Government maturity structure twists
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Abstract

This paper examines the impact of government debt maturity restructuring on inflation and the real economy using a New Keynesian model that features a time-varying maturity structure of nominal debt and allows for changes in the monetary/fiscal policy mix. The irrelevance of open market operations changing the duration of government liabilities (holding the total market value of debt constant) is violated when the slope of the yield curve is nonzero in a fiscally-led policy regime. When the yield curve is downward-sloping, shortening the maturity structure increases the government discount rate, which generates fiscal inflation and an expansion in output. The opposite results obtain when the yield curve is upward-sloping. Conditional maturity restructuring policies depending on the slope of the yield curve can smooth macroeconomic fluctuations and offer substantial welfare benefits. In short, this paper highlights the importance of bond risk premia, in conjunction with the government debt valuation equation, as a transmission channel for open market operations.

Keywords: Fiscal theory of the price level, Government debt, Inflation, Bond risk premia, Markov-switching DSGE, Nonlinear solution methods

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1 Introduction

During the global financial crisis, central banks, constrained by the zero lower bound (ZLB) on nominal interest rates, conducted open market operations on an unprecedented scale that significantly altered the maturity structure of government debt. For example, in the second round of Quantitative Easing (QE) in 2010, the Federal Reserve announced the purchase of $600 billion of long-term Treasuries financed by selling an equivalent amount of short-term assets. The series of open market operations between 2008 and 2014 and the expansion in excess reserves reduced the average duration of U.S. government liabilities by over 20% (from 4.6 years to 3.6 years). While there is some evidence for the effectiveness of these policies in flattening the yield curve in short-run, the longer-term effects on yields are uncertain. Further, the effects of these operations on the real economy are even more controversial. This paper contributes to this debate by demonstrating how bond risk premia, in conjunction with the equilibrium restrictions imposed by the government debt valuation equation, provide a potentially important transmission channel for maturity restructuring policies to the macroeconomy.

To quantitatively examine the impact of government debt restructuring policies, we construct a New Keynesian model with several distinguishing features. First, households have recursive preferences (e.g., Epstein and Zin (1989)) which allows the model to generate realistic bond risk premia (e.g., Piazzesi and Schneider (2007), Bansal and Shaliastovich (2013), Rudebusch and Swanson (2012), and Kung (2014)). Second, the supply of nominal government bonds over various maturities is time-varying (e.g., Cochrane (2001) and Greenwood and Vayanos (2014)). Third, the monetary/fiscal policy mix are subject to recurrent stochastic changes between monetary- and fiscally-led regimes (e.g., Davig and Leeper (2007a), Davig and Leeper (2007b), Bianchi and Ilut (2013), and Bianchi and Melosi (2013)). In the monetary-led regime, the Taylor principle is satisfied and the monetary authority controls inflation while the fiscal authority is committed to stabilizing the value

\[1\] The Fed has been paying interest on reserves since 2008, so that reserves are effectively the same as short maturity treasuries (i.e., Cochrane (2014)).


\[3\] See, for example, the comments from Cochrane (2011a).

\[4\] See Williams (2014) for an survey on the effects of QE.
of debt by adjusting primary surpluses. In the fiscally-led regime, the fiscal authority determines the price level through the government budget constraint while the monetary authority passively stabilizes debt and anchors expected inflation. Leeper (1991) refers to the monetary-led regime as Active Monetary/Passive Fiscal (AM/PF) and the fiscally-led regime as Passive Monetary/Active Fiscal (PM/AF). Lastly, we solve the model using global projection methods to capture bond risk premia and account for the ZLB constraint jointly with rational expectations in an extension of the benchmark model.

In this framework, zero cost shocks to the maturity structure of nominal government debt affect inflation dynamics breaks Wallace (1981) neutrality when the nominal yield curve is nonzero and monetary/fiscal policy is characterized by the PM/AF regime.\(^5\) In this policy mix, the fiscal authority is not committed to adjusting surpluses to stabilize changes in the value of debt.\(^6\) For example, an increase in the government discount rate reduces the market value of debt (without expectations of higher future taxes). Consequently, households reduce their debt holdings by increasing demand for consumption goods, which translates to a rise in the price level (e.g., Cochrane (2011b)). In particular, the price level is determined by the ratio of nominal debt to the present value of surpluses, which is an equilibrium condition in this regime rather than a constraint that Cochrane (2005) refers to as the government debt valuation equation. Further, with sticky prices, restructuring polices impact the real economy.

We find that when the average yield curve is downward-sloping in the PM/AF regime, shortening the maturity structure, while holding the total market value of debt fixed initially, in the PM/AF regime generates fiscal inflation and an expansion in output. Increasing the relative weight on short term debt when the yield curve is downward sloping raises the return on the government bond portfolio. A higher government discount rate implies that aggregate demand increases and the price level increases via the government debt valuation equation. The effect on inflation is persistent due to a positive supply of longer maturity debt, which spreads the rise in the price level over several periods (Cochrane (2001)). With sticky prices, an increase in inflation expectations decreases the real rate and stimulates higher output. With a similar logic, shortening the maturity

\(^{5}\)Given concerns of advanced economies in reaching their fiscal limits during the global recession, expectations of entering a PM/AF regime appears to be a stronger possibility (e.g., Leeper (2013)).

\(^{6}\)See Leeper and Walker (2012) for a survey on monetary/fiscal policy interactions.
structure, when the average yield curve is upward-sloping, decreases the government discount rate, generates deflation and a contraction in output.

When the nominal yield curve is flat (e.g., risk neutrality holds or an interest rate peg) in the PM/AF regime, maturity restructuring operations, holding market value constant, leave the government discount rate unchanged, and, therefore, inflation and real variables are unaffected. Further, when the yield curve is nonzero in the AM/PF (monetary-led regime) without policy regime shifts, maturity restructuring affects government discount rates as in the PM/AF regime, which alters debt values. However, in the AM/PF regime, the fiscal authority is committed to adjusting future surpluses to absorb fluctuations in debt values, which leaves aggregate demand unchanged. More broadly, Ricardian equivalence holds in the AM/PF regime, so that households are insulated from any fiscal disturbances. In short, the slope of the nominal yield curve, in conjunction with the PM/AF policy regime, plays a key role in determining the effects of debt maturity restructuring.

To quantitatively evaluate the impact of the maturity restructuring shocks, it is important that the benchmark model generates a realistic term structure. The benchmark model produces sizeable bond risk premia through a similar mechanism as in Kung (2014) which generates countercyclical real marginal costs, and, therefore, a negative relation between expected consumption growth and inflation.⁷ With recursive preferences, these consumption and inflation dynamics lead to a positive and sizeable average nominal term spread (e.g., Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013)). The model can explain the level and persistence of nominal yields for various maturities. Also, the slope of the yield curve can forecast output growth and inflation.

Even when the yield curve is upward-sloping, on average, shortening the maturity structure in the PM/AF regime conditional on a temporarily downward-sloping yield curve has similar inflationary and stimulative effects as unconditionally reducing the maturity structure when the average yield curve is downward-sloping. For example, high levels of nominal debt generates persistent fiscal inflation to satisfy the government debt valuation equation. An increase in inflation leads the monetary authority to increase the short rate. A temporary increase in the short rate reduces

⁷Kung (2014) endogenizes the low-frequency component in productivity, which in the present model is assumed to be exogenous.
the slope of the yield curve by the expectations hypothesis. If the increase in inflation is sharp enough, the yield curve slopes downward. Additionally, the onset of a deep recession (e.g., due to a very bad productivity shock) is associated with a downward sloping yield curve due to the negative inflation-growth link. Thus, in the PM/AF regime, shortening the maturity structure, in deep recessions or under fiscal stress (high debt), can stimulate the economy and raise inflation expectations. Further, conditional maturity restructuring policies that shorten the maturity structure when the slope is downward sloping and lengthen the maturity structure when the yield curve is upward sloping smooths macroeconomic fluctuations and enhances welfare.

