Do Hot Hands Persist Among Hedge Fund Managers?
An Empirical Evaluation*

Ravi Jagannathan† Alexey Malakhov‡ Dmitry Novikov§
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Abstract

In this paper we empirically demonstrate that both hot and cold hands among hedge fund managers tend to persist. While measuring performance, we use statistical model selection methods for identifying style benchmarks for a given hedge fund and allow for the possibility that hedge fund net asset values may be based on stale prices for illiquid assets. We are able to eliminate the backfill bias by deleting all the backfill observations in our dataset. We also take into account the self-selection bias introduced by the fact that both successful and unsuccessful hedge funds stop reporting information to the database provider. The former stop accepting new money and the latter get liquidated. We find statistically as well as economically significant persistence in the performance of funds relative to their style benchmarks. It appears that half of the superior or inferior performance during a three year interval will spill over into the following three year interval.

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†Kellogg School of Management, Northwestern University and the National Bureau of Economic Research. Email: rjaganna@kellogg.northwestern.edu.

‡Kenan-Flagler Business School, The University of North Carolina at Chapel Hill. Email: alexey_malakhov@unc.edu.

§Goldman, Sachs & Co. Email: dmitry.novikov@gs.com.

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1 Introduction

“Investors have made a trillion-dollar bet that hedge funds will bring them rich returns” claims a recent article in The Economist. Indeed, the hedge fund industry doubled in size and in the number of funds between 1998 and 2004, bringing the total assets under management to almost $1 trillion by the end of 2004.¹

While it seems that investment professionals have enthusiastically embraced hedge funds as an investment vehicle, and are especially eager to invest in hedge funds that have exhibited outstanding past returns, there is little consensus in the empirical finance literature as to whether there is performance persistence among hedge funds. Due to the unregulated nature of hedge funds, any rigorous research about hedge fund performance has to overcome numerous biases and irregularities in the available data. In this paper we study the relative performance persistence among hedge fund managers, while correcting for the backfill, serial correlation, and self-selection biases in the data. We calculate a relative performance measure, alpha, for hedge fund managers. Using this measure, we find that it is possible to identify managers with superior skills. We find performance persistence over three year horizons, i.e. that managers with higher estimated alphas in one three year period tend to have higher estimated alphas in the following three year period. We also demonstrate the importance of proper correction for backfill, serial correlation, and self-selection biases in analyzing hedge fund relative performance.

There are no legal requirements for hedge funds to report performance numbers. However, there are several different databases, to which hedge funds provide information about themselves on a voluntarily basis.² Several papers discuss the issues related to hedge fund data, for example Ackerman, McEnally, and Ravenscraft (1999), Liang (2000), Fung and Hsieh (2000) and Fung and Hsieh (2002). An important feature of a hedge fund database is backfill bias - the case when hedge funds bring all the history with them when they join a database. Since only funds with relatively superior historical performance enter a database, when possible backfilling of data is ignored, this procedure introduces a bias toward mistakenly assigning superior ability to managers of funds in their earlier years. Since our HFR data contains the information on when funds actually joined the database, we are able to eliminate the backfill bias by deleting all the backfill observations in our dataset. Moreover, our data is survivorship bias free, since the HFR database retains all hedge funds, including those that ceased to exist. To further illustrate the importance of the backfill bias we compare our persistence results with and without correcting for the backfill bias.

²Among them are MAR, TASS and HFR (we use the HFR database in the paper).
Another issue with hedge fund analysis is that hedge fund returns exhibit substantial serial correlation, a feature that is extensively investigated in Getmansky, Lo, and Makarov (2004) and Okunev and White (2003). They showed that the presence of illiquid assets in hedge fund portfolios are the primary source for the serial correlation. If serial correlation is not accounted for properly, the manager’s performance measure will be biased.\textsuperscript{3} Notice that when hedge fund returns exhibit serial correlation due to the presence of illiquid assets in the portfolio, benchmark style index factor returns will also exhibit such serial correlation. We assume that unobserved “true” returns on assets are serially uncorrelated, and identify them using the MA2 approach suggested by Getmansky, Lo, and Makarov (2004).

Finally, some hedge funds stop reporting to the database before the end of the sample period used in the study.\textsuperscript{4} Therefore, estimating performance persistence by regressing future alpha on past alpha would produce a biased estimate of alpha persistence. Further, we do not observe hedge fund alphas and have to estimate them. Ignoring the measurement error in estimated alphas would also lead to biased estimate of the persistence in alphas. There is no consensus in the literature on the terminology for this bias,\textsuperscript{5} and we refer to it as a self-selection bias.

We evaluate hedge fund performance persistence by comparing the alphas over consecutive nonoverlapping three year intervals. This is a fairly long time period relative to the time periods examined in the literature reviewed in the following section. Considering a three-year period allows us to accurately capture relative alphas for individual funds, and it also provides us with a better sense of investor returns considering a lockup period. Lockup periods vary among different funds, but a three-year period in not unusual.\textsuperscript{6} We use a model that addresses measurement errors and self-selection bias simultaneously. In our database, the reason a hedge fund stopped reporting is recorded for some funds but not for all funds. We assume that hedge funds that stop reporting but do not give a reason are drawn from the same distribution as funds that continue to report or stop reporting but tell us why. We assume that hedge funds that are liquidated are more likely to be ones with low past performance and those that are closed are more likely to be ones with high past performance. With these assumptions, which we show are reasonable, we develop a novel GMM estimation method that estimates all parameters in the model simultaneously and produces a consistent

\textsuperscript{3}For example, Asness, Krail, and Liew (2001) showed that style index alphas tend to be lower after controlling for serial correlations.

\textsuperscript{4}Notice that the fact of nonreporting to a database does not mean fund liquidation. For example, a fund may stop reporting after it has been closed for new investors. Such a hedge fund will continue to manage funds of current investors.

\textsuperscript{5}For example, Baquero, Ter Horst, and Verbeek (2005) refer to it as a “look-ahead” bias.

\textsuperscript{6}For example, in 1996, LTCM allowed to withdraw one third of investor’s capital in years 2, 3, and 4 (Perold (1999)).
estimate of performance persistence. Our approach is also consistent with the observation in Brown, Goetzmann, and Park (2001) and Liang (2000) that hedge funds with low past performance are primary candidates for liquidation.

We find that relative performance tends to persist among hedge fund managers. The performance persistence parameter is about 56-57%, i.e., a hedge fund that outperformed its benchmark by 100 basis points in the past will on average continue to outperform its benchmark by 56 to 57 basis points in the future. In comparison, a simple regression of future alphas on past alphas gives a downward biased and statistically insignificant estimate of only 6.5-8.5% for alpha persistence.

This rest of the paper is organized as follows. The next section provides a connection to the existing hedge fund performance persistence literature. Section 3 describes the methodology for empirical testing. The model of hedge fund performance is introduced, factor selection, return smoothing and self-selection bias issues are discussed there. Tests for performance persistence are also explained. Section 4 contains data description, along with estimation of hedge fund performance persistence. We discuss the importance of the backfill correction in Section 5. Section 6 concludes.

2 Related Literature

There are several papers in the literature that examine hedge fund managers’ performance persistence. Brown, Goetzmann, and Ibbotson (1999) estimated the offshore hedge fund performance using raw returns, risk adjusted returns using the CAPM, and excess returns over self reported style benchmarks. They found little persistence in relative performance across managers. On the contrary, Agarwal and Naik (2000a) and Agarwal and Naik (2000b) when using both offshore and onshore hedge funds found significant quarterly persistence - that is hedge funds with relatively high returns in the current quarter tend to earn relatively high returns in the next quarter. They used the return on a hedge fund in excess of the average return earned by all funds that follow the same strategy as a measure of performance.\footnote{They also examined the standardized measure of performance, i.e., the excess return dividend by its standard deviation.} They used both parametric and nonparametric tests for performance persistence. In their case the persistence was driven mostly by “losers”. Edwards and Caglayan (2001) considered an eight-factor model to evaluate hedge fund performance. They found the evidence of performance persistence over one and two year horizons. They also showed that the persistence holds among both “winners” and “losers”.

More recently, Bares, Gibson, and Gyger (2003) applied a non-parametric approach
to individual funds, as well as an eight-factor APT model to fund portfolios with a conclusion of performance persistence only over one to three month horizons. Capocci and Hübner (2004) followed the methodology of Carhart (1997), discovering no evidence of performance persistence for best and worst performing funds, but providing limited evidence of persistence for middle decile funds. Boyson and Cooper (2004) have found no evidence of performance persistence if only common risk and style factors are used in estimation, but discovered quarterly persistence when manager tenure was taken into consideration. Baquero, Ter Horst, and Verbeek (2005) concentrated on accounting for the self-selection look-ahead bias in evaluating hedge fund performance. Comparing raw and style-adjusted performance of performance-ranked portfolios they found evidence of positive persistence at the quarterly level. Finally, Kosowski, Naik, and Teo (2005) used a seven-factor model and applied a bootstrap procedure, as well as Bayesian measures to estimate hedge fund performance. Considering performance-ranked portfolios they found evidence of performance persistence over a one year horizon.

This paper contributes to the above literature in three ways. First, we develop a novel GMM procedure that deals with measurement errors and the self-selection bias simultaneously. Second, to our knowledge, this paper is first to account for all three major biases in hedge fund data, i.e. backfill, serial correlation, and self-selection biases. Third, we present evidence of hedge fund managers’ performance persistence over three year horizons.

*Accounting for these three biases in the literature is summarized in Table 1.
3 Econometric Methodology

In this section we describe the estimation of hedge fund performance and then we propose a method to check for performance persistence.

3.1 Modeling the Relative Performance of a Hedge Fund

Hedge fund returns have several distinctive features. This can make the analysis of hedge funds’ performance different from the analysis of performance of other assets like stocks and mutual funds.

First, hedge funds are not required to reveal their financial information including their returns. This raises a question about the selectivity of returns in hedge fund databases. We should take into account possible reasons for a hedge fund to reveal its performance information. One possible explanation is that some hedge funds need to raise funds. Reporting their returns could be a way to advertise themselves. This implies that we will probably not find the most and the least successful hedge funds in the database. The most successful funds most likely have enough clients without any additional promotions. The least successful funds probably would not reveal their information to a broad set of investors.

