Predictive Regressions: A Present-Value Approach*

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Abstract

We propose a latent-variables approach within a present-value model to estimate the expected returns and expected dividend growth rates of the aggregate stock market. This approach aggregates information contained in the whole history of the price-dividend ratio and dividend growth rates to obtain predictors for future returns and dividend growth rates. We find that both returns and dividend growth rates are predictable with R-squared values ranging from 8.2-8.9 percent for returns and 13.9-31.6 percent for dividend growth rates. Both expected returns and expected dividend growth rates have a persistent component, but expected returns are more persistent than expected dividend growth rates.

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We propose a latent-variables approach within a present-value model to estimate the time series of expected returns and expected dividend growth rates of the aggregate stock market. We treat conditional expected returns and expected dividend growth rates as latent variables that follow an exogenously-specified time-series model. We combine this model with a Campbell and Shiller (1988) present-value model to derive the implied dynamics of the price-dividend ratio. We subsequently use a Kalman filter to construct the likelihood of our model and we estimate the parameters of the model by means of maximum likelihood. We find that both expected returns and expected dividend growth rates are time-varying and persistent, but expected returns are more persistent than expected dividend growth rates. The filtered series for expected returns and expected dividend growth rates are good predictors of realized returns and realized dividend growth rates, with R-squared values ranging from 8.2-8.9 percent for returns and 13.9-31.6 percent for dividend growth rates.

We consider an annual model to ensure that the dividend growth predictability we find is not simply driven by the seasonality in dividend payments. Using an annual dividend growth series does however imply that we need to take a stance on how dividends received within a particular year are reinvested. Analogous to the way in which different investment strategies lead to different risk-return properties of portfolio returns, different reinvestment strategies for dividends within a year result in different dynamics of dividend growth rates. We study two reinvestment strategies in detail. First, we reinvest dividends into a 30-day T-bill, which we call cash-invested dividends. Second, we reinvest dividends into the aggregate stock market. We will refer to these dividends as market-invested dividends. Market-invested dividends have been studied widely in the dividend-growth and return-forecasting literature. We find that the reinvestment strategy matters for the time series properties of dividend growth. For instance, the volatility of market-invested dividend growth is twice as high as the volatility of cash-invested dividend growth. Within our model, we derive the link between the time-series models of dividend growth rates for different reinvestment strategies. This analysis demonstrates that if expected cash-invested dividend growth follows a first-order autoregressive process, then expected market-invested dividend growth has both a first-order autoregressive and a moving-average component.

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1See also Cochrane (1994) and Lettau and Ludvigson (2005).
2For instance, the risk-return properties of the end-of-period capital will be different if an investor allocates its capital to stocks instead of either Treasury bonds or real estate. By the same token, the properties of dividend growth rates depend on the reinvestment strategy chosen for dividends that are received within a particular year.
The main assumptions that we make in this paper concern the time-series properties for expected returns and expected dividend growth rates, which are the primitives of our model. We consider first-order autoregressive processes for expected *cash-invested* dividend growth and returns, and then derive the implied dynamics for expected *market-invested* dividend growth rates. Using this specification, we find that both returns and dividend growth rates are predictable, regardless of the reinvestment strategy. We can reject the null hypothesis that either expected returns or expected growth rates are constant at conventional significance levels. For both reinvestment strategies, we find, using a likelihood-ratio test, that expected returns are more persistent than expected growth rates. Also, innovations to both processes are highly positively correlated. Even though we find that future growth rates are predictable, we find that most of the unconditional variance in the price-dividend ratio stems from variation in discount rates, consistent with, for instance, Campbell (1991). If we decompose the conditional variance of stock returns, we find that innovations to expected growth rates can account for 20% of this variance.

As is the case in any linear state-space model, our model, in which we consider low-order autoregressive processes for expected returns and expected dividend growth rates, admits an infinite-order VAR representation in terms of dividend growth rates and price-dividend ratios. Cochrane (2008) rigorously derives the link between our model in Section 1.1 and the VAR representation. This insightful analysis also demonstrates why our approach can improve upon predictive regressions that include only the current price-dividend ratio to predict future returns and dividend growth rates. Our latent-variables approach aggregates the whole history of price-dividend ratios and dividend growth rates to estimate expected returns and expected growth rates. This implies that we expand the information set that we use to predict returns and dividend growth rates. However, instead of adding lags to a VAR model, which increases the number of parameters to be estimated, we suggest a parsimonious way to incorporate the information contained in the history of price-dividend ratios and dividend growth rates. As Cochrane (2008) shows, our model introduces moving-average terms of price-dividend ratios and dividend growth rates, in addition to the current price-dividend ratios, and we find these moving-average terms to be relevant in predicting future returns and dividend growth rates.

The insight that return predictability and dividend growth rate predictability are best studied jointly has already been pointed out by Cochrane (2007), Fama and French (1988), and Campbell and Shiller (1988). The main contribution of our paper is to model

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4Pástor and Stambaugh (2006) show a similar result for return predictability. They abstract, however, from dividend growth predictability and do not impose the present-value relationship.
expected returns and expected dividend growth rates as latent processes and use filtering techniques to uncover them. Fama and French (1988) point out that the price-dividend ratio is only a noisy proxy for expected returns when the price-dividend ratio also moves due to expected dividend growth rate variation. The reverse argument also holds: the price-dividend ratio is only a noisy proxy for expected dividend growth when the price-dividend ratio also moves due to expected return variation. Our framework explicitly takes into account that the price-dividend ratio moves due to both expected return and expected dividend growth rate variation and the filtering procedure assigns shocks to the price-dividend ratio to either expected return and/or expected dividend growth rate shocks.

One may wonder why we choose an AR(1)-process to model expected cash-invested dividend growth as opposed to expected market-invested dividend growth. First note that each reinvestment strategy for dividends corresponds to a different time-series model for expected returns and expected dividend growth rates. It could have been the case that, in fact, expected market-invested dividend growth is well described by an AR(1)-process. Given that most of the literature on return and dividend-growth-rate predictability has focused on market-invested dividend growth rates, this might seem like a more sensible first pass. In Section 5.4 we explore this (counterfactual) specification and find that the persistence coefficient of expected market-invested dividend growth is negative in this case. By fixing the persistence parameter of expected dividend growth in our estimation, and maximizing over all other parameters, we show that the model’s likelihood is bi-modal. This suggests that a simple first-order autoregressive process for expected dividend growth is too restrictive for market-invested dividends. We then argue that if our model in Section 1 which models expected cash-invested dividend growth as an AR(1)-process, is correctly specified, we mistakenly ignore a component that adds negative autocorrelation to expected market-invested dividend growth rates. We then perform a formal specification test and find that the model in Section 1 in which expected cash-invested dividend growth follows an AR(1)-process and expected market-invested dividend growth an ARMA(1,1)-process, is preferred over a model in which expected market-invested dividend growth follows an AR(1)-process.

Our paper is closely related to the recent literature on present-value models, see Cochrane (2007), Lettau and Van Nieuwerburgh (2006), Pástor and Veronesi (2003), Pástor and Veronesi (2006), Pástor, Sinha, and Swaminathan (2007), Bekaert, Engstrom, and Grenadier (2001), Bekaert, Engstrom, and Grenadier (2005), Burnside (1998), Ang and Liu (2004), and Brennan and Xia (2005). All of these papers provide expressions for the price-dividend or market-to-book ratio. However, in cases of Bekaert, Engstrom,
and Grenadier (2001), Pástor and Veronesi (2003), Pástor and Veronesi (2006), Ang and Liu (2004), and Brennan and Xia (2005), the price-dividend ratio is an infinite sum, or indefinite integral, of exponentially quadratic terms, which makes likelihood-based estimation and filtering computationally much more involved. Bekaert, Engstrom, and Grenadier (2001) and Ang and Liu (2004) estimate the model by means of GMM and model expected returns and expected growth rates as an affine function of a set of instruments. Brennan and Xia (2005) use a two-step procedure to estimate their model and use long-term forecasts for expected returns to recover an estimate of the time series of (instantaneous) expected returns, in turn. Alternatively, Lettau and Van Nieuwerburgh (2006) set up a linearized present-value model and recover structural parameters from reduced-form estimators. They subsequently test whether the present-value constraints are violated. They impose, however, that the persistencies of expected returns and expected growth rates are equal.

Our paper also relates to Brandt and Kang (2004), Pástor and Stambaugh (2006), and Rytchkov (2007) who focus on return predictability using filtering techniques. We contribute to this literature by focusing on the interaction between return and dividend growth predictability, and by showing that the reinvestment strategy of dividends has an impact on the specification of the present-value model.

The paper proceeds as follows. In Section 1, we present the linearized present-value model and discuss the two reinvestment strategies that we study. In Section 2, we discuss the data and our estimation procedure. In Section 3, we present our estimation results and compare our empirical results to predictive regressions. Section 4 discusses hypothesis testing including the tests for (the lack of) return and dividend growth rate predictability. Section 5 discusses several additional implications and some robustness checks and Section 6 concludes.

1 Present-value model

In Section 1.1 we explain our present-value model for the case in which expected returns and expected growth rates follow first-order autoregressive processes. In Section 1.2, we then show how to link the dynamics of expected growth rates for cash-invested and market-invested dividends.

5Under these assumptions, no filtering is required to uncover expected returns and expected dividend growth rates. We test, using a likelihood ratio test, whether the persistence of expected returns and expected dividend growth rates is equal, and we reject this hypothesis.
1.1 Theoretical model

In this section we present a log-linearized present-value model in the spirit of Campbell and Shiller (1988). We assume that both expected returns and expected dividend growth rates are latent variables. We first consider the model in which both latent variables follow an AR(1)-process. However, we note that it is straightforward to allow for higher-order VARMA representations for these variables, some of which we further explore when we study different reinvestment strategies.