When there are regime shifts, then fiscal disturbances are non-neutral in the AM/PF regime due to expectations of possibly entering the PM/AF regime. In particular, shortening the maturity structure has similar qualitative effects on inflation and macroeconomic quantities as in the PF/AM regime. Quantitatively, if regimes are very persistent (i.e., small probability of switching), then the responses to fiscal shocks in the AM/PF regime are significantly weaker than in the PF/AM regime. Similarly, if the PM/AF regime is persistent, then, both quantitatively and qualitatively, the impact of maturity structuring shocks are similar to a fixed PM/AF regime. Bianchi and Ilut (2013) find that the AM/PF and PM/AF regimes are indeed very persistent via structural estimation.

To examine the impact of maturity restructuring shocks in a liquidity trap, we augment the benchmark model with preference shocks and a zero lower bound constraint on nominal interest rates (i.e., Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), and Bianchi and Melosi (2013)). Further, we fully capture the nonlinear effects of the ZLB and rational expectations using global projection methods (e.g., Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012) and Aruoba and Schorfheide (2013)). To connect to the extant literature, we focus on the AM/PF regime for conducting the policy experiments at the ZLB. We find that lengthening the maturity structure at the zero lower bound creates inflationary pressure and stimulates output. A large preference shock drives the short term nominal rate to zero, which makes the yield curve steeper and upward-sloping. Lengthening the maturity structure therefore increases the government discount rate, and, in turn, creates inflation pressure and stimulates output. In contrast, shortening the maturity structure at the ZLB exacerbates the deflationary pressure and output losses from the liquidity trap.
1.1 Related Literature

This paper relates to the literature examining how the interactions between monetary and fiscal policy determine the price level. This literature begins with Sargent and Wallace (1981) who show that permanent fiscal deficits have to eventually be financed by seignorage when the government only issues real debt. Further, the money creation leads to inflation. Building on this paper, the fiscal theory of the price level (FTPL) shows that when the government issues nominal debt and does not provide the necessary fiscal backing, deficits are linked to current and expected inflation through the government debt valuation equation, without necessarily relying on seignorage revenues (e.g., Leeper (1991), Sims (1994), Woodford (1994), Woodford (1995), Woodford (2001), Schmitt-Grohé and Uribe (2000), Cochrane (1999), Cochrane (2001), and Cochrane (2005)).

The paper connects to the literature examining the role of long-term public debt for policy. Angeletos (2002) and Buera and Nicolini (2004) solve for the optimal maturity structure of government debt and demonstrate how a portfolio of non-state-contingent debt of different maturities can replicate state-contingent debt. Leeper and Zhou (2013) highlight the importance of the maturity structure in determining optimal monetary/fiscal policy mix. Our paper is most closely related to Cochrane (2001) in emphasizing the importance of the maturity structure for determining inflation dynamics in the FTPL. We build on Cochrane (2001) by quantitatively examining debt maturity structure shocks on both inflation and the real economy in a New Keynesian framework featuring endogenous bond risk premia and stochastic shifts between the AM/PF and PM/AF regimes. A distinguishing feature of our paper is that we highlight the importance of the sign and magnitude of the yield curve slope for determining the effects of debt maturity twists.

The Markov-switching Dynamic Stochastic General Equilibrium (DSGE) framework builds on Davig and Leeper (2007a), Davig and Leeper (2007b), Farmer, Waggoner, and Zha (2009), Bianchi and Ilut (2013), and Bianchi and Melosi (2013). In particular, Bianchi and Ilut (2013) and Bianchi and Melosi (2013) also consider stochastic shifts between AM/PF and PM/AF policy regimes. However, our focus is on how PM/AF policy regime (or expectations of entering the regime) propagate maturity restructuring shocks.

Our paper relates to the theoretical literature studying the effects of unconventional monetary policy and transmission channels. Curdia and Woodford (2010), Gertler and Karadi (2011), and


The paper is organized as follows. Section 2 outlines the benchmark model. Section 3 explains the economic mechanism. Section 4 explores the quantitative implications of the model. Section 4 concludes.

2 Model

This section presents the benchmark model.
2.1 Households

The representative household is assumed to have Epstein-Zin preferences over streams of consumption \( C_t \) and labor \( L_t \):

\[
U_t = \left\{ (1 - \beta) \left( C_t^* \right)^{1-1/\psi} + \beta \left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\psi}} \right\}^{\frac{\theta}{\gamma}} \\
C_t^* = C_t \left( \frac{\bar{L}}{L_t} \right)^{\tau}
\]

where \( \gamma \) is the coefficient of risk aversion, \( \psi \) is the elasticity of intertemporal substitution, \( \theta \equiv \frac{1-\gamma}{1-1/\psi} \) is a parameter defined for convenience, \( \beta \) is the subjective discount rate, and \( \bar{L} \) is the agent’s time endowment. The time \( t \) budget constraint of the household is

\[
P_t C_t + B_{t+1} = P_t D_t + W_t L_t + R_t B_t - T_t,
\]

where \( P_t \) is the aggregate price level, \( B_t \) is the nominal market value of a portfolio of government bonds, \( D_t \) represents real dividends received from the intermediate firms, \( R_t \) is the gross nominal interest rate on the bond portfolio, \( W_t \) is the nominal competitive wage, and \( T_t \) are lump sum taxes from the government. The household chooses sequences of \( C_t, L_t, \) and \( B_t \) to maximize lifetime utility subject to the budget constraints.

The household’s intertemporal condition is

\[
E_t \left[ \frac{M_{t+1}}{\Pi_{t+1}} R_{t+1} \right] = 1
\]

where \( \Pi_{t+1} \) is the inflation rate between \( t \) and \( t + 1 \), and

\[
M_{t+1} = \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{1-\frac{1}{\psi}} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{U_{t+1}^{1-\gamma}}{E_t \left[ U_{t+1}^{1-\gamma} \right]} \right)^{1-\frac{1}{\psi}}
\]

is the real stochastic discount factor. The intratemporal labor condition is,

\[
\frac{W_t}{P_t} = \frac{\tau C_t}{L_t}.
\]
2.2 Firms

Production in our economy is comprised of two sectors: the final goods sector and the intermediate goods sector.

Final Goods A representative firm produces the final consumption goods $Y_t$ in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{it}$ as input in a constant elasticity of substitution (CES) production technology:

$$ Y_t = \left( \int_0^1 (X_{i,t})^{\frac{\nu-1}{\nu}} \right)^{-\frac{\nu}{\nu-1}} $$

where $\nu$ is the elasticity of substitution between intermediate goods. The profit maximization problem of the final goods firm yields the following isoelastic demand schedule\(^8\) with price elasticity $\nu$:

$$ X_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\nu} $$

where $P_t$ is the nominal price of the final goods and $P_{i,t}$ is the nominal price of the intermediate goods $i$. The inverse demand schedule is

$$ P_{i,t} = P_t Y_t^\frac{1}{\nu} X_{i,t}^{-\frac{1}{\nu}} $$

Intermediate Goods The intermediate goods sector is characterized by a continuum of monopolistic firms. Each intermediate goods firm produces $X_{i,t}$ using labor $L_{i,t}$:

$$ X_{i,t} = Z_t L_{i,t} - \Phi Z_t, $$

---

\(^8\)See the appendix for derivations.
where $Z_t$ represents an aggregate productivity shock common across firms, and is composed of both transitory and permanent components (e.g., Croce (2014) and Kung and Schmid (2014)):

\[
\ln(Z_t) = z^* + a_t + n_t \\
\quad a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{at} \\
\quad \Delta n_t = \rho_n \Delta n_t + \sigma_n \epsilon_{nt}
\]

where $z^*$ is the unconditional mean of $\log(Z_t)$, $\Delta n_t = n_t - n_{t-1}$, $\epsilon_{at}$ and $\epsilon_{nt}$ are standard normal shocks with a contemporaneous correlation equal to $\rho_{an}$. The low-frequency component in productivity, $\Delta n_t$, is used to generate long-run risks and sizeable risk premia (i.e., Bansal and Yaron (2004)). The fixed cost of production $\Phi$ is multiplied by $Z_t$ to ensure that it does not become trivially small along the balanced growth path.