Second, hedge fund strategies produce returns that cannot be well explained by standard factors, and they also exhibit option-like features. The usual way to estimate the performance in such a case is to include options on factors in addition to these factors, following the suggestion made by Glosten and Jagannathan (1994).

Third, hedge funds often hold illiquid securities in their portfolios. Usually, it is difficult to obtain current prices for such securities. In this case, managers use past prices to estimate the current value of assets. Therefore, we may observe serial correlation in returns. If we completely ignore this issue, then we will get inconsistent estimates of hedge fund performance. Scholes and Williams (1977) proposed a simple way to account for stale prices. They used lags of factors along with factors in estimating the asset performance. These lags control for the serial correlation in returns. Asness, Krail, and Liew (2001) using this technique showed that the performance of indices may not be as attractive as it appears from a regular regression without including any lags. Lo (2002) showed that annualized Sharpe ratios can be significantly overstated if the serial correlation in returns is not taken into account.

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9 According to SEC regulation 13F institutional investors with assets under management more than $100M are supposed to reveal their long position holdings on quarterly basis.

10 See Fung and Hsieh (1997).


12 In the case of Hedge Fund Research style indices.

Fourth, the history of hedge funds is relatively short. Even for long-livers the reliable data in most cases does not exceed ten years. This creates a problem in analyzing hedge fund risks. The hedge fund return history may simply be too short for a high risk (low probability) event to happen. Weisman (2002) explains several simple strategies\textsuperscript{13} that can be successful for a relatively long period of time (several years), but finally lead to bankruptcy. Those strategies will not be correlated with systematic factors. Pastor and Stambaugh (2002b), Pastor and Stambaugh (2002a), and Ben Dor, Jagannathan, and Meier (2003) developed techniques for dealing with short histories. Ben Dor, Jagannathan, and Meier (2003) used two stage regressions; Pastor and Stambaugh (2002b) and Pastor and Stambaugh (2002a) used Bayesian analysis. Kosowski, Naik, and Teo (2005) applied Bayesian technique to the hedge fund performance analysis.

Finally, the life of hedge funds can be pretty short. Hedge funds can be liquidated or closed for new investments. Even if a database is survivorship bias free (that is, it stores all the liquidated and closed funds), there is the issue of how these hedge funds should be taken into account when analyzing performance persistence.

While analyzing the performance of hedge funds and performance persistence, we will try to control for the above features of hedge fund returns. We follow Getmansky, Lo, and Makarov (2004) in designing an appropriate model for the estimation of hedge fund performance.

Let the true equilibrium (unobserved) returns follow:

\[
R_{i,t}^{un} - r_{f,t} = \alpha_i + X_t \beta_i + \varepsilon_{i,t}
\]

(1)

where \(X_t\) is the vector of excess returns on factor portfolios \((T \times l)\), \(\varepsilon_{it}\) are i.i.d. We define \(\alpha_i\) as the performance of the hedge funds. We assume that the observed returns (as reported by the hedge fund managers) are smoothed. Hence we observe the following returns

\[
R_{i,t} = \theta_0^i R_{i,t}^{un} + \ldots + \theta_s^i R_{i,t-s}^{un}
\]

(Note, \(s\) may be different for different hedge funds). For identification purposes we will use

\textsuperscript{13}Consider for example a strategy from St. Petersburg Paradox. You place one dollar on a coin to be tossed heads. If you lose, then you double your bets (if you do not have your own capital then you have to borrow). If you play long enough, then with probability one you will face a borrowing constraint.
the following normalization on the parameters:

\[ \theta_i^0 = 1 \text{ for any } i \]

Combining with equation (1) we can write the observed returns as follows:

\[ R_{i,t} - r_{f,t} = \tilde{\alpha}_i + X_t\theta_i^0\beta_i + \ldots + X_{t-s}\theta_s^i\beta_i + u_{i,t} \]  

(2)

where

\[ \tilde{\alpha}_i = \alpha_i(\theta_0^i + \ldots + \theta_s^i) - r_{f,t} + \theta_0^i r_{f,t} + \ldots + \theta_s^i r_{f,t-s} \]  

(3)

\[ u_{i,t} = \theta_0^i \varepsilon_{i,t} + \ldots + \theta_s^i \varepsilon_{i,t-s} \]  

(4)

As we see from (4), the error term \( u_{i,t} \) follows an \( MA(s) \) process. Notice that \( \tilde{\alpha}_i \) in (2) is misspecified, since it contains a time-dependent variable \( r_{f,t} \). However, we argue that this misspecification is not critical for the following reasons. First, if we follow the specification from (3) and (4) exactly, we would need to add \( s \) additional factors (\( r_{f,t}, \ldots, r_{f,t-s} \)) to the model (2), plus an additional constraint that their regression coefficients must coincide with \( MA(s) \) coefficients from (4). This may result in an overly specified model. Second, there is not much variation in \( r_{f,t} \) compared to other variables, and it would be reasonable to approximate \( r_{f,t} \) by the average risk-free rate \( r_f \) over the estimation period. Approximating \( r_{f,t} \) by the average risk-free rate \( r_f \) over the estimation period then yields

\[ \tilde{\alpha}_i = \alpha_i(\theta_0^i + \ldots + \theta_s^i) + r_f((\theta_0^i + \ldots + \theta_s^i) - 1) \]  

(5)

Then the true relative performance alpha is approximated by

\[ \alpha_i = \frac{\tilde{\alpha}_i - r_f((\theta_0^i + \ldots + \theta_s^i) - 1)}{(\theta_0^i + \ldots + \theta_s^i)} \]  

(6)

The next step is to choose appropriate factors for the model given by (2), (4), and (5).

### 3.2 Factor Selection

While selecting factors we control for the following criteria:

1) The number of factors should be relatively small as we do not have a long time series of observations on hedge fund returns. This also avoids overparametrization.

2) Factors should reflect the non-linear (option-like) strategies used by hedge funds.

Given this, we choose the following three factors.
Therefore, $X_t' = [R_{mkt}^{r,t} - r_{f,t}, I_{J,\text{self}}^{J,\text{self}} - r_{f,t}, I_{K,\text{aux}}^{K,\text{aux}} - r_{f,t}]$. We use only one factor (excess return on the market portfolio) from the Fama-French three-factor model (Fama and French (1993)), as the other two factors SMB and HML do not add explanatory power to our regressions (this fact can be established by using the Schwarz’s Bayesian criterion (SBC)).

The other factors are style indices. Style indices are defined as an equally weighted average of returns for all hedge funds with the same strategy. The hedge funds themselves provide information about strategies they use. The list of strategies\textsuperscript{14} defined in the database can be found in table 2.

Style indices are good proxies for non-linear strategies of hedge funds, however there are problems with self reported styles. For all hedge funds in the database we can find the styles that were reported by hedge funds themselves. However, hedge funds may change their styles over time, and this may not be reflected in the database. We observe only one style per hedge fund and we do not know if a hedge fund has been using this style lately or some time ago (it may depend on the willingness of a hedge fund to report any changes in its style). To account for this “unpleasant” feature, we are going to add one more style index in addition to the self reported index to try to capture changes in hedge fund styles. This additional style index can be chosen by SBC (details are provided in the next subsection).

The second problem is with style indices as factors. We know that the reported hedge fund returns are smoothed. By definition, a style index is the (equally weighted) average of returns for all hedge funds with the same self-reported strategy. Therefore, we should expect style indices to display serial correlations (or be “smoothed”) as well. To deal with this problem, we consider the following model of “smoothed” indices (again we follow here Getmansky, Lo, and Makarov (2004)):

$$I_t^J = \gamma_0^J \eta_t^J + \ldots + \gamma_l^J \eta_{t-l}$$ \hfill (7)

where $\eta_t^J$ represents the unobservable “true” factor $J$ at time $t$. Let us assume that $\eta_t^J \sim N(\mu_J, \sigma_J^2)$. Equation (7) is a moving average process of order $l$. To identify this process, as before we assume $\gamma_0^J + \ldots + \gamma_l^J = 1$. From equation (7) we see that $I_t^J$ follow an $MA(l)$. Hence, the true factors $\eta_t^J$ can be estimated from (7) by maximum likelihood. For this

\textsuperscript{14}For the official definition of self reported index, please refer to the web page of Hedge Fund Research at http://www.hedgefundresearch.com/pdf/HFR_Strategy_Deadlines.pdf.
<table>
<thead>
<tr>
<th>#</th>
<th>HFR Strategy Style Index</th>
<th>#</th>
<th>HFR Strategy Style Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Convertible Arbitrage</td>
<td>17</td>
<td>Fund of Funds: Conservative</td>
</tr>
<tr>
<td>2</td>
<td>Distressed Securities</td>
<td>18</td>
<td>Fund of Funds: Diversified</td>
</tr>
<tr>
<td>3</td>
<td>Emerging Markets: Asia</td>
<td>19</td>
<td>Fund of Funds: Market Defensive</td>
</tr>
<tr>
<td>4</td>
<td>Emerging Markets: E. Europe/CIS</td>
<td>20</td>
<td>Fund of Funds: Strategic</td>
</tr>
<tr>
<td>5</td>
<td>Emerging Markets: Global</td>
<td>21</td>
<td>Macro</td>
</tr>
<tr>
<td>6</td>
<td>Emerging Markets: Latin America</td>
<td>22</td>
<td>Market Timing</td>
</tr>
<tr>
<td>7</td>
<td>Equity Hedge</td>
<td>23</td>
<td>Merger Arbitrage</td>
</tr>
<tr>
<td>8</td>
<td>Equity Market Neutral</td>
<td>24</td>
<td>Regulation D</td>
</tr>
<tr>
<td>9</td>
<td>Equity Non-Hedge</td>
<td>25</td>
<td>Relative Value Arbitrage</td>
</tr>
<tr>
<td>10</td>
<td>Event-Driven</td>
<td>26</td>
<td>Sector: Energy</td>
</tr>
<tr>
<td>11</td>
<td>Fixed Income: Arbitrage</td>
<td>27</td>
<td>Sector: Financial</td>
</tr>
<tr>
<td>12</td>
<td>Fixed Income: Convertible Bonds</td>
<td>28</td>
<td>Sector: Health Care/Biotechnology</td>
</tr>
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<td>13</td>
<td>Fixed Income: Diversified</td>
<td>29</td>
<td>Sector: Miscellaneous</td>
</tr>
<tr>
<td>14</td>
<td>Fixed Income: High Yield</td>
<td>30</td>
<td>Sector: Real Estate</td>
</tr>
<tr>
<td>15</td>
<td>Fixed Income: Mortgage-Backed</td>
<td>31</td>
<td>Sector: Technology</td>
</tr>
<tr>
<td>16</td>
<td>Fund of Funds (Total)</td>
<td>32</td>
<td>Short Selling</td>
</tr>
</tbody>
</table>

Table 2: Style indices in Hedge Fund Research database.

estimation we set \( l = 2 \) (i.e. we assume that indices are smoothed up to two lags\(^{15}\)). We will use \( \eta_t^l - r_{f,t} \) as factors in (2).

