Margin-Based Asset Pricing and Deviations from the Law of One Price

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Motivation: Margin-Based Asset Pricing and LoOP

- All agents face margin constraints
- These constraints can become binding; e.g., since 2007:
  - Many traditional liquidity providers have become forced sellers
  - Interest-rate spreads increased dramatically
  - Central banks have actively tried to facilitate funding
- One remarkable consequence: Failure of Law of One Price
  - Corporate-bond basis: price gap between bond and CDS
  - Covered interest-rate parity
- Key question: How do margins affect asset prices?
What We Do

- Standard Lucas economy, extended in minimal way:
  - with 2 two agents
  - facing margin constraints
- Derive equilibrium
- Quantify effects of margin
- Addressing the ability to explain:
  - CDS-bond basis
  - Failure of covered interest-rate parity (CIP)
  - The pricing of the Fed’s lending facilities
  - The incentive for regulatory arbitrage

Results: Theory

- (C)CAPM adjusted for margin constraints
  \[ E_t(r^i) - r^C_t = \lambda_t^{i} \beta_t^i + \psi_t x_t m_t^i \]
- Shadow cost of capital \( \psi_t \) can be captured by
  - interest-rate spreads (LIBOR minus GC repo).
- Binding constraints, \( \psi_t > 0 \) (e.g., since August 2007):
  - occur following bad fundamental shocks
  - increase Sharpe market ratio: \( SR = \hat{SR} + f(x_t) \left( \frac{SR}{\hat{SR}} - \frac{1}{m} \right)^i \)
- Basis: can arise due to difference in margins
  \[ E_t(r^i) - E_t(r^k) = \left( \beta_t^{C,i} - \beta_t^{C,k} \right) + \psi_t (m_t^i - m_t^k) \]
- High-margin assets have high sensitivity to funding risk
Results: Applications

- Calibrate model using standard parameters: consumption growth, discount rate, risk aversion, observed margins
  - Large pricing effect of binding constraints
    - Collateralized interest rates drop
    - Interest-rate spreads blow out
    - Margin premium rises
  - High margin assets have high sensitivity to funding risk
    - higher beta
    - higher comovement with each other
- Consistent with model, CDS-bond basis related to:
  - credit tightness (time series)
  - relative margin requirements (cross section)
- Relate interest-rate spread to failure of covered interest parity
- Transmission of unconventional monetary policy:
  - Compute effect of Fed’s lending facilities on asset values
- Quantify banks’ incentives to loosen capital requirements

Related Literature

- Direct evidence from Fed that bids depend significantly on haircuts: Ashcraft, Garleanu, and Pedersen (2009)
Model: Assets

- Continuous-time endowment economy
- Multiple assets in positive supply, characterized by
  - dividend stream: $\delta_t^i$
  - margin requirement: $m_t^i$
  - endogenous price: $dP_t^i - \left( \mu_t^i P_t^i - \delta_t^i \right) dt + P_t^i (\sigma_t^i)^\top dB_t$
- Multiple "derivatives":
  - derivative $i_k$ has the same payoffs $\delta_t^i$ as asset $i$
  - smaller margin: $m_t^k < m_t^i$
- Two types of risk-free lending/borrowing:
  - collateralized (rate $r_t^c$)
  - uncollateralized (rate $r_t^u$)

Model: Agents

- Two types of agents $g = a, b$:
  - Risk averse: $\gamma^a > 1$
  - Risk tolerant (brave): $\gamma^b = 1$ (i.e., log)
- Utility: constant relative risk aversion
  \[
  \max_{C^g, \theta^g, \eta^g} E_0 \int_0^\infty e^{-\rho s} \frac{\left( C_s^g \right)^{1-\gamma^g}}{1 - \gamma^g} ds
  \]
- Constraints:
  - Solvency: $W_t \geq 0$
  - Funding constraint: $\sum_i m_t^i |\theta_t^i| + \eta_t^u \leq 1$
  - Agent $a$
    - Does not lend uncollateralized
    - Faces derivative-trading restrictions
Shadow Cost of Capital

Agent $b$ solves

$$\max_{\theta_t^b, m_t^i} \left\{ \frac{r_t^b + \eta_t^b (r_t^u - r_t^c) + \sum_i \theta_t^b (\mu_t^i - r_t^c) - \frac{1}{2} \sum_{i,j} \theta_t^b \theta_t^i \sigma_t^i (\sigma_t^i)^\top}{\sum_i m_t^i |\theta_t^i| + \eta_t^b} \right\}$$

subject to $\sum_i m_t^i |\theta_t^i| + \eta_t^b \leq 1$.

