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The Value of Information in Matching Markets

Abstract

This paper explores the role of private information in matching markets. Agents with private information about their characteristics choose the accuracy of the information that they reveal about themselves to determine who they are matched with. Ironically, we find that better informed agents don't necessarily lead to better matches particularly when the agents are relatively unskilled. In contrast, matching efficiency is higher in markets where agents are skilled and well informed about their abilities. Moreover requiring the agents to certify the accuracy of their disclosures leads to more efficient matching as one might expect.

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1 Introduction

The most important function that a market or a trading organization can serve is to bring together agents from different sides to create a match that maximizes joint surplus. For instance markets that pair students with schools or interns with hospitals facilitate the crucial process of learning and training. Marriage and dating markets are important institutions for facilitating the formation of social relationships and families. Of course the matching of individuals in social and commercial settings doesn't occur automatically or without effort. In many instances intermediaries or match makers facilitate the selection of agents from different sides of the market to form a match that is based on the preferences and abilities that the individuals possess.

Despite the obvious importance of the matching process in creating value, economists know relatively little about how the pairing of agents with complementary preferences and skills occurs in practice and how individuals behave to influence the quality of the partner they are matched with. Moreover most analyses of matching, with a few notable exceptions, assume that agents' characteristics are known and observable at the time a match is determined. In practice, however, match makers devote considerable effort to gathering and verifying information about the agents that they pair together. For after all, the quality of the match is only as good as the quality of the information that market maker has to work with.

In this paper we attempt to understand how the matching process is affected by the actions of agents to gather and disclose private information about themselves to the market. In particular we examine the social and private value of information for matching markets and how the availability of information about market participants affects the quality of matches. Of course the issue of how private and public information affects market efficiency has previously been studied by economists in many different contexts. For instance Hirshleifer (1971) was one of the first studies to demonstrate that more public information can be harmful by eliminating mutually beneficial trades among differently informed agents in wager and insurance markets. In contrast, Hart and Moore (1988) have discovered that public information may increase efficiency in commercial markets by providing more complete contracts. With regards to private information, Akerlof (1970) and Myerson and Satterwaite (1983) show how it can prevent efficient exchanges of property unless prior trading agreements are made. And with respect to the value of information for the purposes of planning, Radner and Stiglitz (1984) show that there is a fundamental non convexity in collecting information and that sufficiently large amounts of information are required before it becomes useful for planning.

While all these analyses have some application to matching markets, the essential role of information in determining match efficiency, has not been systematically studied, at least not to our knowledge. We address the role of information in matching markets by considering a setting in which agents have imperfect and private information about their matching characteristics when they decide how much information to disclose to the market. Our model of the matching is essentially the celebrated "marriage market" that Becker (1973) and others to follow have studied as a reference paradigm for social matches and partnerships. We posit the existence of a match maker that matches agents together to maximize total match surplus based on their disclosed characteristics. Agents differ in their abilities and the value of a match exhibits complementarity in skill, so that the highest match surplus arises when agents of similar abilities are paired together. The match value is shared equally by each agent and transfers between the agents are not possible.

In our setting, matching is costless and instantaneous. Hence, the efficiency of the matching process is only limited by the quality of information that the agents disclose to the market about their type. We model the interaction between agents and the intermediary as a type of signaling game in which agents choose the accuracy of their disclosure and the intermediary creates matches conditional on this information. In equilibrium agents select a disclosure strategy that depends on their type *and* on the distribution of different types present in the market. Hence unlike the usual signaling equilibrium where agents' strategies are independent of the distribution of agent types in the matching setting we variations in the *distribution* of types causes the equilibrium signaling strategies to shift in predictable ways, as we demonstrate below. Moreover, it is this feature, that signaling behavior is based on the distribution of types, that distinguishes matching markets from other types of information markets that have been previously analyzed.

The analysis of our formal model reveals the importance of complementarity in payoff values in determining how agents reveal themselves to the market and how efficiently agents are matched in the market. When match value exhibits complementarity between agent types, we show that positive assortative matching by the *public scores* of agents' ability is optimal for the intermediary. This finding extends the optimality of positive assortative matching, established by Becker (1973) and others, to settings where the quality of public information about agents is imperfect and endogenously determined as part of the signaling equilibrium. The actual matches resulting in equilibrium, are not perfectly assortative however, because information about agents is imperfect at the time matching occurs. Rather, similar to Smith (2006) and Shimer and Smith (2000) actual matching is assortive in a set value sense, in that the average ability of agents that a high ability type matches with is greater than the average ability of the agents that a low ability agent is matched with.

Given the optimality of assortative matching by public signal, we find that agents' preferred strategy of disclosure is naturally ordered by their private signal of ability: that is, if an agent with a bad quality signal weakly prefers to accurately disclose his information then all agents with good quality signals will strictly prefer to accurately disclose. This ordering property reflects a fundamental complementarity between the agent's ability and preferences for disclosure such that agents with higher perceived ability are more likely to benefit from accurate disclosure than agents with lower abilities. We demonstrate that this property leads directly to a simple characterization of signaling equilibrium, wherein either all agents pool and choose the same accuracy of disclosure or the agents separate with the high quality types selecting to disclose accurately and the low quality types opting for the inaccurate disclosure.

What is significant about the derived signaling equilibrium is not that agents disclose based on their type, but rather the fact that disclosure also depends on *the distribution* of agents types in the market. We find that perceived ability and disclosure are complementary in that all agent types choose to disclose accurately in a *skilled market* where the proportion of high ability agents on both side of the market is large. In contrast, in an *unskilled* market where most agents are low ability, all agents decide to disclose inaccurately. The difference in disclosure strategies for skilled and unskilled markets arises because of the different expectations that the agents have about their ability and their matching possibilities. In a skilled market, each individual believes he is likely to be a high type and therefore selects accurate disclosure to maximize his chances of being paired with another high type. In contrast, in an unskilled market each individual believes he is most likely a low type and therefore discloses inaccurately in the hope he will be mistakenly paired with a high type agent.

The complementarity between perceived ability and disclosure also appears in moderate skilled markets as well. In that case a separating equilibrium exists where agents who receive a good quality signal choose to accurately disclose and the agents with a bad signal disclose inaccurately. The rationale for this behavior is that agents with good signals perceive they are likely to be high quality and therefore should disclose accurately in order to be paired with high ability types. For agents who receive a bad signal, the reverse line of reasoning implies they should choose to signal inaccurately.

The value of information, like the incentives to disclose, depends on the type of market where matching occurs. In both skilled and unskilled markets, the public and private value of information about an agent's type is zero, at least on the margin. This occurs because a small amount of information has no affect on how the agents disclose and consequently how they are matched together in equilibrium. It is only when an agent acquires sufficient new information to cause him to select a different disclosure strategy that information becomes valuable. This finding that only discrete improvements in information are valuable is consistent with Radner and Stiglitz (1983) However, unlike Radner and Stiglitz (1983) and MacDonald (1980) we find that the public value of discrete quantities of information may either be positive or negative, depending on the skill level in the market. That is, the arrival of new information in a skilled market is harmful because it reduces disclosure accuracy, whereas the arrival of new information in an unskilled market is beneficial because it causes an increase in disclosure accuracy.

Another important factor in determining match efficiency is whether the accuracy of the disclosure can be observed and certified by the intermediary. For some applications it's reasonable to expect the matchmaker can certify the accuracy of the agent's disclosure, as would be the case, for example, when the agent takes a standardized test that reveals his skill. In those instances the choice of the type of test by itself reveals information about the agent's type which is useful for matching agents together. Moreover, its more difficult for unskilled agents to hide their type by choosing an uninformative test when the accuracy of their disclosure can be observed. In contrast, when the accuracy of disclosure is not certified, only the outcome of the disclosure, and not the type of disclosure, provides information to the intermediary for matching agents. This suggests, and our analysis confirms, that certifiable disclosure is more informative than non certified disclosure for pairing agents together in matching markets.