The autocorrelations of orders from 1 to 12 for the original database indices \( I_t^l \) are presented in figure 1. We can see that several indices have significant\(^{16}\) first and second order autocorrelation. The examples of such strategies are “convertible arbitrage”, “distressed securities”, “emerging markets”, etc. These strategies involve heavy trading in illiquid securities. Figure 2 displays the autocorrelations of orders from 1 to 12 for unsmoothed indices \( \eta_t^l \). None of the unsmoothed indices \( \eta_t^l \) has statistically significant autocorrelations, and their autocorrelations are substantially smaller than corresponding autocorrelations in figure 1.

3.3 Estimation procedure

In order to check for performance persistence we have to have at least two periods with performance estimates, see figure 3. For every period, we run the following regression based

\(^{15}\)Getmansky, Lo, and Makarov (2004) use two lags to estimate the smooth model of hedge fund returns.

\(^{16}\)At the a 5% significance level.
Figure 1: The autocorrelation functions for style indices are presented in this figure. The style indices used are *before* the adjustment for smoothing (i.e. as they were presented in the original database). The autocorrelations were computed for lags from 1 to 12. The thin horizontal lines around the horizontal axes represent 95% confidence intervals. Style index names can be retrieved from table 2. For example, index #1 stands for Convertible Arbitrage index.
Figure 2: The autocorrelation functions for style indices are presented in this figure. The style indices used are after the adjustment for smoothing (\( \eta_t \) from (7)). The autocorrelations were computed for lags from 1 to 12. The thin horizontal lines around the horizontal axes represent 95% confidence intervals. Style index names can be retrieved from table 2. For example, index #1 stands for Convertible Arbitrage index.
Figure 3: This diagram shows the timeline for the estimation of hedge fund alphas. In this paper $k$ is equal to 36 months. That is evaluation and prediction periods are 3 years. The hypotheses is tested if alphas from the evaluation period can explain alphas from the prediction period.

$$R_{i,t} - r_{f,t} = \bar{\alpha}_z + X_t\delta_{0,i} + ... + X_{t-s}\delta_{s,i} + u_{i,t}$$ (8)

$$u_{i,t} = \theta_{0}^{i}\varepsilon_{i,t} + ... + \theta_{s}^{i}\varepsilon_{i,t-s}$$ (9)

where $z$ is either 0 or 1, depending on if $T \leq t < T + k$ or $T + k \leq t < T + 2k$; $X_t$ is the vector of factors described in the previous subsection. We find hedge fund performance $\alpha_{zi}$ following (6), i.e.

$$\alpha_{zi} = \frac{\bar{\alpha}_z - r_f((\theta_{0}^{i} + ... + \theta_{s}^{i}) - 1)}{(\theta_{0}^{i} + ... + \theta_{s}^{i})}$$ (10)

We estimate the alphas by Maximum Likelihood. We also take into account the fact that the error term $u_{i,t}$ follows moving average process of order $s$. As a result of the maximum likelihood estimation procedure, we obtain consistent and asymptotically efficient estimators.

For every hedge fund we have to determine how many lags $s$ to include and which additional indices are to be used in (8). We use Schwarz’s Bayesian Criterion (Schwarz (1978)) to select the best model:

$$SBC = -2\log (L) + l \times \log (n)$$

where $L$ is the likelihood function, $l$ is the number of parameters and $n$ is the number of observations. Given a hedge fund, we estimate several models like (8) that will be different in the number of lags and additional style indices. We then pick the model with the highest value of the Schwarz’s Bayesian Criterion. For different hedge funds we may have different
number of lags in regression (8) and different additional indices.

We use primary and additional style indices as factors in estimation of hedge fund performance. Therefore, we look at the relative performance of hedge funds with respect to hedge funds that follow similar investment strategies. We do not compare hedge funds to other asset classes. Therefore, a negative alpha for a hedge fund does not indicate that this hedge fund has poor return performance. It only means that the performance of the hedge fund is worse than the performance of an average hedge fund following a similar investment strategy. However, the return for this hedge fund can be larger than that of the S&P500 for example. Vice versa, hedge funds with positive alphas perform better than average funds with similar investment strategies. Their returns however, may be lower than the return on the market portfolio.

3.4 Testing Hedge Fund Performance Persistence

Here we provide an econometric framework for testing a hypothesis of performance persistence.

3.4.1 Simple (Naive) Regressions

Suppose we have obtained the hedge fund alphas for two periods $\alpha_{0i}$ and $\alpha_{1i}$. Then we can run a simple regression

$$\alpha_{1i} = a + b\alpha_{0i} + \varepsilon_{i}$$

(11)

The persistence would mean that the slope coefficient $b$ is statistically different from zero. However, a statistically insignificant slope coefficient would not necessarily mean the absence of persistence. That is because the slope estimate can be biased toward zero due to measurement errors and self-selection. In the next subsection we consider a model that incorporates both of these features.

3.4.2 Self-Selection Bias and Measurement Errors

While estimating alphas in the prediction period, one can notice that some hedge funds, which were available in the evaluation period, disappeared from the database. A hedge fund can be liquidated or closed.$^{17}$ Closed funds typically stop reporting to the database, since they do not need to attract any additional investments. In the HFR database, hedge funds that opt out of the database may indicate the reason (liquidated fund or closed for new investments fund). For some hedge funds this information is missing.

$^{17}$A hedge fund is called closed if it is closed for new investors. It continues to manage capital of its current investors.
We build the following model. Suppose that the hedge fund performance is measured by alphas: \( \alpha_{0i} \) - alpha in the evaluation period and \( \alpha_{1i} \) - alpha in the prediction period. We can observe \( \alpha_{0i} \) for all funds in our sample during the evaluation period, but we can only observe \( \alpha_{1i} \) for funds that were not liquidated or closed during the prediction period. We can also observe whether a hedge fund was liquidated or closed for new investments. We model the above pattern in hedge funds’ performance and reporting as follows:

\[
\begin{align*}
\alpha_{1i}^* &= a + b\alpha_{0i}^* + \varepsilon_i \\
\alpha_{0i} &= \alpha_{0i}^* + u_i \\
\alpha_{1i} &= \begin{cases} 
\text{liquidated,} & \text{with probability } p_0(\alpha_{0i}^*) \\
\alpha_{1i}^*, & \text{with probability } p_1(\alpha_{0i}^*) \\
\text{closed} & \text{with probability } p_2(\alpha_{0i}^*)
\end{cases}
\end{align*}
\]

where \( p_0(\alpha_{0i}^*) + p_1(\alpha_{0i}^*) + p_2(\alpha_{0i}^*) = 1 \).

This model implies that we observe noisy\(^{18}\) variables of hedge fund performance, however the decision on hedge fund liquidation, or closing is based on the “true” \( \alpha_{0i}^* \) measure of performance.

The noise in this model follows

\[
\begin{align*}
\varepsilon_i &\sim N(0, \sigma^2_{\varepsilon}) \\
u_i &\sim N(0, \sigma^2_u)
\end{align*}
\]

and these random variables are independent.

We assume that hedge fund alphas are normally distributed as well.

\[ \alpha_{0i}^* \sim N(\mu_\alpha, \sigma^2_{\alpha^*}) \]

and

\[ \alpha_{0i} \sim N(\mu_\alpha, \sigma^2_\alpha) \]

One can easily establish the relationship between the variance of \( \alpha_{0i}^* \) and \( \alpha_{0i} \):

\[ \sigma^2_\alpha = \sigma^2_{\alpha^*} + \sigma^2_u \] \hspace{1cm} (12)

For notational convenience, we consider \( \sigma_{\alpha^*} \) as an unknown parameter, which is to be estimated (instead of \( \sigma_u \)), then \( \sigma_u \) can be easily found from (12).

\(^{18}\)The measurement error can be attributed for example to the incomplete set of factors in the performance estimation regression.
In the following theorem we prove that the model parameters are identified for the particular case of probability functions \( p_z(\alpha_{0i}^*) \), \( z = 0, 1, 2 \). Assume that if the “true” alpha \( \alpha_{0i}^* \) is less than some threshold \( \gamma_0 \) then hedge fund \( i \) will be liquidated, if \( \alpha_{0i}^* \) is larger than some other threshold \( \gamma_1 \) then hedge fund \( i \) will be closed.

