Imperfect financial integration, uncovered interest parity and central bank foreign exchange reserves

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PRELIMINARY

Abstract

This paper explores the implications of costly international portfolio adjustment by private agents, on the effects of central bank interventions on interest rates, and on short and long-horizon uncovered interest parity (UIP). Using an incomplete-markets two-country monetary model with bonds in positive net supply, we show that with portfolio adjustment costs, surprise central bank interventions can have a substantial negative effect on foreign interest rates. In addition, central bank interventions and monetary policy shocks cause deviations from UIP, or a “forward premium anomaly”. As the time horizon increases, the model-implied regression coefficients become more consistent with the long-run evidence in favor of UIP, as in the data for the major currencies. A crucial element of the analysis is the interaction between the central bank balance sheet, the agents budget constraints, and market clearing conditions in both bond markets.

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1 Introduction

In most countries, the monetary authority or central bank is the key player in the domestic short-term money market, controlling the level of an instrument which is usually an interest rate. Recently, central banks have also become key players in the international bond and currency markets, as a consequence of their desire to influence the level of the exchange rate. Even though market participants and the financial press suspect that central bank actions can have important effects on prices, it is still a matter of debate as to whether they actually do influence prices and, if so, what economic mechanism or market structure would give rise to such an effect.

With respect to market structure, there is an important stylized fact about the behavior of private agent’s positions in the foreign exchange market, documented in Bacchetta and van Wincoop (2008), namely that “there is little active currency management over horizons relevant to medium-term excess return predictability”. They attribute this feature to the fees charged to investors for changing their foreign bond positions: for example, they document that “at 20% risk, a typical fee for a currency fund is a 1% management fee plus 20% of profits” 1. This points to the existence of an important impediment or friction for private agents to modify their foreign exchange portfolios in response to shocks.

This paper studies the impact of central bank accumulation of foreign reserves on interest rates and the predictions for uncovered interest parity in a world characterized by imperfect financial integration where private agents face significant costs to adjust positions in the international bond market. It presents an incomplete-markets two-country monetary model in which the central bank intervenes in the foreign exchange market and conducts domestic monetary policy using an interest rate feedback rule that targets domestic CPI inflation. Open market operations are given by the central bank balance sheet. Bonds in positive net supply are issued by a treasury department. Finally, behavioral constraints are imposed on private agents by the interaction of their budget constraints with the central banks balance sheets and the bond market clearing conditions.

There are two main findings in this paper. One is that when private agents face important costs in adjusting their foreign bond holdings, a purely nominal shock like an innovation in domestic central bank holdings of foreign exchange reserves has a significant impact on foreign interest rates. Instrumental in generating this result is the inability of the agent to neutralize adverse changes in real income when his international portfolio holdings are costly to change.

The other set of results concerns the predictions of the model for uncovered interest parity. In this respect, the paper finds that both monetary policy and foreign exchange intervention shocks can cause a “forward premium anomaly”, that is, a short-horizon deviation from UIP. Additionally, the model is more consistent with UIP at long horizons, as the evidence in the data suggest, giving rise to an upward sloping “term structure” of Fama regression coefficients. For maturities up to 3 years, the model can reproduce the upward sloping term structure of UIP regression coefficients without foreign exchange holdings shocks. This suggests that although part of the story, foreign exchange intervention shocks are not necessary to reproduce the salient features of the data, as argued in Mark and Moh (2007). In fact, when there are no central bank intervention shocks, policy shocks alone are enough to cause

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1 Yes, 20% of profits.
sizeable deviations from UIP.

At the heart of the model’s ability to produce deviations from UIP is the behavior of private agents international bond holdings. In most open economy general equilibrium models in which the behavior of net foreign assets is analyzed, it is necessary to induce stationarity by modifying the basic frictionless model. Among other options, this can be accomplished by making the interest rate on foreign currency deposits depend on the level of lending. In this way an additional variable is introduced that in principle can create deviations from UIP. However, it is an open issue whether the deviations from UIP caused in this way are of the form observed in the data. In this paper it is shown that when financial frictions are important, the deviations from UIP generated by the model are consistent with the data on exchange rate changes and interest rates differentials.

The two papers that are closest in spirit to the present are Bacchetta and van Wincoop (2008), and Canzoneri, Cumby, Diba and Lopez-Salido (2008). Bacchetta and van Wincoop analyze the implications for the forward premium anomaly of introducing infrequent portfolio decisions into an OLG model augmented with liquidity traders to match the observed volatility of exchange rates. Infrequent portfolio decisions are modeled by assuming that investors make only one portfolio decision when born. Their model assumes that bonds are in positive net supply, which allows them to derive implications for optimal portfolio rebalancing using market clearing conditions. They are able to account for UIP deviations in the data, as well as the delayed overshooting result documented in Eichenbaum and Evans (1995). The main differences between their model and the one used in the present paper is that theirs is one good model with no production or labor markets in which the international transmission mechanism of changes in relative prices cannot be analyzed, and central banks are modeled asymmetrically as one sticks to a fixed money supply rule while the other’s interest rate follows an exogenous AR(1) process. Further, they do not have implications for the effects of surprise interventions on yields or on UIP, because in their model central banks do not accumulate reserves.

The model in this paper has many similarities with Canzoneri, Cumby, Diba and Lopez-Salido (2008) (CCDL). They study the effects of a sudden sell-off of reserves by a foreign central bank in an extension of the New Open Economy Model designed to study the implications of a key currency in world trade. Their model introduces stationary foreign bond holdings by assuming that effective transactions balances are produced by a Cobb-Douglas function that has as arguments money and government bond holdings; this renders their model non Ricardian, and shocks that change asset supplies or values have important effects on interest rates and UIP deviations. They find that a sell-off of foreign country’s reserves of U.S. Treasury securities calibrated to equal the reserve buildup since 2002 significantly decreases home consumption and increases home interest rates. The main differences of their model with the one in this paper are that consumers and central banks are modeled asymmetrically (the key currency assumption). In their model, home households hold domestic financial assets only, while the foreign agent must in addition hold domestic bonds; on the other hand, the central bank of the country that provides the key currency does not hold foreign reserves. The first feature implies that the private international bond market is not symmetric, while the second prevents their model from having implications regarding the effects in the national bond market of central bank’s foreign exchange activities. Also, their model can generate significant deviations from UIP triggered by changes in the holdings of
home and foreign bonds, but they do not specifically discuss whether the interest rate differential is negatively correlated with the future depreciation, as required by the empirical evidence, or if simulated data from their model can reproduce a forward premium anomaly. Another difference with the model in the present paper is that the mechanism that generates deviations from UIP is a transactions technology together with a cash-in advance constraint, while in this paper is the budget constraint of the agent.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the calibration of parameters and the numerical solution method. Section 4 presents results concerning the response of foreign interest rate to central bank holdings of foreign reserves. Section 5 shows that the model is consistent with both the short run and the long run evidence on UIP. Section 6 concludes.

2 The Model

Consider a world economy consisting of two countries, “home” and “foreign”. In each country, there is a representative household, a continuum of monopolistically competitive firms indexed by \( j \in [0, 1] \), a central bank, and a treasury department. The model used in this paper is a two-country version of the small open economy framework outlined in Galí and Monacelli (2005) augmented to include bonds in positive net supply, a central bank that intervenes in the foreign exchange market, and a constraint on central bank behavior given by its balance sheet. We will see that the constraint on open market operations imposed by the central bank balance sheet makes the effect of official intervention in one country critically depend on the response of consumers on both countries.

