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Intangible Risk?*

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1 Introduction

Accounting for the asset values by measured physical capital and other inputs arguably omits *intangible* sources of capital. This intangible or unmeasured component of the capital stock may result because some investments from accounting flow measures are not eventually embodied in the physical capital stock. Instead there may be scope for valuing ownership of a technology, for productivity enhancements induced by research and development, for firm specific human capital, or for organizational capital.

For an econometrician, intangible capital becomes a residual needed to account for values. In contrast to measurement error, omitted information or even model approximation error, this residual seems most fruitfully captured by an explicit economic model. It is conceived as an input into technology whose magnitude is not directly observed. Its importance is sometimes based on computing a residual contribution to production after all other measured inputs are accounted for. Alternatively it is inferred by comparing asset values from security market data to values of physical measures of firm or market capital. Asset market data is often an important ingredient in the measurement of intangible capital. Asset returns are used to convey information about the marginal product of capital and asset values are used to infer magnitude of intangible capital.

In the absence of uncertainty, appropriately constructed investment returns should be equated. With an omitted capital input, constructed investment returns across firms, sectors or enterprises will be heterogeneous because of mis-measurement. As argued by Telser (1988) and many others, differences in measured physical returns may be “explained by the omission of certain components of their ‘true’ capital.” McGrattan and Prescott (2000) and Atkeson and Kehoe (2002) are recent macroeconomic examples of this approach. Similarly, as emphasized by Hall (2001) and McGrattan and Prescott (2000), asset values should encode the values of both tangible and of intangible capital. Provided that physical capital stock can be measured there is scope for asset market data to be informative about the intangible component of the capital stock.

Following Hall (2001) we find it fruitful to consider the impact of risk in the measurement of intangible capital. Although not emphasized by Hall, there is well documented heterogeneity in the returns to equity of different types. In the presence of uncertainty, it is well known that use of a benchmark asset return must be accompanied by a risk adjustment. Historical averages of equity returns differ in systematic ways. Inferences about the intangible capital stock using security market returns necessarily must confront risk considerations or some competing interpretation for the heterogeneity in security market returns. Similarly, asset *values* reflect beliefs about the future prospects for firms, but the also reflect the riskiness of the implied cash flows.

In what follows we review the relevant investment theory (see Section 2). In section 3 we review and reproduce some of the findings in the asset pricing literature by Fama and French (1992) on return heterogeneity, and in section 4 we review some of the asset valuation theory used to explain the return heterogeneity by the existence of interpretable risk premia.

Risk premia can be characterized in terms of return risk or dividend or cash flow risk. We

follow some recent literature in finance by exploring dividend risk. Since equity ownership of securities entitles an investor to future claims to dividends in all subsequent time periods, quantifying dividend risk requires a time series process. We consider measurements of dividend risk using vector autoregressive (VAR) characterizations. Since asset valuation entails the study of a present-value relation, long run growth components of dividends can play an important role in determining asset values. In section 5 we reproduce the present-value approximation used in the asset pricing literature and use it to define a long-run measure of risk as a discounted impulse response. In section 6, we study how restricting long-run growth of dividends can alter the dividend-risk measures that have been advocated in the literature. Our discussion of dividend risk is of independent interest beyond the question of how to measure intangible capital.

The literatures on intangible capital and asset return heterogeneity to date have been largely distinct. Our disparate discussion of these literatures will inherit some of this separation. In section 7 we conclude with some discussion of how to understand better lessons from asset pricing for the measurement of intangible capital.

2 Adjustment Cost Model

We begin with a discussion of adjustment costs and physical returns. Grunfeld (1960) shows how the market value of firm is valuable in the explanation of corporate investment. Lucas and Prescott (1971) developed this point more fully by producing an equilibrium model of investment under uncertainty. Hayashi (1982) emphasized the simplicity that comes with assuming constant returns to scale. We exploit this simplicity in our development that follows.

Consider the following setup:

2.1 Production

Let n_t denote a variable input into production such as labor, and suppose there are two types of capital, namely $k_t = (k_t^m, k_t^u)$ where k_t^m is the measured capital and k_t^u is unmeasured or intangible capital stock. Firm production is given by

$$f(k_t, n_t, z_t)$$

where f displays constant returns to scale in the vector of capital stocks and the labor input n_t . The random variable z_t is a technology shock at date t .

Following the adjustment cost literature, there is a nonlinear evolution for how investment is converted into capital.

$$k_{t+1} = g(i_t, k_t, x_t) \tag{1}$$

where g is a two-dimensional function displaying constant returns to scale in investment and capital and x_t is a specific shock to the investment technology. We assume that there are two components of investment corresponding to the two types of capital. This technology

may be separable in which case the first coordinate of g depends only on i_t^m and k_t^m while the second coordinate depends only on i_t^u and k_t^u .

Example 2.1. *A typical example of the first equation in system (1) is:*

$$k_{t+1}^m = (1 - \delta_m)k_t^m + i_t^m - g_m(i_t^m/k_t^m, x_t)k_t^m$$

where δ_m is the depreciation rate and g_m measures the investment lost in making new capital productive.

In the absence of adjustment costs, the function g is linear and separable.

Example 2.2. *A common specification that abstracts from adjustment costs is:*

$$g(k, i, z) = \begin{bmatrix} 1 - \delta_m & 0 \\ 0 & 1 - \delta_u \end{bmatrix} k_t + i_t$$

2.2 Firm Value

Each time period the firm purchases investment goods and produces. Let p_t denote the vector of investment good prices and w_t the wage rate. Output is the numeraire in each date. The date-zero firm value is:

$$E \left(\sum_{t=0}^{\infty} S_{t,0} [f(k_t, n_t, z_t) - p_t \cdot i_t - w_t n_t] \mid \mathcal{F}_0 \right) \quad (2)$$

The firm uses market determined stochastic discount factors to value cash flows. Thus $S_{t,0}$ discounts the date t cash flow back to date zero. This discount factor is stochastic and varies depending on the realized state of the world at date t . As a consequence $S_{t,0}$ not only discounts known cash flows but it adjusts for risk, see Harrison and Kreps (1979) and Hansen and Richard (1987).¹ The notation \mathcal{F}_0 denotes the information available to the firm at date zero.

Form the Lagrangian:

$$E \left(\sum_{t=0}^{\infty} S_{t,0} [f(k_t, n_t, z_t) - p_t \cdot i_t - n_t w_t - \lambda_t \cdot [k_{t+1} - g(i_t, k_t, x_t)]] \mid \mathcal{F}_0 \right) \quad (3)$$

where k_0 is a given initial condition for the capital stock. First-order conditions give rise to empirical relations and valuation relations that have been used previously.

Consider the first-order conditions for investment:

$$p_t = \frac{\partial g}{\partial i}(i_t, k_t, x_t)' \lambda_t \quad (4)$$

¹This depiction of valuation can be thought of assigning state prices, but it also permits certain forms of market incompleteness.

Special cases of this relation give rise to the so called q theory of investment. Consider for instance the separable specification in Example 2.1. Then

$$\frac{\partial g_m}{\partial i}(i_t^m/k_t^m, x_t) = 1 - \frac{p_t^m}{\lambda_t^m}. \quad (5)$$

This relates the investment capital ratio to what is called *Tobin's q* ($q_t = \frac{\lambda_t^m}{p_t^m}$). The Lagrange multiplier λ_t^m is the date t shadow value of the measured capital stock that is productive at date t . There is an extensive empirical literature that has used (5) to study the determinants of investment. As is well known, $\lambda_t^m = p_t^m$ and Tobin's q is equal to one in the absence of adjustment costs as in Example 2.2.

Consider next the first-order condition for capital at date $t + 1$:

$$\lambda_t = E \left(S_{t+1,t} \left[\frac{\partial f}{\partial k}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1} \right] \mid \mathcal{F}_t \right) \quad (6)$$

where $S_{t+1,t} \equiv S_{t+1,0}/S_{t,0}$ is the implied one-period stochastic discount factor between dates t and $t + 1$. This depiction of the first-order conditions is in the form of a one-period pricing relation. As a consequence, the implied returns to investments in the capital goods are:

$$\begin{aligned} r_{t+1}^m &\equiv \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^m}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^m} \\ r_{t+1}^u &\equiv \frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^u}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^u}. \end{aligned}$$

The denominators of these shadow returns are the marginal costs to investing an additional unit capital at date t . The numerators are the corresponding marginal benefits reflected in the marginal product of capital and the marginal contribution to productive capital in future time periods. The shadow returns are model-based constructs and are not necessarily the same as the market returns to stock or bond holders.

In the separable case (Example 2.1), the return to the measurable component of capital is

$$r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m + \lambda_{t+1}^m k_{t+2}^m - p_{t+1}^m i_{t+1}^m}{\lambda_t^m k_{t+1}^m}$$

An alternative depiction can be obtained by using the investment first-order conditions to substitute for λ_t^m and λ_{t+1}^m as in Cochrane (1991). In the absence of adjustment costs (Example 2.2), the return to tangible capital is:

$$r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + (1 - \delta_m)p_{t+1}^m}{p_t^m}. \quad (7)$$

The standard stochastic growth model is known to produce too little variability in physical returns relative to security market counterparts. In the one-sector version, the relative price p_t^m becomes unity. As can be seen in (7), the only source of variability is the marginal

product of capital. Inducing variability in this term by through variability in the technology shock process z_{t+1} generates aggregate quantities such as output and consumption that are too variable.

The supply of capital is less elastic when adjustment costs exist, hence models with adjustment costs can deliver larger return variability than the standard stochastic growth model. This motivated Cochrane (1991) and Jermann (1998) to include adjustment costs to physical capital in their attempts to generate interesting asset market implications in models of aggregate fluctuations. As an alternative, Boldrin, Christiano, and Fisher (2001) study a two sector model with limited mobility of capital across technologies. In our environment, limited mobility between physical and intangible capital could be an alternative source of aggregate return variability.

By the restricting the technology to be constant-returns-to-scale, the time zero firm value is:

$$f(k_0, n_0, z_0) - i_0 \cdot p_0 - w_0 n_0 + k_1 \cdot \lambda_0 = k_0 \cdot \left[\frac{\partial f}{\partial k}(k_0, n_0, z_0) + \frac{\partial g}{\partial k}(i_0, k_0, x_0)' \lambda_0 \right] \quad (8)$$

This relation is replicated over time. Thus the date t firm value is given by the cash flow (profit) plus the *ex-dividend* price of the firm. Equivalently it is the value of the date zero vector of capital stocks taking account of the marginal contribution of this capital to the production of output and to capital in subsequent time periods. Thus asset market values can be used to impute $k_{t+1} \cdot \lambda_t$ after adjusting for firm cash flow. When the firm has unmeasured *intangible* capital, this additional capital is reflected in the asset valuation of the firm.