The foregoing results that link preferences for accurate disclosure to average ability of agents in the market leads to the discovery that match surplus increases disproportionately with increases in the average quality of agents in the market. The link between average quality and matching performance works as follows. As the average quality of agents increases the average surplus of all existing matches increases directly. But in addition there is an indirect affect from a quality increase that causes match efficiency among the agents to increase as well. As the average quality of agents improves each individual is more likely to receive a good signal of his ability and is therefore more likely to believe he has high ability. Agents with higher perceived ability are more likely to disclose accurately and matching efficiency increases with more accurate disclosure. Hence there are increasing returns from quality in matching markets. This finding has important implications for designing matching markets to accommodate the skill levels of the agent population.

Our analysis is not the first to consider how private information affects matching behavior. For instance the issue of private information in matching markets has recently received some attention in the literature. Damiano and Li (2007) establish necessary and sufficient conditions for efficient matching to be revenue-maximizing for a monopoly matchmaker who uses prices to elicit private information. Hopkins (2005) studies costly signaling in a two-sided matching market and finds that agents are assortatively matched based on their signals. Coles and Niederle (2006) ask whether private information about agents' preferences can be credibly signaled with a round of cheap talk before matches are formed in a decentralized market. In contrast to these earlier studies the present paper focuses on how privately informed agents disclose information about themselves to a market intermediary who matches agents to maximize total surplus. The agents can not distort their signals as in Damiano and Li (2007) and Hopkins (2005) but rather they can control the accuracy of their disclosures by the type of public signal they select.

The remainder of our paper is organized as follows. Section 2 presents a simple base-line model of a matching market with non certified disclosure. Section 3 characterizes the equilibria in this model and Section 4 explores the value of information for different types of matching markets. In Section 5 we extend and apply our findings to matching markets with certified disclosure. Section 6 concludes with some reflections on the role of skill and ability in determining matching surplus and efficiency.

2 The baseline model: non certified signaling

We begin our analysis in the context of a simple baseline model in order to fix ideas and establish the basic properties of matching markets. Consider a symmetric two-sided matching market where the population of agents on each side of the market is the same with a mass that is normalized to 1. For concreteness we envision that the market is divided into two sides, a male side and a female side.

2.1 Agent types, information and disclosure

Each agent, whether it be a male or female, is one of two two quality types, T, which is either *high ability* (H) or *low ability* (L). The agent's ability is measured by its ability to perform a particular job or perhaps to provide some service. Higher ability agents can perform all tasks more efficiently than lower ability agents.[1] An agent's type is not directly observable but it is public knowledge that there is a proportion $\pi \in [0, 1]$ of H types on both sides of the market. While an agent can't know its type for sure it receives a private bi-variate signal, $s \in \{g, b\}$ which is either g for *good* or b for *bad*. The signal informs the agent about its true type so that the signal probabilities are assumed to be given by

$$\Pr(g \mid H) = \Pr(b \mid L) = p \geq \frac{1}{2},$$

where the parameter p represents the quality of the private signal. The interesting special case where agents are privately uninformed about their ability arises when $p = \frac{1}{2}$.

Agents on both sides of the market are matched together by an intermediary who observes a bi-variate public score of each agent's type. This public score $t \in \{h, l\}$ is either h which is *high* or l which is *low*. The score provides an indication of the agent's type for the matchmaker to use when deciding on which agents to pair together. The public scores, t and the private signals s are assumed to be independent, conditional on the agent's type. The public score is generated from a test that provides an unbiased prediction of the agent's type. The accuracy the agent selects, denoted by τ, may either be accurate with $\tau = a$ and

$$\Pr(h \mid H) = \Pr(h \mid L) = \lambda^a,$$

or non-accurate with $\tau = n$ and

$$\Pr(h \mid H) = \Pr(h \mid L) = \lambda^n,$$

where $\lambda^a > \lambda^n \geq \frac{1}{2}$. While an agent can not bias the test results in its favor, it can choose the

[1] This is in contrast to MacDonald (1980) who allows agents to be horizontally differentiated in that they have different absolute advantages in working on different jobs. This distinction is important as it leads to different roles for private information in supporting matching.

accuracy of the test that it takes without knowledge of the matchmaker.

It's worth noting that our model is special in several respects in the way that we model private information and disclosure. Some of our assumptions, like the bi-variate private and public signals and the independence of public and private signals, are less restrictive than they seem. We obtain essentially the same results, albeit with more effort, in a more general model that allows for multiple private and public signals which may be correlated with each other. Moreover the assumption that agents transmit unbiased information about themselves is also not necessary for our results, as the matchmaker can readily extract the biases from the public signals that he receives to make inferences about the agents' types. [2]However the assumption that the accuracy of the agents' disclosure is non certifiable is an important restriction on the information that agents are able to disclose and it does affect the efficiency of matching that arises in equilibrium. This assumption is relaxed later on in Section 5 where we analyze certifiable disclosure and compare it with non certifiable disclosure in matching markets.

2.2 Matching payoffs and preferences for pairwise matching

The matching of a type T agent with a type T' agent from different sides of the market produces a common surplus value of $v_{TT'}$ for each agent. For instance, the surplus might represent the value of the services produced when the agents combine their resources together to supply a product. Match value is increasing in the ability types of the agents so that,

$$v_{HH} > v_{HL} > v_{LL} > 0.$$

Most importantly, the match value exhibits complementarity: the marginal match value of a high ability agent is strictly greater when the agent is matched with another high ability rather than a low ability type such that,

$$\frac{v_{HH} - v_{HL}}{v_{LH} - v_{LL}} = \gamma > 1.$$

In models where matching is frictionless, complementarity in match payoffs is what drives the optimality of assortative matching. This appears to extend to settings, for instance see Chade (2006) where information about agent types is imperfect as we show below.[3]

[2] Details and analyses for these extensions of the current model are available from authors

[3] For instance Chade (2006) finds that there is stochastic positive assortative matching in a two sided market with a time cost of search where agents are privately informed about their types.

We assume that the preference of the intermediary is to maximize total match value when it pairs agents together. While we don't model the behavior of the intermediary explicitly, one might imagine that this is induced by competition in the market for intermediation, for instance. The matchmaker is unable to commit to a matching policy in advance of the agents' disclosure of their types, but rather he matches agents based on their expost public signals to maximize total surplus. Further we assume the intermediary treats all male agents (and respectively, all female agents) with the same public score, symmetrically. From this it follows that the intermediary must pair agents either by positive or by negative assortative matching. Let $\beta_t(G)$ be the mass of gender $G = F, M$ agents with public score t for $t \in \{h, l\}$. Then under positive assortative matching the probability that a gender G agent with public score t matches with a gender G' agent with the same score t is $\min\{1, \beta_t(G')/\beta_t(G)\}$ and the probability that G matches with a G' agent with a different score t' is the complementary probably, $1 - \min\{1, \beta_t(G')/\beta_t(G)\}$. Under negative assortative matching the probability that a gender G agent with public score t matches with a gender G' agent with a different score t' is $\min\{1, \beta_{t'}(G')/\beta_t(G)\}$ with the complementary probability, $1 - \min\{1, \beta_{t'}(G')/\beta_t(G)\}$, representing the likelihood that a gender G matches with a gender G' having the same score t.

2.3 The matching game with ex post signals

The matching of agents with private information about their type is determined by a three stage disclosure game. In the first stage of the game, the agent receives a private signal, s, of its ability, which has already been determined. In stage two each agent type and gender privately chooses a disclosure strategy, $\sigma_S^\tau(G)$ which is the probability it selects the test with accuracy τ. In stage three, the intermediary observes all of the public scores and it chooses between a positive and a negative assortative matching strategy, in order to maximize the expected total surplus from the matches. In what follows we characterize the Nash Equilibrium best responses for this game and determine the private and public value of the agent's information in different settings.