Using the inverse demand function from the final goods sector, nominal revenues for intermediate firm $i$ can be expressed as

\[
\mathcal{P}_{i,t} X_{i,t} = \mathcal{P}_t Y_t^{\frac{1}{\nu}} [Z_t L_{i,t} - \Phi Z_t]^{1-\frac{1}{\nu}}
\]

The intermediate firms face a cost of adjusting the nominal price à la Rotemberg (1982), measured in terms of the final good as

\[
G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t) = \frac{\phi_R}{2} \left( \frac{\mathcal{P}_{i,t}}{\Pi_{ss} \mathcal{P}_{i,t-1}} - 1 \right)^2 Y_t
\]

where $\Pi_{ss} \geq 1$ is the steady-state inflation rate and $\phi_R$ is the magnitude of the costs.

The source of funds constraint is

\[
\mathcal{P}_t D_{i,t} = \mathcal{P}_{i,t} X_{i,t} - \mathcal{W}_t L_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t)
\]

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder’s value $V_t^{(i)} = V^{(i)}(\cdot)$ taking the pricing kernel $M_t$, the competitive nominal wage $\mathcal{W}_t$, and
the vector of aggregate state variables $\Upsilon_t = (P_t, Z_t, Y_t)$ as given:

$$V_t^{(i)}(P_{t-1}; \Upsilon_t) = \max_{P_i,t,L_i,t} \left\{ D_{i,t} + E_t \left[ M_{t+1} V^{(i)}(P_{i+1}; \Upsilon_{t+1}) \right] \right\}$$

subject to:

$$D_{i,t} = \frac{P_{i,t} X_{i,t}}{P_t} - W_t L_t - G(P_{i,t}, P_{i,t-1}; P_t, Y_t)$$

$$\frac{P_{i,t}}{P_t} = \left( \frac{X_{i,t}}{Y_t} \right)^{-\frac{1}{\nu}}$$

The corresponding first order conditions are derived in the appendix.

**Government** The flow budget constraint of the government is given by:

$$\sum_{i=1}^{N} B_t^{(i)} = \sum_{i=1}^{N} R_t^{(i)} B_t^{(i)} - S_t$$

where $B_t^{(i)}$ is the nominal debt of maturity $i$ issued at the end of period $t$, $R_t^{(i)}$ is the nominal interest paid on debt of maturity $i$, $S_t$ denotes the nominal value of primary surpluses. Following Bianchi and Melosi (2013), we assume that the government only levies lump-sum taxes and government expenditures are excluded. Thus, the primary surplus equals lump-sum taxes. Denoting the total market value of public debt by $B_t$ and scaling the budget constraint by nominal output $P_t Y_t$,

$$b_{t+1} = \frac{R_t^g}{\Pi_t \Delta Y_t} b_t - s_t$$

(1)

where $b_{t+1} \equiv B_{t+1}/(P_t Y_t)$, $s_t \equiv S_t/(P_t Y_t)$ and $R_t^g = \sum_{i=1}^{N} w_t^{(i)} R_t^{(i)}$ is the nominal gross interest paid on the portfolio of government debt. The government issues nominal debt at $N$ different maturities and we assume that each period the government retires outstanding debt and issues new debt over the $N$ maturities. The proportion of the debt financed with bonds of maturity $i$ is given by:

$$w_t^{(i)} = \bar{w}^{(i)} + \beta^{(i)} x_{mt},$$

(2)
where the constants $\bar{w}^{(i)}$'s determine the steady state maturity structure of debt and the $\beta^{(i)}$'s determine the sensitivity to $x_{mt}$, a stochastic process drives the dynamics of the maturity structure.

The evolution of $x_{mt}$ is given by:

$$x_{mt} = \rho_m x_{mt-1} + \sigma_m \epsilon_{mt}, \quad (3)$$

subject to $\sum_{i=1}^{N} \bar{w}^{(i)} = 1$ and $\sum_{i=1}^{N} \beta_t^{(i)} = 0, \forall t$. The latter condition ensures that the restructuring shocks do not change the total market value of debt initially so as to isolate the effects of change in maturity.

**Monetary and Fiscal Rules**

The central bank follows an interest rate feedback rule:

$$\ln \left( \frac{\mathcal{R}_t^{(1)}}{\mathcal{R}^{(1)}} \right) = \rho_r \ln \left( \frac{\mathcal{R}_{t-1}^{(1)}}{\mathcal{R}^{(1)}} \right) + (1 - \rho_r) \left( \rho_{\pi,\varrho_t} \ln \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \ln \left( \frac{\hat{Y}_t}{Y} \right) \right) + \sigma_r \epsilon_{rt}, \quad (4)$$

where $\mathcal{R}_{t+1}$ is the gross one-period nominal interest rate, $\Pi_t$ is inflation, $\hat{Y}_t$ is detrended output, and $\epsilon_{rt}$ is a normal i.i.d. shock. Note that the coefficient $\rho_{\pi,\varrho_t}$ is indexed by $\varrho_t$, which determines the policy mix at time $t$.

The fiscal authority adjusts primary surpluses according to the following rule:

$$s_t - s = \rho_s (s_{t-1} - s) + (1 - \rho_s) \delta_{b,\varrho_t} (b_t - b) + \sigma_s \epsilon_{st}. \quad \text{The coefficient } \delta_{b,\varrho_t} \text{ is also indexed by } \varrho_t \text{ and is therefore depends on the policy mix at time } t. \quad \text{The other determinacy region is the Passive Monetary/Active Fiscal (PM/AF) regime. The}$$

**Monetary/Fiscal Policy Mix**

Leeper (1991) distinguishes four policy regions in a model with fixed policy parameters. Two of the parameter regions admit a unique bounded solution for inflation. One of the determinacy regions is the Active Monetary/Passive Fiscal (AM/PF) regime, which is the familiar textbook case (e.g., Woodford (2003) and Galí (2008)). The Taylor principle is satisfied ($\rho_\pi > 1$) and the fiscal authority adjusts taxes to stabilize debt ($\delta_b > \left( \beta \Delta Y^{1-\frac{1}{r}} \right)^{-1} - 1$). In this policy mix, monetary policy determines inflation while fiscal policy passively provides the fiscal-backing to accommodate the inflation targeting objectives of the monetary authority.

The other determinacy region is the Passive Monetary/Active Fiscal (PM/AF) regime.
fiscal authority is not committed to stabilizing debt ($\delta_b < (\beta \Delta Y^{1-\frac{1}{\psi}})^{-1} - 1$), but instead the monetary authority passively accommodates fiscal policy ($\rho_\pi < 1$) by allowing the price level to adjust (to satisfy the government budget constraint). In this setting, fiscal policy determines inflation while monetary policy stabilizes debt and anchors expected inflation. Importantly, in this regime, fiscal disturbances, including non-distortionary taxation, have a direct impact on the price level via the government budget constraint because households know that changes in taxes will not be offset by future tax changes.\(^9\)

When both the fiscal and monetary authorities are active (AM/AF), no stationary equilibrium exists. When both authorities are passive, there exist multiple equilibria. Thus, in our regime-switching specification, we follow Bianchi and Melosi (2013) and assume that the policy mix alternates between AM/PF and PM/AF regimes according to a two-state Markov chain with the following transition matrix:

$$
\mathcal{M} = \begin{pmatrix}
    p_{MM} & 1 - p_{FF} \\
    1 - p_{MM} & p_{FF}
\end{pmatrix}
$$

where $p_{ij} = Pr(\varrho_{t+1} = i | \varrho_t = j)$ and $M$ denotes the monetary-led (AM/PF) regime and $F$ denotes the fiscally-led (PM/AF) regime.

3 Results

This section presents the results from the model. First, the calibration of the model is discussed and is followed by description of both the qualitative and quantitative implications.

