Primary paper to be presented

USC FBE APPLIED ECONOMICS WORKSHOP
presented by Lars Stole
FRIDAY, Dec. 3, 2004
1:30 pm – 3:00 pm, **Room: HOH-601K**

A theory of common agency with common screening devices, with applications to public goods, price discrimination and political influence*

David Martimort†

Lars A. Stole††

November 29, 2004

Preliminary and incomplete notes;
prepared for seminar at U.S.C., December 3, 2004

† University of Toulouse (IDEI, GREMAQ) and Institut Universitaire de France.
††University of Chicago, GSB.

1 Introduction

This paper considers the general problem of multiple principals contracting with a privately-informed common agent when the entire set of screening variables are shared by the principals. We seek to understand the economic consequences of commonly utilized screening devices when each principal individually exploits the agent to his own advantage. The basic setting is one in which the agent has private information at the time of contracting. Multiple principals, $i = 1, \ldots, n$, each simultaneously offer a menu of payments, $T_i(q)$, that are conditional on the agent's choice, $q \in \mathcal{Q}$. The agent, in turn, chooses the action which maximizes his direct utility of action plus the sum of transfers.

This seemingly narrow class of contracting games is economically significant, although largely unexplored. Consider the common price discrimination setting in which one firm's screening device is used by another. For example, Intel crafts its product line to segment commercial users from occasional PC users. In the late 1980s and early 1990s, Intel accomplished this by offering co-processor upgrades to its 386 line, and later low- and high-end 486 chips (i.e., the 486SX and the 486DX). More recently, Intel has used its variety of 32-bit and 64-bit offerings to extract more surplus from commercial users while still serving the very profitable occasional PC user. The basic intuition of Intel's product line design is well understood as a textbook example of second-degree price discrimination. That said, there is another effect operating in the background. Software companies sometimes use Intel's product line to discriminate among its own customers of software. For example, Wolfram Resarch offers its flagship software program, Mathematica, for 64-bit Linux (Itanium II) systems at a commercial price that is 67 percent higher than that of its 32-bit Linux version. If most software vendors acted similarly, how would this affect Intel's product line design? As another example, suppose that two upstream wholesalers selling to a common downstream retailer can condition their wholesale price not just on the amount of their own product that is sold, but also on the total sales revenue at that retail outlet. How will this affect the nature of the wholesale contracts? And consider political lobbying. Suppose that a lobbyist's contributions depend not only on the politician's support of the lobbyist's cliental, but also on the politician's votes on other issues of less interest. When competing lobbyists are peddling influence, does the ability of a lobbyist to condition contributions on tangential issues make the politician more or less responsive to his (or his constituents') preferences? We address these and other questions in the applications sections of this paper.

While the class of public contracting games with private information has been largely unstudied, the theory of public contracts with *complete* information has been developed and widely applied. The seminal paper in this literature is by Bernheim and Whinston (1986) on the theory of menu auctions. Their setting is identical to our present one except that the agent's preferences are commonly known. They demonstrate, among other things, that there always exists an equilibrium in which the principals offer transfers that reflect their marginal valuation for the agent's choice (referred to as "truthful strategies") and the agent chooses q to maximize the sum of the principals' and agent's surpluses. There are typically other equilibria, but Bernheim and Whinston argue that these truthful strategies are coalition proof and attractive in their own right, unlike the other strategies. In part because of their simple and powerful results, their complete information approach has been fruitfully applied to other economic fields. In the case of international trade, for example, the theory has been used to explain the nature of observed protectionist policies. Grossman and Helpman (1994) model the domestic politician as an agent who must choose a trade policy while the various principals represent special interest groups willing to make contributions in exchange for favorable trade policy. Although trade policy is welfare maximizing for the groups at the table, it is not efficient once one includes the absent parties (e.g., domestic consumers). This application of Bernheim and Whinston's (1986) model allows the construction of a positive theory of international trade policy, albeit in a setting of complete information.

In the present paper, we consider another source of inefficiency – incomplete information.[1] In what follows, we develop a very general class of incomplete information games with public contracting variables. (Section 2.) We show that any equilibrium outcome in the noncooperative game corresponds to the unique equilibrium outcome in the cooperative case when the agent's rent has a weight of n attached to it. (Section 3.) This finding, in short, argues that the presence of common contracting variables with n principals introduces an n-fold marginalization distortion. Note that this is not related to the standard free-rider problem, but instead arises from the presence of incomplete information. Indeed, in the case of public good provision, we will show that the presence of a common agent eliminates any free-rider problem under complete information; this is an application of Bernheim and Whinston (1986). When incomplete information exists, however, each principal introduces an independent distortion to extract the agent's information rents. Remarkably, these independent distortions aggregate perfectly as though a single entity had an n-times greater desire to reduce the agent's infor-

[1] As an aside, we also find that in the presence of incomplete information, equilibrium transfers are never "truthful," even in the limit as information becomes complete. This is of independent interest for the theory of complete information contracting. We take up this issue briefly in this paper and in more detail in Martimort and Stole (2004).

mation rents. We then apply this theorem to very different contexts: public goods (section 4), price discrimination (section 5) and political influence (section 6).

2 The Model

Consider a public common agency with n principals and a single agent. The principals and the agent have linear preferences over money and general preferences over the vector of activity, $q \in \mathcal{Q} \subset \Re^m$, where $\mathcal{Q}$ is compact. We denote principal i's preferences as

$$v_i(q) - t_i,$$

given the agent chooses q and the principal pays the agent t_i in return. The agent's preferences are

$$u(q, \theta) + \sum_{i=1}^{n} t_i,$$

given the agent chooses q and receives the aggregate payment of $\sum_i t_i$ from the principals. The agent's private information is represented by a parameter $\theta \in \Theta$, where Θ is discrete (for simplicity) and the parameter is distributed according to the probability function, $\phi : \Theta \mapsto (0, 1)$. Θ may be a multidimensional space; we place no restrictions on Θ beyond finiteness. We assume that the preference functions are continuous and differentiable over $\mathcal{Q}$.

We refer to the contracting game as a *public* common agency game, meaning that the agent's activity vector, q, is fully contractible by every principal.[2]

In the agency game, the principals simultaneously offer transfer schedules to the privately-informed agent, $T_i : \mathcal{Q} \mapsto \Re_+$. We restrict attention to transfer functions which are Lipschitz-continuous over the interior of $\mathcal{Q}$ and denote this restriction by requiring that $T_i \in \mathcal{T}$. Wherever we refer to equilibria of the game, we will be implicitly restricting our attention to such piecewise-smooth equilibria. The agent's strategy is a function mapping from transfers and type to choice of q. Specifically, $q_0 : \mathcal{T}^n \times \Theta \mapsto \mathcal{Q}$.

To be precise, the timing of the game is as follows. In the first stage, the agent learns his

[2] In contrast, a *private* agency game allows each principal to contract over a mutually-exclusive dimension of $\mathcal{Q}$. In the simplest private common-agency case, $m = n$, and each principal i contracts over only the component q_i of $q = (q_1, \ldots, q_n)$. See Martimort (1992,1996) and Stole (1991) for results regarding these games.

type, $\theta \in \Theta$, which is drawn by nature in accord with the probability function $\phi(\theta)$. In the second state, each principal $i \in \mathcal{N} \equiv \{1, \ldots, n\}$, simultaneously offers a transfer menu, $T_i \in \mathcal{T}$. Third, the agent decides with which subset of principals, $N \subseteq \mathcal{N}$, it wishes to contract and chooses an action, $q \in \mathcal{Q}$. In the final stage the contracting principals pay according to their contracts and the agent collectively receives $\sum_{i \in N} T_i(q)$.

A pure-strategy profile in this game is an $n+1$ tuple, $\{T_1, \ldots, T_n, q_0\}$. A pure-strategy equilibrium is a strategy profile where each component is part of the player's best-response mapping, given the other players' strategic choices. The equilibrium outcome is a vector of payoffs and an allocation of q, conditional on θ: $\{q(\theta), U(\theta), \pi_1, \ldots, \pi_n\}$, where

$$q(\theta) = q_0(T_1, \ldots, T_n, \theta),$$

$$U(\theta) = \max_{q \in \mathcal{Q}} u(q, \theta) + \sum_{i=1}^{n} T_i(q),$$

$$\pi_i = \sum_{\theta} \left(v_i(q(\theta)) - T_i(q(\theta)) \right) \phi(\theta).$$

We make only two additional restrictions on preferences for our main result. Define the joint surplus function, $S(q, \theta) \equiv u(q, \theta) + \sum_{i=1}^{n} v_i(q)$.

Assumption 1 – Concave programming.