2.1 Households

There is a representative consumer in each country that consumes a composite consumption good \( C_t \), demands real balances \( m_t \) of (domestic) currency and supplies hours of labor \( N_t \) to the domestic firms in order to maximize

\[
\mathbb{E}_t \sum_{i=0}^{\infty} \frac{(C_{t+i} - \theta C_{t+i-1})^{1-\gamma}}{1-\gamma} + V(m_{t+i}) - \frac{\ell(N_{t+i})^{1+\phi}}{1+\phi}. \tag{1}
\]

The composite consumption index is defined as

\[
C_t \equiv \left[ (1-\alpha)^{1/\eta}C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{1/\eta}C_{F,t}^{\frac{\eta-1}{\eta}} \right]^\frac{\eta}{\eta-1}
\tag{2}
\]

where \( C_{H,t} \) is an index of consumption of domestic goods given by

\[
C_{H,t} = \left( \int_0^1 c_{H,t}(j) \frac{\varepsilon-1}{\varepsilon} dj \right)^\frac{\varepsilon}{\varepsilon-1}
\tag{3}
\]

and where \( C_{F,t} \) is an index of consumption of imported goods given by

\[
C_{F,t} = \left( \int_0^1 c_{F,t}(j) \frac{\varepsilon-1}{\varepsilon} dj \right)^\frac{\varepsilon}{\varepsilon-1}
\tag{4}
\]
The parameter \( \alpha \in [0, 1] \) measures the degree of home-bias in preferences; we will assume that \( \alpha < 0.5 \) and this will enable the model to feature real exchange rate fluctuations, i.e. PPP will not hold. The parameter \( \eta > 0 \) is the elasticity of substitution between domestic and foreign bundles of goods, while \( \varepsilon > 1 \) is the elasticity of substitution between individual varieties of goods, and is assumed to be the same for both domestic and foreign varieties. The definitions of the composite, domestic and foreign consumption indexes for the foreign household \( C^*_t, C^*_H,t \) and \( C^*_F,t \) are analogous, and they are defined using the same elasticities \( \eta \) and \( \varepsilon \), but we allow for a different home-bias parameter \( \alpha^* \). In each period, from the optimal allocation of a fixed level of expenditures between domestic and foreign consumption indexes we obtain the following demand curves:

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t
\]

where

\[
P_t \equiv \left[ (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^\frac{1}{1-\eta}
\]

is the Consumer Price Index (CPI),

\[
P_{H,t} \equiv \left( \int_0^1 p_{H,t}(j)^{1-\varepsilon} \, dj \right)^\frac{1}{1-\varepsilon}
\]

is the domestic Producer Price Index (PPI), and

\[
P_{F,t} \equiv \left( \int_0^1 p_{F,t}(j)^{1-\varepsilon} \, dj \right)^\frac{1}{1-\varepsilon}
\]

is the price index for imported goods in domestic currency. Foreign price indexes \( P^*_t, P^*_H,t \) and \( P^*_F,t \) are defined in a similar way; \( P^*_F,t \) would be the price index for imported goods in the foreign economy in foreign currency units. The cost-minimizing demands for domestic and foreign varieties of goods are

\[
c_{H,t}(j) = \left( \frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad c_{F,t}(j) = \left( \frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}
\]

Here, \( p_{H,t}(j) \) is the price in domestic currency of variety \( j \) produced at “home”, while \( p_{F,t}(j) \) is the price in domestic currency of variety \( j \) produced abroad. Denote the price in foreign currency units of variety \( j \) as \( p^*_t(j) \). In this paper, it will be assumed that the Law of One Price (LOOP) holds for individual varieties of goods:

\[
p_{F,t}(j) = E_t p^*_H,t(j) \quad \quad p^*_F,t(j) = \frac{1}{E_t} p_{H,t}(j).
\]

\( E_t \) is the nominal exchange rate, the price of foreign currency in terms of home currency. Then, the LOOP in turn implies that Purchasing Power Parity (PPP) holds between the level of PPI’s across countries:

\[
P_{F,t} = E_t P^*_H,t \quad \quad P^*_F,t = \frac{1}{E_t} P_{H,t}
\]
Note that this does not mean that PPP will hold at the level of CPI’s, simply because of home-bias in preferences. To see this, some notation is in order. Define the real exchange rate as

\[ Q_t = \frac{E_t P^*_t}{P_t}, \]  

(12)

which is interpreted as units of domestic good per unit of foreign good. Also, define the Terms of Trade as

\[ S_t = \frac{P^F_t}{P^H_t}, \]

which is the relative price of a country’s imports in terms if its exports. Then, using the home (and foreign) equation for the CPI (7), the PPI/CPI index ratio \( a_t = \frac{P^H_t}{P_t} \) can be related to \( S_t \) as

\[ a_t^{\eta-1} = (1 - \alpha) + \alpha S_t^{1-\eta} \]  

(13)

and

\[ (a_t^*)^{\eta-1} = (1 - \alpha^*) + \alpha^* S_t^{\eta-1} \]  

(14)

Then, it can be shown that the real exchange rate can be expressed as:

\[ Q_t = \frac{S_t a_t}{a_t^*} = \left[ \frac{\alpha + (1 - \alpha)S_t^{\eta-1}}{(1 - \alpha^*) + \alpha^* S_t^{\eta-1}} \right]^{\eta} \]  

(15)

As can be seen, PPP at the CPI level will hold (i.e. \( Q_t = 1 \) \( \forall t \)) only if \((1 - \alpha) = \alpha^*\), or if one of the countries does not feature home bias. Hence, we will assume home-bias in preferences which give rise to endogenous real exchange rate fluctuations.

It is assumed that markets are complete domestically, but not internationally. The only financial assets available to households of both countries are a nominal bond denominated in domestic and foreign currency, issued by the corresponding treasury departments. The budget constraint of the household is given by:

\[ C_t + \left( b_t - \frac{b_{t-1}R_{t-1}}{\Pi_t} \right) + Q_t \left( b_t^F - \frac{b_{t-1}^F(R_{t-1}^* - \delta b_{t-1}^F)}{\Pi_t^*} \right) + \left( m_t - \frac{m_{t-1}}{\Pi_t} \right) \leq w_t N_t + \frac{1}{P_t} \int_0^1 \Gamma_t(j) dj - \tau_t + \frac{\delta^*(b_{t-1}^F)^2}{\Pi_t}. \]  

(16)

Here, \( b_t \) and \( b_t^F \) are real holdings of the “home” and “foreign” one-period bonds, respectively. \( R_t \) and \( R_t^* \) are the gross-one period nominal interest rates. \( N_t = \int_0^1 N_t(j) dj \) is the total amount of labor supplied by the household, and \( w_t \) is the real wage. \( \Gamma_t(j) \) is the nominal profit of domestic firm \( j \), and \( \tau_t \) is a lump-sum transfer from the “home” treasury department. Finally, \( \Pi_t = P_t/P_{t-1} \) and \( \Pi_t^* = P_t^*/P_{t-1}^* \) are the home and foreign gross CPI inflation rates.

Following Kollmann (2002), a debt-elastic interest rate\(^2\) on foreign bond holdings is used to induce stationarity of \( b_t^F \). Notice that the interest rate at which domestic households can borrow or lend in the international bond market is equal to the foreign nominal short interest rate plus a “spread” \( \delta b_{t-1}^F \) that is decreasing in the level of the household’s foreign bond

\(^2\)This type of friction to induce stationary Net Foreign Assets has also been used by Schmitt-Grohés and Uribe (2001), (2003), Tuladhar (2003). A variant of the debt-elastic interest rate mechanism, which has the same implications, is a multiplicative cost function that affects the purchasing price of the bond, as in Benigno (2007), DePaoli (2007).
holdings. The financial friction is such that if the household lends to the foreign country (i.e. \(b_t^F > 0\)) then the gross return \((R_t^* - \delta b_t^F)\) is less than the foreign short-rate \(R_t^*\); conversely, if the domestic household borrows in the international market (i.e. \(b_t^F < 0\)), it will have to repay more than \(R_t^*\) for each unit of currency received. It is assumed that the proceeds from such “intermediation” activities are received by the household; this is reflected in the last term of the right hand side of (16). The parameter \(\delta\) controls the quantitative importance of the friction and, as we will see later, effectively determines the response of bond holdings and yields to an exogenous innovation in foreign central bank holdings of domestic bonds.

Maximization of (1) subject to (16) gives the following first order conditions:

\[
\lambda_t = (C_t - \theta C_{t-1})^{-\gamma} - \theta \beta \mathbb{E}_t \left[ (C_{t+1} - \theta C_t)^{-\gamma} \right] \tag{17}
\]

\[
1 = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right] R_t \tag{18}
\]

\[
1 = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{Q_{t+1}} \right] \left( R_t^* - \delta b_t^F \right) \tag{19}
\]

\[
w_t = \ell N_t^\phi \lambda_t \tag{20}
\]

\[
\frac{R_t - 1}{R_t} = \frac{V'(m_t)}{\lambda_t} \tag{21}
\]

The first equation defines the marginal utility of consumption with internal habit formation of the difference form; the second is the Euler equation for optimal holdings of domestic bond; the third is the Euler equation for optimal holdings of the foreign bond; the fourth is the FOC for optimal labor supply; and the last one defines optimal holdings of currency. The function \(V(\bullet)\) is not specified, since in the numerical solution of the model, a linear money demand equation is used:

\[
\ln m_t = \kappa + \kappa_c \ln C_t - \kappa_R \ln R_t \tag{22}
\]

The parameters of the above specification are estimated using an OLS regression from U.S. data. The reason why the money demand equation will be assumed instead of derived endogenously is that the numerical algorithm used to solve the model can give negative nominal interest rates, as in any nonlinear model whose numerical solution around the steady state is represented as a VAR with Gaussian disturbances. The simple linear money demand function (22) is introduced to avoid the explosive effect that a negative interest rate can have on the theoretical money demand equation (21). This is important because one of the key features of the model is that domestic open market operations \(b_t^{CB}\) are derived endogenously using the central bank balance sheet identity. Thus, any instability in money demand will necessarily cause a perverse behavior in the holdings of domestic bonds by the central bank.

