

Reputation Effects in Portfolio Management

Heber Farnsworth*

August 21, 2003

Abstract

This paper analyzes a model of moral hazard in portfolio management. Managers wish to earn the higher fees associated with active management but are averse to the effort of identifying superior trading strategies through research. Previous research has focused on contracts which offer explicit incentives. In this paper I address optimal contracting between an investor and a portfolio manager when reputation building is possible. I model reputation in a somewhat different manner than some previous research in which both contracting parties are unsure of the agent's ability. Here only the investor is unsure about the agent's skill. No restrictions are made on the form of the contracts. The model predicts that larger funds, or those with higher reputations, will be more likely to give performance bonuses to managers.

*Heber Farnsworth, John M. Olin School of Business, Washington University, Campus Box 1133, One Brookings Drive, St. Louis, MO 63130-4899, tel: (314) 935-4221, fax: (314) 935-6359, <mailto:farnsworth@olin.wustl.edu>

1 Introduction

Consider a portfolio manager who wishes to earn the high fees associated with active management but is averse to the costly effort necessary to uncover superior investment opportunities. The effort choice of the manager is not observed by investors and so it is possible for a manager to claim to be an active manager while expending no effort. Investors rationally anticipate such behavior and are therefore reluctant to invest with an active manager without some assurance that research efforts are being undertaken. Previous work on delegated portfolio management has focused primarily on contracts which provide explicit incentives for the manager to take effort. However performance bonuses which might provide such incentives do not appear to be widely used. This would seem to imply that somehow managers are able to commit to high effort. In this paper I consider the effect of reputation as a commitment mechanism for the portfolio manager.

The literature on reputation begins¹ with Kreps and Wilson (1982), Milgrom and Roberts (1982), and Kreps, Milgrom, Roberts, and Wilson (1982) who showed how uncertainty about the type of one or more players in a game can lead to cooperative behavior. The prime example is the finitely repeated prisoners dilemma. In each period each player faces the choice to cooperate (mum) or not (rat). The payoffs are such that in each stage game (rat,rat) is the only Nash equilibrium. Now suppose player 2 has some uncertainty about player 1's payoffs or motivations. In particular suppose that player 2 places some small probability on player 1 being of a type that always plays cooperatively. Player 2 decides to play a tit-for-tat strategy, cooperating until the first time player 1 rats and then ratting in each period thereafter. Player 1 may not really be of this cooperative nature but, knowing of player 2's beliefs, player 1 will decide to cooperate, mimicking the cooperative type. Repeated cooperation makes player 2 more certain that player 1 is, in fact cooperative, and so player 1 develops a reputation for cooperation.

In the repeated prisoners dilemma the past play of each player is common knowledge. This need not be the case for reputation to have an effect. If outcomes depend on strategies plus some noise then players can still use past outcomes to try and infer what strategy is being played by the other

¹There are a few earlier papers such as Dybvig and Spatt (1980) which differs from more recent work on reputation in that there is no uncertainty about the payoffs or types of the players.

player and, hence, what type the other player is.² The key to reputation is a belief that the other player may be of a type which always plays a certain strategy. This “commitment type” need not be dedicated to cooperation or high morals. Mailath and Samuelson (2001) consider the case of a firm which can invest in costly high quality or not. Consumers feel that there is some possibility that the firm is an incompetent type which cannot choose high quality. Firms who can choose high quality will do so to build a reputation for competence.

In this paper I follow Mailath and Samuelson (2001) in assuming two possible types for the portfolio manager: competent or incompetent. If a competent manager chooses to expend effort then a signal is generated which is informative about the future payoffs of assets in the market. This signal can be used in making informed portfolio choices. If no effort is expended then a signal is received which is uninformative in the sense that it is independent of future asset values. An incompetent manager receives an uninformative signal regardless of effort choice. A competent manager, then is one who can make an informed portfolio choice.

In a standard reputation model the rules by which the players play is taken to be exogenous. The prisoner’s dilemma game is what it is: the form of the game is not a choice of either player. In a game of reputation for quality the interaction of the players is limited to an exchange of money for a good. The point of such games is to see if reputation building leads to “good” behavior given the exogenously specified rules. In this paper I shall extend the reputation literature by endogenizing the rules of the game, i.e. allowing contracting between the investor and the manager.

One interesting feature of reputation models is that reputation effects break down when reputation gets very good. Eventually the cost of building a reputation outweighs the benefits which would come from having a better reputation. At this point cooperation breaks down and both players play the strategy which is optimal in a one-period game. A similar phenomenon will arise in this context except that in the contracting framework it is the contract which changes and not the behavior of the manager. As reputation increases the incentive compatibility constraint begins to bind and explicit incentives must be provided to take the place of the incentives previously provided by reputation effects.

Since there is more than one type of manager in this model the ques-

²See Fudenberg and Levine (1992).

tion must be asked why standard methods for separating types cannot be used? Why can managers not signal their skill? Or why can some sort of screening mechanism as in Bhattacharya and Pfleiderer (1985) or Heinkel and Stoughton (1994) not be used to identify competent managers? Clearly if these tools are available then sorting by types is possible and reputation will have no role. Reputation is widely believed to be an important factor in contracting relationships. But any model which incorporates learning and reputation must have some feature which makes sorting by types impossible. In this paper I will assume that while competent managers know their type, incompetent managers are convinced that they are competent and nothing can persuade them otherwise.³ So screening or signaling is impossible.

