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On the Robustness of Truthful Equilibria in Common Agency Games[1]

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First Version

Abstract: We analyze the set of payoffs achieved at truthful equilibria of a delegated common agency game under complete information and the set of equilibrium payoffs achieved with differentiable contributions for the same game under asymmetric information. Those sets depends on how equilibrium contribution schedules are extended off the equilibrium. We show that, as the asymmetric information shrinks to zero, the set of equilibrium payoff under asymmetric information may converge or not towards the set of payoffs achieved at truthful equilibria. We show that this convergence fails if the contributions are extended in a natural way off the equilibrium outputs. This casts some doubt on focusing on the whole set of truthful equilibria under complete information.

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1 Introduction

In a seminal paper, Bernheim and Whinston (1986) have analyzed the equilibrium outcomes of delegated common agency games under complete information. In such a game, several principals design non-cooperatively contribution schedules for a common agent. The latter first chooses which contributions to accept and second which decision should be taken on behalf of those principals. Bernheim and Whinston (1986) convincingly argue that one should focus on the class of *truthful Nash equilibria* of these games. In such equilibria, each principal offers a contribution which reflects at the margin his individual preferences for the decision taken by the common agent.

Truthful Nash equilibria are attractive for several reasons. The first one is that they yield an efficient outcome. Since each principal's marginal contributions reflect his own marginal preferences, the agent is made residual claimant for the whole aggregate surplus. The decentralized contracting process aggregates those preferences efficiently. The second reason why truthful Nash equilibria have attracted modelers in some applied works is that the distribution of the first-best aggregate surplus in any such equilibrium can be easily characterized by means of simple inequalities.[1] Lastly, truthful equilibria have also been found popular because they are coalition-proof Nash. They are immune to deviations by coalitions of principals, which would themselves also be robust to further deviations by subcoalitions and so on.

Even though truthful contributions schedules exhibit all these interesting properties, one may question why those strategies would be more attractive than simple forcing contracts in a world of complete information. The conventional wisdom is indeed that schedules are valuable to adapt to changes in the environment or to cope with asymmetric information issues.[2] However, and quite surprisingly, very little work has been so far devoted to give informational foundations to the notion of truthful equilibria. In this paper, we first extend the delegated common agency framework of Bernheim and Whinston (1986) to the case where principals remain uninformed on a parameter of the agent's utility function that the latter privately knows. Adverse selection justifies then in the first place the use of contribution schedules for screening purposes. Truthfulness is no longer imposed a priori on the contribution schedules. It is replaced by the more natural condition that the slopes of those schedules satisfy the agent's incentive compatibility constraint. Then, we ask whether the equilibrium payoffs and strategies converge towards those achieved at truthful equilibria of the complete information game as the support of the adverse selection parameter shrinks.

[1] In the field of political economy, this feature of the solution may be particularly interesting to understand how the political process of lobbying redistributes wealth among lobbying groups and allows or not entry of groups in the political arena. See for instance Grossman and Helpman (1994) and Mitra (1998).

[2] For a similar discussion, see Martimort and Stole (2003a).

A first step is thus to solve for the equilibrium of the delegated common agency game under adverse selection. We do that in the simple case of two principals since it suffices for our purpose. This analysis is of independent interest for the reader interested in delegated common agency games under adverse selection. Under adverse selection, each principal exerts a negative contractual externality on the other. Indeed, each of them has to extract information rent from the privately informed agent but only enjoys a fraction of the overall benefits of doing so. There is an excessive downward distortion in the agent's decision with respect to the case of complete information. The principals' contribution schedules no longer reflect their own marginal valuations but are strictly less for screening purposes. Importantly, the set of equilibrium payoffs achieved depends on how equilibrium contribution schedules are extended off the equilibrium. Those extensions matter since they determine the reservation payoff that the agent can get by refusing one of the principals' contract. This impacts on the principals' equilibrium payoffs for a given size of the uncertainty on types.

We then look at the *limit equilibria* as the adverse selection problem disappears, i.e., as the support of the adverse selection parameter (drawn uniformly) shrinks to the most efficient type. Those limit equilibria are of course efficient. Indeed, even though under adverse selection contribution schedules are no longer truthful, the marginal contribution of a principal reflects his marginal valuation *at the efficient output* in the limiting case where the adverse selection shrinks to the most efficient realization of the agent's type.[3] This convergence may no longer true elsewhere, i.e., for other outputs. It depends in fact on how equilibrium schedules are extended off the equilibrium.

We distinguish two different kinds of extensions which lead each to different payoffs if the agent refuses one of the contract and thus to different equilibrium payoffs for the principals. The so-called *natural* equilibria are such that we hold the same marginal contribution off and on the equilibrium path for *any* output. Note that this property is satisfied by the truthful schedules under complete information. These equilibria are robust in the sense that an increase in the support of the distribution keeps the marginal contribution of each principal unchanged.[4] The so-called *simple* extensions are instead such that the marginal contribution for outputs lower than those on the equilibrium path remains constant.

Quite surprisingly, the set of natural equilibrium payoffs in the limit where the support of the adverse selection parameter shrinks is a *strict subset* of the set of truthful equi-

[3] This is a consequence of the standard *"no distortion at the top"* result of the screening literature. See Laffont and Martimort (2002, Chapter 2) among others.

[4] Of course, because the support changes, the fixed-fees in the contribution which are derived from the condition that the least efficient type's information rent is fully extracted also change. See also Martimort and Stole (2003a) for an analysis of another delegated common agency game in a different context which relies on the use of natural equilibria.

librium payoffs. Moreover, the equilibrium strategies which sustain these payoffs are not everywhere truthful. The rough intuition here is as follows. Even in the limit, the contribution schedules of natural equilibria still keep track of the incentive externality between principals which takes place under adverse selection. Those schedules remain always non-truthful. This non-truthfulness modifies and indeed increases the reservation payoff that the agent can get by contracting with only one principal. Overall, each principal cannot decrease his own contribution to the agent as much as in a truthful equilibrium. Not each payoff achieved with truthful equilibria can be rationalized by embedding the game in an adverse selection framework *if* one focuses on natural equilibria.[5] [6]

Although simple equilibria seems less attractive, because they fail to be robust to natural modifications of the support of types, they generate equilibrium payoffs which instead converge towards the truthful ones.