3 Analysis of equilibrium and the value of information

In a matching market, each agent no matter what its type, seeks to be matched with another agent of high ability. While all agents desire the same outcome, the strategies they select to achieve it will vary according to their type and the matching process utilized by the intermediary. This section characterizes what these strategies are and how they depend on the agent's type.

3.1 Positive assortative matching by public score.

It's well known that positive assortative matching maximizes match surplus when payoffs exhibit complementarity and when agents' characteristics are observable. The optimality of positive assortative matching continues to hold when agents can control what characteristics are disclosed to the matchmaker as is the case in our matching game.

LEMMA 1. *In any equilibrium of the matching game there is only positive assortative matching by public scores.*

PROOF: First we show that an agent's public score is informative, no matter what accuracy it is. Recall that $\beta_t(G)$ is the mass of gender G with score t, and let $\beta_{Tt}(G)$ be the probability that a type T agent on the G side of the market receives the public score t. It follows that these probabilities are given by,

$$\beta_{Hh}(G) = p\left[\sigma_g^a(G)\lambda^a + \left(1-\sigma_g^a(G)\right)\lambda^n\right] + (1-p)\left[\sigma_b^a(G)\lambda^a + \left(1-\sigma_b^a(G)\right)\lambda^n\right];$$

$$\beta_{Hl}(G) = 1 - \beta_{Hh}(G);$$

$$\beta_{Lh}(G) = p\left[\sigma_b^a(G)(1-\lambda^a) + \left(1-\sigma_b^a(G)\right)(1-\lambda^n)\right] + (1-p)\left[\sigma_g^a(G)(1-\lambda^a) + \left(1-\sigma_g^a(G)\right)(1-\lambda^n)\right];$$

$$\beta_{Ll}(G) = 1 - \beta_{Lh}(G).$$

Notice regardless of the agent's strategy $\sigma_b^a(G)$ and $\sigma_g^a(G)$ for each $s, s' \in \{g, b\}$ we have

$$\sigma_s^a(G)\lambda^a + \left(1-\sigma_s^a(G)\right)\lambda^n \geq \lambda^n > 1-\lambda^n \geq \sigma_{s'}^a(G)(1-\lambda^a) + \left(1-\sigma_{s'}^a(1-\lambda^n)\right)$$

It then follows that $\beta_{Hh}(G) > \beta_{Lh}(G)$ and $\beta_{Hl}(G) > \beta_{Ll}(G)$. Moreover, for each $t \in \{h, l\}$

we have

$$\beta_h(G) = \pi\beta_{Hh}(G) + (1-\pi)\beta_{Lh}(G),$$

and

$$\beta_l(G) = 1 - \beta_h(G).$$

Denoting $\mu_{Tt}(G)$ as the probability that an agent on the G side of the market receiving a public score of t is of type T, we have

$$\mu_{Ht}(G) = \frac{\pi\beta_{Ht}(G)}{\beta_t(G)},$$

and

$$\mu_{Lt}(G) = 1 - \mu_{Ht}(G).$$

Since $\beta_{Hh}(G) > \beta_{Lh}(G)$ and $\beta_{Hl}(G) < \beta_{Ll}(G)$ it follows that

$$\mu_{Hh}(G) > \pi > \mu_{Hl}(G)$$

regardless of $\sigma_b^a(G)$ and $\sigma_g^a(G)$. Since this holds for each $G \in \{M, F\}$, an agent with a public score of h is more likely to be an H type and an agent with public score l is more likely to be an L type. Consequently positive assortative matching maximizes match value when match payoffs are complementary. *QED*

Lemma 1 is a variant on a standard result from sampling theory; namely the result from any informative test, no matter what accuracy it is, will be cause for the matchmaker to update its priors on the agent's type. Therefore agents with high public scores will be considered more likely to be high ability types which will therefore be matched together to create greater surplus. It is worth noting that the intermediary does not commit to matching assortatively, but rather that he finds it optimal to do so, independent of the disclosure strategies of the agents.[4] Moreover lemma 1 implies that no agent can escape the tyranny of testing as eventually all agents are evaluated by their test scores. Agents can only control the accuracy of the score they submit, but not who they are matched with by that score.

[4] As to whether the intermediary might wish to commit to a non- positive assortative matching policy ex ante is an open issue that we have not resolved.

3.2 Ordering of agent preferences for accurate disclosure

In order to characterize the equilibrium disclosure strategies of different agent, it is important to determine which agent types benefit most from accurate disclosure. With assortive matching by public scores, a agent who scores high is more likely to be matched with a high ability agent. This raises the issue of how a particular agent type can score high with the greatest probability. A bad type agent has a greater likelihood of being low ability and is therefore more likely to benefit from an inaccurate test that conceals his true ability from the intermediary. In contrast a good type agent with a greater likelihood of being high ability benefits more from an accurate test which reveal his true type to the matchmaker. This intuition is confirmed by the following lemma.

LEMMA 2: In any equilibrium if an agent with a private signal s=b finds it weakly optimal to choose $\tau = a$ then it is strictly optimal for an agent on the same side with $s = g$ to choose $\tau = a$.

PROOF. For $G \neq G' \in \{M, F\}$ and each $t \in \{h, l\}$ let

$$x_t(G) = \min\{1, B_t(G')/B_t(G)\}$$

be the probability that a gender type G matches with a type t from gender G'. Also, for $\tau \in \{a, n\}$ let

$$\tilde{\lambda}^{\tau} = \lambda^{\tau} x_h(G) + (1 - \lambda^{\tau})(1 - x_l(G))$$

be the probability that a gender G agent who is an H type that employs test τ matches with a gender G' agent with a test score of h. Given positive assortative matching, for each private signal $s \in \{g, b\}$, the payoff to the gender G agent with signal s from choosing the public test τ is given by

$$\begin{aligned}
&\Pr(H \mid s)\left[\tilde{\lambda}^{\tau}\left(\mu_{Hh}(G')v_{HH} + \mu_{Lh}(G')v_{HL}\right) + \left(1 - \tilde{\lambda}^{\tau}\right)\left(\mu_{Hl}(G')v_{HH}\right) + \mu_{Ll}(G')v_{HL}\right] \\
&+ \Pr(L \mid s)\left[\left(1 - \tilde{\lambda}^{\tau}\right)\left(\mu_{Hh}(G')v_{HL} + \mu_{Lh}(G')v_{LL}\right) + \tilde{\lambda}^{\tau}\left(\mu_{Hl}(G')v_{HHL} + \mu_{Ll}(G')v_{LL}\right)\right],
\end{aligned}$$

where $\Pr(T \mid s)$ is the probability that the agent's type is T conditional on private signal s. The agent with private signal s prefers $\tau = a$ to $\tau = n$ if and only if the derivative of the above expression with respect to λ^{τ} is positive , holding the variables $x_t(G)$ and $\mu_{Tt}(G')$ constant. This is equivalent to

$$(\mu_{Hh}(G') - \mu_{Hl}(G'))(x_h(G) + x_l(G) - 1)[\Pr(H \mid s)(v_{HH} - vL) - \Pr(L \mid s)(v_{HL} - v_{LL})] > 0.$$

We have

$$\Pr(H \mid g) = \frac{\pi p}{\pi p + (1-\pi)(1-p)} > \pi > \frac{\pi(1-p)}{\pi(1-p) + p(1-\pi)} = \Pr(H \mid b)$$

Note that

$$\mu_{Hh}(G') > \mu_{Hl}(G'),$$

and

$$x_h(G) + x_l(G) - 1 = \min\{x_h(G), x_l(G)\}.$$

The lemma follows immediately. *QED*

Lemma 2 reveals two properties that play an important role in our analysis. The first is a complementarity between agent ability and accurate disclosure; that is agents with higher perceived ability find it more profitable to test accurately. This complementarity shows up later in our analysis when we discover that agents with greater true ability, as opposed to perceived ability, are also more likely to prefer accurate disclosure. This finding has important implications for the beneficial role of skill in matching markets as we disclose in Section 5.