There is a branch of the contracting literature which deals with a similar but distinct issue which is variously called reputation or career concerns. This strand of the literature includes papers such as Harris and Holmström (1982), Holmström and Costa (1986), and Gibbons and Murphy (1992). These papers examine contracts when none of the contracting parties is sure of the type of the agent so once again signaling or screening are not possible. An important feature of such models is that the agents wish to be insured against the risk of discovering that they are of the low type. I will refer to models of this type as “career concerns” models rather than reputation models and I will illustrate how the implications of these two classes of models differ in the current context.

I will begin by deriving optimal contracts assuming that effort is contractible. Then I will drop the assumption of observable effort and ask how reputation affects the form of the optimal contract. Throughout it is assumed that the information uncovered by the manager’s research efforts is simultaneously observed by both the investor and the manager.⁴

The layout of the paper is as follows. The next section presents one-period contracts where there is uncertainty about types and also results with

³There is some question as to whether this reflects irrationality on the part of managers. Formally speaking these managers do update rationally but because their priors are dogmatic their updated beliefs are always equal to their prior beliefs. How they come to hold these dogmatic priors is another issue altogether.

⁴This assumption is necessary for tractability but is unfortunate. This assumption effectively guarantees that if the manager will choose the portfolio strategy which is “correct” given the information gathered. In fact it turns out that for the long-term contracts we examine the manager would tend to be overly conservative in investing if the signal were private.

multiple periods in which new contracts are negotiated each period. Section 3 derives multi-period contracts in the presence of reputation effects. Section 4 examines the effect of such contracts in the presence of moral hazard. Section 5 concludes.

2 Contracting with Asymmetric Beliefs

There are two contracting parties: an investor and a portfolio manager. The manager may be one of two types. A competent manager has access to an information production technology which produces a signal s for a utility cost of c . The joint density of the market state⁵ and the signal is $f^I(\omega|s)f^s(s)$ where $f^s(s)$ is the marginal distribution of the signal. If a competent manager chooses to expend no effort a signal will be received which is independent of the market state such that the joint density is $f^\omega(\omega)f^s(s)$ where $f^\omega(\omega)$ is the marginal distribution of the market state. An incompetent manager receives a signal drawn from this “uninformative” distribution regardless of effort choice.

I shall assume that both the investor and the portfolio manager are risk averse with log utility and that trading takes place in a frictionless and complete market⁶ with a state-price density $p(\omega)$. This setup is similar to that of Dybvig, Farnsworth, and Carpenter (2001). Although that paper presents a one-period model with symmetric beliefs the tractability which is in evidence in that paper carries over into the current context. I will assume that these distributions and state prices are the same each period. This is not important for the results but allowing for time varying distributions would complicate the notation without adding any economic insights. I will also assume that the signal and market state in any period are independent of signal and market states in other periods so that this period’s signal is only good for one period. This means that there is a moral hazard problem in every period and it simplifies the analysis.

Both types of managers have dogmatic beliefs that they are competent and act as though their signals are informative. The investor’s initial assess-

⁵Throughout the paper I shall be vague about what is meant by a market state. The variable ω need not have an interpretation as the payoff on a particular asset. Instead ω is simply a way to index outcomes that may be observed in the market.

⁶By this I mean complete over states distinguished by security payoffs. The market is not complete over signal states.

ment of the probability that the manager is competent is π_0 . An investor with beliefs π_0 takes the joint distribution of the manager's signal and the market state to be

$$\pi_0 f^I(\omega|s) f^s(s) + (1 - \pi_0) f^\omega(\omega) f^s(s).$$

As time goes by and returns are realized the investor updates beliefs such that at the beginning of period $t + 1$ the probability that the manager is competent, as assessed by the investor, is π_t . However I do not restrict the contract in each period to be a function of this belief. Contracts may depend on initial beliefs and subsequent realizations in any way.

2.1 One-period contracts

I shall begin by examining the nature of the investment problem which arises from the different beliefs of the contracting parties. Because the manager and investor have different beliefs about how informative the signal is they also have different optimal investment policies they would pursue based on observing the signal. The question is whether the investor is willing to invest with a manager who is so confident of their own ability. The key to resolving this problem is to allow both parties to trade freely. Most contracting models assume that all trading must be done by the manager. If this assumption is relaxed then the investor may commit only some of their wealth to the manager and may invest the rest in some other position.

To illustrate this one need only look at a one-period model. The setup of the problem is that of a direct mechanism. The manager reports the signal which determines (according to some pre-specified rule) the state-contingent payoffs to manager and agent. I denote the manager's payoff by $\phi(s, \omega)$ and the investor's payoff by $V(s, \omega)$. These two payoffs must satisfy the following budget constraint.

$$(\forall s) \int (\phi(s, \omega) + V(s, \omega)) p(\omega) d\omega = Y_0 \tag{1}$$

where Y_0 is the initial wealth of the investor.