### 2.1.1 Incomplete markets and risk-sharing conditions

Using the analogous first order condition for optimal holdings of foreign and home bonds by the foreign household, together with the assumption of incomplete markets at the interna-
tional level, we obtain two distinct risk-sharing conditions:

\[ 1 = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1} Q_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right] \left( \frac{1}{\beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\Pi_{t+1}^*} \right]} - \delta b_t^F \right) \]

\[ 1 = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}^* Q_t}{\lambda_t^*} \frac{1}{\Pi_{t+1}} \right] \left( \frac{1}{\beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right]} - \delta^* b_t^{F^*} \right) \]

where, as before, the star* denotes foreign variables. These are the typical “expectational” risk-sharing conditions, as in Chari, Kehoe and McGrattan (2002) that obtain when markets are incomplete, augmented to reflect the financial friction imposed on the cross-border trading of bonds. Under incomplete markets, the level of financial frictions is important in determining the dynamic properties of equilibrium interest rates.

2.2 Firms

In each country, there is a continuum of firms indexed by \( j \in [0, 1] \) that produce a perfectly tradeable “variety” using the technology:

\[ y_t(j) = A_t N_t(j). \] (23)

\( A_t \) is an technology shock, different across countries, but the same for all firms within a country. It is assumed that the log-technology shock follow the AR(1) process:

\[ \ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \epsilon_t^A, \] (24)

where \( \epsilon_t^A \sim N(0, \sigma_A^2) \). Firms are monopolistic competitors that set the price of their product but face real adjustment costs in doing so; this renders their problem dynamic. At any period \( t \), nominal profits of the generic firm \( j \) are given by

\[ \Gamma_t(j) = p_{H,t}(j) y_t(j) - W_t N_t - \frac{\vartheta}{2} \left( \frac{p_{H,t}(j)}{p_{H,t-1}(j)} - 1 \right)^2 P_{H,t}. \] (25)

The last term on the right hand side of (25) defines the price adjustment costs, as in Rotemberg (1982). It formalizes the notion of “sticky prices” in this paper. Quadratic price adjustment costs are a tractable way to obtain an expectational Phillips curve, which captures the real effects of nominal variables. Additionally, up to a first order approximation it is similar to the expectations-augmented Philips curve obtained under the Calvo (1983)-Yun (1996) model of sticky prices. The demand curve facing the monopolist is given by

\[ y_t(j) = \left( \frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} \] (26)

\( y_t(j) \) is total world demand for variety \( j \) produced in the home country, while \( Y_{H,t} \) is total aggregate demand in the home country: they are obtained by adding the demand for product \( j \) of domestic and foreign households, using (5) and (9).
Since firms are owned by the households and their problem is dynamic, they discount their cash-flows using the nominal pricing kernel of the representative household, $M^S_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}}$. Then, generic firm $j$ chooses its selling price $p_{H,t+1}(j)$ to

$$\max_{\{p_{H,t}(j)\}} \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}} \Gamma_{t+i}(j)$$

subject to (26) and (23). The first order condition is:

$$p_{H,t} - p_{H,t}(j) Y_{H,t} \left( \frac{\varepsilon}{p_{H,t}(j)} A_t + (1-\varepsilon) \right) - \vartheta \left( \frac{p_{H,t}(j)}{p_{H,t-1}(j)} - 1 \right) \frac{P_{H,t}}{P_{H,t-1}(j)}$$

$$+ \vartheta \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left( \frac{p_{H,t+1}(j)}{p_{H,t+1}(j)} - 1 \right) \frac{P_{H,t+1}}{P_{H,t+1}(j)} \right] = 0$$

In a symmetric equilibrium, all firms charge the same price $p_{H,t}(j) = P_{H,t}$, and we have $y_t(j) = Y_{H,t}$ and $N_t(j) = N_t$. Then, the expectations-augmented Phillips curve or aggregate supply (AS) equation can be expressed as

$$\vartheta (\Pi_{H,t} - 1) = \vartheta \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \Pi_{H,t+1} (\Pi_{H,t+1} - 1) \right] + \varepsilon Y_{H,t} \left( \frac{w_t}{A_t a_t} - \frac{\varepsilon - 1}{\varepsilon} \right)$$

(27)

When prices are perfectly flexible, $\vartheta = 0$ and we get that $P_{H,t} = \frac{\varepsilon W_t}{(\varepsilon - 1) A_t}$; that is, prices are set as a constant mark-up above nominal marginal cost. The Phillips curve (27) says that changes in expected future international prices, via $a_{t+1}/a_t$, or movements in expected future domestic inflation $\Pi_{H,t+1}$, or in today’s deviation of the price-marginal cost ratio from the constant mark-up, or in aggregate demand $Y_{H,t}$, will cause changes in domestic PPI inflation today.

### 2.3 Treasury department

The government of each country imposes lump-sum transfers to the representative agent, in order to fulfill the debt valuation equation, given an exogenous process for the stock of real government debt.

$$-\tau_t = \left( s b_t - s b_{t-1} R_{t-1} \right) \frac{R_{t-1}}{\Pi_t} + \frac{(R_{t-1} - 1) b_{CB}^{CB}}{\Pi_t}$$

(28)

Notice that the treasury department does not enjoy seigniorage revenues, but does not pay interest on debt held by the central bank. It is assumed that log-bond supply follows an AR(1) process:

$$\ln s b_t = (1 - \rho_{sb}) \ln s b_{SS} + \rho_{sb} \ln s b_{t-1} + \epsilon_t^{sb},$$

(29)

where $\epsilon_t^{sb} \sim N(0, \sigma_{sb}^2)$. 

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2.4 Central bank

2.4.1 Inflation targeting

The central bank (CB) of each country conducts monetary policy using an interest rate feedback rule that targets domestic CPI inflation:

$$\ln R_t = (1 - \rho_R) \ln R_{t-1} + \rho_R [\ln \Pi_t - \ln \Pi_{SS}] + \epsilon_t^{MP}. \quad (30)$$

$$\epsilon_t^{MP} \sim N(0, \sigma_{MP}^2)$$ is the monetary policy shock. Notice that we allow for policy “inertia” by including the term $\ln R_{t-1}$; this will control the persistence of the short-rate.

2.4.2 Foreign exchange (FX) reserve accumulation

It is assumed that another entity, perhaps the treasury secretary, instructs the central bank to intervene in the foreign exchange (FX) market to support the domestic currency. The CB purchases foreign currency to maintain the real exchange rate close to an exogenously predetermined target, and its subject to random disturbances, which formalizes the notion of a CB foreign holdings shock, or intervention shock. The reaction function takes the standard form in the literature:\(^3\) The central bank intervenes in the FX market according to the rule:

$$\ln f_{xt} = \ln f_{xss} + \rho_{fx} \ln f_{x,t-1} - \alpha_2 (\log Q_t - \bar{Q}) + \epsilon_t^{fx}, \quad (31)$$

where $\Delta e_t \equiv \ln E_t - \ln E_{t-1}$. $\epsilon_t^{fx} \sim N(0, \sigma_{fx}^2)$ is the exogenous innovation to central bank reserve accumulation. One of the objectives of the paper is precisely to understand its effects on the level of bond holdings and interest rates.

2.4.3 Balance sheet identity

Finally, given the FX reserve accumulation policy and the level of real money balances desired by the public as a function of the domestic interest rate, the central bank purchases domestic bonds $b_t^{CB}$ through open market operations to fulfill its balance sheet identity

$$Q_t \left( f_{xt} - f_{x,t-1} \frac{R_{t-1}}{\Pi^*_t} \right) + \left( b_t^{CB} - b_{t-1}^{CB} \frac{1}{\Pi_t} \right) = \left( m_t - m_{t-1} \frac{1}{\Pi_t} \right) \quad (32)$$

The form of the “flow” budget constraint\(^4\) of the central bank used in this paper simply extends that used in Jeanne and Svensson (2007) for the case where the bank holds FX reserves, as in Escudé (2007).

Notice the assumption that the central bank does not receive interest payments on Treasury bonds.\(^5\) This, a common feature of modern central banks, helps the model achieve an endogenous and stationary $b_t^{CB}$ variable. If this were not the case, then given that both real money demand $m_t$ and real foreign exchange reserves $f_{xt}$ are stationary, equation (32)

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\(^3\)For a comprehensive survey on academic research on the effects of central bank intervention, see Sarno and Taylor (2002).

\(^4\)More like an Income Statement, and ignoring changes in Net Worth.

\(^5\)Or equivalently, that interest payments on Treasury debt are returned to the Treasury Dept. as “dividend”.
through the term $b_{t-1}^R\frac{R_{t-1}}{\Pi_t}$ would define an explosive process, given that real interest rates are assumed positive.

