A Theory of the Evolution of Executive Labor Markets*

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Abstract

We present a model of the market for executives that is consistent with many empirical facts about executive compensation, pay inequality, turnover, and mobility across firms. A novel feature of our model is the existence of two different equilibria in executive markets. This feature fits well with the existing evidence that executive labor markets have become structurally different in more recent times, as the empirical literature suggests. The model allows us to provide a unified narrative of the evolution of executive labor markets.

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1. Introduction

We present a model of executive labor markets that is consistent with many empirical facts about executive compensation, pay inequality, turnover, and mobility across firms. A novel feature of our model is the existence of two different equilibria in executive markets. In the first equilibrium, which we call the old economy, executive mobility is nonexistent, the link between firm size and executive compensation is weak, pay inequality among executives is low, and executive turnover in large firms is low. In contrast, in the second equilibrium, which we call the new economy, we observe a high degree of mobility, a strong link between firm size and compensation, high levels of pay inequality, and high turnover at the largest firms.

Our choice of labels is not accidental. The old economy equilibrium resembles the market for executives in the US from the 1940s to mid 1970s (Frydman, 2005; Frydman and Saks, 2010; Jenter and Frydman, 2010). The new economy equilibrium has characteristics that are normally associated with the market for executives since the mid 1970s. Our parsimonious framework provides a unified narrative of the evolution of the market for executives. As the equilibrium is always unique for a given set of parameters, we can study how changes in regimes occur. The model explains a number of puzzling empirical facts, has many testable predictions, and has social welfare implications that are relatively easy to analyze.

The main novel insight of the model is that drastic changes in the nature of the market for executives may occur as a consequence of small changes in the underlying characteristics of the economy. This result is important because it shows that rapid structural changes in the market for executives do not need to be associated with major changes in technology, competition, institutions or regulations. The differences between older and more recent times are some of the main empirical puzzles in the market for executives in the US, as documented by Frydman and Saks (2010). In the model, a transition from the old equilibrium to the new equilibrium may be triggered by gradual increases in the dispersion of firm profitability, in the relative importance of general skills, in the skewness of talent distribution, or in the death rate of firms.

The model implies a link between executive pay inequality and allocative efficiency in executive labor markets. The new economy equilibrium is more efficient than the old economy equilibrium in a number of aspects. Compared to the old economy, in the new economy the quality of manager-firm matches is improved and inefficient turnover is reduced. A change from the old economy to the new economy also has distributional consequences. In the new economy, provided talent is scarce, there is more pay inequality between top executives and
other executives. Another insight from the model is that the marked increase in the slope of the relation between firm size and executive pay, which occurred in the end of the 1970s, does not constitute evidence against assignment models in which more talented CEOs are matched to larger firms (Gabaix and Landier, 2008; Tervio, 2008). Such a change is in fact an implication of such models, once they are augmented to include turnover costs.

Our analysis builds upon some of the existing theoretical ideas on executive markets. As in firm-CEO assignment models, managers and firms are heterogeneous (Edmans, Gabaix, and Landier, 2009; Eisfeldt and Kuhnen, 2011; Gabaix and Landier, 2008; Tervio, 2008). As in Frydman (2005) and Murphy and Zabojnik (2004, 2006), managers are endowed with both firm-specific and general skills. As in Edmans and Gabaix (2011), the matching of managers to firms is distorted by informational frictions. This paper has much in common with this earlier body of work. It departs in that we consider the possibility of different regimes in the market for executives. Our results concerning the triggers of structural changes and their implications for market efficiency are unique to our setup; they cannot be easily replicated by extending previous models.

Our model is also related to a class of labor market models in which workers and firms gradually learn about the workers’ abilities (Gibbons and Murphy, 1992; Jovanovic, 1979a and 1979b; Harris and Holmstrom, 1982; MacDonald, 1982 and 1988; Murphy, 1986; Taylor, 2012). In our model, jobs are “experience goods,” that is, one can only learn about the ability of a worker by observing the worker on the job for a period of time (Jovanovic, 1979a). In contrast with most papers in this literature, we assume that (i) workers’ abilities in a job are only partially transferrable to different jobs and, most crucially, that (ii) both the worker and its firm acquire non-public information about the worker’s ability. These two assumptions are motivated by our specific application, which is the market for executives. Assumption (i) is meant to capture the fact that top executives are likely to be generalists and possess skills that can be transferred across firms and industries. However, executives also acquire firm-specific skills, and the relative importance of general versus specific skills varies over time (Custodio, Ferreira, and Matos, 2012; Frydman, 2005; Murphy and Zabojnik, 2004 and 2006). Thus, the general ability of managers is only partially transferrable. Assumption (ii) implies asymmetric learning: as time passes, both the manager and its firm (i.e. shareholders or the board of directors) privately learn about the manager’s ability. Other firms, which are potential employers for the manager, only learn from public information, and are thus less well informed about the manager’s ability. This assumption is quite realistic; as there are many complex dimensions of what makes a good executive, those who have worked closely
with an executive (for example, the board of directors) are likely to be better informed about the executive’s ability than those who did not have that experience.

In this introduction, we only give a partial and informal description of the results; later in the paper, we explain the arguments in more detail.

We assume that some firms are inherently more profitable than others. If all else is kept constant, high-talent managers should work for high-profitability firms, because managerial talent and firm quality are complements. However, if a manager that currently works for a low-profitability firm moves to a more profitable firm, her productivity falls due to a loss of firm-specific skills. Thus, for executive mobility to occur in equilibrium, the profitability differential between different types of firms must be sufficiently large. Alternatively, for a given gap in profitability, as firm-specific skills become less important, executive mobility eventually becomes possible.

Consider now the case of a firm and its incumbent manager. After the firm learns about the ability of its manager, it chooses whether to retain the manager or to hire (or promote internally) a new manager. Because the firm’s retention and firing decisions are observable, firms competing for managerial talent rationally infer that managers who are retained by their firms have above-average abilities. A firm that is unhappy with the ability of its current manager would want to poach a manager that is retained by another firm. When faced with a poaching attempt, a firm has two choices: it may choose to fight, and thus match the offer that the poaching firm makes to the manager, or it may choose to let the manager go without a fight. These two alternatives give rise to two different equilibria. In the old economy equilibrium, firms always fight poaching attempts, which implies that no actual poaching occurs in equilibrium. In this case, there is no managerial mobility across firms. Consequently, the quality of the match between firms and managers is poor; high-ability managers only end up in high-profitability firms by chance. Therefore, in the old economy equilibrium there is no clear link between firm quality and managerial talent, and there is only a tenuous link between firm quality and executive pay. Because firm profitability is positively related to firm survival and growth (e.g. Foster, Haltiwanger, and Syverson, 2008; Syverson 2011), firm size and profitability are positively related. Thus, the model predicts that the link between firm size and executive pay should be weak in the old equilibrium.