Since investor and manager have log utility the optimal contract maximizes

$$\iint \log(V(s, \omega)) (\pi_0 f^I(\omega|s) + (1 - \pi_0) f^\omega(\omega)) f^s(s) d\omega ds \tag{2}$$

subject to the budget constraint (1) and the manager's participation constraint

$$\iint \log(\phi(s, \omega)) f^I(\omega|s) f^s(s) d\omega ds - c = u_0 \quad (3)$$

where u_0 is the reservation utility level of the manager.

Proposition 1 *The solution to the one-period problem is*

$$V(s, \omega) = B^P [\pi_0 R^I + (1 - \pi_0) R^B] \quad (4)$$

$$\phi(s, \omega) = B^A R^I \quad (5)$$

where

$$R^I = \frac{f(\omega|s)}{p(\omega)},$$

$$R^B = \frac{f^\omega(\omega)}{p(\omega)}$$

and the constants B^P and B^A are the budget shares of the investor and the manager respectively are given by

$$B^P = \frac{Y_0}{1 + \lambda}$$

$$B^A = \frac{\lambda Y_0}{1 + \lambda}$$

PROOF See appendix

I shall call the portfolio with return R^B the “benchmark” portfolio because it represents the portfolio the investor would have chosen if there were no manager. The portfolio R^I is the portfolio a log investor would choose who was convinced that the signal was informative. The manager's fee is paid in terms of this latter portfolio because the manager is convinced that the signal is informative so this is the most inexpensive way to deliver the reservation utility. The most ready interpretation of this proposition is that R^I is the return on the managed portfolio. The investor wishes to hedge against the risk that the manager is incompetent. This proposition delivers a kind of two-fund theorem for log investors which says that the optimal way

to hedge against this uncertainty is to commit only a part of the invested wealth to the managed portfolio and put the rest in a benchmark. Notice that neither the benchmark nor the managed portfolio depend upon beliefs π_0 . Therefore the existence of the benchmark allows any investor to invest with this manager, regardless of their beliefs, without wanting the fund to be managed in a different way.

In Dybvig and Spatt (1986) and Dybvig, Farnsworth, and Carpenter (2001) the absence of moral hazard leads to a contract which is a proportional sharing rule over the total invested assets. This is interesting because it corresponds to actual industry practice. Because of different beliefs the optimal contract in this case is not a proportional sharing rule. However this does not mean that the contract does not look like those in practice. In practice the manager gets a proportion of “assets under management”, that is the assets that are invested in the managed portfolio, at the end of the period rather than a proportion of the investor’s total assets. End-of-period assets under management is given by

$$B^A R^I + B^P \pi_0 R^I = (\lambda + \pi_0) B^P R^I$$

of which the manager’s fee

$$B^A R^I = \lambda B^P R^I$$

is a fraction $\lambda/(\lambda + \pi_0)$. At first glance this may seem odd because it implies that a manager with higher perceived ability will get a smaller proportion of assets under management. However this is made up for by the fact that a more highly regarded manager will be managing more assets and so will get a smaller piece of a bigger pie.

The proposition above assumed that the signal was truthfully reported or equivalently that the manager chose the investment strategy from the allowable menu⁷ which was correct given the signal. In fact this assumption is not binding.

Corollary *In the absence of moral hazard the manager will have no incentive to misreport the signal.*

⁷See Dybvig, Farnsworth, and Carpenter (2001) for a discussion of the relation between menus of investment strategies and reporting games in the investment management contract.

The proof is immediate upon recognizing that R^I is the optimal portfolio for a log investor who believes the signal is informative.

If the assumption that effort is contractible is dropped then it is not clear that this contract is optimal because the optimal contract must now satisfy incentive compatibility of effort. However it is not clear that such a constraint would bind. Since the manager shares in the returns of the managed portfolio there is some maximum level of effort which the manager is willing to expend even if effort is unobservable. As long as c is less than this maximum then incentive compatibility will not bind and the contract above will be the optimal contract. Denote this maximum utility cost by \bar{c} . This cost will make the manager just indifferent between taking the costly effort and shirking

$$\iint \log(B^A R^I) f^I(\omega|s) f^s(s) d\omega ds - \bar{c} = \iint \log(B^A R^I) f^\omega(\omega) f^s(s) d\omega ds.$$

Rewriting this yields an expression for \bar{c} .

$$\iint \log(R^I) (f^I(\omega|s) - f^\omega(\omega)) f^s(s) d\omega ds = \bar{c}$$

Notice that this expression depends only on the quality of the manager's signal (through the difference between f^I and f^ω) and not on the investor's beliefs. This is because beliefs only enter through B^A and this drops out because of log utility and the fact that $f^I - f^\omega$ integrates to zero.

In a one-period setting there is no reputation building. I will now turn to multi-period settings to see how the contracting problem changes and whether reputation effects give more incentives to take effort than this one-period contract.

2.2 Dynamically Optimal Contracts

I will now show that a dynamic setting by itself does not give rise to reputation effects. Consider the problem of dynamic optimal contracting. The contract is chosen each period based on the information currently available to the contracting parties. In this case this information includes the past history of performance and the revised beliefs of the investor based on this past history.