Also it is necessary to point out that, in the way the model is specified, central bank intervention would actually be unsterilized, that is, money demand would also change. However, quantitatively the response of money demand to an innovation in foreign reserves is almost negligible, so that for practical purposes the intervention can be thought of as “sterilized”.

2.5 Market clearing conditions

2.5.1 Aggregate supply

The representative firm supplies variety $j$ to satisfy demand. Some of its output is lost as a consequence of the adjustment costs. In equilibrium, aggregate supply is given by:

$$Y_t = A_tN_t - \frac{\theta}{2} (\Pi_{H,t} - 1)^2$$  \hspace{1cm} (33)

A similar condition holds for the foreign country, replacing the variables with their * counterparts.

2.5.2 Aggregate demand

Adding the demands of domestic and foreign households for variety $j$, we obtain the following demand-side market clearing conditions:

$$Y_t = a_t^{\eta} \left[ (1 - \alpha)C_t + \alpha_Q^* Q_t^* \right]$$

$$Y_t^* = a_t^{*\eta} \left[ (1 - \alpha^*)C_t^* + \alpha_Q^* Q_t^* \right]$$

Notice how the real exchange rate affects inversely foreign demand for domestic output, while the PPI/CPI ratio affects domestic demand in the same way.

2.5.3 Bond market

Both households and central banks invest in bonds of the two currencies. Therefore, the market clearing conditions for domestic and foreign bonds are:

$$b_t + b_t^{CB} + fx_t^* + b_t^{F*} = sb_t$$

$$b_t^* + b_t^{CB*} + fx_t + b_t^F = sb_t^*$$  \hspace{1cm} (34)

2.6 Current account dynamics

If we substitute the treasury department’s debt valuation equation (28) into the consumer’s budget constraint (16) and additionally use (34) to substitute for $sb_t$, we obtain the current account (CA) dynamics equation

$$Q_t \left( b_t^F + fx_t \right) - (b_t^{F*} + fx_t^*) = a_t Y_t - c_t + Q_t \left( b_t^{F-1} + fx_{t-1} \right) \frac{R_{t-1}^*}{\Pi_t^*} - (b_t^{F*})^2 \frac{R_{t-1}}{\Pi_t}$$

$$+ \frac{\delta^* (b_t^{F*})^2}{\Pi_t^*} - \frac{\delta (b_t) (b_t-1)^2}{\Pi_t^*}.$$  \hspace{1cm} (35)
This equation says that the change in net foreign assets comes from net exports and from surpluses in interest payments and transactions costs.

3 Solution method and calibration

3.1 Solution method

The model is solved using a first order approximation of the equilibrium conditions around the steady state, also known as first order perturbation method. The software used is dynare++, version 1.3.6. For details of the solution method, see Kamenik (2007).

3.2 Calibration

The values assigned to the parameters in the simulations are presented in Table 1. Unless otherwise noted, the same values are assumed for both countries. On the consumer side, \( \beta \) is set equal to 0.9926, which corresponds to an annual steady state real interest rate of about 3\%. The relative risk-aversion parameter is set to a “low” 1.5, to avoid an excessively small elasticity of intertemporal substitution. The home-bias parameter \( \alpha \) is set equal to 0.4 as in Galí and Monacelli (2005). The habit formation parameter is set equal to 0.66 as in Rudebusch and Swanson (2008), henceforth RS. For labor supply, \( \ell = 4.74 \), \( \phi = 1.5 \) as in RS. The elasticity of substitution between domestic and foreign consumption indexes \( \eta \) and between individual varieties \( \varepsilon \) are set equal to 1.5 and 7.5, respectively, as in Faia and Monacelli (2008).

On the supply side, the Rotemberg adjustment cost parameter \( \vartheta \) is set equal to 75, as in Faia and Monacelli (2008); this is obtained by mapping estimated parameters for the price adjustment frequency in the Calvo model with the coefficient on marginal cost on the Phillips curve under the Rotemberg model. The log-Technology shock parameters \( \bar{A}, \rho_A, \sigma_A^2 \) are set equal to 1, 0.9 and 0.012, respectively.

To calibrate the bond supply processes \( s_b \) and \( s_b^* \), an AR(1) process is estimated using the log-HP filtered real \(^6\) total amount outstanding of marketable treasury bills collected from the Monthly Statement of the Public Debt reports\(^7\) for the period 1978:02-2008:02. \( \rho_{sb} \) and \( \sigma_{sb} \) are set equal to 0.8031 and 0.0361, while \( s_b^{ss} \) is set equal to the unconditional mean of the ratio of total t-bills outstanding to Personal Consumption Expenditures\(^8\), which is about 0.10. Notice that \( R_t \) in the model refers to the 3 month treasury rate, and not to all short term bills, which means that the strategy used to calibrate \( s_b \) is only an approximation. The parameters of the Taylor rule are standard in the literature: \( \phi_R = 0.9 \), \( \phi_\Pi = 1.5 \) and \( \sigma_{MP} = 0.004 \).

Canzoneri, Cumby, Diba and Lopez-Salido (2008), henceforth CCDLS, calibrate the parameters of the rule (31) using data on foreign official institutions (FOI) holdings of U.S. Treasury securities from the Flow of Funds accounts of the United States. This number

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\(^6\)All nominal quantities in this paper are deflated using the Price Index for Personal Consumption Expenditures from the FRED Database at the Federal Reserve bank of St.Louis website: http://research.stlouisfed.org/fred2/

\(^7\)Which can be found at: http://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm

\(^8\)This ratio has fluctuated from 0.08 to 0.15 during the 1978-2002 period.
Table 1: Benchmark case parameter values

<table>
<thead>
<tr>
<th>Category</th>
<th>Preferences</th>
<th>Technology</th>
<th>Financial Friction</th>
<th>Treasury Department</th>
<th>Taylor Rule</th>
<th>FX reserve policy</th>
<th>Bond Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta = (1.03)^{-0.25}$, $\theta = 0.66$, $\gamma = 1.5$</td>
<td>$\vartheta = 75$, $\tilde{A} = 1$, $\rho_A = 0.9$, $\sigma_A = 0.01$</td>
<td>$\delta = 0.0034$ and $\delta = 10.4$</td>
<td>$sb_{ss} = 0.1$, $\rho_{sb} = 0.8031$, $\sigma_{sb} = 0.0361$</td>
<td>$\rho_R = 0.9$, $\rho_{\Pi} = 1.5$, $\sigma_{MP} = 0.004$</td>
<td>$fx_{ss} = 0.02$, $\rho_{fx} = 0.8091$, $\alpha_2 = 0.016$, $\sigma_{fx} = 0.0663$</td>
<td>$\rho_{sb} = 0.8031$, $\sigma_{sb} = 0.0361$, $sb_{ss} = 0.10$</td>
</tr>
</tbody>
</table>

Unless otherwise noted, the values are the same for both countries.

evidently includes holdings of bills, notes and bonds. However, since in this paper the short rate refers to the 3 month rate and the market we model is specifically the market for short-term treasury securities, instead of calibrating $fx_t$ to FOI’s holdings of all types of treasury securities, we use data on their holdings of short-term instruments from the Treasury International Capital System (TIC) website \(^9\), for the period 1978:02-2008:02. This provides a more accurate, but still imperfect, measure of the variable whose effects we are trying to trace. Estimating an AR(1) process on log-HP filtered real holdings, we obtain the following estimates: $fx_{ss} = 0.02$, $\rho_{fx} = 0.8091$, $\alpha_2 = 0.016$, $\sigma_{fx} = 0.0663$. The estimated process is less persistent but 1.5 times more volatile than foreign holdings of all types of treasuries estimated by CCDLS. Parameters are summarized in Table 1.

A key parameter in the model is the level of financial friction, $\delta$. A first order approximation of the risk-sharing conditions gives

$$i_t - i^*_t = \mathbb{E}_t[\Delta c_{t+1}] - \varphi b^F_t,$$

where $\varphi = \frac{\delta \beta}{\Pi_{ss}}$. In Kollmann (2002) a similar equation describes the relationship between the interest rate differential and net foreign assets (NFA) normalized by net exports; evidently, that variable has a different interpretation than $b^F_t$ in this model, which corresponds to a country’s private sector holdings of foreign country’s short-term treasury bills. To obtain $\delta$ in (36), Kollmann uses an estimate taken from Lane and Milesi-Ferreti (2001).