Dispersion in profitability is a widely documented fact, even within narrowly defined industries. A large strategy literature attributes profitability dispersion to monopoly profits explained by barriers to entry or the ownership of unique resources (McGahan and Porter, 1997; Rumelt, 1991). Even in industries with free entry, equilibrium (ex post) profitability dispersion could be explained by the accumulation of organization capital (Atkeson and Kehoe, 2005). For a recent review of the literature on productivity dispersion, see Syverson (2011).
In the new economy equilibrium, a low-profitability firm never fights a poaching attempt. Thus, in equilibrium, high-profitability firms often poach the best managers from low-profitability firms. Executive mobility is substantial, which implies a better matching between firm quality and managerial talent than the one that is observed in the old economy. If high-ability managers are short supply, the link between compensation and firm size also becomes stronger.

2. Model Setup

2.1. Firms and Managers

We consider an infinite-horizon economy populated by a continuum of infinitely-lived firms. Firms are risk-neutral and share a common discount factor $\delta \in [0, 1)$ (we use “firms” as a shortcut for either shareholders or boards of directors that are fully aligned with shareholders). Each firm is uniquely identified by a real number $j$. Firms are of two types: high-profitability (type $i = h$) or low-profitability (type $i = l$). The mass of $h$-firms is $H$ and the mass of $l$-firms is $L$. A type-$i$ firm is endowed with a technology represented by a profitability parameter $i \in \{h, l\}$. We set $h = \theta > 1 = l$. That is, the profitability parameter of an $l$-firm is normalized to one, therefore $\theta$ can also be interpreted as the profitability advantage of $h$-firms relative to $l$-firms (or the profitability differential, for short).

We do not model the determinants of profitability. Empirically, differences in profitability across firms can be explained by idiosyncratic productivity, idiosyncratic demand, or a combination of both (Foster, Haltiwanger, and Syverson, 2008). Our approach can accommodate these different interpretations. We also do not define industries or model product market competition. One may think of the firms in our model as producers of similar but differentiated goods. Barriers to entry and product differentiation can explain positive profits in equilibrium. Alternatively, the firms in our model could be interpreted as the whole set of firms in the economy. These different interpretations are possible because executive ability is transferrable, although imperfectly, across industries.\(^2\)

All firms need a manager (e.g. a CEO) to operate their technologies. Managers live for two periods: young age and old age. At each period $t$ ($t = 0, 1, 2, ...$) a mass $M$ of young managers enter the labor market. Young managers are in excess supply: $M > H + L$. For

\(^2\)Recent evidence suggests that the demand for executives with multi-industry experience has been increasing over time (Custodio, Ferreira, and Matos, 2012).
consistency, we also assume that a mass $M$ of old managers exists at time $t = 0$.\(^3\)

Managers are risk-neutral and have an outside option of working as non-executives for an exogenously given compensation, which we normalize to zero. We assume that managers are protected by limited liability; they must be paid a non-negative (per period) wage $w$. Young managers have zero personal wealth. These assumptions simplify the analysis, as they rule out cases in which young managers pay fees to work, and cases in which old managers pay fines if they quit from their firms. These assumptions jointly imply that firms can only offer one-period compensation contracts to managers.

Managers are endowed with some general managerial skills, which are represented by a real number $\tau \in [0, \bar{\tau}]$, where $\bar{\tau} < \infty$. We refer to $\tau$ as a manager’s general talent, or simply as talent. Young managers enter the labor market with unknown general talent. All managers and firms know that the talent $\tau$ of a young manager is drawn from a cumulative distribution function $F(\tau)$ on the bounded support $[0, \bar{\tau}]$. For tractability, we assume that $F$ is twice differentiable.

If a firm hires a young manager at the beginning of period $t$, both the firm and the manager expect the manager’s talent to be $\mu \equiv \int_{[0, \bar{\tau}]} \tau dF(\tau)$. If the same (now old) manager is still employed by the firm at the beginning of period $t + 1$, both the firm and the manager learn $\tau$. One interpretation is that knowledge of $\tau$ is a by-product of the relationship between the firm and the manager. In Jovanovic’s (1979a) terminology, managerial jobs are “experience goods”: firms learn about the manager’s ability by observing the manager on the job. In contrast to Jovanovic (1979a), we assume that $\tau$ is a characteristic of the manager, and not of the match. Thus, $\tau$ is transferrable to other firms.

Our learning technology differs from labor market models in the tradition of Harris and Holmstrom (1982) in two aspects. First, Harris and Holmstrom (1982), and most of the models that followed them, assume symmetric learning: all agents in the economy learn about the worker’s ability from the same public signals. In contrast, we assume asymmetric learning, in the sense that only the firm and its incumbent manager learn about the latter’s ability. Everyone else remains uninformed about $\tau$. This assumption is important for some of our results and their interpretations, so we discuss it in more detail in Subsection 5.1. The second difference is that we assume that learning reveals the manager’s ability perfectly after one period. We make this assumption for simplicity; it is not important for any of the results.

The technology requires two managerial inputs: general talent and firm-specific skills.

\(^3\)Economies with infinitely-lived firms and overlapping generations of workers can also be found in related models, such as Harris and Holmstrom (1982).
In an extension of our model (in Subsection 5.2), we also consider effort as a third input. Firm-specific skill is measured by $s \in \{\gamma, 1\}$. As in Murphy and Zabojnik (2004, 2006) a manager who has been working for a given firm has firm-specific skill $s = 1$ in her own firm, but would have firm-specific skill $s = \gamma \in (0,1)$ if she was to move to a different firm. The acquisition of firm-specific skills also requires one unit of time. If a firm hires a new manager (young or old) in period $t$, this manager has a firm-specific skill parameter of $\gamma$. If a young manager stays with the same firm until her old age, she then acquires firm-specific skills of $1 > \gamma$.

Models of investment in firm-specific skills have a long tradition in economics, dating back at least to Becker (1962). Thus we find it natural to interpret parameter $s$ as a measure firm-specific skills. We note however that alternative interpretations are also possible. A larger $\gamma$ implies that newly-hired managers are more productive, thus $\gamma$ could also be interpreted as (the inverse of) the cost of hiring an external manager. This cost could arise endogenously from search costs (Rogerson, Shimer, and Wright, 2005) or from any other unspecified exogenous source.

Production is stochastic and (conditional on the characteristics of the firm and of the manager) independently distributed across firms and across time. There are only two possible revenue states: *high*, in which the firm receives one unit of the numeraire, and *low*, in which the firm receives zero. At time $t$, if firm $j$, which has type $i_j$, employs a manager with firm-specific skill $s_{jt}$ and general talent $\tau_{jt}$, its period-$t$ revenue is given by

$$R_{jt} (i_j, s_{jt}, \tau_{jt}) = \begin{cases} 1, & \text{with probability } \min \{i_j s_{jt} \tau_{jt}, 1\} \\ 0, & \text{with probability } 1 - \min \{i_j s_{jt} \tau_{jt}, 1\} \end{cases}$$

For simplicity of exposition, we assume that we are always in the region where $i_j s_{jt} \tau_{jt} < 1$, thus we can ignore the $\min$ operator above. Under this assumption, the expected output of a firm endowed with $(i_j, s_{jt}, \tau_{jt})$ is simply $i_j s_{jt} \tau_{jt}$. When there is no risk of confusion, we drop the subscripts $j$ and $t$ to simplify the notation. This probabilistic interpretation places some restrictions on the parameters. First, we need $\tau < 1$. Second, we also require $\theta \leq \bar{\theta} \equiv \frac{1}{\tau}$. These assumptions can be seen as simple normalizations; they have no empirical implications.