3.1 Calibration

Table 1 presents the quarterly calibration. Panel A reports the values for the preference parameters. The elasticity of intertemporal substitution $\psi$ is set to 1.5 and the coefficient of relative risk aversion

\(^9\)In this regime, the government budget constraint is an equilibrium condition (rather than a constraint that has to hold for any price path), which Cochrane (2005) refers to as the government debt valuation equation.
\( \gamma \) is set to 10.0, which are standard values in the long-run risks literature (e.g., Bansal and Yaron (2004)). The subjective discount factor \( \beta \) is calibrated to 0.9945 to be consistent with the average return on the government bond portfolio (see Panel A of Table 2). The relative preference for leisure \( \tau \) is set to so that the household works one-third of the time in the steady-state.

Panel B reports the calibration of the technological parameters. The price elasticity of demand \( \nu \) is set to 2. The fixed cost of production \( \Phi \) is set such that dividend is zero in the deterministic steady state. The price adjustment cost parameter \( \phi_R \) is set to \( 10^{10} \). The mean growth rate of productivity \( z^* \) is set to obtain a mean growth rate of output of 2\%. The parameters \( \rho_a \) and \( \sigma_a \) are set to be consistent with the standard deviation and persistence of output growth, respectively (see Panel B of Table 2). The parameters \( \rho_n \) and \( \sigma_n \) are set to match the the standard deviation and persistence of expected productivity growth.

For parsimony, we assume that shocks to the short-run and long-run components of productivity are perfectly correlated \( (\epsilon_{a,t} = \epsilon_{n,t}) \). Indeed, Kung and Schmid (2014). Kung (2014) show that a stochastic endogenous growth framework produces a very strong positive correlation between these components (i.e., around 0.98). Kung (2014) illustrates that these productivity dynamics help to generate countercyclical real marginal costs, which implies a negative relation between inflation and expected growth. Further, these inflation and growth dynamics imply an upward sloping nominal yield curve (see Table 3). Overall, the model does a good job in matching the level and persistence of nominal yields. The volatility of yields falls a bit short of the empirical moments, however, Kung (2014) shows that incorporating conditional heteroscedastic productivity shocks help to fit the second moments better. Further, the model can explain the joint dynamics between bond yields and real variables. Table 4 shows the slope of the nominal yield curve can positively forecast consumption and output growth while negatively forecast inflation as in the data. Generating a realistic term structure is important given that it plays a central role for the propagation of the maturity restructuring shocks.

\[ ^{10} \text{For example, in a log-linear approximation, the parameter } \phi_R \text{ can be mapped directly to a parameter that governs the average price duration in a Calvo pricing framework. In this calibration, } \phi_R = 10 \text{ corresponds to an average price duration of 3.7 quarters, a standard value in the macroeconomics literature (e.g. Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012)).} \]

\[ ^{11} \text{In a separate appendix (available upon request), we relax the perfect correlation assumption and show our key term structure results hold for a wide a range of correlations.} \]
Panel C reports the calibration of the policy rule parameters. We set the steady state debt-to-GDP ratio to match the empirical average. The persistence and volatility parameters, $\rho_s$ and $\sigma_s$, are chosen to match primary surplus dynamics. The surplus rule parameter, $\delta_s$, is set to 0.05 and 0.00 in the AM/PF and PM/AF regimes, respectively. The interest rate rule parameter, $\rho_\pi$, is set to 1.5 and 0.4 in the AM/PF and PF/AM regimes, respectively. The calibration of these policy parameters, conditional on regime, are consistent with structural estimation evidence from Bianchi and Ilut (2013). The persistence of the interest rate rule $\rho_R$ is calibrated to 0.5. For parsimony, we abstract from monetary policy shock and output smoothing. Steady-state inflation $\Pi_{ss}$ is calibrated to match the average level of inflation. Following Bianchi and Melosi (2013), we assume that the transition matrix governing the dynamics of the policy/mix is symmetric: $p_{MM} = p_{FF} = p$ is set to 0.9875, implying that the economy stays on average 20 years in a given regime.

Panel D reports the calibration of the government bond supply dynamics. Fig. 1 plots the average maturity of net government liabilities held by households from Q1:2005 to Q3:2013. Note that the three QE operations and the Maturity Extension Program (MEP), show up quite visibly as each of these operations significantly shortened the maturity structure of debt.\footnote{Details on data construction are in the data appendix.} We calibrate the bond supply process to capture salient feature of maturity structure dynamics. We set $N = 40$, so that we include bonds up to a maturity of 10 years. The steady state maturity structure $\{\bar{w}^{(i)}\}$ is set to match the sample average. To calibrate the dynamics of the process driving the duration of government liabilities, $x_{mt}$, we proceed as follow. First, we run a principal component analysis on the panel data of maturity structure. Next, we extract the first principal component ($PC_1$) and fit the time series to an AR(1) process.\footnote{The first principal component explains about 62% of the cross-sectional dynamics of the debt maturity structure.} The estimates for $\rho_m$ and $\sigma_m$ are 0.9513 and 1.28%, respectively. The loadings $\{\beta^{(i)}\}$ for bonds of each maturity are also obtained from the first principal component.

### 3.2 Fixed Regimes

To illustrate the economic mechanisms more clearly, we begin the analysis by first assuming fixed policy parameters (i.e., $p_{MM} = p_{FF} = 1$) before moving to the benchmark model with regime
shifts. In the PM/AF regime, zero cost maturity structure shocks impact inflation when the slope of the yield curve is non-zero at the time of the shock. A nonzero slope implies that the changes in the composition of the government bond portfolio affect the portfolio return. Changes in the government discount rate, in turn, affect inflation through the government debt valuation equation, which can be seen by iterating on Eq. (1) and rewriting in present value form (see the appendix for derivations):

\[
b_t = E_t \left[ \sum_{i=0}^{\infty} \frac{s_{t+i}}{\Pi_{t+j}^{-1} \Delta Y_{t+j}^{-1} R_{t+j}^2} \right], \tag{5}
\]

and note that total debt \( b_t \) is chosen at time \( t - 1 \). Importantly, Eq. (5) illustrates that any net changes to current or expected surpluses, discount rates, growth rates are absorbed by inflation in this regime. Since there is long-term debt outstanding, changes in inflation are spread out over multiple periods, and the timing is determined by the relative proportion of bonds at each maturity (i.e., Cochrane (2001)). This inflation timing channel can be seen more clearly by rearranging Eq. (5):

\[
E_t \left[ \sum_{i=0}^{\infty} M_{t,t+i} \frac{Y_{t+i}}{Y_{t-1}} s_{t+i} \right] = \left( \frac{b_t^{(1)}}{P_t^{(1)}} \right) \frac{1}{\Pi_t} + \left( \frac{b_t^{(2)}}{P_t^{(2)}} \right) E_t \left[ \frac{M_{t+1}}{\Pi_t \Pi_{t+1}} \right] + \cdots + \left( \frac{b_t^{(n)}}{P_t^{(n)}} \right) E_t \left[ \frac{\prod_{j=1}^{n-1} M_{t+j}}{\prod_{j=0}^{n-1} \Pi_{t+j}} \right], \tag{6}
\]

where \( M_{t,t+i} \) is the discount factor between \( t \) and \( t + i \) and \( P_t^{(i)} \) is the price at time \( t \) of a nominal bond of maturity \( i \). Note that a shorter maturity structure implies less inflation smoothing.

