

Preliminary Version

Information, Diversification, and Cost of Capital *†

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Abstract

Information, Diversification, and Cost of Capital

We study the pricing implications of information in a noisy rational expectations model with a factor structure for multi-asset payoffs. There are two classes of price taking investors in our model; informed investors who receive private signals on asset payoffs, and uninformed investors who draw imperfect inferences about those signals from prices. We solve the equilibrium explicitly. We show that only information about systematic factors matters in determining asset risk premiums when the number of the risky assets is large. Idiosyncratic risk as well as the asymmetric information risk associated with idiosyncratic factors is fully diversifiable.

1 Introduction

This paper employs a noisy rational expectation equilibrium model to study the relationship between information and cost of capital. In this model, there are multiple risky assets for which payoffs have a factor structure. There are two classes of investors; informed investors who observe signals on systematic and idiosyncratic components of asset payoffs, and uninformed investors who do not observe the signals, but draw inferences from prices. We solve the model explicitly.

An important insight from the modern finance is that, in large economies, because idiosyncratic risks can be eliminated by forming diversified portfolios there will be no premium associated with those risks. We extend that insight by demonstrating that while information associated with idiosyncratic risks has pricing implications in economies with a finite number of assets, similar to idiosyncratic risks themselves, this information has no effect on cost of capital in economies with an infinite number of assets.

However, it is reasonable to envision that information often has components related to systematic factors affecting asset payoffs. Even information at the firm level such as cash flows or earnings might be expected to contain systematic components. Accordingly, one might conjecture that information of this nature will have an impact on cost of capital. We confirm this conjecture by characterizing systematic risk premiums under both symmetric information (when all investors are informed) and asymmetric information (when only some investors are informed) in economies with either a finite or infinite number of assets.

Information in our model takes the form of signals for each asset that include components that are informative of systematic factors generating asset payoffs (systematic components) as well as an idiosyncratic component. We derive a number of interesting results by varying the aggregate precision of the systematic components. On one hand, if the aggregate precision of the systematic components increases with the number of the assets, then the cost of capital will decrease. In the extreme case where the aggregate precision increases without bound, then investors will learn the factor perfectly and the cost of capital of all the risky assets will be the risk-free rate. On the other hand, if the aggregate precision of the systematic components decreases with the number of the assets, then the cost of capital will increase as less uncertainty about systematic factors is resolved. In the extreme case where the aggregate precision goes to zero, the cost of capital will be unaffected by information.

Notably, when the aggregate precision of information is neither zero or infinite in the limit, the cost of capital will be reduced but not to the risk-free rate. Moreover, the closed form solution of the model in this case allows us to analyze the speed of convergence as the number of assets becomes large.

In our model, the un-informed agent does not observe the signal but he can infer it from the equilibrium price. The inference is imperfect because the supply of the assets is random. One novel feature of our model is that the random supply has both systematic and

idiosyncratic components. The equilibrium price will reveal the price in the limit of large number of assets, if the random supply of the assets has no systematic component. The fact that there are systematic components of random supply of assets are consistent with both IPO waves and recent empirical findings of systematic liquidity by Huberman and Hulka (1998) and commonality in liquidity by Chordia, Roll, and Subramanyam (2000).

Stated broadly, our principal contribution lies in examining the interplay between information, diversification, and cost of capital in large economies when information has systematic as well as idiosyncratic components. While Admati (1982, 1985) considers the role of information in a multi-asset noisy rational expectations framework similar to ours, her focus is on characterizing the aggregation of diverse information equilibrium prices. Her principal analysis assumes that asset payoffs are distributed as multi-variate normal random variables and that infinitely many agents receive uncorrelated signals. Notwithstanding that Admati (1982) recognizes the advantages of imposing a factor structure to asset payoffs in considering market-wide information as well as firm-specific information, she is unable to derive an explicit solution in that case. As a consequence, Admati does not study the consequences of diversification and information on both systematic and idiosyncratic factors.

Easley and O'Hara (2004) also model a multi-asset economy in a noisy rational expectations framework. Similar to the information structure in our study, they capture the notion of asymmetric information by assuming two classes of investors; informed investors, who receive identical private signals of asset payoffs, and uninformed investors, who receive no private signals. However, asset payoffs in their model are driven only by idiosyncratic factors and the number of assets is constrained to be finite. Generally speaking, their model can be viewed as a special case of ours from two perspectives. First, if the factor loadings are zero in our model, our model reduces to Easley and O'Hara's model. In this case, we show that the cost of capital is the riskfree rate when the number of assets goes to infinity. Another way is to view Easley and O'Hara's assets as diversified portfolios that correspond to the factors in our model. By setting all idiosyncratic risks equal to zero and assuming all the factors are independent, our model again reduces to that Easley and O'Hara.

The assumption that investors receive private signals at the firm level that are informative of systematic factors has empirical support. Both Seyhun (1992) and Lakonishok and Lee (2001) report that aggregate trading by corporate insiders appears to be predictive of market movements. Such market timing ability of insiders suggests that they are able to extract information on systematic factors from private firm-level data. Another interpretation of informed investors is that these are sophisticated traders who engage in market-wide information acquisition and processing activities such as some institutional traders.

Our results are closely related to an emerging empirical literature that seeks to provide evidence consistent with the pricing of risks posed by information asymmetries. A number of studies show a correlation between proxies for the firm's information environment and its cost of capital. Proxies for the firm's information environment include the Associ-

ation for Investment and Research’s assessment of corporate disclosure practices (Botosan, 1997, and Botosan and Plumlee, 2002), dispersion in analysts’ earnings forecasts (Botosan, Plumlee, and Xie, 2004), analysts’ coverage (Healy, Hutton, and Palepu, 1999), abnormal accruals in accounting earnings (Francis, LaFond, Olsson, and Schipper, 2002), and a market microstructure-based measure (Easley, Hvidkjaer, and O’Hara, 2002). Aboody, Hughes and Liu (2004) further test whether the cost of capital effect of information asymmetry is associated with trading by corporate insiders. While Aboody, Hughes and Liu (2002) examine the systematic nature of asymmetric information, the rest of the studies use firm characteristics undifferentiated between idiosyncratic and systematic to proxy for the information risk. Our analysis questions the theoretical underpinnings of specifications used by studies that do not address this issue because of the diversification effects we demonstrate.

The rest of the paper is organized as follows: Section 2 studies the economy with finite number of risky assets; Section 3 studies the limit of large economy when the number of the risky assets is infinite; and Section 4 concludes.

2 The Finite Economy

In this section, we present a noisy rational expectation model in which the asset payoffs, signals, and the random supply of the assets all have factor structures. We solve the equilibrium in closed form. We then give some examples.

2.1 The Setup

We assume that there is a risk-less asset with return R_f . There are N risky assets that have payoff ν which is generated by a factor structure of the form

$$\nu = \bar{\nu} + \beta F + \Sigma^{1/2} \epsilon. \quad (1)$$

The mean of asset payoffs $\bar{\nu}$ is a $N \times 1$ constant vector, the factor F is a $K \times 1$ vector of mean normal random variables with covariance matrix Σ_F , the factor loading β is a $N \times K$ constant matrix, and the idiosyncratic risk ϵ is a vector of standard normal random variables.

The supply of risky assets, x , is a vector of $N \times 1$ random variables with mean vector \bar{x} ,

$$x = \bar{x} + \beta_x F_x + \Sigma_x^{1/2} \eta_x, \quad (2)$$

covariance matrix Σ_x and η_x is a standard normal random variable. The noisiness of the supply is a necessary assumption to prevent prices from being fully revealing of the informed investors private signal (defined below) and can be interpreted as caused by trading for liquidity reasons. The presence of a systematic component is based on the reasonable assumption that liquidity trading is influenced by market-wide forces that may or may not correspond

to factors influencing risky asset payoffs. Without a systematic component in the random supply, then in the limiting case as the number of risky assets becomes large (implying an infinite number of independent asset specific signals) prices would be fully revealing of the informed investors private signals. In other words, noisy supply is necessary but not sufficient to ensure that asymmetric information is not a moot issue in large economies; there also needs to be a systematic component. We further assume that F_x is independent of the factors generating asset payoffs. This independence assumption is made for simplicity. We will comment on effects of relaxing this assumption later.

We assume that there are two classes of investors, informed and uninformed, with each class containing an infinite numbers of identical agents. The informed investors all receive signal s on asset payoffs and the uninformed can only (imperfectly) infer the signal from market prices. This specification is used by Grossman and Stiglitz (1980) and Easley and O'Hara (2004). In Admati (1985), there are infinitely many agents each of whom receives independent signals. It can be argued that our assumption and Admati's are two special cases of a general information structure where investors have both diverse and asymmetric information: while we emphasize asymmetry, Admati emphasizes diversity. Technically speaking, the correlation between the private signals across informed investors is perfect in our model and zero in Admati's model. The implication of the difference in assumptions is profound. While in our analysis price will be a function of informed Investors private information, price is a function of the actual asset payoffs in Admati's case due to the elimination of signal noise through aggregation of signals across assets.

Rational expectation equilibrium models with many assets having a factor structure have been considered in Caballe and Krishnan (1994), Daniel, Hirshleifer, and Subrahmanyam (2001), Kodres and Pritsker (2002). As assumed above, in our model, there are also systematic factors in the random supply.

We assume all investors have mean-variance utility

$$U = \mu - \frac{A}{2}\sigma^2, \quad (3)$$

where μ is the mean portfolio payoff, σ^2 is the variance of the portfolio payoff, and A is the investors absolute risk aversion coefficient. Each investor faces a budget constraint:

$$W_1 = W_0R_f + D'(\nu - R_f p) \quad (4)$$

where W_0 is the investor's initial wealth, W_1 is the investor's terminal wealth, and D is a vector containing the number of shares invested in risky assets. Given mean-variance utility, the investors' portfolio choice problem is

$$\begin{aligned} \max_D \quad & E[W_1|J] - \frac{A}{2}\text{var}[W_1|J], \\ \text{s.t.} \quad & W_1 = W_0R_f + D'(\nu - R_f p), \end{aligned}$$

where J represents the investor's information set. The first-order condition implies optimal demand takes the following form:

$$D_J^* = \frac{1}{A} \Sigma_{\nu|J} E[\nu - R_f p | J], \quad (5)$$

When asset payoffs do not depend on systematic factors, $\beta = 0$, it is easy to show investors demands for securities are increasing in expected asset payoffs and the precision of information about asset payoffs, and decreasing in relative risk aversion. In the more general case where asset payoffs do depend on systematic factors, $\beta \neq 0$, the demand for asset i depends not only on investors posterior precision of information on asset i , but also on their posterior information on other assets. The informed and the uninformed have differential demand schedules because they condition on different information sets J .