Since it would be preferable to use an empirical measure of $b^F_t$ that is close to foreign private sector holdings of 3-month treasury bills instead of NFA, I estimate $\delta$ from an OLS regression of an interest rate differential variable \(^1^0\) on real foreign non-official holdings of

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\(^9\)The data can be downloaded from: https://treas.gov/tic/bltype\_history.txt

\(^1^0\)Constructed as the difference between the U.S. 3-month T-Bill rate, and a trade-weighted average of the 3 month rates from the currencies included in the Trade-Weighted exchange rate index of the dollar against major currencies (AUS,SWE,GBP,EUR,JPY,CAN,CHF)
short term treasury securities. I obtain an estimate of 0.0034, close to Kollman’s estimate of 0.0019, or Selaive and Tuesta (2003) GMM estimate of 0.0044, using Australian data. The obvious difference with their estimate is that these papers calibrate their measure of financial friction $\delta$ using data on a country’s Net Foreign Asset position.

However, even when estimated in the manner described above and approximately consistent with the values found in other studies, there are still two problems with the estimate of $\delta$. The first one is that the data available does not correspond to foreign private holdings of 3-month Treasury Bills specifically. The second is that given that by construction $b_F^t$ and $b_F^t*$ are zero-mean variables, in practice, when the model is simulated using $\delta = 0.0034$, a 95% confidence interval for the average effective spread $\delta b_F^t$ is $\left(-3e^{-4}, 3e^{-4}\right)$. This means that most of the time, the spread is negligible. Therefore, for practical purposes, the friction is unexistent and will not affect the model dynamics.

Bacchetta and van Wincoop (2008) document that, in the real world, ”there is little active currency management over ... medium term horizons”. They attribute this feature to the fees charged to investors for changing their foreign bond positions: for example, they document that “at 20% risk, a typical fee for a currency fund is a 1% management fee plus 20% of profits”. Their theoretical analysis shows that “a small asset management cost discourages investors from actively exploiting the predictability”, and their model is able to reproduce the forward premium anomaly. However, their model is different from the workhorse Open Economy DSGE model used in this paper in almost every feature (for example, monetary policy is totally exogenous in both countries), and the conclusions of both models cannot be compared easily, specially the shock transmission mechanisms. Since their model is substantially different from the one employed in this paper, it is no surprise that in the present one a “small” transaction fee has no impact whatsoever on the dynamics of the model. Therefore, in order to be able to introduce the observed sluggishness of changes in FX private portfolios to market conditions, I choose to study in addition to the baseline case of $\delta = 0.0034$, a case of “high” costs to take positions in the international bond market, where $\delta = 10.4$. As we will see later, at this level of friction, the model features less responsive $b_F^t$ and $b_F^t*$ and has interesting insights into the effects of CB interventions, and deviations from UIP.

Notice that since the same level of friction is assumed for both countries, and preferences are symmetric across consumers, on average neither country looses as a consequence of the transaction costs. However, its effects are crucial for the dynamic response of the world economy to shocks.

4 The effects of central bank accumulation of foreign reserves on interest rates

Figure 1 presents the impulse response function of the foreign interest rate to an exogenous shock in domestic central bank holdings of foreign treasury’s bonds, for two cases: one in which the level of financial friction is $\delta = 0.0034$ (top row), and another in which $\delta = 10.4$ (bottom row). Since both countries are symmetric, I only concentrate on the case of the foreign short-rate being shocked by a unitary standard deviation innovation in domestic central bank holdings of foreign bonds.
The plots in the top row of Figure 1 show the impulse response of the foreign interest rate $R^*_t$ to a positive one standard deviation innovation in $\varepsilon^{FX}_t$. As can be seen in the left box, the effect is negative but very small with a magnitude of $-5.77^{-7}$. The response is negligible when compared to other widely studied economic sources of time variation in bond yields, namely, technology and monetary policy (MP) shocks, as the box in the right column shows. The reason for this result is simply that after an exogenous innovation in $\varepsilon^{FX}_t$, both foreign CPI-inflation $\Pi^*_t$ and foreign marginal utility $\lambda^*_t$ barely respond to the shock; $-0.9^{-6}$ and $-4.5^{-6}$, respectively. In turn, $\Pi^*_t$ does not change because the shock fails to cause quantitatively important changes in the relative prices $Q_t$, $a^*_t$ and $\Pi^*_{H,t}$, as boxes (c) and (e) of Figure 3 show. $\lambda^*_t$ changes by a greater magnitude than foreign CPI-inflation, but it is also very small in absolute terms, as box (c) in Figure 2 shows; this is to be expected since the change in consumption $c^*_t$ is minimal, as the bottom left box in Figure 5 shows. At the heart of the almost absent change in consumption is the ability of foreign households to neutralize changes in their domestic bond market with opposite positions in the foreign bond market, to satisfy their budget constraint. This can clearly be seen in the bottom left box on Figure 5, which plots the impulse responses of the components of the foreign agent’s budget constraint to the $\varepsilon^{FX}_t$ shock: all the agent needs to do is to substitute domestic bonds with foreign ones.

However, things change when the level of the financial friction is “high”, $\delta = 10.4$. The bottom left plot in Figure 1 shows that in this case, the foreign interest rate decreases by about 2.1 bp after a positive innovation to $\varepsilon^{FX}_t$. The plot on the right indicates that now a foreign holding’s shock has an effect that is approximately half of the decrease in interest rates following a technology shock. The bigger negative impulse in $R^*_t$ has two components. One is the positive expected growth in marginal utility (or a negative expected growth in consumption) that will follow the shock, which is depicted in box (d) of Figure 2; this tends to decrease interest rates. The other is the response in expected CPI inflation $\Pi^*_{t+1}$ which is positive, as can be seen in the same plot; this tends to increase interest rates. As can be seen, the marginal utility effect dominates. Finally, notice that now the response in foreign CPI-inflation $\Pi^*_t$ overshoots its steady state value in period 2 after the shock, and therefore causes a positive expected inflation next period $E_t[\Pi^*_{t+1}]$. This happens because PPI inflation in the foreign economy $\Pi^*_{H,t+1}$ will be less negative than $\Pi^*_{H,t}$, while at the same time $a^*$ experiences a negative growth from $t$ to $t+1$. The total effect then follows from the definition $\Pi^*_t = \Pi^*_{H,t} + a_{t+1}^*$.

Now, the interesting question is: how does increasing the level of financial frictions changes the effect of the foreign holdings shock on interest rates? Since the shock has effects on the bond, currency, and goods markets brought by movements in relative prices and inflation, it is necessary to analyze in some detail how the effects of the shock propagate through both economies in each market. This is what is done next.

4.0.1 Effects on bond and currency markets

First, when a positive innovation to $\varepsilon^{FX}_t$ hits the economy, the balance sheet of the central bank (32) requires that if $f x_t$ increases, then $b^*_t$ must decrease. Then, there are two effects on the bond markets of both countries: in the foreign country’s bond market, an increase in the foreign reserves of the home central bank $f x_t$ necessarily induces a market clearing
decrease of the same magnitude in \((b_t^* + b_t^F)\), the sum of the holdings of bonds of the private sector; in the domestic country’s bond market, a decrease in the holdings of treasury bills of the domestic central bank \(b_t^{CB}\) necessarily induces a market clearing increase of the same magnitude in \((b_t + b_t^{F*})\). This can be seen in boxes (c) and (e) on Figure 3.

If domestic and foreign bonds were perfect substitutes and there were no financial frictions, since both representative agents are perfectly symmetric, we would have that \(\Delta b_t^* = \Delta b_t^F = \frac{1}{2} \Delta f_{t}\), and similarly \(\Delta b_t = \Delta b_t^{F*} = \frac{1}{2} \Delta b_t^{CB}\), where \(\Delta x_t = x_t - x_{t-1}\). Also, it is important to notice that we will have \(\Delta f_{x_t} \simeq \Delta b_t^{CB}\), as the change in \(Q_t\) is small in magnitude when compared to the changes in the balance sheet items: this implies that the size of the intervention is \(|\Delta f_{x_t}|\) or equivalently \(|\Delta b_t^{CB}|\).

Now, because there is a cost to taking positions in the foreign bond market for both representative agents, we will have that \(\Delta b_t^{F*} < \frac{1}{2} \Delta b_t^{CB}\) and \(\Delta b_t^F < \frac{1}{2} \Delta f_{x_t}\). This implies that \((\Delta b_t^{F*} + \Delta b_t^F) < \Delta f_{x_t} = |\Delta b_t^{CB}|\). In fact, \(\Delta b_t^{F*}\) and \(\Delta b_t^F\) will be smaller than \(\frac{1}{2} \Delta b_t^{CB}\) and \(\frac{1}{2} \Delta f_{x_t}\), respectively, the higher \(\delta\) is. This can clearly be seen from the risk-sharing equation

\[
b_t^F = \frac{1}{\varphi} \left( \mathbb{E}_t[\Delta e_{t+1}] + (R_t^* - R_t) \right),
\]

where \(\varphi = \frac{\beta \delta}{\pi s}.\) When the level of friction \(\delta\) is low, we have that \(1/\varphi\) is high and \(b_t^{F*}\) responds strongly to even small changes in expected nominal depreciation and the interest rate differential, and vice versa when \(\delta\) is high. Or, equivalently, when \(\delta\) is high, significant changes in the exchange rate and interest rates are needed to induce even small changes in portfolios.