To illustrate the failure of multi-period models it suffices to consider a two-period case. For notational convenience I will employ an abuse of notation

and let

$$f(\pi) = (\pi f^I(\omega|s) + (1 - \pi)f^\omega(\omega)) f^s(s),$$

denote the joint distribution of signal and market state given the investor's beliefs. The distribution given the manager's beliefs is $f(1)$. Assume that the investor receives a deterministic income stream Y_0, Y_1 and in each period invests the entire amount and consumes all the proceeds. Because a log investor consumes a deterministic proportion of remaining wealth in each period this assumption is harmless. Allowing for saving would only complicate the notation without adding any insights.

In the second (last) stage the problem is identical to the one-period problem already considered. The reporting problem in the first stage is to choose ϕ_1 and V_1 to maximize

$$\iint \left\{ \log(V_1) + \delta \iint \log(V_2) f(\pi_1) d\omega_2 ds_2 \right\} f(\pi_0) d\omega_1 ds_1$$

subject to

$$(\forall s_1) \int (V_1 + \phi_1) p(\omega_1) d\omega_1 = Y_0$$

and

$$\iint \left\{ \log(\phi_1) + \delta \iint \log(\phi_2) f(1) d\omega_2 ds_2 \right\} f(1) d\omega_1 ds_1 - c - \delta c = u_0 + \delta u_1$$

Interestingly the solution to this problem is identical in form to the solution of the one-period problem.⁸ The solutions are

$$V_t = B_t^P [\pi_{t-1} R_t^I + (1 - \pi_{t-1}) R_t^B]$$

and

$$\phi_t = B_t^A R_t^I$$

for $t = 1, 2$ where

$$B_t^A = \lambda_t B_t^P,$$

⁸The proof is nearly identical to the proof of the one-period model and is omitted.

$$B_t^P = \frac{Y_{t-1}}{1 + \lambda_t}$$

and λ_t is the Lagrange multiplier for the participation constraint in period t .

The maximum effort the manager would be willing to expend in the first period is

$$\iint (\log(R_1^I) + \delta \log(B_2^A)) (f^I(\omega_1|s_1) - f^\omega(\omega_1)) f^s(s_1) d\omega_1 ds_1 = \bar{c}_1.$$

Since B_2^A depends only on the constants Y_1 and λ_2 this term disappears and \bar{c}_1 is identical to \bar{c} so that there are no reputation effects in this model.

The intuition behind this result is straight-forward. In each period the contract gives only the reservation utility level to the manager. Since current actions cannot affect future utility levels a dynamically optimal contract provides no extra incentives over one-period contracts.

This will be true as long as second-period reservation utility u_1 is known at the beginning of the first period. An alternative view would be that the manager's reservation utility is a function of beliefs and so the associated Lagrange multiplier might depend on past signals and market states. In this case there may be some extra incentives which could be called reputation effects. However in the next section I shall show that this is not optimal as long as full commitment contracts are possible.

3 Full Commitment Contracts

What is missing from the multi-period model above is a notion of commitment. A full commitment contract in a multi-period model is one in which the sequence of contracts $\{V_1, \phi_1\}, \{V_2, \phi_2\}, \dots, \{V_T, \phi_T\}$ is chosen at time 0. Each contract may depend on the entire history of play up to and including outcomes in the current period, e.g. $\phi_t(s_1, \dots, s_t, \omega_1, \dots, \omega_t)$.

The notion of full commitment is familiar from games in which one of the contracting parties has private knowledge about their type. A lack of commitment prevents this player from revealing the information because the opponent will use this knowledge against them in future rounds. Full commitment allows for truthful reporting and so the revelation principle applies.

In this case the motivation is somewhat different since this is a case of moral hazard rather than hidden type. Instead, commitment allows managers and investors to bet on their beliefs about the future. To the investor,

beliefs about whether the manager is competent form a martingale. Because managers are convinced they are competent they view the investor's beliefs as a sub-martingale which converges in the limit to 1. Hence a manager will be willing to accept low pay in early rounds of play as long as higher pay is assured if a successful track record is established. The investor is willing to accept this arrangement as long as poor performance leads to lower pay in future periods. This is advantageous for the investor because allowing the manager to bet on a successful future is, from the investor's perspective, the "cheapest" way to give the manager their reservation utility. Full commitment cannot make the investor worse off because it involves a choice over a richer set of contracts. The investor could choose to commit to the dynamically optimal set of contracts outlined above.

To illustrate the effects of full commitment it suffices once again to consider the two period case. The problem is to choose $V_1(s_1, \omega_1)$, $V_2(s_1, s_2, \omega_1, \omega_2)$, $\phi_1(s_1, \omega_1)$, and $\phi_2(s_1, s_2, \omega_1, \omega_2)$ to maximize

$$\iint \left\{ \log(V_1) + \delta \iint \log(V_2) f(\pi_1) d\omega_2 ds_2 \right\} f(\pi_0) d\omega_1 ds_1 \quad (6)$$

subject to

$$(\forall s_1) \int (V_1 + \phi_1) p(\omega_1) d\omega_1 = Y_0, \quad (7)$$

$$(\forall s_1, s_2, \omega_1) \int (V_2 + \phi_2) p(\omega_2) d\omega_2 = Y_1, \quad (8)$$

and

$$\iint \left\{ \log(\phi_1) + \delta \iint \log(\phi_2) f(1) d\omega_2 ds_2 \right\} f(1) d\omega_1 ds_1 - c - \delta c = u_0 + \delta u_1. \quad (9)$$