The presence of intangible capital alters how we interpret Tobin's q . In effect there are now multiple components to the capital stock. Tobin's q is typically measured as a ratio of values and not as a simple ratio of prices. While the market value of a firm has both contributions, a replacement value constructed by multiplying the price of new investment goods by the measured capital stock will no longer be a simple price ratio. Instead we would construct:

$$\frac{\lambda_t \cdot k_{t+1}}{p_t^m k_{t+1}^m}.$$

Heterogeneity in q across firms or groups of firms reflects in part different amounts of intangible capital not simply a price signal to conveying the profitability of investment.

The dynamics of the ex-dividend price of the firm are given by:

$$\lambda_t \cdot k_{t+1} = E(s_{t+1,t} [f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - n_{t+1} w_{t+1} + k_{t+2} \cdot \lambda_{t+1}] | \mathcal{F}_t).$$

The composite return to the firm is thus

$$\begin{aligned} r_{t+1}^c &= \frac{f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - n_{t+1} w_{t+1} + k_{t+2} \cdot \lambda_{t+1}}{\lambda_t \cdot k_{t+1}} \\ &= \frac{\lambda_t^m k_{t+1}^m r_{t+1}^m + \lambda_t^u k_{t+1}^u r_{t+1}^u}{\lambda_t \cdot k_{t+1}} \end{aligned} \quad (9)$$

Recall that k_{t+1} is determined at date t (but not its productivity) under our timing convention. The composite return is a weighted average of the returns to the two types of capital with weights given by the relative values of the two capital stocks.

Firm ownership includes both bond and stock holders. The market counterpart to the composite return is a weighted average of the returns to the bond holders and equity holders with portfolio weights dictated by the amount of debt and equity of the firm.

2.3 Imputing the Intangible Capital Stock

These valuation formulas have been used by others to make inferences about the intangible capital stock. First we consider a return-based approach. We then consider a second approach based on asset values.

Following Atkeson and Kehoe (2002) and others, we exploit the homogeneity of the production function and Euler's Theorem to write:

$$y_{t+1} = \frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m + \frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1} + \frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u.$$

where $y_{t+1} = f(k_{t+1}, n_{t+1}, z_{t+1})$ is output. Thus a measure of the contribution of intangible capital to output:

$$\frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u}{y_{t+1}} = 1 - \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}}.$$

To make this operational we require a measure of the labor share of output given by compensation data and a measure of the share of output attributed to measured component of capital. Using formula (7) from Example 2.2 and knowledge of the return and the depreciation rate, we can construct

$$\frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})}{p_t^m} = r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m}.$$

Thus

$$\frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} = \left[r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m} \right] \frac{p_t^m k_{t+1}^m}{y_{t+1}}. \quad (10)$$

This formula avoids the need to directly measure rental income to measured capital, but it instead requires measures of the physical return, physical depreciation scaled by value appreciation, and the relative value of tangible capital to income.

The physical return to measured capital is not directly observed. Even if we observed the firm's, (or industry's or aggregate) return from security markets, this would be the composite return (9) and would include the contribution to intangible capital. As a result, a time series of return data from security markets is not directly usable. Instead Atkeson and Kehoe (2002) take a steady state approximation implying that returns should be equated to measure the importance of intangible capital in manufacturing. Income shares and price appreciation are measured using time series averages. Given the observed heterogeneity in average returns, as elsewhere in empirical studies based on the deterministic growth model, there is considerable ambiguity as to which average return to use. To their credit, Atkeson

and Kehoe (2002) document the sensitivity of their intangible capital measure to the assumed magnitude of the return.² We will have more to say about return heterogeneity subsequently.

To infer the value of the intangible capital relative to output using return data, we combine equation (10) with its counterpart for intangible capital to deduce that:

$$1 - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}} = r_{t+1}^c \frac{p_t \cdot k_{t+1}}{y_{t+1}} - (1 - \delta_m) \left(\frac{p_{t+1}^m}{p_t^m} \right) \left(\frac{p_t^m k_{t+1}^m}{y_{t+1}} \right) - (1 - \delta_u) \left(\frac{p_{t+1}^u}{p_t^u} \right) \left(\frac{p_t^u k_{t+1}^u}{y_{t+1}} \right). \quad (11)$$

To use this relation we must not only use the return r_{t+1}^c but also the growth rate in the investment prices for the two forms of capital and the depreciation rates. From this we may produce a measure of $\frac{p_t^u k_{t+1}^u}{y_{t+1}}$ using (11). McGrattan and Prescott (2000) use a similar method along with steady state calculations and a model in which $p_t^u = p_t^m = 1$ to infer the intangible capital stock.³ Instead of using security market returns or historical averages of these returns, they construct physical returns presuming that the noncorporate sector does not use intangible capital in production. Rather than making this seemingly hard to defend restriction, the return r_{t+1}^c could be linked directly to asset returns as in Atkeson and Kehoe (2002). Although the practical question of which security market return to use would still be present.⁴

In contrast to Atkeson and Kehoe (2002) and McGrattan and Prescott (2000), uncertainty is central in the analysis of Hall (2001). For simplicity Hall considers the case in which there is effect a single capital stock and a single investment good, but only part of capital is measured. Equivalently, the capital stocks k_t^m and k_t^u are perfect substitutes. Thus the production function is given by:

$$y_t = f^a(k_t^a, n_t, z_t)$$

where $k_t^a = k_t^m + k_t^u$. Capital evolves according to:

$$k_{t+1}^a = g^a(k_t^a, i_t^a) \quad (12)$$

with x_t excluded. The first-order conditions for investment are now given by:

$$\frac{\partial g^a}{\partial i}(k_t^a, i_t^a) = \frac{p_t^a}{\lambda_t^a}, \quad (13)$$

²Atkeson and Kehoe (2002) are more ambitious than what we describe. They consider some tax implications and two forms of measured capital: equipment and structures. Primarily they develop and apply an interesting and tractable model of organizational capital.

³McGrattan and Prescott (2000) also introduce tax distortions and a noncorporate sector. They also consider uncertainty, but with little gain. They use a minor variant of the standard stochastic growth model, and that model is known to produce physical returns with little variability.

⁴The measurement problem is made simpler by the fact that it is the composite return that needs to be computed and not the individual return on measured capital. The implied one-period returns to equity and bond-holders can be combined as in Hall (2001), but computing the appropriate one-period returns for bond-holders can be problematic.

and

$$v_t = \frac{\lambda_t^a k_{t+1}^a}{p_t^a} \quad (14)$$

is measured from the security markets using the firm value relation (8) and taking investment to be numeraire. For a given k_t^a , relations (12), (13) and (14) are three equations in the three unknowns $\frac{\lambda_t^a}{p_t^a}$, k_{t+1}^a , i_t^a . In effect they provide a recursion that can be iterated over time with the input of firm market value v_t . Instead of returns, Hall (2001) uses asset values to deduce a time series for the aggregate capital stock and the corresponding shadow valuation of that stock.⁵

While Hall (2001) applies this method to estimate a time series of aggregate capital stocks, we will consider some evidence from empirical finance on return heterogeneity that indicates important differences between returns to the tangible and intangible components of the capital stocks. This suggests the consideration of models in which intangible capital differs from tangible capital in ways that might have important consequences for measurement. This includes models that outside the adjustment cost models described here.

3 Evidence for Return Heterogeneity

We now revisit and reconstruct results from the asset pricing literature. Since the work of Fama and French (1992) and others, average returns to portfolios formed on the basis of the ratio of book value to market value are constructed. While the book to market value is reminiscent of the q measure of the ratio of the market value of a firm *vis. a. vis.* the replacement cost of its capital, here the book to market value is computed using only the equity-holders stake in the firm. Capital held by bond holdings is omitted from the analysis.

Recall from section 2 that intangible capital is reflected in only the market measure of assets but is omitted from the book measure. We are identifying firms with high intangible capital based on high book equity-to-market equity (BE/ME). It is difficult to check this identification directly because the market value of debt at the firm level is not easily observed. As a check on our interpretation of the portfolios as reflected different levels of intangible capital we examined whether our portfolio construction would be different if we included debt. We used the book value of debt as an approximation to the market value and considered rankings of firms based on book assets-to-market assets. This resulted in essentially the same rankings of firms. In fact the rank correlation between book assets-to-market assets and BE/ME averaged 0.97 over the 53 years of our sample. This gives us confidence in identifying the high BE/ME portfolio as containing firms with low levels of intangible and the low BE/ME portfolio as containing firms with high levels of intangibles.

Fama and French form portfolios based on the ratio of book equity-to-market equity (BE/ME), and estimate the mean return of these groups. They find that low BE/ME have low average returns. Fama and French (1992) view a low BE/ME as signaling sustained

⁵Hall (2001) establishes the stability of this mapping for some adjustment cost specifications, guaranteeing that impact initializing k_0^a of the recursion the recursion at some arbitrary k_0^a decays over time.

high earnings and/or low risk. While we follow Fama and French (1992) in constructing portfolios ranked by BE/ME ratios, we use a coarser sort than they do. We focus on five portfolios instead of ten, but this does not change the overall nature of their findings. Each year listed firms are ranked by their BE/ME using information from Compustat. Firms are then allocated into five portfolios and this allocation is held fixed over the following year. The weight placed on a firm in a portfolio is proportional to its market value each month.⁶

Firms may change groups over time and the value weights are adjusted accordingly. In effect the BE/ME categories are used to form five portfolio dividends, returns and values each time period. This grouping is of course different in nature than the grouping of firms by industry SEC codes, an approach commonly used in the Industrial Organization (IO) literature. For instance firms in the low BE/ME category may come from different industries and the composition may change through time. On the other hand this portfolio formation does successfully identify interesting payout heterogeneity at the firm level as we demonstrate below.

Figure 1 plots the market value relative to book value of 5 portfolios of US stocks over the period 1947 to 2001. Notice that there is substantial heterogeneity in the market value relative to book value of these portfolios. This potentially reflects substantial differences in intangible capital held by the firms that make up the portfolios. Further the value of market equity to book equity fluctuates dramatically over time.

These fluctuations can reflect changes in the relative composition of the capital stock between tangible and intangible capital. They may also reflect changes in the relative valuation of the two types of capital. Changes in valuation reflect changes in conjectured productivity of the different types of capital but may also reflect changes in how the riskiness is perceived and valued by investors.