**Theorem 1** Suppose that the following conditions are satisfied:

\[
p_0(\alpha_{0i}^*) = \begin{cases} 
0, & \text{if } \alpha_{0i}^* \geq \gamma_0 \\
1, & \text{if } \alpha_{0i}^* < \gamma_0 
\end{cases}
\]

\[
p_2(\alpha_{0i}^*) = \begin{cases} 
0, & \text{if } \alpha_{0i}^* < \gamma_1 \\
1, & \text{if } \alpha_{0i}^* \geq \gamma_1 
\end{cases}
\]

Then all the parameters \( P = (a, b, \gamma_0, \gamma_1, \sigma_z, \sigma_{\alpha^*}) \) in model (M) are identified

**Proof.** To establish identification, first let us look at the expected value of the depended variable.

\[
\mu_1 = E(I\{\alpha_{1i} = \text{liquidated}\} | P) = E(I\{\alpha_{0i}^* < \gamma_0\} | P)
\]

\[
= \Pr\{\alpha_{0i} - u_i < \gamma_0\} = \Pr\{u_i > \alpha_{0i} - \gamma_0\}
\]

\[
= \Phi\left(\frac{\gamma_0 - \alpha_{0i}}{\sigma_u}\right) = \Phi\left(\frac{\gamma_0 - \alpha_{0i}}{\sqrt{\sigma_{\alpha}^2 - \sigma_{\alpha^*}^2}}\right)
\]

where \( I\{\cdot\} \) is the indicator function and \( \Phi(\cdot) \) is the c.d.f of the standard normal random variable.

\[
\mu_2 = E(I\{\alpha_{1i} = \text{closed}\} | P) = E(I\{\alpha_{0i}^* \geq \gamma_1\} | P)
\]

\[
= \Pr\{\alpha_{0i} - u_i \geq \gamma_1\} = \Pr\{u_i \leq \alpha_{0i} - \gamma_1\}
\]

\[
= \Phi\left(\frac{\alpha_{0i} - \gamma_1}{\sigma_u}\right) = \Phi\left(\frac{\alpha_{0i} - \gamma_1}{\sqrt{\sigma_{\alpha}^2 - \sigma_{\alpha^*}^2}}\right)
\]

where \( \phi(\cdot) \) is the p.d.f of the standard normal random variable.

Next, we compute the probability of an incorrect prediction due to measurement errors (i.e. prediction mistake). For example, suppose we observe a hedge fund alpha which is
below the threshold $\gamma_0$, however this fund was not liquidated.

$$\Pr(\text{prediction mistake}) = \Pr \left( \begin{array}{c} \alpha^*_{0i} \geq \gamma_0 \\ \alpha_{0i} < \gamma_0 \end{array} \right)$$

$$= \left\{ \begin{array}{ccc} \alpha^*_{0i} \leq \gamma_0 & \text{or} & \alpha^*_{0i} < \gamma_1 \\
\alpha_{0i} \geq \gamma_0 & \text{or} & \alpha_{0i} < \gamma_1 \\
\alpha_{0i} \geq \gamma_1 & \text{or} & \alpha_{0i} \leq \gamma_1 \end{array} \right.$$  \hspace{1cm} (16)

$$= \int_{S} \phi(x, y, \bar{\mu}, \Sigma) \, dx \, dy$$ \hspace{1cm} (17)

where the integral is taken over the two-dimensional region (shaded area in figure 4), and $\phi(\cdot, \cdot, \bar{\mu}, \Sigma)$ is the density of the bivariate normal distribution with known mean vector $\bar{\mu} = (\mu_\alpha, \mu_\alpha)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} \sigma^2_{\alpha^*} & \sigma^2_{\alpha^*} \\ \sigma^2_{\alpha^*} & \sigma^2_{\alpha} \end{pmatrix}$$

The above equations allow us to identify the two thresholds $\gamma_0, \gamma_1$ and the standard deviation of the measurement error $\sigma_u$. In fact, from (14) and (15) we can find $\gamma_0$ and $\gamma_1$ as functions of $\sigma_u$. When we know the probability of incorrect prediction regarding whether a fund will be liquidated, then using (16) we can get the value of $\sigma_u$.

To identify the other parameters $a, b$ and $\sigma_\varepsilon$, we can look at the following relationships:

$$E(\alpha_{1i}|\alpha_{1i} \text{ is observable}, P) = E(\alpha_{1i}|\gamma_0 \leq \alpha^*_{0i} < \gamma_1, P)$$

$$= E \left( a + b(\alpha_{0i} - u_i) + \varepsilon_i | \alpha_{0i} - u_i < \gamma_1, P \right)$$

$$= E \left( a + b \left( \alpha_{0i} - \gamma_1 \right) - u_i \right) \left( \alpha_{0i} - u_i < \gamma_1, P \right)$$

$$= a + b \left[ \alpha_{0i} - \sigma_u \times g_1(\frac{\alpha_{0i} - \gamma_1}{\sigma_u}, \frac{\alpha_{0i} - \gamma_0}{\sigma_u}) \right]$$

$$Var(\alpha_{1i}|\alpha_{1i} \text{ is observable}, P) = Var \left( a + b(\alpha_{0i} - u_i) + \varepsilon_i | \gamma_0 \leq \alpha_{0i} - u_i < \gamma_1, P \right)$$

$$= \sigma^2_\varepsilon + \sigma^2_u + b^2 \sigma^2_u \times g_2 \left( \frac{\alpha_{0i} - \gamma_1}{\sigma_u}, \frac{\alpha_{0i} - \gamma_0}{\sigma_u} \right)$$ \hspace{1cm} (19)

where functions $g_1(\cdot, \cdot)$ and $g_2(\cdot, \cdot)$ are defined in Lemma 2.

Now we can see that the slope ($b$) and the intercept ($a$) can be found from (18) and the variance $\sigma^2_\varepsilon$ can be found from (19).

**Lemma 2** Suppose that $z$ is a random variable with standard normal distribution. Let us

$$\text{cov}(\alpha_{0i}^*, \alpha_{0i}) = \text{cov}(\alpha_{0i}^*, \alpha_{0i}^* + u_i) = \text{cov}(\alpha_{0i}^*, \alpha_{0i}) = \sigma^2_\alpha.$$
Figure 4: The shaded region in this graph indicates the event of incorrect prediction due to measurement error. For example, the true alpha ($\alpha^*_0$) may be less than the threshold $\gamma_0$ (hedge fund $i$ was liquidated) but we observe $\alpha_{0i}$ greater than $\gamma_0$. This example corresponds to some point at the upper left corner of the graph.
define functions \( g_1 \) and \( g_2 \) as follows

\[
\begin{align*}
g_1 (c_1, c_2) &= E [z|c_1 \leq z < c_2] \\
g_2 (c_1, c_2) &= Var [z|c_1 \leq z < c_2]
\end{align*}
\]

The expressions for \( g_1 \) and \( g_2 \) are given by

\[
\begin{align*}
g_1 (c_1, c_2) &= \frac{\phi (-c_1) - \phi (-c_2)}{\Phi (-c_1) - \Phi (-c_2)} \\
g_2 (c_1, c_2) &= \frac{\Phi (c_2) - c_2 \phi (c_2) - (\Phi (c_1) - c_1 \phi (c_1))}{\Phi (c_2) - \Phi (c_1)} - [g_1 (c_1, c_2)]^2
\end{align*}
\]

Proof. Let \( z \) be the standard normal random variable. Then,

\[
\begin{align*}
E (z|c_1 < z < c_2) &= \int_{c_1}^{c_2} z \phi (z) \, dz \\
&= \frac{\int_{c_1}^{c_2} z \phi (z) \, dz}{\Phi (c_2) - \Phi (c_1)} \\
&= \frac{\phi (c_1) - \phi (c_2)}{\Phi (c_2) - \Phi (c_1)} = \frac{\phi (-c_1) - \phi (-c_2)}{\Phi (-c_1) - \Phi (-c_1)}
\end{align*}
\]

\[
\begin{align*}
E (z^2|c_1 < z < c_2) &= \int_{c_1}^{c_2} z^2 \phi (z) \, dx \\
&= \frac{\int_{c_1}^{c_2} z^2 \phi (z) \, dx}{\Phi (c_2) - \Phi (c_1)} \\
&= \frac{\Phi (c_2) - c_2 \phi (c_2) - (\Phi (c_1) - c_1 \phi (c_1))}{\Phi (c_2) - \Phi (c_1)}
\end{align*}
\]

Hence, by definition

\[
\begin{align*}
g_1 (c_1, c_2) &= E (z|c_1 < z < c_2) \\
g_2 (c_1, c_2) &= E (z^2|c_1 < z < c_2) - [E (z|c_1 < z < c_2)]^2
\end{align*}
\]
3.4.3 Estimation

The proof of the identification theorem leads us to the GMM estimation of the parameters of the model (M). Moment conditions for estimating model (M) parameters are presented below.

1) Probability of liquidation

\[ \Pr (\alpha^*_0 < \gamma_0) = \Phi \left( \frac{\gamma_0 - \mu}{\sigma^*_\alpha} \right) \]  \hspace{1cm} (20)

2) Probability of closing

\[ \Pr (\alpha^*_0 \geq \gamma_1) = \Phi \left( \frac{\mu - \gamma_1}{\sigma^*_\alpha} \right) \]  \hspace{1cm} (21)

3) Probability of incorrect prediction (for example a hedge fund was liquidated but its alpha \( \alpha_0 \) was above the threshold)

\[ \Pr \left( \begin{array}{c} \alpha^*_0 < \gamma_0 \\
\alpha_0 \geq \gamma_0 \\
\alpha_0 < \gamma_0 \\
\alpha_0 \geq \gamma_1 \\
\alpha_0 < \gamma_1 \end{array} \right) = \int \int \phi(x,y,\mu,\Sigma) \, dx \, dy \]  \hspace{1cm} (22)

4) Expected value of \( \alpha_1i \)

\[ E(\alpha_{1i}|\alpha_{1i} \text{ is observable}) = E(\alpha^*_{1i}|\gamma_0 \leq \alpha^*_0 < \gamma_1) = E(a + b\alpha^*_0 + \varepsilon_i|\gamma_0 \leq \alpha^*_0 < \gamma_1) = a + bE(\alpha^*_0|\gamma_0 \leq \alpha^*_0 < \gamma_1) = a + b\sigma^*_\alpha \times g_1 \left( \frac{\gamma_0 - \mu}{\sigma^*_\alpha}, \frac{\gamma_1 - \mu}{\sigma^*_\alpha} \right) \]  \hspace{1cm} (23)

5) Variance of \( \alpha_{1i} \)

\[ Var(\alpha_{1i}|\alpha_{1i} \text{ is observable}) = Var(a + b\alpha^*_0 + \varepsilon_i|\gamma_0 \leq \alpha^*_0 < \gamma_1) = \sigma^2_\varepsilon + b^2\sigma^2_{\alpha^*} \times g_2 \left( \frac{\gamma_0 - \mu}{\sigma^*_\alpha}, \frac{\gamma_1 - \mu}{\sigma^*_\alpha} \right) \]  \hspace{1cm} (24)
6) Covariance between $\alpha_{1i}$ and $\alpha_{0i}$

\[
\text{cov} (\alpha_{1i}, \alpha_{0i} | \alpha_{1i} \text{ is observable}) = \text{cov} (a + b\alpha_{0i}^* + \epsilon_i, \alpha_{0i}^* + u_i | \gamma_0 \leq \alpha_{0i}^* < \gamma_1)
\]

\[
= b\text{Var} (\alpha_{0i}^* | \gamma_0 \leq \alpha_{0i}^* < \gamma_1)
\]

\[
= b\sigma_{\alpha^*}^2 \times g_2 \left( \frac{\gamma_0 - \mu_\alpha}{\sigma_{\alpha^*}}, \frac{\gamma_1 - \mu_\alpha}{\sigma_{\alpha^*}} \right)
\]

The above conditions specify the exactly identified case. Notice that the two thresholds $\gamma_0$, $\gamma_1$ and the standard deviation of the true alpha $\sigma_{\alpha^*}$ can be obtained by solving the system of first three equations (20), (21), and (22). The slope can be found from (25), the intercept can be computed from (23), and the variance $\sigma_{\epsilon}^2$ can be obtained from (24). The parameters and standard errors can be estimated by two step GMM.