The first period payoffs are just as in the one-period world

$$V_1 = \frac{Y_0}{1 + \lambda} [\pi_0 R_1^I + (1 - \pi_0) R_1^B]$$

$$\phi_1 = \frac{\lambda Y_0}{1 + \lambda} R_1^I$$

but in the second period the situation is quite different. The first-order conditions for V_2 and ϕ_2 respectively are

$$\frac{\delta f(\pi_1)f(\pi_0)}{V_2} = \mu(s_1, s_2, \omega_1)p(\omega_2) \quad (10)$$

and

$$\frac{\delta f(1)f(1)}{\phi_2} = \mu(s_1, s_2, \omega_1)p(\omega_2) \quad (11)$$

where $\mu(s_1, s_2, \omega_1)$ is the multiplier of the second period budget constraint. Multiplying both sides of (10) by V_2 and integrating with respect to ω_2 gives the following expression for this multiplier.

$$\mu(s_1, s_2, \omega_1) = \frac{\delta f^s(s_2)f(\pi_0)}{B_2^P} \quad (12)$$

Plugging this back into (10) gives (after some manipulation)

$$V_2 = B_2^P [\pi_1 R_2^I + (1 - \pi_1) R_2^B]. \quad (13)$$

A similar procedure for ϕ_2 gives

$$\phi_2 = B_2^A R_2^I \quad (14)$$

and

$$B_2^A = \lambda \frac{f(1)}{f(\pi_0)} B_2^P. \quad (15)$$

The usefulness of this last expression is not apparent until one considers the updating process of the investor. Each period there are two pieces of information revealed: the signal, s , and the market state, ω . The probability of observing a given pair (s, ω) conditional on the the manager being competent is $f^I(\omega|s)f^s(s) = f(1)$. The unconditional probability of this pair is $\pi f^I(\omega|s)f^s(s) + (1 - \pi)f^\omega(\omega)f^s(s) = f(\pi)$. So the posterior probability of the manager being competent is $\pi f(1)/f(\pi)$. With this observation (15) can be rewritten as

$$B_2^A = \lambda B_2^P \frac{\pi_1}{\pi_0}.$$

Combining this with $B_2^P + B_2^A = Y_1$ gives

$$B_2^P = \frac{Y_1}{1 + \lambda \frac{\pi_1}{\pi_0}}$$

and

$$B_2^A = \frac{\lambda \frac{\pi_1}{\pi_0} Y_1}{1 + \lambda \frac{\pi_1}{\pi_0}}.$$

In the above analysis past performance affects contracts only through beliefs. Further the effect on the manager's payoff is only through the budget share. This two-period example generalizes in a straightforward way to T periods. At time t the budget shares satisfy

$$B_t^A = \lambda B_t^P \frac{\pi_{t-1}}{\pi_0} \tag{16}$$

so budget shares always depend on only λ , current beliefs, and initial beliefs.

The total assets under management at the end of period t are

$$[B_t^A + B_t^P \pi_{t-1}] R_t^I.$$

Using (16) one can see that the manager's fee is a fraction

$$\frac{\lambda}{\lambda + \pi_0}$$

of this amount. This fraction depends only on initial (not current) beliefs and λ . In fact this fraction is the same fraction of assets under management that can be computed using the first period's contracts. Although this fraction is constant the managers pay is not. An increasing reputation will lead to higher expected pay because of inflows into the fund. These inflows come from the investor who commits a larger and larger percentage of wealth to the manager as reputation improves. So a constant proportional sharing rule is seen to be an optimal contract with reputation effects with no moral hazard and observable signals.

4 Moral Hazard and Reputation

In the previous section I ignored moral hazard and focused on showing how updating leads to inflows to the fund. The question now is whether such inflows can take the place of explicit incentives in the presence of moral hazard.

Obviously if the cost of effort is less than \bar{c} then incentive compatibility will never bind and so the contract of the previous section is optimal. Would a similar contract be optimal if effort were unobservable and the cost of effort, c , was strictly greater than \bar{c} ?

To answer this it suffices to consider the 2-period setting again. There are two additional constraints in this problem which are the incentive compatibility constraints for each period.

$$\iint \left\{ \log(\phi_1) + \delta \iint \log(\phi_2) f(1) d\omega_2 ds_2 \right\} (f_1(1) - f_1(0)) d\omega_1 ds_1 \geq c \quad (17)$$

and

$$\iint \log(\phi_2) (f_2(1) - f_2(0)) d\omega_2 ds_2 \geq c \quad \forall s_1, \omega_1 \quad (18)$$

The question is whether the effect of reputation will cause the first of these constraints to be slack.

If a solution to this problem exists⁹ then the investor's payout is identical (in form) to what was presented in the previous section. The manager's payout in the first period is

$$\phi_1 = B_1^A \left[R_1^I + \frac{\gamma_1}{\lambda} (R_1^I - R_1^B) \right]$$

where γ_1 is the multiplier on the first period IC constraint (see the appendix for details). This result is similar to that obtained in Dybvig, Farnsworth, and Carpenter (2001) in a one-period setting.

