chain of inequalities shows that implementing e_1^*, e_1^* is cheaper than implementing $e_1^*, \mathbf{e}_2^*$.

Let us now show that $\mathbf{e}_2^*, \mathbf{e}_2^*$ can be implemented more cheaply than $e_1^*, \mathbf{e}_2^*$. The first order conditions with respect to e_{2h} from problem (27) read

$$\sum_{j=1}^N \mu_{hj} \frac{p_j'\left(e_{2h}^*\right)}{c\left(e_{2h}^*\right)} = 1.$$

The cost of implementing $e_1^*, \mathbf{e}_2^*$ is

$$\begin{aligned}
& \sum_h p_h\left(e_1^*\right) \sum_j \mu_{hj} p_j\left(e_{2h}^*\right) \\
= & \sum_h p_h\left(e_1^*\right) \sum_j \mu_{hj} \frac{p_j'\left(e_{2h}^*\right)}{c\left(e_{2h}^*\right)} \widetilde{p}_j\left(e_{2h}^*\right) \\
= & \sum_h p_h\left(e_1^*\right) \sum_j \mu_{hj} \frac{p_j'\left(e_{2h}^*\right)}{c\left(e_{2h}^*\right)} \widetilde{p}_{k+}\left(e_{2h}^*; e_{2h}^*\right) \\
\geq & \sum_h p_h\left(e_1^*\right) \widetilde{p}_{k+}\left(e_{2h}^*; e_{2h}^*\right).
\end{aligned}$$

To verify the inequality split the argument into two parts. When $\widetilde{p}_j > 0$ then $p_j' > 0$ and since $\widetilde{p}_{k+}\left(e_{2h}^*; e_{2h}^*\right) \leq \widetilde{p}_j\left(e_{2h}^*; e_{2h}^*\right)$, the inequality is verified. When $\widetilde{p}_j < 0$ then $p_j' < 0$ and therefore substituting $\widetilde{p}_{k+} > 0$ for $\widetilde{p}_j$ gives rise to a negative term where previously there was a positive one, again consistent with the inequality. This chain of inequalities shows that $e_1^*, \mathbf{e}_2^*$ is more expensive to implement than a lottery which implements e_{2h}^*, e_{2h}^* with probability $p_h\left(e_1^*\right)$.

Let us conclude the proof by showing that purely in terms of value of effort, the principal must prefer either e_1^*, e_1^* or $\mathbf{e}_2^*, \mathbf{e}_2^*$ to $e_1^*, \mathbf{e}_2^*$. The value to the principal of implementing $e_1^*, \mathbf{e}_2^*$ net of the cost is

$$\begin{aligned}
& U\left(e_1^*\right) + \delta \sum_h p_h\left(e_1^*\right) U\left(e_{2h}^*\right) \\
\leq & \left(1+\delta\right) \max \left\{ U\left(e_1^*\right), \sum_h p_h\left(e_1^*\right) U\left(e_{2h}^*\right) \right\}.
\end{aligned}$$

The value to the principal of $\mathbf{e}_2^*, \mathbf{e}_2^*$ net of the cost is

$$\left(1+\delta\right) \sum_h p_h\left(e_1^*\right) U\left(e_{2h}^*\right).$$

The value to the principal of implementing e_1^*, e_1^* net of the cost is

$$\left(1+\delta\right) U\left(e_1^*\right).$$

This proves that, purely in terms of value of effort, the principal must prefer either e_1^*, e_1^* or $\mathbf{e}_2^*, \mathbf{e}_2^*$ to $e_1^*, \mathbf{e}_2^*$. ∎

Quadratic 3 periods: no revelation > revelation after second period > revelation after first >? full revelation

Meg's Example: no revelation < revelation after second period < revelation after first <? full revelation

6 References

Abreu, Dilip, Paul Milgrom, and David Pierce (1991) "Information and Timing in Repeated Partnerships," *Econometrica*, Vol. 59, No. 6, November, pp. 1713-1733.

Anil, Jonathan Glover, and Shyam Sunder (1998), "Earnings Management and the Revelation Principle", *Review of Accounting Studies*, Vol. 3, Nos. 1-2, pp. 7-34.

Chiappori, Pierre Andre, Ines Macho, Patrick Rey and Bernard Salanié (1994), "Repeated moral hazard: memory, commitment, and the access to credit markets", *European Economic Review*, 1527-53.

Fudenberg, Drew, Bengt Holmstrom and Paul Milgrom (1990), "Short-Term Contracts and Long-Term Agency Relationships", *Journal of Economic Theory*, 51, 1-32.

Gibbs, Michael (1991), "Economic Approach to Process in Pay and Performance Appraisals", manuscript, University of Chicago, Graduate School of Business.

Holmstrom, Bengt and Paul Milgrom (1987), "Aggregation and Linearity in the Provision of Intertemporal Incentives", *Econometrica*, Vol. 55, No. 2, pp. 303-328.

Kahn, Charles and Gur Huberman (1988), "Two-Sided Uncertainty and "Up-or-Out" Contracts", *Journal of Labor Economics*, Vol. 6, No. 4, pp. 423-444.

Krishna, Vijay and John Morgan (1998), "The Winner-Take-All Principle in Small Tournaments", in *Advances in Applied Microeconomics*, Vol. 7, pp. 61-74.

Lambert, Richard (2001), "Contracting Theory and Accounting", *Journal of Accounting and Economics*, Vol. 32, Nos. 1-3, pp. 3-87.

Lazear, Edward (1999), "Keeping Salaries Confidential", manuscript, Stanford University, Graduate School of Business.

Malcolmson, James (1984), "Work Incentives, Hierarchy, and Internal Labor Markets", *The Journal of Political Economy*, Vol. 92, No. 3, pp. 486-507.

Meyer, Margaret (1992), " Biased Contests and Moral Hazard: Implications for Career Profiles", *Annales d'Economie et de Statistique*, Nos. 25/6, pp. 165-187.

Moldovanu, Benny and Aner Sela (2001), “The Optimal Allocation Prizes in Contests”, *American Economic Review*, Vo. 91, No. 3, pp. 542-558.

Moldovanu, Benny and Aner Sela (2002), “Contest Architecture”, Working Paper, University of Mannheim, January.

Prendergast, Canice (1992), “Career Development and Specific Human Capital Acquisition”, *The Journal of the Japanese and International Economies*, 6, pp. 207-27.

Rogerson, William (1985), “Repeated Moral Hazard”, *Econometrica*, Vol. 53, No. 1, pp. 69-76.