Table 1 presents sample statistics for these portfolios of stocks. For comparison, the column labeled “Market” gives statistics for the CRSP value weighted portfolio. Consistent with figure 1 there are substantial differences in the average value of BM/ME for these portfolios. Notice that the portfolios with lower BM/ME (high market value relative to book value of equity) are also the ones with the highest level of R&D relative to sales. This is consistent with the idea that large R&D expenditures will ultimately generate high cash flows in the future thus justifying high current market values. Also the high level of R&D by firms with high market valuation relative to book value may reflect substantial investment in intangibles.

While the five BE/ME portfolios are likely to have different compositions of capital, these portfolios also imply different risk-return tradeoffs. As in Fama and French (1992), the low BE/ME portfolios have lower mean returns but not substantially different volatility than high BE/ME portfolios. The mean returns differ and the means of implied excess returns scaled by volatility (Sharpe ratios) also differ. High BE/ME portfolios have higher Sharpe ratios. In particular, the highest BE/ME portfolio has a Sharpe ratio that is higher than that of the overall equity market. A portfolio with an even larger Sharpe ratio can be constructed by taking a long position in the high BE/ME portfolio and offsetting this

⁶See Fama and French (1992) for a more complete description of portfolio construction.

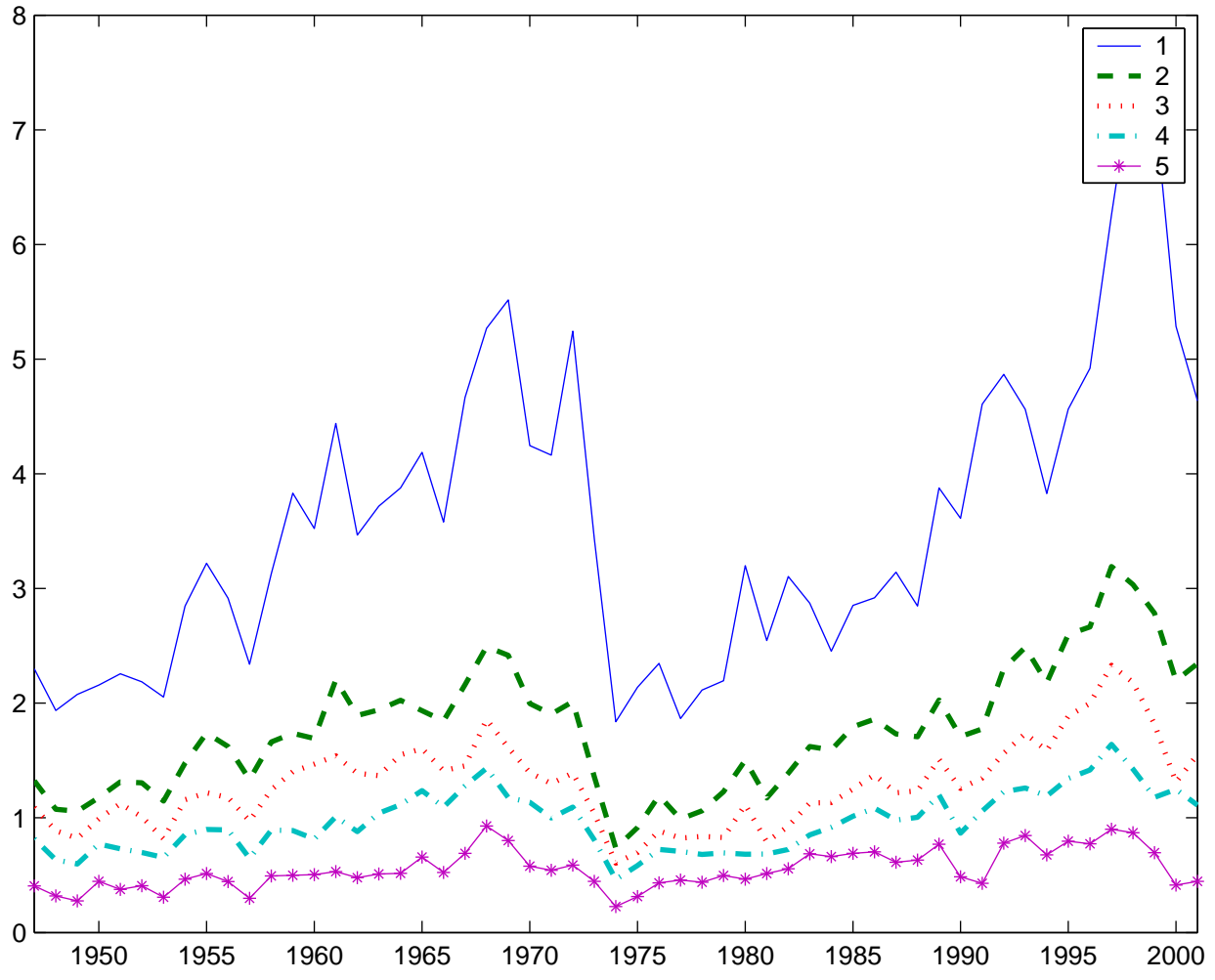


Figure 1: Market-to-Book value of equity for 5 portfolios of stocks

Table 1: Properties of Portfolios Sorted by Book-to-Market Value

	Portfolio					Market
	1	2	3	4	5	
Avg. Return (%)	6.48	6.88	8.90	9.32	11.02	7.23
Std. Return %	37.60	32.76	29.64	31.66	35.50	32.94
Avg. B/M	0.32	0.62	0.84	1.12	2.00	0.79
Avg. R&D/Sales	0.12	0.06	0.04	0.03	0.03	0.08
Sharpe Ratio	0.18	0.20	0.28	0.28	0.30	0.21

Portfolios formed by sorting portfolios into 5 portfolios using NYSE breakpoints from Fama and French (1993). Portfolios are ordered from lowest to highest average book-to-market value. Data from 1947 Q1 to 2001 Q4 for returns and B/M ratios. R&D/Sales ratio is from 1950 to 2001. Returns are converted to real units using the implicit price deflator for nondurable and services consumption. Average returns and standard deviations are calculated using the natural logarithm of quarterly gross returns multiplied by 4 to put the results in annual units. Average book-to-market are averaged portfolio book-to-market or the period computed from Compustat. Average R&D/Sales also computed from Compustat. The Sharpe Ratio is based on quarterly observations.

with a short position in the low BE/ME portfolios. This occurs because there is substantial positive correlation across the portfolios. The spectacular Sharpe ratios that are possible have been noted by many authors. See MacKinlay (1995), for example.

Differences in BE/ME are partially reflected in differences in future cash flows. Table 2 presents some basic properties of the dividend cash flows from the portfolios. These dividends imputed from the Center for Research in Securities Prices (CRSP) return files. Each month and for each stock CRSP reports a return without dividends, denoted $R_{t+1}^{wo} \equiv P_{t+1}/P_t$ and a total return that includes dividends, denoted $R_{t+1}^w \equiv (P_{t+1} + D_{t+1})/P_t$. The dividends yield D_{t+1}/P_t is then imputed as:

$$D_{t+1}/P_t = R_{t+1}^w - R_{t+1}^{wo} .$$

Changes in this yield along with the capital gain in the portfolio are used to impute the growth in portfolio dividends. This construction has the interpretation of following an initial investment of \$1 in the portfolio and extracting the dividends while reinvesting the capital gains. From the monthly dividend series we compute quarterly averages. Real dividends are constructed by normalizing nominal dividends on a quarterly basis by the implicit price deflator for nondurable and service consumption take from the National Income and Product accounts. Finally some adjustment must be done to quarterly dividends because of the pronounced seasonal patterns in corporate dividend payouts. Our measure of quarterly

Table 2: Cash Flow Properties of Portfolios Sorted by Book-to-Market Value

	Portfolio					Market
	1	2	3	4	5	
Avg. (log) Div. Growth %	1.78	1.68	3.13	3.54	4.48	3.09
Std. (log) Div. Growth %	13.50	17.09	11.71	12.05	17.76	23.99
Avg. log(D/P)	-3.78	-3.41	-3.23	-3.11	-3.15	n/a
Avg. P/D	49.12	33.01	27.00	23.96	24.82	n/a

dividends is constructed by taking an average of the logarithm of dividends in a particular quarter and over the previous three quarters. We average the logarithm of dividends because our empirical modelling will be linear in logs. Table 2 reports statistics for this constructed proxy of log dividends.

Notice from Table 2 that the low BE/ME portfolios also have low dividend growth. Just as there is considerable heterogeneity in the the measures of average returns, there is also considerably heterogeneity in growth rates. An important measurement question that we will explore is whether these *ex post* sample differences in dividend growth is something that is fully perceived *ex ante*, or whether some of this heterogeneity is the outcome of dividend processes with low frequency components. We suspect that much of the observed heterogeneity in dividend growth was known *a priori* by investors and hence this heterogeneity will reflect potential differences in risk. Some of our calculations that follow will treat this heterogeneity as reflecting in part differences in long-run risk. In the section 5, we turn to a discussion of risk measurement for these cash flows.

Given the selection rule for portfolio construction, it may not seem surprising that the implied dividend process have different growth rates. The capital gain component in our dividend construction can play an important role, as we now illustrate.

Example 3.1. *Consider a deterministic model with two production processes. Each production process alternates between high growth and low growth. When one process is in the high growth state the other one is in the low growth state. There is no capital accumulation and dividends coincide with production. Prices are discounted dividends and for simplicity supposes that the discount rate constant. Returns will be equated across the two securities, but the price-dividend ratio for each security will alternative over time. This ratio will be lower for the security with low dividend growth in the current period.*

Consider constructing two portfolios. One always picks the security with low dividend growth and the other with high dividend growth. While one might expect the constructed dividend growth rates to be lower on average for the first security, the capital gains used in the dividend construction for the portfolio work against this conclusion. Instead there are no permanent differences in the growth rates between the two portfolios.

4 Asset Pricing

Models of asset pricing link investor preferences and opportunities to deduce equilibrium relations for returns and prices. These models aim to explain return heterogeneity by the existence of risk premia. Investors require larger expected returns as compensation for holding risky portfolios. Alternative asset pricing models imply alternative risk-return tradeoffs. Equivalently [*e.g.*, see Hansen and Richard (1987)] they imply an explicit model of a stochastic discount factors, the variables $S_{t+1,t}$ used by investors to value one-period and hence multiple period assets.

There remain considerable discussion within the asset pricing literature about the feasibility of constructing an economically meaningful model of stochastic discount factors and hence risk premia. Nevertheless in this section we find it useful to consider one such model that, by design, leads to tractable restrictions on economic time series. This model is rich enough to help us return heterogeneity as it relates to risk.