The second property revealed by Lemma 2 is given by the following inequality in the proof

$$\begin{aligned} & (\mu_{Hh}(G') v_{HH} + \mu_{Hl}(G'))(x_h(G) + x_l(G) - 1)\Pr(H \mid s)(v_{HH} - v_{HL}) \\ > \quad & (\mu_{Hh}(G') v_{HH} + \mu_{Hl}(G'))(x_h(G) + x_l(G) - 1)\Pr(H \mid s)(v_{HL} - v_{LL}). \end{aligned}$$

The left hand side of the above inequality represents the benefit for a type s agent of choosing $\tau = a$ rather than n if it turns out to be a high ability agent It reflects the fact that an accurate test increases the probability of matching with H types of agents from the other side of the market. The right hand side of the inequality represents the cost of choosing $\tau = a$ rather than n if the agent turns out to be a low ability It reflects the fact that a more accurate test reduces the probability of matching with H types of agents. Our ordering property establishes how the benefit and cost comparison depends on the agent's private signal, s. It corresponds to the single crossing condition in a standard signaling model. More important it limits the number of different equilibria that may exist to two, as we demonstrate below.

3.3 Equilibrium testing in matching markets

The aforementioned ordering and complementarity properties that are implied by Lemma 2 are all that we require to characterize equilibrium in matching markets. First, by the ordering property of Lemma 2 we know that for each $G \in \{M, F\}$ in any equilibrium when $\sigma_b^a(G) \geq 0$ this implies $\sigma_g^a(G) = 1$. Moreover since this condition holds for $G = M$ if and only if it holds for $G = F$ it follows that only symmetric equilibrium exists. Hence we can now drop the dependence on G in all of the notation to follow. It follows from this that only pooling equilibrium where both types choose either to test accurately or inaccurately and separating equilibrium where g type agents signal accurately and b type agent test inaccurately are possible. The following proposition is immediate.[5]

PROPOSITION 1: Given a matching market characterized by (π, p, γ), *there exists ability probabilities* $\pi^u(p, v) = \left(\frac{1-p}{p}\gamma + 1\right)^{-1}$ and $\pi^s(p, v) = \left(\frac{p}{1-p}\gamma + 1\right)^{-1}$ *such that*

(i) For $\pi < \pi^u(p, v)$ *there is a unique pooling equilibrium where* $\sigma_g^a = 0, \sigma_b^a = 0$ *and all agents disclose inaccurately.*

(ii) For $\pi^u(p, v) < \pi < \pi^s(p, v)$ *there is a unique separating equilibrium with* $\sigma_g^a = 1, \sigma_b^a = 0$ *where all g agents disclose accurately and all b agents disclose inaccurately.*

(iii) For $\pi^s(p, v) < \pi$ *there is a unique pooling equilibrium with* $\sigma_g^a = 1, \sigma_b^a = 1$ *where all agents disclose accurately.*

An immediate implication of Proposition 1 is that there is *stochastic* positive assortative matching of agents in equilibrium. The matching of identical types together is not perfect, of course, because the h and l groups from which agents are paired do not perfectly represent the H and L agent types respectively. Nonetheless the likelihood of there being an H type agent in group h

[5] The equilibrium are generically unique. At $\pi = \pi^u$ and $\pi = \pi^s$ a pooling and separating equilibrium simultaneously exists.

is greater than in group l. Hence an H type agent who is more likely to be paired with an agent from group h will therefore matched with an agent group that has a higher percent of H types. In contrast, a L type will more likely be matched with agents from group l who are lower ability agents on average.

Proposition 1 reveals that agents' incentives to signal their ability is determined partially by the average skill of the market population. In the relevant regions of the probability space $\pi \in (0, 1)$ different signaling equilibrium occur. For instance in the skilled region, the probability of being a high type is so large all agents choose test a rather than n. No matter what their private signal is, each agent believes it is likely to be a high type and should therefore select the accurate test. Similarly in the unskilled region, the prior probability of being a low type is so high that, regardless of their private signal, all agents benefit more from testing n rather than a. It is only in the moderate skilled region, where agents use their private signal to select their test. Good types choose test a and bad types choose test n.

Proposition 1 reflects the complementarity between ability and preference for accurate disclosure that we have previously identified. The characterization of equilibrium above implies that the greater is the average skill of the population , π, the more likely it is that agents choose the test a rather than n. Moreover Proposition 1 also implies that the greater is γ, the complementarity in payoffs, the more likely it is that an agent will choose the accurate test. The greater benefit there is from matching high ability agents together, the greater is the relative benefit for an agent to disclose accurately.

Whereas increases in π or γ result in a greater likelihood of accurate disclosure, an increase in the quality of the private signal, p, has non monotone effects on the equilibrium probability of any given agent disclosing accurately. Indeed, an increase in p makes it more likely that the equilibrium is separating rather than pooling in either a or in n. The intuition for this is clear from part (ii) of Proposition 1 which reflects the fact that each agent learns more about his real type from observing a more accurate private signal. Consequently an agent with a good signal perceives that it benefits from selecting the accurate test and the agent with a bad signal believes that ut benefits from selecting the inaccurate signal.

4 The private and social value of information in matching markets

This section examines when private information about an agent's ability is socially valuable and when it is personally valuable for the agent to have this information. Our analysis necessarily begins with a discussion of how to measure matching efficiency in markets.

4.1 Measuring matching efficiency

The following lemma establishes a simple criteria for measuring the surplus and the degree of efficiency achieved in matching markets. In this lemma we refer to W as the social surplus from matching which is defined by

$$W = N_{HH}v_{HH} + N_{HL}v_{HL} + N_{LL}v_{LL}$$

where $N_{TT'}$ is the mass of TT' matches in equilibrium.

LEMMA 3: In any matching equilibrium,

(i) The total matching surplus W is uniquely determined by N_{HH}.

(ii) Match surplus, W, is locally increasing in some parameter α provided:

$$\frac{dN_{HH}}{d\alpha} = (\mu_{Hh} - \mu_{Hl}) \left[\pi \left(2 - (\mu_{Hh} + \mu_{Hl})\right) \frac{d\beta_{Hh}}{d\alpha} - (1-\pi)\left(\mu_{Hh} + \mu_{Hl}\right) \frac{d\beta_{Lh}}{d\alpha} \right] > 0.$$

PROOF: See the appendix.

Let W^a be the total match value under the assumption that $\sigma_g^a = \sigma_b^a = 1$ and let W^n be the total match value when $\sigma_g^a = \sigma_b^q = 0$. Since β_{Hh} increases and β_{Lh} decreases in σ_g^a and σ_b^a, from the equation in Lemma 3 we find that N_{HH} is bounded from above by the value it attains when all agents choose test a and it is bounded below by the value it attains when all agents choose n. Thus, W^a is the maximal match surplus and W^n is the minimal match surplus.

Lemma 3 reveals that the number of HH matches is a sufficient statistic for measuring matching surplus. This implies that an increase in the quality of private information that increases the frequency of matches between high ability types will cause total match surplus to increase, for instance.

4.2 The social value of private information

The social value of private information varies by the characteristics of the matching markets including the average skill level, the degree of complementarity in matching and the relative accuracy of the public scores. For instance, when $p = \frac{1}{2}$ so that agents effectively have no private information about their type, they must rely on public information about the average skill level in the market to decide which test to take. If the prior probability of being a high type, π, is sufficiently large so that $\pi > \frac{1}{1+\gamma}$ then the agent will select the a test. Otherwise if $\pi < \frac{1}{1+\gamma}$ the agent will select the low test. In the former case when $\pi > \frac{1}{1+\gamma}$ we refer to the market as being *skilled* and in the latter case when $\pi < \frac{1}{1+\gamma}$ we call the market *unskilled*. Moreover once the quality of private information is sufficient high, agents will begin to base their testing decision on their private signal. At that stage the equilibrium shifts from a pooling to a separating equilibrium. We denote $p^u(\pi, \gamma)$ to be the lowest quality private signal at which the agent chooses the test a or n based on his private information in an *unskilled* market and we define $p^s(\pi, \gamma)$ similarly for a *skilled* market. Finally we measure the accuracy of the a test relative to the n test by $\xi = \frac{2\lambda^a - 1}{2\lambda^n - 1}$.