2.2 The Informed Investors

The informed investors receive private signal s which takes the form

$$s = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2} \eta. \quad (6)$$

The $N \times 1$ constant vector b reflects the relative information content of the signal with respect to the systematic factors and η is a $N \times 1$ standard normal random variable. To conform with the interpretation of factor models, we will assume that F , ϵ , η , and η_x are jointly normal and independent and the matrices Σ and Σ_x are diagonal.

Our specification of the payoffs is distinct from an alternative specification where asset payoffs do not follow a factor structure, but satisfy a general variance-covariance matrix (e.g., Admati, (1985)). Though a factor structure such as (1) implies a specific variance-covariance matrix, a general variance-covariance matrix does not imply a corresponding factor structure. Admati (1985) entertains such constructions and concludes that a factor structure is a natural way to capture the idea that asset payoffs have systematic and idiosyncratic components upon which private information may be obtained.

The signal s for each risky asset specified in equation (6) is a linear combination of information about the systematic components of the asset's payoff and information about the idiosyncratic component of that payoff. The signal s can also be interpreted as a combination of two signals: a signal about the idiosyncratic component of asset payoffs, $s_1 = \Sigma^{1/2} \epsilon + (bF + \Sigma_s^{1/2} \eta)$, where $bF + \eta$ as a whole can be interpreted as noise; and a signal about the systematic components, $s_2 = bF + (\Sigma^{1/2} \epsilon + \Sigma_s^{1/2} \eta)$, where $(\Sigma^{1/2} \epsilon + \Sigma_s^{1/2} \eta)$ is interpreted as noise. The assumption that informed investors receive information not only about the idiosyncratic component, but also about the systematic components of payoffs, although uncommon in the theoretical literature, is intuitive. Informed investors such as corporate insiders are likely to know more than the general public about the firm's fundamentals such

as earnings and cash flows. To the extent that the fundamentals are generated by a factor structure, the private information is likely to contain both components. Consistent with this assumption, Seyhun (1992) and Lakonishok and Lee (2001) show that aggregated trading by corporate insiders is predictive of future market returns.

Our specification of signals differs from Admati's (1982). Besides signals in our model being perfectly correlated across informed investors, the "two signals" constructively received by informed investors in our model are correlated with covariance matrix Σ_s conditional on ν and F , whereas the two signals for a given investor in Admati (1982) are uncorrelated. Assuming the two signals are uncorrelated as in Admati's signal specification changes the expressions, but does not affect either the structure of the explicit solution or the qualitative results that follow from that solution.

To calculate the conditional expectations and covariance matrixes, we need to derive the joint density function of ν and F conditional on information s .

Lemma 1 *The moments conditional on signal s is given by*

$$\begin{aligned} \mathbb{E}[\nu|s, F] &= \bar{\nu} + \beta F + \Sigma_{\nu|s, F} \Sigma_s^{-1} (s - bF), \\ \mathbb{E}[F|s] &= \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s, \\ \Sigma_{\nu|s, F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1} \\ \Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b' (\Sigma + \Sigma_s)^{-1} b \\ \hat{\Sigma}_s &= \Sigma + b \Sigma_F b' + \Sigma_s. \end{aligned}$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal s , the payoff is of the form

$$\nu = \bar{\nu} + \Sigma_{\nu|s, F} \Sigma_s^{-1} s + (\beta - \Sigma_{\nu|s, F} \Sigma_s^{-1} b) F + \Sigma_{\nu|s, F}^{1/2} \epsilon_{\nu|s, F}, \quad (7)$$

where, condition on s and F , $\epsilon_{\nu|s, F}$ is a standard normal random variable. We note that from the perspective of an informed investor the factor loading on the systematic factors has become $\beta_s = \beta - \Sigma_{\nu|s, F} \Sigma_s^{-1} b$. The precision matrix of the factors has increased from Σ_F^{-1} to $\Sigma_{F|s} = \Sigma_F^{-1} + b' (\Sigma + \Sigma_s)^{-1} b$.

From equation (7), the expectation of ν conditional on s is

$$\mathbb{E}[\nu|s, F] = \bar{\nu} + \Sigma_{\nu|s, F} \Sigma_s^{-1} s + (\beta - \Sigma_{\nu|s, F} \Sigma_s^{-1} b) \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s \quad (8)$$

and the variance of ν conditional on s

$$\Sigma_{\nu|s} = \Sigma_{\nu|s, F} + (\beta - \Sigma_{\nu|s, F} \Sigma_s^{-1} b) \Sigma_{F|s} (\beta - \Sigma_{\nu|s, F} \Sigma_s^{-1} b)'. \quad (9)$$

Equations (8) and (9) can be substituted into demand function to calculate the investors demand D_j^* for risky assets:

$$D_s^* = \frac{1}{A} \Sigma_{\nu|s}^{-1} (\bar{\nu} + \Phi_s s - R_f p). \quad (10)$$

where

$$\Phi_s = \Sigma_{\nu|s,F} \Sigma_s^{-1} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1}.$$

2.3 The Uninformed Investors

The uninformed investors do not observe the signal s , but can infer s imperfectly from the equilibrium price.

We conjecture that the equilibrium prices have the following form:

$$p = C + B(s - \lambda(x - \bar{x})),$$

where C is a $N \times 1$ vector and B and λ are two $N \times N$ matrices. We will assume that B are invertible. Therefore, observing the price p is equivalent to observing θ which is defined as

$$\theta = B^{-1}(p - C) = s - \lambda(x - \bar{x}).$$

Substituting equations (2) and (6), we can write

$$\theta = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2} \eta + \lambda \beta_x F_x + \lambda \Sigma_x^{1/2} \eta_x. \quad (11)$$

Therefore, we can interpret θ as another signal which has sensitivity b to the factor F and idiosyncratic shocks. The “idiosyncratic” shocks of θ have the covariance matrix Σ_θ where

$$\Sigma_\theta = \Sigma_s + \lambda(\beta_x \Sigma_{F_x} \beta_x' + \Sigma_x) \lambda'.$$

Note that signal θ is less informative than signal s , i.e., its conditional variance-covariance matrix is larger than that of s , i.e., $\Sigma_\theta = \Sigma_s + \lambda \Sigma_x \lambda' \geq \Sigma_s$. We should remark that λ is in general non-diagonal; the “idiosyncratic” shocks $\Sigma_s^{1/2} \eta + \lambda \Sigma_x^{1/2} \eta_x$, although independent of F , are not independent of each other.

When systematic factors in random supply are uncorrelated with systematic factors in asset payoffs, as we assumed, the signal s is a sufficient statistic for (s, θ) (θ is a garbling of s). However, it is plausible that the two systematic factors are correlated. In this case, the signal s is no longer a sufficient statistic for (s, θ) . While the uninformed will continue to condition on only θ , the informed agent will now condition on both s and θ , a departure from the above analysis in which the informed only conditioned on s . We make the uncorrelated assumption for the tractability although we believe that our methodology can be extended to accommodate the case of correlated factors. Furthermore, we conjecture our results are robust with respect to the relaxation of the assumed independence between F and F_x . The crucial aspect for cost of capital to be affected by asymmetric information is whether the informed investors learn more about systematic factors that influence asset payoffs than uninformed investors in equilibrium, this can be modeled with or without the correlation between the two classes of systematic factors.

If we interpret the random supply are due to liquidity effect, then our assumption of systematic components in random supply are supported by empirical studies show that there are systematic components of liquidity, for example, Huberman and Hulka (1998) and Chordia, Roll, and Subrahmanyam (2000). Another potential empirical support is the IPO waves.

To calculate the conditional expectations and covariance matrixes, we need to derive the joint density function of ν and F conditional on information θ .

Lemma 2 *The moments conditional on the signal θ are*

$$\begin{aligned} E[\nu|\theta, F] &= \bar{\nu} + \beta F + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} (\theta - bF), \\ E[F|\theta] &= \Sigma_{F|\theta} b' (\Sigma + \Sigma_{\theta})^{-1} \theta, \\ \Sigma_{\nu|\theta, F}^{-1} &= \Sigma^{-1} + \Sigma_{\theta}^{-1} \\ \Sigma_{F|\theta}^{-1} &= \Sigma_F^{-1} + b' (\Sigma + \Sigma_{\theta})^{-1} b \\ \hat{\Sigma}_{\theta} &= \Sigma + b \Sigma_F b' + \Sigma_{\theta}. \end{aligned}$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal θ , the payoff is of the form

$$\nu = \bar{\nu} + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} \theta + (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b) F + \Sigma_{\nu|\theta, F}^{1/2} \epsilon_{\nu|\theta, F}, \quad (12)$$

where $\epsilon_{\nu|\theta, F}$ is a standard normal random variable. We note that from the perspective of an uninformed investor the factor loading on the systematic factors has become $\beta + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b$. The precision matrix of the factors has increased from Σ_F^{-1} to $\Sigma_{F|\theta} = \Sigma_F^{-1} + b' (\Sigma + \Sigma_{\theta})^{-1} b$.