1. *$S(q, \theta)$ is strictly concave over $\mathcal{Q}$ for any $\theta \in \Theta$,*
2. *$u(q, \theta) - u(q, \tilde{\theta})$ is linear over $\mathcal{Q}$ for any $\theta, \tilde{\theta} \in \Theta$.*

These conditions guarantee that the Kuhn-Tucker necessary conditions for the cooperative program are sufficient for determining the optimum. The first condition is stronger than it need be; pseudo concavity and strict quasi-concavity are sufficient. The second requirement implies that the constraint set is defined by linear inequalities, thereby generating a concave programming problem and satisfying a constraint qualification condition. The second requirement also implies a variation of a multi-dimensional single-crossing. Note, however, that the sign of the single-crossing property may vary by the dimension of $\mathcal{Q}$. One may want to drop this assumption in economic settings where the nature of single-crossing is unknown or in multi-dimensional settings where the nature of the binding incentive constraints is not a priori known. Without this second restriction our main result must be weakened whenever the cooperative program fails to be concave and the Kuhn-Tucker conditions become insufficient. The economic intuition for the main result, however, appears robust to a loss of global concavity, and so we believe these regularity conditions are largely for mathematical simplicity.

3 A characterization of pure-strategy noncooperative equilibria

With our concavity assumption maintained, we proceed by first characterizing the solution to a class of cooperative optimization problems. Our main result will establish an equivalence between equilibrium outcomes in a subset of these problems and the noncooperative game.

3.1 A useful class of cooperative programming problems

It is useful to first characterize the equilibrium outcome when the n principals can form a collective and jointly determine an aggregate transfer, $T(q)$, to present to the agent. For our purposes, it is worthwhile to characterize this program for an arbitrary outside option of the agent, $\underline{u}(\theta)$, which may be type dependent, and for an arbitrary weighting of the agent's utility, γ in the joint-principals' program. Thus, a cooperative program is identified by $\{\gamma, \bar{u}(\theta)\}_{\theta \in \Theta}$.

Specifically, we consider the following program (defined in the dual space of payoffs and allocations):

$$\max_{\{U(\theta), q(\theta)\}_{\theta \in \Theta}} \sum_{\theta} \left(S(q(\theta), \theta) - \gamma U(\theta) \right) \phi(\theta),$$

subject to

$$U(\theta) - U(\hat{\theta}) \geq u(q(\hat{\theta}), \theta) - u(q(\hat{\theta}), \hat{\theta}), \;\; \forall \theta, \hat{\theta} \in \Theta,$$

$$U(\theta) \geq \underline{u}(\theta), \;\; \forall \theta \in \Theta.$$

The first set of constraints are the general, global incentive constraints. Let the associated KT-Lagrange multipliers for these constraints be denoted $\lambda(\theta, \hat{\theta})$. The second set of constraints assures the agent's participation; let the associated multipliers be represented by $\mu(\theta)$. Notice that for $\gamma = 0$, we typically have the planner's program of maximizing aggregate surplus,

$S(q,\theta)$; for $\gamma = 1$ we have the standard program in which the cooperative maximizes only its own aggregate payoffs and is indifferent regarding the agent's rent. Given our maintained assumption, the cooperative program is strictly concave for any γ.

In general, we do not know which constraints are binding without assuming more than single-crossing on preferences. The cooperative solution could contain regions of pooling or regions of types where the outside option is binding. We leave these questions aside for now and instead simply provide the characterizing necessary and sufficient Kuhn-Tucker conditions. The following result is an immediate application of well-known results from nonlinear programming.

Lemma 1 *Given assumption 1, for a given* $\{\gamma, \underline{u}(\theta)\}_{\theta\in\Theta}$*, a solution to the cooperative principal's program exists and is unique. In particular, the solution is characterized by a unique vector,* $\{q(\theta), U(\theta), \lambda(\theta,\hat{\theta}) \geq 0, \mu(\theta) \geq 0\}_{\theta,\hat{\theta}\in\Theta}$ *such that*

$$\nabla_q S(q(\theta),\theta)\phi(\theta) = \sum_{\hat{\theta}} \lambda(\hat{\theta},\theta)\left(\nabla_q u(q(\theta),\hat{\theta}) - \nabla_q u(q(\theta),\theta)\right), \quad \forall\theta\in\Theta, \tag{1}$$

$$-\gamma\phi(\theta) + \sum_{\hat{\theta}}\left(\lambda(\theta,\hat{\theta}) - \lambda(\hat{\theta},\theta)\right) + \mu(\theta) = 0, \quad \forall\theta\in\Theta, \tag{2}$$

$$\lambda(\theta,\hat{\theta})\left(U(\theta) - U(\hat{\theta}) - u(q(\hat{\theta}),\theta) + u(q(\hat{\theta}),\hat{\theta})\right) = 0, \quad \forall\theta,\hat{\theta}\in\Theta, \tag{3}$$

$$\mu(\theta)\left(U(\theta) - \underline{u}(\theta)\right) = 0, \quad \forall\theta\in\Theta. \tag{4}$$

Existence follows from our underlying assumptions of continuity and compactness. Assumption 1 guarantees the program is strictly concave, and hence the given Kuhn-Tucker conditions are necessary and sufficient, and that the solution is unique.

3.2 Our main theorem of n-marginalizations

We now turn to the game of central interest: n principals simultaneously offer continuously piecewise-differentiable transfer functions to a common agent in order to influence the agent's choice of $q \in \mathcal{Q}$.

First, note that from principal i's perspective, it is as if he is the only principal but he is contracting with an agent with Lipschitz-continuous preferences given by

$$\tilde{u}_i(q,\theta) \equiv u(q,\theta) + \sum_{j \neq i} T_j(q).$$

(For the time being, we are assuming that the agent accepts each principal's offer.) The agent's outside option vis-a-vis this principal depends upon what the agent is allowed to choose absent a relationship with principal i. If, for example, the agent is entirely unrestrained and transfers are required to be nonnegative, then the relevant outside option is

$$\underline{u}_i(\theta) \equiv \max_{q \in \mathcal{Q}} \tilde{u}_i(q,\theta).$$

This may make sense in the case of the private provision of public goods: anyone can offer to pay the agent to build a public good and, absent a relationship with citizen i, the agent may still obtain rents and produce some amount of the public facility. We address this case below in section 4. If, alternatively, all principals must be present for the agent to obtain any returns from trade, the agent's outside option vis-a-vis principal i is $\underline{u}_i(\theta) \equiv \underline{u}(\theta)$. This may be the appropriate assumption when the principals are bribe-maximizing bureaucrats, each of whose authorization is needed for the agent to proceed with his own surplus-generating project. A simple example of this setting is found in Shleifer and Vishny (1993). In all cases, the economic environment dictates the appropriate $\underline{u}_i(\theta)$ which each principal i must satisfy to achieve acceptance from the agent.

Finally, given that the agent always has the option to disregard any subset of principals' offers, it is without loss of generality to consider equilibria in which full participation arises. To see this, suppose that there is an equilibrium in which some subset of types, $\Theta_i \subset \Theta$, does not accept principal i's offer and that $q(\theta)$ is implemented. Principal i can replace his original offer with a new transfer that offers $T_i(q(\theta)) = 0$ for all $\theta \in \Theta_i$ and leave his payoffs unchanged. If the agent always accepts this modified contract, the agent's payoffs are also unchanged. Hence, these artificial strategic modifications induce full participation without affecting the equilibrium outcome. We can therefore require each principal to satisfy the agent's participation constraint for every $\theta \in \Theta$. It is, of course, still interesting to study the various boundaries of agent non-activity and principal zero-transfers in the set of Θ.

We are now able to define principal i's programming problem. Define $S_i(q,\theta) \equiv v_i(q) +$

$\tilde{u}_i(q, \theta)$. Then principal i's program is simply

$$\max_{\{U(\theta), q(\theta)\}_{\theta \in \Theta}} \sum_{\theta} \left(S_i(q(\theta), \theta) - U(\theta)\right) \phi(\theta),$$

subject to

$$U(\theta) - U(\hat{\theta}) \geq \tilde{u}_i(q(\hat{\theta}), \theta) - \tilde{u}_i(q(\hat{\theta}), \hat{\theta}), \ \ \forall \theta, \hat{\theta} \in \Theta,$$

$$U(\theta) \geq \underline{u}_i(\theta), \ \ \forall \theta \in \Theta.$$

Similar to before, let the associated KT-Lagrange multipliers for the incentive constraints be denoted $\lambda_i(\theta, \hat{\theta})$ and those for the participation constraints be denoted by $\mu_i(\theta)$. Although we do not necessarily have a strictly concave programming problem (this depends upon the shape of the aggregate transfers offered by the rival principals), the Kuhn-Tucker conditions are still necessary for any best response by principal i when the aggregate transfers are differentiable. In the present setting, these transfers are not necessarily differentiable and may contain kinks, such as those that arise in pooling equilibria. Fortunately, modified Kuhn-Tucker conditions for non-smooth optimization are available which differ only in the weakening of the requirement that the Lagrangian has zero derivative at $q(\theta)$ to a requirement that 0 is in the subdifferential of the Lagrangian at $q(\theta)$. See, for example, Clarke (1983, Theorem 6.1). Applying this programming result, we have an immediate characterization of principal i's best-response correspondence.