Truthful equilibria have recently been also criticized by Kirchsteiger and Prat (2001) who argued that simple forcing contracts may be as well attractive for real-world players. They ran experiments which confirmed their insights. Laussel and Lebreton (1998) have instead rationalized the use of truthful schedules by introducing uncertainty on the agent's cost function with finite support and ex ante contracting. We will modify their framework by considering adverse selection and ex post contracting. With that timing, non-cooperating principals exert on each other an incentive externality which modifies their contributions away from truthfulness even in the limit of a small degree of adverse selection. Martimort and Moreira (2003) offer another criticism of the "truthfulness criterion" and replace also this notion by incentive compatibility in a models where principals are now privately informed and use contribution schedules both to signal their preferences and learn those of others. In Martimort (2004), the truthfulness criterion is instead rationalized by considering a moral hazard environment where principals observe only a noisy signal of the agent's action.

Section 2 presents the model. Section 3 reminds the characterization of the truthful equilibria under complete information. Section 4 solves the delegated common agency game under adverse selection without specifying off the equilibrium extensions of the schedules. Section 5 fully characterizes equilibria for natural and simple extensions off the equilibrium path. Section 6 analyzes limit equilibria as the support of the adverse selection

[5]This robustness check incidentally casts also some doubt on the often heard claim that truthful equilibria are interesting because they are coalition-proof. Some coalition-proof equilibria under complete information are not limits of natural equilibria of the game under adverse selection. Another criticism of coalition-proof equilibria is given by Lebreton, Konishi and Weber (1999) who show that coalition-proof equilibria may be Pareto-dominated by other Nash equilibria.

[6]Following Laussel and Lebreton (2001), we can identify the truthful equilibrium payoffs of the common agency game analyzed in this paper with the core of a public good game. Introducing asymmetric information and focusing on natural equilibria provides us with a valuable tool to select within this set a strict subset of payoffs.

parameter shrinks and shows that the convergence towards the thruthful equilibrium payoffs fails for natural equilibria whereas it holds for simple ones. Section 7 briefly concludes. All proofs are relegated to an Appendix.

2 The Model

To make our argument simpler, we focus on the case of two otherwise symmetric principals $(i = 1, 2)$ whose utility functions are $V_i = S_i(q) - t_i$ where t_i is the monetary transfer made to a common agent who chooses the level of a public good q on the principals' behalf. We assume that $S_i(\cdot)$ satisfies the Inada Conditions $S_i'(0) = +\infty$, $S_i'(\infty) = 0$ with $S(0) = 0$ to ensure interior optima under all circumstances. We will also impose the condition $2S_i'' < S_{-i}''$ for $i = 1, 2$. It is satisfied when principals have symmetric preferences but also for relatively small asymmetries as well.[7]

The common agency game unfolds as follows. First, the principals offer nonlinear contribution schedules $t_i(q)$ to the agent. Second, the latter chooses which contracts to accept. Third, the agent chooses the quantity $q(\theta)$ and the corresponding payments are made.

We are interested in describing the sets of the principals' equilibrium payoffs in two cases. In the first setting, there is complete information as in Bernheim and Whinston (1986). In the second setting, there is asymmetric information and none of the principals knows the cost parameter θ. This parameter is drawn on a uniform distribution on $[\underline{\theta}, \bar{\theta}]$. We are particularly interested in looking at the limit of the equilibrium payoffs of that game under adverse selection when the support of the distribution shrinks to zero.

3 Complete Information

The outcome to the delegated common agency game is well-known in that case. Provided that we focus on *truthful* equilibria, the characterization of those equilibria is easy.

Let us suppose that principal 2 offers a truthful schedule which perfectly reflects his own marginal valuation for the public good when it is positive, namely $t_2(q) = \max\{S_2(q) - c_2, 0\}$ where c_2 is a constant.[8] For expositional purposes, it is best to begin with the characterization of principal 1's best response to such a nonlinear scheme when principal 1's offer induces a common representation. We show in the Appendix that

[7]This condition ensures that each equilibrium schedule is concave under adverse selection; an important step to compute the optimal payoff of the agent if he chooses only one contract.

[8]Note in particular that $t_i(q)$ is piecewise differentiable.

principal 1 does not want to deviate towards an exclusive representation by inducing the agent to take only his contract because $t_2(q) \geq 0$ and the agent always want to accept such positive reward.

Under a common agency representation, principal 1's contractual best response is solution to the following problem:

$$(P_1) : \max_{\{q,t_1\}} \; S_1(q) - t_1$$

subject to

$$t_1 + t_2(q) - \theta q \;\geq\; \max_{q \geq 0} \{t_2(q) - \theta q\}, \tag{1}$$

$$t_1 + t_2(q) - \theta q \;\geq\; 0. \tag{2}$$

The participation constraints (1) and (2) stipulate that the agent prefers taking both contracts rather than contracting with principal 2 only (equation (1)) or not participating at all and getting his reservation payoff normalized at zero.

Either (1) or (2) is necessarily binding at the optimum, depending on c_2. Then, the optimal $q^*(\theta)$ solves the Lindahl-Samuelson condition

$$\sum_{i=1}^{2} S_i'(q^*(\theta)) = \theta. \tag{3}$$

This is the standard Lindahl-Samuelson rule. Output is efficient in any *truthful equilibrium.*

Only the optimal pair $\{q^*(\theta), t_1\}$ in the support of principal 1's best-response is determined at the solution of (P_1).[9] Of course, many nonlinear functions $\{t_1(q)\}$ support that outcome.[10] Within that best-response correspondence, we may select, as suggested by Bernheim and Whinston (1986), a truthful schedule which reflects principal 1's preferences as long as it is positive, namely $t_1(q) = \max\{S_1(q) - c_1, 0\}$ for some constant c_1. Importantly, this specification extends also the nonlinear schedule for outputs off the equilibrium path. With a truthful schedule, the principal's preferences are not only reflected at the equilibrium point but also elsewhere.

Using the participation constraints (1) and (2), it is easy to see that the pair of constants (c_1, c_2) which also correspond to the principals' equilibrium payoffs in a truthful equilibrium is obtained as a solution to the system:

$$\max_q \left\{ \sum_{i=1}^{2} S_i(q) - \theta q \right\} - \sum_{i=1}^{2} c_i = \max \left\{ 0, \max_q \{S_j(q) - \theta q\} - c_j \right\}, \tag{4}$$

[9]It is unique since the objective is strictly concave.

[10]Following Peters (2001) and Martimort and Stole (2002), there is indeed no loss of generality in looking at competition between principals with nonlinear menus of this kind.

for $j = 1, 2$.