From equation (12), the expectation of ν conditional on θ is

$$E[\nu|\theta, F] = \bar{\nu} + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} \theta + (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b) \Sigma_{F|\theta} b' (\Sigma + \Sigma_{\theta})^{-1} \theta \quad (13)$$

and the variance of ν conditional on θ

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|\theta, F} + (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b) \Sigma_{F|\theta} (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b)'. \quad (14)$$

Equations (13) and (14) can be substituted into demand function to calculate the investors demand D_j^* for risky assets:

$$D_{\theta}^* = \frac{1}{A} \Sigma_{\nu|\theta}^{-1} (\bar{\nu} + \Phi_{\theta} \theta - R_f p), \quad (15)$$

where

$$\Phi_{\theta} = \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} + (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b) \Sigma_{F|\theta} b' (\Sigma + \Sigma_{\theta})^{-1}.$$

2.4 The Equilibrium

Imposing the market clearing condition, that the total demand from the informed and the uninformed investors equals the supply, we obtain the following equation

$$x = \frac{\mu}{A} \Sigma_{\nu|s}^{-1} (\bar{\nu} + \Phi_s s - R_f p) + \frac{1-\mu}{A} \Sigma_{\nu|\theta}^{-1} (\bar{\nu} + \Phi_\theta \theta - R_f p),$$

where μ is the proportion of informed investors. Defining $\bar{\Sigma}_\nu = \left(\mu \Sigma_{\nu|s} + (1-\mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1}$, we derive an expression for the prices of risky assets:

$$\begin{aligned} p &= \frac{1}{R_f} \left(\bar{\nu} + \bar{\Sigma}_\nu \left(\mu \Sigma_{\nu|s}^{-1} \Phi_s s + (1-\mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \theta - Ax \right) \right) \\ &= \frac{1}{R_f} (\bar{\nu} - \bar{\Sigma}_\nu A \bar{x}) + \frac{1}{R_f} \bar{\Sigma}_\nu \mu \Sigma_{\nu|s}^{-1} \Phi_s \left(s - \left(\mu \Sigma_{\nu|s}^{-1} \Phi_s \right)^{-1} A(x - \bar{x}) \right) \\ &\quad + \frac{1}{R_f} \bar{\Sigma}_\nu (1-\mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta (s - \lambda(x - \bar{x})). \end{aligned} \quad (16)$$

Comparing the above expression to the conjectured form of the price p , it must be true that

$$\lambda = (\mu \Sigma_{\nu|s}^{-1} \Phi_s)^{-1} A. \quad (17)$$

Note that λ is solved in terms of the parameters of the model. The matrices $\Sigma_{\nu|\theta}$, Φ_θ , and $\bar{\Sigma}_\nu$ are expressed in terms of λ as well as the parameters of the model; they are solved once λ is solved.

Proposition 1 *Given that the informed investors receive a private signal informative about both the idiosyncratic component and the systematic components of the asset payoffs, a partially revealing noisy rational expectations equilibrium exists, and prices of risky assets satisfy*

$$p = \frac{1}{R_f} \bar{\nu} - \frac{1}{R_f} \bar{\Sigma}_\nu A \bar{x} + \frac{1}{R_f} \bar{\Sigma}_\nu \left(\mu \Sigma_{\nu|s}^{-1} \Phi_s + (1-\mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right) (s - \lambda(x - \bar{x})). \quad (18)$$

This equation confirms the conjectured form of the price

$$p = C + B(s - \lambda(x - \bar{x})),$$

with $C = \frac{1}{R_f} (\bar{\nu} - \bar{\Sigma}_\nu A \bar{x})$ and $B = \frac{1}{R_f} \bar{\Sigma}_\nu \left(\mu \Sigma_{\nu|s}^{-1} \Phi_s + (1-\mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right)$.

The average price, \bar{p} , is given by

$$\bar{p} = \frac{1}{R_f} (\bar{\nu} - A \bar{\Sigma}_\nu \bar{x});$$

and the average risk premium of assets satisfies

$$\bar{\nu} - R_f \bar{p} = A \bar{\Sigma}_\nu \bar{x} = A \left(\mu \Sigma_{\nu|s}^{-1} + (1-\mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} \bar{x}. \quad (19)$$

Proof: The price p and the expressions for B and C are derived by combining the equations (16) and (17). Average price before the signal s is revealed is obtained by taking the unconditional average of the equation (18). The equation for the average risk premium immediately follows.

The first term in the price p is the expected payoff without signals discounted by the risk-free return. This is the price if the investors are risk-neutral ($A = 0$) and there are no signals in the economy. The second term is the average discount in price associated with risk when there are no signals in the economy. The sum of the first two terms, $\frac{1}{R_f}\bar{v} - \frac{1}{R_f}\bar{\Sigma}_\nu A\bar{x}$, is the average price. The third term is associated with signals and noisy supply. The price of an asset will be higher than its average if either there is a positive signal ($s > 0$) or a below-average supply ($x < \bar{x}$).

The risk premium is determined by the geometric average of the covariance matrices of asset payoffs conditional on s and θ , $\Sigma_{\nu|s}$ and $\Sigma_{\nu|\theta}$. That is, the risk premium compensates the average of the risks conditional on s and θ . Two properties of the risk premium follows. First, from equation (9), $\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'$ and similarly for $\Sigma_{\nu|\theta}$, the average risk includes idiosyncratic risk $\Sigma_{\nu|s,F}$ and $\Sigma_{\nu|\theta,F}$. Therefore, idiosyncratic risks are priced. Second, the average covariance matrix, $\bar{\Sigma}_\nu$ depends on β nonlinearly, thus the risk premium depends on β nonlinearly.

To present more concrete picture of the equilibrium properties, we next give some concrete examples.

2.5 Special Cases

2.5.1 No Information: $\mu = 0$

There are only uninformed agents when $\mu = 0$. In this case, $\lambda \rightarrow \infty$, the inferred signal θ is infinitely more noisy than s thus is not informative at all. It follows immediately that the covariance matrix conditional on θ , $\Sigma_{\nu|\theta}$ is the same as Σ and the factor covariance matrix conditional on θ , $\Sigma_{F|\theta}$, is the same as Σ_F . Furthermore, beta conditional on θ does not change. From Proposition 1, the risk premium is

$$\bar{v} - R_f\bar{p} = \Sigma_{\nu|\theta}^{-1}A\bar{x} = \left(\Sigma + \beta\Sigma_F\beta'\right)A\bar{x}. \quad (20)$$

The above is the risk premium for the economy with no signal, private or contained in price, and thus no updating of beliefs, as expected. The first term in the parentheses is the risk premium for idiosyncratic risk and the second term is the risk premium for the systematic risk. In this case, the idiosyncratic risk is price but β appears linearly in the risk premium.

2.5.2 Symmetric Information: $\mu = 1$

All agents are informed when $\mu = 1$. An application of Proposition 1 implies that the risk premium for this case is

$$\bar{\nu} - R_f \bar{p} = \Sigma_{\nu|s}^{-1} A \bar{x} = \left(\Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b)' \right) A \bar{x}.$$

Similar to the previous case, the idiosyncratic risk is priced. However, the risk premium depends on the beta conditional on s , $\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b$. In such an economy, an econometrician who observes the return but not the signal will conclude that the risk premium depends on β as well as some firm specific characteristics, $\Sigma_{\nu|s,F} \Sigma_s^{-1} b$. Thus Firms with the same β but different $\Sigma_{\nu|s,F} \Sigma_s^{-1} b$ may have different expected returns. This economy seems to potentially provide a theory for the empirical findings of Daniel and Titman (1998).

2.5.3 Identical Assets

In this case, we allow for the presence of both informed and uninformed investors and assume identically distributed risky asset payoffs and related signals; i.e., the covariance matrices of the payoffs, signals, and the random supply are all proportional to the identity matrix, the betas of all risky asset payoffs are equal, and the sensitivities of the signals to the factor (we assume for convenience that the number of factors is one) are equal. The case of identical assets allows explicit computations while preserving most of the intuition applicable to more general cases.

Let $1_{N \times M}$ denote the $N \times M$ matrix with all elements being 1 and I_N denote the identity matrix of dimension N . We will abuse the notation and denote: $\bar{\nu} = \bar{\nu} 1_{N \times 1}$, $\beta = \beta 1_{N \times 1}$, $b = \frac{k}{\sqrt{N}} 1_{N \times 1}$, $\beta_x = \beta_x 1_{N \times 1}$, $\Sigma = \sigma^2 I_N$, $\Sigma_s = \sigma_s^2 I_N$, $\Sigma_x = \sigma_x^2 I_N$, $\Sigma_F = \sigma_f^2$. In this example, we expressed parameters with some scaling of powers of N , this is for convenience for taking the large N limit: the large N limit of these parameters will be the corresponding powers. For example, in the large N , we expect β is independent of N but b will go to zero as $\frac{1}{\sqrt{N}}$.

We will present the various formula for the identical asset case. The derivation of these formula are given in the Appendix for the proof of Corollary 1 below.

The beta conditional on s is $\beta_s = \beta - \frac{\sigma^2}{\sigma^2 + \sigma_s^2} b$, which can be larger or smaller than β , depending on the sign b . The covariance matrices conditional on s are

$$\begin{aligned} \Sigma_{\nu|s,F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\ \Sigma_{F|s} &= (\sigma_f^{-2} + N b^2 (\sigma^2 + \sigma_s^2)^{-1})^{-1} = (\sigma_f^{-2} + k^2 (\sigma^2 + \sigma_s^2)^{-1})^{-1} \equiv \sigma_{fs}^2. \end{aligned}$$

The conditional covariance matrix $\Sigma_{\nu|s,F}$ is same as the one in the standard case of no correlation. The covariance matrix of the factor, $\Sigma_{F|s}$, conditional on s is smaller than the factor covariance matrix without information, σ_f^2 . Keeping b fixed, the more assets the

smaller the conditional factor covariance matrix. The explicit N dependence is due to the assumptions of identical assets.

The covariance matrix of the asset payoff conditional on s , $\Sigma_{\nu|s}$, is given by

$$\begin{aligned}\Sigma_{\nu|s} &= (\sigma^{-2} + \sigma_s^{-2})^{-1}I_N + (\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1}\sigma_s^{-2}b)^2\Sigma_{F|s}1_{N \times 1}1_{1 \times N} \\ &= S_0I_N + S_11_{N \times N},\end{aligned}$$

with

$$\begin{aligned}S_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1}, \\ S_1 &= \frac{\left(\beta - \frac{1}{\sqrt{N}}\frac{\sigma^2}{\sigma^2 + \sigma_s^2}k\right)^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1}k^2} = \sigma_{fs}^2\beta_s^2.\end{aligned}$$

S_0 is the idiosyncratic variance conditional on s and $(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1}k^2)^{-1}$ is the factor variance conditional on s . Both are smaller than their counterparts with no information. However, the total systematic risk which is the product of the beta and factor risk conditional on s , given by $S_1 = \sigma_{fs}^2\beta_s^2$, can be greater than the total systematic risk without information, given by $\sigma_f^2\beta^2$, if the $\beta_s^2 \geq \beta^2$. For example, this happens if $\beta = 0$ but $k \neq 0$. In this case, there is no factor risk in the payoffs thus the systematic risk is zero without information; the signal introduces the factor risk into the payoffs conditional on the signal if the signal has a factor component.