Let $r_{t+1}$ denote the total log return on the aggregate stock market:

$$r_{t+1} \equiv \log \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right).$$

let $PD_t$ denote the price-dividend ratio of the aggregate stock market:

$$PD_t \equiv \frac{P_t}{D_t},$$

and let $\Delta d_{t+1}$ denote the aggregate log dividend growth rate:

$$\Delta d_{t+1} \equiv \log \left( \frac{D_{t+1}}{D_t} \right).$$

We model both expected returns ($\mu_t$) and expected dividend growth rates ($g_t$) as an AR(1)-process:

$$
\begin{align*}
\mu_{t+1} &= \delta_0 + \delta_1 (\mu_t - \delta_0) + \varepsilon^\mu_{t+1}, \\
g_{t+1} &= \gamma_0 + \gamma_1 (g_t - \gamma_0) + \varepsilon^g_{t+1},
\end{align*}
$$

where:

$$
\begin{align*}
\mu_t &\equiv E_t [r_{t+1}], \\
g_t &\equiv E_t [\Delta d_{t+1}].
\end{align*}
$$

The distribution of the shocks $\varepsilon^\mu_{t+1}$ and $\varepsilon^g_{t+1}$ will be specified shortly. The realized dividend

\footnote{Many authors have argued that expected returns are likely to be persistent, including Fama and French (1988), Campbell and Cochrane (1999), Ferson, Sarkissian, and Simin (2003), and Pástor and Stambaugh (2006). Further, it has been argued that expected dividend growth rates have a persistent component, see for example Bansal and Yaron (2004), Menzly, Santos, and Veronesi (2004), and Lettau and Ludvigson (2005).}
growth rate is equal to the expected dividend growth rate plus an orthogonal shock:

\[
\Delta d_{t+1} = g_t + \varepsilon_{t+1}^D.
\]

Defining \(pd_t \equiv \log(PD_t)\), we can write the log-linearized return as:

\[
r_{t+1} \simeq \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t,
\]

with \(\overline{pd} = E[pt]\), \(\kappa = \log(1 + \exp(\overline{pd})) - \rho \overline{pd}\) and \(\rho = \frac{\exp(\overline{pd})}{1 + \exp(\overline{pd})}\), as in Campbell and Shiller (1988). If we iterate upon this equation, it is straightforward to derive that (see also Appendix A):

\[
pt = A - B_1 (\mu_t - \delta_0) + B_2 (g_t - \gamma_0),
\]

with \(A = \kappa + \frac{\gamma_0 - \delta_0}{1 - \rho}, B_1 = \frac{1}{1 - \rho \gamma_1}, \text{ and } B_2 = \frac{1}{1 - \rho \gamma_1}\). Note that the log price-dividend ratio is linear in the expected return \(\mu_t\) and the expected dividend growth rate \(g_t\). The loading of the price-dividend ratio on expected returns and expected dividend growth rates depends on the relative persistence of these variables (\(\delta_1\) versus \(\gamma_1\)). The three shocks in the model, which are shocks to expected dividend growth rates (\(\varepsilon_{t+1}^g\)), shocks to expected returns (\(\varepsilon_{t+1}^\mu\)), and realized dividend growth shocks (\(\varepsilon_{t+1}^D\)), have mean zero, covariance matrix

\[
\Sigma \equiv \text{var}
\begin{bmatrix}
\varepsilon_{t+1}^g \\
\varepsilon_{t+1}^\mu \\
\varepsilon_{t+1}^D
\end{bmatrix}
=
\begin{bmatrix}
\sigma_g^2 & \sigma_g \mu & \sigma_g D \\
\sigma_g \mu & \sigma_{\mu}^2 & \sigma_{\mu} D \\
\sigma_g D & \sigma_{\mu} D & \sigma_D^2
\end{bmatrix},
\]

and are independent and identically-distributed over time. Further, in the maximum likelihood estimation procedure, we assume that the shocks are jointly normally distributed.

### 1.2 Reinvesting dividends and modeling growth rates

In this section, we analyze the impact of the reinvestment strategy of dividends on the time-series model for dividend growth rates. To illustrate why the reinvestment strategy is potentially important for the time-series model of dividend growth rates, we present the following extreme example. Consider the case where year-\(t\) prices are recorded on December 31st and year-(\(t + 1\)) dividends are all paid out one day later, on January 1st.\[1\]

\[3\]In Section 5.3 we further elaborate on the accuracy of this log linear approximation.
Denote by $D_{t+1}$ the dividends paid out on January 1st. Assuming (for ease of exposition) that the one-year interest rate is 0, the end-of-year cash-invested dividends are simply given by $D_{t+1}$. However, the end-of-period market-invested dividends are given by:

$$D^M_{t+1} = D_{t+1} \exp (r_{t+1}),$$  \hspace{1cm} (1)$$

where $r_{t+1}$ denotes the aggregate stock market return.\footnote{It is important to note here that even though realized dividend growth rates are strongly dependent on the reinvestment strategy, the aggregate stock market return does not. The correlation between cum-dividend returns where dividends are reinvested in the market and cum-dividend returns where dividends are reinvested in the risk free rate is 0.9999, see also Figure 3. As such, from an empirical perspective, these two series can be used interchangeably. To save on notation and (hopefully) avoid confusion, we will therefore denote the return on the market by $r_t$, regardless of the reinvestment strategy.} The observed market-invested dividend growth rates are then given by:

$$\Delta d^M_{t+1} = \log \left( \frac{D^M_{t+1}}{D^M_t} \right) = \log \left( \frac{D_{t+1}}{D_t} \right) + r_{t+1} - r_t.$$  \hspace{1cm} (2)$$

First note that this expression suggests that the past return on the market is a candidate predictor of market-invested dividend growth rates.\footnote{See also Fama and French (1988).} If the return on the market in period $t$ is high, this increases the dividend growth rate at time $t$, but it implies a lower dividend growth rate at time $t + 1$ relative to cash-invested dividends. Second, the expression suggests that reinvesting dividends in the market can add substantial volatility to dividend growth rates.

In reality, dividends are paid out throughout the year, for example at the end of each quarter. To better understand the impact of reinvesting dividends in the aggregate stock market, we consider the following parsimonious reduced-form representation:

$$D^M_{t+1} = D_{t+1} \exp(\varepsilon^M_{t+1}),$$  \hspace{1cm} (3)$$

in which $D_{t+1}$ denotes the cash-invested dividend. We assume that $\varepsilon^M_{t+1}$ is i.i.d. over time with mean zero and standard deviation $\sigma_M$. Further, we allow for correlation between $\varepsilon^M_{t+1}$ and aggregate market returns:

$$\rho_M = \text{corr}(\varepsilon^M_{t+1}, \varepsilon^r_{t+1}),$$  \hspace{1cm} (4)$$

with $\varepsilon^r_{t+1} \equiv r_{t+1} - \mu_t \approx -B_1 \rho \varepsilon^M_{t+1} + B_2 \rho \varepsilon^g_{t+1} + \varepsilon^B_{t+1}$. In our previous example, where all dividends were paid out at the beginning of the year, this correlation is close to 1 and $r_{t+1} \approx \varepsilon^M_{t+1}$. If dividend payments are made throughout the year, we expect a positive...
value for $\rho_M$, but not necessarily close to one. Using this model, we can decompose $\varepsilon_t^M$ into a part that is correlated with $\varepsilon_t^r$ and a part that is orthogonal to $\varepsilon_t^r$:

$$
\varepsilon_t^M = \beta_M \varepsilon_t^r + \varepsilon_t^M_\perp,
$$

(5)
in which $\beta_M = \rho_M\sigma_M/\sigma_r$, $\sigma_r = \sqrt{\text{var}(\varepsilon_t^r)}$ and $\varepsilon_t^M_\perp$ is orthogonal to $\varepsilon_t^r$. To keep the model parsimonious, we assume that all correlation between $\varepsilon_t^M$ and the structural shocks in our model, that is, $\varepsilon_t^g$, $\varepsilon_t^\mu$, and $\varepsilon_t^D$, comes via the aggregate market return. The latter assumption implies that $\text{cov}(\varepsilon_t^M_\perp, \varepsilon_t^g) = \text{cov}(\varepsilon_t^M_\perp, \varepsilon_t^D) = \text{cov}(\varepsilon_t^M_\perp, \varepsilon_t^\mu) = 0$.

Suppose that expected growth rates for cash-invested dividends follow an AR(1)-process. Based on this model,

$$
\Delta d_{t+1}^M = \Delta d_{t+1}^r + \varepsilon_t^M - \varepsilon_t^M = g_t + \varepsilon_t^D + \varepsilon_t^M - \varepsilon_t^M,
$$

(6)
where $g_t \equiv E_t[\Delta d_{t+1}]$. This implies that $g_t^M \equiv E_t[\Delta d_{t+1}^M] = g_t - \varepsilon_t^M$.

Next, we derive the time-series process for expected market-invested dividends, assuming that cash-invested dividends follow an AR(1)-process:

$$
g_{t+1}^M = g_{t+1} - \varepsilon_t^M
= \gamma_0 + \gamma_1(g_t - \gamma_0) + \varepsilon_t^g + \varepsilon_t^M - \varepsilon_t^M
= \gamma_0 + \gamma_1(g_t^M - \gamma_0) + \gamma_1\varepsilon_t^M + \varepsilon_t^g - \varepsilon_t^M,
$$

(7)
which shows that expected market-invested dividend growth is not a first-order autoregressive process, but instead an ARMA(1,1)-process. In Section 5 we show how approximating the expected market-invested dividend growth rate with an AR(1)-process, while in reality expected cash-invested dividend growth follows an AR(1)-process, can lead to a downward-biased estimate for the persistence coefficient of expected market-invested dividend growth.