This observation has important implications for the impact of the foreign exchange intervention on the equilibrium exchange rate. Notice that the home central bank is demanding foreign currency when \(f_{x_t}\) increases, and this puts upward pressure on the nominal-exchange rate \(E_{t};\) that is, it tends to depreciate the home currency. But, on the other hand, since private agent’s positions of the other country’s bonds respond in the opposite direction, that is \(\Delta b_t^F < 0\) (sell foreign currency) when \(f_{x_t} > 0\) and \(\Delta b_t^{F*} > 0\) (buy home currency) when \(b_t^{CB} < 0\), then \(\Delta b_t^F\) and \(\Delta b_t^{F*}\) tend to appreciate the home country currency. However, given the existence of a financial friction, the change in \(b_t^{F*}\) and \(b_t^F\) will be smaller than \(\frac{1}{2} \Delta b_t^{CB}\) and \(\frac{1}{2} \Delta f_{x_t}\), which implies that the combined pressure to appreciate the home country currency is smaller than the tendency to depreciate it. This increase in \(f_{x_t}\) will depreciate the home country currency, but the magnitude critically depends on how much \(b_t^F\) and \(b_t^{F*}\) respond to the shock, which in turn depends on \(\delta\). As can be seen in boxes (e) and (f) of Figures 2, we observe a greater change in relative prices when the financial friction parameter \(\delta\) is high; in particular, notice that the nominal depreciation is much greater when \(\delta\) is high, as box (b) on Figures 3 shows. The exchange rate effect gives rise to most of the other changes.

### 4.0.2 Effects on relative prices and real allocations

Now, consider the effects on relative prices and real allocations of the nominal depreciation of the home country currency. Since demand depends on relative prices, we will analyze first what happens to \(a_t, a_t^*\). In the present case of a positive innovation in \(\varepsilon_t^{FX}\), we have that the resulting depreciation of the home country currency reduces \(a_t = P_{H,t}/P_t\) and increases \(a_t^* = P_{H,t}^*/P_t\). The first result can be explained as follows. \(a_t = P_{H,t}/P_t\) goes down because \(P_t\)
increases by more than $P_{H,t}$. There is one force that puts upward pressure on $P_{H,t}$, namely, real wages: as the change in real exchange rates increases world demand for domestic output relative to foreign output, domestic firms must increase the real wage $w_t$ to induce the agent to work more, i.e., increase $N_t$. Then, the increase in real wages in the domestic economy causes an increase in domestic PPI $P_{H,t}$. Next, recall that the domestic CPI $P_t$ is a weighted average of the domestic PPI $P_{H,t}$ and the foreign PPI in units of domestic currency $P_{F,t}$. The CPI-index $P_t$ increases by more than $P_{H,t}$ simply because the impact effect on nominal depreciation is about 19 times greater than the increase in domestic inflation: this means that $P_{F,t}$ increases by more than $P_{H,t}$. This explains why $a_t$ goes down. Finally, the increase $a^*_t$ follows from the obvious opposite mechanism in the foreign labor market and the exchange rate appreciation in foreign currency units. It can be seen in boxes (e) and (f) of Figures 2 that the impact change in $a_t$ and $a^*_t$ is magnified when $\delta$ increases.

The effects of the shock on output are that $Y_t$ increases while $Y^*_t$ decreases. The first result can be explained as follows. Domestic demand for domestic output depends inversely on $a_t$, which in this case decreases; this explains the increase in domestic demand for the domestic good. On the other hand, foreign demand for domestic goods depends inversely on $(a_t/Q_t)$. And as boxes (b) of Figure 3 and (f) of Figure 2 show, the nominal depreciation induces a real depreciation. Then, $Q_t$ increases and we have that the term $(a_t/Q_t)$ decreases by more than $a_t$ alone, expanding world demand for domestic output. As world demand for domestic output expands, $Y_t$ increases. Again, notice that the increase in $Y_t$ is greater when $\delta$ is higher, as can be seen in the plots of the agent’s budget constraints on Figure 5.

The explanation of the decrease in $Y^*_t$ follows the same logic, but with the effects reversed. Foreign demand for foreign output depends inversely on $a^*_t$, which in this case increases; this explains the decrease in foreign demand for their local good. On the other hand, home country demand for foreign output depends inversely on $(a^*_tQ_t)$, which increases by more than $a^*_t$ alone; this explains why home country demand for foreign output decreases. In the end, the result is a fall in world aggregate demand for the foreign country’s good, which causes the decrease in $Y^*_t$. The decrease in $Y^*_t$ is greater when $\delta$ is higher, as can be seen in Figure 5.

### 4.0.3 Effects on consumption through the agent’s budget constraint

Now we need to explain what is the mechanism that causes $c_t$ to decrease while $Y_t$ increases, and at the same time $c^*_t$ to increase when $Y^*_t$ decreases. The answer lies in the changes that the foreign exchange intervention by the domestic central bank induces on both agents’ budget constraints. Since the explanation centers on the foreign interest rate $R^*_t$, we will use the foreign agent’s budget constraint, which is reproduced here in a slightly different way for ease of exposition:

$$c^*_t + b^*_t + \frac{1}{Q_t} b^F_t + m^*_t = a^*_t Y_t + \Omega^*_t$$

(37)

where

$$\Omega^*_t = \frac{1}{\Pi^*_t} \left[ b^*_{t-1} R^*_t + m^*_{t-1} + \delta(b^F_{t-1})^2 \right] - \tau^*_t + \frac{1}{Q_t \Pi^*_t} b^F_t (R_t - \delta^* b^F_{t-1})$$

(38)

defines real financial wealth. On impact $m^*_t$ changes very little, at least when compared to the other components of the balance sheet of the central bank; this can be seen in boxes (a)
and (b) of Figure 4. Because of this, we concentrate the discussion of the impacts of the shock on the remaining terms: $c_t^*, b_t^*, \frac{1}{Q_t} b_F^*, a_t^* Y_t^*$ and $\Omega_t^*$.

First, notice that box (e) of Figure 2 shows that when $\delta = 0.0034$, the shock to $\varepsilon_t F X$ causes negligible changes in relative prices. Then, it is no surprise to find that after an $\varepsilon_t^{FX}$ shock, the only variables of the budget constraint that respond are the bond positions variables $b_t^*$ and $b_F^*$; this can be seen in the bottom left box of Figure 5. They change simply to maintain equilibrium in the bond market. The key here is to notice that $b_F^*$ changes as much as $b_t^*$, a direct consequence of the low friction.

When $\delta$ is high, $c_t^*$ increases more and the answer as to why it does can be seen in the bottom right box in Figure 5, which plots the response of the components of the foreign agent’s budget constraint to the $\varepsilon_t^{FX}$ shock. On the asset side, we know that $Y_t^*$ decreases, but it is actually $a_t^* Y_t^*$ what matters, since the nominal profits of the firms are paid at PPI prices, not CPI prices. And since $a_t^*$ increases after the shock, we have that the “real income” of the foreign agent does not decrease as much as $Y_t^*$. Also, real financial wealth $\Omega_t^*$ decreases, mainly from an increase in real taxes that comes with the impulse decrease in inflation $\Pi_t^*$. Therefore, the main “income” components of the foreign agent balance sheet decrease after the shock.

With real income and real financial wealth decreasing at impact, it might seem counterintuitive to have increasing consumption, but the key is the change in the portfolio composition of the agent. The big negative change in $b_t^*$ forced by the small change in $b_F^*$ because of the financial friction, causes the remaining term $c_t^*$ to increase, given a mitigated decrease in $a_t^* Y_t^*$. In essence, what happens is that the decrease in bond holdings is so big that with a not-so-big decrease in income, consumption must increase. Finally, the obvious opposite effect explains why $c_t$ decreases. The increase in $c_t^*$ gives rise to an impact decrease in $\lambda_t^*$ that is associated with a positive expected marginal utility growth $E_t[\lambda_{t+1}^*] - \lambda_t^*$, as can be seen in box (d) of Figure 2. Since the growth in expected marginal utility is greater in magnitude than expected inflation, the interest rate decreases.

5 The forward premium anomaly and UIP

We have seen that the financial friction can have important effects on the dynamics of interest rates and exchange rates. It is natural then to ask what are its implications for uncovered interest parity, one of the cornerstone conditions in equilibrium international finance models. Special attention will be paid to the short and long-run evidence on UIP. Before explaining the implications of the model for the theory of UIP, it will be useful to briefly review the theory, the way it is tested, and the available empirical evidence. This will help to put the results into context and establish some terminology that will be used when discussing the numerical results of the model. The reader acquainted with the theory of UIP is invited to skip the next section and jump directly to section 5.2.