One can verify that the matrix Φ_s is given by

$$\Phi_s = \frac{\sigma^2}{\sigma^2 + \sigma_s^2}I_N + \frac{1}{\sqrt{N}}\sigma_{fs}^2\beta_s k 1_{N \times N}.$$

The λ matrix is given by

$$\lambda = A\mu^{-1}\Phi_s^{-1}\Sigma_{\nu|s} = \frac{1}{N}\lambda_0I_N + \frac{1}{\sqrt{N}^3}\lambda_1I_{N \times N},$$

where

$$\begin{aligned}\lambda_0 &= NA\mu^{-1}\sigma_s^2 = \gamma\mu^{-1}\sigma_s^2 \\ \lambda_1 &= \gamma\mu^{-1}\frac{\beta_s(\sigma^2 + \sigma_s^2) - \frac{1}{\sqrt{N}}\sigma_s^2k}{k + \frac{1}{\sqrt{N}}\frac{\sigma^2}{\sigma_{fs}^2\beta_s}}.\end{aligned}$$

We used the notation $\gamma = \frac{A}{N}$. The signal θ can be written as

$$\begin{aligned}\theta &= s - \lambda(x - \bar{x}) \\ &= s - \left(\frac{\lambda_0}{N} + \frac{\lambda_1}{\sqrt{N}}\right)\beta_x 1_{N \times 1}F_x - \left(\frac{\lambda_0}{N}\sigma_x\eta_x + \frac{\lambda_1}{\sqrt{N}}\sigma_x\hat{\eta}_x I_{N \times 1}\right),\end{aligned}$$

where $\hat{\eta}_x \equiv \frac{1'}{N}\eta_x$ is the sample average of η_x . The covariance matrix of θ is given by

$$\Sigma_\theta = \sigma_{\theta 0}^2 I_N + \frac{1}{N} \sigma_{\theta 1}^2 1_{N \times N},$$

where

$$\begin{aligned} \sigma_{\theta 0}^2 &= \sigma_s^2 + \frac{\lambda_0^2}{N^2} \sigma_x^2, \\ \sigma_{\theta 1}^2 &= \left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{fx}^2 \beta_x^2 + \left(\frac{\lambda_1^2}{N} + \frac{2\lambda_0\lambda_1}{N^{3/2}} \right) \sigma_x^2. \end{aligned}$$

Note that $\sigma_{\theta 0}^2$ idiosyncratic variance conditional on (θ, F) ; it is larger than σ_s because of idiosyncratic random supply term in θ , $-\frac{\lambda_0}{N}\sigma_x\eta_x$. Note that Σ_θ is not diagonal due to the systematic component in the random supply. The covariance matrix of the payoff conditional on (θ, F) also has systematic terms, $\frac{1}{N}\sigma_{\theta 1}^2 1_{N \times N}$, due to the systematic component, F_x , in random supply which gives rise to $\left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{fx}^2 \beta_x^2$, as well as the idiosyncratic component η_x . The idiosyncratic component η_x creates correlation between assets from the variance of $\hat{\eta}_x = \frac{1}{N}1'\eta_x$ (the $\frac{\lambda_1^2}{N}\sigma_x^2$ term) and its correlation with η_x (the $-\frac{2\lambda_0\lambda_1}{N^{3/2}}\sigma_x^2$ term).

The covariance matrix of payoffs conditional on (θ, F) is given by

$$\Sigma_{\nu|\theta, F} = (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left(I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2 (\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} 1_{N \times N} \right).$$

This covariance matrix is also non-diagonal due to the systematic component in random supply. The factor covariance conditional on θ is given by

$$\Sigma_{F|\theta} = (\Sigma_F^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)^{-1} = \left(\sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^{-1} \equiv \sigma_{f\theta}^2.$$

Because $\sigma_s^2 < \sigma_{\theta 0}^2$, the variance of the payoff factor conditional on θ is larger than that conditional on s , as expected. The beta conditional on θ is given by

$$\beta_\theta = \beta - \Sigma_{\nu|\theta, F} \Sigma_\theta^{-1} b = \left(\beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1}.$$

We note that β_θ can be smaller or greater than β , depending on the sign of k . Furthermore, the magnitude of $\beta_\theta - \beta$ is smaller $\beta_s - \beta$, because the un-informed is less confident about the signal than the informed. The covariance matrix of the asset payoffs conditional on θ is given by

$$\Sigma_{\nu|\theta} = \Theta_0 I_N + \Theta_1 I_{N \times N},$$

where

$$\begin{aligned}\Theta_0 &= (\sigma^{-2} + \sigma_{\theta_0}^{-2})^{-1}, \\ \Theta_1 &= \sigma_{f\theta}^2 \left(\beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta_0}^2 + \sigma_{\theta_1}^2} \right)^2 + \frac{1}{N} \frac{\sigma^4 \sigma_{\theta_1}^2}{(\sigma^2 + \sigma_{\theta_0}^2)(\sigma^2 + \sigma_{\theta_0}^2 + \sigma_{\theta_1}^2)}.\end{aligned}$$

Note that the idiosyncratic covariance conditional θ , Θ_0 , is greater than the idiosyncratic covariance conditional on s , S_0 , as expected. However, the systematic covariance conditional on θ , Θ_1 , may be smaller or greater than the systematic covariance conditional on s , even though the factor risk conditional on θ , $\sigma_{f\theta}^2$, is always greater than the factor risk conditional on s .

Corollary 1 *Under the assumptions for identical assets, the risk premium is given by*

$$\begin{aligned}\bar{v} - R_f \bar{p} &= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\times \left(\frac{1 + N \frac{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}}{\mu S_1^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_1^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} \right) \bar{x} I_{N \times 1}.\end{aligned}$$

The proof is given in the appendix. Same as the two previous cases, the idiosyncratic risks are priced. Furthermore, the risk premium depends on β in a highly non-linear way.

Figure 1 plots the average risk premium against the fraction of the informed agents. The risk premium decrease with N , this is due to the diversification effect. The risk premium depends more sensitively on the fraction of the informed agents for larger number of assets. When the fractional of informed agents is increased, the risk premium is decreased. Since the economy for the case of no informed agents is the same as the economy of no information, this graph also implies that the information always reduces the cost of capital. This latter point is not completely obvious. Although the factor covariance conditional on a signal is smaller than the factor covariance without a signal, the beta conditional on a signal can be greater than the beta without information. The systematic risk conditional on a signal, which is the product of the factor covariance and the beta conditional on a signal, can be greater the its counterpart without information. Despite this, the risk premium conditional a signal is smaller than the risk premium without information.

In an economy of finite number of assets, we conclude that: 1) idiosyncratic risk as well as information associated with it are priced; 2) beta conditional on the signal can change which can lead to risk premium depending on characteristics and depending on beta nonlinearly. As we will show next, in the limit of infinite number of assets, neither properties survives.

3 The Diversification Limit

In this section, we study the effects of asymmetric information on price and the cost of capital when the economy is large in the sense that the number N of the risky assets is large; i.e., in the limit as $N \rightarrow \infty$, which we call the diversification limit. We will show that only the systematic risk component and asymmetric information risk associated with that component are priced in this limit and discuss various scenarios the information can affect the cost of capital in the diversification limit.

In order to address caveats concerning the implications of constant absolute risk aversion for risk premiums in the diversification limit, we will begin with the case of no information, when $\mu = 0$. From equation (20), we get

$$\bar{v} - R_f \bar{p} = (\sigma^2 + \beta^2 \sigma_f^2 N) A \bar{x}.$$

There are two problems with the above risk premium. First, the risk premium depends on the idiosyncratic volatility σ^2 ; in other words, the idiosyncratic risk is priced. The more serious problem is that the risk premium goes to infinity as $N \rightarrow \infty$. The root of these problems is that the investors are assumed to have the constant absolute risk aversion. The investors in this case are so risk averse that even the “small” idiosyncratic risk is priced with a non-zero risk premium and the “big” systematic risk is priced with an infinite risk premium. The fact that investors with constant relative risk aversion price idiosyncratic risk demand a finite risk premium is pointed out by Dybvig (1983) and Grinblatt and Titman (1983).

For idiosyncratic risk not to be priced, the absolute risk aversion coefficient of the investors should approach zero when the wealth is high. In our formulation, the only way to have a finite and non-zero factor risk premium is for the absolute risk aversion coefficient A to be inversely proportional to N ,

$$A(N) = \gamma/N,$$

where the constant γ is proportional to the relative risk aversion coefficient.

Under this assumption, the risk premium for the case of no information is given by

$$\bar{v} - R_f \bar{p} = \gamma \beta^2 \sigma_f^2 \bar{x}.$$

The above risk premium does not price the idiosyncratic risk and produces a finite result in the diversification limit.

3.1 Information on Purely Idiosyncratic Factors

We now study the case when the informed investors receive an imperfect private signal about just the idiosyncratic components of risky asset payoffs. In this case, $b = \beta_x = 0$ and the

signal can be written as

$$s = \nu - \bar{\nu} - \beta F + \Sigma_s^{1/2} \eta. \quad (21)$$

Note that when $\beta \neq 0$, the asset payoffs are correlated. In the special case where all assets are uncorrelated, i.e., $\beta = 0$, this structure reduces to the case considered by Easley and O'Hara (2004). It is easy to see that, for finite N , the information asymmetry about idiosyncratic factors matters because of the terms $\Sigma_{\nu|J,F}$. This result, similar to that of Easley and O'Hara (2004), is not surprising because all idiosyncratic risk matters if we do not take the diversification limit.