Adopting this parameterization to our analysis of the value of information, we can identify market settings in which private information is socially beneficial as well as settings where it is socially costly. First consider a unskilled market where the average skill level is low.

PROPOSITION 2: In an unskilled market with $\pi < \frac{1}{1+\gamma}$,

(i) *For* $p < p^u(\pi, \gamma)$ *private information has no social value and the surplus is minimized with* $W(p) = W^n$.

(ii) *For* $p > p^u(\pi, \gamma)$, *the surplus is greater with* $W(p) > W^n$ *and,*

$$\frac{dW}{dp} = \begin{cases} < 0 \text{ for } p^u(\pi, \gamma) < p < \hat{p}(\pi, \xi) \\ > 0 \text{ for } \hat{p}(\pi, \xi) < p \leq 1 \end{cases}$$

where $\hat{p}(\pi, \xi) = \min\left[1, \frac{\xi - \pi(1+\xi)}{\xi - 1}\right]$.

PROOF: To determine the social value of private information, we first note that

$$\frac{d\beta_{Hh}}{dp} = \frac{d\beta_{Lh}}{dp} = (\lambda^a - \lambda^n)\left(\sigma_g^a - \sigma_b^a\right)$$

It therefore follows from Lemma 3 part (ii) that

$$\frac{dN_{HH}}{dp} = (\mu_{Hh} - \mu_{Hl})(\lambda^a - \lambda^n)\left(\sigma_g^a - \sigma_b^a\right)(2\pi - \mu_{Hh} - \mu_{Hl}).$$

In an $n-$pooling equilibrium which occurs for $p < p^u(\pi,\gamma)$ we have $\sigma_g^a = \sigma_b^a = 0$, implying that $\frac{dN_{HH}}{dp} = 0$. This establishes part (i) of the proposition.

For $p > p^u(\pi,\gamma)$ a separating equilibrium occurs where $\sigma_g^a = 1$ and $\sigma_b^a = 0$. Since β_{Hh} is increasing in σ_g^a and β_{Lh} is decreasing in σ_g^a, it is readily verified that $W(p) > W^n$. Moreover since $\mu_{Hh} > \mu_{Hl}$, the sign of $\frac{dN_{HH}}{dp}$ and hence $\frac{dW}{dp}$ is the same as the sign of $(2\pi - \mu_{Hh} - \mu_{Hl})$, which can be shown to have the same sign as $\beta_h - \beta_l$. Therefore, in a separating equilibrium, $\frac{dW}{dp} > 0$ if and only if

$$\frac{\pi}{1-\pi} > \frac{p + (1-p)\xi}{p\xi + (1-p)}.$$

Note that the right hand side of the previous equation is decreasing with p. This implies that as p varies the sign of $\frac{dW}{dp}$ can change at most once and only from negative to positive for

$$p > \hat{p}(\pi,\xi) = \frac{\xi - \pi(1+\xi)}{\xi - 1},$$

where $\hat{p}(\pi,\xi)$ is such that $\frac{dW}{dp} = 0$. Moreover, note that

$$p^u(\pi,\gamma) = \frac{1-\pi}{1-\pi+\gamma\pi}$$

so that at $p = p^u(\pi,\gamma)$ we have

$$\frac{p^u + (1-p^u)\xi}{p^u\xi + (1-p^u)} = \frac{1-\pi+(\gamma\pi)\xi}{(1-\pi)\xi + (\gamma\pi)} > \frac{\gamma\pi(1+\xi)}{(1-\pi)(1+\xi)} > \frac{\pi}{1-\pi}.$$

Since $\pi\gamma < (1-\pi)$ in an unskilled market as we've assumed,

$$\frac{1-\pi+(\gamma\pi)\xi}{(1-\pi)\xi + (\gamma\pi)} > \frac{\gamma\pi(1+\xi)}{(1-\pi)(1+\xi)} > \frac{\pi}{1-\pi}.$$

Hence, for p close to p^u it follows that $\frac{dW}{dp} < 0$. As p increases $\frac{dW}{dp}$ may turn positive for $p > \hat{p}(\pi, \xi)$ provided $\hat{p}(\pi, \xi) < 1$, the maximum value that p can assume. *QED*

Proposition 2 highlights the anomalous impact of private information on matching efficiency. Apparently, private information in small amounts has no impact on matching efficiency until it reaches a threshold level whereupon matching surplus increases discontinuously to a higher level. But beyond that level further increases in information reduce matching surplus on the margin, at least until information becomes sufficiently accurate where upon matching surplus may once again increase. The non monotonic effects of private information on surplus seems to be a feature peculiar to matching markets, whereas the value of discrete improvements in information that arises in matching markets also arises in other settings as Radner and Stiglitz (1983) have pointed out.

The rationale for this anomalous effect of information comes from the discrete changes in equilibrium behavior arising as the quality of information improves. For instance when the quality of private information is low, all agents even those with a good signal believe that they are likely to be a low type and are therefore better off selecting the inaccurate test. This results in minimal match surplus and moreover small increases in private information have no effect on the testing behavior of the agents and therefore no effect on matching efficiency. However once private information increases to a critical level the equilibrium behavior of agents change, with the agents receiving good signals now believing that they are sufficiently likely to be high types that they benefit from selecting the accurate test. This change from a n pooling equilibrium to a separating equilibrium, produces a discrete increase in the matching surplus as a portion of the population now tests accurately, enabling the intermediary to match more efficiently.

To understand the rationale for how, in a separating equilibrium, the total match surplus can initially decrease and then increase with the quality of information requires us to look more closely at the effects of improving information quality. First as p increases the relative size and matching efficiency of the two pools of agents with public scores of h and l will change. More H types will score h because they will be more likely to receive a good signal and will select the accurate test that will yield a public score of h. However at the same time, more L types will score h as well, because they will receive a bad signal and take the inaccurate test that will increase the chances they score h. Hence there is an *allocative effect* from better information that causes the pool of agents with public scores of h to increase relative to the pool of agents with public score l. At the same time, there is a *matching effect* in that the efficiency of matching truly H types with each other will deteriorate in both pools. The allocative effect causing the h pool to grow relative to l pool will increases the number of HH matches since $\mu_{Hh} > \pi > \mu_{Hl}$, implying that match efficiency is always higher in the h pool than in the l pool. However the *matching* effect causes a decline in matching efficiency in both pools thus producing a smaller number of HH matches.

Which of the two effect predominates depends on various parameters of the market including p, π and ξ. The higher π and ξ the more likely it is that matching efficiency will increase because the allocative effect will be strong. When p is relatively small, the private signal is not very informative, so the number of true H types who signal accurately and obtain a public score of h is small, as is the number of true L types who signal inaccurately and obtain a public score of h. Hence the allocative effect will be weak in this case leading to a deterioration in overall matching efficiency. However, once p becomes sufficiently large there will be a strong allocative effect as the agent's private signal will accurately direct H and L types to obtain more h public scores so that the allocative effect will be strong leading to greater match efficiency.

The effect of information on matching efficiency in skilled markets is summarized in the following proposition.

PROPOSITION 3: In a skilled market with $\pi > \frac{1}{1+\gamma}$,

(i) For $p < p^u(\pi,\gamma)$ *private information has no social value and surplus is maximized at* $W(p) = W^a$.

(ii) For $p > p^u(\pi,\gamma)$, *surplus is smaller with* $W(p) < W^a$ *and*

$$\frac{dW}{dp} = \begin{cases} < 0 \text{ for } p^u(\pi,\gamma) < p < \hat{p}(\pi,\xi) \\ > 0 \text{ for } \hat{p}(\pi,\xi) < p \leq 1 \end{cases}$$

where $\hat{p}(\pi,\xi) = \min\left[1, \frac{\xi-\pi(1+\xi)}{\xi-1}\right]$.