\footnote{To develop the argument, we assume (for now) that $\varepsilon_t^M$ is observable at time $t$ and no filtering is required to infer its values. In our estimation procedure, we do not observe $\varepsilon_t^M$ and we therefore filter its value.}
2 Data and estimation

2.1 Data

We obtain the with-dividend and without-dividend monthly returns on the value-weighted portfolio of all NYSE, Amex, and Nasdaq stocks for the period 1946-2007 from the Center for Research in Security Prices (CRSP). We then use these data to construct our annual data for aggregate dividends and prices. We consider two reinvestment strategies. First, we consider dividends reinvested in 30-day T-bills and compute the corresponding series for dividend growth, the price-dividend ratio, and returns. Data on the 30-day T-bill rate is also obtained from CRSP. Secondly, we consider dividends reinvested in the aggregate stock market and compute the corresponding series for dividend growth, the price-dividend ratio, and returns. The latter reinvestment strategy is commonly used in the return predictability literature. It causes annual dividend growth to be highly volatile with an annual unconditional volatility of 12.3% versus a volatility of 6.2% for cash-reinvested dividend growth. In the next two subsections we present our estimation procedure. In Section 2.2 we discuss our estimation procedure for the model where dividends are reinvested cash, and in Section 2.3 we discuss our estimation procedure when dividends are reinvested in the market.

2.2 State space representation: cash-invested dividends

Our model features two latent state variables, \( \mu_t \) and \( g_t \). We assume that each of these is an AR(1)-process. The de-meaned state variables are:

\[
\hat{\mu}_t = \mu_t - \delta_0,
\]
\[
\hat{g}_t = g_t - \gamma_0.
\]

The model has two transition equations:

\[
\hat{g}_{t+1} = \gamma_1 \hat{g}_t + \varepsilon^g_{t+1},
\]
\[
\hat{\mu}_{t+1} = \delta_1 \hat{\mu}_t + \varepsilon^\mu_{t+1},
\]
and two measurement equations:\(^{11}\)

\[
\Delta d_{t+1} = \gamma_0 + \hat{g}_t + \varepsilon_{t+1}^D, \\
pd_t = A - B_1\hat{\mu}_t + B_2\hat{g}_t.
\]

Because the second measurement equation contains no error term, we can substitute the equation for \(pd_t\) into the transition equation for de-meaned expected returns, to arrive at the final system that has just one transition and two measurement equations:

\[
\hat{g}_{t+1} = \gamma_1\hat{g}_t + \varepsilon_{t+1}^g, \quad (8) \\
\Delta d_{t+1} = \gamma_0 + \hat{g}_t + \varepsilon_{t+1}^D, \quad (9) \\
pd_{t+1} = (1 - \delta_1) A + B_2(\gamma_1 - \delta_1)\hat{g}_t + \delta_1pd_t - B_1\varepsilon_{t+1}^\mu + B_2\varepsilon_{t+1}^g. \quad (10)
\]

As all equations are linear, we can compute the likelihood of the model using a Kalman filter (Hamilton (1994)). We then use unconditional maximum likelihood estimation (MLE) to estimate the vector of parameters:

\[
\Theta \equiv (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_D, \rho_g\mu, \rho_gD, \rho_\muD).
\]

The details of this estimation procedure are described in Appendix\(^{12}\). We maximize the likelihood using simulated annealing. This maximization algorithm is designed to search for the global maximum (see Goffe, Ferrier, and Rogers (1994)).

### 2.3 State-space representation: market-invested dividends

As before, we define the two de-meaned state variables as:

\[
\mu_t = \delta_0 + \hat{\mu}_t, \\
g_t = \gamma_0 + \hat{g}_t.
\]

Again, the model has two transition equations:

\[
\hat{g}_{t+1} = \gamma_1\hat{g}_t + \varepsilon_{t+1}^g; \\
\hat{\mu}_{t+1} = \delta_1\hat{\mu}_t + \varepsilon_{t+1}^\mu;
\]

\(^{11}\)It may be surprising that there is no measurement equation for returns. However, the measurement equation for dividend growth rates and the price-dividend ratio implies the measurement equation for returns, rendering the latter measurement equation redundant.
and two measurement equations:

\[
\begin{align*}
\Delta d_{t+1}^M &= \gamma_0 + \hat{\gamma}_t + \varepsilon_{t+1}^D + \varepsilon_{t+1}^M \varepsilon_t^M, \\
pd_t^M &= A - B_1 \hat{\mu}_t + B_2 \hat{\gamma}_t - \varepsilon_t^M.
\end{align*}
\]

(11) \hspace{1cm} (12)

Note that we are now using dividends reinvested in the market to compute the log dividend growth rate and the log price-dividend ratio. As before, we can substitute for one latent variable to arrive at the final system consisting of two measurement equations and one transition equation:

\[
\begin{align*}
\Delta d_{t+1}^M &= \gamma_0 + \hat{\gamma}_t + \varepsilon_{t+1}^D + \varepsilon_{t+1}^M - \varepsilon_t^M, \\
pd_{t+1}^M &= (1 - \delta_1) A + B_2 (\gamma_1 - \delta_1) \hat{\gamma}_t + \delta_1 pd_t^M - B_1 \varepsilon_{t+1}^\mu + B_2 \varepsilon_{t+1}^g - \varepsilon_{t+1}^M + \delta_1 \varepsilon_t^M, \\
\hat{\gamma}_{t+1} &= \gamma_1 \hat{\gamma}_t + \varepsilon_{t+1}^g.
\end{align*}
\]

(13) \hspace{1cm} (14) \hspace{1cm} (15)

As all equations are still linear, we can compute the likelihood of the model using the Kalman filter, and use unconditional MLE to estimate the set of parameters.

2.4 Identification

It is straightforward to derive that the first four parameters in the set \( \Theta \) are identified. Identification is harder to establish for the elements of the covariance matrix. As it turns out, all but one of the parameters in the covariance matrix are identified.\(^{12}\) We therefore assume that the correlation between realized dividend growth shocks \( (\varepsilon_{t+1}^D) \) and expected dividend growth shocks \( (\varepsilon_{t+1}^g) \) is zero.\(^{13}\)

3 Results

3.1 Estimation results: cash-invested dividends

Table 3 shows the maximum likelihood estimates of the parameters of the present-value model described in equations (8)-(10) where we use dividend data reinvested in the risk-free rate. We estimate the unconditional expected log return equal to \( \delta_0 = 9.0\% \) and the unconditional expected log growth rate of dividends to be \( \gamma_0 = 6.2\% \). Further,
we find that expected returns are highly persistent, consistent with Fama and French (1988), Campbell and Cochrane (1999), Ferson, Sarkissian, and Simin (2003), and Pástor and Stambaugh (2006) with an annual persistence coefficient ($\delta_1$) of 0.933. Further, the persistence of expected dividend growth rates equals 0.354. Even though it seems that expected dividend growth rates are persistent, they are less persistent than expected returns.\footnote{Note that an annual persistence coefficient of 0.354 corresponds to a monthly persistence coefficient of 0.917, which is somewhat lower than assumed in the long-run risk literature started by Bansal and Yaron (2004).}

We test for this more formally with a likelihood ratio test in Section 4. Note further that shocks to expected returns and expected dividend growth rates are positively correlated.\footnote{This is consistent with Menzly, Santos, and Veronesi (2004) and Lettau and Ludvigson (2005).} In our state-space model we compute the $R^2$ values for returns and dividend growth rates as (see also Harvey (1989))

$$R^2_{\text{Ret}} = \frac{\hat{\text{var}}(r_{t+1} - \mu_t^F)}{\text{var}(r_t)},$$

$$R^2_{\text{Div}} = \frac{\hat{\text{var}}(\Delta d_{t+1} - g_t^F)}{\text{var}(\Delta d_{t+1})},$$

where $\text{var}$ is the sample variance, $\mu_t^F$ is the filtered series for expected returns ($\mu_t$), and $g_t^F$ is the filtered series for expected dividend growth rates ($g_t$). The $R^2$ value for returns is equal to 8.2% and for dividend growth rates it equals 13.9%.

### 3.2 Estimation results: market-invested dividends

Table 4 shows maximum likelihood estimates of the parameters of the present-value model described in equation (13)-(15). First, note the similarities of our parameter estimates with those presented in Table 1. We now find that the average expected log return over our sample period is $\delta_0 = 8.6\%$ and the average expected log dividend growth rate equals $\gamma_0 = 6.0\%$. Further, we find that the persistence coefficient of expected returns ($\delta_1$) equals 0.957 and the persistence of expected dividend growth rates equals 0.638, indicating, as before, that expected returns are more persistent than expected dividend growth rates.

We compute two $R^2$ values for dividend growth rates. One excludes the filtered value of $\varepsilon_t^M$, denoted by $\varepsilon_t^{M,F}$, which is in the information set of the investor at time $t$ and the
other includes it. We thus compute the following three $R^2$ values:

\[ R^2_{\text{Ret}} = \frac{\text{var}(r_{t+1} - \mu^F_t)}{\text{var}(r^M_t)}, \]

\[ R^2_{\text{Div}} = \frac{\text{var}(\Delta d_{t+1} - g^F_t)}{\text{var}(\Delta d^M_{t+1})}, \]

\[ R^2_{\text{DivM}} = \frac{\text{var}(\Delta d_{t+1} - g^F_t + \varepsilon^M,F_t)}{\text{var}(\Delta d^M_{t+1})}, \]

where $\text{var}$ is the sample variance, $\mu^F_t$ is the filtered series for expected returns ($\mu_t$), $g^F_t$ is the filtered series for expected dividend growth rates ($g_t$), and $\varepsilon^M,F_t$ is the filtered value for $\varepsilon^M_t$. For returns, we find an $R^2$ value of 8.9% and for dividend growth rates, we find $R^2$-values of respectively $R^2_{\text{Div}} = 20.4\%$ and $R^2_{\text{DivM}} = 31.6\%$, depending on whether we include the filtered value of $\varepsilon^M_t$ to predict future dividend growth rates. Further, the standard deviation of the shock $\varepsilon^M_t$ equals 5.4% and the correlation between $\varepsilon^M_t$ and the unexpected return on the aggregate market is $\rho_M = 0.6$. Note that, if all dividends would have been paid out at the beginning of the year, $\varepsilon^M_t$ would closely resemble the market return and we would expect a standard deviation $\sigma_M$ equal to that of the aggregate market and the correlation to be close to 1. When all dividend payments are paid out at the end of the year, then we would expect a value of $\sigma_M$ close to zero and a correlation close to 0. When dividends are paid out throughout the year, as they are in our data set, it is reassuring to find values of $\sigma_M$ and $\rho_M$ in between these two extreme cases. It suggests that our reduced-form model indeed captures, at least partially, the reinvestment of dividends in the aggregate market.