Lemma 2 *For a given $\{\tilde{u}_i, \underline{u}_i\}$, a solution to principal i's program exists and is characterized by a vector, $\{q(\theta), U(\theta), \lambda_i(\theta, \hat{\theta}) \geq 0, \mu_i(\theta) \geq 0\}_{\theta, \hat{\theta} \in \Theta}$ such that, for $\varepsilon > 0$ sufficiently small,*

$$\nabla_q S_i(q(\theta) - \varepsilon, \theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda_i(\hat{\theta}, \theta) \left(\nabla_q u(q(\theta) - \varepsilon, \hat{\theta}) - \nabla_q u(q(\theta) - \varepsilon, \theta)\right) \geq 0 \geq$$

$$\nabla_q S_i(q(\theta) + \varepsilon, \theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda_i(\hat{\theta}, \theta) \left(\nabla_q u(q(\theta) + \varepsilon, \hat{\theta}) - \nabla_q u(q(\theta) + \varepsilon, \theta)\right), \ \ \forall \theta \in \Theta, \quad (5)$$

$$-\phi(\theta) + \sum_{\hat{\theta}} \left(\lambda_i(\theta, \hat{\theta}) - \lambda_i(\hat{\theta}, \theta)\right) + \mu_i(\theta) = 0, \ \ \forall \theta \in \Theta, \quad (6)$$

$$\lambda_i(\theta, \hat{\theta}) \left(U(\theta) - U(\hat{\theta}) - u(q(\hat{\theta}), \theta) + u(q(\hat{\theta}), \hat{\theta})\right) = 0, \ \ \forall \theta, \hat{\theta} \in \Theta, \quad (7)$$

$$\mu_i(\theta) \left(U(\theta) - \underline{u}_i(\theta)\right) = 0, \ \ \forall \theta \in \Theta. \quad (8)$$

Notice that the presence of $\tilde{u}_i$ is absent from the result above. This follows because of the agent's marginal utility of transfers is independent of θ, which is implied by the assumption that preferences are quasi-linear in money. Also note that the aggregate transfers of the rival principals are embedded in S_i; the possibility that S_i may not be differentiable in q requires the inequality statement in (5). At $q(\theta)$ such that $\nabla_q S(q(\theta), \theta)$ is defined, (5) reduces to the requirement that the Lagrangian has zero derivative at $q = q(\theta)$; otherwise the kink must be locally quasi-concave.

We are now in a position to state our main result regarding the equivalence between the noncooperative outcome and the overly-distorted cooperative program with $\gamma = n$.

Theorem 1 *Given assumption 1 holds, the equilibrium outcome of every pure-strategy, noncooperative equilibrium with Lipschitz-continuous transfer schedules, $\{q(\theta), U(\theta)\}$, is equal to the outcome of the cooperative principal setting in which the agent's utility weight is $\gamma = n$ and the outside option is*

$$\underline{u}(\theta) \equiv \max\{\underline{u}_1(\theta), \ldots, \underline{u}_n(\theta)\}.$$

Proof: We need only verify that the necessary conditions for the noncooperative equilibrium imply the sufficient conditions for the cooperative program. Given assumption 1, the cooperative program has a unique equilibrium outcome $\{q(\theta), U(\theta)\}$, so the theorem follows.

To verify that the sufficient conditions of the cooperative program are satisfied, sum the necessary conditions in (5)-(7) across $i = 1, \ldots, n$. Letting $\lambda(\theta, \hat{\theta}) \equiv \sum_{i=1}^n \lambda_i(\theta, \hat{\theta})$ and $\mu(\theta) \equiv \sum_{i=1}^n \mu_i(\theta)$, we immediately obtain

$$\sum_{i=1}^n \nabla_q S_i(q(\theta) - \varepsilon, \theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda(\hat{\theta}, \theta) \Big(\nabla_q u(q(\theta) - \varepsilon, \hat{\theta}) - \nabla_q u(q(\theta) - \varepsilon, \theta)\Big) \geq 0 \geq$$
$$\sum_{i=1}^n \nabla_q S_i(q(\theta) + \varepsilon, \theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda(\hat{\theta}, \theta) \Big(\nabla_q u(q(\theta) + \varepsilon, \hat{\theta}) - \nabla_q u(q(\theta) + \varepsilon, \theta)\Big), \quad \forall \theta \in \Theta, \quad (9)$$

$$-n\phi(\theta) + \sum_{\hat{\theta}} \Big(\lambda(\theta, \hat{\theta}) - \lambda(\hat{\theta}, \theta)\Big) + \mu(\theta) = 0, \quad \forall \theta \in \Theta, \quad (10)$$

$$\lambda(\theta, \hat{\theta}) \Big(U(\theta) - U(\hat{\theta}) - u(q(\hat{\theta}), \theta) + u(q(\hat{\theta}), \hat{\theta})\Big) = 0, \quad \forall \theta, \hat{\theta} \in \Theta. \quad (11)$$

Note, however, that at points of differentiability,

$$\sum_{i=1}^{n} \nabla_q S_i(q,\theta) = \sum_{i=1}^{n} \nabla_q v_i(q) + n\nabla_q u(q,\theta) + (n-1)\sum_{i=1}^{n} \nabla_q T_i(q) = \nabla_q S(q,\theta).$$

Using the envelope theorem, (9) further simplifies to

$$\nabla_q S(q(\theta)-\varepsilon,\theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda(\hat{\theta},\theta)\left(\nabla_q u(q(\theta)-\varepsilon,\hat{\theta}) - \nabla_q u(q(\theta)-\varepsilon,\theta)\right) \geq 0 \geq$$
$$\nabla_q S(q(\theta)+\varepsilon,\theta)\phi(\theta) - \sum_{\hat{\theta}} \lambda(\hat{\theta},\theta)\left(\nabla_q u(q(\theta)+\varepsilon,\hat{\theta}) - \nabla_q u(q(\theta)+\varepsilon,\theta)\right), \quad \forall \theta \in \Theta,$$

or, after taking the limit as $\varepsilon \to 0$ and using the fact that $S(q,\theta)$ *is* differentiable, simply

$$\nabla_q S(q(\theta),\theta)\phi(\theta) = \sum_{\hat{\theta}} \lambda(\hat{\theta},\theta)\left(\nabla_q u(q(\theta),\hat{\theta}) - \nabla_q u(q(\theta),\theta)\right). \tag{12}$$

(10)-(11) and (12) correspond to the Kuhn-Tucker conditions for the cooperative program with $\gamma = n$; namely, (1)-(3).

To verify that the fourth condition, (4), of the cooperative program holds, suppose that for all $i \in E(\theta)$, we have $\mu_i(\theta) > 0$ and for $i \notin E(\theta)$ we have $\mu_i(\theta) = 0$. Then it follows that

$$U(\theta) = \max_{j \in E(\theta)} \{\underline{u}_j(\theta)\} > \max_{j \in \neq E(\theta} \{\underline{u}_j(\theta)\},$$

which implies that $U(\theta) = \max_{j\in\mathcal{N}}\{\underline{u}_j(\theta)\}$ if $E(\theta) \neq \emptyset$ and $U(\theta) > \max_{j\in\mathcal{N}}\{\underline{u}_j(\theta)\}$ otherwise. Now suppose that $\underline{u}(\theta) \equiv \max_{j\in\mathcal{N}}\{\underline{u}_j(\theta)\}$ and that $\mu(\theta) \equiv \sum_{i=1}^{n} \mu_i(\theta)$. If $E(\theta) = \emptyset$, then $\mu_i(\theta) = 0$ for all $i = 1, \ldots, n$ and therefore $\mu(\theta) = 0$; in this case, $U(\theta) > \underline{u}(\theta)$. If $E(\theta)$ is not empty, then some $\mu_i(\theta) > 0$ and therefore $\mu(\theta) > 0$; $U(\theta) = \underline{u}(\theta)$. Hence, (4) holds:

$$\mu(\theta)\left(U(\theta) - \underline{u}(\theta)\right) = 0, \quad \forall \theta \in \Theta.$$

Having verified (1)-(4) hold, it follows that there is a unique equilibrium outcome, $\{q(\theta), U(\theta)\}_{\theta\in\Theta}$ which corresponds to the equilibrium outcome under the cooperative program when agent utility is weighted n-fold. □

This result is remarkable: for any pure-strategy equilibrium with piecewise-smooth transfer functions, the outcome is equivalent to a distorted cooperative problem with an n-fold marginalization of the agent's rent term. Significantly, this result holds even in settings with a complex web of non-locally binding incentive constraints. Note that the single-crossing portion of assumption 1 is used only to guarantee that the necessary conditions for the cooperative program are also sufficient. It is not used to characterize the nature of the cooperative solution. More generally, the above result will hold whenever the necessary conditions for the cooperative program are also sufficient – regardless of concavity and single-crossing properties. Fundamentally, the result follows from the equivalence of the necessary conditions in the two settings, which, in turn, follows from our assumption of quasi-linear preferences and an application of the envelope theorem. Only in the peculiar case where the non-cooperative equilibrium satisfies the Kuhn-Tucker necessary conditions of the n-weighted cooperative program but fails to be the maximizer is it possible for the conclusion of the theorem to fail.