For further reference, denote $W_{\{12\}}(\theta) = \max_q \left\{\sum_{i=1}^{2} S_i(q) - \theta q\right\}$ and $W_i(\theta) = \max_q\{S_i(q) - \theta q\}$. Those payoffs induce a cooperative game between principals with a characteristic function $W_S(\theta)$ for any subset $S \subseteq \{1, 2, \{1, 2\}\}$. We show in the Appendix that this game is convex and has a non-empty core.[11]

Proposition 1 *: The set of truthful equilibrium payoffs of the principals* (V_1, V_2) *in the delegated common agency game under complete information is characterized by the following linear constraints:*

$$V_1 + V_2 = W_{\{12\}}(\theta) \tag{5}$$

$$W_i(\theta) \leq V_i \leq W_{\{12\}}(\theta) - W_{-i}(\theta), \quad i = 1, 2. \tag{6}$$

Principal i gets at least his marginal contribution to the aggregate surplus of the grand-coalition he forms with the other principal and the agent and at most what he could gain by standing alone with the agent. Conditions (5) and (6) show that the set of truthful equilibrium payoffs can be identified with the core of the cooperative game between principals with characteristic function $W_i(\theta)$.[12]

Note also that, at an equilibrium, each principal offers a positive contribution since the following inequality holds:

$$t_i(q^*(\theta)) = S_i(q^*(\theta)) - c_i \geq W_i(\theta) - (S_i(q^*(\theta)) - \theta q^*(\theta)) > 0.$$

4 Asymmetric Information

Let turn now to the case where there is asymmetric information. The common agent knows the value of θ before contracting with the principal. Let denote by $U_i(\theta)$ the agent's information rent if he takes only principal i's contract:

$$U_i(\theta) = \max_{q \geq 0} \{t_i(q) - \theta q\}$$

where $t_i(\cdot)$ is the nonlinear contribution schedule offered by principal i. We will denote $q_i(\theta)$ the corresponding optimal output. This is assuming strict concavity of the objective function, i.e., that $t_i(\cdot)$ is concave. It turns out that this is the case for small uncertainties $\bar{\theta} - \underline{\theta}$ or, more generally, when $2S_i'' < S_{-i}''$ as assumed.

[11] See Shapley (1971).

[12] On this identification see Laussel and Lebreton (2001) and more recently Milgrom (2004, p.319).

Again, we will focus on equilibria with piecewise differentiable schedules.[13]

Let also $U(\theta)$ be the agent's rent when taking both contracts and $q(\theta)$ be the corresponding output:

$$U(\theta) = \max_{q\geq 0}\left\{\sum_{i=1}^{2} t_i(q) - \theta q\right\}.$$

Using the Envelope Theorem, we get the standard expression

$$\dot{U}(\theta) = -q(\theta), \tag{7}$$

where $q(\theta)$ is given by the first-order condition

$$\sum_{i=1}^{2} t_i'(q(\theta)) = \theta. \tag{8}$$

Moreover, standard revealed preference arguments show also that $q(\cdot)$ must be monotonically decreasing and thus almost everywhere differentiable

$$\dot{q}(\theta) \leq 0. \tag{9}$$

We will again start by supposing that a common representation emerges in equilibrium. We show in the Appendix that this is what happens for $\bar{\theta}-\underline{\theta}$ small and that deviations by a principal to induce the agent to contract only with him are dominated when contributions are again positive.

Under a common representation, principal 1's problem under asymmetric information becomes:

$$(P_1)^{AS}: \max_{\{q(\cdot),U(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} \left[S_1(q(\theta)) + t_2(q(\theta)) - \theta q(\theta) - U(\theta)\right] \frac{d\theta}{\bar{\theta}-\underline{\theta}}$$

subject to (7)-(9) and

$$U(\theta) \geq U_2(\theta), \quad \forall \theta \in \Theta, \tag{10}$$

$$U(\theta) \geq 0, \quad \forall \theta \in \Theta. \tag{11}$$

We have now to consider the *ex post* participation constraints which stipulate that, whatever his type θ, the agent should prefer to take both contracts rather than only that offered by principal 2 (constraint (10)) and refusing all contracts (constraint (11)).

[13] Under complete information we focus on equilibria with truthful schedules which are piecewise differentiable. It is thus a minor restriction to impose piecewise differentiability on the schedules used under adverse selection given that we are interested in the limit of the equilibrium sets as the support of the type distribution goes to zero. It is nevertheless well known that non-differentiable equilibria can be sustained under common agency (see Laffont and Tirole (1993, Chapter 17) for instance).

A priori the solution to $(P_1)^{AS}$ may be quite complex, depending on which and where constraints (10) and (11) bind. We do not plan to complete such a full-fledged analysis here.[14] Remember that we are mainly interested in the case where $\Delta\theta = \bar{\theta} - \underline{\theta}$ is small enough. This significantly simplifies the equilibrium characterization and allows us to focus only on (11) which is actually binding at the upper bound of the distribution $\bar{\theta}$ as shown in the Appendix.

For further reference, consider the schedule $q^c(\theta)$ obtained as the unique solution to

$$\sum_{i=1}^{2} S_i'(q^c(\theta)) = 3\theta - 2\underline{\theta}. \tag{12}$$

As we will see below, this turns out to be the unique output in any equilibrium of the common agency game under asymmetric information.[15] Different equilibrium payoffs may be sustained by varying the fixed-fees in each principal's schedule but all these equilibria correspond to the same output $q^c(\theta)$.

Because $S_i''(\cdot) < 0$, $q^c(\theta)$ is monotonically decreasing. Let denote $\theta^c(q)$ the inverse function defined over the range $[q^c(\bar{\theta}), q^c(\underline{\theta})]$. We have:

$$\theta^c(q) = \frac{1}{3}\left\{\sum_{i=1}^{2} S_i'(q) + 2\underline{\theta}\right\}.$$

When $\bar{\theta} - \underline{\theta}$ is small, (11) binds at $\bar{\theta}$ only, and thus:

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx + \max\{U_2(\bar{\theta}), 0\}. \tag{13}$$

Inserting this expression into the maximand of $(P_1)^{AS}$, integrating by parts and optimizing pointwise with respect to $q(\theta)$ yields the first-order condition:

$$S_1'(q^c(\theta)) + t_2'(q^c(\theta)) = 2\theta - \underline{\theta}. \tag{14}$$

Summing this condition with a similar one obtained from principal 2's best response and using the agent's incentive constraint (8), we get (12) and thus $q^c(\theta)$ is the equilibrium output.

Finally, still using (8), we obtain the expression of the marginal nonlinear schedule offered by principal 1:

$$t_1'(q^c(\theta)) = S_1'(q^c(\theta)) - (\theta - \underline{\theta}), \tag{15}$$

for any θ in Θ. Integrating and keeping the positive part of the schedule so obtained, we get (20).

[14] Tsai (2003) does so but, unfortunately, his analysis suffers from some shortcomings.

[15] Of course, this output remains positive when $\bar{\theta} - \underline{\theta}$ is small enough.