### 3.3 Comparison with OLS regressions

As a benchmark for our latent-variables approach, we also report results from the corresponding predictive OLS regressions:\footnote{These regressions have been studied widely in the literature, and an incomplete list of references includes Cochrane (2007), Lettau and Van Nieuwerburgh (2006), and Stambaugh (1999).}

\[
\begin{align*}
  r_{t+1} &= \alpha_r + \beta_r pd_t + \varepsilon^r_{t+1}, \\
  \Delta d_{t+1} &= \alpha_d + \beta_d pd_t + \varepsilon^d_{t+1}.
\end{align*}
\]

For market-invested dividends, the return regression has a predictive coefficient of $\beta_r = -0.10$ with an $R^2$ value of 7.96\% and a t-statistic of -2.19, where we use OLS
standard errors to compute the t-statistic. The dividend growth rate regression results in a predictive coefficient of $\beta_d = -0.04$, with an $R^2$ value of 1.56% and a t-statistic of -0.91. Note that the dividend growth rate regression has an insignificant coefficient, which seems to have the wrong sign, in the sense that a high price-dividend ratio predicts a low expected dividend growth rate as opposed to a high expected dividend growth rate. The higher $R^2$ values that we obtain are achieved by using a larger information set and by aggregating this expanded information set in a parsimonious way. The predictive regressions above only use the price-dividend ratio at time $t$ to obtain predictions for returns and dividend growth rates in period $t+1$. In our filtering approach, we use the whole history of dividend growth rates and the whole history of the price dividend ratio up to time $t$ to obtain a prediction of returns and dividend growth rates at time $t+1$. Our approach should not be confused with simply adding lags of the price-dividend ratio and realized dividend growth rates in both regressions. The present-value model we propose in combination with the Kalman filter, allows us to expand the information set without increasing the number of parameters.

For cash-invested dividends, the return regression has a predictive coefficient of $\beta_r = -0.10$ with an R-squared value of 8.20% and a t-statistic of -2.32. The dividend growth rate regression results in a predictive coefficient of $\beta_d = -0.01$, with an $R^2$ value of 0.01% and a t-statistic of -0.91.

We have argued that the reinvestment strategy matters for realized dividend growth and that market-invested dividend growth is more volatile than cash-invested dividend growth due to the volatility of stock returns. Further, we have argued that apart from this added volatility, realized market-invested dividend growth can be well described by an ARMA(1,1) process. To further explore this argument we present results for several OLS regressions for market-invested dividends. The results are summarized in Table 5. When we include in the regression a constant term and an AR(1)-term, we find a negative coefficient which is significant at the 10% level. The $R^2$ value is low and equal to 5.0%. Further, when we include a constant term, an AR(1)-term and an MA(1) term, none of the latter two coefficients shows up significantly in the regression and the $R^2$ value remains low at 6.3%.

However, when we control for the lagged return ($r_{t-1}$) in this regression the results

\[ \frac{\sigma_d^2}{1-\gamma_1^2} \leq \frac{B_1}{B_2} \frac{\sigma_{eq}}{1-\gamma_1 \delta_1}. \]

\[ ^{17} \]Given that for values of $\gamma_1$ smaller than $1/\rho$, the coefficient $B_2$ is bigger than zero, we would expect a positive sign in this regression. Note that the price-dividend ratio is a noisy proxy for expected dividend growth rates when the price-dividend ratio also moves due to expected return variation, which can lead to the wrong sign in the regression. Binsbergen and Koijen (2008) show that the price-dividend ratio relates negatively to expected growth rates if: $\frac{\sigma_d^2}{1-\gamma_1^2} \leq \frac{B_1}{B_2} \frac{\sigma_{eq}}{1-\gamma_1 \delta_1}$. 


substantially change, with an AR(1) coefficient value of 0.782 and an MA(1) coefficient of -0.979, both statistically significant at the 1% level. Further, the lagged return enters significantly, as expected, with a negative coefficient of -0.313. The $R^2$ value of the latter regression equals 27.5%. In fact, including the lagged return as the sole regressor already leads to an $R^2$ value of 22.3%.\footnote{See also Fama and French (1988).} Note that the $R^2$ value of 27.5% is still lower than the 31.6% that we achieve by filtering, even though the OLS regressions allow for an additional degree of freedom compared to the specification for dividend growth in the filtering procedure. To increase the $R^2$ further we need to include the information contained in the price-dividend ratio, which in the OLS regressions above, we have not yet explored. However, including the lagged price-dividend ratio in the regression does not lead to a higher $R^2$ value, the coefficient is not significant and the coefficient has the wrong sign, consistent with the OLS regression above, where the price-dividend ratio is the only regressor. This is not very surprising. When the price-dividend ratio moves both due to expected returns and expected dividend growth rates, the price-dividend ratio is only a noisy proxy for expected dividend growth rates. Our filtering approach explicitly takes into account that the price-dividend ratio also moves due to expected return movement, which allows us to filter out the relevant expected dividend growth rate information and achieve an $R^2$ value for dividend growth equal to 31.6%.

4 Hypothesis testing

Our estimates reveal several important properties of expected returns and expected dividend growth rates. In particular, both expected returns and expected growth rates seem to vary over time, expected returns seem to be more persistent than expected growth rates, and both seem to contain a persistent component. In this section, we perform a series of hypothesis tests to establish the statistical significance of these results.

Our likelihood-based estimation approach facilitates a straightforward way to address these questions using the likelihood-ratio (LR) test. Denote the log-likelihood that corresponds to the unconstrained model by $\mathcal{L}^1$. The log-likelihood that follows from estimating the model under the null hypothesis is denoted by $\mathcal{L}^0$. The likelihood-ratio test statistic is given by:

$$LR = 2(\mathcal{L}^1 - \mathcal{L}^0),$$

which is asymptotically chi-squared distributed with the degrees of freedom equal to
the number of constrained parameters. We perform our test for both market-reinvested dividend growth rates and cash-invested dividend growth rates.

First, we test for a lack of return predictability. The associated null hypothesis is:

\[ H_0 : \delta_1 = \sigma_\mu = \rho_{\mu g} = \rho_{\mu D} = 0, \]

whose LR statistic has a \( \chi^2 \)-distribution. Under this null hypothesis, all variation in the log price-dividend ratio comes from variation in expected growth rates. In this case, we can uncover expected dividend growth rates through an OLS regression of dividend growth rates on the lagged price-dividend ratio.

Secondly, we test for the lack of dividend growth rate predictability. The null hypothesis that corresponds to this test for cash-reinvested dividends reads:

\[ H_0 : \gamma_1 = \sigma_g = \rho_{\mu g} = 0, \]

whose LR statistic follows a \( \chi^2 \)-distribution. If dividend growth is unpredictable, we can uncover expected returns through an OLS regression of dividend growth rates on the lagged price-dividend ratio. For market-reinvested dividends, the null hypothesis is:

\[ H_0 : \gamma_1 = \sigma_g = \rho_{\mu g} = \sigma_M = \rho_M = 0, \]

and the LR statistic has a \( \chi^2 \)-distribution. The absence of dividend growth predictability also requires that \( \sigma_M \) and \( \rho_M \) are zero. If not, \( \varepsilon_{t+1} \) correlates with returns and forecasts subsequent dividend growth rates. Under this null hypothesis, all variation in the log price-dividend ratio comes from variation in expected returns.

Third, we test whether the persistence coefficient of expected dividend growth rates equals zero. The null hypothesis that corresponds to this test is:

\[ H_0 : \gamma_1 = 0, \]

where the LR statistic has a \( \chi^2 \)-distribution. The question of whether expected dividend growth rates are time-varying, and what their persistence is, plays an important role in general-equilibrium models with long-run risk (Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008)).

Fourth, we test whether the persistence coefficients of expected dividend growth rates and expected returns are equal, which has been used by Cochrane (2007) and Lettau and
Van Nieuwerburgh (2006) for analytical convenience. The null hypothesis for this test is:

\[ H_0 : \gamma_1 = \delta_1, \]  

(26)

and the LR statistic has a \( \chi^2 \)-distribution. Under the null hypothesis of equal persistence coefficients, the price-dividend ratio is an AR(1)-process, which has been used as a reduced-form model by many authors. Under the alternative hypothesis, the price-dividend ratio is not an AR(1)-process, as the sum of two AR(1)-processes is an ARMA(2,1)-process.

Finally, we test whether the inclusion of \( \varepsilon^M_t \) adds significantly to the fit of the model. The null hypothesis for this test is:

\[ H_0 : \sigma_M = \rho_M = 0, \]  

(27)

and the LR statistic has a \( \chi^2 \)-distribution.

We summarize the LR statistics of all these tests in Table 6. The critical values at the 5% and 1% significance levels for the \( \chi^2 \) with \( N \) degrees of freedom are summarized in table 7. The tables show that all the null hypotheses stated above can be rejected at the 5% level. This suggests, in the context of our model, that both returns and dividend growth rates are predictable. Furthermore, it seems that expected returns are more persistent than expected dividend growth rates, given that (i) we find a lower value of \( \gamma_1 \) than for \( \delta_1 \) in our unconstrained estimates and (ii) the hypothesis that these two coefficients are equal can be rejected at the 1% level. Finally, the inclusion of the term \( \varepsilon^M_t \) in our specification for market-invested dividend growth seems to add significantly to the fit of the model. The correlation between returns and \( \varepsilon^M_t \) is significantly different from 0 and positive, lending further support to our interpretation of \( \varepsilon_M \) as a reduced-form model for reinvesting dividends in the market throughout the year.