Our main theorem has been proven in the case of discrete Θ. This simplification allows us to use very powerful and straightforward techniques in nonlinear programming. The theorem holds for a countably infinite Θ, but a measure-theoretic modification of the proof is required to extend the result to the continuous-type domain. Rather than pursue that generalization here, we find it more fruitful to construct several examples with continuous-type spaces that illustrate the general validity of the theorem and that provide some important economic insights.

It is also interesting to note that when there are no asymmetries in information, the cooperative n-weight solution is first-best efficient because the agent's rent is forced to the participation level for all θ. Hence, the non-cooperative equilibrium with piecewise-smooth transfer functions is also efficient.

Corollary 1 *Suppose that $|\Theta| = 1$. Then all pure-strategy, Lipschitz-continuous equilibria of the noncooperative contracting game are efficient: i.e., $q(\theta) = \max_{q \in \mathcal{Q}} S(q, \theta)$.*

This is one of the findings of Bernheim and Whinston (1986), which we will discuss in more detail below.

One final note: Although the above results state that the n principal public agency game induces an n-fold marginalization of the agent's rent extraction, this statement is conditioned on a particular set of equilibrium outside options. What remains to be understood is the equilibrium nature of the participation constraints and how these outside options may also affect allocations and the agent's rent. This, in turn, requires that we more carefully describe the economic environment and consider refinements on the set of equilibria. We consider a few examples to illustrate the variety of possibilities.

4 Application: Private provision of public goods

We consider a simple setting with two citizens (i.e., principals), $i = 1, 2$, each interested in a one-dimensional public good – a public park, for concreteness – of size $q \in \mathcal{Q} \subset \Re_+$. The principals in this story have preferences $v_i(q) = \alpha_i q - \frac{1}{2}q^2$, while the contractor (i.e., agent) has a cost of construction equal to θq, with θ distributed uniformly on $\Theta \equiv [\underline{\theta}, \overline{\theta}]$.[3] As a reference point, the first-best level of public good is $q^{fb}(\theta) = \frac{1}{2}(\alpha_1 + \alpha_2 - \theta)$.

Before continuing with the private information setting, it is useful to set out the full-information benchmark. Recall that Bernheim and Whinston (1986) have demonstrated that there always exists an equilibrium which maximizes the sum of the agent's and principals' utilities. This may be surprising in the present context since we are apt to error in assuming every public goods provision setting has a fundamental free-riding problem. This is not true when there is a common agent who may externalize the free-riding externality. Consider the following principal strategies.

$$T_i(q) = \max\{0, \alpha_i q - \frac{1}{2}q^2 - k_i\}, \quad i = 1, 2,$$

where $k_1 + k_2 = S(q^{fb}(\theta), \theta)$ and $k_i \geq \alpha_i q^{fb}(\theta) - \frac{1}{2}q^{fb}(\theta)^2$. By construction, these transfers are globally nonnegative and strictly positive to the right of $q^{fb}(\theta)$. As a consequence, the agent always participates and chooses q to maximize $T_1(q) + T_2(q) - \theta q$, which requires $q = q^{fb}(\theta)$.

So why is there no free-rider problem in this public goods model? It is precisely because a common agent externalizes the free-riding externality in our example. In contrast, consider the setting in which each principal i contracts with his own exclusive agent (of known type θ) for q_i, but each principal enjoys the aggregate public good $q = q_1 + q_2$. In this setting, principal 1 takes his rival's provision as fixed and solves

$$\max_{q_1} \alpha_1(q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2 - \theta q_1,$$

[3] We assume that $\sup_q \mathcal{Q} < \min\{\alpha_1, \alpha_2\}$ so that principal preferences are increasing over $\mathcal{Q}$.

yielding a first-order condition that $\alpha_1 - (q_1 + q_2) - \theta = 0$. Combining this with a the analogous condition for principal 2, we observe the classic free-rider problem in the unique Nash equilibrium:

$$q = q_1 + q_2 = \frac{1}{2}(\alpha_1 + \alpha_2 - 2\theta) < q^{fb}(\theta).$$

It is as if the return to production is half as high when provision is through private agents. Common agency solves the free-rider problem that would otherwise exist under exclusive agency. Because we focus on cases of common agency, when we introduce asymmetric information it should be clear that the n-fold distortions which arise are not driven by the classic free-rider problem on q but by some other contracting externality.[4]

Now return to the case of private information in which θ is privately known by the agent and is uniformly distributed on $[\underline{\theta}, \overline{\theta}]$ and let's suppose that the agent's outside option if he contracts with no one is $\underline{u}(\theta) \equiv 0$. The cooperative solution with $\gamma = 2$ and $\underline{u}(\theta) = 0$ is straightforward to derive using standard techniques of nonlinear pricing. The simplified program is

$$\max_{q(\theta) \geq 0, U(\theta), \theta^o} \int_{\underline{\theta}}^{\theta^o} \left((\alpha_1 + \alpha_2 - \theta) q(\theta) - q(\theta)^2 - 2(\theta - \underline{\theta}) q(\theta) - \underline{u}(\theta^o) \right) d\theta,$$

subject to $q(\theta)$ nonincreasing. Ignoring the monotonicity constraint and maximizing pointwise in $q(\theta)$ yields

$$q(\theta) = \max \left\{ 0, \frac{1}{2}(\alpha_1 + \alpha_2 - 3\theta + 2\underline{\theta}) \right\} = \max \left\{ 0, q^{fb}(\theta) - 2(\theta - \underline{\theta}) \right\}.$$

This is, indeed, nonincreasing. Hence, the level of public good is distorted downward from the first-best by *twice* the inverse hazard rate. The utility-maximizing cooperative would instead choose to distort by only half as much.[5]

[4]In a manner of speaking, there is another, more subtle, free-riding problem on the over-utilization of common screening devices – a tragedy of the commons – that we will return to below.

[5]This doubly-distorted outcome arises in the privatization model of Laffont and Tirole (1991). There, they consider the case in which a firm is controlled noncooperatively by both the government and the shareholders. The presence of two principals doubles the traditional information-rent distortion.

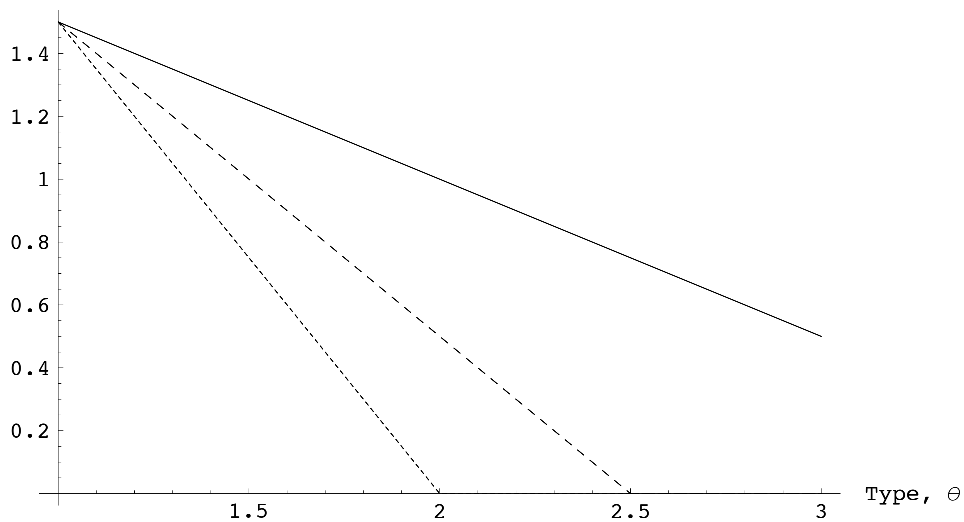

Figure 1: Public good provision for the case of $v_1(q) = v_2(q) = 2q$ and $\Theta = [1, 3]$. The solid line represents $q^{fb}(\theta)$, the long-dashed line represents the choice of a utility-maximizing cooperative, and the short-dashed line represents the noncooperative choice of $q(\theta)$. Notice that although it is efficient for all types to produce, only half of the types produce in the noncooperative game, while three-quarters produce under the cooperative solution.

The optimal cutoff level of type (i.e., the highest type that is asked to produce) is given by differentiation with respect to θ_0, noting that $\underline{u}(\theta) \equiv 0$. Hence, the critical type θ_0 is given by precisely the highest type such that $q(\theta) = 0$, or $\theta_0 = \frac{1}{3}(\alpha_1 + \alpha_2 + 2\underline{\theta})$. The agent's utility is given by the requirements of incentive compatibility:

$$U(\theta) = \int_{\theta}^{\theta^o} q(z)dz = \int_{\theta}^{\overline{\theta}} q(z)dz.$$

Let $\theta(q)$ be the inverse of $q(\theta)$ and let $\mathcal{Q}^*$ be the equilibrium range of outputs. Then the collective transfer is simply

$$T(q) = U(\theta(q)) - u(q, \theta(q)$$

defined over $\mathcal{Q}^*$.