Adverse selection *rationalizes* thus the use by the modeler of a nonlinear contribution schedule.[16] Any output q in the range of $q^c(\cdot)$ corresponds now to a value of the marginal contribution made by principal 1 given by:

$$\begin{aligned} t_1'(q) &= S_1'(q) - \theta^c(q) + \underline{\theta}, \\ &= \frac{2S_1'(q) - S_2'(q) + \underline{\theta}}{3}. \end{aligned} \tag{16}$$

Note that the marginal contribution of principal 1 in any differentiable equilibrium is uniquely defined by (16). Although equilibrium nonlinear prices may differ through the fixed-fee used to reward the agent depending on the reservation payoff $U_2(\bar{\theta})$, they all have the same margin. This uniqueness shows also that, in any equilibrium of the delegated common agency game, the agent necessarily produces $q^c(\theta)$.

However, those contributions are not everywhere truthful; i.e., $t_1'(q) \neq S_1'(q)$ at any q except for $q = q^c(\underline{\theta})$. The marginal contribution of principal 1 is below his marginal valuation for screening purposes except, of course, for the highest equilibrium output; a standard "no distortion at the top result".

Integrating (16) yields for *any* equilibrium output:

$$t_1(q) = \frac{1}{3}\{2S_1(q) - S_2(q) + \underline{\theta}q\} - \tilde{c}_1 \tag{17}$$

for some constant $\tilde{c}_1$ as long as this contribution remains positive.[17]

Importantly, the set of possible values of the fixed-fee c_1 is determined by the reservation payoff $U_2(\bar{\theta})$ which of course depends on how the nonlinear price $t_2(q)$ is extended for off equilibrium outputs $q \leq q^c(\bar{\theta})$. Different specifications of the equilibrium schedule for outputs off the equilibrium path yield different values of $U_2(\bar{\theta})$ and thus different distributions of the principals' payoffs in equilibrium for a given size of the uncertainty on cost $\bar{\theta} - \underline{\theta}$. Therefore different specifications may lead to different selections within the set of truthful equilibrium outcomes when the support of the adverse selection parameter shrinks to zero. The purpose of the next sections is precisely to characterize those different sets for different *plausible* extensions and analyze their limits when uncertainty on types shrinks.

For the time being, we content ourselves with summarizing the common features of any equilibrium in the next proposition leaving to the next sections the characterization of the set of equilibrium fixed-fees.

Proposition 2 *: Assume that $\bar{\theta} - \underline{\theta}$ is small enough, then, at any equilibrium of the delegated common agency game under asymmetric information with piecewise differen-*

[16] On this argument, see also Martimort and Stole (2003b).

[17] A similar expression is obtained for principal 2.

tiable schedules, principals offer nonlinear contribution schedules $\{t_i(q)\}_{1\le i\le 2}$ *which are not truthful everywhere:*

$$t_i(q) = \max\left\{\frac{1}{3}\{2S_i(q) - S_{-i}(q) + \underline{\theta}q\} - \tilde{c}_i, 0\right\}, \tag{18}$$

for any equilibrium output $q \in [q^c(\bar{\theta}), q^c(\underline{\theta})]$ *for some constants* $\tilde{c}_i$ $(i = 1,2)$*. Those schedules correspond all to the same marginal contribution.*

In any such equilibrium, the agent with type θ *produces an output* $q^c(\theta)$*.*

In any such equilibrium, the constants $(\tilde{c}_1, \tilde{c}_2)$ *satisfy the following system of linear equations*

$$\max_{q\ge q^c(\bar{\theta})}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\} - \sum_{i=1}^{2}\tilde{c}_i = \max\left\{0, \max_{q\le q^c(\bar{\theta})}\{t_j(q) - \bar{\theta}q\}\right\}, \tag{19}$$

for $j = 1,2$*.*

From (18), principal 1's contribution is lower as principal 2 values the good more and vice versa. Each contribution reflects now both principals' preferences. This contrasts sharply with the case of complete information where the truthful contribution of each principal depends only on its preferences.

To understand this effect, notice that adverse selection introduces a negative externality between the principals. Indeed, in a common agency environment where principals contract separately with the privately informed agent, each principal designs his own contribution with an eye on the information rent grasped by the agent. Starting from the cooperative outcome where principals would share equally the cost of getting information from the agent, a given principal has an incentive to induce less production to make the other one think that the agent is less efficient. By doing so this latter principal pays more for the agent's services and the deviating principal can reduce his own contribution.

To induce less production from the agent, a given principal reduces thus his marginal contribution below what he would offer under complete information for any equilibrium output below $q^c(\underline{\theta}) = q^*(\underline{\theta})$. Formally, we have indeed:

$$\begin{aligned} S_1'(q) - t_1'(q) &= \frac{1}{3}[S_1'(q) + S_2'(q) - \underline{\theta}] \text{ for any } q \in [q^c(\bar{\theta}), q^c(\underline{\theta})], \\ &\ge \frac{1}{3}[S_1'(q^c(\underline{\theta})) + S_2'(q^c(\underline{\theta})) - \underline{\theta}] = 0. \end{aligned}$$

This expression shows that principal 1's marginal contribution is closer to his own marginal valuation when principal 2 is less ready to pay for the good. When principal 2's preferences are more pronounced, principal 1 has indeed less incentives to contribute at the margin.

5 Equilibrium Sets

5.1 Natural Equilibria

At this stage, (17) suggests the expression of the contribution not only for any output produced on the equilibrium path by one type θ but also for any out-of-equilibrium output, i.e., outputs which are not produced by any type. They thus keep track of the existence of the incentive externality both for on and off equilibrium outputs.

This extension of the mechanism is, in our view, the most *natural* one. In particular, it consists in keeping the same expression of the marginal contribution even if outputs are lower than the minimal equilibrium output $q^c(\bar{\theta})$. To justify this focus on *natural* equilibria, one may think of the principals as insisting on mechanisms which are robust to perturbations of the information structure. Indeed, if the uniform distribution of types is extended over an increasingly large support (increasing $\bar{\theta}$), the marginal price given by (16) remains the same.

We are now ready to characterize the natural equilibria of the delegated common agency game.

Proposition 3 *: Assume that $\bar{\theta} - \underline{\theta}$ is small enough, then, at any natural equilibrium of the delegated common agency game under asymmetric information, principals offer nonlinear contribution schedules $\{t_i(q)\}_{1\leq i\leq 2}$ which are not truthful:*

$$t_i(q) = \max\left\{\frac{1}{3}\{2S_i(q) - S_{-i}(q) + \underline{\theta}q\} - \tilde{c}_i, 0\right\}, \tag{20}$$

for some constants $\tilde{c}_i$ $(i = 1, 2)$ and for any output q both on and off the equilibrium path.