4.1 Preferences

We follow Epstein and Zin (1989) by depicting preferences recursively. We use a Cobb-Douglas recursion:

$$V_t = (C_t)^{(1-\beta)} \mathcal{R}_t(V_{t+1})^\beta \quad (15)$$

where C_t is date t consumption and V_t is the date t continuation value associated with the consumption process. The function \mathcal{R}_t adjusts the continuation value for risk via:

$$\mathcal{R}_t(V_{t+1}) = \left[E(V_{t+1})^{1-\theta} | \mathcal{F}_t \right]^{\frac{1}{1-\theta}} \quad (16)$$

where \mathcal{F}_t is the current period information set.

There is an alternative more convenient way to represent this utility recursion. Take logarithms of the continuation value and write:

$$v_t = (1 - \beta)c_t + \frac{\beta}{1 - \theta} \log E(\exp[(1 - \theta)v_{t+1}] | \mathcal{F}_t), \quad (17)$$

where $c_t = \log C_t$ and $v_t = \log V_t$. Recursions (15) and (17) give equivalent depictions of preferences. Recursion (17) is a version of the *risk-sensitive* recursive preference assumption used by Hansen and Sargent (1995) and Tallarini (1998). An important limiting case occurs when $\theta = 1$. In this case preferences are logarithmic and separable over time and states of the world with discount factor β . Under perfect foresight, the risk adjustment transformation \mathcal{R}_t is inconsequential and preferences are also time additive with a logarithmic period utility functions and a discount factor β . The parameter θ gives rise to an extra risk adjustment.

Our logarithmic specification is made for convenience. Campbell (1996) argues for less intertemporal substitution and Bansal and Yaron (2002) assume more, but both use comparable utility recursions.

4.2 Shadow Valuation

Consider the shadow valuation associated with a given consumption process. The utility recursion gives rise to a corresponding valuation recursion and implies a stochastic discount factor used to represent this valuation.

The first utility recursion (15) is homogeneous of degree one in consumption and the future continuation utility. Use Euler's Theorem to write:

$$V_t = (MC_t)C_t + E[(MV_{t+1})V_{t+1}|\mathcal{F}_t] \quad (18)$$

where

$$\begin{aligned} MC_t &= (1 - \beta) \frac{V_t}{C_t} \\ MV_{t+1} &= \beta \frac{(V_{t+1})^{-\theta} V_t}{[\mathcal{R}_t(V_{t+1})]^{1-\theta}} \end{aligned}$$

The right-hand side of (18) measures the shadow value of consumption today and the continuation value of utility tomorrow.

Let consumption be numeraire, and suppose for the moment that we value claims to the future continuation value V_{t+1} as a substitute for future consumption processes. Divide both sides of (18) by MC_t and use marginal rates of substitution to compute shadow values. Then the value of the consumption plan is given by:

$$\begin{aligned} W_t = \frac{V_t}{MC_t} &= C_t + E \left[\left(\frac{MV_{t+1}}{MC_t} \right) V_{t+1} | \mathcal{F}_t \right] \\ &= C_t + E \left[\left(\frac{MV_{t+1} MC_{t+1}}{MC_t} \right) W_{t+1} | \mathcal{F}_t \right] \\ &= C_t + \lambda E \left(\left[\frac{(V_{t+1})^{1-\theta}}{\mathcal{R}_t(V_{t+1})^{1-\theta}} \right] \left[\frac{C_t}{C_{t+1}} \right] W_{t+1} | \mathcal{F}_t \right) \\ &= \frac{C_t}{1 - \beta}. \end{aligned} \quad (19)$$

The final equality of (19) gives the familiar formula for logarithmic preferences that consumption and wealth are proportional.⁷

The stochastic discount factor:

$$S_{t+1,t} \equiv \beta \left(\frac{C_t}{C_{t+1}} \right) \left[\frac{(V_{t+1})^{1-\theta}}{\mathcal{R}_t(V_{t+1})^{1-\theta}} \right] \quad (20)$$

is used to value claims to future wealth W_{t+1} expressed in units of next period consumption. This stochastic discount factor is the product of two terms. One term $\beta C_t/C_{t+1}$ is the usual

⁷The constancy of the consumption-wealth ratio prevents our use of this model to interpret directly the findings of Lettau and Ludvigson (2001b) and Lettau and Ludvigson (2001a).

intertemporal marginal rate of substitution for logarithmic preferences and the other is a risk adjustment from recursive utility. Notice that the term associated with the risk-adjustment satisfies

$$E \left[\frac{(V_{t+1})^{1-\theta}}{\mathcal{R}_t(V_{t+1})^{1-\theta}} | \mathcal{F}_t \right] = 1$$

and can thus be thought of as distorting the probability distribution.

4.3 Computing Continuation Values

To make our formula for the marginal rate of substitution operational, we need a formula for V_{t+1} computed using the equilibrium consumption process. Suppose that the first-difference of the logarithm of equilibrium consumption has a moving-average representation:

$$c_t - c_{t-1} = \gamma(L)w_t + \mu_c$$

where $\{w_t\}$ is a vector, *iid* standard normal process and

$$\gamma(z) = \sum_{j=0}^{\infty} \gamma_j z^j$$

where γ_j is a row vector and

$$\sum_{j=0}^{\infty} |\gamma_j|^2 < \infty.$$

This linear times series representation is adopted to help us relate to some of the time series evidence that we will discuss subsequently. Log-linear approximations are often used in macroeconomic modelling. In what follows we will take the log-linear specification as being correct.

Guess a solution:

$$v_t - c_t = v(L)w_t + \mu_v.$$

Rewrite recursion (17) as:

$$v_t - c_t = \frac{\beta}{1-\theta} \log E (\exp [(1-\theta)(v_{t+1} - c_t)] | \mathcal{F}_t).$$

Thus v must solve:

$$zv(z) = \beta[v(z) - v(0) + \gamma(z) - \gamma(0)],$$

which in particular implies that

$$v(0) + \gamma(0) = \gamma(\beta).$$

Solving for v and μ_v :

$$v(z) = \beta \frac{\gamma(z) - \gamma(\beta)}{z - \beta}$$

$$\mu_v = \frac{\beta}{1-\beta} \left[\mu_c + \frac{(1-\theta)}{2} \gamma(\beta) \cdot \gamma(\beta) \right].$$

The formula $v(z)$ is the solution to the forecasting problem:

$$v(L)w_t = \sum_{j=1}^{\infty} \beta^j E(c_{t+j} - c_{t+j-1} | \mathcal{F}_t)$$

familiar from the rational expectations literature on the permanent income model of consumption. The risk parameter θ enters only the constant term of continuation value process. The term $\gamma(\beta)$, which enters the formula for μ_v is the *discounted impulse response* of consumption growth rate to a shock.

The logarithm of the stochastic discount factor can now be depicted as:

$$s_{t+1,t} \equiv \log S_{t+1,t} = -\delta - \gamma(L)w_{t+1} - \mu_c + (1-\theta)\gamma(\beta)w_{t+1} - \frac{(1-\theta)^2\gamma(\beta) \cdot \gamma(\beta)}{2}$$

where $\beta = \exp(-\delta)$. The term $\gamma(\beta)w_{t+1}$ is the solution to

$$\sum_{j=0}^{\infty} \beta^j [E(c_{t+j} | \mathcal{F}_{t+1}) - E(c_{t+j} | \mathcal{F}_t)].$$

The stochastic discount factor includes both the familiar contribution from contemporaneous consumption plus a forward-looking term that discounts the *impulse responses* for consumption growth. The innovation to the logarithm $s_{t+1,t}$ of the stochastic discount factor is:

$$[-\gamma(0) + (1-\theta)\gamma(\beta)]w_{t+1}$$

shows how a shock at date $t+1$ alters the stochastic discount factor. This term determines the magnitude of the risk premium.

For simplicity, consider the pricing of the normally distributed shock vector w_{t+1} :

$$\begin{aligned} \pi_t(w_{t+1}) &= E(S_{t+1,t}w_{t+1} | \mathcal{F}_t) \\ &= E(S_{t+1,t} | \mathcal{F}_t) [-\gamma(0) + (1-\theta)\gamma(\beta)] \\ &= \left(\frac{1}{R_{t+1}^f} \right) [-\gamma(0) + (1-\theta)\gamma(\beta)] \end{aligned}$$

where R_{t+1}^f is the return on a one-period risk free asset. The premia for bearing risk is encoded in the price vector $\pi_t(w_{t+1})$ of the underlying mean zero shocks. The vector depends on the coefficient vector $\gamma(0)$ that dictates how w_{t+1} influences date $t+1$ consumption growth and it depends on the discounted income response vector $\gamma(\beta)$. The first component is familiar from the work of Hansen and Singleton (1983). Large values of the risk parameter θ enhance the importance of the latter component. This latter effect is featured in the analysis

of Bansal and Yaron (2002).⁸ While this model has a simple and usable characterization of how temporal dependence in consumption growth alters risk premia, it has the counterfactual implication of risk premia that are time invariant. Other authors, including Campbell and Cochrane (1999) argue that risk premia vary over the business cycle.

5 Dividend Risk

In asset pricing it is common to explore risk premia by characterizing how returns co-vary with a benchmark return as in the CAPM or more generally how returns co-vary with a candidate stochastic discount factor. The focus of the resulting empirical investigations are on return risk, in contrast to dividend or cash-flow risk.

Recently there has been an interest in understanding cash-flow risk using linear time series methods. Examples include the work of Bansal, Dittmar, and Lundblad (2002a), Bansal, Dittmar, and Lundblad (2002b), and Cohen, Polk, and Vuoteenaho (2002). We follow this literature by using linear time series methods to motivate and construct a measure of dividend risk.

To use linear time series methods requires a log-approximation for present-discounted value formulas as developed by Campbell and Shiller (1988a, 1988b).⁹ Write the one-period return in an equity as:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{(1 + P_{t+1}/D_{t+1})D_{t+1}/D_t}{P_t/D_t}$$

where P_t is the price and D_t is the dividend. Take logarithms and write

$$r_{t+1} = \log(1 + P_{t+1}/D_{t+1}) + (d_{t+1} - d_t) - (p_t - d_t)$$

where lower case letters denote the corresponding logarithms. Next approximate:

$$\log(1 + P_{t+1}/D_{t+1}) \approx \log[1 + \exp(\overline{p - d})] + \frac{1}{1 + \exp(\overline{d - p})}(p_{t+1} - d_{t+1} - \overline{p - d})$$

where $\overline{p - d}$ is the average logarithm of the price dividend ratio. Use this approximation to write:

$$r_{t+1} - (d_{t+1} - d_t) = \chi + \rho(p_{t+1} - d_{t+1}) - (p_t - d_t) \quad (21)$$

⁸Anderson, Hansen, and Sargent (2003) suggest a different interpretation for the parameter θ . Instead of risk, this parameter may reflect model misspecification that investors confront by not knowing the precise riskiness that they must confront in the marketplace. As argued by Anderson, Hansen, and Sargent (2003), under this alternative interpretation, $|(1 - \theta)\gamma(\beta)|$ is measure of model misspecification that investors have trouble disentangling because this misspecification is disguised by the underlying shocks that impinge on investment opportunities.