The second-period payoff for the manager is somewhat more complicated. From the first-order conditions (after some manipulation) we have

$$\phi_2 = B_2^P(s_1, \omega_1, s_2) \left[L(\pi_1, \pi_0) R_2^I + (R_2^I - R_2^B) \frac{\gamma_2(s_1, \omega_1)}{\delta f_1(\pi_0)} \right]$$

where $\gamma_2(s_1, \omega_1)$ is the Lagrange multiplier on the IC constraint in the second period and $L(\pi_1, \pi_0)$ is the function

$$L(\pi_1, \pi_0) = \lambda \frac{\pi_1}{\pi_0} + \gamma_1 \left(\frac{\pi_1}{\pi_0} - \frac{1 - \pi_1}{1 - \pi_0} \right).$$

⁹Unfortunately there is no guarantee that a solution will exist, especially if c is very large.

Multiplying by state prices and integrating with respect to ω_2 gives

$$B_2^A = B_2^P L(\pi_1, \pi_0)$$

which, combined with the fact that the two budget constraints must add up to Y_1 , gives

$$B_2^P = \frac{Y_1}{1 + \lambda L(\pi_1, \pi_0)}$$

so once again budget shares do not depend on s_2 and they depend on first-period results only through beliefs. The fact that second period budget shares depend only on information available at the end of the first period allows us to identify the multiplier $\gamma_2(s_1, \omega_1)$.

Since the second period is the last period and $c > \bar{c}$ we know that the IC constraint in this period will bind regardless of what happens in the first period. Substituting into the expression for this constraint gives

$$\iint \log \left(B_2^A \left[R_2^I + \frac{\gamma_2(s_1, \omega_1)}{\delta f_1(\pi_0) L(\pi_1, \pi_0)} (R_2^I - R_2^B) \right] \right) (f_2(1) - f_2(0)) d\omega_2 ds_2 = c$$

Because the budget share is known at the end of the first period it drops out. Since this equation must hold for all s_1 and ω_1 it must be the case that

$$\frac{\gamma_2(s_1, \omega_1)}{\delta f_1(\pi_0) L(\pi_1, \pi_0)} = k$$

where k is some constant.

Now that we have an expression for ϕ_2 we can plug it back into the first period IC constraint and show under what conditions it does not bind. The cost which makes the agent just indifferent between putting forth effort and shirking is

$$\begin{aligned} & \iint \log \left(B_1^A \left[R_1^I + \frac{\gamma_1}{\lambda} (R_1^I - R_1^B) \right] \right) (f_1(1) - f_1(0)) d\omega_1 ds_1 + \\ & \delta \iiint \log \left(B_2^A \left[R_2^I + k(R_2^I - R_2^B) \right] \right) f_2(1) (f_1(1) - f_1(0)) d\omega_2 ds_2 d\omega_1 ds_1 \end{aligned}$$

Because of log utility we can divide the utility of the agent's fee in both periods into the log of the budget share and the log of return based component. In the second period this return-based portion integrates to zero because it does not depend on s_1 or ω_1 . The log of the budget share can be further decomposed into the log of Y_1 and a part which depends only on $L(\pi_1, \pi_0)$ to give

$$\iint \log \left(R_1^I + \frac{\gamma_1}{\lambda} (R_1^I - R_1^B) \right) + \delta \log \left(\frac{L(\pi_1, \pi_0)}{1 + L(\pi_1, \pi_0)} \right) (f_1(1) - f_1(0)) d\omega_1 ds_1. \quad (19)$$

The second term is the incentive effect of reputation for this contract which, not surprisingly, is positive.

Although incentive effects are always positive they are not independent of beliefs. The derivative of (19) with respect to π_0 is negative.¹⁰ So the incentive effects of reputation are decreasing in initial beliefs. For a cost of effort sufficiently close to \bar{c} the first-period IC constraint will not bind so that γ_1 will be zero and there will be no bonus offered in the first period. Bonuses must increase as the incentive effects of reputation decrease.

This dynamic behavior of contracts also seems to conform to industry practice. A relatively small percentage of funds get any incentive fees. Among those which do get incentive fees the form is of "fulcrum" type meaning that a portion of the fee is linear in the excess return over a benchmark as do the contracts above. Cuoco and Kaniel (2001) point out that while the numbers of such contracts is small it is the larger funds which have such fees. This is also implied by this model. Smaller funds are small because their reputations are not high. The anticipation of inflows provides all the necessary incentives. Larger funds are large because they have good reputations. Incentive fees take the place of the reputation effect in order to induce effort.

4.1 Comparison with Career Concerns

In a career concerns model, unlike the reputation model, beliefs are homogeneous at each point in time. This means that the doubts the investor holds about the managers skills are also held by the manager. For this reason the manager does not want to be compensated in terms of the funds performance.

¹⁰This is true whether or not the first-period IC constraint binds.

In fact, in a first-best world the manager will receive a fee which looks like the investor's consumption.

Proposition 2 *In the career concerns model with no moral hazard the optimal contract is*

$$V_t = \frac{Y_{t-1}}{1 + \lambda} [\pi_{t-1} R_t^I + (1 - \pi_{t-1}) R_t^B]$$

$$\phi_t = \frac{Y_{t-1} \lambda}{1 + \lambda} [\pi_{t-1} R_t^I + (1 - \pi_{t-1}) R_t^B]$$

where λ is a positive constant.