The constants $\tilde{c}_i$ $(i = 1, 2)$ satisfy the following linear constraints:

$$\sum_{i=1}^{2} \tilde{c}_i = \max_{q\geq 0}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\}, \tag{21}$$

$$\max_{q\geq 0}\left\{\frac{1}{3}\left(2S_i(q) - S_{-i}(q) + \underline{\theta}q\right) - \bar{\theta}q\right\} \leq \tilde{c}_i$$

$$\leq \max_{q\geq 0}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\} - \max_{q\geq 0}\left\{\frac{1}{3}\left(2S_{-i}(q) - S_i(q) + \underline{\theta}q\right) - \bar{\theta}q\right\} \text{ for } i = 1, 2. \tag{22}$$

5.2 Simple Equilibria

Consider the following continuously differentiable and concave extensions such that the marginal contributions for out of equilibrium outputs $q \leq q^c(\bar{\theta})$ are given by:

$$t_i'(q) = S_i'(q) - \bar{\theta} + \underline{\theta}. \tag{23}$$

For out of equilibrium outputs $q \geq q^c(\underline{\theta})$, we keep the same expression as in the natural equilibrium.

The main feature of these extensions is that, for sufficiently low outputs, the marginal contributions are, up to a constant that depends on the size of the uncertainty, equal to the marginal valuation of the principals.

Proposition 4 *: Assume that $\bar{\theta} - \underline{\theta}$ is small enough, then, at any simple equilibrium of the delegated common agency game under asymmetric information, principals offer continuously differentiable contribution schedules $\{t_i(q)\}_{1\leq i\leq 2}$ which are not truthful everywhere:*

$$t_i(q) = \max\left\{\frac{1}{3}\{2S_i(q) - S_{-i}(q) + \underline{\theta}q\} - \tilde{c}_i, 0\right\}, \tag{24}$$

for $q \geq q^c(\bar{\theta})$ and

$$t_i(q) = \max\left\{S_i(q) - (\bar{\theta} - \underline{\theta})q - \tilde{k}_i\} - \tilde{c}_i, 0\right\}, \tag{25}$$

for $q \leq q^c(\bar{\theta})$; for some constants $\tilde{c}_i$ and $\tilde{k}_i$ $(i = 1, 2)$ such that

$$\tilde{k}_i = \tilde{c}_i + S_i(q^c(\bar{\theta})) - (\bar{\theta} - \underline{\theta})q^c(\bar{\theta}) - \frac{1}{3}\{2S_i(q^c(\bar{\theta})) - S_{-i}(q^c(\bar{\theta})) + \underline{\theta}q^c(\bar{\theta})\}.$$

The constants $\tilde{c}_i$ $(i = 1, 2)$ satisfy the following linear constraints:

$$\sum_{i=1}^{2} \tilde{c}_i = \max_{q\geq q^c(\bar{\theta})}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\}, \tag{26}$$

$$\max_{q\leq q^c(\bar{\theta})}\left\{S_i(q) - (2\bar{\theta} - \underline{\theta})q\right\} - \left\{S_i(q^c(\bar{\theta})) - (\bar{\theta} - \underline{\theta})q^c(\bar{\theta}) - \frac{1}{3}\{2S_i(q^c(\bar{\theta})) - S_{-i}(q^c(\bar{\theta})) + \underline{\theta}q^c(\bar{\theta})\}\right\} \leq \tilde{c}_i$$

$$\leq \max_{q\geq q^c(\bar{\theta})}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\} - \max_{q\leq q^c(\bar{\theta})}\left\{S_{-i}(q) - (2\bar{\theta} - \underline{\theta})q\right\}$$

$$+S_{-i}(q^c(\bar{\theta})) - (\bar{\theta} - \underline{\theta})q^c(\bar{\theta}) - \frac{1}{3}\{2S_{-i}(q^c(\bar{\theta})) - S_i(q^c(\bar{\theta})) + \underline{\theta}q^c(\bar{\theta})\} \text{ for } i = 1, 2. \tag{27}$$

6 The Limiting Case of Small Uncertainty

Let us now turn to the limiting case where $\bar{\theta} - \underline{\theta}$ converges to zero. We need to describe the payoffs for the principals in the common agency game under asymmetric information, both with natural and simple equilibria, when $\bar{\theta} - \underline{\theta}$ is small and compare these payoffs with those achieved with truthful schedules (characterized by equations (5) and (6)).

6.1 Natural Equilibria

In the limit, natural equilibrium contributions are given by:

$$t_i(q) = \frac{1}{3}\{2S_i(q) - S_{-i}(q) + \underline{\theta}q\} - \tilde{c}_i, \text{ for any } i = 1, 2,$$

where $\tilde{c}_i$ satisfy now (21) and (22) when $\underline{\theta} = \bar{\theta}$. Of course, facing those nonlinear contributions, the agent with type $\underline{\theta}$ still chooses the efficient output $q^*(\underline{\theta})$ just like with truthful schedules. The limit of natural equilibrium schedules are however only truthful at the equilibrium point, not elsewhere.

The equilibrium payoff of principal i in the limit is thus:

$$\tilde{V}_i = S_i(q^*(\underline{\theta})) - t_i(q^*(\underline{\theta})) = \tilde{c}_i + \frac{W_{\{12\}}(\underline{\theta})}{3}.$$

Proposition 5 *: The set of the principals' payoffs $(\tilde{V}_1, \tilde{V}_2)$ at limits of natural equilibria of the delegated common agency of the principals is a strict subset of the equilibrium payoffs achieved with truthful strategies under complete information.*

$$\tilde{V}_1 + \tilde{V}_2 = W_{\{12\}}(\underline{\theta}) \tag{28}$$

$$W_i(\underline{\theta}) < \frac{1}{3}W_{\{12\}}(\underline{\theta}) + \frac{1}{3}\max\{2S_i(q) - S_{-i}(q) - 2\underline{\theta}q\} \leq \tilde{V}_i$$
$$\leq \frac{2}{3}W_{\{12\}}(\underline{\theta}) - \frac{1}{3}\max\{2S_{-i}(q) - S_i(q) - 2\underline{\theta}q\} < W_{\{12\}}(\underline{\theta}) - W_{-i}(\underline{\theta}), \quad i = 1, 2. \tag{29}$$

Equilibrium schedules in the limit are not truthful.

Both the truthful equilibria and the limit of natural equilibria of the game under asymmetric information are such that principals extract all the agent's rent. The sum of the principals' payoffs is always equal to the first-best aggregate surplus in both cases. The difference in the set of equilibrium payoffs comes thus from the extent by which the reservation payoff that the agent gets by contracting only with one principal constrains what the other principal can get.

In a truthful equilibrium, each principal can at most get a share of the first-best surplus which is what his own participation brings to the grand-coalition he forms with the agent and the other principal (equation (6)). As principal 2 values more the good, the share of the aggregate surplus which goes to principal 1 is necessarily lower.