⁹Santos and Veronesi (2001) suggest a models for studying cash flow risk that avoids linear approximation by instead adopting a nonlinear model of income shares.

where

$$\rho \equiv \frac{1}{1 + \exp(\overline{d - p})}.$$

As shown by Campbell and Shiller (1988a), this approximation is reasonably accurate in practice.

Treat (21) as a difference equation in the log price dividend ratio and solve this equation forward:

$$p_t - d_t = \sum_{j=0}^{\infty} \rho^j (r_{t+1+j} - d_{t+1+j} + d_{t+j}) - \frac{\chi}{1 - \rho}.$$

This relation says that a time $t + 1$ shock to current and future dividends must be offset by the same shock to returns in the sense of a present-discounted value. The discount factor ρ will differ depending on the average logarithm of the dividend/price ratio for the security or portfolio. The present discounted value restriction is mathematically the same as that developed by Hansen, Roberds, and Sargent (1991) in their examination of the implications of present-value budget balance.

To understand this restriction, posit a moving-average representation for the dividend growth process and the return process:

$$\begin{aligned} d_t - d_{t-1} &= \eta(L)w_t + \mu_d \\ r_t &= \kappa(L)w_t + \mu_r. \end{aligned}$$

Since $p_t - d_t$ depends only on date t information, future shocks must be present-value neutral:

$$\kappa(\rho) - \eta(\rho) = 0. \quad (22)$$

For instance if returns are close to being iid, but not dividends then

$$\kappa(0) \approx \eta(\rho)$$

The discounted dividend response should equal the return response. Whereas Hansen and Singleton (1983) looked at return risk, Bansal, Dittmar, and Lundblad (2002a) and Bansal, Dittmar, and Lundblad (2002b) look at the discounted dividend risk. We follow Bansal, Dittmar, and Lundblad (2002a) and treat $\eta(\rho)$ as a measure of risk in dividend growth. We refer to this measure as discounted dividend risk. As ρ tends to 1, we refer to the limit $\eta(1)$ as long run risk.

To evaluate the magnitudes of the discounted dividend responses, we use the discounted dividend response to deduce an implied value of θ . Under the assumption that the return to each portfolio is iid, (22) implies that the return to portfolio j satisfies:

$$r_{t+1}^j = \eta^j(\rho^j)w_{t+1} + \mu_r^j.$$

Our model of the stochastic discount predicts that the expected return for portfolio j , μ_r^j satisfies:

$$E[r_{t+1}^j | \mathcal{F}_t] - r_{t+1}^f = \mu_r^j - r_{t+1}^f = -\frac{\sigma_{t,j}^2}{2} + [\gamma(0) + (\theta - 1)\gamma(\beta)] \cdot \eta^j(\rho^j) \quad (23)$$

where $\sigma_{t,j}^2$ is the variance of return of portfolio j . Recall that $\eta^j(\rho^j)$ is the discounted dividend response for portfolio j . For a fixed return variance and discounted dividend response, risk premia are larger for larger value of θ . For any given value of the discount rate β ,¹⁰ we can compute an implied value of θ from the difference in the risk premia between any two portfolios. In what follows we use the risk premium of each portfolio relative to that of portfolio 5 to calculate θ . This calculation clearly ignores the restriction that the same value of θ should be used in explaining the entire cross-section of average returns. Our goal is merely to provide a convenient metric to evaluate the quantitative significance of observed differences in discounted dividend responses.

We will not include returns in our vector autoregressions for the reasons explained by Hansen, Roberds, and Sargent (1991).¹¹ We will sometimes include dividend/price ratios in the vector autoregressive systems. These ratios are known to be informative about future dividends. Write implied moving-average representation as:

$$p_t - d_t = \xi(L)w_t + \mu_p.$$

We may then back out a return process (approximately) as:

$$r_t = \kappa(L)w_t + \mu_r$$

where

$$\kappa(z) = (\rho - z)\xi(z) + \eta(z).$$

It follows from this formula for κ that the present-value-budget balance restriction (22) is satisfied by construction and is not testable.

To summarize, we use $\eta(\rho)$ as our measure of discounted dividend risk. When dividend-prices ratios are also included in the VAR system, the present-value-budget-balance restriction (22) is automatically satisfied. By construction, discounted return risk and discounted dividend risk coincide.

6 Measuring Dividend Risk Empirically

In this section we evaluate the riskiness of the five BE/ME portfolios using the framework of sections 4 and 5. Riskiness is measured by the sensitivity of portfolio cash flows and prices to different assumptions made to identify aggregate shocks. Since we are interested in the long-run impact of aggregate shocks we consider several VAR specifications that make different assumptions about the long-run relationships between consumption, portfolio cash flows and prices. In particular we examine the effects of moving from the assumption of little long-run relationship between aggregates and portfolio cash flows to the assumption that there is a cointegrated relationship between aggregate consumption and cash flows.

¹⁰We choose $\beta = 0.9873$ which corresponds to an annual discount rate of 0.95. We also conduct the calculation with different values of β , and find that the implied values of θ is not sensitive to the choice of β .

¹¹Hansen, Roberds, and Sargent (1991) show that when returns are included in a VAR, restrictions (22) cannot be satisfied for the shocks identified by the VAR unless the VAR system is stochastically singular.

6.1 Data Construction

For our measure of aggregate consumption we use aggregate consumption of nondurables and services taken from the National Income and Product Accounts. This measure is quarterly from 1947 Q1 to 2002 Q4, is in real terms and is seasonally adjusted. Portfolio dividends were constructed as discussed in section 3. For portfolio prices in each quarter we use end of quarter prices.

Motivated by the work of Lettau and Ludvigson (2001b) and Santos and Veronesi (2001), in several of our specifications we allow for a second source of aggregate risk that captures aggregate exposure to stock market cash flows. This is measured as the share of corporate cash flows in aggregate consumption and is measured as the ratio of corporate earnings to aggregate consumption. Corporate earnings are taken from NIPA.

In all of the specifications reported below the VAR models were fit using one year (four quarters) of lags.¹²

6.2 Bivariate Results

First we follow Bansal, Dittmar, and Lundblad (2002b) and consider bivariate regressions that include aggregate consumption and the dividends for each portfolio separately. Table 3 reports these initial results. We examine several different assumptions about the long-run properties of consumption and dividend cash flows. In the first the state variable x_t is given by:

$$x_t = \begin{bmatrix} c_t - c_{t-1} \\ d_t^j - d_{t-1}^j \end{bmatrix}, \quad (24)$$

where d_t^j is the logarithm of the dividend cash flow to portfolio j at time t .¹³ For notational convenience we do not display the dependence of x_t on j . In this specification no long-run relationship between consumption and dividends is imposed. We refer to this specification as simply *bivariate VAR*.

For this specification we estimate γ and η by fitting a vector autoregression to growth rates in consumption and dividends net of a constant growth rate. We ignore any restrictions on the growth rates. Consumption and dividends can grow at different rates. We estimate:

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \dots + A_\ell x_{t-\ell} + B w_t$$

where consumption growth is the first entry of x_t and dividend growth the second entry. We normalize B to be lower triangular with positive entries on the diagonal. We call the first shock the consumption shock and the second one the dividend shock.

Form:

$$A(z) = I - A_1 z - A_2 z^2 - \dots - A_\ell z^\ell.$$

¹²We also conducted some runs with eight lags. With the exception of the results for portfolio 1 when using aggregate earnings, the results were not greatly effected.

¹³Notice that we consider separate specifications of the state variable for each portfolio. Ideally estimation with all of the portfolio cash flows would be interesting but because of data limitations this is not possible.

Then we form $\gamma(\beta)$ as the first row of $A(\beta)^{-1}B$ and $\eta(\rho)$ as the second row of $A(\rho)^{-1}B$. We produce these calculations for each B/M portfolio.

We also compute the limiting values when $\beta = \rho = 1$ and the corresponding frequency zero spectral regression of dividend growth onto consumption growth as a long-run measure of risk. The frequency zero spectral density matrix for the state vector process is:

$$S_x(0) = A(1)^{-1}BB'A(1)^{-1'}$$

Thus by computing the long-run responses, we are using the inputs into spectral density matrix at a given frequency.

Statistical theory (and intuition) suggests that this estimation problem is a hard one. Estimation based on finite-order vector autoregressions can be viewed as statistically consistent approximations to underlying infinite-order systems. Under this view, lag lengths must increase to ensure the consistent approximation, but the resulting rate of convergence of $A(1)^{-1}B$ and the implied spectral density estimation is slower than the standard parametric rate. In this sense the estimation problem is more difficult than a standard parametric one. Strictly speaking, discounting speeds up the convergence rate but this gain should be small for discount factors close to unity.

In the VAR system just described, consumption and dividends are allowed to have different growth rates and they are allowed to have long responses to shocks that are distinct. Dividends and consumption are not restricted to be co-integrated. We also explore systems in which growth is restricted.

In one of these systems we continue to subtract separate sample means from consumption growth and dividend growth. We then let:

$$x_t = \begin{bmatrix} \hat{c}_t - \hat{c}_{t-1} \\ \hat{d}_t^j - \hat{c}_t \end{bmatrix},$$

where the $\hat{\cdot}$'s are included because of the initial subtraction of sample means. If the resulting $A(z)$ is stable (is nonsingular for $|z| \leq 1$) then any shock that has permanent impact on consumption must also have a permanent impact on dividends. We refer to this specification as a *demeaned cointegrated VAR*.

Finally we consider the *cointegrated VAR* specification in which:

$$x_t = \begin{bmatrix} c_t - c_{t-1} \\ d_t^j - c_t \end{bmatrix}.$$

In this specification we do not remove the mean growth rate in dividends *a priori* but we do include a constant term in the VAR. Provided that $A(z)$ is stable, consumption and dividends share a common growth rate and any permanent shock to dividends must be offset by a permanent shock to consumption. We perform this calculation in part because we suspect the heterogeneity in measured dividend growth rates is classifying false some long run risk as deterministic dividend growth.