Fig. 2 plots impulse response functions to a positive maturity restructuring shock (\( \epsilon_{mt} > 0 \)), which shortens the maturity structure, when the average nominal yield curve is upward-sloping (solid line), flat (thin line with squares), and downward-sloping (dashed line). Recall that the maturity structure variable, \( x_{mt} \), follows an independent stochastic process. Thus, when the average yield curve is upward-sloping (the benchmark case), shortening the maturity structure, on average, decreases the government discount rate. A lower discount rate drives down the price level and inflation through the government debt valuation equation. The intuition is that a lower discount rate raises the value of debt, so households increase demand for bonds and reduce demand for

15
consumption goods. A fall in aggregate demand for consumption goods drives down prices and inflation. When prices are flexible, the prices will fall such that the households will be satisfied with their original consumption plan. However, when prices are sticky, the price adjustment is sluggish so that prices are too high relative to the flexible price case. Consequently, production falls and the real rate rises. Also, a fall in inflation lowers the short nominal rate (FFR) due to the interest rate rule, which increases the slope of the yield curve.

When the average slope of the yield curve is downward-sloping the responses to a positive maturity restructuring shock are reversed from the upward-sloping case. Shortening the maturity in this case raises the government discount rate, which lowers the value of debt. Consequently, households want to unload their bond positions by increasing demand for consumption goods. Higher aggregate demand creates inflationary pressure. The presence of sticky prices implies that prices are too low relative to flexible prices, so production increases and the real rate falls. A rise in inflation increases the short nominal rate, which decreases the slope of the yield curve.

When the average yield curve is flat, zero cost maturity restructuring shocks have no average effect on inflation or real economic variables. Changes in the maturity structure in this case do not affect the return on the government bond portfolio. Also, the maturity restructuring shocks are neutral in the AM/PF regime, even when the slope is nonzero. In this regime, any changes to the present value of surpluses (or equivalently to the value of debt) are offset by adjustments in future taxes. For example, if the government discount rate falls, the value of debt increases and the fiscal authority increases taxes to stabilize debt.

Fig. 3 plots the responses to debt restructuring for high (solid line) and low (dashed line) uncertainty cases in the benchmark model. Higher uncertainty raises risk premia and steepens the slope of the yield curve. A steeper slope implies that shortening the maturity structure reduces the government discount rate more,

Even when the average nominal yield curve is upward-sloping, conditional maturity restructuring can produce similar responses to unconditional restructuring shocks when the average yield curve is downward-sloping. Fig. 4 plots impulse response functions to a positive maturity structure

\(^{14}\)A flat average yield curve can be obtained, for example, by assuming that agents are risk neutral. For the flat yield curve case, we assume a nominal interest peg which implies a flat and constant term structure. Thus, restructuring shocks are always neutral (and not only average).
shock for high debt (dashed line) and low debt (solid line) from the benchmark model with an upward-sloping average yield curve. In the PM/AF regime, high debt (without offsetting future taxes) makes agents feel wealthier. This wealth effect drives up aggregate demand and creates inflationary pressure. Higher inflation increases the short nominal rate, which lowers the slope of the yield curve. If the level of debt is sufficiently high, the yield curve is downward sloping. Thus, shortening the maturity structure when debt is high (e.g., during fiscal stress) increases the discount rate of the government. An increase in the discount rate generates more inflation and an expansion in output, as with unconditional restructuring shocks when the average yield curve is downward-sloping.

As in the case with high debt, at the onset of a large recession, the nominal yield curve is downward-sloping. In the model, the negative inflation-growth link implies that low expected growth is associated with a rise in inflation. An increase in inflation leads to a rise in the short term nominal rate, and the yield curve is downward-sloping if the recession is large enough. Thus, shortening the maturity structure when expected growth is inflationary and expansionary. Fig. 5 plots the impulse response functions to a positive maturity shock at the onset of a recession (dashed line) and at the steady-state (solid line). In short, shortening the maturity structure can be an effective tool in stimulating the economy during large recessions and in times of fiscal stress.

Motivated by the conditional restructuring examples relating to high debt and low expected growth, we also consider a maturity restructuring policy that depends directly on the slope of the nominal yield curve:

$$x_{m,t} = \rho_{m}x_{m,t-1} + \rho_{m,ys} \left( y_t^{\delta Y} - y_t^{1Q} \right) + \sigma_m \epsilon_{m,t}$$  \hspace{1cm} (7)

A positive coefficient ($\rho_{m,ys} > 0$) implies that the government shortens the maturity structure when yield curve is upward-sloping and lengthens the maturity structure when the yield curve is downward-sloping. A negative coefficient implies the opposite policy. Fig. 6 plots comparative statistics for varying $\rho_{m,ys}$ from -1 to 1. Note that more negative values of $\rho_{m,ys}$ smooth macroeconomic fluctuations, reduce risk premia, and improves welfare. In contrast, more positive values of $\rho_{m,ys}$ increase consumption and inflation volatility, and, in turn, increases welfare costs. Negative values
of $\rho_{m,ys}$ shorten the maturity structure when the yield curve is downward sloping, which stimulates the economy and generates fiscal inflation exactly during low expected growth states. Using similar logic, positive values for $\rho_{m,ys}$ deepen recessions and increase deflationary pressure.

### 3.3 Regime-Switching

With stochastic and recurrent policy shifts, fiscal shocks are no longer neutral in the AM/PF regime due to the possibility of entering the PM/AF regime. Fig. 7 plots impulse response function to a positive surplus shock ($\epsilon_{st} > 0$) conditional on being in the AM/PF (solid line) and PM/AF (dashed line) regimes. Note that the surplus shock is less persistent in the AM/PF regime due to the dependence of the surplus rule on debt that smooths the shock. In PF/AM regime, a positive surplus shock (without the expectation of lower future taxes), makes agents feel poorer. This negative wealth effect makes agents demand less consumption goods, which lowers prices. Due to sticky prices, the price level is too high relative to the flexible price case, so production falls and the real rate rises. A fall in inflation also lowers the short-term nominal rate, which steepens the slope of the yield curve. The responses to the surplus shock in the AM/PF regime are qualitatively similar due to agents expectations of possibly entering in the PM/AF. However, since regimes are persistent and the probability of switching is small, quantitatively, the responses are significantly smaller.

As agents are less insulated from fiscal shocks in the in the PM/AF regime, macroeconomic volatility is higher than in the AM/PF regime (see Panel A in Table 2). Since the fiscal shocks induce positive correlation between inflation and macroeconomic quantities, the negative correlation (induced by the productivity shocks) between inflation and consumption growth is weaker in the PM/AF regime. Fig. 8 plots impulse response functions to a positive productivity shock. A weaker correlation reduces bond risk premia (see Panel B in Table 2).

Fig. 9 plots impulse response functions for a positive maturity shock in the AM/PF (solid line) and PM/AF (dashed line) regimes. The responses from the PM/AF with regime shifts are quite similar to the fixed regime cases given that the regimes are persistent. Although the responses are somewhat smaller, the magnitudes are quantitatively significant. Shortening the maturity structure by 0.18 years (as in QE2), reduces output by just less than one percentage point. The responses
in the AM/PF regime are qualitatively similar to those in the PM/AF regime, but significantly smaller. Fig. 10 illustrates that shortening the maturity structure conditional on high debt in the PM/AF regime stimulates output as in the fixed regime case. The debt-to-GDP in the high debt case (dashed line) is set to be 50% higher than in the steady-state case (solid line), as in the onset of the financial crisis in 2008 and reducing the maturity structure by 0.18 years increases output by around 1%.

To address the efficacy of maturity restructuring policies in a liquidity trap, we augment the benchmark model with regime shifts to include preference shocks and a zero lower bound (ZLB) constraint on the short-term nominal rate. To capture preference shocks, the time discount factor of the agent is assumed to follow:

$$\ln(\beta_t) = (1 - \rho_\beta) \ln(\beta) + \rho_\beta \ln(\beta_{t-1}) + \sigma_\beta \epsilon_\beta.$$  

The ZLB constraint is given by

$$\ln\left(\frac{R_t^{(1)}}{R^{(1)}}\right) = \max\left\{0, \rho_r \ln\left(\frac{R_{t-1}^{(1)}}{R^{(1)}}\right) + (1 - \rho_r)\left(\rho_{\pi,\phi} \ln\left(\frac{\Pi_t}{\Pi}\right) + \rho_y \ln\left(\frac{\hat{Y}_t}{Y}\right)\right) + \sigma_r \epsilon_{rt}\right\}.$$  

We focus on the AM/PF regime to relate to the literature on the zero lower bound and also because the ZLB rarely binds in the PM/AF regime.\textsuperscript{15} Fig. 11 plots impulse response functions to a negative maturity restructuring shock at the ZLB (dashed line) and away from the ZLB (solid line). Away from the ZLB (i.e., in steady state), the average yield curve is upward-sloping, so a maturity restructuring shock reduces inflation and output as discussed above. To reach the ZLB, we assume a large preference shock that drives nominal rates to zero, which binds, on average, for around four quarters. This shock steepens the slope of the yield curve, which amplifies the responses relative to being away from the ZLB, and further exacerbates the problems in a liquidity trap.