We did not choose this technology on the basis of our prior beliefs or empirical evidence. Much in the same spirit as Edmans, Gabaix, and Landier (2009), we see our model as an exploration of the implications of multiplicative technologies. Importantly, the multiplicative technology implies a complementarity between managerial talent and firm profitability.
Such a complementarity implies that assortative matching—high-talent managers working for high-profitability firms—is efficient (Becker, 1973).

A simplifying assumption that we adopt for most of this paper is that there are no public signals about $\tau$. In particular, we assume that the past performance of a firm provides no public information about $\tau$. This is in contrast with e.g. Harris and Holmstrom (1982), in which learning is based only on a public signal of past performance. We adopt this assumption only for simplicity; in Subsection 5.3 we briefly discuss the case in which a public and a private signal of ability coexist, and we show that the qualitative results are not fundamentally altered.

Define
\[
E[\tau \mid \tau \geq \tilde{\tau}] = \frac{\int_{\tilde{\tau}}^{\infty} \tau dF(\tau)}{1 - F(\tilde{\tau})}. 
\]

Our final technical restriction on the function $F$ is

Assumption T1 $\frac{\partial E[\tau \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq \tau$.

Assumption T1 is not necessary for any of the important results in this paper. It is however a convenient assumption that simplifies some of the proofs and guarantees equilibrium uniqueness in most of the cases that we consider. We also note that Assumption T1 is never a necessary condition for equilibrium existence or uniqueness. Equilibrium existence is trivially guaranteed for any continuous $F$. Assumption T1 is sufficient (but not necessary) for uniqueness. We provide an additional discussion of this assumption in the Appendix.

3. Equilibrium Characterization

To build intuition, we first solve the model in the special case in which $\delta = 0$, that is, assuming that firms only care about current profits. This case is cleaner and easier to follow, and delivers most (but not all) of the crucial results of our model. In Section 4, we analyze the general case in which $\delta \geq 0$.

Footnote 4: We note that the case in which $\delta = 0$ is not exactly identical to a fully static model. For example, in a static, one-period version of our model, the equilibrium managerial turnover rate is indeterminate. This is so because the number of firms that start a period without an incumbent manager is a free parameter in a one-period model. In the dynamic case (regardless of the value of $\delta$), however, the proportion of firms without a manager at the beginning of each period is uniquely determined by how the game was played in the preceding period.
3.1. Equilibrium conditions

In this subsection we state and discuss the necessary conditions for any equilibrium. In Subsections 3.2 and 3.3 we formally define each of the two possible types of equilibrium.

To state the equilibrium conditions, we first introduce some notation. At each period $t$, there is a mass of $2M$ managers that can be hired, with half of them being old. Let $m \in [0, 2M]$ denote a uniquely defined manager (young or old). For each $m$, the market sets a retention wage $w_m$, which is the wage that a firm would pay to retain $m$ as an incumbent manager, and a poaching wage $\hat{w}_m$, which is the wage that a firm would need to pay to poach manager $m$ from a different firm if this manager is retained by her firm. Without loss of generality, we assume that poaching wages are never lower than retention wages, $\hat{w}_m \geq w_m$.

Let $m^y$ denote any young manager. Our assumption that there exist many young managers ($M > L + H$) implies that young managers have no bargaining power. In equilibrium, young managers are paid their reservation wage, which we normalize to zero for simplicity. Because young managers are all identical ex ante and are in excess supply, we set $w_m = \hat{w}_{m^y} = 0$. Similarly, as it will never be rational for a firm to hire an old manager who has been fired by another firm, without loss of generality we adopt the convention that $\hat{w}_m = 0$ if $m$ is an old manager who has been fired.

Firms compete for managers. We assume that competition is perfect, in the sense that firms and managers are price takers in managerial labor market. That is, they take wages as given.

**Assumption E1 (Competition):** Firms maximize profits taking wages as given.

We now describe the sequence of dates within each period $t$.

**Time line:**

**Date 1.** The market sets retention wages $w_m$ and poaching wages $\hat{w}_m$ for all $m \in [0, 2M]$.

**Date 2.** Firms with incumbent managers decide whether to retain or fire their managers.

**Date 3.** All firms without managers choose between hiring a young manager or an old manager.

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5 Any equilibrium in which $\hat{w}_m < w_m$ can also be sustained with $\hat{w}_m = w_m$.

6 We do not model the wage setting process explicitly as a game. The set of fully micro-founded wage setting mechanisms that we could in principle adopt is very large. We choose instead a simpler approach, in which wages are set by a fictitious Walrasian auctioneer, who selects wage profiles so that a set of equilibrium conditions are satisfied. These conditions are analogous to those that require that prices clear markets in more traditional models of perfect competition, in the sense that there is no excess demand or excess supply in equilibrium.
**Date 4.** Firms that lost their managers to poachers hire young managers.

Implicit in this time line is the assumption that retention and poaching wages may reflect only publicly available information, i.e. the identities of the manager and of her employer. The fictitious price setter does not know the talent $\tau_m$ of each manager $m$, thus private information about $\tau_m$ does not directly affect $w_m$ and $\hat{w}_m$. The fact that some firms have private information about the talent of their incumbent managers nevertheless affects wages indirectly, because equilibrium wages must be such that the optimal choices of firms are consistent with those wages.

Profit maximization implies that any equilibrium must have a threshold property, in which all old managers whose talents are revealed to be above a given threshold are retained, while those below are fired. That this threshold property must hold is easy to see: Suppose that a firm retains its incumbent manager $m$ when the retention wage is $w_m$ and the manager’s talent is $\tau_m$. If the firm is of type $i \in \{1, \theta\}$, its expected profit if the manager is retained is $i\tau_m - w_m$. Thus, if it is worth retaining a manager with talent $\tau_m$, then it is worth retaining any manager $m'$ with talent $\tau_{m'} \geq \tau_m$, as long as $w_{m'} = w_m$, which must hold because retention wages cannot depend on the unobserved talent of a given manager.

To state the equilibrium conditions, we need a bit more of notation. Let $j \in [0, L + H]$ denote a unique firm. In equilibrium, unique matches of firms and managers $(j, m)$ will be formed (some managers will not be matched to firms because $2M > L + H$). Let $\pi_j (m, w)$ denote the equilibrium expected profit of firm $j$ if it hires manager $m$ at wage $w$. The arguments $(m, w)$ need not be equilibrium values, that is, $\pi_j (m, w)$ is the profit that firm $j$ would expect if it hired $m$ at wage $w$ and all other firms played their equilibrium choices. The equilibrium set of retention and poaching wages is denoted by $(w_m, \hat{w}_m)$, for all $m \in [0, 2M]$. Manager $m$’s equilibrium wage is denoted by $w^*_m$, where $w^*_m = w_m$ if the manager is retained in equilibrium, $w^*_m = \hat{w}_m$ if the manager is poached in equilibrium, and $w^*_m = 0$ if the manager is fired or is young.