We explore how changes in the growth configuration in the VAR change the long-run risk measure for dividends. Table 3 reports the discounted response of dividends to several specifications of the shocks. Of particular interest is the discount response to a consumption shock. To the extent that this discount response is large the consumer expects a large average return. In particular recall from Table 1 that as we move from portfolio 1 through portfolio 5 the average return of the portfolios increases substantially. To explain this relationship we need exposure to aggregate shocks to increase substantially as we move from portfolio 1 through portfolio 5.

The first row of panel A is quite striking in that the discounted response to a consumption shock increases more than ten times in comparing portfolio 1 and portfolio 5. This “success” was noted by Bansal, Dittmar, and Lundblad (2002a). Notice, however, that much of the increase occurs as we move from portfolio 3 to 4 and then dramatically from portfolio 4 to 5. To illustrate this more fully consider figure 2. Portfolio 5 has a substantially different response to aggregate consumption both in the short- and long-run. Specifically, the long-run (infinite lag) responses of portfolio 4 and 5 are also considerably higher than responses for the other portfolios.

As Table 3 demonstrates, when dividends and consumption are restricted to respond the same way to permanent shocks, the discounted risk measures become much more similar across portfolios. This change effectively eliminates the explanation of Bansal, Dittmar, and Lundblad (2002a) for return heterogeneity. For example, the discounted response to a consumption shock in the case of portfolio 5 in the cointegrated system is a third of the response in the bivariate VAR.

In Bansal, Dittmar, and Lundblad (2002a) a specification is considered in which individual dividend cash flows do not Granger cause consumption. This restriction is often imposed in asset pricing models. Imposition of this restriction makes it easier to compare estimation across runs because the the law of motion for consumption is held fixed when we change portfolios. As a check on the plausibility of this restriction we perform Granger-causality tests for our different specifications of the dynamics of consumption and dividend cash flows. We use a standard Likelihood Ratio test. The P-value of these tests is reported in Table 4. Notice that the first two rows of Table 4 implies that there is little evidence that the individual cash flows Granger cause consumption. When the log price-dividend ratio is included in the regressions, however, we do tend to reject the Granger-causality test because in this case the test also examines whether portfolio prices Granger cause consumption. The forward-looking behavior of stock prices make it difficult to support this restriction.

For these reasons in Table 3 (panels C and D), we also report results when dividends are excluded from the consumption growth evolution. The results are very similar to those in which we do not restrict the consumption evolution equation. The discounted risk measure for portfolio five with cointegration does increase to be about double that of the other portfolios, but it is still much smaller than when cointegration is not imposed.

In figure 3 we depict the impulse responses when cointegration is imposed and consumption is restricted. Comparing the impulse responses to a consumption shock in this figure to those in figure 2, we see that measured response of portfolios 4 and 5 dividends are similar

**Table 3: Discounted Responses of Portfolio Dividends to Shocks
Bivariate Specifications**

	Portfolio				
	1	2	3	4	5
Discount Factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Consumption Shock</i>					
Bivariate VAR	0.33	0.60	0.51	1.63	4.03
Demeaned cointegrated VAR	0.68	0.81	0.82	1.25	1.21
Cointegrated VAR	0.46	0.78	0.79	1.19	1.32
<i>Panel B: Dividend Shock</i>					
Bivariate VAR	5.64	3.64	3.87	3.69	5.59
Demeaned cointegrated VAR	1.65	2.23	1.99	1.42	0.69
Bivariate cointegrated VAR	3.40	3.30	2.66	1.23	1.04
<i>Panel C: Consumption Shock (Restricted Dynamics)</i>					
Bivariate VAR	0.35	0.60	0.50	1.58	4.15
Demeaned cointegrated VAR	0.77	0.86	0.84	1.15	1.49
Cointegrated VAR	0.59	0.86	0.82	1.11	2.02
<i>Panel D: Dividend Shock (Restricted Dynamics)</i>					
Bivariate VAR	5.54	3.60	3.84	3.66	6.31
Demeaned cointegrated VAR	1.06	2.47	1.50	1.09	1.24
Bivariate cointegrated VAR	2.65	2.56	2.02	0.99	2.28
<i>Panel E: Permanent Consumption Shock</i>					
Demeaned cointegrated VAR	1.64	2.26	1.88	1.65	0.78
Cointegrated VAR	3.33	3.29	2.58	1.47	0.04
<i>Panel F: Transitory Consumption Shock</i>					
Demeaned cointegrated VAR	0.69	0.71	1.04	0.92	1.15
Cointegrated VAR	0.83	0.83	1.02	0.89	1.68

Table 4: P-value of Granger-Causality Test (%)

	Portfolio				
	1	2	3	4	5
Bivariate VAR	22.07	64.14	3.41	40.93	4.26
Demeaned cointegrated VAR	13.62	16.91	1.32	46.13	1.10
Cointegrated VAR	29.94	26.29	1.77	43.71	3.25
Cointegrated VAR with P/D ratio	1.23	1.48	0.27	1.71	0.06

Table 5: Limiting Responses of Dividends to Consumption Shock

	Portfolio				
	1	2	3	4	5
<i>Panel A: Consumption Shock, Unrestricted Consumption Dynamics</i>					
Bivariate VAR	0.31	0.61	0.54	1.70	4.18
Cointegrated VAR	0.60	0.68	0.80	0.91	0.49
<i>Panel B: Consumption Shock, Restricted Consumption Dynamics</i>					
Bivariate VAR	0.33	0.60	0.54	1.64	4.29
Cointegrated VAR	0.83	0.83	0.83	0.83	0.83

across figures for up to ten quarters. After ten quarters, the dividend responses come down to the long-run consumption responses in the cointegrated system but they stay high in the original bivariate system estimated in growth rates. The ten period responses appear to be quite robust across runs but not the discounted responses. The high discount factors apparently make the discounted dividend responses very sensitive to changes in the growth configuration assumed in the VAR.

Table 5 further illustrates the sensitivity of the results to the assumed value of the discount factor and our assumptions about growth. This table reports the limiting response of dividends to a consumption shock. For example, consider the results for the unrestricted Cointegrated VAR systems. The order of the limiting response across portfolios is reversed relative to the order seen in Table 3. This is dramatic evidence of the importance of the assumed nature of long-run risk.

To summarize, the intriguing findings of Bansal, Dittmar, and Lundblad (2002a), at least

as reproduced here, rely on the presumption of that there are permanent shocks that shift dividends without also moving consumption in a one-to-one manner. Once permanent shocks are presumed to move consumption and dividends in the same manner, the discounted risk measures based on consumption shocks are still ordered the same way, but their magnitudes are much more similar to each other.

For the two cointegrated systems, we consider an alternative identification scheme. We do not impose Granger causality, but we instead identify a permanent and transitory shock following an approach suggested by Blanchard and Quah (1989). The long-run response to consumption is given by the first row of $A(1)^{-1}B$. Since the covariance matrix of the one-step-ahead forecast errors is given by BB' , the matrix B is only identified up to a post-multiplication by an orthogonal matrix. Previously, we assumed that B was lower triangular. We now replace this assumption with a restriction that $A(1)^{-1}B$ be lower triangular. As a consequence, the first shock has a permanent effect on consumption and the second shock has only a transitory effect on consumption. In the absence of cointegration the second shock can have a permanent effect on dividends, however. The results are reported in panels E and F of Table 3. Surprisingly, the discounted responses to the permanent consumption shock differ from the response to the previously identified consumption shocks. For instance, abstracting from cointegration, while portfolio 5 has the largest discounted response, portfolio 1 has the second largest response.

To summarize the results so far, the results in the first rows of panels A and C of Table 3 replicate closely the results of Bansal, Dittmar, and Lundblad (2002a). They show the largest differences in the discounted risk across portfolios. We find, however, substantial sensitivity to how we restrict the long growth of the VAR system and how we use the system to identify shocks. In particular, the results in the last row of panel C of Table 3 restrict consumption not to be Granger-caused by dividends and restrict the long-run impulse response of dividends to be the same as the long-run response of consumption to dividends. Here we find the same qualitative impact as Bansal, Dittmar, and Lundblad (2002a), but the quantitative magnitudes are much smaller.

To assess the magnitudes of the discounted dividend responses we examine the implied value of θ needed to reconcile the dividend responses and observed average returns. As discussed earlier we do this based on each portfolio's mean return and discounted dividend response relative to the mean return and discounted dividend response of portfolio 5. The results are reported in Table 6 based on Bivariate and Cointegrated VAR's where the models are restricted such that dividends do not Granger cause consumption. Notice first that the implied values of θ are quite large. The discounted dividend responses do imply higher returns for the higher book-to-market portfolios but the magnitude of the risks are small. As a result, we calculate high implied coefficients of relative risk aversion. Similar observations have been made using the equity premium.¹⁴ Second, notice that the implied values of θ increase dramatically as we move from the Bivariate VAR to the Cointegrated VAR models. Although the discounted responses reported in Table 3 do identify differences in risk, the magnitudes are significantly smaller in the Cointegrated VAR systems than the in the Bivariate VAR

¹⁴See, for example, Mehra and Prescott (1985).