To alleviate deflationary pressure and output losses associated with being at ZLB, instead requires a lengthening of the maturity structure rather than shortening. Fig. 12 plots impulse

\textsuperscript{15}In the PM/AF regime, the interest rule is less sensitive to changes in inflation, so it is less likely that the deflationary shock sends the short rate to the ZLB.
response functions to a negative maturity restructuring shock at the ZLB (dashed line) and away from the ZLB (solid line). At the ZLB, lengthening the maturity structure by 0.18 years raises expected inflation by 4 basis points and output by over 20 basis points at the peak of the responses.

4 Conclusion

This paper examines the impact of nominal government debt maturity restructuring shocks in a DSGE model that features realistic bond risk premia and stochastic changes in the monetary/fiscal policy mix. We show that zero cost maturity restructuring polices are non-neutral when the slope of the nominal yield curve is nonzero in (or the possibility of entering in) a fiscally-led policy regime. When the slope is nonzero, maturity restructuring changes the government discount rate, which affects the price level through the government debt valuation equation. Conditional restructuring policies that shorten the maturity structure when the yield curve is downward-sloping, such as during times of fiscal stress or at the onset of a recession, creates fiscal inflation and an expansion in output. Market timing policies that shorten the maturity structure when the yield curve is downward-sloping and lengthen the maturity structure when the yield curve is upward-sloping smooth macroeconomic fluctuations and improve welfare. In a liquidity trap, lengthening the maturity structure can be effective in attenuating deflationary pressure and output losses. In short, this paper highlights the importance of bond risk premia, in conjunction with the monetary/fiscal policy mix, for open market debt maturity restructuring policies.
Appendix A. Numerical Procedure

**Single Regime**  For each of the regimes the model is solved using a global method following Judd, Maliar, and Maliar (2012) and Judd, Maliar, Maliar, and Valero (2013). A subset of the policy functions are approximated by standard ordinary polynomials of the state variables. The state variables are:

\[
S_t = (r_{t-1}, a_{t-1}, s_{t-1}, b_{t-1}, x_{mt}, Y_{t-1}, \Delta n_{t-1}, \beta_t, + \epsilon_{at}, \epsilon_{nt}, \epsilon_{st}, \{P_{t-1}^{(n)}\}^N_{i=2}),
\]

where \(r_{t-1}\) is the nominal one-period risk-free rate, \(a_{t-1}\) is the transitory productivity shock, \(s_{t-1}\) is the governments’ surplus, \(b_{t-1}\) is the total debt of the government, \(x_{mt}\) is the stochastic process driving the maturity structure, \(Y_{t-1}\) is the final consumption goods, \(\Delta n_{t-1}\) is the permanent productivity shock, \(\beta_t\) is the subjective discount rate, \(\epsilon_{at}\) is the innovation to the transitory productivity shock, \(\epsilon_{nt}\) is the innovation to the permanent productivity shock, \(\epsilon_{st}\) is the innovation to the government’s surplus, and \(\{P_{t-1}^{(n)}\}^N_{i=2}\) are the nominal bond bond prices. The approximated policy functions are:

\[
G = (F_t, C_t, U_t, \{P_{t-1}^{(n)}\}^N_{i=2}),
\]

where

\[
F_t = \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right) \frac{\Pi_t}{\Pi_{ss}},
\]

\(C_t\) is the household’s consumption, \(U_t\) is the household’s utility, and \(\{P_{t-1}^{(n)}\}^N_{i=2}\) are the nominal bond prices.

The model is solved by finding the set of polynomial coefficients \(\Theta\) that minimizes the mean squared residuals for the approximated decision rules over a fixed grid. For each point \(j\) on the
grid the residuals are calculated as:

\[ R^j_1 = \phi R^j_t Y^j_t - E^j_t \left[ M_{t+1} \phi R^j_{t+1} Y_{t+1} \right] - \Lambda_t, \]

\[ R^j_2 = E^j_t \left[ M_{t+1} R^W_{t+1} \right] - 1, \]

\[ R^j_3 = U^j_t - \left\{ (1 - \beta) \left( C^j_t \right)^{1-1/\psi} + \beta \left( E^j_t \left[ U^{1-\gamma}_{t+1} \right] \right)^{\frac{1}{3}} \right\}^{\frac{1}{1-1/\psi}}, \]

\[ R^j_{i+2} = E^j_t \left[ \frac{M_{t+1}}{P_{t+1}} P^{(i-1)}_{t+1} \right] - P^{(i)}_t \forall i = 2..N. \]

\( R^j_1 \) is calculated using the first order condition (in equilibrium) of the firms in the intermediate goods sector, \( R^j_2 \) is calculated using the Euler equation for the return on the wealth portfolio, \( R^j_3 \) is computed using the value function equation. Finally, \( \{R^j_i\}_{i=4}^N \) are computed using the Euler equations for the nominal bonds.

The grid on the state variables space is calculated in 4 steps. First, the model is solved using a second order perturbation approximation. The solution is used to find an initial guess for the set of coefficients \( \Theta \). Second, the model is simulated and the principal components of the state variables are calculated. Third, an auxiliary grid on the principal components space is calculated using the Smolyak algorithm. Finally, the grid on the state variables space is calculated by performing a linear transformation of the auxiliary grid calculated in the previous step.

The Smolyak algorithm is used for the auxiliary grid because it is a highly efficient method to calculate a sparse grid in a hypercube. The drawback of the Smolyak algorithm is that the points are not chosen to maximize the number of points on the region of the state space where the model’s ergodic distribution is located. The Smolyak algorithm is improved by adapting it to the characteristics of the model using the principal components transformation.

Second order polynomials are used for each of the regimes. The minimization is done using a numerical optimizer. To improve the speed of the code analytical gradient and monomial integration are used. The mean square error is of the order of \( 10^{-7} \) for the monetary-led regime and \( 10^{-6} \) for the fiscally-led regime.

**Regime Switching** The regime switching model is also solved using the global approximation method. In this case the decision rules are approximated by a piece-wise polynomial, as in Aruoba
and Schorfheide (2013). Let $\mathcal{L}$ be a policy function, the policy function is approximated as:

$$\hat{\mathcal{L}} = 1_F p_F + 1_M p_M,$$

(A.1)

where $1_j$ is an indicator function that takes value one in regime $j$ and zero otherwise, and $p_j$ is a polynomial. Equation A.1 shows that for each regime a different polynomial is used, $p_F$ for the fiscally-led regime and $p_M$ for the monetary-led regime. The use of piece-wise polynomials allows for a more flexible structure to fit the model. The initial guesses for the polynomial coefficients are the solutions for each of the single regime models. The grid is calculated as the union of the two single regime grids (fiscally-led regime and monetary-led regime). As in the single regime case, second order standard ordinary polynomials are used for each of the regime-specific polynomials. A numerical optimizer with analytical gradient is used in this case as well. The mean square error is of the order of $10^{-6}$, in line with the mean square errors for the single regime models.