Although players take wages as given, we require wages to be set so that there is no room for renegotiation. We assume that firms and managers can always make a take-it-or-leave-it offer to another party. These offers need to be credible. A credible offer is one in which, if refused, the party making the offer is no worse than in the status quo. For example, if firm $j$ proposes $w' < w_m$ to its incumbent manager, then there must exist some manager $m''$ who would accept to work for firm $j$ for a wage $w''$ such that $\pi_j (m'', w'') \geq \pi_j (m, w_m)$. Similarly, if manager $m$ threatens to quit, there must be another firm willing to offer $w' \geq w_m$ to manager $m$. 
We can now state the key conditions that define an equilibrium wage profile:

**Definition 1 (Equilibrium wage profile)** A set \( \{(w_m, \hat{w}_m) : m \in [0, 2M]\} \) of retention and poaching wages is a (renegotiation-proof) competitive equilibrium wage profile if:

1. There is no firm \( j \) that would accept a credible take-it-or-leave-it offer \( w' \) by manager \( m' \) such that \( w' > w^*_m \), where \( w^*_m \) is manager \( m \)’s equilibrium wage.

2. There is no manager \( m' \) that would accept a credible take-it-or-leave-it offer \( w' \) by firm \( j \) such that \( \pi_j (m', w') > \pi_j (m, w^*_m) \), where \( (j, m) \) is an equilibrium match.

In short, we require firms to maximize profits taking wages as given and that wages are renegotiation-proof.

Profit maximization and the two conditions in the definition above are sufficient for the characterization of equilibria. As an equilibrium selection condition, we further require stationarity, that is, we only focus on equilibria in which equilibrium actions and outcomes are the same in every period. Stationarity is an intuitive requirement for equilibrium selection in our analysis, because we want to characterize the long-run properties of the executive labor market. Thus, changes in the nature of stationary equilibria can be interpreted as structural, lasting changes to the nature of this market. We describe the different types of equilibria in the next two subsections, where we also state the necessary stationarity conditions for each case.

### 3.2. No-poaching Equilibrium

In this subsection, we define, characterize, and prove the existence of the first of the two possible types of equilibrium in our model: an equilibrium in which managers do not move from one firm to another. We call this equilibrium a *no-poaching equilibrium*.

We start by characterizing the stationarity conditions.\(^7\) Let \( \hat{\tau}_i \) denote the retention threshold for a firm of type \( i \in \{h, l\} \). That is, at date 2 a type-\( i \) firm retains all managers with talent equal to or greater than \( \hat{\tau}_i \). Given such a threshold, of all young managers working for type-\( i \) firms at time \( t \), a fraction \( 1 - F(\hat{\tau}_i) \) is retained at \( t + 1 \).\(^8\)

\(^7\)We always restrict ourselves to symmetric equilibria, in which all firms of a given type \( i \) choose the same strategies. Non-trivial asymmetric equilibria may only exist for a constellation of parameters that has measure zero. We thus ignore these non-generic cases.

\(^8\)Recall that there is a continuum of firms of each type, thus the probability that a randomly selected manager is retained by a type-\( i \) firm (which is \( 1 - F(\hat{\tau}_i) \)) is the same as the fraction of young managers working for type-\( i \) firms who end up being retained at the beginning of the next period.
Define $\alpha_i \in [0, 1]$ as the proportion of type-$i$ firms that retain old managers at period $t$ and $\beta_i = 1 - \alpha_i$ as the proportion type-$i$ firms that hire young managers at period $t$. With this convention, these proportions at times $t$ and $t+1$ are

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<td>firms with retained managers</td>
<td>$\alpha_i$</td>
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<td>firms with young managers</td>
<td>$\beta_i$</td>
<td>$\alpha_i + \beta_i F(\tilde{\tau}_i)$</td>
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for $i \in \{l, h\}$. To guarantee that the equilibrium is stationary, we need these fractions to be the same in both periods. The stationarity conditions then write:

\[
\begin{align*}
\alpha_i &= \beta_i(1 - F(\tilde{\tau}_i)) \\
\beta_i &= \alpha_i + \beta_i F(\tilde{\tau}_i) \\
\alpha_i + \beta_i &= 1
\end{align*}
\]

which then imply $\beta_i = \frac{1}{2 - F(\tilde{\tau}_i)}$. Notice that $\beta_l$ and $\beta_h$ correspond to the steady state turnover rates in type-$l$ and type-$h$ firms, respectively.

We can now define a no-poaching equilibrium:

**Definition 2 (No-poaching Equilibrium)** A no-poaching (stationary) equilibrium is a (time-invariant) set of retention wages, poaching wages, retention thresholds, and turnover rates $\{(w_l, \hat{w}_l, \tilde{\tau}_l, \beta_l), (w_h, \hat{w}_h, \tilde{\tau}_h, \beta_h)\}$ such that:

- $(w_l, w_h, \hat{w}_l, \hat{w}_h)$ is an equilibrium wage profile;
- (Profit maximization): Assumption E1 holds; in particular, a firm of type $i \in \{l, h\}$ retains an incumbent manager with talent $\tau_i$ if and only if $\tau_i \geq \tilde{\tau}_i$;
- (No poaching): Firms without managers at date 3 always hire young managers; and
- (Stationarity): The fraction of type-$i$ firms that hire young managers (the turnover rate) is $\beta_i = \frac{1}{2 - F(\tilde{\tau}_i)}$, $i = l, h$.

### 3.2.1. Equilibrium Existence and Characterization

We now describe the derivation of a no-poaching equilibrium. At date 2, a firm of type $i$ that retains an old manager with talent $\tau_i$ and pays a wage $w_i$ has an expected profit $\pi_i^o(\tau_i)$ given by

\[
\pi_i^o(\tau_i) = i\tau_i - w_i,
\]
where \( i \in \{ h = \theta, l = 1 \} \). As discussed above, if the firm chooses instead to hire a young manager, it pays \( w_{my} = 0 \). Thus, the profit \( \pi^y_i \) from replacing the old manager with a young manager is

\[
\pi^y_i = i\gamma\mu. \tag{6}
\]

Note that this profit takes into account the loss in firm-specific skills, which is represented here by the parameter \( \gamma < 1 \).

Under no poaching, profit maximization implies that the retention decision of firm \( i \) is given by comparing \( \pi^o_i (\tau_i) \) with \( \pi^y_i \), which leads to the following rule:

\[
\begin{align*}
\text{If } i\tau_i - w_i &\geq i\gamma\mu, \text{ retain the manager,} \\
\text{if } i\tau_i - w_i &< i\gamma\mu, \text{ fire the manager.}
\end{align*} \tag{7, 8}
\]

This rule implies a threshold decision (given \( w_i \)) to retain the manager only if \( \tau_i \geq \hat{\tau}_i \), where

\[
\hat{\tau}_i = \gamma\mu + \frac{w_i}{i}, \quad i = l, h. \tag{9}
\]

This condition links the equilibrium thresholds to retention wages.

We need another condition to pin down both \( w_i \) and \( \hat{\tau}_i \). To derive this condition, we need to verify that no firm strictly prefers to poach a retained manager at date 3. Here we only need to consider the cases in which the potential poachers are type-\( h \) firms, because if a poaching attempt is profitable for a type-\( l \) firm, it is also profitable for a type-\( h \) firm.