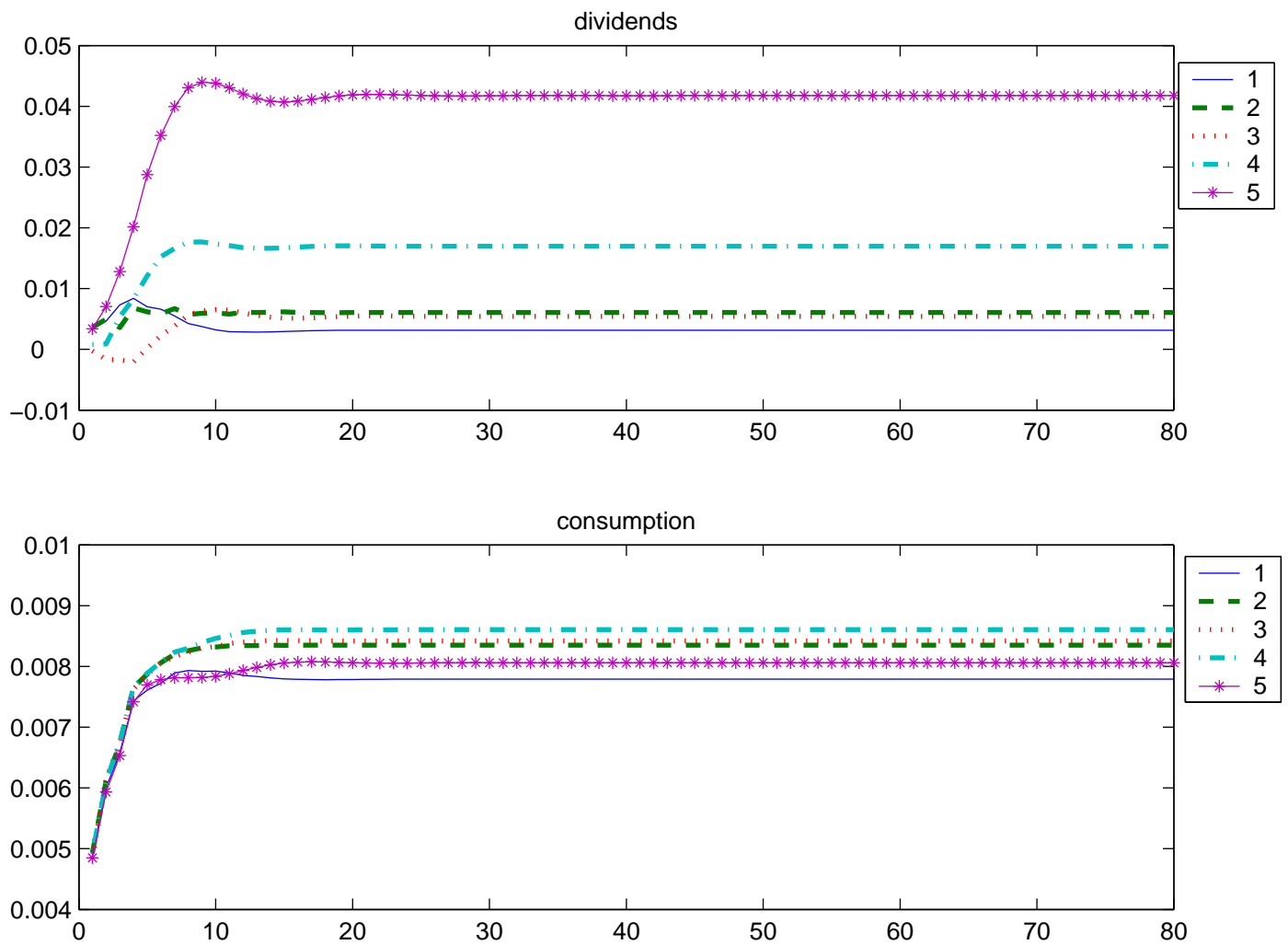


Figure 2: Impulse Response to a Shock to a Consumption Shock, Bivariate VAR

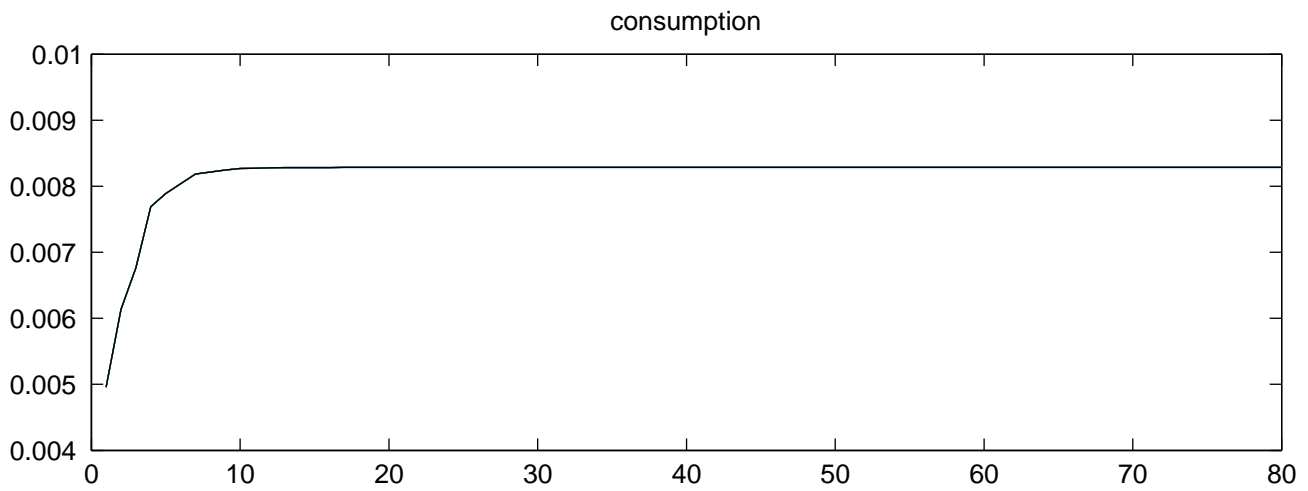
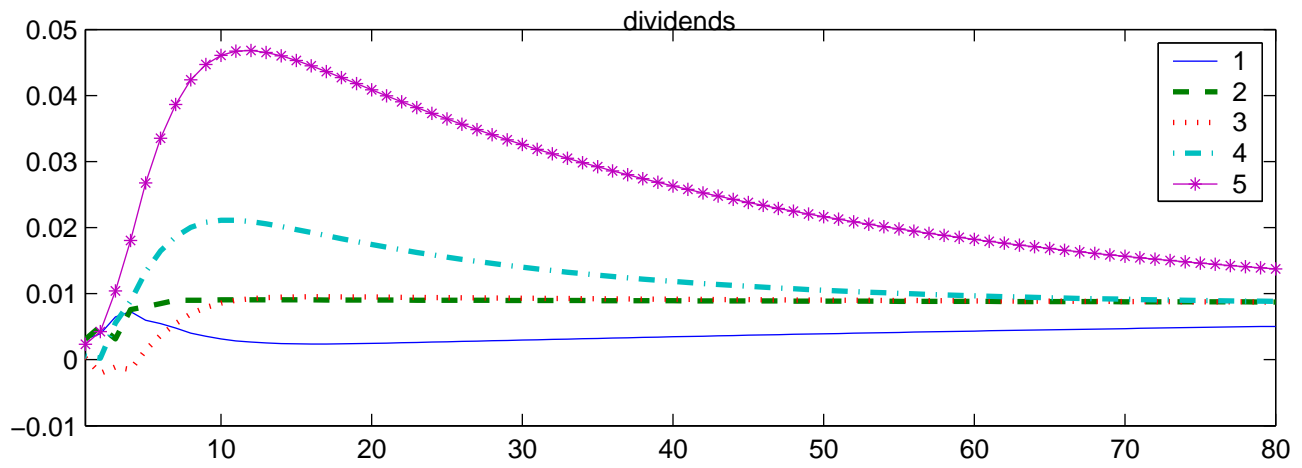


Figure 3: Impulse Response to a Shock to a Consumption Shock, Cointegrated VAR

Table 6: Implied Value of Risk Aversion Coefficient θ

	Portfolio			
	1	2	3	4
Bivariate VAR	36.27	38.97	22.74	25.12
Cointegrated VAR	94.99	116.82	67.68	69.38

systems. Different assumptions about the long-run result in dramatic differences in implied risk exposure confirming our interpretation of the results of Table 3.

6.3 Results with Prices

We now consider results where the log price-dividend ratio is included in each regression as well. Dividend price ratios are known to be useful in forecasting future dividends and returns. Recall from section 5 that the discounted return risk and the discounted dividend risk will coincide provided that we approximate returns using formula (21). In addition to reporting the discounted impulse responses for dividends, we also consider the discounted responses for prices using the decomposition:

$$p_{t+1} - p_t = (p_{t+1} - d_{t+1}) - (p_t - d_t) + (d_{t+1} - d_t).$$

Since the price appreciation is related to the first-difference in the dividend price ratio, the undiscounted ($\rho = 1$) dividend and price responses will coincide.

We study cointegrated VAR's with the logarithm of the dividend price ratio as a third variable. Table 7 reports the discounted responses. We also include results in which dividends and prices do not Granger-cause consumption. As before the response of dividends to a consumption shock is not dramatically different across portfolios. With the Granger-causality restriction, the results are very similar to what we obtained previously, except that the portfolio one responses are higher. The discounted price response increases across portfolios when the consumption evolution is restricted, but these responses are not monotone without this restriction.

6.4 Results with Additional Aggregate Shock

In our final VAR specification we consider what happens when an additional aggregate shock is added. In this specification x_t is given by:

$$x_t = \begin{bmatrix} c_t - c_{t-1} \\ e_t - c_t \\ p_t - d_t \\ d_t - c_t \end{bmatrix}, \quad (25)$$

**Table 7: Discounted Responses of Individual Dividends and Prices to a Consumption Shock
Regressions with Prices**

	Portfolio				
	1	2	3	4	5
Discount Rate	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Unrestricted Consumption Dynamics</i>					
Dividend Response	0.80	0.53	0.68	1.16	1.18
Price Response	0.69	0.68	1.11	0.67	1.34
<i>Panel B: Restricted Consumption Dynamics</i>					
Dividend Response	0.92	0.66	0.79	1.15	1.89
Price Response	0.79	0.86	1.29	1.66	2.14

where e_t is corporate profits at time t . To avoid parameter proliferation, we restrict the dynamics of the aggregate variables $c_t - c_{t-1}$ and $e_t - c_t$ to not be Granger-caused by the individual portfolio dividends and ratios of price to dividends. We consider the discounted response to a shock to consumption and to a shock to the share of corporate profits in aggregate consumption. We are led to consider this latter variable by the empirical investigations of Lettau and Ludvigson (2001b) and Santos and Veronesi (2001). These authors argue for the addition of an aggregate share variable to help account for asset values. For example Santos and Veronesi (2001) argue that exposure to stock market risk is affected by the contribution of corporate payouts to aggregate consumption. We restrict B as a lower-triangular matrix with positive entries on the diagonal. We refer the first shock as the consumption shock and the second one as the aggregate profits shock.

The implied responses from this four-variable system are reported in Table 8. The impulse response of prices and portfolio dividends to a consumption shock are displayed in figure 4, and the responses to an aggregate earnings shock are displayed in figure 5. We no longer find that the high book-to-market portfolios have larger discounted responses to a consumption shock.

The aggregate shock in this system is a shock to the relative size of earnings. This shock has a positive long-run effect on aggregate consumption. For example the discounted response of consumption to this shock is 0.56, assuming a value of β of 0.99. For comparison, the corresponding discounted consumption response to the consumption shock is 0.97. As a result the the second shock appears to be an important source of aggregate risk. Notice, however, that just as with a consumption shock we do not find a monotone relationship with respect to an aggregate profits shock.