In the limit of a natural equilibrium instead, the contribution of one principal, say principal 1, is everywhere deflated below his own valuation $S_1(q)$ by an amount $\frac{1}{3}(S_1(q) + S_2(q) - \underline{\theta}q)$. This can be viewed as an externality tax paid by the agent to each principal. It captures the fact that, at equilibrium, the marginal contributions of the principals are lower than for a truthful schedule so that output is reduced everywhere except at the very best types under adverse selection due to the contractual externality appearing under asymmetric information.[18] In the limit of natural equilibria, everything happens thus as if each principal could systematically reduce his own contribution by one third of the first-best aggregate surplus, namely $\frac{1}{3}W_{\{12\}}(\underline{\theta})$. Because the whole agent's rent ends up being extracted in the limit equilibria anyway, the sum of the fees $\tilde{c}_1$ and $\tilde{c}_2$ cannot exceed the residual of the overall surplus that accrues to the agent, i.e., the last third of the aggregate surplus $W_{\{12\}}(\underline{\theta})$.

If instead the agent decides to contract only with principal 2, he would produce also less than in a truthful equilibrium since $t_2(q)$ is at the margin less than $S_2(q)$. Both the aggregate surplus available for rent extraction if the agent contracts with both principals and the stand-alone payoff that the agent gets if he contracts only with principal 2 decrease at the same time when one moves from truthful to limit equilibria. However, the second of these terms decrease less. Intuitively, the incremental value of having principal 1 contracting with the agent is less in the limit of natural equilibria since those equilibria "keep memory" from the fact that, under asymmetric information, principal 1's bilateral contracting with the agent exerts a negative externality on principal 2, reducing thereby the value of bringing principal 1 in. Formally, by contracting with both principals in a limit of a natural equilibrium, the agent has to incur twice the externality tax above whereas it is incurred only once by contracting with a single principal.

What is quite spectacular on this example is the fact that this externality has still a non-zero impact when the uncertainty goes to zero. This is due to the fact that as the support of the adverse selection parameter shrinks, the equilibrium schedule (at least at the margin) remains unchanged and keeps the "memory" of the negative externality between principals that would exist under adverse selection for larger supports.

[18] Of course, the marginal "tax" is zero at the equilibrium output $q = q^*(\underline{\theta})$.

6.2 Simple Equilibria

For a simple equilibrium, principal i's payoff in the limit is again given by:

$$\tilde{V}_i = S_i(q^*(\underline{\theta})) - t_i(q^*(\underline{\theta})) = \tilde{c}_i + \frac{1}{3}W_{\{12\}}(\underline{\theta})$$

exactly as for natural equilibria.

In the limit when the uncertainty on types shrinks to zero, simple equilibrium contributions off the equilibrium path converges towards truthful contributions. Hence, not surprisingly, the limit of the equilibrium set of payoffs of simple equilibria converges towards the truthful outcomes.

Proposition 6 *: The set of the principals' payoffs $(\tilde{V}_1, \tilde{V}_2)$ at limits of simple equilibria of the delegated common agency of the principals corresponds to the full set of the equilibrium payoffs achieved with truthful strategies under complete information.*

7 Conclusion

One interpretation of the results of this paper is that one has to be cautious in invoking an underlying game of asymmetric information to justify the use of nonlinear truthful contribution schedules in games of complete information. The set of equilibrium payoffs of the delegated common agency game under asymmetric information depends on how the equilibrium schedules are extended off the equilibrium. Convergence towards all equilibrium payoffs achieved with truthful schedules may fail.

Of course, quite natural extensions/generalizations of the model should investigate the case of more general preferences, generalize the analysis to other distribution of types or check also the robustness to the introduction of informational asymmetries on the principals' preferences.

Our results raise also some more conceptual issues. First, the significant role of off the equilibrium extensions on the equilibrium characterization suggests that more work should be devoted to pin-down what are the most relevant conjectures. We feel comfortable with the natural extension of the equilibrium schedules because of the robustness of such schedules to changes in the support of the distribution. More work should be devoted to make this argument more formal or to extend it beyond the case of a uniform distribution. Second, since truthful equilibria payoffs correspond to coalition-proof payoffs and to the core of a cooperative game between the principals, our analysis suggests that those notions might have to be significantly modified under asymmetric information. Such modifications are likely to depend precisely on the information structures and the kind of informational asymmetries postulated. We hope to investigate some these issues in future research.

References

Bernheim, D. and M. Whinston, (1986), "Menu Auctions, Resource Allocations and Economic Influence", *Quarterly Journal of Economics*, 101: 1-31.

Grossman, G. and E. Helpman, (1994), "Protection for Sale", *American Economic Review*, 84: 833-850.

Kirchsteiger, G. and A. Prat, (2001), "Inefficient Equilibria in Lobbying", *Journal of Public Economics*, 82: 349-375.

Laffont, J.J. and D. Martimort, (2002), *The Theory of Incentives: The Principal-Agent Model,* Princeton University Press, Princeton.

Laffont, J.J. and J. Tirole, (1993), *A Theory of Incentives in Regulation and Procurement,* MIT Press, Cambridge.

Laussel, D. and M. Lebreton, (1998), "Efficient Private Production of Public Goods under Common Agency", *Games and Economic Behavior*, 25: 194-218.

Laussel, D. and M. Lebreton, (2001), "Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency", *Journal of Economic Theory*, 100: 93-128.

Lebreton, M., H. Konishi and S. Weber, (1999), "On Coalition-Proof Nash Equilibria in Common Agency Games", *Journal of Economic Theory*, 85: 122-139.

Martimort, D., (1992), "Multi-Principaux avec Anti-Selection", *Annales d'Economie et de Statistiques*, 28: 1-38.

Martimort, D., (2004), "Delegated Common Agency under Moral Hazard and The Formation of Interest Groups", mimeo IDEI Toulouse.

Martimort, D. and H. Moreira, (2004), "Common Agency with Informed Principals", mimeo IDEI Toulouse.

Martimort, D. and L. Stole, (2002), "The Revelation and Delegation Principles in Common Agency Games", *Econometrica*, 70: 1659-1674.

Martimort, D. and L. Stole, (2003a), "Contractual Externalities and Common Agency Equilibria", *Advances in Theoretical Economics*, Vol. 3, Issue 1, Article 4. http://www.bepress.com/bejte.

Martimort, D. and L. Stole, (2003b), "Market Participation under Delegated and Intrinsic Common Agency Games", mimeo University of Chicago and IDEI Toulouse.

Milgrom, P., (2004), *Putting Auction Theory to Work,* Cambridge University Press, Cambridge.