### 3.4.4 Biases in the Simple (Naive) Model

The OLS slope estimator from the naive regression (11) is equal to

\[
\hat{b}_{OLS} = \frac{\text{cov} (\alpha_{1i}, \alpha_{0i})}{\text{Var} (\alpha_{0i})},
\]

and the consistent GMM estimator can be obtained from (25) as

\[
\hat{b}_{GMM} = \frac{\text{cov} (\alpha_{1i}, \alpha_{0i})}{\sigma_{\alpha^*}^2 \times g_2 \left( \frac{\gamma_0 - \mu_\alpha}{\sigma_{\alpha^*}}, \frac{\gamma_1 - \mu_\alpha}{\sigma_{\alpha^*}} \right)}.
\]

Notice that $g_2 (\cdot, \cdot)$ is always less than one, as, by definition, it is the variance of the truncated standard normal distribution. Therefore

\[
\text{Var} (\alpha_{0i}^* | \gamma_0 \leq \alpha_{0i}^* < \gamma_1) = \sigma_{\alpha^*}^2 \times g_2 \left( \frac{\gamma_0 - \mu_\alpha}{\sigma_{\alpha^*}}, \frac{\gamma_1 - \mu_\alpha}{\sigma_{\alpha^*}} \right) < \text{Var} (\alpha_{0i}^*).
\]

In order to compare $\hat{b}_{OLS}$ and $\hat{b}_{GMM}$ estimators we have to account for the two types of estimation bias:

1) Measurement bias: $\text{Var} (\alpha_{0i}) > \text{Var} (\alpha_{0i}^*)$.
2) Self-selection bias: $\text{Var} (\alpha_{0i}^*) > \text{Var} (\alpha_{0i}^* | \gamma_0 \leq \alpha_{0i}^* < \gamma_1)$.

The combined effect of the above biases is that $\text{Var} (\alpha_{0i}) > \text{Var} (\alpha_{0i}^* | \gamma_0 \leq \alpha_{0i}^* < \gamma_1)$, which results in

\[
|\hat{b}_{OLS}| < |\hat{b}_{GMM}|.
\]
<table>
<thead>
<tr>
<th>year</th>
<th>total</th>
<th>entered</th>
<th>left</th>
<th>attrition</th>
<th>mean return</th>
<th>median return</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>1132</td>
<td>1132</td>
<td>92</td>
<td>8.13%</td>
<td>0.59%</td>
<td>0.61%</td>
<td>5.07%</td>
</tr>
<tr>
<td>1997</td>
<td>1341</td>
<td>301</td>
<td>152</td>
<td>11.33%</td>
<td>1.16%</td>
<td>0.88%</td>
<td>5.27%</td>
</tr>
<tr>
<td>1998</td>
<td>1469</td>
<td>280</td>
<td>203</td>
<td>13.82%</td>
<td>-0.17%</td>
<td>0.24%</td>
<td>7.94%</td>
</tr>
<tr>
<td>1999</td>
<td>1543</td>
<td>277</td>
<td>195</td>
<td>12.64%</td>
<td>1.51%</td>
<td>0.69%</td>
<td>7.93%</td>
</tr>
<tr>
<td>2000</td>
<td>1612</td>
<td>264</td>
<td>250</td>
<td>15.51%</td>
<td>-0.33%</td>
<td>0.15%</td>
<td>7.26%</td>
</tr>
<tr>
<td>2001</td>
<td>1925</td>
<td>563</td>
<td>218</td>
<td>11.32%</td>
<td>0.16%</td>
<td>0.26%</td>
<td>4.64%</td>
</tr>
<tr>
<td>2002</td>
<td>2141</td>
<td>434</td>
<td>833</td>
<td>38.91%</td>
<td>-0.10%</td>
<td>0.13%</td>
<td>4.31%</td>
</tr>
</tbody>
</table>

Table 3: Yearly distribution of hedge funds. The table presents the total number of funds that reported during a year, the number of funds that entered and left the database, and mean, median, and standard deviation of monthly excess returns. A year represents the time period from May of that year until April of the next year.

This means that the naive regression OLS slope estimator (26) is biased toward zero compared to the GMM slope estimator (27).

4 Estimation Results

In this section we present the data and the results of the estimation of all the models proposed in the last section.

4.1 Data Description

The data for this research was generously provided by Hedge Fund Research. The database contains the history of monthly hedge fund returns beginning in 1990. However, the information about when a fund actually joined the database is only available since May 1996. Hence, we consider the time period from May 1996 until May 2003. We consider only hedge funds with dollar returns (both offshore and onshore), which report their returns as net of all fees. The yearly summary statistics is presented in table 3.

When a hedge fund joins the HFR database, it is given an option to select one strategy from the HFR list. These strategies are used in computation of monthly self reported style indices. The indices are computed as returns on equally weighted portfolios of all funds using the same strategy.

---

\(^{20}\)For some funds, the history goes back to 1980s.

\(^{21}\)Only hedge funds with dollar returns reported on monthly basis, net of all fees are used in the computation of self reported indices.
4.2 Data Biases, Model Selection and Distribution of Alphas

In this section we demonstrate empirically how the distribution of hedge fund alphas is affected by different biases. In particular, we estimate three different models, eliminating one by one the problems related to the hedge fund data and then observe the differences in the distributions of alphas. Stale prices and changes in hedge fund strategies are considered. We run the following three regressions.

1. Stale prices are not taken into account:

\[ R_{i,t} - r_{f,t} = \alpha_i + \beta_i \left( R_{t}^{mkt} - r_{f,t} \right) + \gamma_i \left( \eta_{t}^{J,\text{self}} - r_{f,t} \right) + \varepsilon_{i,t} \]  

(28)

We assume that residuals \((\varepsilon_{i,t})\) are i.i.d., so that the data is exposed to stale prices. To estimate hedge fund performance we use a market index, and a self declared style as benchmarks.

2. Now we take into account the stale prices. To do this we run a different regression:

\[
\begin{align*}
R_{i,t} - r_{f,t} &= \hat{\alpha}_i + \beta_{0,i} \left( R_{t}^{mkt} - r_{f,t} \right) + \ldots + \beta_{s,i} \left( R_{t-s}^{mkt} - r_{f,t-s} \right) \\
&\quad + \beta_{0,i}^{\text{self}} \left( \eta_{t}^{J,\text{self}} - r_{f,t} \right) + \ldots + \beta_{s,i}^{\text{self}} \left( \eta_{t-s}^{J,\text{self}} - r_{f,t-s} \right) \\
&\quad + \beta_{0,i}^{\text{aux}} \left( \eta_{t}^{K,\text{aux}} - r_{f,t} \right) + \ldots + \beta_{s,i}^{\text{aux}} \left( \eta_{t-s}^{K,\text{aux}} - r_{f,t-s} \right) + u_{i,t} \\
\end{align*}
\]  

(29)

In this regression we include lags of the benchmarks, and assume that the error term \((u_{i,t})\) follows MA(s) process, \((\varepsilon_{i,t})\) are i.i.d. The number of lags is selected by SBC (Schwartz - Bayesian Criterion). \(\alpha_i\) is then obtained from (10). For the details of the regression estimations see subsection 3.3.

3. To account for hedge funds changing their strategies over time, we add an additional index into the regression (29). The additional index and the number of lags are selected by SBC. The regression equation is as follows:

\[
\begin{align*}
R_{i,t} - r_{f,t} &= \hat{\alpha}_i + \beta_{0,i} \left( R_{t}^{mkt} - r_{f,t} \right) + \ldots + \beta_{s,i} \left( R_{t-s}^{mkt} - r_{f,t-s} \right) \\
&\quad + \beta_{0,i}^{\text{self}} \left( \eta_{t}^{J,\text{self}} - r_{f,t} \right) + \ldots + \beta_{s,i}^{\text{self}} \left( \eta_{t-s}^{J,\text{self}} - r_{f,t-s} \right) \\
&\quad + \beta_{0,i}^{\text{aux}} \left( \eta_{t}^{K,\text{aux}} - r_{f,t} \right) + \ldots + \beta_{s,i}^{\text{aux}} \left( \eta_{t-s}^{K,\text{aux}} - r_{f,t-s} \right) + u_{i,t} \\
\end{align*}
\]  

(30)

\[ u_{i,t} = \theta_{0,i} \varepsilon_{i,t} + \ldots + \theta_{s,i} \varepsilon_{i,t-s} \]
In our estimation of the above regressions, we only consider hedge funds that had at least two years of observations. This leaves us with 1760 hedge funds. The brief summary statistics of alphas for the above three models are presented in table 4. Since we use HFR indices in our regressions, and these indices are equally weighted averages of returns for all hedge funds within the same strategies, we expect the mean and the median of all alphas to be approximately equal to zero and the number of positive alphas to be about fifty percent. However from table 4 we can clearly see that our alpha estimations in models 1 and 2 suffer from a positive bias. When we take into account stale prices, the percentage of positive alphas decreases from 55.11% to 51.93% (monthly basis). Finally, when we take into consideration stale prices along with an additional style index, the percentage of positive alphas goes down to 50.57%. These results provide us with an preliminary indication of the accuracy of our approach to estimating relative alphas.