5 Additional implications and robustness

5.1 Comparing the filtered series

In this section, we compare the filtered series for both expected returns and expected dividend growth rates for both reinvestment strategies. In Figure 4, we plot the filtered series for \( \mu_t \) as well as the realized log return when dividends are reinvested in the risk-

\[ \text{See for example Stambaugh (1999), Lewellen (2004), and Lettau and Van Nieuwerburgh (2006).} \]
free rate. We compare it to the fitted return series from an OLS regression of realized log returns on the lagged price-dividend ratio. The figure shows that the two expected return series are almost identical, consistent with the comparable $R^2$ values we find for both approaches. In Figure 3 we then plot the same series when dividends are reinvested in the market. Note that, in this case, the expected return series of our filtering procedure is different from the OLS series. The filtered series is lower in the eighties and higher by the end of the nineties. Consequently, the OLS regression predicts a negative return in the nineties, whereas the filtered series remains positive.

In Figure 6 we plot the filtered series for $g_t$ when dividends are reinvested in the risk-free rate as well as the fitted value from an OLS regression of realized log dividend growth rates (again reinvested in the risk-free rate) on the lagged price-dividend ratio. The difference between the two series is large. The filtered series picks up much more variation of realized dividend growth than the fitted values from the OLS regression does. Further, it seems that expected dividend growth has a positive autocorrelation, but its persistence is not as large as the price-dividend ratio. The price-dividend ratio is mainly driven by expected returns, which are more persistent than expected dividend growth rates, as we formally tested in Section 4.

In Figure 7 and 8, we plot the same series, but now for the reinvestment strategy that reinvests dividends in the market. Figure 7 plots the filtered series for $g_t$ and $g_t - \varepsilon_M^t$, and Figure 8 plots the fitted value from an OLS regression of realized dividend growth on the lagged price-dividend ratio. Note that the filtered series pick up a large fraction of the variation of market-reinvested dividend growth rates. In other words, a large fraction of realized dividend growth seems to be predictable.

### 5.2 Variance decompositions

We now derive a variance decomposition of both the price-dividend ratio and of returns in this model. The variance decomposition of the price-dividend ratio is defined as:

$$
var(pd_t) = B_1^2 \text{var}(\mu_t) + B_2^2 \text{var}(g_t) - 2B_1B_2 \text{cov}(\mu_t, g_t)
= \frac{(B_1 \sigma_\mu)^2}{1 - \delta_1^2} + \frac{(B_2 \sigma_g)^2}{1 - \gamma_1^2} - \frac{2B_1B_2 \sigma_{\mu g}}{1 - \delta_1 \gamma_1}.
$$

(28)

---

20Campbell and Thompson (2007) suggest to impose the restriction that the equity risk premium is always positive in predictive regressions, which, as they show, enhances the out-of-sample predictability of stock returns.
We decompose the variance of unexpected stock returns as in Campbell (1991) into variance due to discount rate news and cash flow news as well as their covariance, and refine it by decomposing the cash flow news:

$$\text{var}(r_{t+1}) = \text{var}(\rho B_1 \varepsilon_{\mu t+1} + \rho B_2 \varepsilon_{g t+1}) - 2\text{cov}(\rho B_1 \varepsilon_{\mu t+1}, \varepsilon_{t+1}^D + \rho B_2 \varepsilon_{g t+1})$$

$$= \rho^2 B_1^2 \sigma_\mu^2 + \sigma_D^2 + \rho^2 B_2^2 \sigma_g^2 + 2\rho B_2 \sigma_{gD} - 2\rho B_1 \sigma_{\mu D} - 2\rho^2 B_1 B_2 \sigma_{g\mu}. \quad (29)$$

In the results that we report below, we use sample covariances and we standardize all terms on the right-hand side of (28) and (29) by the left-hand side, so that the sum of the terms is 100%.

Using the parameter estimates described in Section 3, we can now compute the variance decompositions described above. For market reinvested dividends, we exclude the contribution of $\varepsilon^M_{t+1}$ and focus on the relative contribution of discount rates, expected dividend growth rates, and unexpected dividend growth shocks. We do this for the following reasons. First, for unexpected returns, the influence of $\varepsilon^M_{t+1}$ is small. Second, to understand the dynamics and decomposition of the price-dividend ratio, we are interested in the economic contribution of discount rates and expected dividend growth rates, in line with the previous literature (see for instance Campbell (1991)). The influence of the reinvestment strategy only distracts from this broader economic point.

The decomposition of the price dividend ratio is reported in the Table 1.

<table>
<thead>
<tr>
<th>Reinvestment strategy</th>
<th>$\frac{(B_1 \sigma_\mu)^2}{1-\delta_1^2}$</th>
<th>$\frac{(B_2 \sigma_g)^2}{1-\gamma_1^2}$</th>
<th>$\frac{2B_1 B_2 \sigma_{g\mu}}{1-\delta_1 \gamma_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>104.6%</td>
<td>4.6%</td>
<td>-9.2%</td>
</tr>
<tr>
<td>Market</td>
<td>117.7%</td>
<td>2.7%</td>
<td>-20.4%</td>
</tr>
</tbody>
</table>

Table 1: Variance Decomposition of the Price-Dividend Ratio

For both reinvestment strategies, most variation of the price-dividend ratio is related to expected return variation. Table 2 below reports the decomposition of unexpected returns. In the table, each of the terms in the return decomposition are named after their relevant shocks. For example the first term is named $\varepsilon^\mu_{t+1}$, and the last term is named $\varepsilon^\mu_{t+1}, \varepsilon^g_{t+1}$.

One notes that the variance decomposition of unexpected returns can be quite different across reinvestment strategies. This difference is caused by the difference in the correlation

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21The unexpected return is given by: $\varepsilon^\mu_{t+1} = r_{t+1} - \mu_t \approx -B_1 \rho \varepsilon^\mu_{t+1} + B_2 \rho \varepsilon^g_{t+1} + \varepsilon^D_{t+1} + (1-\rho)\varepsilon^M_{t+1}$. Given that $\rho$ equals 0.97 the contribution of this term is very small.
Reinvestment strategy | $\varepsilon_{\mu_{t+1}}$ | $\varepsilon_{g_{t+1}}$ | $\varepsilon_{D_{t+1}}$ | $\varepsilon_{\mu_{t+1}, \varepsilon_{g_{t+1}}, \varepsilon_{D_{t+1}}}$
--- | --- | --- | --- | ---
Cash | 118.4% | 34.6% | 0.0% | 0.4% | -53.4%
Market | 216.8% | 34.0% | 16.4% | 4.3% | -171.6%

Table 2: Variance Decomposition of Unexpected Returns

between $\varepsilon^\mu$ and $\varepsilon^g$, which is higher in the case of market-invested dividends and the difference in the persistence of expected dividend growth rates, $\gamma_1$, which is higher in the case of market-invested dividends. Finally, the decomposition of unexpected returns suggests that there is a substantial role for expected dividend growth variation when explaining unexpected returns.

5.3 Robustness to log-linearizations

In deriving the expression for the log price-dividend ratio in Section 1.1, we use the approximation to the log total stock return in equation (1). In Binsbergen and Koijen (2008), we study a non-linear present-value model within the class of linearity-inducing models developed by Menzly, Santos, and Veronesi (2004) and generalized by Gabaix (2007). Because the transition equation is non-linear in this model, we use non-linear filtering techniques to estimate the time series of expected returns. More specifically, we use an unscented Kalman filter (Julier and Uhlmann (1997)) and a particle filter. We find that the main results that we report in this paper are not sensitive to the linearization of log total stock returns. Both expected returns and expected growth rates are persistent processes, but expected returns are more persistent than expected growth rates. Innovations to expected returns and expected growth rates are highly positively correlated, and we find that the filtered series are good predictors of future returns and dividend growth rates.

5.4 Reinvestment strategy and model specification

We have assumed that the conditional expected dividend growth rate is an AR(1)-process if dividends are reinvested in the risk-free rate. We subsequently derive the implied dynamics for market-reinvested dividends. We stress again that there is a present-value

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We refer to Fernández-Villaverde and Rubio-Ramírez (2004), Fernández-Villaverde, Rubio-Ramírez, and Santos (2006), and Fernández-Villaverde and Rubio-Ramírez (2006) for theoretical results related to maximum-likelihood estimation in linearized models as well as applications to DSGE models in macroeconomics.
model for each reinvestment strategy of dividends, reflected in the time series properties of expected returns and expected dividend growth rates. To illustrate this further, suppose that the present-value model in equations (8)-(10) is correctly specified for the strategy in which dividends are reinvested in the risk-free rate, but we apply this model without adaptation to the data series in which dividends are reinvested in the aggregate stock market. In that case, the model is misspecified, because we have mistakenly imposed that $\sigma_M = 0$. However, it may be instructive to show what would happen to the first-order autocorrelation of expected dividend growth rates in this case:

$$\frac{\text{cov}(g_{t+1}^M, g_t^M)}{\text{var}(g_t^M)} = \gamma_1 + \frac{\text{cov}(\varepsilon_t^M, g_t^M)}{\text{var}(g_t^M)}$$

$$= \gamma_1 + \frac{\text{cov}(\varepsilon_t^M, g_t - \varepsilon_t^M)}{\text{var}(g_t^M)}$$

$$= \gamma_1 + \gamma_1 \frac{\sigma_M^2}{\text{var}(g_t^M)} + \gamma_1 \beta_M \frac{\text{cov}(\varepsilon_t^M, g_t)}{\text{var}(g_t^M)}$$

$$= \gamma_1 - \gamma_1 \frac{\sigma_M^2}{\text{var}(g_t^M)} + \gamma_1 \beta_M \frac{\text{cov}(\varepsilon_t^M, g_t)}{\text{var}(g_t^M)}$$

$$= \gamma_1 - \gamma_1 \frac{\sigma_M^2}{\text{var}(g_t^M)} + \gamma_1 \beta_M \frac{\text{cov}(-B_1 \rho \varepsilon_t^M + B_2 \rho \varepsilon_t^g + \varepsilon_t^D, \varepsilon_t^g)}{\text{var}(g_t^M)}$$

$$= \gamma_1 - \gamma_1 \frac{\sigma_M^2}{\text{var}(g_t^M)} + \gamma_1 \beta_M \frac{\text{cov}(\varepsilon_t^M, g_t)}{\text{var}(g_t^M)} + \gamma_1 \beta_M \left( -B_1 \rho \sigma_g \mu + B_2 \rho \sigma_g^2 \right) \left( -B_1 \rho \sigma_g \mu + B_2 \rho \sigma_g^2 \right), \quad (30)$$

in which the first equality uses $g_t^M = g_t - \varepsilon_t^M$ as well as the implied dynamics of $g_t^M$ in equation (7). The third equality uses the decomposition in equation (5) and the fact that $\text{cov}(\varepsilon_t^M, \varepsilon_t^g) = 0$.