Table 8: Discounted Responses of Individual Dividends and Prices to a Aggregate Consumption and Aggregate Profit Share

	Portfolio				
	1	2	3	4	5
Discount Rate	0.9943	0.9918	0.9902	0.9889	0.9894
<i>Panel A: Consumption Shock</i>					
Dividend Response	0.74	1.27	0.43	1.40	1.17
Price Response	0.79	1.46	1.60	0.74	1.74
<i>Panel B: Shock to Profit Share</i>					
Dividend Response	0.59	1.50	1.85	0.99	-0.24
Price Response	0.49	1.30	0.90	0.45	0.42

These results and the ones previously reported clearly show that the predicted covariation between consumption and the returns to these portfolios is sensitive to the variables included and the long-run restrictions. A robust finding across all of the runs is the distinctive response of dividends from portfolio 5 to a consumption shock in the first ten time periods. Apparently VAR methods are better able to measure transient responses such as this one than discounted responses with high discount factors. While understanding long run growth rate risk may be vital for understanding heterogeneity in asset returns or values, plausible economic growth restrictions appear to be needed to produce meaningful estimates.

7 Conclusions

In this paper we reviewed two findings pertinent for using asset market data to make inferences about the intangible capital stock. We presented evidence familiar from the empirical finance literature that returns are heterogeneous when firms are grouped according to their ratio of market equity to book equity. This evidence suggests that there are important differences in the riskiness of investment in measured capital *vis a vis* intangible capital. This has potentially important ramifications for how to build explicit economic models to use in constructing measurements of the intangible capital stock.

A risk-based interpretation of return heterogeneity requires more than just a model with heterogeneous capital. It also requires a justification for the implied risk premia. There has been much interest recently in the finance literature on using vector autoregressive (VAR) methods to understand riskiness of serially correlated cash flows or dividends. Low frequency or long run risk is often conjectured to be important in understanding asset values. In this paper we document the sensitivity of long-run measures of dividend risk based on dis-

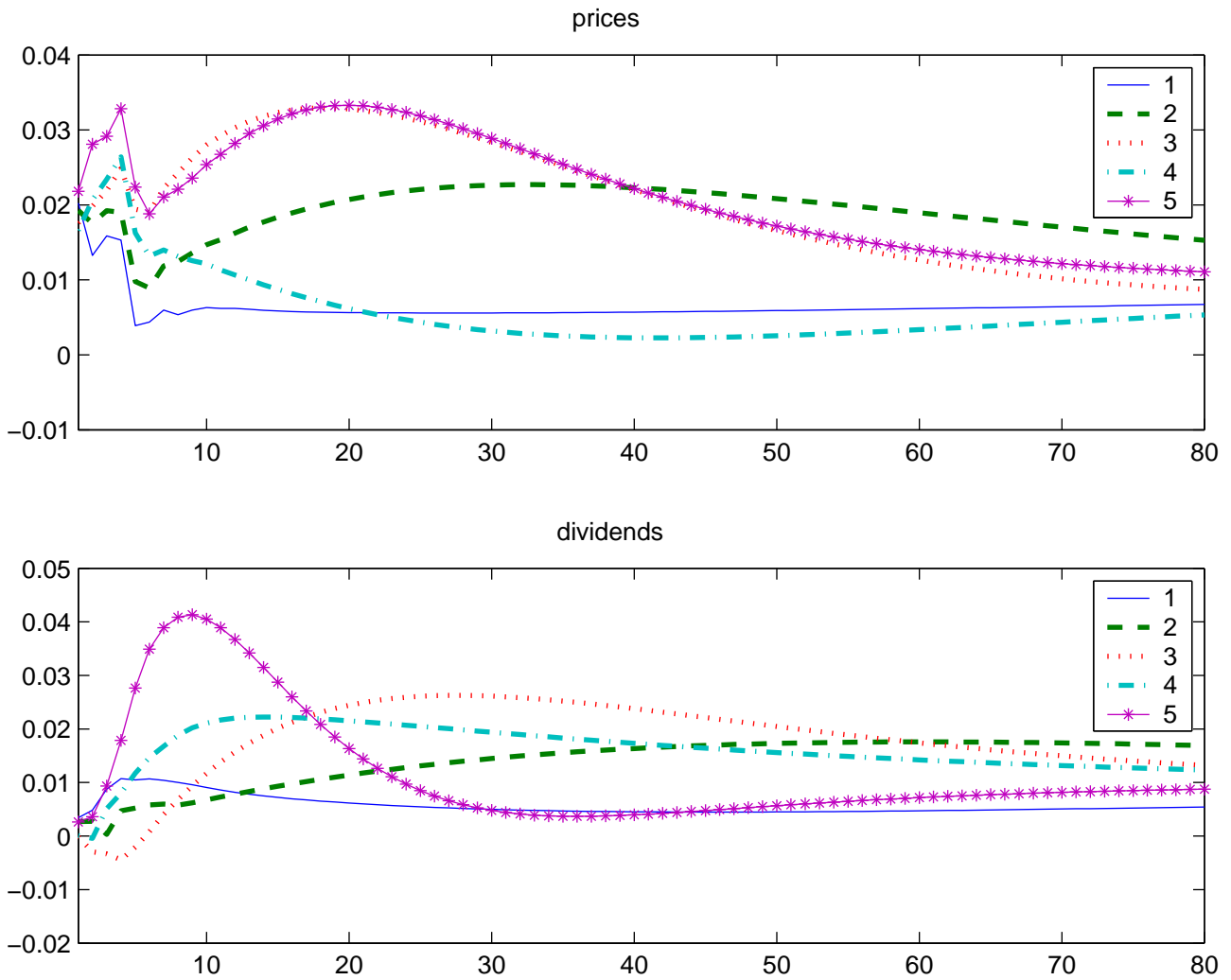


Figure 4: Impulse Response to a Shock to a Consumption Shock, Cointegrated VAR

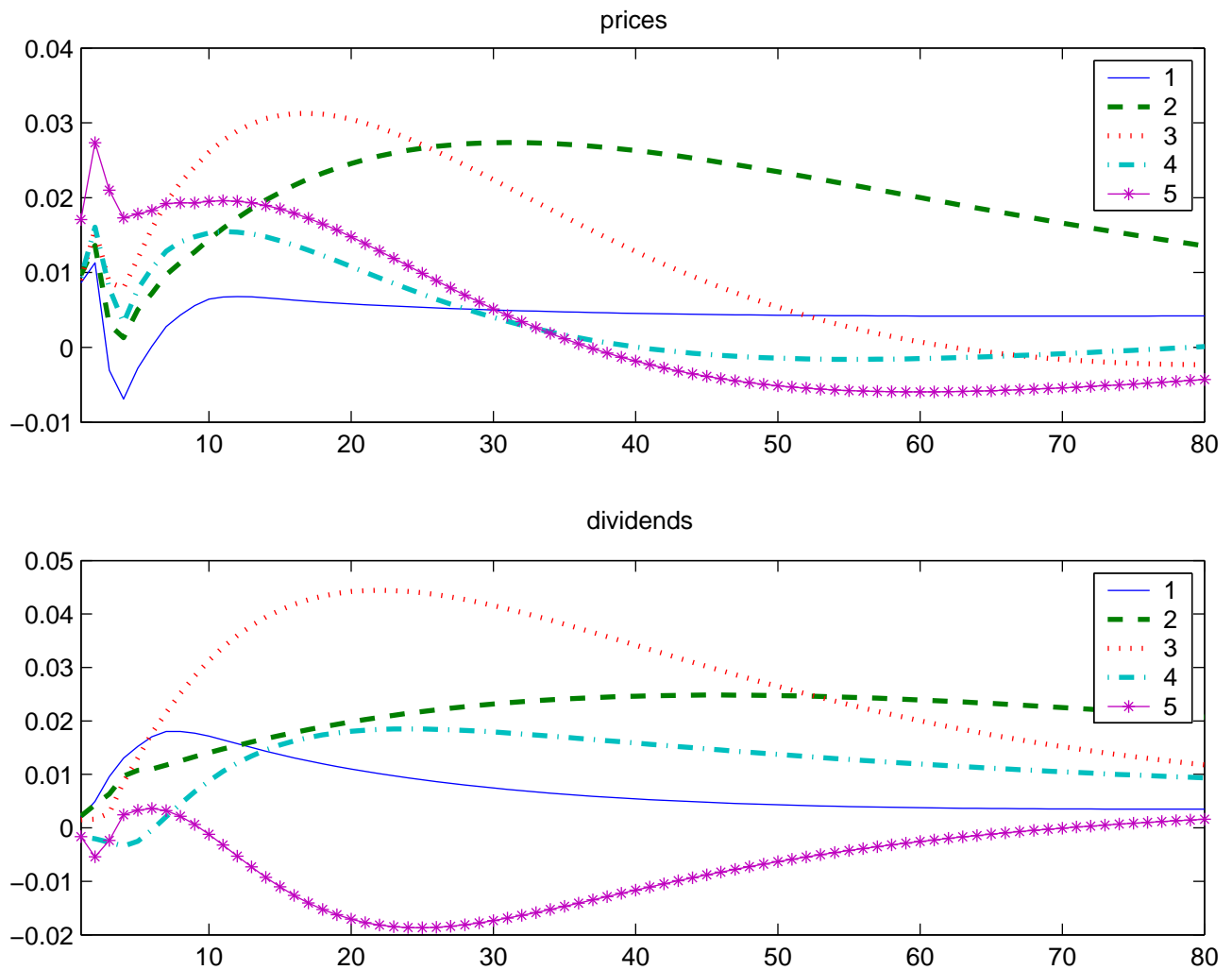


Figure 5: Impulse Response to a Shock to Earnings Share, Cointegrated VAR

counted impulse responses to changes in underlying specification of growth. Adding arguable plausible structure to the measurement method changes the measurements in quantitatively important ways. This is not a general indictment against VAR methods, because some features of the impulse responses remain similar with or without restricting how time series grow. On the other hand our findings of sensitivity to specification are of more general interest beyond concerns with measuring the intangible capital stock. VAR methods that do not restrict growth in economically meaningful ways may produce misleading measures of long-run risk. Adding *a priori* reasonable structure to the stochastic growth opportunities of firms, enterprises or productive components of the economy is necessary for obtaining reliable risk measures of dividends or cash flows.

The empirical evidence we report follows the finance literature by focusing on the claims of equity-holders. As emphasized by Hall (2001), what is pertinent for measurement purposes is the combined claims of bond-holders and equity-holders. It is the overall value of the firm or enterprise that is pertinent. Similarly, this analysis focuses on dividends as the underlying claims of equity-holders and not on overall cash flows of the firms. The risk associated with broader-based cash flow measures are of considerable interest for future research.

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