We are ultimately interested in the noncooperative outcome. If the noncooperative equilibrium has $\max_i \{\underline{u}(\theta)\} = 0$, then from Theorem 1 we know that the noncooperative outcome is equal to that of the cooperative program with $\{\gamma = 2, \underline{u}(\theta) \equiv 0\}$. To proceed, we conjecture that the noncooperative choice of $q(\theta)$ is the same as the $\gamma = 2$ program, and then we calculate the resulting noncooperative outside options to determine if our conjecture was valid.

The noncooperative outside options are simply

$$\underline{u}_i(\theta) \equiv \max_{q \in \mathcal{Q}} -\theta q + T_j(q).$$

Let $\underline{q}_j(\theta)$ represent the maximizer of the right-hand side. It follows that $\frac{d}{d\theta}\underline{u}_i(\theta) = -\underline{q}_j(\theta)$ using the envelope theorem. If the transfer function of principal i is nondecreasing in q, then it follows that $U'(\theta) = -q(\theta) < -\underline{q}_j(\theta)$. Hence, if transfer functions are nondecreasing, the inside utility can bind with the outside option for only an upper interval of types. Moreover, the highest productive type in equilibrium (the type θ^o who is indifferent about producing, $U(\theta^o) = 0$) must do weakly worse by participating with only one principal, so $U(\theta^o) = 0 \geq \max\{\underline{u}_1(\theta), \underline{u}_2(\theta)\}$. Because indirect utilities are monotonic in type, this relationship must hold for all $\theta \geq \theta^o$. Now, suppose that the noncooperative equilibrium implements the allocation $q(\theta)$ from the $\gamma = 2$ program. The marginal productive type is θ^o defined by $q(\theta^o)$. Providing transfers are constructed so that $U(\theta^o) = 0$, it will be the case that each principal's participation constraint is slack for all $\theta < \theta^o$. Finally, note that the solution of the $\gamma = 2$ cooperative program using $\underline{u}(\theta) = \max\{u_1(\theta), \underline{u}_2(\theta)\} \geq 0$ instead of $\underline{u}(\theta) = 0$ is unchanged because the higher constraint still remains below the agent's surplus until θ^o, at which point the constraints coincide with value of 0.

What remains is the construction of nondecreasing transfer functions that implement $q(\theta)$ with $U(\theta) = 0$ for all $\theta \geq \theta^o$, and that are simultaneously best responses to each other. It is helpful to explicitly write each principal's first-order condition in equilibrium. At all points of differentiability, we have

$$\alpha_1 - q(\theta) + T_2'(q(\theta)) - \theta = (\theta - \underline{\theta}),$$

$$\alpha_2 - q(\theta) + T_1'(q(\theta)) - \theta = (\theta - \underline{\theta}).$$

Note that if we sum these equations and use the fact that $T_1'(q(\theta)) + T_2'(q(\theta)) = \theta$, we obtain $q(\theta) = \frac{1}{2}(\alpha_1 + \alpha_2 - 3\theta + 2\underline{\theta})$, which is a key component in the proof of Theorem 1. Next, consider the first relation and use the fact that $T_1'(q(\theta)) + T_2'(q(\theta)) = \theta$ to substitute out $T_2'(q(\theta))$, in order to obtain

$$T_1'(q(\theta)) = \alpha_1 - q(\theta) - 2\theta + \underline{\theta}.$$

Using the inverse of $q(\theta)$, which is $\theta(q) = \frac{1}{3}(\alpha_1 + \alpha_2 + 2\underline{\theta} - 2q)$, we can reduce this marginal transfer to a function of q:

$$T_1'(q) = v_1(q) - 2\theta(q) + \underline{\theta} = \frac{2}{3}v_1'(q) - \frac{1}{3}v_2'(q) + \frac{1}{3}\underline{\theta}.$$

Integrating (and imposing nonnegativity without a loss of generality), we obtain the transfer function

$$T_1(q) = \max\left\{0, \frac{1}{3}\left(2v_1(q) - v_2(q) + \underline{\theta}q\right) - k_1\right\}.$$

Similarly, for principal 2, we have

$$T_2(q) = \max\left\{0, \frac{1}{3}\left(2v_2(q) - v_1(q) + \underline{\theta}q\right) - k_2\right\}.$$

Finally, we require that the k_i are chosen such that the agent types $\theta \geq \theta^o$ obtain zero rent.

Note that although these transfer schedules implement the first-best when $\overline{\theta} = \underline{\theta}$ (as argued in corollary 1, these schedules are not "truthful" in the sense of Bernheim and Whinston (1986), even at the limit. This raises some concerns that the refinement of "truthfulness" may be inappropriate (or at least non-robust) when there exists small amounts of uncertainty. We argue elsewhere in Martimort and Stole (2004) that the set of limit payoffs that arise under the incomplete information setting can be a proper subset of the equilibrium truthful payoffs in Bernheim and Whinston (1986). We leave a further discussion to that paper, and instead proceed to a more complex application with multi-dimensional q.

5 Application: Nonlinear pricing by duopolists

We now turn to the setting mentioned in the introduction in which two firms sell their products using nonlinear prices that condition on the amounts purchased from each principal. This setting describes the case of wholesellers who can observe their own sales and those of other products (or at least a crude measure such as aggregate storewide sales). This setting also describes the case in which the software vendor can condition pricing on the consumer's choice of computer hardware and the hardware vendor can condition his price on the consumer's choice of software.[6]

In what follows, we assume that there are two firms, $i = 1, 2$, and that firm i offers a pricing schedule, $P_i(q_1, q_2) = -T_i(q_1, q_2)$. Each firm's preferences are to maximize profits, where costs in the present setting are simply $v_i(q_i) = -c_i q_i$. Consumers are heterogeneous and have preferences given by

$$u(q_1, q_2, \theta) = \theta(q_1 + q_2) - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 + \rho q_1 q_2,$$

where $\rho < 0$ (resp., $\rho > 0$) implies that the goods are substitutes (resp., complements) and $\rho = 0$ implies the goods are independent. The first-best output levels are, $i \neq j$,

$$q_i(\theta) = \max\left\{\frac{1}{1-\rho^2}(\theta(1+\rho) - c_i - \rho c_j), 0\right\}.$$

We need to make a decision regarding the modeling of outside options to proceed. Suppose that the exogenous outside option when the agent purchases from no one is simply 0. Furthermore, suppose that a consumer who does not purchase from firm i can enter into any contractual relationship with firm $j \neq i$ that both find jointly acceptable. Let $P_i(q_i)$ be the price schedule offered to consumers who purchase exclusively from firm i. Then we have

$$\underline{u}_1(\theta) \equiv \max_{q_2} u(0, q_2, \theta) - P_2(q_2),$$

$$\underline{u}_2(\theta) \equiv \max_{q_1} u(q_1, 0, \theta) - P_1(q_1).$$

Unlike the case of public good provision where the outside option was (in part) constrained by the equilibrium transfer schedule, here the outside option has its own transfer schedule that is independent of the chosen $P_i(q_1, q_2)$ functions.

To fully describe the noncooperative equilibrium, we need to have a model for these exclusive-dealing price offers. This is especially problematic when all consumer types purchase from both firms in equilibrium because such exclusive price offers arise off the equilibrium path. If we be-

[6] Note that this last aspect is less than realistic and is chosen for the present paper out of simplicity. A more realistic setting in which the software firm (firm 1) offers prices $P_1(q_1, q_2)$ and the hardware manufacturer (firm 2) offers prices conditioning only on the hardware $P_2(q_2)$ can be analyzed without much more difficulty by combining results from this paper and the private common agency literature originating with Martimort (192,1996) and Stole (1991).

lieve that competition is fierce, we might argue that these should be zero-profit price schedules; if competition is very lax, we might alternatively expect the choice of $P_i(q) = \infty$, effectively shutting down exclusive sales. We remain silent on this issue in the present paper, but rather take the values of $\underline{u}_i(\theta)$ as given and focus on the effects upon $\{q_1(\theta), q_2(\theta)\}$ for those consumers who participate.

As before, we solve for the $\gamma = 2$ cooperative solution for a given conjectured $\underline{u}(\theta)$, and apply Theorem 1. We then check ex post that the resulting noncooperative equilibrium indeed generates this outside option when the outputs correspond to those in the $\gamma = 2$ program. If the outside option utilities of the noncooperative game satisfy the requirement of the theorem that $\underline{u}(\theta) = 0 = \max_i\{\underline{u}_i(\theta)\}$, then we have found our noncooperative equilibrium outcome.