Proposition: The shadow cost of the margin constraint is

$$\psi_t = r_t^u - r_t^c$$

Proposition: If agent $b$ is long asset $i$, its excess return is

$$\mu_t^i - r_t^c = \beta_t^{C, i} \psi_t m_t^i$$

where $\beta_t^{C, i} = \text{cov}_t \left( \frac{dC}{C}, \frac{dP^i}{P^i} \right)$

CCAPM with Margins

Suppose that agent $a$ is unconstrained w.r.t. asset $i$ and let

$$\frac{1}{\gamma_t} = \frac{1}{\gamma^a} \frac{C_t^a}{C_t} + \frac{1}{\gamma^b} \frac{C_t^b}{C_t}$$

$$x_t = \frac{C_t^b}{\gamma^b} + \frac{C_t^a}{\gamma^a}$$

$$\beta_t^{C, i} = \text{cov}_t \left( \frac{dC}{C}, \frac{dP^i}{P^i} \right)$$

Proposition:

$$\mu_t^i - r_t^c = \gamma_t \beta_t^{C, i} + x_t \psi_t m_t^i$$
CAPM with Margins

Let $q$ be the portfolio with highest correlation with aggregate consumption and

$$
\beta^i_t = \frac{\text{cov}_t \left( \frac{dP^i}{P^i}, \frac{dP^q}{P^q} \right)}{\text{var}_t \left( \frac{dP^q}{P^q} \right)}
$$

Proposition:

$$
\mu^i_t - r^c_t = \lambda_t \beta^i_t + x_t \psi_t m^i_t
$$

Basis Trades

Proposition:

- If agent $b$ is long asset $i$ and derivative $i_k$

$$
\mu^i_t - \mu^{i_k}_t = \psi_t \left( m^i_t - m^{i_k}_t \right) \mid \left( \beta_t^{C^b,i} - \beta_t^{C^b,i_k} \right)
$$

- If he is long $i$ and short $i_k$, then

$$
\mu^i_t - \mu^{i_k}_t = \psi_t \left( m^i_t + m^{i_k}_t \right) + \left( \beta_t^{C^b,i} - \beta_t^{C^b,i_k} \right)
$$

- The derivative price $P^{i_k}$ decreases with $m^{i_k}$.
Explicit Equilibrium

Specializing the setup for tractability to consider explicit equilibrium and calibration:

- Aggregate consumption $C$ is geometric Brownian motion
- Continuum of underlying assets with dividend $\delta^i = Cs^i$, where $s^i$ independent martingales
- All underlying assets have the same margin $m^i = m$
- Derivatives with $m^k \leq m$ traded only by $b$

Solving Explicitly

- It suffices to calculate equilibrium as if there were one underlying paying $C$ and derivatives on it
- State variables: $C$ and $c^b = C^b / C$
- Pricing kernel for underlying assets: Agent $a$ is marginal:
  \[
  \xi_t = e^{-\rho t} (C^a)^{-\gamma^a} \\
  d\xi_t = \xi_t \left( \mu^\xi dt + \sigma^\xi dw_t \right)
  \]
- Collateralized interest rate:
  \[
  r^\xi_t = -\mu^\xi = -\frac{D \left( e^{-\rho t} (C^a)^{-\gamma^a} \right)}{e^{\rho t} (C^a)^{-\gamma^a}}
  \]
- Market price of aggregate wealth $P_t = C_t \zeta (c^b_t)$:
  \[
  P_t \xi_t = E_t \int_t^\infty C_s \xi_s ds
  \]
Solution

Proposition:

- Agent $b$'s margin constraint binds iff
  \[ \frac{\mu - r^c}{\sigma^2} = \frac{SR}{\sigma} \geq \frac{1}{m} \]

- The price-to-dividend ratio $P_t/C_t - \zeta(c_t^b)$ is given as the solution to an ODE and all other endogenous variables are explicit functions of $\zeta$.

- Binding margin constraint increases the Sharpe Ratio:
  \[ SR = \tilde{SR} + \frac{x}{1-x} \frac{\tilde{\sigma}}{1 - \frac{\zeta c^b}{m\zeta}} \left( \frac{SR}{\tilde{\sigma}} - 1 \right)^+ \]
  where $\tilde{SR} = \gamma \sigma^C$ and $\tilde{\sigma}$ are the Sharpe and return volatility without constraints.