### Appendix B. Data

We obtain quarterly data for consumption, and output from the Bureau of Economic Analysis (BEA). Consumption is measured as real personal consumption expenditures (DPCERX1A020NBEA). Output is measured as real gross domestic product (GDPC1). Inflation is computed by taking the log return on the Consumer Price Index for All Urban Consumers (CPIAUCSL), obtained from the Bureau of Labor Statistics (BLS). Monthly yield data are from CRSP. Nominal yield data for maturities of 4, 8, 12, 16, and 20 quarters are from the CRSP Fama-Bliss discount bond file. The one-quarter nominal yield is from the the CRSP Fama risk-free rate file. Finally we build our bond supply maturity structure data using the same methodology as Doepke and Schneider (2006) and Greenwood and Vayanos (2014). In particular each month we collect the complete history of U.S. government bonds issued from the CRSP historical bond database. We then break the stream of each bonds cash flows into principal and coupon payments. Summing the streams from each outstanding bond vintage over their respective maturity give us the monthly maturity structure of government debt. The sample period runs from Q1-1964 to Q3-2013.
Appendix C. Household Problem

The time-$t$ Lagrangian writes

$$U_t = \left\{ (1 - \beta) (C_t^*)^{1-1/\psi} + \beta \left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\psi}} \right\}^{\frac{1}{1-\psi}} + \lambda_t \left[ D_t + \frac{W_t}{P_t} L_t + R_t \frac{B_t}{P_t} + T_t - C_t - \frac{B_{t+1}}{P_t} \right]$$

The first order conditions are

$$[C_t]: (1 - \beta) U_{t+1}^{1/\psi} (C_t^*)^{-1/\psi} \left( \frac{\bar{L}}{L_t} \right)^\tau = \lambda_t \quad \text{(C.2)}$$

$$[B_{t+1}]: U_{t+1}^{1/\psi} \beta E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{\psi} - 1} E_t \left[ U_{t+1}^{1-\gamma} \frac{P_t}{P_t} R_{t+1} \right] = \lambda_t \quad \text{(C.3)}$$

$$[L_t]: (1 - \beta) \tau U_{t+1}^{1/\psi} (C_t^*)^{-1/\psi} C_t \left( \frac{\bar{L}}{L_t} \right)^{\tau - 1} \bar{L} = \lambda_t E_t \frac{M_t}{P_t} \quad \text{(C.4)}$$

Dividing both sides of (C.3) by $(\lambda_t/U_{t+1}^{1/\psi})$ and using (C.2) to replace $\lambda_{t+1}$, we get

$$\beta E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{\psi} - 1} E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} U_{t+1}^{1-\gamma} \frac{P_t}{P_t} R_{t+1} \right] = 1$$

or

$$E_t \left[ \frac{M_{t+1}}{P_{t+1}} R_{t+1} \right] = 1$$

where

$$M_{t+1} = \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{1-\frac{1}{\psi}} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{U_{t+1}^{1-\gamma}}{E_t U_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\psi}}$$

The intratemporal decision is obtained by plugging (C.2) into (C.4).

Appendix D. Monopolistic firm problem

The maximization problem of the individual firm is

$$V_t^{(i)} (P_{i,t-1}; Y_t) = \max_{P_i, t, L_{i,t}} \left\{ D_{i,t} + E_t \left[ M_{t+1} V^{(i)} (P_{i,t}; Y_{t+1}) \right] \right\}$$
subject to:

\[ D_{i,t} = \frac{P_{i,t}}{P_t} X_{i,t} - \frac{W_t}{P_t} L_{i,t} - G(P_{i,t}, P_{t,t-1}; P_t, Y_t) \]

\[ \frac{P_{i,t}}{P_t} = \left( \frac{X_{i,t}}{Y_t} \right)^{-\frac{1}{\nu}} \]

After plugging the definition of \( D_{i,t} \), the Lagrangian of the problem is

\[ \mathcal{L}_t = Y_t^{\frac{1}{\nu}} \left[ Z_t L_{i,t} - \Phi Z_t \right]^{1-\frac{1}{\nu}} - \frac{W_t}{P_t} L_{i,t} - \frac{\phi_R}{2} \left( \frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right)^2 Y_t \]

\[ + \Lambda_{i,t} \left( \frac{P_{i,t}}{P_t} - Y_t^{\frac{1}{\nu}} \left[ Z_t L_{i,t} - \Phi Z_t \right]^{-\frac{1}{\nu}} \right) \]

\[ + E_t \left[ M_{t+1} V^{(i)}_{t+1} (P_{i,t}; Y_{t+1}) \right] \]

The first order conditions are

\[ \frac{\Lambda_{i,t}}{P_t} = \phi_R \left( \frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss} P_{i,t-1}} - E_t \left[ M_{t+1} \phi_R \left( \frac{P_{i,t+1}}{\Pi_{ss} P_{i,t}} - 1 \right) \frac{Y_{t+1} P_{i,t+1}}{\Pi_{ss} P_{i,t}^{\nu}} \right] \]

\[ \frac{W_t}{P_t} = \left( 1 - \frac{1}{\nu} \right) Y_t^{\frac{1}{\nu}} X_{i,t}^{\frac{1}{\nu}} Z_t + \Lambda_{i,t} \left( \frac{1}{\nu} \right) Y_t^{\frac{1}{\nu}} X_{i,t}^{\frac{1}{\nu}} Z_t \]

This specification yields a symmetric equilibrium in which \( P_{i,t} = P_t \), \( X_{i,t} = X_t \), \( L_{i,t} = L_t \), \( D_{i,t} = D_t \), and \( V^{(i)}_t = V_t \). The equilibrium condition for the economy are:

\[ \frac{W_t}{P_t} = \left( 1 - \frac{1}{\nu} \right) Z_t + \Lambda_{i,t} \left( \frac{1}{\nu} \right) Z_t \frac{Y_t}{Y_t} \]

\[ \Lambda_t = \phi_R \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right) \frac{\Pi_t Y_t}{\Pi_{ss}} - E_t \left[ M_{t+1} \phi_R \left( \frac{\Pi_{t+1}}{\Pi_{ss}} - 1 \right) \frac{Y_{t+1} \Pi_{t+1}}{\Pi_{ss}} \right] \]

where \( \Lambda_t \) is the Lagrange multipliers on the inverse demand constraint.

**Appendix E. Present value of surpluses**

The flow budget constraint of the government (1) can be written as

\[ b_t = \frac{\Pi_t \Delta Y_t}{\Pi_{ss} R_g} (b_{t+1} + s_t) \]  

(E.5)
where \( s_t = \tau_t - g_t \) is the government surplus. Leading expression (E.5) for one period and taking the conditional expectation at time \( t \),

\[
E_t[b_{t+1}] = E_t \left( \frac{\Pi_{t+1} \Delta Y_{t+1} b_{t+2}}{R^g_{t+1}} \right) + E_t \left( \frac{\Pi_{t+1} \Delta Y_{t+1}}{R^g_{t+1}} s_{t+1} \right) \tag{E.6}
\]

Iterating forward on \( E_t[b_{t+1}] \) using the law of iterated expectation and assuming a transversality condition on real debt,

\[
E_t[b_{t+1}] = E_t \left( \sum_{i=1}^{\infty} \frac{P_{t+i} Y_{t+i}}{P_{t-1} Y_{t-1} \prod_{j=1}^{i} \frac{1}{R^g_{t+j}}} s_{t+i} \right) \tag{E.7}
\]

Using equations (E.5) and (E.7) together, the present value of government surpluses is

\[
b_t = E_t \left[ \sum_{i=0}^{\infty} \frac{s_{t+i}}{\prod_{j=0}^{i} \Pi_{t+j} \Delta Y_{t+j} R^g_{t+j}} \right] \tag{E.8}
\]
References


Table 1: Quarterly Calibration

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<td>$\rho_s$</td>
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<td>1.5 / 0.4</td>
</tr>
<tr>
<td>$p$</td>
<td>Switching probability</td>
<td>0.9875</td>
</tr>
<tr>
<td>D. Bond Supply</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Steady state Debt-to-GDP ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Persistence of $x_{mt}$</td>
<td>0.958</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Volatility of $e_{mt}$</td>
<td>1.28%</td>
</tr>
</tbody>
</table>