4.3 Estimation of Hedge Fund Alphas

As described in the econometrics methodology section, in order to test for the persistence in hedge fund returns, we first estimate alphas $\alpha_0$ in the evaluation period, then estimate alphas $\alpha_1$ in the prediction period for the same hedge funds (if available) and proceed with a cross-section of hedge fund alphas (future and past alphas) which is tested for persistence. We form two overlapping cross-sections with three year evaluation and prediction periods using the seven years of available backfill bias free data. The first cross-section covers the evaluation period of May 1996 to April 1999, and the prediction period of May 1999 to April 2002. The second cross-section covers the evaluation period of May 1997 to April 2000 and the prediction period of May 2000 to April 2003. Figure 5 shows the timeline for the estimation of alphas.

Notice that we cannot compute alphas $\alpha_1$ for hedge funds that disappear from the database by the end of the evaluation period.
Figure 5: Timeline for evaluation and prediction periods

4.4 Performance Persistence

4.4.1 Simple (Naive) Regression

The first approach to check for persistence is to run the naive regression (11):

$$\alpha_{1i} = a + b\alpha_{0i} + \varepsilon_i.$$ 

This regression is estimated only for hedge funds with observed returns in the prediction period. Hence, it does not take into account the fact that some hedge funds disappeared from the database due to different reasons. The results of the naive regression for both cross-sections are presented in table 5. The slope coefficient $b$ is not consistently significant in both cross-sections. However, because the estimations of the naive regressions are biased, we cannot make conclusions about the persistence at this point. We investigate this question in the next section.

4.4.2 Self-Selection Bias and Non-Reporting Funds

During the prediction period, a hedge fund can either remain in the database or disappear from it due to liquidation, closing, or stop reporting for unknown reasons. The distribution of hedge funds according to this decomposition is presented in tables 6 and 7 for the first
and for the second cross-sections correspondingly.

The non-reporting funds comprise 19.76% of the data in the first cross-section, and 18.35% of the data in the second cross-section. Can we use these funds in our further performance analysis? The answer to this question lies in the distribution of observable characteristics of the non-reporting funds during the evaluation period. We may attempt to classify the non-reporting funds as closed or liquidated on the basis of their evaluation period performance $\alpha_0$. Such classification would be consistent with assumptions of the model (M) and the specification (13), but only if the distribution of the relative performance measure $\alpha_0$ for non-reporting funds resembles the distributions of $\alpha_0$ for funds that stopped reporting, but indicated a reason for doing so (i.e. liquidated and closed funds). Unfortunately, Kolmorogov-Smirnov test for distribution closeness indicates the closest fit for the non-reporting funds distribution with the combined distribution of all reporting funds (i.e. observable, liquidated, and closed funds).

Hence we conclude that classifying non-reporting funds as either closed or liquidated would result in model (M) misspecification. Finally, we conclude that treating non-reporting funds as missing data provides us with the most accurate estimates, since the distribution of non-reporting funds closely resembles the distribution of all reporting funds.

4.4.3 GMM Estimation

Here we take into account the self-selection bias and measurement errors by estimating parameters in the model (M) with the specification (13). The estimates from the GMM procedure described in subsection 3.4.3 are provided in table 9. The GMM estimates of the slope coefficients $b$ are significant and consistent in value in both cross-sections. This is indicative of performance persistence among hedge fund managers. We interpret the value of the slope coefficient (0.56 in the first cross-section, and 0.57 in the second cross-section) as evidence that a hedge fund manager that outperformed his style benchmark by 100 basis points in a three year evaluation period will on average outperform his style benchmark by 56 to 57 basis points during the next three year period.

4.4.4 Distribution of Alphas

We assumed normality of the distribution of alphas in the model (M). Distributions of $\alpha_0$ for the first and for the second cross-sections are presented in figures 6 and 7 correspondingly.

---

22 See table 8 for Kolmorogov-Smirnov test results.
23 We also excluded one extreme outlier with $\alpha_1 = -168.4355$ in the second cross-section in the future analysis.
24 At the 5% significance level.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-statistics</td>
</tr>
<tr>
<td>a</td>
<td>0.0599</td>
<td>0.60</td>
</tr>
<tr>
<td>b</td>
<td>0.0651</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Table 5: Naive regression results. Persistence is captured by the slope coefficient $b$, which is not consistently statistically significant.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Liquidated</th>
<th>Closed</th>
<th>Non-Reporting</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of hedge funds</td>
<td>523</td>
<td>159</td>
<td>65</td>
<td>184</td>
</tr>
<tr>
<td>percent</td>
<td>56.18%</td>
<td>17.08%</td>
<td>6.98%</td>
<td>19.76%</td>
</tr>
<tr>
<td>$\alpha_0$ mean</td>
<td>-0.0959</td>
<td>-0.3867</td>
<td>0.2210</td>
<td>-0.1765</td>
</tr>
<tr>
<td>$\alpha_0$ median</td>
<td>0.0399</td>
<td>-0.3241</td>
<td>0.4162</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_0$ std. dev.</td>
<td>2.4888</td>
<td>1.4804</td>
<td>1.1500</td>
<td>1.4429</td>
</tr>
<tr>
<td>$\alpha_0$ min</td>
<td>-30.5407</td>
<td>-6.2725</td>
<td>-4.2407</td>
<td>-8.0258</td>
</tr>
<tr>
<td>$\alpha_0$ max</td>
<td>17.8603</td>
<td>4.2145</td>
<td>3.1838</td>
<td>4.1612</td>
</tr>
<tr>
<td>assets ($\text{M}$) mean</td>
<td>230.69</td>
<td>35.25</td>
<td>51.06</td>
<td>96.84</td>
</tr>
<tr>
<td>assets ($\text{M}$) median</td>
<td>50.85</td>
<td>10.00</td>
<td>12.15</td>
<td>25.38</td>
</tr>
<tr>
<td>assets ($\text{M}$) std. dev.</td>
<td>670.47</td>
<td>87.85</td>
<td>75.89</td>
<td>228.24</td>
</tr>
<tr>
<td>assets ($\text{M}$) min</td>
<td>0.15</td>
<td>0.00</td>
<td>0.88</td>
<td>0.10</td>
</tr>
<tr>
<td>assets ($\text{M}$) max</td>
<td>9327.00</td>
<td>701.55</td>
<td>390.51</td>
<td>2000.00</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Observable</th>
<th>Liquidated</th>
<th>Closed</th>
<th>Non-Reporting</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of hedge funds</td>
<td>524</td>
<td>167</td>
<td>70</td>
<td>171</td>
</tr>
<tr>
<td>percent</td>
<td>56.22%</td>
<td>17.92%</td>
<td>7.51%</td>
<td>18.35%</td>
</tr>
<tr>
<td>$\alpha_0$ mean</td>
<td>-0.0305</td>
<td>-0.6239</td>
<td>0.4146</td>
<td>1.4938</td>
</tr>
<tr>
<td>$\alpha_0$ median</td>
<td>0.1634</td>
<td>-0.1531</td>
<td>0.4693</td>
<td>0.1052</td>
</tr>
<tr>
<td>$\alpha_0$ std. dev.</td>
<td>2.5037</td>
<td>2.2464</td>
<td>2.156</td>
<td>13.8676*</td>
</tr>
<tr>
<td>$\alpha_0$ min</td>
<td>-39.3374</td>
<td>-16.1324</td>
<td>-5.0266</td>
<td>-19.9643</td>
</tr>
<tr>
<td>$\alpha_0$ max</td>
<td>14.3163</td>
<td>3.8540</td>
<td>5.7029</td>
<td>144.8887*</td>
</tr>
<tr>
<td>assets ($\text{M}$) mean</td>
<td>227.71</td>
<td>27.56</td>
<td>40.07</td>
<td>98.16</td>
</tr>
<tr>
<td>assets ($\text{M}$) median</td>
<td>57.45</td>
<td>6.93</td>
<td>12.40</td>
<td>19.15</td>
</tr>
<tr>
<td>assets ($\text{M}$) std. dev.</td>
<td>665.56</td>
<td>66.71</td>
<td>66.91</td>
<td>223.45</td>
</tr>
<tr>
<td>assets ($\text{M}$) min</td>
<td>0.11</td>
<td>0.00</td>
<td>0.15</td>
<td>0.34</td>
</tr>
<tr>
<td>assets ($\text{M}$) max</td>
<td>9327.00</td>
<td>660.00</td>
<td>390.51</td>
<td>2000.00</td>
</tr>
</tbody>
</table>

Table 7: Distribution of hedge funds in the prediction period from the second cross-section: 1997-2000 - 2000-2003.

* The relatively high standard deviation of $\alpha_0$ for non-reporting funds is caused by two extreme outliers ($\alpha_0 = 144.8887$ and $\alpha_0 = 100.6640$). If we eliminate these outliers, then StdDev($\alpha_0$) = 3.405, which is in line with the distribution of reporting funds.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KSa statistic</td>
<td>p-value</td>
</tr>
<tr>
<td>Observable funds</td>
<td>0.8092</td>
<td>0.5292</td>
</tr>
<tr>
<td>Liquidated and closed funds</td>
<td>1.1510</td>
<td>0.1413</td>
</tr>
<tr>
<td>All reporting funds</td>
<td>0.4344</td>
<td>0.9916</td>
</tr>
</tbody>
</table>

Table 8: Kolmogorov-Smirnov tests for closeness of alpha 0 distributions. KSa statistic denotes the asymptotic Kolmogorov-Smirnov statistic, and the p-value is provided for the test of the null hypothesis that there is no difference between two distributions. The non-reporting funds distribution is compared to the observable funds distribution, liquidated and closed funds distribution, and to the all reporting funds (i.e. observable, liquidated, and closed funds) distribution.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-statistic</td>
</tr>
<tr>
<td>$a$</td>
<td>0.1078</td>
<td>1.3433</td>
</tr>
<tr>
<td>$b$</td>
<td><strong>0.5636</strong></td>
<td><strong>3.3311</strong></td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>-1.3104</td>
<td>-14.2468</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.8817</td>
<td>17.0388</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>2.2313</td>
<td>9.3768</td>
</tr>
<tr>
<td>$\sigma_{\alpha}$</td>
<td>1.4807</td>
<td>38.4223</td>
</tr>
</tbody>
</table>

Table 9: Results for the GMM procedure. The procedure takes into account measurement error and self-selection bias. Persistence is captured by the slope coefficient $b$, which is statistically significant at the 5 percent level. Persistence estimates are consistent for the first and the second cross-sections.
Figure 6: $\alpha_0$ distribution for the 1996-1999 - 1999-2002 cross-section. The superimposed normal distribution follows the GMM specification ($\mu = \bar{x}_{\alpha_0} = -0.13, \sigma = \sigma_{\alpha^*} = 1.4807$).