We have the following corollary summarizing the limiting behavior of the risk premium:

Corollary 2 *Given that the informed investors receive a private signal about the idiosyncratic components of asset payoffs as specified in (21), in the limit as $N \rightarrow \infty$, the risk premium of assets satisfies*

$$\bar{\nu} - R_f \bar{p} = \gamma \beta \Sigma_F \beta' \bar{x} / N. \quad (22)$$

The proof is given in the Appendix. Note that $\beta' \bar{x}$ is of the order N thus $\beta' \bar{x} / N$ is of order 1, when $N \rightarrow \infty$. Thus we have a finite risk premium.

The above corollary shows that when the private signal is only about the idiosyncratic component of a risky asset's payoff, the asset's risk premium is unaffected by the information asymmetry. In other words, the risk posed by asymmetric information on purely idiosyncratic factors is fully diversifiable. It is easy to verify that the risk premium in (22) is no different from the risk premium obtained in the standard setting where investors have homogeneous beliefs. Furthermore, in the case studied by Easley and O'Hara (2004), $\beta = 0$, and the risk premium is reduced to zero, i.e., $\bar{\nu} - R_f \bar{p} = 0$.

More generally, we expect that the same results will hold as long as $b'(\Sigma + \Sigma_s)^{-1}b \rightarrow 0$ and $b'(\Sigma + \Sigma_\theta)^{-1}b \rightarrow 0$ when $N \rightarrow \infty$. Intuitively, diversification works at the power of $1/N$, if the systematic component of the signal has a power less than $1/N$, it will be eliminated by the diversification.

We remark that although the information in this case does not change the ex-ante risk premium, it does change the price of the assets as the price is linear in signal s . Similarly, the information also affects the portfolio holdings and expected utility of both informed and un-informed agents.

3.2 Information on Total Risky Asset Payoffs

We now consider the case when the informed investors receive a private signal about total asset payoffs. In this case, $b = -\beta$ and the signal can be written as

$$s = \nu - \bar{\nu} + \Sigma_s^{1/2} \eta. \quad (23)$$

This is the special case of Admati (1985), when the covariance matrix of the assets has a form of the factor structure and the signals between different assets is uncorrelated.

In this case, $\Sigma_{F|s}^{-1} = \Sigma_F^{-1} + \beta'(\Sigma + \Sigma_s)^{-1}\beta$, which goes to infinity as $N \rightarrow \infty$. Therefore, we have

$$\Sigma_{F|s} = 0.$$

Similarly, $\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + (\beta + \beta_x)'(\Sigma + \Sigma_\theta)(\beta + \beta_x)$, which also goes to infinity as long as $\beta + \beta_x$ goes to a constant as $N \rightarrow \infty$; thus we also have

$$\Sigma_{F|\theta} = 0.$$

It is easy to show that the above two equations imply that the risk premium is zero.

The intuition is clear. The infinitely many signals about the payoffs reveal the systematic factor F completely and thus eliminate the risk associated with that factor. Therefore, conditional on s or θ , all the risks are idiosyncratic and the cost of capital in this case is the riskfree rate. More generally, as long as $b'(\Sigma + \Sigma_s)b \rightarrow \infty$ and $b'(\Sigma + \Sigma_\theta)b \rightarrow \infty$, the cost of capital will be the riskfree rate.

3.3 Information on Systematic and Idiosyncratic Factors

We have considered the cases where $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b) \rightarrow 0$ and $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b) \rightarrow \infty$. We now consider the cases where the limit of $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b)$ is a nonzero finite constant. This happens, for instance, if the elements of either $\sqrt{N}b$ or $\sqrt{N}b$ go to a non-zero constant when $N \rightarrow \infty$.

In this case, in addition to information about the idiosyncratic component of firm's asset payoffs, informed investors receive private information about the systematic factor. We will show that the risk premium will be affected by the information. Intuitively, the private signal is informative about both the systematic and the idiosyncratic component of asset payoffs. While any risk associated with the private information about the idiosyncratic component is fully diversified, the private information about the systematic factor has an impact on the risk premium in equilibrium. Since the effect on the equilibrium risk premium by the informed is different from the one by the uninformed investors, the fraction of the informed investors in the economy plays an important role in the determination of risk premium.

3.3.1 Special Case: Identically Distributed Risky Asset Payoffs

For concreteness, we will first consider the case of identically distributed risky asset payoffs. In this case, all asset payoffs have the same β , same sensitivities b and β_x , and same idiosyncratic variance. Therefore, all assets have the same distribution; however, these distributions are not independent if either $\beta \neq 0$, or $b \neq 0$, or $\beta_x \neq 0$. We can take the $N \rightarrow \infty$ limit using

the results of subsection 2.5.3. In this case,

$$\begin{aligned}\Sigma_{\nu|s,F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\ \Sigma_{F|s} &= (\sigma_f^{-2} + k^2(\sigma^2 + \sigma_s^2)^{-1})^{-1},\end{aligned}$$

Therefore,

$$\Sigma_{\nu|s} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N + \frac{\beta^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2} 1_{N \times N}.$$

It follows that

$$\Phi_s = \frac{\sigma^2}{\sigma^2 + \sigma_s^2} I_N + \frac{1}{\sqrt{N}} \frac{\beta k}{(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2)(\sigma^2 + \sigma_s^2)} 1_{N \times N}.$$

Therefore,

$$\lambda = \frac{1}{N} \gamma \mu^{-1} \sigma_s^2 I_N + \frac{1}{\sqrt{N^3}} \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} I_{N \times N}.$$

The signal θ can be written as

$$\theta = s - \frac{1}{\sqrt{N}} \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \beta_x F_x 1_{N \times 1},$$

where terms involving η_x become negligible. The covariance matrix conditional on θ is

$$\Sigma_\theta = \sigma_s^2 I_N + \frac{1}{N} \left(\gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \right)^2 \sigma_{fx}^2 \beta_x^2 I_{N \times N}.$$

Therefore, the uncertainty of θ due to idiosyncratic components are the same as that of s . The covariance matrix of payoffs conditional on (θ, F) is given by

$$\Sigma_{\nu|\theta,F} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N.$$

This covariance matrix is diagonal since the non-diagonal due to the systematic component is not big enough.

The factor covariance conditional on θ is given by

$$\Sigma_{F|\theta} = \left(\sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_s^2 + \left(\gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \beta_x \sigma_{fx} \right)^2} \right)^{-1}.$$

Note that the variance of the payoff factor conditional on θ is larger than that conditional on s , as expected. The beta conditional on θ is β , that is, it is unchanged in the diversification limit. The covariance matrix of the asset payoffs conditional on θ is given by

$$\Sigma_{\nu|\theta} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N + \left(\sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_s^2 + \left(\gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \right)^2 \sigma_{fx}^2 \beta_x^2} \right)^{-1} \beta^2 I_{N \times N}.$$

The risk premium in the large N limit is

$$\gamma \sigma_f^2 \beta^2 \bar{x} \left(1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2} \left(\mu + \frac{1 - \mu}{1 + (\sigma^2 + \sigma_s^2) \left(\frac{\gamma \beta \sigma_{fx} \beta_x}{\mu k} \right)^2} \right) \right)^{-1} I_{N \times 1}.$$

The first factor is the risk premium without information. The risk premium when all agents are informed is given by $\gamma \sigma_f^2 \beta^2 \bar{x} \left(1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2} \right)$. The factor $\mu + \frac{1 - \mu}{1 + (\sigma^2 + \sigma_s^2) \left(\frac{\gamma \beta \sigma_{fx} \beta_x}{\mu k} \right)^2}$ decrease the effect of asymmetric information.

It is straightforward to check that the cost of capital increases when either the fraction of the uninformed or the accuracy of the information, σ_s^2 , increases. These results are quite intuitive. When $k = 0$, the information is only idiosyncratic, the risk premium reduces to that of the case with no information, $\gamma \sigma_f^2 \beta^2 \bar{x}$, even if $\beta_x \neq 0$ and $\sigma_{fx} \neq 0$.

We should also remark that parameters that characterize the idiosyncratic properties of asset payoffs or information, such as σ^2 or σ_s^2 , enter into the factor risk premium. However, we emphasize that the idiosyncratic risk and idiosyncratic information are not priced.

3.3.2 General Case

For the general case of finite aggregation information, the risk premium is given by the following corollary.

Corollary 3 *Under the assumption of finite aggregate information, the average risk premium is*

$$\gamma \beta \left(\mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}$$

and the factor premium is given by

$$\gamma \left(\mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}.$$

The proof is given in the appendix.

In the limit of large number of assets, neither the idiosyncratic risk nor idiosyncratic information is price. The risk premium is determined complete by beta. In fact, it is proportional to beta, as is the case of no information. Systematic information, from signal as well as random supply of assets, changes the covariance of the factors. The risk premium is proportional to the geometric average of the factor covariance matrices conditional on s and θ , $\Sigma_{F|s}$ and $\Sigma_{F|\theta}$. For the case of finite aggregation information case, the beta of the assets does not change in the large N limit. Since $\Sigma_{F|s} < \Sigma_{F|\theta}$, the cost of capital decreases with the fraction of the informed; in particular, the cost of capital with information is always smaller than the cost of capital without information.

4 Conclusion

Recent empirical studies have sought to assess the extent to which cost of capital may be influenced by a firm's information environment (eg., Botosan, Plumlee, and Xie, 2004; Francis, LaFond, Olsson, and Schipper, 2002; Easley, Hvidkjaer, and O'Hara, 2002; and Aboody, Hughes and Liu, 2004). One potential explanation for findings of an association is that asymmetric information risks may be priced. An oft-cited result of Easley and O'Hara (2004) that associates a risk premium with proportions of investors that are privately informed or uninformed about idiosyncratic factors that affect asset payoffs in a multi-asset setting has lent theoretical support to such findings. However, Easley and O'Hara's (2004) result depends crucially on whether the number of assets is sufficiently small for risk premiums to be detected.

This paper provides a noisy rational expectations model of economies in which asset payoffs are influenced by systematic factors and for which some investors obtain private information on those factors as well as on idiosyncratic components of asset payoffs. The principal results are that information on idiosyncratic components of asset payoffs does not affect asset risk premiums, while information on systematic factors underlying those payoffs does affect asset risk premiums except in less interesting cases where the factor is fully revealed.