PROOF See appendix

Once again there are several ways to interpret this result. One interpretation would be that the manager is paid a fixed fraction of assets under management and the way in which the fund is managed changes through time. When beliefs are π a portion π of the fund is invested according to the signal with the remainder invested in the benchmark¹¹. The problem with this interpretation is that this would imply there are no performance related flows: the investor always invests all of that period's income in the fund. The other interpretation, similar to the interpretation we used for the reputation model, is that the manager invests all of the fund assets according to the signal and assets under management varies with π . While this interpretation seems more consistent with the common view of the relationship between flows and performance it implies a contract in which the manager's pay does not depend in any simple way on assets under management. Also it implies that the manager will be somewhat hedged against bad performance by the fund. If the fund does poorly the manager may still receive a large fee if the benchmark does well. This does not seem to conform to industry practice.

With regard to bonuses recall that in the reputation model managers of larger funds (those with better reputations) received performance bonuses in the first period. Managers with moderate or low reputations did not because the reputation effect led them to choose high effort in order to get higher payouts in the second period. With career concerns note that a manager with a poor reputation has an extra disincentive to exert effort. Low effort will produce a useless signal but the manager is quite unsure whether high effort

¹¹Berk and Green (2002) contains a model along these lines.

will produce an informative signal. The choice will depend on the difference between the perceived distribution under high effort and no effort, $f(\pi) - f(0)$. When π is very low this difference is very small. So the cross-sectional implication would work the other way in a career concerns model. Managers of small funds would be more likely to receive bonuses than managers of large funds in order to induce them to take effort. Larger funds would be less likely to get bonuses.

5 Conclusion

I have presented a model of optimal contracting which takes account of reputation effects. The model suggests that contracts which offer a fixed percentage of assets under management may be optimal in that they provide the correct incentives to undertake costly effort. As a manager's reputation becomes more established it is more likely that the manager will receive a bonus for performance (measured by excess return). This prediction seems consistent with current industry practice.

Throughout the paper I have assumed that any information uncovered by the agent becomes public knowledge so that it is contractible and cannot be misreported. In one period contracts this assumption is not needed. In the full commitment contract it is needed. Dybvig, Farnsworth, and Carpenter (2001) showed that excess return based incentive fees cause managers to be overly conservative in reporting the signal. In this case the reporting problem will exist even in the absence of such incentive fees. The reputation effect itself gives the manager incentive to manipulate the updating process by misreporting. Since contracts such as those presented in this paper are observed in practice and since reputation effects seem to be important in this market the problem of potential over-conservatism may in fact exist. I will leave this question to future research.

Appendix

Proof of Proposition 1

The one-period contract maximizes (2) subject to (3) and (1). The first-order conditions are

$$\frac{(\pi_0 f^I(\omega|s) + (1 - \pi_0) f^\omega(\omega)) f^s(s)}{V} = \mu(s) p(\omega) \quad (\text{A.1})$$

$$\lambda \frac{f^I(\omega|s) f^s(s)}{\phi} = \mu(s) p(\omega) \quad (\text{A.2})$$

where λ is the multiplier of the participation constraint and $\mu(s)$ is the multiplier of the (infinitely many) budget constraints. Multiplying both sides of (A.1) by V and integrating with respect to ω , the market state, gives

$$f^s(s) = \mu(s) B^P(s)$$

where $B^P(s)$ is the investor's budget share. Solving for $\mu(s)$ and substituting into equation (A.2) gives (after some rearranging)

$$\phi = \lambda B^P(s) \frac{f^I(\omega|s)}{p(\omega)} \quad (\text{A.3})$$

which can be written as $B^P(s) R^I$ as noted in the text. Multiplying both sides of this equation by $p(\omega)$ and integrating with respect to ω gives the following expression for the manager's budget share

$$B^A(s) = \lambda B^P(s).$$

I have written both budget shares as functions of the signal but the equation above, taken together with the fact that the sum of the budget shares is Y_0 , allows us to solve for both budget shares and demonstrate that they are, in fact, constants. ■

The reputation model with moral hazard

The derivation of the second-period payoffs is given in the text. Here I derive the form of the first-period payoffs. The problem is to choose V_1 and ϕ_1 so

maximize (6) subject to (9), (7), and (17). The first-order conditions are

$$\frac{f(\pi_0)}{V_1} = \mu_1(s_1)p(\omega_1) \quad (\text{A.4})$$

$$\lambda \frac{f^I(\omega_1|s_1)f^s(s_1)}{\phi_1} + \gamma_1 \frac{(f^I(\omega_1|s_1) - f^\omega(\omega_1))f^s(s_1)}{\phi_1} = \mu_1(s_1)p(\omega_1) \quad (\text{A.5})$$

where $\mu_1(s_1)$ is the multiplier of the first-period budget constraint and γ_1 is the multiplier of the incentive compatibility constraint. As above we have that

$$\mu_1(s_1) = \frac{f^s(s_1)}{B_1^P(s_1)}$$

Substituting this in and rearranging and using the definitions of R^I and R^B gives

$$\phi_1 = B_1^P(s_1) [\lambda R^I + \gamma_1(R^I - R^B)]. \quad (\text{A.6})$$

Multiplying by state prices and integrating with respect to ω_1 gives

$$B_1^A = \lambda B_1^P$$

from which we can obtain that the budget shares are of the same form as above.