In the last expression of equation (30), we find that the second term ($-\gamma_1 \sigma_M^2 / \text{var}(g_t^M)$) is negative. Further, the expression $(-B_1 \rho \sigma_g \mu + B_2 \rho \sigma_g^2)$ is negative for the point estimates that follow from our MLE procedure. This implies that if expected cash-invested dividend growth is a first-order autoregressive process, and if we subsequently approximate expected market-invested growth by an AR(1)-process, then we expect the persistence parameter to be considerably lower.

These results still obtain under the assumption that cash-invested dividends follow an AR(1)-process. As a next step, we estimate a specification in which expected growth rates of *market-invested* dividends are modeled as an AR(1)-process. The parameter estimates of this model are presented in Table 8. The table shows that the estimated value of $\gamma_1$ is not only lower than in the model in which cash-invested expected dividend growth is an

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23 We test the hypothesis that $\sigma_M = 0$ in Section 4 and reject it.
AR(1)-process, it is in fact estimated to be negative. Despite this negative value for $\gamma_1$, we still find relatively high $R^2$-values for both returns and dividend growth rates.

To further explore this evidence of a negative estimated value for $\gamma_1$ in this misspecified model, we construct a grid of possible levels of $\gamma_1$. For each point in the grid, we optimize over the other parameters and record the associated likelihoods and parameter estimates, shown in Table 9. The main results are summarized in Panel A of Figure 9 where we plot the likelihood as a function of $\gamma_1$. The picture shows that the likelihood has two peaks, of which one is positive; the other is negative. Panels B and C show plots of the $R^2$ values for returns and dividend growth rates as a function of $\gamma_1$. Note that the $R^2$ value for dividend growth rates also exhibits a bi-model shape, and, perhaps surprisingly, the $R^2$ value is higher for the positive root than for the negative root of $\gamma_1$. Furthermore, the $R^2$ value for returns is also higher for the positive root of $\gamma_1$. The pictures therefore illustrate that maximizing $R^2$ values is not necessarily equivalent to maximizing the likelihood.

6 Conclusion

We propose a new approach to predictive regressions by assuming that conditional expected returns and conditional expected dividend growth rates are latent, following an exogenously specified ARMA-model. We combine this with a Campbell and Shiller (1988) present-value model to derive the implied dynamics of the price-dividend ratio, and use filtering techniques to uncover estimated series of expected returns and expected dividend growth rates. The filtered series turn out to be good predictors for future returns and for future dividend growth rates.

We find that the high volatility of realized dividend growth rates is partially due to reinvesting dividends in the aggregate market. This induces a saw-tooth pattern in dividend growth rates, which makes it harder to uncover the persistent component of expected dividend growth rates. We propose a simple extension of our baseline model to help uncover this persistent component. Alternatively, one could use a different data set where dividends are reinvested in the risk-free rate. When we apply our baseline model to this dataset, we easily uncover the persistent component of expected dividend growth rates.

Finally, our likelihood setup allows for straightforward hypothesis testing using the likelihood ratio test. We can statistically reject the hypotheses that returns and dividend growth rates are unpredictable. Further, we can reject the hypothesis that expected dividend growth rates are unpredictable. Further, we can reject the hypothesis that expected dividend growth rates are unpredictable.

24See also Harvey (1989).
returns and expected dividend growth rates are equally persistent and we find that expected dividend growth rates are less persistent than expected returns.
References


A Derivations of the present-value model

We consider the model:

\[ \Delta d_{t+1} = g_t + \varepsilon_{t+1}^D, \]
\[ g_{t+1} = \gamma_0 + \gamma_1 (g_t - \gamma_0) + \varepsilon_{t+1}^g, \]
\[ \mu_{t+1} = \delta_0 + \delta_1 (\mu_t - \delta_0) + \varepsilon_{t+1}^\mu, \]

with:

\[ \Delta d_{t+1} \equiv \log \left( \frac{D_{t+1}}{D_t} \right), \]
\[ \mu_t \equiv E_t[r_{t+1}], \]
\[ r_{t+1} \equiv \log \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right). \]

We also define: \( pd_t = \log(PD_t) \). Now consider the log-linearized return, with \( \bar{pd} = E[pd_t] \):

\[ r_{t+1} = \log (1 + \exp (pd_{t+1})) + \Delta d_{t+1} - pd_t \]
\[ \approx \log (1 + \exp (\bar{pd})) + \frac{\exp (\bar{pd})}{1 + \exp (\bar{pd})}pd_{t+1} + \Delta d_{t+1} - pd_t \]
\[ = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t, \]

or, equivalently, we have:

\[ pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}, \]

in which:

\[ \kappa = \log (1 + \exp (\bar{pd})) - \rho \bar{pd}, \]
\[ \rho = \frac{\exp (\bar{pd})}{1 + \exp (\bar{pd})}. \]

By iterating this equation we find:

\[ pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1} \]
\[ = \kappa + \rho ( \kappa + \rho pd_{t+2} + \Delta d_{t+2} - r_{t+2} ) + \Delta d_{t+1} - r_{t+1} \]
\[ = \kappa + \rho \kappa + \rho^2 pd_{t+2} + \Delta d_{t+1} - r_{t+1} + \rho ( \Delta d_{t+2} - r_{t+2} ) \]
\[ = \sum_{j=0}^{\infty} \rho^j \kappa + \rho^\infty pd_{\infty} + \sum_{j=1}^{\infty} \rho^{j-1} ( \Delta d_{t+j} - r_{t+j} ) \]
\[ = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} ( \Delta d_{t+j} - r_{t+j} ), \]
assuming that $\rho^{\infty} p d_{\infty} = \lim_{j \to \infty} \rho^{j} p d_{t+j} = 0$ (in expectation would suffice for our purpose). Next, we take expectations conditional upon time-$t$:

$$pd_t = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t [\Delta d_{t+j} - r_{t+j}]$$
$$= \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t [g_{t+j-1} - \mu_{t+j-1}]$$
$$= \frac{\kappa}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j E_t [g_{t+j} - \mu_{t+j}],$$

$$= \frac{\kappa}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j \left( \gamma_0 + \gamma_1^j (g_t - \gamma_0) - \delta_0 - \delta_1^j (\mu_t - \delta_0) \right)$$
$$= \frac{\kappa}{1 - \rho} + \frac{\gamma_0 - \delta_0}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j \left( \gamma_1^j (g_t - \gamma_0) - \delta_1^j (\mu_t - \delta_0) \right)$$
$$= \frac{\kappa}{1 - \rho} + \frac{\gamma_0 - \delta_0}{1 - \rho} + \frac{g_t - \gamma_0}{1 - \rho \gamma_1} - \frac{\mu_t - \delta_0}{1 - \rho \delta_1},$$

which uses:

$$E_t [x_{t+j}] = \alpha_0 + \alpha_1^j (x_t - \alpha_0),$$

provided that:

$$x_{t+1} = \alpha_0 + \alpha_1 (x_t - \alpha_0) + \varepsilon_{t+1}.$$

## B Kalman filter

In this section we discuss the Kalman filtering procedure of our model. We discuss the most general case in which dividends are reinvested in the market. The other models that we discuss are special cases of this general setup.

We first reformulate the model in standard state-space form. Define an expanded state vector:

$$X_t = \begin{bmatrix} \hat{g}_{t-1} \\ \varepsilon_t^D \\ \varepsilon_t^\theta \\ \varepsilon_t^\mu \\ \varepsilon_t^M \\ \varepsilon_{t-1}^M \end{bmatrix},$$

which satisfies:

$$X_{t+1} = FX_t + \Gamma \varepsilon_{t+1}^X,$$
with

\[
F = \begin{bmatrix}
\gamma_1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix},
\]

\[
\Gamma = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

and where

\[
\varepsilon_{t+1}^X = \begin{bmatrix}
\varepsilon_t^D \\
\varepsilon_t^g \\
\varepsilon_t^\mu \\
\varepsilon_t^M
\end{bmatrix},
\]

which we assume to be jointly normally distributed.