PROOF: The proof follows Proposition 2 and is therefore omitted.

Proposition 3 reveals that in contrast to an unskilled market, large amounts of private information can be harmful in a skilled market. The reason is that agents shift from an accurate test pooling equilibrium to a separating equilibrium once private information exceeds a threshold level. As a result matching efficiency declines as some agents select the inaccurate test which causes a reduction in matching efficiency and surplus. Once the market enters into a separating equilibrium the comparative static effects of information on match efficiency are similar in the skilled and unskilled markets.

4.3 Personal value of private information

Its interesting to contrast the social value with the personal value of private information. Consider an agent's incentives to acquire private information in different equilibrium of our model. This may be accomplished by computing an agent's ex ante (prior to observing his signal) payoff from having a private signal of accuracy $\tilde{p}$ in an equilibrium where all other agents have some arbitrary fixed quality signal p. In this setting it's clear that the personal value of private information must be non negative since an increase in the accuracy of a single agent's signal has no affect on equilibrium and therefore can not harm the agent. Moreover by revealed preference an individual who uses private information to make a decision must strictly benefit from better information. By Proposition 1 this implies an agent strictly benefits from private information when $\tilde{p} > p^s(\pi, \gamma)$ in skilled markets or when $\tilde{p} > p^u(\pi, \gamma)$ in unskilled markets since its in those situations that the agent uses its private signal to select between tests. We note that this holds for any equilibrium in which there is positive assortative matching. These arguments lead to the following:

PROPOSITION 4: The personal value of private information is always non negative and it is strictly positive when $\tilde{p} > p^s(\pi, \gamma)$ in skilled markets and when $\tilde{p} > p^u(\pi, \gamma)$ in unskilled markets.

Proposition 4 should not be surprising in light of the fact that agents are able to make better personal decisions when their private information improves. However, in a setting where agents must expend resources to acquire private information, Proposition 4 implies that agents might not collect additional information because it's only valuable in discrete amounts. Its also unsurprising that the personal and social value of private information diverges. So while the acquisition of private information by a single agent might be personally beneficial it wouldn't necessarily improve overall efficiency in the market.

5 Certified signaling in matching markets

The foregoing analysis has highlighted the role of private information and public scores in the pairing of agents in matching markets. Our major findings that agents with good private signals are more apt to test accurately and that the social and personal value of private information don't always coincide are derived in a simple setting that abstracts from several important issues. Among those issue which we ignore is the fact that the accuracy of the agent's test may be observed by the match maker in some realistic settings. For instance in a labor market, a worker's credentials and record of previous employment may serve as a certifiable public score of his ability whose accuracy can be verified by the matchmaker.

The setting where signals are certified introduces an additional factor for the matchmaker to consider when selecting among agents to pair together. Now agents are observed to have two characteristics which include the type of test they submit, denoted by $\tau \in \{a, n\}$ and the public score they achieve, denoted, as before by $t \in \{h, l\}$. If we restrict attention to symmetric test taking strategies, as we do here, this means that the intermediary maximizes the total match value by selecting agents from the two sides based on both their test choices and their test scores. There are therefore a maximum of four pools of agents to draw from, denoted by (τ, t) for $\tau = a, n$ and $t = h, l$.

Aside from the fact that there are more pools of agents to select from here as compared with the non certifiable signaling case, our analysis of this setting proceeds in the same way as for the baseline setting. Beginning with some notation, let β_t^τ represent the mass of agents who signal in the (τ, t) pool and denote by β_{Tt}^τ the probability that a T type agent will signal in the (τ, t) pool for $\tau = a, n$ and $t = h, l$. Finally let μ_{Tt}^τ represent the probability that an agent in the (τ, t) pool is of type T. Then employing this notation we can characterize the conditions that determine the agents preferences for disclosure in the following,

LEMMA 4: Each agent prefers test a to test n if and only if

$$\Pr(H \mid s)(v_{HH} - v_{HL}) k_H \left(\sigma_g^a, \sigma_b^a\right) > \Pr(L \mid s)(v_{HL} - v_{LL}) k_L \left(\sigma_g^a, \sigma_b^a\right),$$

where

$$
\begin{aligned}
k_H\left(\sigma_g^a, \sigma_b^a\right) &= \left[\lambda^a \mu_{Hh}^a + (1-\lambda^a)\,\mu_{Hl}^a\right] - \left[\lambda^n \mu_{Hh}^n + (1-\lambda^n)\,\mu_{Hl}^n\right], \\
k_L\left(\sigma_g^a, \sigma_b^a\right) &= \left[(1-\lambda^n)\,\mu_{Hh}^n + \lambda^n \mu_{Hl}^n\right] - \left[(1-\lambda^a)\,\mu_{Hh}^a + \lambda^a \mu_{Hl}^a\right]
\end{aligned}
$$

are assumed to be well-defined. Moreover if $k_H(0,1), k_L(1,0) > 0$, *then in any equilibrium if an agent with a private signal* $s = b$ *finds it weakly optimal to choose* $t = a$ *then it is strictly optimal for an agent on the same side with* $s = g$ *to choose* $\tau = a$.

PROOF: See the appendix.

The first part of Lemma 4 duplicates the results we found for the non certified signaling case, namely that an agent prefers to disclose accurately if the benefits of choosing a rather than n are greater than the cost of choosing n rather than a, both in terms of a greater probability of matching with an H type instead of a L type. Moreover, provided λ^a is sufficiently large, to ensure that k_H and k_L are strictly positive for all σ_g^a and σ_b^a, the preferences for disclosing are ordered such that the good type agent is more likely to prefer accurate disclosure than the bad type. These findings lead to a complete characterization of the signaling equilibria which is provided in the following proposition.

PROPOSITION 5: When testing is certifiable there exist skill probabilities:

$$
0 < \pi^u(p,\gamma) < \pi^{mu}(p,\gamma) < \pi^m(p,\gamma) < \pi^{ms}(p,\gamma) < \pi^s(p,\gamma) < 1
$$

such that

(i) For $\pi < \pi^u(p,v)$ *there is a unique n-pooling equilibrium where* $\tilde{\sigma}_g^a = \tilde{\sigma}_b^a = 0$ *and all agents disclose inaccurately.*

(ii) For $\pi^u(p,v) < \pi < \pi^{mu}(p,v)$ *there is an n-semi-pooling equilibrium with* $\tilde{\sigma}_g^a \in (0,1), \tilde{\sigma}_b^a = 0$ *where g types mix between test a and n and b types select test n.*

(iii) For $\pi^{mu}(p,v) < \pi < \pi^{ms}(p,v)$ *there is a separating equilibrium where* $\tilde{\sigma}_g^a = 1, \tilde{\sigma}_b^a = 0$ *where g types choose test a and b types choose test n.*

(iv) For $\pi^{ms}(p,v) < \pi < \pi^{\ s}(p,v)$ *there is an a-semi pooling equilibrium with* $\tilde{\sigma}_g^a = 1, \tilde{\sigma}_b^a \in (0,1)$ *where g types choose test a and b mixes between selecting test a and test n.*

(v) For $\pi^s(p,v) < \pi$ *there is a unique a-pooling equilibrium with* $\tilde{\sigma}_g^a = \tilde{\sigma}_b^a = 1$ *where all agents choose test a.*

PROOF: See the appendix.