Poaching when \( \hat{w}_i \geq w_i \) can be profitable only if poaching when \( \hat{w}_i = w_i \) is profitable. Thus, to find a no-poaching equilibrium, it is sufficient to consider the case in which \( \hat{w}_i = w_i \), that is, there is no difference between retention and poaching wages.\(^9\)

If a poaching attempt is successful, firm \( h \)'s profit is

\[
\hat{\pi}_h (\hat{\tau}_i) = \theta\gamma E[\tau \mid \tau \geq \hat{\tau}_i] - w_i. \tag{10}
\]

In a no-poaching equilibrium, this profit must not be larger than the one obtained by hiring a young manager. Thus, we have:

\[
\theta\gamma E[\tau \mid \tau \geq \hat{\tau}_i] - w_i \leq \theta\gamma\mu. \tag{11}
\]

\(^9\)In a no-poaching equilibrium, the poaching wage \( \hat{w}_i \) is indeterminate, in the sense that any \( \hat{w}_i \geq w_i \) will lead to the same equilibrium outputs. This non-uniqueness is observationally irrelevant, because no manager is paid wage \( \hat{w}_i \) in equilibrium.
Finally, we note that renegotiation-proofness implies that (11) cannot hold with slack in equilibrium. To see this, consider a firm of type $i$ that has an incumbent manager with talent equal to its retention threshold (we call this firm the marginal firm). Given wage $w_i$, this firm is indifferent between retaining its incumbent manager or hiring a young one. Thus, this firm would like to offer its manager a take-it-or-leave-it wage $w_i - \varepsilon$, with $\varepsilon > 0$ small. This offer is credible because the firm could hire a young manager and earn the same profit as in the status quo. If (11) does not bind, the incumbent manager has no option but to accept the offer. But then $w_i$ cannot be an equilibrium retention wage. Thus, (11) must bind, which implies

$$w_i = \theta \gamma (E[\tau | \tau \geq \hat{\tau}_i] - \mu),$$

which, together with condition (9) yields

$$i(\hat{\tau}_i - \gamma \mu) = \theta \gamma (E[\tau | \tau \geq \hat{\tau}_i] - \mu),$$

which determines the equilibrium pairs $(\hat{\tau}_i, \hat{\tau}_h)$. The next proposition states a necessary and sufficient condition for the existence of a unique solution to the equation above. All proofs are in the Appendix.

**Proposition 1 (Existence of no-poaching equilibrium)** A no-poaching equilibrium exists and is unique if and only if

$$\Delta \leq 1,$$

where

$$\Delta \equiv \frac{\theta \gamma (\tau - \mu)}{\tau - \gamma \mu}.$$

### 3.2.2. Equilibrium Properties

There is no simple analytical solution unless we impose a specific functional form on $F$. However, without imposing any further structure, we can prove a number of interesting properties of the no-poaching equilibrium.

The key implications of the no-poaching equilibrium for compensation and turnover are summarized in the next proposition.

**Proposition 2 (Turnover and Compensation)** In a no-poaching equilibrium:

- The retention threshold is higher in type-$l$ firms: $\hat{\tau}_l > \hat{\tau}_h$;
- The turnover rate is higher in type-$l$ firms: $\beta_l > \beta_h$;
- The retention wage is higher in type-$l$ firms: $w_l > w_h$. 

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The intuition for this proposition is as follows. Retention requires offering a high wage to the incumbent manager. Because of the complementarity between profitability and managerial quality, a manager with talent $\tau$ is always less valuable to a low-profitability firm than to a high-profitability firm. Thus, in equilibrium, the talent retention threshold for low-profitability firms has to be larger than the retention threshold for high-profitability firms. This in turn implies higher turnover and higher retention wages in low-profitability firms.

It is important to note that Proposition 2 shows only that the wage of a retained manager in type-$l$ firms is higher than that of a retained manager in a type-$h$ firm. Empirically, the average wage between retained and newly hired managers is often a more useful measure:

$$\text{Average wage across firms of type } i = \frac{1 - F(\tilde{\tau}_i)}{2 - F(\tilde{\tau}_i)}w_i. \quad (15)$$

In a no-poaching equilibrium, the average wage in type-$h$ firms can be higher or lower than the average wage in type-$l$ firms. This ambiguity arises because of two opposing effects. As shown in Proposition 2, type-$l$ firms offer higher wages to their retained managers. However, the same proposition also shows that type-$l$ firms retain fewer managers than do type-$h$ firms. Thus, type-$h$ firms pay positive wages to a larger fraction of managers.

Empirically, our analysis suggests that the cross-sectional relation between profitability (and thus size) and executive pay is ambiguous in the old economy equilibrium. This ambiguity arises despite the fact that managerial ability and firm profitability are complements. This complementarity is however not strong enough to generate assortative matching, and thus there is no obvious correlation between managerial ability and firm quality. Furthermore, because low-profitability firms defend against poaching attempts by increasing wages, it is even possible for wages to be negatively correlated with firm profitability.

**Proposition 3 (The profitability differential)** In type-$l$ firms, the turnover rate $\beta_l$ increases with $\theta$, while in type-$h$ firms turnover is independent of $\theta$. Compensation for retained managers $w_i$ increases with the profitability differential in both types of firms.

As firms become more heterogeneous ($\theta$ increases), low-profitability firms find it harder to compete with high-profitability firms in the market for executives. To sustain a no-poaching equilibrium, low-profitability firms need to offer higher wages to retain their managers. This in turn forces them to increase turnover by firing more managers.

Proposition 3 also implies that, as $\theta$ increases, the average wage in type-$h$ firms must eventually become larger than in that in type-$l$ firms. This happens because the average
wage in type-\(l\) firms converges to zero as the turnover rate converges to one.

**Proposition 4 (Firm-specific skills)** In both types of firms, both compensation \(w_i\) and turnover \(\beta_i\) (weakly) increase with \(\gamma\).

A larger \(\gamma\) represents a situation in which firm-specific skills become less important. Turnover increases for two reasons. First, turnover increases because the value of a newly-hired manager is larger when firm-specific skills are less important. This effect is present even if firms face no competition for managers. When there is competition in the executive labor market, changes in the importance of firm-specific skills have an additional, indirect effect on turnover: an increase in \(\gamma\) makes poaching more attractive, which in turn requires firms to pay larger wages to their retained managers to prevent poaching. As discussed above, higher wages imply higher turnover because firms can no longer afford to keep mediocre managers.

### 3.3. Poaching Equilibrium

In this subsection, we define, characterize, and prove the existence of the second of the two possible types of equilibrium in our model: an equilibrium with managerial mobility across firms. We call this equilibrium a *poaching equilibrium*.

Unlike in the no-poaching equilibrium, here we have two qualitatively distinct possibilities. The first case happens if the number of poachers is larger than the number of qualified poaching targets. In this case, we say that qualified old managers are in short supply. We expect managers to benefit significantly from the existence of equilibrium poaching in this case. The second case happens if old managers are in excess supply, in which case we expect type-\(h\) firms to benefit substantially from their ability to poach.

There is no theoretical reason to choose one case over the other. Thus, we characterize both cases and discuss their somewhat different empirical implications. Here we establish the necessary conditions for each case to arise.