Although both distributions fail the Kolmogorov-Smirnov test of normality, they do not suffer from excessive skewness, and they also closely resemble normal distributions that are implied by the GMM estimation. This confirms the validity of our model assumptions of normality.

4.5 Robustness Checks

In order to get a better understanding of what drives the persistence results in the previous subsection, we perform the above analysis for three truncated data sets. In the first data set we remove funds within the top 1% and the bottom 1% of the estimation period alphas (i.e. $\alpha_{0i}$). In the second set, we remove funds only within the bottom 1% of the estimation
Figure 7: $\alpha_0$ distribution for the 1997-2000 - 2000-2003 cross-section. The superimposed normal distribution follows the GMM specification ($\mu = \bar{x}_{\alpha_0} = -0.12, \sigma = \sigma_{\alpha^*} = 1.7274$).
period alphas. In the third set we remove funds only within the top 1% of the estimation period alphas. The results are summarized in tables 10 and 11 for the first cross-section, and in tables 12 and 13 for the second cross-section.

Notice that we cannot completely rely on the results of the GMM correction applied to truncated data sets, since the model (M) becomes misspecified after the truncation of the data. For example, a drop in significance of the performance persistence coefficient $b$ in the dataset truncated on both ends could be attributed to the loss of normality in error distribution specified in model (M), as well as to a conjecture of stronger evidence of performance persistence in the tails of the $\alpha_0$ distribution. However, we can use the above procedure to get a rough idea about performance persistence among the top and bottom 1% of hedge funds by comparing the significance of the coefficient $b$ for different truncated versions of the data. In the first cross-section including either the bottom or the top 1% of $\alpha_0$ distribution results in higher p-values for $b$, compared to the p-value for $b$ in the distribution truncated on both ends. In the second cross-section the p-value dynamics is reversed, indicating no evidence of either weaker or stronger performance persistence in the tails.

### 4.6 Portfolio Performance Interpretation

In this section we attempt to interpret the significance of the main results about performance persistence. We construct portfolios of hedge funds based on their past performance in the evaluation period, and then track their performance during the prediction period. However, the fact that some hedge funds disappear in the prediction period gives rise to the self-selection bias. Hence, it is impossible to make an unbiased portfolio performance comparison during the prediction period. Here we attempt to estimate the persistence magnitude using all the hedge funds that are available in the evaluation period.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.1078</td>
<td>1.3433</td>
<td>0.1796</td>
<td>0.1467</td>
<td>1.4219</td>
<td>0.1555</td>
</tr>
<tr>
<td>$b$</td>
<td>0.5636</td>
<td>3.3331</td>
<td>0.0009</td>
<td>2.1955</td>
<td>2.8732</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>-1.3104</td>
<td>-14.2468</td>
<td>0.0000</td>
<td>-0.5988</td>
<td>-19.1328</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.8817</td>
<td>17.0388</td>
<td>0.0000</td>
<td>0.8045</td>
<td>15.0836</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>2.2313</td>
<td>9.3768</td>
<td>0.0000</td>
<td>2.1563</td>
<td>8.7540</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_{\alpha^*}$</td>
<td>1.4807</td>
<td>38.4223</td>
<td>0.0000</td>
<td>0.6571</td>
<td>26.9570</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 10: Results for the GMM procedure in the first cross-section (1996-1999 - 1999-2002). The slope persistence coefficient is significant throughout the truncated data.
Table 11: Results for the GMM procedure for the first cross-section (1996-1999 - 1999-2002). The slope persistence coefficient is significant throughout the truncated data. However, it is less significant upon bringing back the top and the bottom 1 percent of evaluation period alphas. The influence of the top 1 percent of alphas is more pronounced than the influence of bottom 1 percent of alphas.

Table 12: Results for the GMM procedure for the second cross-section (1997-2000 - 2000-2003). The slope persistence coefficient loses significance for the truncated data.

Table 13: Results for the GMM procedure for the second cross-section (1997-2000 - 2000-2003). The slope persistence coefficient is more significant upon bringing back the top and the bottom 1 percent of evaluation period alphas. The influence of the bottom 1 percent of alphas is more pronounced than the influence of top 1 percent of alphas.
We sort all the hedge funds by their evaluation period performance.\textsuperscript{25} We compose an inferior portfolio of all hedge funds in the bottom third, a neutral portfolio of all funds in the middle third, and a superior portfolio of all funds in the top third. We then invest one dollar to every portfolio in the beginning of the prediction period. One dollar is equally split among all the hedge funds in a given portfolio.

We consider two scenarios. Under a pessimistic scenario we assume that the money invested into disappeared hedge funds cannot be recovered at all. That is, if a hedge fund disappears from our database, we lose all the money invested there, regardless of the reason the hedge fund disappeared. The pessimistic scenario is modeled by assigning -100\% return to a fund during the month after its disappearance from the database, and zero returns thereafter. Under a neutral scenario, we assume that we can take all the money from a disappeared hedge fund and invest it into the HFR total index.\textsuperscript{26} The neutral scenario is modeled by assigning HFR total index returns to a fund after its disappearance from the database. For each scenario we calculate each portfolio performance as an equally weighted average of individual fund alphas. This is summarized in figure 8.

Under the assumptions of our model, liquidated hedge funds performed poorly, and closed funds performed well, relative to other hedge funds in the evaluation period. In the pessimistic scenario we may significantly underestimate the performance of every portfolio, since, in reality, some money can be recovered even from liquidated funds. In the neutral scenario, the relationship of the estimated performance to the actual performance is more ambiguous. The performance of the inferior portfolio is probably overestimated, as liquidation would be the main reason for fund disappearance. The performance of the superior portfolio is most likely underestimated, as the main reason for a fund to disappear is to close for new investors. One can expect that such hedge funds will perform better than average funds in the prediction period. It is difficult to make any preliminary conclusions for the neutral portfolio. These results are summarized in table 14.

We report the performance of the three portfolios in the evaluation and prediction periods in tables 15 and 16. The performance for the evaluation period is presented in column ‘Past Alpha’. We also use our model (M) to make prediction period alpha estimates $\hat{\alpha}_1$ based on alpha values $\alpha_0$ from the evaluation period, i.e.

$$\hat{\alpha}_1 = \hat{a} + \hat{b}\alpha_0,$$

where $\hat{a}$ and $\hat{b}$ are GMM estimates of the model (M) parameters. The results are presented


\textsuperscript{26}HFR total index is an equally weighted average of returns for all hedge funds in the HFR database.
Figure 8: Test portfolio formation process. All portfolios are tested under 100% loss and 100% recovery with reinvestment into the HFR Total Index scenarios.

Table 14: Performance comparison under pessimistic and neutral scenarios to the actual case

<table>
<thead>
<tr>
<th></th>
<th>Pessimistic Scenario</th>
<th>≼</th>
<th>Actual Portfolio</th>
<th>≼</th>
<th>Neutral Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inferior</td>
<td>⋅</td>
<td>&lt;</td>
<td>⋅</td>
<td>&lt;</td>
<td>⋅</td>
</tr>
<tr>
<td>Neutral</td>
<td>⋅</td>
<td>&lt;</td>
<td>⋅</td>
<td>?</td>
<td>⋅</td>
</tr>
<tr>
<td>Superior</td>
<td>⋅</td>
<td>&lt;</td>
<td>⋅</td>
<td>&gt;</td>
<td>⋅</td>
</tr>
</tbody>
</table>
Table 15: Out of sample performance of three portfolios. Portfolios are formed and ranked according to the previous relative alpha performance in the evaluation period. Then portfolio alphas in the prediction period are calculated under pessimistic and neutral scenarios, as well as the GMM estimates of prediction period alphas. First cross-section: Evaluation period 1996-1999, prediction period 1999-2002

<table>
<thead>
<tr>
<th>Portfolios \ Performance</th>
<th>Past Alpha</th>
<th>Pessimistic Scenario</th>
<th>Model Predicted Alpha</th>
<th>Neutral Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inferior (lowest performance in the past)</td>
<td>-1.312</td>
<td>-1.673</td>
<td>-0.605</td>
<td>-0.289</td>
</tr>
<tr>
<td>Neutral</td>
<td>0.051</td>
<td>-0.836</td>
<td>0.133</td>
<td>0.076</td>
</tr>
<tr>
<td>Superior (highest performance in the past)</td>
<td>1.277</td>
<td>-1.224</td>
<td>0.797</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 16: Out of sample performance of three portfolios. Portfolios are formed and ranked according to the previous relative alpha performance in the evaluation period. Then portfolio alphas in the prediction period are calculated under pessimistic and neutral scenarios, as well as the GMM estimates of prediction period alphas. Second cross-section: Evaluation period 1997-2000, prediction period 2000-2003

<table>
<thead>
<tr>
<th>Portfolios \ Performance</th>
<th>Past Alpha</th>
<th>Pessimistic Scenario</th>
<th>Model Predicted Alpha</th>
<th>Neutral Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inferior (lowest performance in the past)</td>
<td>-1.678</td>
<td>-1.944</td>
<td>-1.175</td>
<td>-0.383</td>
</tr>
<tr>
<td>Neutral</td>
<td>0.166</td>
<td>-1.101</td>
<td>-0.239</td>
<td>-0.011</td>
</tr>
<tr>
<td>Superior (highest performance in the past)</td>
<td>1.446</td>
<td>-1.700</td>
<td>0.410</td>
<td>-0.163</td>
</tr>
</tbody>
</table>

As we see from tables 15 and 16, portfolio performances in pessimistic and neutral scenarios are in line with predictions from table 14. The inferior portfolio’s performance predicted by the model falls between performance estimates in pessimistic and neutral scenarios. The superior portfolio’s performance predicted by the model is higher than performance estimates in both the neutral and pessimistic scenarios.