The measurement equation, which has the observables \( Y_t = (\Delta d_t, pd_t) \), is:

\[
Y_t = M_0 + M_1 Y_{t-1} + M_2 X_t,
\]

with

\[
M_0 = \begin{bmatrix}
\gamma_0 \\
(1 - \delta_1) C
\end{bmatrix},
\]

\[
M_1 = \begin{bmatrix}
0 & 0 \\
0 & \delta_1
\end{bmatrix},
\]

\[
M_2 = \begin{bmatrix}
1 & 1 & 0 & 0 & 1 & -1 \\
B_2(\gamma_1 - \delta_1) & 0 & B_2 & -B_1 & -1 & \delta_1
\end{bmatrix}.
\]
The Kalman procedure is then straightforward, and given by:

\[
X_{0|0} = E[X_0] = 0_{4 \times 1},
\]

\[
P_{0|0} = E[X_0'X_0'],
\]

\[
X_{t|t-1} = FX_{t-1|t-1},
\]

\[
P_{t|t-1} = FP_{t-1|t-1}F' + \Gamma \Sigma \Gamma',
\]

\[
\eta_t = Y_t - M_0 - M_1 Y_{t-1} - M_2 X_{t|t-1},
\]

\[
S_t = M_2 P_{t|t-1} M_2',
\]

\[
K_t = P_{t|t-1} M_2' S_t^{-1},
\]

\[
X_{t|t} = X_{t|t-1} + K_t \eta_t,
\]

\[
P_{t|t} = (I - K_t M_2) P_{t|t-1}.
\]

The likelihood is based on prediction errors (\(\eta_t\)) and their covariance matrix (\(P_{t|t-1}\)), which is subject to change in every iteration:

\[
L = - \sum_{t=1}^T \log(\det(S_t)) - \sum_{t=1}^T \eta_t' S_t^{-1} \eta_t.
\]

Finally, the covariance matrix of the shocks is:

\[
\Sigma = \text{var} \left( \begin{bmatrix} \varepsilon^g_{t+1} \\ \varepsilon^\mu_{t+1} \\ \varepsilon^D_{t+1} \\ \varepsilon^M_{t+1} \end{bmatrix} \right) = \begin{bmatrix} \sigma_g^2 & \sigma_{g\mu} & \sigma_{gD} & \sigma_{gM} \\ \sigma_{g\mu} & \sigma_\mu^2 & \sigma_{\muD} & \sigma_{\muM} \\ \sigma_{gD} & \sigma_{\muD} & \sigma_D^2 & \sigma_{DM} \\ \sigma_{gM} & \sigma_{\muM} & \sigma_{DM} & \sigma_M^2 \end{bmatrix},
\]

Recall that we have assumed that:

\[
\varepsilon^M_{t+1} = \beta_M \varepsilon^\mu_{t+1} + \varepsilon^M_{t+1}, 
\]

in which \(\beta_M = \rho_M \sigma_M / \sigma_r\) and \(\sigma_r = \sqrt{\text{var}(\varepsilon^r_{t+1})}\) and:

\[
\varepsilon^r_{t+1} \equiv r_{t+1} - \mu_t \approx -B_1 \rho g_{t+1} + B_2 \rho g_{t+1} + \varepsilon^D_{t+1}. 
\]

Therefore,

\[
\sigma_{gM} = \beta_M \rho B_1 \sigma_{g\mu} + \beta_M \rho B_2 \sigma_g^2, 
\]

\[
\sigma_{\muM} = \beta_M \sigma_{\muD} + \beta_M \rho B_1 \sigma_\mu^2 + \beta_M B_2 \rho \sigma_{\mu g}, 
\]

\[
\sigma_{DM} = \beta_M \sigma_D^2 + \beta_M \rho B_1 \sigma_{\mu D}.
\]

We subsequently maximize the likelihood over the parameters:

\[
\Theta \equiv \left(\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_D, \rho_{g\mu}, \rho_{gD}, \rho_{\mu D}, \sigma_M, \rho_M\right).
\]
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.090</td>
<td>(0.021)</td>
<td>$\gamma_0$</td>
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</tr>
<tr>
<td>$\delta_1$</td>
<td>0.932</td>
<td>(0.159)</td>
<td>$\gamma_1$</td>
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<tr>
<td>$\sigma_\mu$</td>
<td>0.016</td>
<td>(0.011)</td>
<td>$\sigma_g$</td>
<td>0.058</td>
</tr>
<tr>
<td>$\rho_{D\mu}$</td>
<td>-0.147</td>
<td>(0.636)</td>
<td>$\sigma_D$</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rho_{\mu g}$</td>
<td>0.417</td>
<td>(0.216)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Implied present-value model parameters

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>3.571</td>
<td>(0.467)</td>
</tr>
<tr>
<td>$B_1$</td>
<td>10.334</td>
<td>(3.708)</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.523</td>
<td>(1.8608)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.969</td>
<td></td>
</tr>
</tbody>
</table>

Panel C: R-squared values

|       | $R^2_{\text{Returns}}$ | 8.2% | $R^2_{\text{Div}}$ | 13.9% |

Table 3: Estimation results: cash-invested dividends

We present the estimation results of the present-value model in Equations 8-10. The model is estimated by unconditional maximum likelihood using data from 1946 to 2007 on cash-invested dividend growth rates and the corresponding price-dividend ratio. Panel A presents the estimates of the coefficients of the underlying processes (bootstrapped standard errors between parentheses). Panel B reports the resulting coefficients of the present-value model ($pd_t = A + B_1 (\mu_t - \delta_0) + B_2 (g_t - \gamma_0)$). Note that the constants $A$, $B_1$ and $B_2$ are non-linear transformations of the underlying present-value parameters. Therefore, when interpreting the standard errors, it should be taken into account that the distribution of these constants is not symmetric. In Panel C we report the R-squared values for returns and dividend growth rates.
### Panel A: Maximum-likelihood estimates

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.e.</th>
<th></th>
<th>Estimate</th>
<th>S.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.086</td>
<td>(0.039)</td>
<td>$\gamma_0$</td>
<td>0.060</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.957</td>
<td>(0.055)</td>
<td>$\gamma_1$</td>
<td>0.638</td>
<td>(0.170)</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.016</td>
<td>(0.012)</td>
<td>$\sigma_g$</td>
<td>0.033</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\rho_{D\mu}$</td>
<td>-0.036</td>
<td>(0.330)</td>
<td>$\sigma_D$</td>
<td>0.058</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$\rho_{\mu g}$</td>
<td>0.999</td>
<td>(0.206)</td>
<td>$\sigma_M$</td>
<td>0.054</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>0.586</td>
<td>(0.191)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel B: Implied present-value model parameters

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>3.612</td>
<td>(0.953)</td>
<td>$\rho$</td>
<td>0.968</td>
</tr>
<tr>
<td>$B_1$</td>
<td>13.484</td>
<td>(5.626)</td>
<td>$B_2$</td>
<td>2.616</td>
</tr>
</tbody>
</table>

### Panel C: R-squared values

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2_{\text{Returns}}$</td>
<td>8.9%</td>
<td>$R^2_{\text{Div}}$</td>
<td>20.4%</td>
<td></td>
</tr>
<tr>
<td>$R^2_{\text{DivM}}$</td>
<td>31.6%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4: Estimation results: market-invested dividends**

We present the estimation results of the present-value model in Equations 13-15. The model is estimated by unconditional maximum likelihood using data from 1946 to 2007 on market-invested dividend growth rates and the corresponding price-dividend ratio. Panel A presents the estimates of the coefficients of the underlying processes (bootstrapped standard errors in parentheses). Panel B reports the resulting coefficients of the present-value model ($pd_t = A + B_1(\mu_t - \delta_0) + B_2(g_t - \gamma_0)$). Note that the constants $A, B_1$ and $B_2$ are non-linear transformations of the underlying present-value parameters. Therefore, when interpreting the standard errors, it should be taken into account that the distribution of these constants is not symmetric. In Panel C we report the R-squared values for returns and dividend growth rates.
### Table 5: OLS predictive regressions

The table reports the results for several reduced-form specifications of realized log dividend growth estimated with OLS. Dividends are reinvested in the aggregate stock market. One asterisk (*) denotes significance at the 10% level, two asterisks indicates significance at the 5% level and three asterisks indicates significance at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>( \text{constant} )</th>
<th>( \text{AR}(1) )</th>
<th>( \text{MA}(1) )</th>
<th>( r_{t-1} )</th>
<th>( R^2 )</th>
<th>( \text{Adj. } R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0604 ***</td>
<td>-0.5261</td>
<td>0.3652</td>
<td>-</td>
<td>0.0634</td>
<td>0.0311</td>
</tr>
<tr>
<td></td>
<td>(0.0138)</td>
<td>(0.3506)</td>
<td>(0.3937)</td>
<td>(0.0904)</td>
<td>(0.0170)</td>
<td>(0.0340)</td>
</tr>
<tr>
<td></td>
<td>0.0605 ***</td>
<td>-0.2214 *</td>
<td>-</td>
<td>-</td>
<td>0.0501</td>
<td>0.0340</td>
</tr>
<tr>
<td></td>
<td>(0.0127)</td>
<td>(0.1255)</td>
<td>(0.0904)</td>
<td>(0.0170)</td>
<td>(0.0130)</td>
<td>(0.02095)</td>
</tr>
<tr>
<td></td>
<td>0.1007 ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2227</td>
<td>0.2095</td>
</tr>
<tr>
<td></td>
<td>(0.0170)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.02095)</td>
<td>(0.0130)</td>
<td>(0.02358)</td>
</tr>
<tr>
<td></td>
<td>0.0872 ***</td>
<td>-</td>
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<td>0.2746</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.02358)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7817 ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2746</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1148)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.02358)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>-0.9793 ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2746</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0320)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.02358)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>-0.3717 ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.3125</td>
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</tr>
<tr>
<td></td>
<td>(0.0904)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.0904)</td>
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</tr>
<tr>
<td></td>
<td>-0.3125 ***</td>
<td>-</td>
<td>-</td>
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<td>0.3125</td>
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</tr>
<tr>
<td></td>
<td>(0.1162)</td>
<td>-</td>
<td>(0.0904)</td>
<td>(0.0904)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Dependent Variable: Market-invested Dividend Growth |
Table 6: Likelihood-ratio tests

We report the LR statistics for the tests described in Section 4. We do the first four tests for the two specifications that we explore in this paper. “Cash” refers to the system in equations (8)-(10) using the data where dividends are reinvested in the risk free rate. “Market” refers to the system in equations (8)-(10) using the data where dividends are reinvested in the aggregate stock market. Two asterisks (**) denotes that we reject the hypothesis at the 5% level and 3 asterisks (***) indicates that we reject the hypothesis at the 1% level.