**Case 1: Old managers in short supply:** \(H > (1-F(\hat{\gamma}_l))L\). In this case, all incumbent managers from type-\(l\) firms are poached in equilibrium.\(^{10}\) Let \(p\) denote the fraction of the \(h\)-firms that become poachers in equilibrium and, as before, let \(\alpha_h\) denote the fraction of

\(^{10}\)We caution against a literal interpretation of this case. Empirically, we are thinking about a situation in which there are relatively few qualified managers from small firms that could make the jump to a larger firm. That is, there could be many small firms, but only a few are breeding grounds for top managers. We could easily modify our model to fit this interpretation by, for example, assuming that only a fraction of the existing managers are viable poaching targets.
$h$-firms that retain their managers and $\beta_h$ denote the fraction of $h$-firms that hire young managers. At the end of periods $t$ and $t+1$, respectively, we have

<table>
<thead>
<tr>
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<th>$t$</th>
<th>$t+1$</th>
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<tbody>
<tr>
<td>poachers</td>
<td>$p$</td>
<td>$p$</td>
</tr>
<tr>
<td>firms with retained managers</td>
<td>$\alpha_h$</td>
<td>$\beta_h (1 - F(\hat{\tau}_h))$</td>
</tr>
<tr>
<td>firms with young managers</td>
<td>$\beta_h$</td>
<td>$\beta_h F(\hat{\tau}_h) + \alpha_h$</td>
</tr>
</tbody>
</table>

The stationarity conditions write:

$$
\begin{aligned}
\alpha_h &= \beta_h (1 - F(\hat{\tau}_h)) \\
\beta_h &= \alpha_h + \beta_h F(\hat{\tau}_h) \\
\alpha_h + \beta_h + p &= 1 \\
pH &= (1 - F(\hat{\tau}_l))L
\end{aligned}
$$

(16)

The equilibrium fraction type-$h$ firms that hire young managers is $\beta_h = \left[1 - \frac{(1-F(\hat{\tau}_h))L}{H}\right] / (2 - F(\hat{\tau}_h))$. This fraction is positive (i.e. there are some junior hires in equilibrium) only if $H > (1 - F(\hat{\tau}_l))L$, which is a necessary condition for this case to occur.

**Case 2: Old managers in excess supply:** $H \leq L(1 - F(\hat{\tau}_l))$. In this case, only a fraction of old managers retained by $l$-firms are poached, which means that $h$-firms never need to hire young managers (retained old managers are better than young ones on average and, because old managers are in excess supply, they are cheap). Notice that any equilibrium in which some $h$-firms retain their managers can only last for one period, because in the next period all $h$-firms will need to hire new managers. Thus, in a stationary equilibrium, the turnover rate in $h$-firms is 100% and all $h$-firms are poachers.\textsuperscript{11} The fractions of retained managers $\alpha_l$ and young hires $\beta_l$ in $l$-firms are:

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t+1$</th>
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</thead>
<tbody>
<tr>
<td>firms with retained managers</td>
<td>$\alpha_l$</td>
<td>$\beta_l (1 - F(\hat{\tau}_l)) \left[1 - \frac{H}{(1-F(\hat{\tau}_h))L}\right]$</td>
</tr>
<tr>
<td>firms with young managers</td>
<td>$\beta_l$</td>
<td>$\beta_l F(\hat{\tau}_l) + \alpha_l + \frac{H}{L}$</td>
</tr>
</tbody>
</table>

The stationarity conditions write:

$$
\begin{aligned}
\alpha_l &= \beta_l (1 - F(\hat{\tau}_l)) - \frac{H}{L} \\
\beta_l &= \beta_l F(\hat{\tau}_l) + \alpha_l + \frac{H}{L} \\
\alpha_l + \beta_l &= 1
\end{aligned}
$$

(17)

\textsuperscript{11} Again, the interpretation should not be literal; this is simply a case in which turnover at high-profitability firms is very high.
implying $\beta_i = \left(1 + \frac{H}{L}\right) / (2 - F(\hat{\tau}_i))$ and $\alpha_i = \left(1 - F(\hat{\tau}_i) - \frac{H}{L}\right) / (2 - F(\hat{\tau}_i))$. This equilibrium may exist only if $H \leq L(1 - F(\hat{\tau}_i))$.

We can now define a poaching equilibrium:

**Definition 3 (Poaching Equilibrium)** A poaching (stationary) equilibrium is a (time-invariant) set of retention wages, poaching wages, retention thresholds, and turnover rates $\{(w^p_l, \hat{w}^p_l, \hat{\tau}^p_l, \beta^p_l), (w^p_h, \hat{w}^p_h, \hat{\tau}^p_h, \beta^p_h + p)\}$ such that:

- $(w^p_l, w^p_h, \hat{w}^p_l, \hat{w}^p_h)$ is an equilibrium wage profile;
- (Profit maximization): Assumption E1 holds; in particular, a firm of type $i \in \{l, h\}$ retains an incumbent manager with talent $\tau_i$ if and only if $\tau_i \geq \hat{\tau}^p_i$;
- (Poaching): Firms of type $l$ without managers at date 3 always hire young managers; at least some firms of type $h$ without managers at date 3 poach old managers from type $l$ firms; and
- (Stationarity): The stationarity conditions (16) in Case 1 or (17) in Case 2 hold.

### 3.3.1. Equilibrium Existence and Characterization

We now describe the derivation of a poaching equilibrium. Because $l$-firms do not fight poaching attempts by $h$-firms, their retention threshold is determined by the no-poaching condition for $l$-firms:

$$\hat{\tau}^p_l - \gamma E[\tau \mid \tau \geq \hat{\tau}^p_l] = 0.$$  \hspace{1cm} (18)

Notice that, given Assumption T1, a unique solution to this equation always exists, and is the same for both cases of poaching equilibria. Note also that, for a given $\gamma$, the retention threshold for an $l$-firm in a poaching equilibrium is identical to the threshold for an $h$-firm in a no-poaching equilibrium: $\hat{\tau}^p_l = \hat{\tau}_h$ (see equation (79) in the Appendix). The equilibrium wage for retained managers at $l$-firms is thus:

$$w^p_l = \gamma (E[\tau \mid \tau \geq \hat{\tau}^p_l] - \mu).$$  \hspace{1cm} (19)

Given $\hat{\tau}^p_l$, we can now check whether we are in Case 1 or Case 2.

**Case 1:** $H > (1 - F(\hat{\tau}^p_l))L$.

We have shown that, in this case, some $h$-firms without incumbent managers will be forced to hire young managers. Competition will then drive up the wages of poached managers until
$h$-firms are indifferent between poaching an old manager or hiring a young one:

$$\theta \gamma E[\tau | \tau \geq \tilde{\tau}_i^p] - \hat{w}_i^p = \theta \gamma \mu.$$  

(20)

This equation pins down the equilibrium poaching wage:

$$\hat{w}_i^p = \theta \gamma \left( E[\tau | \tau \geq \tilde{\tau}_i^p] - \mu \right).$$  

(21)

The retention threshold for type-$h$ firms is determined by the no-poaching condition:

$$\tilde{\tau}_h^p = \gamma E[\tau | \tau \geq \tilde{\tau}_h^p].$$  

(22)

Notice that (22) is identical to (18), which implies $\tilde{\tau}_h^p = \tilde{\tau}_i^p$ in an equilibrium under Case 1.