Limit Basis

Proposition:

As $c^b \to 0$, the basis between asset $i$ and derivative $i_k$ becomes
\[ \mu^i - \mu^{i_k} = \psi(m^i - m^{i_k}) \]
where
\[ \psi = \frac{(\sigma^C)^2}{m} \left( \gamma^a - \frac{1}{m} \right)^+ \]
In the cross section of asset-derivative pairs,
\[ \frac{\mu^i - \mu^{i_k}}{m^i - m^{i_k}} = \frac{\mu^j - \mu^{i_k}}{m^j - m^{i_k}} \]
Calibration: Parameters

- We use the following parameter values

<table>
<thead>
<tr>
<th>$\mu^C$</th>
<th>$\sigma^C$</th>
<th>$\gamma^a$</th>
<th>$\rho$</th>
<th>$m$</th>
<th>$m^{med}$</th>
<th>$m^{low}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.08</td>
<td>8</td>
<td>0.02</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

- Constraint binds for $c^b \leq 0.22$
- Since $b$ is levered more than $a$, low $c^b$ is the result of bad shocks to fundamentals

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Calibration: Interest Rates

Figure: Interest rates: complete markets, collateralized with constraints ($r^C$), and uncollateralized with constraints ($r^U$).
**Calibration: Bases**

![Graph showing return spreads of high-margin underlying versus low-margin derivative](image)

**Figure:** Return spreads of high-margin underlying versus low-margin derivative (i.e., large margin spread $m_{\text{underlying}} - m_{\text{low}} = 30\%$) and versus intermediate-margin derivative (i.e., small margin spread $m_{\text{underlying}} - m_{\text{medium}} = 10\%$).

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**Calibration: Sharpe Ratios**

![Graph showing Sharpe ratios](image)

**Figure:** Sharpe ratios: complete markets, underlying with constraints, and two derivatives with constraints.
Calibration: Price Premium

![Graph showing the price premium](image)

**Figure:** Price Premium. The figure shows how the price premium, \( \frac{P_{\text{derivative}}}{P_{\text{underlying}}} - 1 \), for three derivatives with identical cash flows and different margins.

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Monetary Policy and Lending Facilities

- **Term Auction Facility (TAF)** – December 2007
- **Term Securities Lending Facility (TSLF)** – March 2008
- **Term Asset-Backed Securities Loan Facility (TALF)** – November 2008

Goal: Improve funding conditions and “help market participants meet the credit needs of households and small businesses by supporting the issuance of asset-backed securities”

The model suggests that when the Fed offers lower margins, required returns go down:

\[
E(r^{\text{Fed}}) - E(r^{\text{no Fed}}) \approx \psi(M^{\text{Fed}} - m^{\text{Fed}}) < 0
\]

- I.e., ABS prices go up, and access to credit eases, helping the real economy
Evidence on Monetary Policy and Margins Affecting Prices (Ashcraft, Garleanu, and Pedersen (2009))

**Figure:** Bid for Super-Senior CMBS with Fed Funding above Cash Bid.

**Figure:** Market reaction to TALF-related announcements.

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**Figure**: The CDS-Bond basis, the LIBOR-GC repo Spread, and Credit Standards.

**Figure**: Investment Grade (IG) and High Yield (HY) CDS-Bond Bases, Adjusted for Their Margins.
Figure: Average Deviation from Covered-Interest Parity and the TED Spread.

- Pressure to free capital by moving assets off the balance sheet or titling portfolios towards low capital-requirement assets
- Basel requirement is similar to the margin constraint
  \[ \sum_i m^{\text{Reg}, i | \psi^i} \leq 1 \]
- Required return increased by \( m^{\text{Reg}, i | \psi^i} \)
Conclusion

• Margin-based general-equilibrium model
  • Strong asset pricing predictions
  • Predicts that a decline in fundamentals leads to
    • Binding constraints
    • Drop in Treasury and GC repo rates
    • Spikes in interest-rate spreads, risk premium, margin premium
    • Basis between securities with identical cash flows, related to margin differences

• Calibrated model predicts large margin premium in crisis

• Applications:
  • CDS-bond basis
  • Covered interest parity
  • Monetary policy, fed lending facilities
  • Banks’ incentives to use off-balance-sheet instruments