Proposition 5 reveals that the introduction of certifiable disclosure induces a greater variety of testing responses in equilibrium as compared to the setting where testing is uncertified In addition to the pooling and separating equilibria that exist in the baseline setting, there are now semi pooling equilibria that may occur when signaling is certified. The impact of certifiable testing is that it enables the matchmaker to group agents by the type of test they select. Hence when agents select different tests conditional on their private types, this provides the matchmaker with more information for deciding which agents to pair together. For example agents with a good private signal may select the accurate test whereas agents receiving a bad signal choose the non accurate test. This selection permits the matchmaker to condition matches on the agent's private informaton. As a result, the ability of agents to obscure information about their type by testing inaccurately, as in the non certifiable signaling case, is reduced when tests are certifiable. The implication of this for testing selection and matching efficiency are recorded in the next proposition. In that proposition we refer to $\tilde{\sigma}_s^\tau(p,\pi)$ as the strategy of a type s agent and $\tilde{W}(p,\pi)$ as the expected match surplus in the certified testing equilibrium.

PROPOSITION 6: As compared to the non certifiable signaling equilibrium, the certified signaling equilibrium has the properties that

(i.) There is more accurate testing with $\tilde{\sigma}_g^a(p,\pi) \geq \sigma_g^a(p,\pi)$ *and* $\tilde{\sigma}_b^a(p,\pi) \geq \sigma_b^a(p,\pi)$ *for all p and* π*. and*

(ii) Greater match efficiency is generated with certified signaling for all p and π with $\tilde{W}(p,\pi) \geq W(p,\pi)$

PROOF: See the appendix.

It is not surprising that with certified signaling there is more accurate testing than with non certified signaling, because now for each agent there is an added cost of choosing the non accurate test because of peer group effects. An agent who chooses the inaccurate signal will be paired with other agents who are typically of low ability thus resulting in a lower surplus for the agent. This not only results in agents choosing to test more accurately but as a result it leads to greater match efficiency in the market. The greater efficiency resulting from a finer partition of agent types is embellished by having more agents choosing the accurate test thus leading to a more reliable public score for matching.

6 The social value of skill in matching markets

The aforementioned results regarding certifiable and non certifiable signaling suggest a strong complementarity between agent skill level and incentives to test accurately. This follows for instance, from the ordering properties of Lemmas 2 and 4 that shows that agents that are more likely to be high skilled are more likely to test accurately. Combining this with the finding that matching efficiency increases when more agents choose accurate testing, implies that markets with higher ability agents not only benefit directly from the population skill, but they also achieve a higher degree of match efficiency given the characteristics of their population. That arises because agents with greater ability have more to gain from testing accurately, thus leading to greater match efficiency and hence greater surplus in the matching market.

To formalize this argument consider a matching market characterized by the parameters $\{\pi, p, \lambda_H(\pi, p), \lambda_L(\pi, p)\}$. The variables $\lambda_H(\pi, p)$ and $\lambda_L(\pi, p)$ represent the average accuracy of the tests selected by H and L type agents respectively in equilibrium when the market has an average skill level of π and quality of private information is p. So for instance when disclosure is non certifiable, the average testing accuracy for types H and L is the same with $\lambda_H(\pi, p) = \lambda_L(\pi, p) = \lambda^a$ in an a pooling equilibrium, whereas in a separating equilibrium one can readily verify that $\lambda_H(\pi, p) = \lambda_H^s = p\lambda^a + (1-p)\lambda^n$ and $\lambda_L(\pi, p) = \lambda_L^s = p\lambda^n + (1-p)\lambda^a$. Moreover, since $\lambda^n < \lambda_L^s < \lambda_H^s < \lambda^a$ Proposition 1 implies that the average testing accuracy for both types of agents is increasing with π as the equilibrium transitions from an $n-$pooling equilibrium, to a separating equilibrium and finally to an $a-$pooling equilibrium as π gets sufficiently large.

Let $W^j(\pi, p, \lambda_H, \lambda_L)$ denote the expected match surplus for a market where signaling is either certifiable ($j = c$) or non certifiable ($j = nc$), and the average test accuracy for types H and L is fixed at λ_H and λ_L. The corresponding equilibrium match surplus for the market is denoted by $W^j(\pi, p, \lambda_H(\pi, p), \lambda_L(\pi, p))$. Then we have

PROPOSITION 7: For all $\pi \in (0,1)$ *and* $p, \lambda_H, \lambda_L \in (1/2, 1)$

(i) $W^j(\pi, p, \lambda_H, \lambda_L)$ *is increasing and convex in* π*;*

(ii) $W^j(\pi, p, \lambda_H, \lambda_L)$ *is increasing in* λ_H *and* λ_L*;*

(iii) $W^j(\pi, p, \lambda^n, \lambda^n) \leq W^j(\pi, p, \lambda_H(\pi, p), \lambda_L(\pi, p)) \leq W^j(\pi, p, \lambda^a, \lambda^a)$.

PROOF: First consider the $j = nc$ case where signaling is non certifiable. Recall an increase in π affects $W(\pi, p, \lambda_H, \lambda_L)$ in the following way

$$dW = 2(v_{HH} + v_{LL} - 2v_{LH})\, dN_{HH}$$

so that changes in W have the same sign as changes in N_{HH} by complementarity, and N_{HH} is given by

$$N_{HH} = \beta_h (\mu_{Hh})^2 + (1 - \beta_h)(\mu_{Hl})^2 .$$

It is straightforward to verify that N_{HH} is increasing in π at an increasing rate for any $\pi \in (0,1)$ and that N_{HH} is increasing in λ_H or λ_L for all $\lambda_H, \lambda_L \in (1/2, 1)$. Furthermore, since $W^j(\pi, p, \lambda^n, \lambda^n)$ and $\lambda_H(\pi, p)$ and $\lambda_L(\pi, p)$ are all increasing in π it follows immediately that $W^j(\pi, p, \lambda_H(\pi, p), \lambda_L(\pi, p))$ is also increasing and that

$$W^j(\pi, p, \lambda^n, \lambda^n) \leq W^j(\pi, p, \lambda_H(\pi, p), \lambda_L(\pi, p)) \leq W^j(\pi, p, \lambda^a, \lambda^a)$$

since $\lambda^n < \lambda_L^s < \lambda_H^s < \lambda^a$.

The proof for the $j = n$, the certifiable signaling case, follows similarly. *QED*

From a positive standpoint Proposition 7 predicts that markets with high ability agents perform better than markets populated by agents with lower skills. The rise in matching efficiency occurring with greater skilled agents reflects two effects. First is the *population density effect* given by part (i), that measures the disproportionate increase in matching efficiency that occurs as the proportion of higher skilled agents in the market grows. Second is the *testing effect,* given by part (ii), that accounts for the greater efficiency of matches that arise when agents test more accurately in order to increase their chances of being paired with a more productive type agent. These two effects combined account for the agglomeration economies that are associated with higher skill levels.

Proposition 7 suggests that performance in markets where there are few skilled agents will not only suffer from a lack of ability but also from inaccurate disclosure of ability types which will make it even more difficult to match the few skilled types together that exist. In contrast, highly skilled markets benefit from more efficient matching of ability types brought about by the accurate disclosure of types. This may account for the unexpectently large descrepancies in output that one finds between relatively skilled and unskilled labor markets for instance. This also suggests that if agents were to invest ex ante to increase their skill levels, that they would likely to invest too little, for they will fail to account for the positive externality of their obtaining a higher skill level on the level of accurate disclosure in the market.