In both cross-sections the superior portfolio outperforms the inferior portfolio under both pessimistic and neutral scenarios. These results indicate that in reality we should expect the superior portfolio to outperform the inferior portfolio.
Table 17: Summary statistics for alpha. Statistics are provided for the data with the backfill bias. There are 2429 funds with at least a two-year history available.

<table>
<thead>
<tr>
<th>model</th>
<th>description</th>
<th>mean</th>
<th>median</th>
<th>percent of positive alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>stale prices</td>
<td>.16621</td>
<td>.18824</td>
<td>63.11%</td>
</tr>
<tr>
<td>2</td>
<td>no stale prices</td>
<td>-.09715</td>
<td>.14613</td>
<td>58.75%</td>
</tr>
<tr>
<td>3</td>
<td>multiple indices</td>
<td>-.13237</td>
<td>.09806</td>
<td>56.44%</td>
</tr>
</tbody>
</table>

5 Backfill Bias: Effect and Correction Methodology

5.1 Backfill Bias Effect

An important feature of a hedge fund database is backfill bias - the case when hedge funds bring their past returns with them when they join a database. Arguably, the major reason a hedge fund would like to publish its returns is for advertising purposes in order to attract additional investments. A hedge fund can indirectly advertise itself by publishing its relatively good past returns. The returns of such a hedge fund could be higher than the average returns of other hedge funds following the same strategy. While it is plausible to conjecture that the backfill bias could result in abnormally high performance for the time period prior to fund’s joining the database, its effect on performance persistence is ambiguous. On one hand, it is possible to conjecture that the stellar backfill performance could be due to extraordinary amount effort by managers during the first years after starting a fund, self-selection, and just pure luck. Since the above reasons do not accurately reflect managerial talent, it is possible that any abnormal performance during the backfill period would disappear after a fund joins the database. On the other hand, we can argue that a stellar backfill performance is a reflection of managerial talent, and thus it is likely that it would continue after a fund joins the database. In this section we check for performance persistence without correcting for the backfill bias, and find no consistent evidence of performance persistence.

We repeat the analysis from subsections 4.2 and 4.4 without correcting for the backfill bias. The distribution of alphas is provided in table 17. As expected, we observe a higher percentage of positive alphas compared to the backfill-free results in the table 4. This confirms the conjecture of a positive effect of the backfill bias on hedge fund performance.

GMM estimates of model (M) parameters for the data with the backfill are presented in table 18. While the results suggest performance persistence in the first cross-section, we do not have consistent evidence of performance persistence in both cross-sections. We also observe lower values of the slope coefficient, $b$, compared to the values from subsection 4.4. This indicates that at least a part of the abnormal performance during the backfill period was

---

27Hedge funds are legally prohibited from advertizing their services to investors.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.1504</td>
<td>1.5125</td>
<td>0.1307</td>
<td>-0.0919</td>
<td>-0.9062</td>
<td>0.3651</td>
</tr>
<tr>
<td>$b$</td>
<td>0.4240</td>
<td>2.6270</td>
<td>0.0087</td>
<td>0.4684</td>
<td>1.6292</td>
<td>0.1036</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>-1.3536</td>
<td>-15.6430</td>
<td>0.0000</td>
<td>-1.0501</td>
<td>-10.5118</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>2.3815</td>
<td>22.7479</td>
<td>0.0000</td>
<td>2.1538</td>
<td>20.8502</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>3.6049</td>
<td>3.9324</td>
<td>0.0001</td>
<td>2.4181</td>
<td>5.7124</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_{\alpha^*}$</td>
<td>1.5817</td>
<td>51.8492</td>
<td>0.0000</td>
<td>1.4062</td>
<td>27.3166</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 18: Results for the GMM procedure for data with the backfill bias. The performance persistence coefficient $b$ is statistically significant in the first cross-section, but it is not significant in the second cross-section.

not due to superior managerial talent. The above analysis also highlights the importance of the backfill correction for accurately estimating performance persistence in our results from Section 4.

### 5.2 Alternative Correction Methodology

To further highlight the importance of the complete backfill correction, we describe an alternative backfill bias correction methodology that could be applied if the exact length of the backfill period is unknown. We then compare our results from Section 4 with the results obtained by applying the estimated backfill correction described below.

One way to deal with the backfill bias is to delete the first few months of observations for each hedge fund in the database. We develop a methodology to estimate the ‘average’ length of backfill bias\(^{28}\) by considering buy and hold abnormal returns (BHAR) for hedge funds against their self-reported styles. More precisely, BHAR represents the return on the portfolio in which you long the hedge fund and short the corresponding style index. The HFR indices do not have the backfill bias problem: when a hedge fund joins the HFR database and brings its history along, the HFR indices are not updated. Given this fact we can do the following: fix the number of months in the backfill period (say 16 months), then delete this number of months of return observations for every hedge fund (that is, for every hedge fund we will throw the first 16 returns written in the database), and then construct our own ‘synthetic’ style indices. We then compare our own ‘synthetic’ indices with the official HFR style indices. The number of months that produces the smallest difference is then interpreted as the average length of backfill bias.

\(^{28}\)We would like to thank Dobrislav Dobrev for the suggested idea.
Formally, we have to minimize the objective function (31)

\[
\min_n \sum_t \sum_J \left[ HFRI^J_t - \frac{1}{|S^J_{n,t}|} \sum_{(i,t) \in S^J_{n,t}} R^J_{it} \right]^2
\]  

by the number of months in the backfill period. In objective function (31) \( J \) corresponds to the style index reported by HFR. \( S^J_{n,t} \) is the set of all hedge fund returns in style \( J \) at time \( t \) that are left after we deleted the first \( n \) months of observations from every hedge fund and \( |S^J_{n,t}| \) is the number of such funds. The value of this objective function for different numbers of months in the backfill bias is presented on figure 9. We can observe that the minimum of this objective function is reached at 25 months. We find that this estimate of the average backfill period is very close to the actual backfill mean of 25.81 months in our data (see figure 10).

We delete the first 25 months of observations for every hedge fund in our database. If the total history of a hedge fund is less than 25 months, we completely exclude this fund.
Table 19: Summary statistics for alpha. Statistics are provided for the data after the 25-month backfill correction. There are 1561 funds with at least a two-year history available, after deleting first 25 months of observations for each fund.

<table>
<thead>
<tr>
<th>model</th>
<th>description</th>
<th>mean</th>
<th>median</th>
<th>percent of positive alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>stale prices</td>
<td>.02489</td>
<td>.10662</td>
<td>58.74%</td>
</tr>
<tr>
<td>2</td>
<td>no stale prices</td>
<td>.0475</td>
<td>.07708</td>
<td>55.48%</td>
</tr>
<tr>
<td>3</td>
<td>multiple indices</td>
<td>-.01259</td>
<td>.03543</td>
<td>52.27%</td>
</tr>
</tbody>
</table>

We then repeat the analysis from subsections 4.2 and 4.4. The distribution of alphas is provided in table 19. We observe that the 25 month backfill correction resulted in positive alpha percentages being closer to 50% than in the data without the backfill correction (see table 17), but still not as close to 50% as the percentages in the backfill-free data (see table 4).

GMM estimates of model (M) parameters for the data with the 25 month backfill correction are presented in table 20. Although results are not statistically significant, they are consistent with our backfill-free results from Section 4. This further confirms the importance of the precise backfill bias correction in estimating performance persistence magnitude and statistical significance.

---

29 After deleting the first 25 months of observations.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.0161</td>
<td>0.2395</td>
<td>0.8108</td>
<td>-0.1893</td>
<td>-1.7124</td>
<td>0.0874</td>
</tr>
<tr>
<td>(b)</td>
<td>0.1041</td>
<td>1.1990</td>
<td>0.2310</td>
<td>0.3618</td>
<td>1.5258</td>
<td>0.1055</td>
</tr>
<tr>
<td>(\gamma_0)</td>
<td>-1.6871</td>
<td>-12.5549</td>
<td>0.0000</td>
<td>-1.5944</td>
<td>-12.3767</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>2.6470</td>
<td>17.0122</td>
<td>0.0000</td>
<td>2.7400</td>
<td>17.7521</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\sigma^e)</td>
<td>1.8172</td>
<td>6.0199</td>
<td>0.0000</td>
<td>1.9934</td>
<td>8.1130</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\sigma_{\alpha^*})</td>
<td>1.297</td>
<td>42.5263</td>
<td>0.0000</td>
<td>1.8392</td>
<td>46.4351</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 20: Results for the GMM procedure for data with the 25 month backfill correction. Performance persistence coefficients are not statistically significant in both cross-sections.

6 Conclusion

Hedge fund managers are given much more flexibility regarding where and how to invest compared to mutual fund managers. The growth of hedge funds, with almost a trillion dollars invested in assets at the end of 2004, may well reflect the need for giving talented managers who know where superior opportunities exist at a given point in time the necessary flexibility to exploit that talent. A natural question that arises is whether it is possible to identify those hedge fund managers who are able to exploit the flexibility given to them better than others.

While the flexibility given to hedge fund managers may help in generating superior returns, it also makes performance evaluation more difficult. Hedge fund returns are unlike returns from standard asset classes, and exhibit option-like features that have to be taken into account when evaluating performance. Further, since hedge funds invest in illiquid assets, care has to be exercised in measuring their systematic risk. In this paper we develop a method for evaluating the performance of a hedge fund manager relative to a suitably constructed peer group. Our method takes into account option-like features in hedge fund strategies and serial correlation in hedge fund returns caused possibly by investments in illiquid assets. We also take into account the backfill bias in our data set and the self-selection bias (i.e. the fact that a hedge fund may be liquidated or closed and exit the data set).

Using our method, we find statistically as well as economically significant persistence in the performance of funds relative to their style benchmarks. It appears that half of the superior or inferior performance during a three year interval will spill over into the following three year interval.

Our analysis highlights the difficulties that arise in predicting how a hedge fund manager will perform in the future relative to his peer group. While the assumptions we had to make in
order to answer the question of performance persistence among hedge fund managers appear reasonable, we need a better understanding of what happened to funds that vanished from publicly available databases to provide a quantitative answer to that question with utmost confidence. We hope that our findings will stimulate research examining how funds that discontinue reporting their performance do subsequently.
References