This table reports the parameter values used in the quarterly calibration of the model. The table is divided into four categories: Preferences, Production, Policy, and Bond Supply parameters.
Table 2: Macroeconomic Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>AM/PF</th>
<th>PM/AF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Means</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Delta y)$ (in %)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$E(y^{(20)} - y^{(1)})$ (in %)</td>
<td>1.02</td>
<td>1.01</td>
<td>1.88</td>
<td>0.14</td>
</tr>
<tr>
<td>$E(r_g)$ (in %)</td>
<td>5.47</td>
<td>5.81</td>
<td>6.18</td>
<td>5.45</td>
</tr>
<tr>
<td><strong>B. Standard Deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta y)$ (in %)</td>
<td>2.22</td>
<td>2.53</td>
<td>2.34</td>
<td>2.72</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>1.64</td>
<td>1.06</td>
<td>0.36</td>
<td>1.40</td>
</tr>
<tr>
<td>$\sigma(\Delta l)/\sigma(\Delta y)$</td>
<td>0.92</td>
<td>0.47</td>
<td>0.13</td>
<td>0.63</td>
</tr>
<tr>
<td>$\sigma(y^{(20)} - y^{(1)})$ (in %)</td>
<td>1.05</td>
<td>0.55</td>
<td>0.26</td>
<td>0.37</td>
</tr>
<tr>
<td>$\sigma(r_g)$ (in %)</td>
<td>4.53</td>
<td>0.93</td>
<td>0.92</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>C. Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr$(\pi, \Delta c)$</td>
<td>-0.56</td>
<td>-0.10</td>
<td>-0.17</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

This table presents the means, and standard deviations for key macroeconomic variables from the data and the model. The model is calibrated at a quarterly frequency and the reported statistics are annualized.

Table 3: Term structure

<table>
<thead>
<tr>
<th></th>
<th>1Q</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
<th>5Y - 1Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal yields</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (Model) (in %)</td>
<td>5.15</td>
<td>5.47</td>
<td>5.74</td>
<td>5.93</td>
<td>6.05</td>
<td>6.16</td>
<td>1.00</td>
</tr>
<tr>
<td>Mean (Data) (in %)</td>
<td>5.03</td>
<td>5.29</td>
<td>5.48</td>
<td>5.66</td>
<td>5.80</td>
<td>5.89</td>
<td>1.02</td>
</tr>
<tr>
<td>Std (Model) (in %)</td>
<td>0.51</td>
<td>0.47</td>
<td>0.42</td>
<td>0.42</td>
<td>0.44</td>
<td>0.48</td>
<td>0.55</td>
</tr>
<tr>
<td>Std (Data) (in %)</td>
<td>2.97</td>
<td>2.96</td>
<td>2.91</td>
<td>2.83</td>
<td>2.78</td>
<td>2.72</td>
<td>1.05</td>
</tr>
<tr>
<td>AC1 (Model)</td>
<td>0.96</td>
<td>0.91</td>
<td>0.92</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>AC1 (Data)</td>
<td>0.93</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.74</td>
</tr>
</tbody>
</table>

This table presents summary statistics for the term structure of interest rates: the annual mean, standard deviation, and first autocorrelation of the one-quarter, one-year, two-year, three-year, four-year, and five-year nominal yields and the 5-year and one-quarter spread from the model and the data. The model is calibrated at a quarterly frequency and the moments are annualized.
Table 4: Forecasts with the Yield Spread

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizon (in quarters)</td>
<td>1</td>
</tr>
<tr>
<td>A. Output</td>
<td>β</td>
<td>β</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>S.E.</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>R²</td>
</tr>
<tr>
<td>β</td>
<td>1.023</td>
<td>0.987</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.306</td>
<td>0.249</td>
</tr>
<tr>
<td>R²</td>
<td>0.067</td>
<td>0.148</td>
</tr>
<tr>
<td>B. Consumption</td>
<td>β</td>
<td>β</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>S.E.</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>R²</td>
</tr>
<tr>
<td>β</td>
<td>0.731</td>
<td>0.567</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.187</td>
<td>0.163</td>
</tr>
<tr>
<td>R²</td>
<td>0.092</td>
<td>0.136</td>
</tr>
<tr>
<td>C. Inflation</td>
<td>β</td>
<td>β</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>S.E.</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>R²</td>
</tr>
<tr>
<td>β</td>
<td>-1.328</td>
<td>-1.030</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.227</td>
<td>0.315</td>
</tr>
<tr>
<td>R²</td>
<td>0.180</td>
<td>0.157</td>
</tr>
</tbody>
</table>

This table presents output growth, consumption growth, and inflation forecasts for horizons of one, four, and eight quarters using the five-year nominal yield spread from the data and the model. The $n$-quarter regressions, $\frac{1}{n}(x_{t+1} + \cdots + x_{t+n}) = \alpha + \beta(y_{t}^{(5)} - y_{t}^{(1Q)}) + \epsilon_{t+1}$, are estimated using overlapping quarterly data and Newey-West standard errors are used to correct for heteroscedasticity.
This figure plots the average maturity structure of government held by the public from Q1-2005 to Q3-2013.
Figure 2: Maturity Restructuring Shocks with Different Slopes

This figure plots impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 ($e_{mt}$) in the PM/AF regime. Results are reported for three shapes of the unconditional term structure: upward sloping (solid line), flat (squares), and downward sloping (dashed line). The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
This figure plots impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 ($\epsilon_{mt}$) in the PM/AF regime. Results are reported for the benchmark calibration (solid line), and when all shock volatilities, except for $\epsilon_{mt}$, are cut in half (dashed line). The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
This figure compares the impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 \((e_{mt})\) in the PM/AF regime when debt-to-GDP is at the steady state (solid line) or higher than the steady state (dashed line). The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
This figure reports the impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 ($e_{mt}$) in the PM/AF regime. The response in a recession (dashed) is obtained by shocking the economy with a negative productivity shock ($e_{at}$) at the time of the restructuring. The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
Figure 6: Market Timing Restructuring

This figure plots the welfare cost, the yield spread (annualized percentage), the standard deviation of consumption growth and the standard deviation of inflation for various debt restructuring policies. The $y$-axis is normalized to the benchmark case of exogenous debt management, i.e. $\rho_{m, yldsprd} = 0$ (differenced for first moments and divided by the standard deviation for second moments)
This figure plots impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a positive one standard deviation surplus shock ($e_{st}$), when the economy is initially in the active monetary (solid line) and fiscal (dashed line) regime. The units of the y-axis are annualized percentage deviations from the steady-state, except for the surplus that is in levels.
This figure plots impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a positive one standard deviation technology shock ($e_{at}$), when the economy is initially in the active monetary (solid line) and fiscal (dashed line) regime. The units of the y-axis are annualized percentage deviations from the steady-state.
This figure reports the conditional impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 ($e_{mt}$) in the AM/PF regime (solid line) and PM/AF regime (dashed line). The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
This figure reports the conditional impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to an increase in the average maturity of government debt of the size of QE2 ($\epsilon_{mt}$) when the economy is initially in the PM/AF regime. The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years.
This figure reports the conditional impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to a decrease in the average maturity of government debt of the size of QE2 ($e_{mt}$) when the economy starts initially in the AM/PF regime. The dashed line represents the response at the Zero Lower Bound and the solid line, the response away from the Zero Lower Bound. The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years and the FFR that is in levels.
This figure reports the conditional impulse response functions of the expected government cost of capital, expected inflation, the 5 years to 1 quarter nominal yield spread, the federal fed fund rate, the one-year nominal yield, the real interest rate, and output to an increase in the average maturity of government debt of the size of QE2 ($e_{mt}$) when the economy starts initially in the AM/PF regime. The dashed line represents the response at the Zero Lower Bound and the solid line, the response away from the Zero Lower Bound. The units of the y-axis are annualized percentage deviations from the steady-state, except for the average duration that is in years and the FFR that is in levels.