The retention wage for type-$h$ firms is determined by

$$w_h^p = \theta \gamma \left( E[\tau | \tau \geq \tilde{\tau}_h^p] - \mu \right).$$  

(23)

Because $\tilde{\tau}_h^p = \tilde{\tau}_i^p$, (23) and (21) imply $w_h^p = \hat{w}_i^p$. Notice that, because $l$-firms offer $w_l^p$ to all “retained” managers and $h$-firms offer $\hat{w}_i^p = w_h^p > w_l^p$ to all managers retained by $l$-firms, $l$-firms are unable to retain any manager.\footnote{Intuitively, sequential rationality is the only reason for $l$-firms to offer positive wages to managers: if, for whatever reason, an $h$-firm does not try to poach those incumbent managers who have talents above the threshold, then an $l$-firm would want to offer $w_l^p$ to prevent their managers from being poached by another $l$-firm.}

**Case 2:** $H \leq (1 - F(\tilde{\tau}_i^p))L$.

In this case, all $h$-firms are poachers. We just need to determine $\hat{w}_i^p$. Because the supply of managers that can be poached is larger than the number of type-$h$ firms, renegotiation-proofness implies that $\hat{w}_i^p = w_l^p$.

We also need to check whether it is rational for an $h$-firm to become a poacher rather than to hire a young manager:

$$\theta \gamma E[\tau | \tau \geq \tilde{\tau}_i^p] - w_h^p = (\theta - 1) \gamma E[\tau | \tau \geq \tilde{\tau}_i^p] + \gamma \mu > \gamma \mu,$$  

(24)

which always holds.

**Existence.** Assuming that a poaching equilibrium is possible, we have shown that, for each case there is only one set of equilibrium thresholds and wages that characterize the
equilibrium. This means that we have proven part of the following proposition:

**Proposition 5 (Existence of poaching equilibrium)** A poaching equilibrium exists and is unique if and only if $\Delta \geq 1$.

Notice that, except for the non-generic case of $\Delta = 1$, poaching and no-poaching equilibria cannot coexist. Thus, Propositions 1 and 5 jointly imply that the equilibrium is generically unique.

### 3.3.2. Equilibrium Properties

The main implications of the poaching equilibrium for compensation and turnover are summarized in the next proposition.

**Proposition 6 (Turnover and Compensation)** In a poaching equilibrium:

*In Case 1,*

- Both type-$h$ and type-$l$ firms have the same retention threshold $\tilde{\tau}_h^p = \tilde{\tau}_l^p$;
- Type-$l$ firms have a higher turnover rate than type-$h$ firms;
- Type-$l$ firms pay lower wages than type-$h$ firms.

*In Case 2,*

- The retention rate for type-$h$ firms is not defined;
- Type-$l$ firms have a lower turnover rate than type-$h$ firms;
- Type-$l$ firms pay lower wages than type-$h$ firms.

To understand the intuition behind these results, we have to consider each case separately. In Case 1 (old managers in short supply), type-$i$ firms only try to prevent poaching by other type-$i$ firms. Thus, the retention thresholds for either type of firm are identical. Despite having the same retention thresholds, type-$l$ firms are unable to retain their managers because these managers receive better offers from type-$h$ firms. Thus, the turnover rate in type-$l$ firms is 100%, while in type-$h$ firms some managers are still retained in equilibrium.

Because turnover in type-$l$ firms is 100%, all managers employed by type-$l$ firms are young and receive the lowest possible wage (which is zero). Thus, the average wage at type-$l$ firms is lower than that of type-$h$ firms. Empirically, this implies a positive relation between
executive pay levels and firm profitability in the cross-section of firms. This unambiguous result contrasts sharply with the ambiguous relation between firm type and average wage found in the old economy equilibrium.

In Case 2, because in a stationary equilibrium type-\(h\) firms never hire young managers, they also never have an opportunity to retain an old manager. Thus the retention threshold is undefined for type-\(h\) firms.\(^{13}\) Now turnover in type-\(h\) firms is 100%. To poach a manager, type-\(h\) firms only need to pay the same wage as that paid by type-\(l\) firms. Thus, all managers in type-\(h\) firms are paid \(w^p_1\). But because some type-\(l\) firms hire young managers for a zero wage, again the average wage in type-\(l\) firms is lower than the average wage in type-\(h\) firms.

**Proposition 7 (The profitability differential)** The profitability differential has no impact on the equilibrium choices of type-\(l\) firms. In Case 1, an increase in \(\theta\) increases the wage offered by type-\(h\) firms to both retained and poached managers. In Case 2, \(\theta\) has no impact on wages.

In Case 1, old managers in \(l\)-firms are in short supply, and thus are able to capture a sizeable part of the surplus created by an increase in \(\theta\), which is reflected in their increased compensation. In Case 2, \(h\)-firms fully capture all of the surplus created by an increase in \(\theta\). Proposition 7 implies that, when talented managers are in short supply, an increase in firm heterogeneity leads to more pay inequality between managers working for the more profitable firms and those working for less profitable firms.

**Proposition 8 (Firm-specific skills)** In both Case 1 and Case 2, wages \(w^p_1\) and \(\ddot{w}_h\) increase with \(\gamma\).

When firm-specific skills are less important (that is, \(\gamma\) is larger), firms want to retain only highly-talented managers, thus they need to pay higher wages to avoid poaching. Similarly, poachers are willing to pay more for old managers when firm-specific skills are less important.

### 3.4. Comparing the two equilibria

To facilitate the interpretation of the results, we now refer to the no-poaching equilibrium as the *old economy equilibrium*, and to the poaching equilibrium as the *new economy equilibrium*.

\(^{13}\)Technically speaking, one could define the off-the-equilibrium-path retention threshold for type-\(h\) firms, but this is unnecessary because the equilibrium outcomes are unaffected by what happens off the equilibrium path.
Running the risk of abusing our freedom to interpret the model, we refer to type-\(l\) firms as small firms, and to type-\(h\) firms as large firms. We do so for better comparability with the existing literature and also for empirical reasons. Many papers have equated firm size with firm productivity (Rosen (1982) is an early example) and, empirically, more profitable firms are more likely to survive and grow (Foster, Haltiwanger, and Syverson, 2008).

**Regime changes in executive markets.** We interpret the two equilibria as different regimes in the market for executives. Before we discuss the differences between the two equilibria, we note that the model identifies a number of potential “triggers” of changes of regime. Equilibrium switches occur when parameters change and make \(\Delta\) cross the threshold of 1.

We focus on three parameters that have intuitive interpretations. The first one is \(\theta\), which could be interpreted as the (cross-sectional) dispersion in firm profitability. Because firm profitability is positively related to firm survival and growth, \(\theta\) is also related to the heterogeneity in firm sizes. The second parameter, \(\gamma\), measures the importance of general skills relative to firm-specific skills. Alternatively, an increase in \(\gamma\) can also be interpreted as a decrease in the cost of recruiting a new manager (e.g. search costs).