<table>
<thead>
<tr>
<th>Parameters under $H_0$</th>
<th>LR</th>
<th>Sign</th>
<th>Log Lik. $H_0$</th>
<th>Log Lik. $H_a$</th>
<th>$\delta_0$</th>
<th>$\delta_1$</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\sigma_\mu$</th>
<th>$\sigma_\gamma$</th>
<th>$\sigma_D$</th>
<th>$\rho_{\mu\mu}$</th>
<th>$\rho_{\mu D}$</th>
<th>$\sigma_M$</th>
<th>$\rho_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test for Lack of Return Predictability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash reinv. dividends</td>
<td>28.67</td>
<td>***</td>
<td>7.0593</td>
<td>7.5218</td>
<td>0.0936</td>
<td>0</td>
<td>0.0637</td>
<td>0.9900</td>
<td>0</td>
<td>0.0065</td>
<td>0.0659</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Market reinv. dividends</td>
<td>22.37</td>
<td>***</td>
<td>6.4773</td>
<td>6.8381</td>
<td>0.0926</td>
<td>0</td>
<td>0.0666</td>
<td>0.9036</td>
<td>0</td>
<td>0.0057</td>
<td>0.0780</td>
<td>0</td>
<td>0</td>
<td>0.0607</td>
<td>0.8521</td>
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<tr>
<td><strong>Test for Lack of Div. Growth Predictability</strong></td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cash reinv. dividends</td>
<td>9.23</td>
<td>**</td>
<td>7.3730</td>
<td>7.5218</td>
<td>0.0882</td>
<td>0.9261</td>
<td>0.0607</td>
<td>0</td>
<td>0.0164</td>
<td>0</td>
<td>0.0617</td>
<td>0</td>
<td>0.3494</td>
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<td>-</td>
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<tr>
<td>Market reinv. dividends</td>
<td>29.59</td>
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<td>6.8381</td>
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<td>0.9514</td>
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<td>0.1222</td>
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<td>0.2973</td>
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<td><strong>Test for Lack of Persistence in Expected Div. Growth</strong></td>
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<tr>
<td>Cash reinv. dividends</td>
<td>8.26</td>
<td>***</td>
<td>7.3886</td>
<td>7.5218</td>
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<td>0.0605</td>
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<td>0.2550</td>
<td>0.2636</td>
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<td>-</td>
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<td>Market reinv. dividends</td>
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<td>0.0501</td>
<td>0.6792</td>
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<tr>
<td><strong>Test whether $\gamma_t$ and $\mu_t$ are Equally Persistent</strong></td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cash reinv. dividends</td>
<td>8.60</td>
<td>***</td>
<td>7.3831</td>
<td>7.5218</td>
<td>0.0867</td>
<td>0.9437</td>
<td>0.0595</td>
<td>0.9437</td>
<td>0.0157</td>
<td>0.0822</td>
<td>0.0617</td>
<td>0.9483</td>
<td>0.3090</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Market reinv. dividends</td>
<td>5.10</td>
<td>**</td>
<td>6.7558</td>
<td>6.8381</td>
<td>0.0782</td>
<td>0.9478</td>
<td>0.0548</td>
<td>0.9478</td>
<td>0.0166</td>
<td>0.0033</td>
<td>0.0764</td>
<td>0.9351</td>
<td>0.3541</td>
<td>0.0631</td>
<td>0.9254</td>
</tr>
<tr>
<td><strong>Test for exclusion of $\epsilon_{M}$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash reinv. dividends</td>
<td>11.00</td>
<td>***</td>
<td>6.6607</td>
<td>6.8381</td>
<td>0.0854</td>
<td>0.9321</td>
<td>0.0591</td>
<td>-0.3254</td>
<td>0.0149</td>
<td>0.0039</td>
<td>0.0635</td>
<td>0.9064</td>
<td>-0.4212</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Market reinv. dividends</td>
<td>6.93</td>
<td>***</td>
<td>6.7264</td>
<td>6.8381</td>
<td>0.0853</td>
<td>0.9321</td>
<td>0.0584</td>
<td>0.4419</td>
<td>0.0269</td>
<td>0.0595</td>
<td>0.0653</td>
<td>0.9945</td>
<td>-0.1048</td>
<td>0.0479</td>
<td>0</td>
</tr>
</tbody>
</table>

We do the first four tests for the two specifications that we explore in this paper. “Cash” refers to the system in equations (8)-(10) using the data where dividends are reinvested in the risk free rate. “Market” refers to the system in equations (8)-(10) using the data where dividends are reinvested in the aggregate stock market. Two asterisks (**) denotes that we reject the hypothesis at the 5% level and 3 asterisks (***) indicates that we reject the hypothesis at the 1% level.
<table>
<thead>
<tr>
<th>Degrees of freedom (N)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2_{N,0.05}$</td>
<td>3.841</td>
<td>5.991</td>
<td>7.815</td>
<td>9.488</td>
<td>11.070</td>
</tr>
<tr>
<td>$\chi^2_{N,0.01}$</td>
<td>6.635</td>
<td>9.210</td>
<td>11.345</td>
<td>13.277</td>
<td>15.086</td>
</tr>
</tbody>
</table>

Table 7: Critical Values of the Likelihood Ratio Tests
Panel A: Maximum-likelihood estimates

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.e.</th>
<th>Estimate</th>
<th>S.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.085</td>
<td>(0.019)</td>
<td>$\gamma_0$</td>
<td>0.059</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.933</td>
<td>(0.148)</td>
<td>$\gamma_1$</td>
<td>-0.324</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.015</td>
<td>(0.014)</td>
<td>$\sigma_g$</td>
<td>0.094</td>
</tr>
<tr>
<td>$\rho_{D\mu}$</td>
<td>-0.422</td>
<td>(0.276)</td>
<td>$\sigma_D$</td>
<td>0.065</td>
</tr>
<tr>
<td>$\rho_{\mu g}$</td>
<td>0.905</td>
<td>(0.076)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Implied present-value model parameters

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>3.596</td>
<td>(0.349)</td>
</tr>
<tr>
<td>$B_1$</td>
<td>10.263</td>
<td>(3.439)</td>
</tr>
</tbody>
</table>

Panel C: R-squared values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2_{\text{Returns}}$</td>
<td>8.6%</td>
</tr>
<tr>
<td>$R^2_{\text{Div}}$</td>
<td>18.7%</td>
</tr>
</tbody>
</table>

Table 8: Estimation results of the model in (8)-(10) using market-invested dividends

We present the estimation results of the present-value model in equations (8)-(10) using market-reinvested dividend data. The model is estimated by maximum likelihood using data from 1946 to 2007 on the dividend growth rate and the price-dividend ratio. Panel A presents the estimates of the coefficients of the underlying processes (bootstrapped standard errors in parentheses). Panel B reports the resulting coefficients of the present-value model ($pd_t = A + B_1(\mu_t - \delta_0) + B_2(g_t - \gamma_0))$. Note that these parameters are non-linear transformations of the original present-value parameters. When interpreting the standard errors, it should be taken into account that the distribution of the coefficients is not symmetric. In Panel C we report the R-squared values for returns and dividend growth rates.
Table 9: Estimating the misspecified model
In the column "\( \gamma_1 > 0 \)" we report the maximum-likelihood estimates of equations (8)-(10), but (mistakenly) using dividends that are reinvested in the market. In the first column, we impose that the persistence coefficient of expected dividend growth rates is positive. We then define a grid for \( \gamma_1 \) between -0.9 and 0.8 with increments of 0.1, and compute for each of these values of \( \gamma_1 \) the likelihood while optimizing over all the other parameters.
Figure 1: Dividend-growth rates: Reinvesting in either the risk-free rate or in the market
The graph plots the log dividend growth rate for two dividend reinvestment strategies: reinvesting in the risk-free rate and reinvesting in the market.

Figure 2: Price-dividend ratio: Reinvesting in either the risk-free rate or in the market
The graph plots the log price dividend ratio for two dividend reinvestment strategies: reinvesting in the risk-free rate and reinvesting in the market.
Figure 3: Cum-dividend returns: Reinvesting in either the risk-free rate or in the market
The graph plots the log cum dividend return ($r_t$) for two dividend reinvestment strategies: reinvesting in the risk free rate and reinvesting in the market.
Figure 4: Filtered series for expected returns for reinvesting in the risk-free rate
The graph plots the filtered series of expected returns ($\mu_t$) when dividends are reinvested in the risk-free rate. The graph also plots the realized return $r_{t+1}$ as well as the expected return obtained from an OLS regression of $r_{t+1}$ on the lagged price-dividend ratio.
Figure 5: Filtered series for expected returns for reinvesting in the market
The graph plots the filtered series of expected returns ($\mu_t$) when dividends are reinvested in the market. The graph also plots the realized return $r_{t+1}$ (again when dividends are reinvested in the market) as well as the expected return obtained from an OLS regression of $r_{t+1}$ on the lagged price-dividend ratio.
Figure 6: Filtered series for expected dividend growth for reinvesting in the risk-free rate

The graph plots the filtered series of expected dividend growth \( (g_t) \) when dividends are reinvested in the risk-free rate. The graph also plots the realized dividend growth \( \Delta d_{t+1} \) (again when dividends are reinvested in the risk-free rate) as well as the expected dividend growth rate obtained from an OLS regression of realized dividend growth \( \Delta d_{t+1} \) on the lagged price-dividend ratio.
Figure 7: Filtered series for expected dividend growth for reinvestment in the market
The graph plots the filtered series of expected dividend growth, $g_t$ (again when dividends are reinvested in the market). The graph also plots $g_t - \varepsilon^M_t$. Finally, the graph plots the realized dividend growth rate $\Delta d_{t+1}^M$ when dividends are reinvested in the market.
Figure 8: OLS forecast for expected dividend growth for reinvesting in the market
The graph plots the fitted OLS value, where log dividend growth rates (reinvested in the market) are regressed on the lagged price-dividend ratio. The graph also plots the realized dividend growth rate $\Delta d_{t+1}^M$ when dividends are reinvested in the market.
Figure 9: Log likelihood and $R^2$ values as a function of $\gamma_1$

The graph plots the log likelihood and the $R^2$ values as a function of the persistence of expected dividend growth, $\gamma_1$, using the system described in Equations 8, 10 and data where dividends are reinvested in the aggregate market (i.e., the misspecified model). We define a grid for $\gamma_1$ between -0.9 and 0.9 with step size 0.1, and compute for each of these grid points the likelihood and the $R^2$ values of the model while optimizing over all the other parameters.