We focus our attention on the simplest equilibrium of $\underline{u}(\theta) \equiv 0$. The simplified $\gamma = 2$ program (ignoring monotonicity issues) is by now familiar. The $\gamma = 2$ cooperative chooses $\{q_1(\theta), q_2(\theta)\}$ to satisfy the first-order conditions

$$\theta - q_1(\theta) + \rho q_2(\theta) - c_1 = \gamma(\overline{\theta} - \theta),$$

$$\theta - q_2(\theta) + \rho q_1(\theta) - c_2 = \gamma(\overline{\theta} - \theta).$$

Jointly, this requires that for $i \neq j$, $\gamma = 2$

$$q_i(\theta) = \max\left\{\frac{1}{1-\rho^2}(3\theta(1+\rho) - c_i - \rho c_j - 2\overline{\theta}(1+\rho)), 0\right\} = \max\left\{q_i^{fb}(\theta) - \frac{2(\overline{\theta} - \theta)}{1-\rho}, 0\right\},$$

which is nondecreasing. In contrast, a profit-maximizing (i.e., $\gamma = 1$) cooperative would choose a lessor distortion, yielding

$$q_i^{coop}(\theta) = \max\left\{q_i^{fb}(\theta) - \frac{(\overline{\theta} - \theta)}{1-\rho}, 0\right\}.$$

If, in equilibrium, each outside option, $\underline{u}_i(\theta)$, binds at some θ_i^o and is slack for all $\theta \geq \theta_i^o$, then we know that the noncooperative outputs will coincide with the $\gamma = 2$ cooperative solution for this nonbinding interval of Θ. If the outside exclusive price schedules are infinite, then $\underline{u}_i(\theta) \equiv 0$, and the outside option is identical to that under the $\gamma = 2$ program that we solved; the equilibrium outcomes are equal. This particular case is a straightforward application of our theorem and is worthy of some discussion so we focus our attention on it immediately.

Graphically, the distortion is familiar to the case of public good provision. We give a simple example in Figure 2 below.

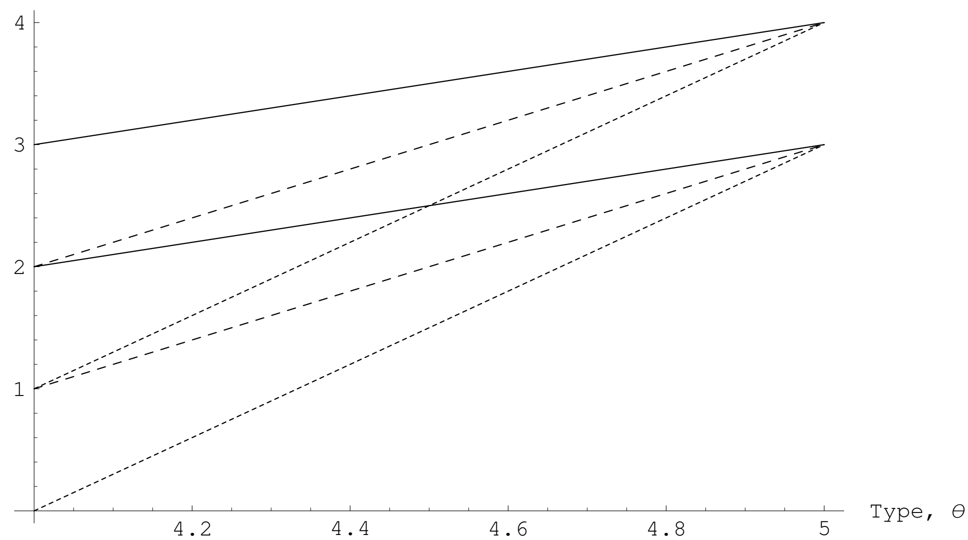

Figure 2: Price discrimination for the case of $v_1(q) = -q_1$, $v_2(q) = -2q_2$, $\rho = 0$ and $\Theta = [4, 5]$. The solid line represents $q^{fb}(\theta)$, the long-dashed line represents the choice of a profit-maximizing cooperative, and the short-dashed line represents the noncooperative choice of $q_1(\theta)$ and $q_2(\theta)$.

As before in the case of public good provision, the output distortion is twice that of a profit-maximizing cooperative relative to the first best. Notice that the example in figure 2 has $\rho = 0$, and so the products are independent in the consumer's preferences. Even so, each firm individually prefers to distort the consumption of his "rival's" product. Fundamentally, each firm is using the other's product as screening device, introducing distortions to reduce consumer rents while ignoring the externality this imposes on the rival's profits. In this sense, there is a free-riding problem on the common screening devices.

It is helpful to consider explicitly the first-order conditions in this noncooperative equilibrium. Each firm maximizes pointwise over q_1 and q_2, yielding the following conditions:

$$-c_1 + \frac{\partial}{\partial q_1} u(q_1(\theta), q_2(\theta), \theta) - \frac{\partial}{\partial q_1} P_2(q_1, q_2) - (\overline{\theta} - \theta) \frac{\partial^2}{\partial \theta \partial q_1} u(q_1(\theta), q_2(\theta), \theta) = 0,$$

$$\frac{\partial}{\partial q_2}u(q_1(\theta), q_2(\theta), \theta) - \frac{\partial}{\partial q_2}P_2(q_1, q_2) - (\overline{\theta} - \theta)\frac{\partial^2}{\partial\theta\partial q_2}u(q_1(\theta), q_2(\theta), \theta) = 0,$$

$$\frac{\partial}{\partial q_1}u(q_1(\theta), q_2(\theta), \theta) - \frac{\partial}{\partial q_1}P_1(q_1, q_2) - (\overline{\theta} - \theta)\frac{\partial^2}{\partial\theta\partial q_1}u(q_1(\theta), q_2(\theta), \theta) = 0,$$

$$-c_2 + \frac{\partial}{\partial q_2}u(q_1(\theta), q_2(\theta), \theta) - \frac{\partial}{\partial q_2}P_1(q_1, q_2) - (\overline{\theta} - \theta)\frac{\partial^2}{\partial\theta\partial q_2}u(q_1(\theta), q_2(\theta), \theta) = 0.$$

Two interesting economic facts emerge from this system of partial differential equations. First, subtracting the first (resp., second) equation from the fourth (resp., third), one obtains

$$\frac{\partial}{\partial q_1}P_1(q_1, q_2) - c_1 = \frac{\partial}{\partial q_1}P_2(q_1, q_2),$$

$$\frac{\partial}{\partial q_2}P_2(q_1, q_2) - c_2 = \frac{\partial}{\partial q_2}P_1(q_1, q_2).$$

Hence, each firm introduces the same marginal distortion on each good, excepting for marginal cost. Second, using the envelope theorem and simplifying the second and fourth equations, we obtain

$$\frac{\partial}{\partial q_2}P_1(q_1, q_2) = (\overline{\theta} - \theta)\frac{\partial^2}{\partial\theta\partial q_2}u(q_1(\theta), q_2(\theta), \theta) > 0,$$

$$\frac{\partial}{\partial q_1}P_2(q_1, q_2) = (\overline{\theta} - \theta)\frac{\partial^2}{\partial\theta\partial q_1}u(q_1(\theta), q_2(\theta), \theta) > 0,$$

and thus each firm charges for the use of the rival's product on the margin, regardless of whether the products are substitutes or complements.

This last result is particularly remarkable. We know that horizontally differentiated duopolists in a Bertrand pricing game will generally introduce greater distortions when competing (relative to cooperation) if and only if the goods are complements; if the goods are substitutes, distortions are smaller relative to a cooperative. This intuition extends to nonlinear pricing as shown in Martimort (1992,1996) and Stole (1991). Here, however, we find that a common agent in tandem with common screening devices generates the double distortion.

To understand the connection between common agency and common screening devices, let's consider common agency with exclusive contracting variables, as in Martimort and Stole (2003a). Consider the contracting problem from the point of view of principal 1, taking principal 2's contract, $P_2(q_2)$, as given. It is, as if, principal 1 was facing an agent with preferences given by

$$\tilde{u}(q_1,\theta) \equiv \max_{q_2} u(q_1,q_2,\theta) - P_2(q_2).$$

Denote $q_2^*(q_1,\theta) \equiv \arg\max_{q_2} u(q_1,q_2,\theta) - P_2(q_2)$. Suppose for the moment that in equilibrium this max-utility function satisfies single-crossing in (q_1,θ). The principal's point-wise first-order condition for $q_1(\theta)$ is (after applying the envelope theorem)

$$v_1'(q_1)+\frac{\partial}{\partial q_1}u(q_1,q_2^*(q_1,\theta),\theta) = (\overline{\theta}-\theta)\left(\frac{\partial^2}{\partial\theta\partial q_1}u(q_1,q_2,\theta)+\frac{\partial^2}{\partial\theta\partial q_2}u(q_1,q_2,\theta)\frac{\partial}{\partial q_1}q_2^*(q_1,\theta)\right).$$

The left hand side represents the marginal joint surplus between the firm and the consumer from an additional increase in q; the right hand side represents the marginal-inframarginal information rent which must be paid in repsonse to an increase in q. Note the the right hand side contains two terms. The first is familiar. The second represents the fact that an increase in q_1 can be expected to change q_2 in a way that may increase or decrease information rents. When the goods are substitutes, $q_2^*(q_1,\theta)$ is decreasing in q_1, and hence the marginal information rent is less than in the profit-maximizing cooperative case. It follows generally that distortions are smaller when the goods are substitutes. The reverse is the case with complements. We can therefore conclude that the basic intuition for the effects of substitutability or complementarity in simple pricing games is equally valid when firms use nonlinear prices and exclusive contracting variables.