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11 The next section on UIP draws heavily from chapter 2 in Sarno and Taylor (2002), and from the discussion in Backus, Foresi and Telmer (2001)
5.1 Brief review of UIP and the forward premium anomaly

If market participants are risk neutral, then in equilibrium the expected payoff from a strategy in which an investor borrows 1 unit of the domestic currency at the log interest rate of \( i_t \), converts it into foreign currency at the spot rate of \( E_t \), invests it at the foreign interest rate of \( i^*_t \), and sells the proceeds at the spot price prevailing at the end of one period \( E_{t+1} \), should be zero:

\[
\mathbb{E}_t[\Delta e_{t+1}] = i_t - i^*_t. \tag{39}
\]

In the last equation, \( \Delta e_{t+1} = e_{t+1} - e_t = \ln(E_{t+1}/E_t) \) is the log-depreciation rate of the home currency, \( i_t = \ln R_t \) is the log nominal interest rate and \( i^*_t = \ln R^*_t \) is the log nominal foreign interest rate. On the other hand, a related proposition that is a direct consequence of the absence of arbitrage opportunities is Covered interest Parity (CIP). It states that if arbitrageurs follow a similar strategy but instead of selling spot, they cover their position in foreign currency by selling the proceeds at the forward price of \( F_t^{(1)} \), the following must hold:

\[
f_t^{(1)} - e_t = i_t - i^*_t, \tag{40}
\]

where \( f_t^{(1)} = \ln F_t^{(1)} \). Therefore, if there are no arbitrage opportunities and market participants are risk neutral, as a consequence of both (40) and (39) we get that

\[
f_t^{(1)} = \mathbb{E}_t[e_{t+1}], \tag{41}
\]

which is known as the unbiasedness hypothesis of the forward rate. UIP is usually tested by running regressions of the form:

\[
\Delta e_{t+k} = \alpha^{(k)} + \beta^{(k)}(i_t^{(k)} - i^*_t^{(k)}) + \varepsilon_t^{(k)} \tag{42}
\]

or

\[
\Delta e_{t+k} = \alpha^{(k)} + \beta^{(k)}(f_t^{(k)} - i_t^{(k)}) + \varepsilon_t^{(k)} \tag{43}
\]

Examples of studies that use specification (43) include Fama (1984), Backus, Foresi and Telmer (2001), Burnside, Eichenbaum, Kleshchelski and Rebelo (2006), and McCallum (1994). Studies that use specification (42) include Chinn (2006), Chinn and Meredith (2004),(2005), and Mark and Moh (2007). In the data, it is generally found that CIP (No Arbitrage) holds, or that its deviations are within bid-ask spread bounds. For the rest of the paper, equation (42) or (43) will be refereed to as a “Fama regression”, and the slope coefficient \( \beta^{(k)} \) will be referred to as a “Fama slope coefficient”. A plot of \( \beta^{(k)} \) against \( k \) will be referred to as the “term structure of Fama slope coefficients”. Alvarez, Atkeson and Kehoe (2005) also refer to \( \beta \) as “the slope coefficient in the Fama Regression”.

As can be seen, regression (42) is actually a test of the joint hypothesis that agents are risk neutral and that their expectations are rational, while (43) adds the requirement that the forward rate be an unbiased predictor of future spot rates. Both UIP and the Unbiasedness Hypothesis require that the estimated coefficients satisfy \( \alpha^{(k)} = 0 \) and \( \beta^{(k)} = 1 \) for all \( k \).
5.1.1 Short-horizon evidence.

A wealth of empirical studies, surveyed in Sarno and Taylor (2002), generally find that estimates of the parameters in both regressions satisfy $\alpha > 0$ and $\beta < 0$, with $\beta$ usually being close to -1. Therefore, the evidence rejects the UIP proposition, and the unbiasedness hypothesis of the forward rate, $f_t^{(1)} \neq E_t[e_{t+1}]$. Interestingly, some of the most influential studies, such as Fama (1984) or Backus, Foresi and Telmer (2001) estimate the regression for short horizons, such as $k = 1$ or 3 months. A common explanation, among others \(^{12}\) is that the rejection of the null $\beta = 1$ reflects the presence of a time-varying risk premium \(^{13}\), or that equivalently, agents are not risk neutral. To see this, decompose the forward premium $(f_t^{(1)} - e_t)$ into the expected depreciation and a risk-premium, as follows

$$f_t^{(1)} - e_t = E_t[\Delta e_{t+1}] + r p_t.$$  

(44)

Since $r p_t$ is imputed into a forward price, it is named a “forward” risk premium. Then, the population slope coefficient in regression (43) can be expressed as

$$\beta = \frac{\text{cov}(f_t^{(1)} - e_t, e_{t+1} - e_t)}{\text{var}(f_t^{(1)} - e_t)} = \frac{\text{cov}(r p_t, \Delta e_{t+1}) + \text{var}(\Delta e_{t+1})}{\text{var}(r p_t + \Delta e_{t+1})}$$

We see that $\beta = 1$ only if $\text{var}(r p_t) = 0$. On the contrary, the finding that $\beta < 0$ implies that $\text{cov}(r p_t, \Delta e_{t+1}) < 0$ and that $\text{var}(r p_t) > \Delta e_{t+1}$. Therefore, this evidence formalizes the notion that foreign exchange risk premiums are time-varying and negatively correlated with subsequent depreciation rates. This constitutes the so called “Forward Premium Anomaly” or the short-run evidence against UIP.

5.1.2 Long-horizon evidence.

Chinn (2006) and Chinn and Meredith (2004), (2005), perform long-horizon regression tests of (42) for maturities greater than 1 year. They cannot reject the null hypothesis $\beta = 1$ for the 5 and 10 year maturities for most of the currencies they study (JPY, DEM, GBP, CAN). Most importantly, they find that the “term structure” of Fama slope coefficients is upward sloping: a plot of $\beta_k$ against time delivers an upward sloped line that starts at values close to -1 and ends at numbers close to 1, for the 10 year horizon. This is the long-horizon evidence in favor of UIP.

5.2 Model implications for UIP and the forward premium anomaly

The model is consistent with both the short-horizon failure of UIP, and an upward sloping term structure of Fama slope coefficients up to the 3-year horizon. Figure 10 plots the model-implied term structure of Fama slope coefficients for several specifications of the model. As can be seen, under the baseline parameterization, the financial friction is the key ingredient

\(^{12}\)Again, see Sarno and Taylor (2002) for a discussion of other approaches to explaining the failure of UIP, most notably irrational expectations, rational learning, peso problems in the data, and information processing.

\(^{13}\)The discussion follows Backus, Foresi and Telmer (2001) closely.
that gives rise to a “forward premium anomaly” at short-horizons, and to an upward-sloping term structure of regression coefficients, consistent with the long-horizon regressions. In Figure 11, the model-implied term structure of UIP coefficients is shown together with estimated coefficients for several currencies, taken from Chinn and Meredith (2004). For maturities up to 12 quarters, the model-implied term structure is close to that for the CAN, DEM, and GBP.

Next, we describe in some detail the mechanisms that generate a forward premium anomaly and an upward sloping term structure of Fama slope coefficients in the theoretical model. Of the 4 structural shocks in the model, only monetary policy and foreign holdings shocks generate deviations from UIP.

5.2.1 Monetary policy shocks

As can be seen on box (b) on Figure 7, when 𝛿 is high a monetary policy shock causes a forward premium anomaly, or short-term deviation from UIP for two reasons: the response of the interest rate differential \( R_t - R^*_t \) is more positive and persistent, while the home currency depreciation is less negative at impact but the effect takes longer time to dissipate. Then, a positive interest rate differential today is associated with a negative home currency depreciation tomorrow, and a regression of \( \Delta \varepsilon_{t+1} \) on \( (R_t - R^*_t) \) will generate a negative slope coefficient.

Compared to the case of low 𝛿, why does increasing the level of financial friction increase the impulse response of the interest rate differential to an \( \varepsilon_{t}^{MP} \) shock, while making more persistent the negative response of \( \Delta \varepsilon_t \)? In order to answer this question, we must examine the effects of the shock on the two variables of interest, and the role the financial market friction plays in the propagation of the shock.