Proof of Proposition 2

This proof is for a two-period version of the model. The extension to more time periods is straightforward. For notational ease let $f(\pi_{t-1})$ denote the joint distribution of signal and market state at date t given the investor's belief at the beginning of the period. The first-order conditions of the contracting problem are

$$\frac{f(\pi_0)}{V_1} = \mu_1(s_1)p(\omega_1) \quad (\text{A.7})$$

$$\frac{\delta f(\pi_1)}{V_2} = \mu_2(s_1, \omega_1, s_2)p(\omega_2) \quad (\text{A.8})$$

$$\lambda \frac{f(\pi_0)}{\phi_1} = \mu_1(s_1)p(\omega_1) \quad (\text{A.9})$$

$$\lambda \frac{\delta f(\pi_1)}{\phi_2} = \mu_2(s_1, \omega_1, s_2)p(\omega_2) \quad (\text{A.10})$$

Proceeding as in the proof of the reputation model we have from (A.7) and (A.8) that $\mu_1 B_1^P = f^s(s_1)$ and $\mu_2 B_2^P = \delta f_1(\pi_0) f^s(s)$. Substituting into (A.7) through (A.10) and rearranging yields

$$V_1 = B_1^P \frac{f_1(\pi_0)}{p(\omega_1) f^s(s_1)} \quad (\text{A.11})$$

$$V_2 = B_2^P \frac{f_2(\pi_1)}{p(\omega_2) f^s(s_2)} \quad (\text{A.12})$$

$$\phi_1 = \lambda B_1^P \frac{f_1(\pi_0)}{p(\omega_1) f^s(s_1)} \quad (\text{A.13})$$

$$\phi_2 = \lambda B_2^P \frac{f_2(\pi_1)}{p(\omega_2) f^s(s_2)} \quad (\text{A.14})$$

It remains to show that $B_1^P(s_1)$ and $B_2^P(s_1, \omega_1, s_2)$ are constants. Multiplying (A.13) and (A.14) by $p(\omega_1)$ and $p(\omega_2)$ respectively and integrating with respect to the market state yields

$$\begin{aligned} B_1^A &= \lambda B_1^P \\ B_2^A &= \lambda B_2^P \end{aligned}$$

which, combined with the observation that the sum of manager and investor budget shares must sum to the total invested wealth of the investor in each period gives

$$\begin{aligned} B_1^P &= \frac{Y_0}{1 + \lambda} \\ B_2^P &= \frac{Y_1}{1 + \lambda} \end{aligned}$$

as was required. ■

References

- Berk, J. B., and R. C. Green, 2002, “Mutual Fund Flows and Performance in Rational Markets,” Working paper, Carnegie-Mellon Graduate School of Industrial Administration.
- Bhattacharya, S., and P. Pfleiderer, 1985, “Delegated Portfolio Management,” *Journal of Economic Theory*, 36(1), 1–25.
- Cuoco, D., and R. Kaniel, 2001, “Equilibrium Prices in the Presence of Delegated Portfolio Management,” Working Paper, University of Texas at Austin.
- Dybvig, P. H., H. K. Farnsworth, and J. Carpenter, 2001, “Portfolio Performance and Agency,” Working paper, Olin School of Business.
- Dybvig, P. H., and C. Spatt, 1986, “Agency and the Market for Mutual Fund Managers: The Principle of Preference Similarity,” Working paper, Graduate School of Industrial Administration, Carnegie Mellon University.
- Dybvig, P. H., and C. S. Spatt, 1980, “Does it Pay to Maintain a Reputation?,” Unpublished Working Paper.
- Fudenberg, D., and D. K. Levine, 1992, “Maintaining a Reputation when Strategies are Imperfectly Observed,” *The Review of Economic Studies*, 59(3), 561–579.
- Gibbons, R., and K. J. Murphy, 1992, “Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence,” *The Journal of Political Economy*, 100(3), 468–505.
- Harris, M., and B. Holmström, 1982, “A Theory of Wage Dynamics,” *The Review of Economic Studies*, 49(3), 315–333.
- Heinkel, R., and N. M. Stoughton, 1994, “The Dynamics of Portfolio Management Contracts,” *The Review of Financial Studies*, 7(2), 351–387.
- Holmström, B., and J. R. I. Costa, 1986, “Managerial Incentives and Capital Management,” *The Quarterly Journal of Economics*, 101(4), 835–860.

Kreps, D. M., P. Milgrom, J. Roberts, and R. Wilson, 1982, "Rational Cooperation in the Finitely Repeated Prisoner's Dilemma," *Journal of Economic Theory*, 27, 245–252.

Kreps, D. M., and R. Wilson, 1982, "Reputation and Imperfect Information," *Journal of Economic Theory*, 27, 253–279.

Mailath, G. J., and L. Samuelson, 2001, "Who Wants a Good Reputation?," *Review of Economic Studies*, 68, 415–441.

Milgrom, P., and J. Roberts, 1982, "Predation, Reputation, and Entry Deterrence," *Journal of Economic Theory*, 27, 280–312.