7 Appendix

PROOF OF LEMMA 3: The total match value in equilibrium, W, is given by

$$W = 2\left(N_{HH}v_{HH} + N_{LL}v_{LL} + N_{LH}v_{LH}\right)$$

where $N_{TT'}$ is the mass of matches between type T and T' agents, which is determined by

$$N_{TT} = \beta_h\left(\mu_{Th}\right)^2 + B_t\left(\mu_{Tt}\right)^2,$$

and

$$N_{HL} = 2\beta_h\mu_{Hh}\mu_{Lh} + 2\beta_l\mu_{Hl}\mu_{Ll}.$$

It is straightforward to verify that the following identities hold:

$$N_{HH} + N_{HL} + N_{LL} = 1$$

and

$$N_{HH} - N_{LL} = 2\pi - 1.$$

Consider how changes in the agents' strategy profile σ_g^a and σ_b^a affect the total match value. We have

$$dW = 2\left(v_{HH} + v_{LL} - 2v_{HL}\right) dN_{HH},$$

which establishes part (i) of the lemma. By the assumption of complementary match values, changes in W have the same sign as changes in N_{HH}. Further straightforward calculations yield

$$dN_{HH} = \left(\mu_{Hh} - \mu_{Hl}\right)\left[\pi\left(2 - \left(\mu_{Hh} + \mu_{Hl}\right)\right) d\beta_{Hh} - (1-\pi)\left(\mu_{Hh} + \mu_{Hl}\right) d\beta_{Lh}\right],$$

which establishes part (ii) of the lemma. *QED*

PROOF OF LEMMA 4: Let β_{Th}^a be the probability that a type $T = H, L$ agent ends up in the pool with test $\tau = a$ and score $t = h$. Then,

$$\beta_{Hh}^a = p\sigma_g^a \lambda^a + (1-p)\lambda^a,$$

$$\beta_{Lh}^a = p\sigma_b^a\left(1 - \lambda^a\right) + (1-p)\sigma_g^a\left(1 - \lambda^a\right).$$

The size of the pool with test a and score h is

$$\beta_h^a = \pi\beta_{Hh}^a + (1-\pi)\beta_{Lh}^a.$$

The proportion of $T = H, L$ type agents in the pools with test a and score h is

$$\mu_{Hh}^a = \frac{\pi\beta_{Hh}^a}{B_h^a}$$

and

$$\mu_{Lh}^a = 1 - \mu_{Hh}^a.$$

Define $\beta_{Tt}^\tau, \beta_t^\tau$, μ_{Tt}^τ for each $T = H, L$, $\tau = a, n$, $t = h, l$ in a similar fashion. Note that these variables are well-defined as long as β_h^a and β_h^n are strictly positive. The following properties can be easily verified: (i) μ_{Hh}^a and μ_{Hl}^a are increasing in σ_g^a and decreasing in σ_b^a, with $\mu_{Hh}^a > \mu_{Hl}^a$;

(ii) μ^n_{Hh} and μ^n_{Hl} are decreasing in σ^a_g and increasing in σ^a_b, with $\mu^n_{Hh} > \mu$.

The expected payoff for an agent with private signal $s = g, b$ taking test $\tau = a, n$ is

$$\begin{aligned}&\Pr(H \mid s)\left[\lambda^\tau\left(\mu^\tau_{Hh}v_{HH} + \mu^\tau_{Lh}v_{HL}\right) + (1-\lambda^\tau)\left(\mu^\tau_{Hl}v_{HH} + \mu^\tau_{Ll}v_{HL}\right)\right] \\ &+ \Pr(L \mid s)\left[(1-\lambda^\tau)\left(\mu^\tau_{Hh}v_{HL} + \mu^\tau_{Lh}v_{LL}\right) + \lambda^\tau\left(\mu^\tau_{Hl}v_{HL} + \mu^\tau_{Ll}v_{LL}\right)\right].\end{aligned}$$

By definition,

$$\begin{aligned}k_H\left(\sigma^a_{g,}\sigma^a_b\right) &= \left[\lambda^a\mu^a_{Hh} + (1-\lambda^a)\mu^a_{Hl}\right] - \left[\lambda^n\mu^n_{Hh} + (1-\lambda^n)\mu^n_{Hl}\right]; \\ k_L\left(\sigma^a_{g,}\sigma^a_b\right) &= \left[(1-\lambda^n)\mu^n_{Hh} + \lambda^n\mu^n_{Hl}\right] - \left[(1-\lambda^a)\mu^a_{Hh} + \lambda^a\mu^a_{Hl}\right].\end{aligned}$$

Then the agent prefers test a to test n if and only if,

$$\Pr(H \mid s)(v_{HH} - v_{HL})\,k_H\left(\sigma^a_g, \sigma^a_b\right) > \Pr(L \mid s)(v_{HL} - v_{LL})\,k_L\left(\sigma^a_g, \sigma^a_b\right).$$

Since μ^a_{Hh} and μ^a_{Hl} increase with σ^a_g and decrease with σ^a_b , while μ^n_{Hh} and μ^n_{Hl} decrease with σ^a_g and increase with σ^a_b, we have that μ^a_{Hh} and μ^a_{Hl} are increasing in σ^a_g and decreasing in σ^a_b. It follows that k_H increases and k_L decreases with σ^a_g, and k_H decreases and k_L increases with σ^a_b. From the assumption of the lemma that $k_H(0,1), k_L(1,0) > 0$, we have that k_H and k_L are strictly positive for all σ^a_g and σ^a_b. The lemma then follows from the assumption of complementarity, $v_{HH} > v_{HL} > v_{LL}$, and that $\Pr(H \mid g) > \Pr(L \mid b)$. *QED*

PROOF OF PROPOSITION 5: By Lemma 4, so long as β^a_h and β^n_h are strictly positive, regardless of the test taking strategies σ^a_g and σ^n_b, if an agent with private signal b weakly prefers a to n then an agent with private signal g strictly prefers a to n. When $\beta^a_h = 0$ or $\beta^n_h = 0$, Lemma 4 implies that the out of equilibrium belief is pinned down following the reasoning of the standard refinement concept D_1 in signaling games (Banks and Sobel, 1987).[6]

Equilibria can now be characterized. (i) If

$$\frac{v_{HL} - v_{LL}}{v_{HH} - v_{HL}} \leq \frac{\pi(1-p)}{(1-\pi)p}\frac{k_H(1,1)}{k_L(1,1)},$$

[6] See Damiano and Li (forthcoming) for a formal treatment of this type of out of equilibrium refinement based on strategic stability of Kolberg and Mertens (1986).

then there is a a-pooling equilibrium in which all agents choose test a. (ii) If

$$\frac{\pi(1-p)}{(1-\pi)p}\frac{k_H(1,1)}{k_L(1,1)} < \frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} < \frac{\pi(1-p)}{(1-\pi)p}\frac{k_H(1,0)}{k_L(1,0)},$$

then there is an a-semipooling equilibrium in which agents with private signal g choose test a and agents with b randomize between a and n with probability $\sigma_b^a \in (0,1)$, uniquely determined by

$$\frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} = \frac{\pi(1-p)}{(1-\pi)p}\frac{k_H(1,\sigma_b^a)}{k_L(1,\sigma_b^a)}.$$

(iii) If

$$\frac{\pi(1-p)}{(1-\pi)p}\frac{k_H(1,0)}{k_L(1,0)} \leq \frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} < \frac{\pi p}{(1-\pi)(1-p)}\frac{k_H(1,0)}{k_L(1,0)},$$

then there is a separating equilibrium in which agents with private signal g choose test a and agents with b choose n. (iv) If

$$\frac{\pi p}{(1-\pi)(1-p)}\frac{k_H(0,0)}{k_L(0,0)} \leq \frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} < \frac{\pi p}{(1-\pi)(1-p)}\frac{k_H(1,0)}{k_L(1,0)},$$

then there is a n-semipooling equilibrium in which agents with private signal b choose test n and agents with g randomize between a and n with probability $\sigma_g^a \in (0,1)$, uniquely determined by

$$\frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} = \frac{\pi p}{(1-\pi)(1-p)}\frac{k_H(\sigma_g^a,0)}{k_L(\sigma_g^a,0)}.$$

(v) Finally, if

$$\frac{v_{HL}-v_{LL}}{v_{HH}-v_{HL}} \geq \frac{\pi p}{(1-\pi)(1-p)}\frac{k_H(0,0)}{k_L(0,0)},$$

then there is a n-pooling equilibrium where all agents choose test n. Note an overlap in the last three cases: if the parameter values fall into case (iv), there are three equilibria: a separating equilibrium, an n-semipooling equilibrium and an n-pooling equilibrium. Outside this overlapping range, the equilibrium is unique. *QED*

PROOF OF PROPOSITION 6: (to be supplied)

8 References

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