**Interest rate differential.** When 𝛿 is low, the interest rate differential increases, but by a small 5 basis points, evidently because interest rates increase in about the same magnitude on both countries. This is accomplished by a stronger negative subsequent marginal utility growth \( E_t[\lambda_{t+1}] \) that overpowers the effect of decreases in expected inflation \( E_t[\Pi_{t+1}] \). When 𝛿 is high, the interest rate differential increases strongly after the shock. This is because the home interest rate \( R_t \) increases 1.5 times more than in the case of low 𝛿, while the foreign interest rate \( R^*_t \) decreases by a factor of about 0.7. As before, the response of the interest rate can be separated into the effect on marginal utility and inflation. In the home country case, expected inflation \( E_t[\Pi_{t+1}] \) is actually more negative; this would in fact contribute to a smaller response of the nominal rate. Therefore, the strong positive increase in \( R_t \) is almost entirely a consequence of a more negative expected growth in marginal utility, which in turn reflects a more negative response in consumption \( c_t \) to the monetary policy shock. And here is where the level of the financial friction is critical: as can be seen in the top right panel of Figure 9, the high level of financial friction prevents the agent from taking a big negative position in the international bond market. Therefore, consumption must decrease by more, and marginal utility increases at impact, thus causing the subsequent fall in marginal utility growth. Notice that the agent cannot just substitute a negative position in \( b^F_t \) with an even more negative position in \( b_t \) because that would require a market-clearing increase in the bond position of some other agent, either the central bank or the foreign consumer. But as can be
seen in boxes (b) and (d) of Figure 8, central banks increase their real holdings of domestic bonds just enough to satisfy their balance sheets (in real terms); this limits the short position that the agent can take. On the other hand, the foreign consumer is as constrained in the international bond market as the domestic one, so its real holdings of home country bonds $bf_t^*$ do increase, but a little. Unable to short bonds to anyone, the home agent is forced to reduce its consumption. A similar mechanism explains why the consumption of the foreign agent does not decrease below its steady state value as much when $\delta$ is high. Basically, when $\delta$ was low, it was easy to sacrifice consumption to buy real bonds and then finance a persistent level of consumption above real income $a_t^*Y_t^*$ in the future, as the bottom left plot of Figure 9 shows, specially 6 quarters after the shock. When $\delta$ is high, the foreign agent cannot take big positive positions in the bond market to invest for the future, so there is no point in sacrificing consumption today, and therefore his decrease in consumption is smaller. This makes both the impact increase in marginal utility and its subsequent negative change much smaller, mitigating the effect on $R_t^*$.

**Currency depreciation** When $\delta$ is low, the home currency appreciates strongly at impact, but only to return close to its steady state level in the next period and stay at it afterwards; that is, although strong, the policy shock has no persistence on $\Delta e_t$. In fact, on the period after the shock, $\Delta e_{t+1}$ is actually more positive than $(R_t - R_t^*)$ and this relationship is maintained for 7 quarters ahead until it dissipates. Because of this, a regression on $\Delta e_{t+1}$ on $(R_t - R_t^*)$ in fact generates a positive slope coefficient, consistent with UIP.

On the other hand, when $\delta$ is high, the home currency’s appreciation is less strong at impact, but critically, is now very persistent. To analyze the behavior of home currency depreciation $\Delta e_t$ when $\delta$ is high, it is useful to use the identity $\Delta e_t = \Delta q_t + \Pi_t - \Pi_t^*$, which decomposes the depreciation into its components. At impact $\Delta e_t$ decreases less than when $\delta$ is low because there is a smaller real depreciation of the domestic currency, a smaller decrease in domestic CPI inflation, and a smaller increase in foreign CPI inflation. But the key is that after the shock, the home currency depreciation remains below its steady state level for many periods, as the appreciation is now persistent. This is in contrast to the case when $\delta$ is low, where impact appreciation is very strong, but almost disappears after the second period. Now, to understand the behavior of $\Pi_t$, we use the identity $\Pi_t = \Pi_{H,t} - \Delta a_t$, where $\Delta a_t = \ln a_t - \ln a_{t-1}$. Because now the change in the relative price $a_t$ is smaller, the response of $\Pi_t$ mimics that of $\Pi_{H,t}$. In turn, the response of domestic inflation $\Pi_{H,t}$ is virtually insensitive to changes in $\delta$, a result that was to be expected given that the contractionary effects on $Y_t$ must necessarily decrease the real wage $w_t$. Most importantly, the response of $\Pi_{H,t}$ is always very persistent: it does not fade away even after 10 quarters. Therefore, the persistent response in $\Pi_t$ can be explained from the fact that its behavior now mimics that of $\Pi_{H,t}$. Finally, since the shock originates in the other country, the effect on foreign CPI inflation $\Pi_t^*$ depends mainly on $a_t^*$, which responds less strongly when $\delta$ is high, because the real exchange rate $Q_t$ responds less. Overall, it is the persistent negative response of domestic PPI inflation together with a weak impact on relative prices that causes a persistent home currency depreciation rate below its long run level.

These two effects, namely, the persistent positive impulse in the interest rate differential
together with a persistent appreciation of the domestic currency explain a negative slope coefficient in a UIP regressions. As the effect dissipates over time, the relationship between these two variables is less strong and regression coefficients decrease in absolute value, thus explaining the upward sloping term structure of UIP regression coefficients.

5.2.2 Central bank FX intervention shocks

In box (b) of Figure 3, the response of the interest rate differential and the rate of home currency depreciation to an innovation in home central bank holdings of foreign short-term treasury securities is shown. The response of the interest rate differential \(R_t - R_t^*\) is positive, while that of the rate of depreciation \(\Delta e_t\) is negative for all quarters after impact. This means that \(\beta^{(k)}\) in regression (42) will be negative. Therefore, in this model, foreign holdings shock also cause short-term deviations form UIP. This feature of the model is consistent with the results in Mark and Moh’s (2007) modified target-zone model in which surprise central bank interventions cause the forward premium anomaly.

The economic mechanism behind UIP deviations after a foreign holdings shock is similar to what was explained in section 4. Specifically, the interest rate differential is more positive than in the case of low \(\delta\) because a high financial friction forces greater changes in relative prices and (domestic) bond portfolios, which translate into greater changes in aggregate demand and consumption, and hence, in marginal utility. Home currency depreciation is negative and persistent after the shock, mainly because domestic CPI inflation \(\Pi_t\) undershoots its steady state level after experiencing a drastic increase at impact due to the nominal depreciation, while foreign inflation does the opposite. Both rates of inflation then slowly return to their steady state levels, which imparts the necessary persistence to \(\Delta e_t\), as can be seen from the identity: \(\Delta e_t = \Delta q_t + \Pi_t - \Pi_t^*\).

6 Conclusion

This paper shows that when there are adjustment costs in taking positions in the international bond market for the private agents of both countries, exogenous innovations in central bank demand for foreign exchange reserves can cause a significant decrease in the foreign interest rate. They can also cause deviations from UIP. Also, when such adjustment costs are present, the effects of monetary policy shocks on both the foreign exchange and the money market changes compared to the case when such costs are virtually zero. Specifically, when there are costs that prevent flexible adjustment of bond positions to shocks, policy shock can cause deviations from UIP at short-horizons, and change the structure of impulse responses in the model in a way that makes the model consistent with the long-run evidence in favor of UIP.

The central feature that allows the model to be consistent with the evidence on UIP and that predicts significant effects on interest rates from central bank interventions is a cost to take positions in the international bond market. This effectively prevents any representative agent from instantaneously taking extreme positions in response to shocks. When the shock is in the bond market, the effect is mainly on relative prices, and there is a greater relative price change when agents are unable to change their positions. When the shock is via interest rates, the effect is mainly on real allocations and can be mitigated if portfolios are flexible, but when they are not, consumption must respond more. If one agent cannot take extreme
positions, this limits the extent to which the other agent can absorb either a large short-sell or purchase, or mitigate real income falls. Effectively, both agents and central banks are linked through bond market clearing conditions.
References


Figure 1: Impulse response of foreign interest rate for $\delta = 0.0034$ (top row), and $\delta = 10.4$ (bottom row). Left panels are the response of $R_t^*$ to an $\varepsilon_{FX}^t$ shock only. Right panels show the impulse responses to all shocks in the model.
Figure 2: Impulse Responses of model variables to $\varepsilon^{FX}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 3: Impulse Responses of model variables to $\varepsilon_{FX}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 4: Impulse Responses of model variables to $\varepsilon^{FX}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 5: Impulse Responses of components of the Budget Constraint for both agents (home and foreign), to a $\varepsilon^{FX}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 6: Impulse Responses of model variables to $\varepsilon^{MP}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 7: Impulse Responses of model variables to $\varepsilon^{MP}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 8: Impulse Responses of model variables to $\varepsilon^{MP}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 9: Impulse Responses of components of budget constraint to an $\varepsilon_{MP}$ shock, when $\delta = 0.0034$ (left column), and $\delta = 10.4$ (right column).
Figure 10: Model-implied term structure of Fama slope coefficients, different market structures.
Figure 11: Term structures of Fama slope coefficients, for different currencies and model. The data is taken from Chinn and Meredith (2004).