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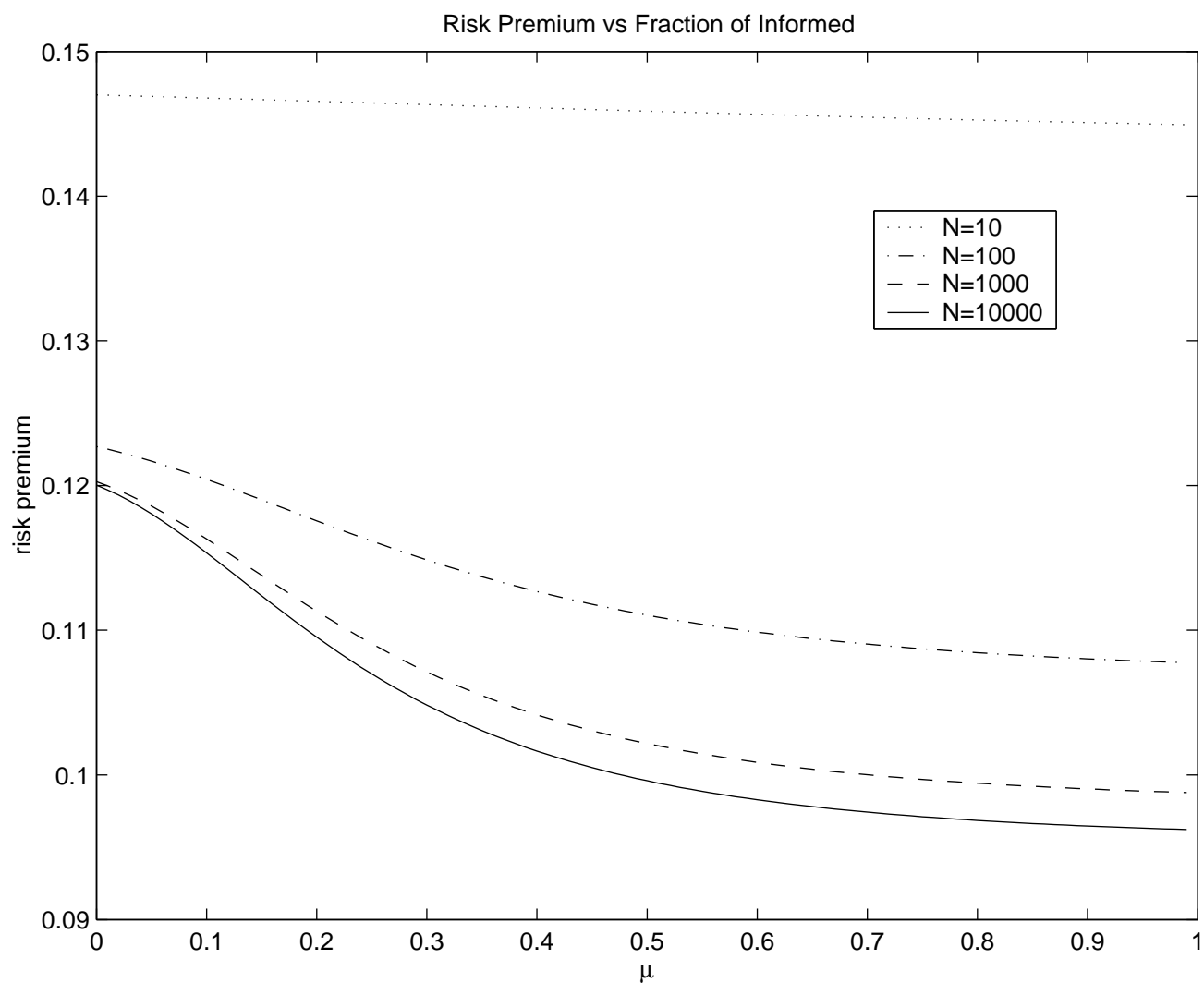


Figure 1. The Risk Premium. This graph plots the average risk premium of the identical asset case against the fraction of the informed agents, for various asset number N . The parameters are: $\gamma = 3$, $\sigma = 30\%$, $\beta = 1$, $\sigma_f = 20\%$, $\sigma_s = 25\%$, $\sigma_{fx} = 30\%$, $\sigma_x = 30\%$, $\beta_x = 1$, and $k = -1$.

Appendix

In the Appendix, we will extensively use the following identity:

$$(\Sigma + \beta\Omega\beta')^{-1} = \Sigma^{-1} - \Sigma^{-1}\beta(\Omega^{-1} + \beta'\Sigma^{-1}\beta)\beta'\Sigma^{-1}.$$

The Proof of Lemma 1.

We solve for the filtering rule given signal s . Our assumptions have specified the distribution of $f(\nu|F, s)$, $f(\nu|F)$ and $f(F)$. Therefore,

$$f(\nu, F, s) = f(s|\nu, F)f(\nu|F)f(F).$$

We need to rewrite it as

$$f(\nu, F, s) = f(\nu|s, F)f(F|s)f(s).$$

Focusing on the exponential terms of the joint normal distribution densities, we obtain

$$\begin{aligned} -\ln f(\nu, F, s) &= -\ln f(s|\nu, F) - \ln f(\nu|F) - \ln f(F) \\ &= \frac{1}{2}(s - (\nu - \bar{\nu} - \beta F) - bF)'\Sigma_s^{-1}(s - (\nu - \bar{\nu} - \beta F) - bF) \\ &\quad + \frac{1}{2}(\nu - \bar{\nu} - \beta F)'\Sigma^{-1}(\nu - \bar{\nu} - \beta F) + \frac{1}{2}F'\Sigma_F^{-1}F \\ &= \frac{1}{2}(\nu - \bar{\nu} - \beta F)'(\Sigma^{-1} + \Sigma_s^{-1})(\nu - \bar{\nu} - \beta F) - (\nu - \bar{\nu} - \beta F)'\Sigma_s^{-1}(s - bF) \\ &\quad + \frac{1}{2}(s - bF)'\Sigma_s^{-1}(s - bF) + \frac{1}{2}F'\Sigma_F^{-1}F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F])\Sigma_{\nu|s, F}^{-1}(\nu - \mathbb{E}[\nu|s, F]) \\ &\quad + \frac{1}{2}(s - bF)'(\Sigma + \Sigma_s)^{-1}(s - bF) + \frac{1}{2}F'\Sigma_F^{-1}F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F])\Sigma_{\nu|s, F}^{-1}(\nu - \mathbb{E}[\nu|s, F]) + \frac{1}{2}s'(\Sigma + \Sigma_s)^{-1}s \\ &\quad + \frac{1}{2}(bF)'(\Sigma + \Sigma_s)^{-1}bF - s'(\Sigma + \Sigma_s)^{-1}bF + \frac{1}{2}F'\Sigma_F^{-1}F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F])\Sigma_{\nu|s, F}^{-1}(\nu - \mathbb{E}[\nu|s, F]) \\ &\quad + \frac{1}{2}(F - \mathbb{E}[F|s])'\Sigma_{F|s}^{-1}(F - \mathbb{E}[F|s]) + \frac{1}{2}s'\hat{\Sigma}_s^{-1}s \\ &= -\ln f(\nu|s, F) - \ln f(F|s) - \ln f(s), \end{aligned}$$

the distribution functions $f(\nu|s, F)$, $f(F|s)$, and $f(s)$ can then identified from the above equation, with

$$\mathbb{E}[\nu|s, F] = \bar{\nu} + \beta F + \Sigma_{\nu|s, F}\Sigma_s^{-1}(s - bF),$$

$$\begin{aligned}
\mathbb{E}[F|s] &= \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s, \\
\Sigma_{\nu|s,F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1}, \\
\Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b' (\Sigma + \Sigma_s)^{-1} b, \\
\hat{\Sigma}_s &= \Sigma + b' \Sigma_F b + \Sigma_s.
\end{aligned}$$

The Proof of Lemma 2.

The structure of the filtering rule given signal θ is the same as s . The proof goes exactly the same.

The Proof of Corollary 1.

The beta conditional on s is $\beta_s = \beta - \frac{\sigma^2}{\sigma^2 + \sigma_s^2} b$. We have

$$\begin{aligned}
\Sigma_{\nu|s,F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\
\Sigma_{F|s} &= (\sigma_f^{-2} + k^2 (\sigma^2 + \sigma_s^2)^{-1})^{-1}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Sigma_{\nu|s} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N - (\beta + (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} b)^2 \Sigma_{F|s} \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N} \\
&\equiv S_0 I_N + S_1 \mathbf{1}_{N \times N},
\end{aligned} \tag{24}$$

with

$$\begin{aligned}
S_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1}; \\
S_1 &= \frac{\left(\beta - \frac{1}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_s^2} k \right)^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2}.
\end{aligned}$$

It follows that

$$\begin{aligned}
\Phi_s &= (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} I_N \\
&\quad + (\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} b) (\sigma_f^{-2} + b^2 (\sigma^2 + \sigma_s^2)^{-1} N)^{-1} b (\sigma^2 + \sigma_s^2)^{-1} \mathbf{1} \mathbf{1}' \\
&\equiv \phi_0 I_N + \frac{\phi_1}{\sqrt{N}} \mathbf{1}_{N \times N},
\end{aligned} \tag{25}$$

with

$$\begin{aligned}
\phi_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} = \frac{\sigma^2}{\sigma^2 + \sigma_s^2}; \\
\phi_1 &= \frac{\left(\beta - \frac{1}{\sqrt{N}} (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} k \right) k}{(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2) (\sigma^2 + \sigma_s^2)}.
\end{aligned}$$

One can readily verify that

$$\Phi_s^{-1} = \phi_0^{-1} \left(I_N - (\sqrt{N}\phi_0\phi_1^{-1} + N)^{-1} 1_{N \times N} \right).$$