Now, return to the present setting in which the contracting variables are commonly controlled. Before the firms could only indirectly influence the consumer's choice of the rival's good via the function $q_j^*(q_i,\theta)$. Now the firms can control these variables directly. It is as if firm 1 controls q_{i1} and firm 2 controls q_{i2}, but physical requirements force $q_{i1} = q_{i2} = q_i$. These physical requirements are mathematically equivalent to supposing that the two control variables are perfect Leontief complements. In this case, regardless of whether the goods are substitutes or complements, we have $\frac{\partial}{\partial q_i}q_j^*(q_i,\theta) = 1$. The distortion that arises is akin to perfect complementarity, even when the goods are independent or substitutes.[7]

The previous analysis focused on the refined set of noncooperative equilibria in which each

[7] As an aside, it should now be clear how one would proceed in a common agency game in which principal 1 is allowed to condition prices on the output vector, $P_1(q_1,q_2)$, but principal 2 is allowed only to price on the basis of his own output: $P_2(q_2)$. For principal 1, the first-order conditions are as presented above for the case of common screening variables. For principal 2, the first-order conditions follow the literature on common agency with exclusive contracting variables. We leave this application for future work.

firm offers an exclusive contract that is unappealing.[8] More generally, we have to consider equilibria that allow for the possibility that $\underline{u}_i(\theta)$ increases sufficiently fast that there is an interval or more of types that are indifferent to exclusive purchasing. This possibility arises if the inside utility (using the $\{q_1(\theta), q_2(\theta)\}$ outputs from the $\gamma = 2$ program) increases less over type than the outside option. Formally, using the envelope theorem, we have an interior interval of binding participation if

$$\frac{d}{d\theta}U(\theta) = q_1(\theta) + q_2(\theta) < \frac{d}{d\theta}\underline{u}_i(\theta) = \arg\max_q \ \ u(0, q_j, \theta) - P_j(q),$$

for some i and interior interval in Θ. Whether such a binding interval arises requires us to take a position on the unmodelled exclusive contracts. When the participation a non-degenerate interval of types is binding in the noncooperative game, however, we can say that it will also be binding for the $\gamma = 2$ cooperative program when $\underline{u}(\theta)$ is replaced with $\max_i\{\underline{u}_i(\theta)\}$. Over such regions of binding participation, the double distortion becomes less economically meaningful as the binding constraints determine the ultimate choices from $\mathcal{Q}$. As a consequence, it is also possible that such binding constraints are as economically signficant as the n-fold marginalization effect on which we have focused. Rather than pursue this issue in more detail, we put this off for future research and continue on to a final application.

6 Application: Lobbying for influence

We now turn to an application of political influence and lobbying, although the model we present could equally well be describing a worker with two bosses, each with differing preferences over how the worker allocates his time. This application will illustrate a setting in which pooling arises in equilibrium.

In our setting, we assume that there are two lobbyists. Lobbyist 1 has increasing preferences for the intensity of policy q_1: $v_1(q_1) = \alpha_1 q$. Lobbyist 2 has similar preferences over policy intensity q_2: $v_2(q_2) = \alpha_2 q_2$. Neither lobbyist directly cares about the other lobbyist's special interests. We assume that the agent is a politician with preferences against any policy intensity (in favor of the easy life), but with a privately known political bias toward either policy 1 or 2.

[8] Technically, this particular class of equilibria has a further requirement that the market is covered.

Specifically,

$$u(q_1, q_2, \theta) = -\frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - \theta q_1 - (1-\theta)q_2,$$

where θ is the private bias of the politician and is uniformly distributed on $[0, 1]$. Each lobbyist can make contributions to the politician that depend upon the entire policy vector, $T_i(q_1, q_2)$, $i = 1, 2$. We assume the politician can always choose the low-intensity policies of $q_i = 0$ for $i = 1, 2$ and obtain $\underline{u}(\theta) \equiv 0$.

The policy choice which maximizes the collective sum of these preferences is

$$q_1^{fb}(\theta) = \alpha_1 - \theta,$$

$$q_2^{fb}(\theta) = \alpha_2 - (1-\theta).$$

As a second reference point, suppose that the lobbyists are restricted to offering contributions that depend only on their own special interests, and not those of rivals. What are the equilibrium intensities? Given that there is no policy interactions in the politician's preferences, this is just a straightforward replication of the nonlinear pricing problem facing a monopolist in which the agent has an outside option of 0. Lobbyist 1, for example, ignores q_2 entirely and considers the politician to have preferences of simply $-\frac{1}{2}q_1^2 - \theta q_1$. The optimal allocation of q_1 is simply $q_1^{mon}(\theta) = \alpha_1 - 2\theta$. Similarly, the optimal allocation for lobbyist 2 is simply $q_2^{mon}(\theta) = \alpha_2 - 2(1-\theta)$. Each lobbyist distorts political intensities downward to reduce the cost of political contributions while still implementing higher margin policies.

As a third reference point, consider the outcome if both lobbyists could join forces (say in a single political foundation) and channel their money jointly to the politician to maximize their collective utility; i.e., they solve $\gamma = 1$ cooperative program. What would the optimal allocation look like? Perhaps surpsing, there will be substantial pooling across moderate types. This is because the the two lobbyists can jointly coordinate to enlarge the region of zero rents to the agent around the point $\theta = \frac{1}{2}$, which is accomplished by introducing pooling. Formally, the cooperative maximizes

$$\max_{q_1, q_2, U} \int_0^1 \left\{ \alpha_1 q_1(\theta) - \frac{1}{2}q_1(\theta)^2 + \alpha_2 q_2(\theta) - \frac{1}{2}q_2(\theta)^2 - \theta q_1(\theta) - (1-\theta)q_2(\theta) - U(\theta) \right\} d\theta,$$

subject to incentive compatibility and individual participation. Using the envelope theorem, it follows that $\frac{d}{d\theta}U(\theta) = q_2(\theta) - q_1(\theta)$. Moreover, one can show using a standard revealed preference argument that incentive compatibility implies that $U(\theta)$ must be convex. Hence, the participation constraint binds in (at most) an interval of Θ. Note that over any such interval it must be the case that $q_1(\theta) = q_2(\theta)$. Let's suppose for the moment that the binding interval is $[\theta_0, \theta_1] \subset \Theta$. The cooperative will set $U(\theta) = \underline{u}(\theta) = 0$ for this interval and the politician's expected rent becomes (after integrating by parts)

$$\int_0^1 U(\theta)d\theta = \int_0^{\theta_0} \theta(q_1(\theta) - q_2(\theta))d\theta + \int_{\theta_1}^1 (1-\theta)(q_2(\theta) - q_1(\theta))d\theta.$$

Substituting for $U(\theta)$ in the cooperatives objective, we obtain the simplified program of a $\gamma = 1$ cooperative:

$$\max_{q_1,q_2,U,\theta_0,\theta_1} \int_0^1 \left\{\alpha_1 q_1(\theta) - \frac{1}{2}q_1(\theta)^2 + \alpha_2 q_2(\theta) - \frac{1}{2}q_2(\theta)^2 - \theta q_1(\theta) - (1-\theta)q_2(\theta)\right\} d\theta$$
$$- \int_0^{\theta_0} \theta(q_1(\theta) - q_2(\theta))d\theta - \int_{\theta_1}^1 (1-\theta)(q_2(\theta) - q_1(\theta))d\theta,$$

subject to $q_2(\theta) - q_1(\theta)$ nondecreasing and $q_1(\theta) = q_2(\theta)$ for all $\theta \in [\theta_0, \theta_1]$. The first-order necessary and sufficient conditions determine the equilibrium allocation.

$$q_1(\theta) = \begin{cases} \alpha_1 - 2\theta \;\; \text{if } \theta \leq \theta_0, \\ q_1^{fb}(\frac{1}{2}) \;\; \text{if } \theta \in (\theta_0, \theta_1), \\ \alpha_1 - \theta - (1-\theta) \;\; \text{if } \theta \geq \theta_1, \end{cases}$$

$$q_2(\theta) = \begin{cases} \alpha_2 - (1-\theta) + \theta \;\; \text{if } \theta \leq \theta_0, \\ q_2^{fb}(\frac{1}{2}) \;\; \text{if } \theta \in (\theta_0, \theta_1), \\ \alpha_2 - 2(1-\theta) \;\; \text{if } \theta \geq \theta_1. \end{cases}$$

Note that these allocations are continuous, exhibit both upward and downward distortions, and involve a significant region of pooling at the average first best for the interval, $q_i^{fb}(\frac{1}{2})$.