Mitra, D., (1999), “Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Policy Formation”, *American Economic Review*, 89: 1116-1134.

Peters, M., (2001), “Common Agency and the Revelation Principle”, *Econometrica*, 69: 1349-1372.

Shapley, L., (1971), “Core of Convex Games”, *International Journal of Game Theory*, 1: 11-26.

Stole, L., (1991), “Mechanism Design under Common Agency”, Mimeo Chicago University.

Tsai, T.S., (2002), “Voluntary Provision of Public Goods under Common Agency,” mimeo Institute of Economics Taipei.

Appendix

• **Proof of Proposition 1:**

Lemma 1 *The cooperative game with characteristic function $W_S(\theta)$ is convex.*

Proof: Denote $\tilde{q} = \max(q_1, q_2)$ for any pair $(q_1, q_2) \in \mathbb{R}^2_+$. Because $S_i'(\cdot) > 0$, we have $S_i(\tilde{q}) > S_i(q_i)$, $\forall i \in \{1, 2\}$ with a strict inequality if $q_1 \neq q_2$ and also $-\theta\tilde{q} \geq -\theta(q_1 + q_2)$. Summing yields:

$$\sum_{i=1}^{2} S_i(\tilde{q}) - \theta\tilde{q} > \sum_{i=1}^{2} S_i(q_i) - \theta q_i,$$

and thus

$$W_{\{12\}}(\theta) = \max_{\tilde{q}} \left(\sum_{i=1}^{2} S_i(\tilde{q}) - \theta\tilde{q} \right) \geq \sum_{i=1}^{2} (S_i(q_i) - \theta q_i).$$

Taking the *max* on the right-hand sides yields:

$$W_{\{12\}}(\theta) > \sum_{i=1}^{2} W_i(\theta).$$

Hence, the game is convex and has thus a non-empty core (Shapley (1971)). ■

Coming back on (4), the only feasible solutions are when the right-hand sides is obtained for $0 \geq \max_q\{S_j(q) - \theta q\} - c_j$. This yields the system (5)-(6) for $c_i = V_i$. The solution to this system can be identified with the core of the cooperative game.

No Exclusive Deviation: Finally, we have to show that principal i does not want to deviate so that the agent takes only his own contract (exclusive representation).

Lemma 2 *Under complete information, deviations by principal i to an exclusive representation are suboptimal.*

Proof: Take $i = 1$ for instance, and consider the best such of such deviations to an exclusive representation. It must solve

$$(P_1^d) : \max_{\{q_1,t_1\}} \; S_1(q_1) - t_1$$

subject to

$$\begin{aligned} t_1 - \theta q_1 &\geq \max_{q\geq 0} \{t_2(q) - \theta q\}\,, &\text{(A1)}\\ t_1 - \theta q_1 &\geq 0. &\text{(A2)}\end{aligned}$$

The harder of these participation constraints is binding. This yields:

$$t_1 = \theta q_1 + \max\{0, \max_q\{S_2(q) - c_2\}\}.$$

The best of such deviations yields principal 1 a payoff

$$W_1(\theta) - \max\{0, \max_q\{S_2(q) - c_2\}\}. \tag{A3}$$

Under a common representation, again the harder participation constraint is binding and

$$t_1 = \theta q - t_2(q) + \max\{0, \max_q\{S_2(q) - c_2\}\}.$$

The best of such deviations yields principal 1 a payoff

$$\max_q\{S_1(q) + \max\{S_2(q) - c_2, 0\} - \theta q\} - \max\{0, \max_q\{S_2(q) - cg_2\}\}. \tag{A4}$$

Comparing (A3) and (A4), exclusive deviations are dominated since $t_2(q) = \max\{S_2(q) - c_2, 0\} \geq 0$. ■

This ends the proof of Proposition 1. ■

• **Proof of Proposition 2:** We solve $(P_1)^{AS}$ in the case of small uncertainty.

Lemma 3 *: $U_2(\cdot)$ is non-increasing in θ. Denote $q_2(\theta) = \arg\max_{q\geq 0}\{t_2(q) - \theta q\}$, then:*

$$\begin{aligned} \dot{U}_2(\theta) &= -q_2(\theta), &\text{(A5)}\\ q_2(\theta) &\ \textit{non-increasing},\ \dot{q}_2(\theta) \leq 0. &\text{(A6)}\end{aligned}$$

Proof: Standard revealed preferences arguments show that

$$t_2(q_2(\theta)) - \theta q_2(\theta) \geq t_2(q_2(\theta')) - \theta q_2(\theta')$$

and

$$t_2(q_2(\theta')) - \theta' q_2(\theta') \geq t_2(q_2(\theta)) - \theta' q_2(\theta)$$

for any pair (θ, θ) with $\theta > \theta'$.

Summing yields $(\theta - \theta')(q_2(\theta) - q_2(\theta')) \leq 0$. Hence, $q_2(\theta)$ is non-increasing, and thus almost everywhere differentiable. (A5) follows immediately. That $U_2(\theta)$ is non-increasing in θ is immediate. ■

The question is now to determine which participation constraints between (10) and (11) binds and where.

Note that, if $t_2''(\cdot) < 0$ (a property which holds in equilibrium for equilibrium outputs and which can be imposed on the extension for off equilibrium outputs), $q_2(\theta)$ is given by the first-order condition:

$$t_2'(q_2(\theta)) = \theta, \tag{A7}$$

if $t_2(q)$ is extended over $\mathbb{R}_+$.

Now, let us turn to the properties of rent profile $U(\theta)$.

Lemma 4 *: $U(\cdot)$ is non-increasing in θ with*

$$\dot{U}(\theta) = -q(\theta),$$
$$q(\theta) \text{ non-increasing, } \dot{q}(\theta) \leq 0.$$

Proof: The proof is similar to that of Lemma 3 and is thus omitted. ■

Because the agent with type θ finds optimal to choose output $q(\theta)$, we have:

$$\sum_{i=1}^{2} t_i'(q(\theta)) = \theta.^{19} \tag{A8}$$

Comparing (A7) and (A8), $q(\theta) > q_2(\theta)$ if $t_1'(\cdot) > 0$. This monotonicity property will also hold in equilibrium.

Since $q(\theta) > q_2(\theta)$, the participation constraint (10), if it binds, is binding at $\bar{\theta}$ only. Similarly, (11), if it binds, is also binding at $\bar{\theta}$ only. In any case we can write

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx + U(\bar{\theta}), \tag{A9}$$

where

$$U(\bar{\theta}) = \max_q \left\{ \sum_{i=1}^{2} t_i(q) - \bar{\theta} q \right\} = \max\{0, U_2(\bar{\theta})\}. \tag{A10}$$

A similar condition would be obtained by looking at principal 2's best response.