Therefore,

$$\begin{aligned} \lambda &= A\mu^{-1}\Phi_s^{-1}\Sigma_{\nu|s} \\ &= A\mu\phi_0^{-1}S_0 \left(I_N + \left(S_0^{-1}S_1 - \frac{1 + S_0^{-1}S_1N}{\sqrt{N}\phi_0\phi_1^{-1} + N} \right) I_{N \times N} \right) \\ &= A\mu^{-1}\phi_0^{-1}S_0 \left(I_N + \left(S_0^{-1}S_1 - \frac{1 + S_0^{-1}S_1N}{\sqrt{N}\phi_0\phi_1^{-1} + N} \right) I_{N \times N} \right) \\ &= A\mu^{-1}\phi_0^{-1}S_0 \left(I_N + \frac{\sqrt{N}S_0^{-1}S_1\phi_0\phi_1^{-1} - 1}{N + \sqrt{N}\phi_0\phi_1^{-1}} I_{N \times N} \right) \\ &= \frac{1}{N}\lambda_0 I_N + \frac{1}{\sqrt{N^3}}\lambda_1 I_{N \times N}, \end{aligned}$$

where

$$\begin{aligned} \lambda_0 &= NA\mu^{-1}\sigma_s^2 = \gamma\mu^{-1}\sigma_s^2 \\ \lambda_1 &= \sqrt{N^3}A\mu^{-1} \frac{\frac{\sqrt{N}(\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1}\sigma_s^{-2}b)(\sigma^2 + \sigma_s^2)}{\sqrt{Nb}} - \sigma_s^2}{N + \sqrt{N}\phi_0\phi_1^{-1}} \\ &= \gamma\mu^{-1} \frac{\left(\beta - \frac{1}{\sqrt{N}}(\sigma^{-2} + \sigma_s^{-2})^{-1}\sigma_s^{-2}k \right) (\sigma^2 + \sigma_s^2) - \frac{1}{\sqrt{N}}\sigma_s^2k}{k + \frac{1}{\sqrt{N}} \frac{\sigma^2(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1}k^2)}{\left(\beta - \frac{1}{\sqrt{N}}(\sigma^{-2} + \sigma_s^{-2})^{-1}\sigma_s^{-2}k \right)}}. \end{aligned}$$

We used the notation $\gamma = \frac{A}{N}$. The signal θ can be written as

$$\begin{aligned} \theta &= s - \lambda(x - \bar{x}) = s - \left(\frac{\lambda_0}{N} I_N + \frac{\lambda_1}{\sqrt{N^3}} I_{N \times N} \right) (\beta_x 1_{N \times 1} F_x + \sigma_x \eta_x) \\ &= s - \left(\frac{\lambda_0}{N} + \frac{\lambda_1}{\sqrt{N}} \right) \beta_x 1_{N \times 1} F_x - \left(\frac{\lambda_0}{N} \sigma_x \eta_x + \frac{\lambda_1}{\sqrt{N}} \sigma_x \hat{\eta}_x I_{N \times 1} \right), \end{aligned} \quad (26)$$

where $\hat{\eta}_x \equiv \frac{1}{N} \eta_x$ is the sample average of η_x . So we have

$$\begin{aligned} \Sigma_\theta &= \sigma_s^2 I_N + \frac{1}{N} \left(\lambda_1 + \lambda_0/\sqrt{N} \right)^2 \sigma_{fx}^2 \beta_x^2 1_{N \times N} + \frac{\lambda_0^2}{N^2} \sigma_x^2 I_N + \left(\frac{\lambda_1^2}{N^2} + 2 \frac{\lambda_0 \lambda_1}{N^{5/2}} \right) \sigma_x^2 I_{N \times N} \\ &= \left(\sigma_s^2 + \frac{\lambda_0^2}{N^2} \sigma_x^2 \right) I_N + \frac{1}{N} \left(\left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{fx}^2 \beta_x^2 + \left(\frac{\lambda_1^2}{N} + \frac{2\lambda_0 \lambda_1}{N^{3/2}} \right) \sigma_x^2 \right) I_{N \times N} \end{aligned} \quad (27)$$

$$= \sigma_{\theta 0}^2 I_N + \frac{1}{N} \sigma_{\theta 1}^2 I_{N \times N}, \quad (28)$$

where

$$\begin{aligned}\sigma_{\theta 0}^2 &= \sigma_s^2 + \frac{\lambda_0^2}{N^2} \sigma_x^2; \\ \sigma_{\theta 1}^2 &= \left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{fx}^2 \beta_x^2 + \left(\frac{\lambda_1^2}{N} + \frac{2\lambda_0 \lambda_1}{N^{3/2}} \right) \sigma_x^2.\end{aligned}\quad (29)$$

The covariance matrix of payoffs conditional on (θ, F) is given by

$$\begin{aligned}\Sigma_{\nu|\theta, F} &= \left(\sigma^{-2} I_N + \left(\sigma_{\theta 0}^2 I_N + \frac{1}{N} \sigma_{\theta 1}^2 \mathbf{1}_{N \times N} \right)^{-1} \right)^{-1} \\ &= \left(\sigma^{-2} I_N + \sigma_{\theta 0}^{-2} \left(I_N + \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma_{\theta 0}^2} \mathbf{1}_{N \times N} \right)^{-1} \right)^{-1} \\ &= \left(\sigma^{-2} I_N + \sigma_{\theta 0}^{-2} \left(I_N - N^{-1} \left(\frac{\sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} \mathbf{1}_{N \times N} \right) \right)^{-1} \\ &= \left((\sigma^{-2} + \sigma_{\theta 0}^{-2}) I_N - N^{-1} \left(\frac{\sigma_{\theta 0}^4}{\sigma_{\theta 1}^2} + \sigma_{\theta 0}^2 \right)^{-1} \mathbf{1}_{N \times N} \right)^{-1} \\ &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left(I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2 (\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} \mathbf{1}_{N \times N} \right).\end{aligned}$$

The matrix $\Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1}$ is

$$\begin{aligned}\Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} &= (\Sigma^{-1} + \Sigma_{\theta}^{-1})^{-1} \Sigma_{\theta}^{-1} = (I_N + \Sigma_{\theta} \Sigma^{-1})^{-1} \\ &= \left(I_N + \frac{\sigma_{\theta 0}^2}{\sigma^2} I_N + \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2} \mathbf{1}_{N \times N} \right)^{-1} \\ &= \left(1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left(I_N - \frac{1}{N} \left(\frac{\sigma^2 + \sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} \mathbf{1}_{N \times N} \right) \\ &= \left(1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left(I_N - \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \mathbf{1}_{N \times N} \right).\end{aligned}$$

The factor covariance conditional on θ is given by

$$\begin{aligned}\Sigma_{F|\theta} &= (\Sigma_F^{-1} + b'(\Sigma + \Sigma_{\theta})^{-1}b)^{-1} = \left(\sigma_f^{-2} + \frac{k^2}{N} \mathbf{1}_{1 \times N} \left((\sigma^2 + \sigma_{\theta 0}^2) I_N + \frac{\sigma_{\theta 1}^2}{N} \mathbf{1}_{N \times N} \right)^{-1} \mathbf{1}_{N \times 1} \right)^{-1} \\ &= \left(\sigma_f^{-2} + \frac{k^2}{N(\sigma^2 + \sigma_{\theta 0}^2)} \mathbf{1}_{1 \times N} \left(I_N + \frac{\sigma_{\theta 1}^2}{(\sigma^2 + \sigma_{\theta 0}^2)N} \mathbf{1}_{N \times N} \right)^{-1} \mathbf{1}_{N \times 1} \right)^{-1} \\ &= \left(\sigma_f^{-2} + \frac{k^2}{N(\sigma^2 + \sigma_{\theta 0}^2)} \mathbf{1}_{1 \times N} \left(I_N - \left(\frac{(\sigma^2 + \sigma_{\theta 0}^2)N}{\sigma_{\theta 0}^2} + N \right)^{-1} \mathbf{1}_{N \times N} \right) \mathbf{1}_{N \times 1} \right)^{-1}\end{aligned}$$

$$= \left(\sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2} \left(1 - \left(\frac{\sigma^2 + \sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} \right) \right)^{-1} = \left(\sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^{-1} \equiv \sigma_{f\theta}^2.$$

The beta conditional on θ is given by

$$\begin{aligned} \beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b &= \beta 1_{N \times 1} - \frac{k}{\sqrt{N}} \left(1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left(I_N - \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} 1_{N \times N} \right) 1_{N \times 1} \\ &= \beta 1_{N \times 1} - \frac{k}{\sqrt{N}} \left(1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left(1 - \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1} = \left(\beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1}. \end{aligned}$$

The covariance matrix of the asset payoffs conditional on θ is given by

$$\begin{aligned} \Sigma_{\nu|\theta} &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left(I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2 (\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} 1_{N \times N} \right) + \sigma_{f\theta}^2 \left(\beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^2 1_{N \times N} \\ &= \Theta_0 I_N + \Theta_1 I_{N \times N}, \end{aligned}$$

where

$$\begin{aligned} \Theta_0 &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1}, \\ \Theta_1 &= \sigma_{f\theta}^2 \left(\beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^2 + \frac{1}{N} \frac{\sigma^4 \sigma_{\theta 1}^2}{(\sigma^2 + \sigma_{\theta 0}^2)(\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)}. \end{aligned}$$

The average of inverse covariance matrix of the asset payoffs is given by

$$\begin{aligned} &\mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \\ &= \mu S_0^{-1} \left(I_N - \left(\frac{S_0}{S_1} + N \right)^{-1} I_{N \times N} \right) + (1 - \mu) \Theta_0^{-1} \left(I_N - \left(\frac{\Theta_0}{\Theta_1} + N \right)^{-1} I_{N \times N} \right) \\ &= (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}) I_N - \left(\mu S_0^{-1} \left(\frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{\Theta_1} + N \right)^{-1} \right) I_{N \times N}. \end{aligned}$$

The geometric average of the covariance matrices is given by

$$\begin{aligned} \bar{\Sigma}_{\nu} &= \left(\mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} \\ &= \left((\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}) I_N - \left(\mu S_0^{-1} \left(\frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{\Theta_1} + N \right)^{-1} \right) I_{N \times N} \right)^{-1} \\ &= (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\quad \times \left(I_N - \left(- \frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{\Theta_1} + N \right)^{-1}} + N \right)^{-1} I_{N \times N} \right). \end{aligned}$$

The average risk premium is given by

$$\begin{aligned}
\bar{\nu} - R_f \bar{p} &= A \bar{\Sigma}_\nu \bar{x} 1_{N \times 1} \\
&= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\
&\times \left(1 - \left(-\frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{\Theta_1} + N \right)^{-1}} + N \right)^{-1} N \right) \bar{x} 1_{N \times 1} \\
&= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\
&\times \left(1 - \left(-\frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} + 1 \right)^{-1} \right) \bar{x} 1_{N \times 1}.
\end{aligned}$$

Note that

$$\begin{aligned}
&1 - \frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} \\
&= \frac{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1} - \mu S_0^{-1} - (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} \\
&= -\frac{1}{N} \frac{\mu S_1^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_1^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}}.
\end{aligned}$$

The risk premium can be written as

$$A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \left(1 + N \frac{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}}{\mu S_1^{-1} \left(\frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_1^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} \right) \bar{x} 1_{N \times 1}.$$

Proof of Corollary 2.