The allocation is much different than the uncoordinated duopoly allocations using exclusive contracting variables.

Last, we turn to the noncooperative equilibrium of the political influence game. As before, we need to be explicit about the outside options. Assuming, as in the price discrimination case, that the politician can continue to contract with a single lobbyist absent joint acceptance, the principal's outside options are

$$\underline{u}_1(\theta) \equiv \max_{q_2} u(0, q_2, \theta) + T_2(q_2),$$

$$\underline{u}_2(\theta) \equiv \max_{q_1} u(q_1, 0, \theta) + T_1(q_1).$$

As before, there is considerable freedom in choosing the exclusive contribution functions, and hence there is considerable freedom in choosing the outside options. We consider the simplest case in this paper – the one where each exclusive contribution schedule is infinity. Hence, $\underline{u}_i(\theta) = 0$. This can be held together as an equilibrium when there is full coverage.[9]

We proceed by solving the cooperative program for $\gamma = 2$ and $\underline{u}(\theta) \equiv 0$, and then applying Theorem 1. This is straightforward and requires only that the previous cooperative solution be amended with second information rent term. The result is the following program:

$$\max_{q_1, q_2, U, \theta_0, \theta_1} \int_0^1 \left\{ \alpha_1 q_1(\theta) - \frac{1}{2} q_1(\theta)^2 + \alpha_2 q_2(\theta) - \frac{1}{2} q_2(\theta)^2 - \theta q_1(\theta) - (1-\theta) q_2(\theta) \right\} d\theta$$
$$- 2\int_0^{\theta_0} \theta(q_1(\theta) - q_2(\theta)) d\theta - 2\int_{\theta_1}^1 (1-\theta)(q_2(\theta) - q_1(\theta)) d\theta,$$

subject to $q_2(\theta) - q_1(\theta)$ nondecreasing and $q_1(\theta) = q_2(\theta)$ for all $\theta \in [\theta_0, \theta_1]$. The first-order

[9] Other equilibria are also interesting to consider, such as the case where each principal offers very competitive exclusive contribution schedules tantamount to $T_i(q_i) = \alpha_i q_i - k_i$. This case is left for future research, but it is reasonable to conjecture that the participation constraint will be binding over a large interval of types leading to more efficient outcomes than the present equilibrium selection.

necessary and sufficient conditions determine the equilibrium allocation.

$$q_1(\theta) = \begin{cases} \alpha_1 - 3\theta \ \text{ if } \theta \leq \theta_0, \\ \\ q_1^{fb}(\frac{1}{2}) \ \text{ if } \theta \in (\theta_0, \theta_1), \\ \\ \alpha_1 - \theta - 2(1-\theta) \ \text{ if } \theta \geq \theta_1, \end{cases}$$

$$q_2(\theta) = \begin{cases} \alpha_2 - (1-\theta) + 2\theta \ \text{ if } \theta \leq \theta_0, \\ \\ q_2^{fb}(\frac{1}{2}) \ \text{ if } \theta \in (\theta_0, \theta_1), \\ \\ \alpha_2 - 3(1-\theta) \ \text{ if } \theta \geq \theta_1. \end{cases}$$

Graphically, the first-best, exclusive-screening duopoly, cooperative and noncooperative solutions are illustrated below for the simple case of $\alpha_1 = \alpha_2 = \frac{3}{2}$.

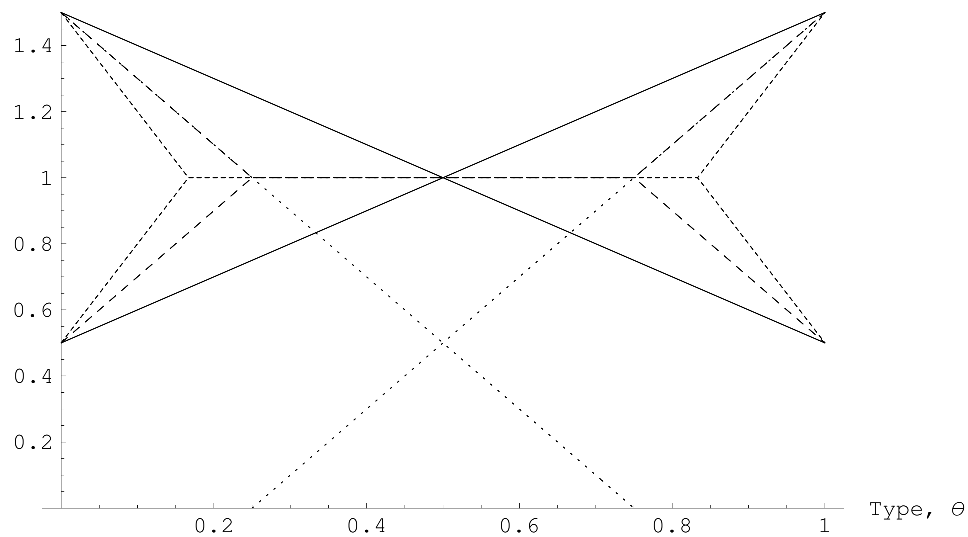

Figure 3: Peddling influence for the case of $\alpha_1(q) = \alpha_2(q) = \frac{3}{2}$. The solid line represents $q^{fb}(\theta)$, the long-dashed line represents the choice of a utility-maximizing cooperative, the short-dashed line represents the noncooperative choice of $q(\theta)$, and the dotted line represents the noncooperative choice of lobbyists contracting over only their own special interests.

two remarkable comparative statics are clear in the graph (and in the more general mathematics of the setting). First, assuming that the lobbyists are able to make their contributions depend upon all aspects of political choice, the presence of competition (relative to a joint institution of coordinated lobbyists) leads to *more pooling* and greater inefficiencies. The presence of double-screening has the effect of muting the politician's (or constituents') underlying preferences and inducing a pooled equilibrium that is only efficient on average. Second, when lobbyists are able to condition their contributions on all political choices rather than on only those that are related to their own special interests, outcomes are more efficient. Here, double-marginalization reduces distortions becuase one lobbyist's marginalization reduces the other's marginalization – they are reducing rather than additive.

7 Concluding Remarks

This paper, in its present version, has only scratched the surface regarding the effects of shared screening devices and n-fold marginalization. Fundamentally, we have presented a new result:

that double-marginalization can be understood as a special case of n-fold applications of common screening variables. This result seems quite general and robust. What remains to be understood (and we hope to address in future versions of this paper or related research) is how the outside options are determined and how they affect the final allocations. Our conjecture is that more competitive outside options will work to offset the n-fold marginalization effects when participation constraints are binding over larger intervals.

References

Bernheim, D. and M. Whinston, 1986, “Common Agency,” *Econometrica*, Vol. 54, pp. 923-943.

Clarke, F., 1983, *Optimization and Nonsmooth Analysis – Canadian Mathematical Society Series of Monographs and Advanced Texts*, John Wiley & Sons.

Grossman, G. and E. Helpman, 1994, “Protection for Sale,” *American Economic Review*, 84(4): 833-850.

Laffont, J.-J. and Tirole, J., “Privatization and Incentives,” *Journal of Law, Economics and Organization*, Vol. 7, pp. 84-105.

Martimort, D., 1992, “Multi-Principaux avec Selection Adverse,” *Annales d’Economie et de Statistique,* Vol. 28, 1992, pp. 1-38.

Martimort, D., 1996, “Exclusive Dealing, common agency, and multi-principals incentive theory,” *RAND Journal of Economics* Vol. 27(1), Spring 1996, pp. 1-31.

Martimort, D., 2004, “Delegated Common Agency under Moral Hazard and the Formation of Interest Groups,” working paper, University of Toulouse (IDEI, GREMAQ).

Martimort, D. and Stole, L., 2003a, “Market Participation under Delegated and Intrinsic Common Agency Games,” working paper, University of Chicago, GSB.

Martimort, D. and Stole, L., 2003b, ‘Contractual Externalities and Common Agency Equilibria,’ Advances in Theoretical Economics, 3(1): Article 4. http://www.bepress.com

Martimort, D. and Stole, L., 2004, “On the robustness of truthful equilibria in common agency games,” working paper, University of Chicago, GSB.

Mezzetti, C., 1997, “Common agency with horizontally differentiated principals,” *RAND Journal of Economics,* 28(2): 323-345.

Mussa, M., and S. Rosen, 1978, “Monopoly and Product Quality,” *Journal of Economic Theory*, 18: 301-17.

Shleifer, A. and Vishny, R., 1993, “Corruption,” *Quarterly Journal of Economics*, 108: 599-617.

Stole, L., 1991, “Common Agency and Mechanism Design,” mimeo.