The optimal output characterized in (14) is then derived using the argument in the text.

Derivation of Equation (19): Equation (19) is obtained using (A10) for principal 1 and a similar condition for principal 2. Note that, when computing $U_i(\theta)$, we have:

$$U_i(\theta) = \max_{q \leq q^c(\bar{\theta})} \{t_i(q) - \bar{\theta} q\}$$

since for outputs greater than $q^c(\bar{\theta})$ the agent whatever his type certainly prefers to take both contracts.

No Exclusive Deviation: We now show that a principal, say principal 1, does not want to induce an exclusive deviation. Let consider a nonlinear scheme $t_1(q)$ under exclusive representation, using the same proof as for Lemma 3, we get:

$$\dot{U}_1(\theta) = -q_1(\theta), \tag{A11}$$

$$q_1(\theta) \text{ non-increasing, } \dot{q}_1(\theta) \leq 0. \tag{A12}$$

The best of such exclusive deviations solves thus the following problem:

$$(P_1)^{dAS} : \max_{\{q_1(\cdot), U_1(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} [S_1(q_1(\theta)) - \theta q_1(\theta) - U_1(\theta)] \frac{d\theta}{\bar{\theta} - \underline{\theta}}$$

subject to (A11)-(A12) and

$$U_1(\theta) \geq U_2(\theta), \quad \forall \theta \in \Theta, \tag{A13}$$

$$U_1(\theta) \geq 0, \quad \forall \theta \in \Theta. \tag{A14}$$

Lemma 5 *Under asymmetric information, deviations by principal i to an exclusive representation are suboptimal when $\bar{\theta} - \underline{\theta}$ is small enough.*

Proof: Take $i = 1$ for instance. Note first that the agent's rent when accepting principal 1's contract inducing an exclusive representation can be written (using (A11), (A13) and (A14)) as:

$$U_1(\theta) = \int_{\theta}^{\bar{\theta}} q_1(x) dx + U_1(\bar{\theta}). \tag{A15}$$

where

$$U_1(\bar{\theta}) \geq \max\{U_2(\bar{\theta}), 0\} \tag{A16}$$

and $q_1(\cdot)$ satisfies (A12) and is such that (A13) holds.

After integrating by parts, principal 1's payoff from an exclusive deviation is thus at most

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[S_1(q_1(\theta)) - (2\theta - \underline{\theta})q_1(\theta) - U_1(\bar{\theta})\right] \frac{d\theta}{\bar{\theta} - \underline{\theta}}. \tag{A17}$$

Since, by construction $t_2(q) \geq 0$, this quantity is less than

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[S_1(q_1(\theta)) + t_2(q_1(\theta)) - (2\theta - \underline{\theta})q_1(\theta) - U_1(\bar{\theta})\right] \frac{d\theta}{\bar{\theta} - \underline{\theta}}, \tag{A18}$$

which would be achieved under a common representation implementing the same quantity $q_1(\cdot)$ and giving the same profile of rent $U(\theta) = U_1(\theta)$ to the agent. ■

This ends the proof of Proposition 2. ■

• **Proof of Proposition 3:** From (15), we have

$$t_1'(q) = S_1'(q) + \underline{\theta} - \theta^c(q),$$

for any output q at a natural equilibrium. Thus $t_1'(q)$ remains positive when $\bar{\theta} - \underline{\theta}$ is small enough. A similar argument holds also for $t_2'(q)$. Because $t_i'(q) > 0$ for $i = 1, 2$, we can use an argument similar to that made in the proof of Proposition 1 to show, using (19), that $\tilde{c}_1$ and $\tilde{c}_2$ satisfy (21) and (22). ■

• **Proof of Proposition 4:** The construction of $t_i'(q)$ is such that it remains always positive when $\bar{\theta} - \underline{\theta}$ is small enough. Because $t_i'(q) > 0$ for $i = 1, 2$, we can use an argument similar to that made in the proof of Proposition 1 to show, using (19), that $\tilde{c}_1$ and $\tilde{c}_2$ satisfy (26) and (27). Using again (19) shows that $\tilde{c}_1$ and $\tilde{c}_2$ satisfy (26) and (27). ■

• **Proof of Proposition 5:** Note first that when $\bar{\theta}$ converges to $\underline{\theta}$, (21) becomes

$$\sum_{i=1}^{2} \tilde{c}_i = \frac{1}{3} \max_{q \geq 0} \left\{ \sum_{i=1}^{2} S_i(q) - \underline{\theta} q \right\} = \frac{W_{\{12\}}(\underline{\theta})}{3}.$$

Therefore, in terms of the principals' payoffs, we get:

$$\sum_{i=1}^{2} \tilde{V}_i = W_{\{12\}}(\underline{\theta}), \tag{A19}$$

exactly as for the truthful equilibria.

Let us turn to (22) when $\bar{\theta}$ converges to $\underline{\theta}$. It becomes:

$$\tilde{c}_i \leq \frac{W_{\{12\}}(\underline{\theta})}{3} - \frac{1}{3}\max_{q\geq 0}\{2S_{-i}(q) - S_i(q) - 2\underline{\theta}q\}.$$

Thus, we get

$$\tilde{V}_i \leq \frac{2}{3}W_{\{12\}}(\underline{\theta}) - \frac{1}{3}\max_{q\geq 0}\{2S_{-i}(q) - S_i(q) - 2\underline{\theta}q\}. \tag{A20}$$

The claim in Proposition 3 is true whenever the right-hand side of (A20) is *strictly* less than that of (6). This holds when:

$$W_{\{12\}}(\theta) > -\max_{q\geq 0}\{2S_{-i}(q) - S_i(q) - 2\underline{\theta}q\} + 3\max_{q\geq 0}\{S_{-i}(q) - \underline{\theta}q\}, \text{ for } i = 1, 2. \tag{A21}$$

Take for instance $i = 1$ and note that:

$$\sum_{i=1}^{2} S_i(q) - \underline{\theta}q + 2S_2(q) - S_1(q) - 2\underline{\theta}q = 3(S_2(q) - \underline{\theta}q), \text{ for all } q \geq 0.$$

Hence

$$W_{\{12\}}(\theta) + 2S_2(q) - S_1(q) - 2\underline{\theta}q > 3(S_2(q) - \underline{\theta}q), \text{ for all } q \geq 0,\ q \neq q^*(\theta),$$

which implies (A21) since $q^*(\theta)$ does not achieve the maximum of $2S_2(q) - S_1(q) - 2\underline{\theta}q$.■

• **Proof of Proposition 6:** Obvious from the text and from making $\bar{\theta} - \underline{\theta}$ converge towards zero into (26) and (27). ■