We have $b = 0$. So we have

$$\begin{aligned}
\Sigma_{F|s} &= \Sigma_F; \\
\Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta \Sigma_F \beta'; \\
\Phi_s &= \Sigma_{\nu|s,F} \Sigma_s^{-1}; \\
\Sigma_{F|\theta} &= \Sigma_F; \\
\Sigma_{\nu|\theta} &= \Sigma_{\nu|\theta,F} + \beta \Sigma_F \beta'.
\end{aligned}$$

The λ matrix is given by

$$\lambda = \frac{1}{N} \gamma \mu^{-1} \Phi^{-1} \Sigma_{\nu|s} = \frac{1}{N} \gamma \mu^{-1} \Sigma_s \left(I_N + \Sigma_{\nu|s,F}^{-1} \beta \Sigma_F \beta' \right).$$

The signal θ is

$$\theta = s - \frac{1}{N} \gamma \mu^{-1} \Sigma_s \left(I_N + \Sigma_{\nu|s,F}^{-1} \beta \Sigma_F \beta' \right) \Sigma_x^{1/2} \eta_x = s - \frac{1}{N} \gamma \mu^{-1} \Sigma_s \left(\Sigma_x^{1/2} \eta_x + \Sigma_{\nu|s,F}^{-1} \beta \Sigma_F \beta' \Sigma_x^{1/2} \eta_x \right).$$

The term $\frac{1}{N} \gamma \mu^{-1} \Sigma_s \Sigma_x^{1/2} \eta_x$ is negligible in the large N limit. The term $\frac{1}{N} \gamma \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \Sigma_F \beta' \Sigma_x^{1/2} \eta_x$ has typical elements of $\frac{1}{N} \beta' \Sigma_x^{1/2} \eta_x$, which goes to a constant in the large N limit. Therefore, in the large N limit, θ is essentially the same as s . Thus

$$\begin{aligned} \bar{\Sigma}_{\nu} &= \left(\mu (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta') + (1 - \mu) (\Sigma_{\nu|\theta,F} + \beta \Sigma_F \beta') \right)^{-1} \\ &\rightarrow \left(\mu (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta') + (1 - \mu) (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta') \right)^{-1} = \Sigma_{\nu|s,F} + \beta \Sigma_F \beta'. \end{aligned}$$

Therefore, the average risk premium is given by

$$\bar{\nu} - R_f \bar{p} = \gamma \frac{1}{N} \bar{\Sigma}_{\nu} \bar{x} \rightarrow \gamma \frac{1}{N} (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta') \bar{x} \rightarrow \gamma \frac{1}{N} \beta \Sigma_F \beta' \bar{x}.$$

Proof of Corollary 3.

For the case of non-identically distributed risky asset payoffs, the leading order terms in the large- N are

$$\begin{aligned} \Sigma_{\nu|s,F} &= (\Sigma^{-1} + \Sigma_s^{-1})^{-1}; \\ \Sigma_{F|s} &= \left(\Sigma_F^{-1} + \frac{1}{N} k' (\Sigma + \Sigma_s)^{-1} k \right)^{-1}. \end{aligned}$$

The variance of ν conditional on s

$$\begin{aligned} \Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta \Sigma_{F|s} \beta' + O(N^{-1/2}); \\ \Phi_s &= \Sigma_{\nu|s,F} \Sigma_s^{-1} + \frac{1}{\sqrt{N}} \beta \Sigma_{F|s} k' (\Sigma + \Sigma_s)^{-1} + O(N^{-1}). \end{aligned}$$

Both first terms in the above equations are diagonal. The second terms are due to factors. We use $O(N^\alpha)$ to denote matrices with all of its element generally non-zero and of order N^α . In the case of identical assets, $O(N^\alpha) \propto N^\alpha \mathbf{1}_{N \times N}$. These terms will be negligible, in the large N limit, as far as the risk premium is concerned. The Φ_s^{-1} matrix is

$$\begin{aligned} \Phi_s^{-1} &= \Sigma_s \left(I_N + \frac{1}{\sqrt{N}} \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \right)^{-1} \Sigma_{\nu|s,F}^{-1} \\ &= \Sigma_s \left(I_N - \Sigma_{\nu|s,F}^{-1} \beta \left(\sqrt{N} \Sigma_{F|s}^{-1} + k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \right) \Sigma_{\nu|s,F}^{-1}, \end{aligned}$$

and

$$\begin{aligned}
\Phi_s^{-1}\Sigma_{\nu|s} &= \Sigma_s \left(I_N - \Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s} \left(\sqrt{N}I_K + k'(\Sigma_s^{-1}\Sigma + I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s} \right)^{-1} k'(\Sigma_s^{-1}\Sigma + I_N)^{-1} \right) \\
&\times \left(I_N + \Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s}\beta' \right) \\
&= \Sigma_s \left(I_N - \Sigma_{\nu|s,F}^{-1}\beta \left(\sqrt{N}\Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1}\Sigma + I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} k'(\Sigma_s^{-1}\Sigma + I_N)^{-1} \right. \\
&\quad \left. - \Sigma_{\nu|s,F}^{-1}\beta \left(\sqrt{N}\Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1}\Sigma + I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} \sqrt{N}\beta' \right) \\
&\rightarrow \Sigma_s \left(I_N + \frac{1}{\sqrt{N}}\Sigma_{\nu|s,F}^{-1}\beta \left(\frac{1}{N}k'\Sigma^{-1}\beta \right)^{-1} \beta' \right).
\end{aligned}$$

Therefore,

$$\lambda = \frac{\gamma}{\mu N}\Phi_s^{-1}\Sigma_{\nu|s} = \frac{1}{\sqrt{N^3}}\gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta \left(\frac{1}{N}k'\Sigma^{-1}\beta \right)^{-1} \beta'.$$

The signal θ is now given by

$$\theta = s - \frac{1}{\sqrt{N}}\gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta \left(\frac{1}{N}k'\Sigma^{-1}\beta \right)^{-1} \frac{\beta'\beta_x}{N}F_x \equiv s - \frac{1}{\sqrt{N}}\Lambda F_x,$$

with $\Lambda = \gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta \left(\frac{1}{N}k'\Sigma^{-1}\beta \right)^{-1} \frac{\beta'\beta_x}{N}$. The idiosyncratic component of the random supply disappears—it is diversified away. The covariance matrix of the payoffs conditional on θ is

$$\Sigma_\theta = \Sigma_s + \frac{1}{N}\Lambda\beta_x\Sigma_{F_x}\beta_x'\Lambda'.$$

Note that Σ_s is a diagonal matrix while $\Lambda\beta_x\Sigma_{F_x}\beta_x'\Lambda'$ is a matrix with all of its matrix elements being of order 1. Therefore, when Σ_θ is multiplied by a vector of 1's from the right, the second term has the same order of magnitude as the first term. We can show that

$$\Sigma_{\nu|\theta,F} = \Sigma_{\nu|s,F} + O(N^{-1}).$$

As will be shown later, the contribution of such terms to the risk premium goes to zero in the limit of $N \rightarrow \infty$. The factor covariance matrix conditional on θ is

$$\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + \frac{1}{N}k' \left(\Sigma + \Sigma_s + \frac{1}{N}\Lambda\beta_x\Sigma_{F_x}\beta_x'\Lambda' \right)^{-1} k.$$

Note that, when multiplied by a vectors of 1's from left and from right, the term $\frac{1}{N}\Lambda\beta_x\Sigma_{F_x}\beta_x'\Lambda'$ produces a $K \times K$ matrix with elements of order N , the same as matrix $\Sigma + \Sigma_s$.

The variance of ν conditional on θ

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|s,F} + \beta \Sigma_{F|\theta} \beta'.$$

The matrix $\Sigma_{\nu|s,F}$ is diagonal, while all the elements of the matrix $\beta \Sigma_{F|\theta} \beta'$ is of order 1. The terms neglected earlier produces matrices with all elements of order N^{-1} .

From the identity,

$$\begin{aligned} & \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \\ = & \Sigma_{\nu|s,F}^{-1} - \Sigma_{\nu|s,F}^{-1} \beta \left(\mu \left(\Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} + (1 - \mu) \left(\Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \right) \beta' \Sigma_{\nu|s,F}^{-1}, \end{aligned}$$

we can write

$$\left(\mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} = \Sigma_{\nu|s,F} + \beta M^{-1} \beta',$$

where

$$\begin{aligned} M &= \left(\mu \left(\Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} + (1 - \mu) \left(\Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \right)^{-1} - \beta' \Sigma_{\nu|s,F} \beta \\ &= \left(\mu \left(\Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} + (1 - \mu) \left(\Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \right)^{-1} \\ &\times \left(\mu \left(\Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left(\Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \Sigma_{F|\theta}^{-1} \right). \end{aligned}$$

In the large N limit, $\beta' \Sigma_{\nu|s,F}^{-1} \beta$ is of order N , therefore, $\Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \rightarrow \beta' \Sigma_{\nu|s,F}^{-1} \beta$. Similarly, $\Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \rightarrow \beta' \Sigma_{\nu|\theta,F}^{-1} \beta$, so

$$\begin{aligned} M &\rightarrow \beta' \Sigma_{\nu|s,F}^{-1} \beta \left(\mu \left(\beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left(\beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \Sigma_{F|\theta}^{-1} \right) \\ &= \mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1}. \end{aligned}$$

The risk premium is given by

$$\gamma \beta \left(\mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}$$

and the factor premium is given by

$$\gamma \left(\mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}.$$