Finally, we also consider \(\eta \equiv \tau/\mu\), which is a measure of “right-tail dispersion” in the distribution of talent. This right-tail dispersion is intuitively related to the skewness of the distribution of talent. To see this, suppose that \(\tau\) increases while \(\mu\) remains constant. For this to happen, some density weight from the right of the mean must be shifted to the left of the mean. Thus, the distribution of talent becomes more positively skewed. We thus refer to \(\eta\) as a measure of skewness in the talent distribution. Skewness in talent and compensation is associated with existence of superstars (Rosen, 1981). The increasing importance of superstar managers can thus be modeled as an increase in \(\eta\): a large \(\eta\) indicates the existence of very few individuals with abilities much above the average.\(^{14}\)

The following remark follows from the definition of \(\Delta\) in (14):

**Remark 1** A transition from the old economy equilibrium to the new economy equilibrium may be triggered by:

1. An increase in the dispersion in firm profitability, \(\theta\);

2. An increase in the relative importance of general skills, \(\gamma\);

\(^{14}\)There is some evidence that superstar CEOs, as identified by the winning of media awards, subsequently underperform (Malmendier and Tate, 2009).
3. An increase in the skewness of the talent distribution, $\eta$.

A unique result of the model is that transitions between the two qualitatively different equilibria do not require large changes to the underlying parameters. As $\theta$, $\gamma$, and $\eta$ change continuously, no major change occurs until such changes make $\Delta$ cross its threshold.

**Old economy vs. New economy (Case 1).**

We now compare the characteristics of the no-poaching equilibrium with those of the Case-1 poaching equilibrium. For low values of $\theta$, $\gamma$, and $\eta$, turnover only occurs due to involuntary departures of less talented managers or because old managers retire. Hence, in the old economy we do not observe executive mobility across firms. Executives are appointed either internally or from a pool of outside managers. As $\theta$, $\gamma$, or $\eta$ increase, poaching becomes possible and so are voluntary departures of managers. In the new economy, more profitable/larger firms poach talented managers from smaller, less profitable firms.

**Remark 2** Executive mobility jumps discontinuously from zero to $(1 - F(\tilde{p}) )L(1 - F(\tilde{p}))$ once either $\theta$, $\gamma$, or $\eta$ become large enough.

In other words, the quality of the executive-firm match is improved once we move from the old economy to the new economy. Intuitively, as upward mobility becomes possible, large firms can free-ride on the information generated by the retention decisions of small firms and thus poach their best managers. Thus, large firms will attract the best managers from small firms. This poaching represents an improvement in the allocation of resources in the economy, because talent is complementary to size (i.e. profitability).

We now focus on comparing the behavior of large firms in each of the equilibria. Most of the available long-run evidence on executive turnover and compensation is from large firms only (e.g. Frydman and Saks, 2010), thus we focus on these results because they can be directly compared with the existing evidence.

One advantage of our model is that we can compare the two different equilibria while keeping constant all the other characteristics of the economy. That is, by considering the case in which $\Delta = 1$, in which both equilibria are possible, we can determine what happens to compensation and turnover when moving form one equilibrium to another, while keeping everything else constant.

We first consider the effect of a change in regime on executive turnover in large firms. Turnover in $h$-firms in a poaching equilibrium (Case 1) is given by

$$\beta_p^h + p = \frac{1 + \frac{L}{\pi}}{2 - F(\tilde{p}^p)} > \frac{1}{2 - F(\tilde{p}^h)} = \beta_h,$$

(25)
which proves the next remark:

**Remark 3** The new economy has a higher rate of executive turnover in large firms than the old economy does.

In the new economy, large firms find executive turnover more desirable, because now they can replace their incumbent executives with the best executives from smaller firms. This in turn increases turnover in large firms, once the new economy regime kicks in.

Next, we consider the effect of a change in regime on executive compensation in large firms. The average wage at type-$h$ firms in the old economy is given by (15), with $i = h$, and the average wage at type-$h$ firms in the new economy is given by

$$
(\alpha^p_h + p) w^p_h = \frac{1 - F(\tilde{\tau}^p_h) + \frac{(1 - F(\tilde{\tau}^p_h))L}{H}}{2 - F(\tilde{\tau}^p_h)} w^p_h. \tag{26}
$$

Comparing these two wages proves the following remark:

**Remark 4** Executives in large firms are paid more in the new economy than in the old economy.

We conclude that mobility, compensation, and turnover in large firms increase discontinuously when the new economy kicks in. The reason for this change is quite simple: now that poaching is feasible, large firms have more options and thus replace their managers more often. The competition for managers bids up the wages of poached managers.

There are also a number of less straightforward results. Recall that $\tilde{\tau}^p_h = \tilde{\tau}_h$. Define the pay-profitability sensitivity as the marginal effect of $\theta$ on the average wage. Comparing this measure for $h$-firms in the old economy

$$
\frac{\partial \alpha_h w_h}{\partial \theta} = \frac{1 - F(\tilde{\tau}_h)}{2 - F(\tilde{\tau}_h)} \gamma \left( E[\tau \mid \tau \geq \tilde{\tau}_h] - \mu \right) \tag{27}
$$

with that in the new economy

$$
\frac{\partial \alpha^p_h w^p_h}{\partial \theta} = \frac{1 - F(\tilde{\tau}^p_h) + \frac{(1 - F(\tilde{\tau}^p_h))L}{H}}{2 - F(\tilde{\tau}^p_h)} \gamma \left( E[\tau \mid \tau \geq \tilde{\tau}^p_h] - \mu \right), \tag{28}
$$

proves the following result:

**Remark 5** In large firms, the positive relation between firm size and executive pay is stronger in the new economy than in the old economy.
This result fits the empirical evidence quite well. The pay-size relationship was relatively flat until the mid 1970s, when it became much steeper (Frydman and Saks, 2010). Interestingly, such an evidence has been considered a challenge to models in which managerial talent is matched to firm productivity/size. Our model reveals that periods of a low correlation between firm size and executive pay can be explained by the equilibrium "costs of poaching" being larger than the "benefits from poaching." As large firms become larger (in model this happens as $\theta$ increases), eventually poaching becomes possible and a new equilibrium arises. In this new equilibrium, the correlation between firm size and executive pay (in large firms) is much stronger than in that in the old equilibrium. Our model also implies that such changes can be quick and dramatic: as $\Delta$ becomes larger than 1, executive pay at the largest firms increases discretely, and its relation with $\theta$ becomes steeper.

Our model also has implications for the differences in the cross-section of firms in each equilibrium. For example, we can show that pay inequality across firms of different sizes is more pronounced in the new economy:

**Remark 6** The firm-size pay premium is larger in the new economy than in the old economy.

This result follows immediately from the fact that the average pay in large firms increases as the new economy kicks in (Remark 4), while pay in small firms does not change.\(^\text{15}\)

**Old economy vs. New economy (Case 2).**

In this case, both Remarks 2 and 3 remain valid. However, Remarks 4 to 6 no longer hold. Intuitively, when large firms are relatively more powerful than managers, the improvement in the quality of matching, which is a characteristic of the new economy, benefits the shareholders of large firms more than their managers. The results of Case 1 seem to fit the evidence better, and also the popular perception that CEOs benefit more from improved profitability than shareholders.

4. The General Case

We now consider the general case in which $\delta \geq 0$. In this case, the dynamics of the model are at play, and firms also compare the costs of hiring a unknown young manager today with the benefits of retaining this manager in the next period. Learning about $\tau$ gives the firm an option to retain exactly those managers who are revealed to have high talent.

\(^{15}\)In fact, this result is even stronger in the more general case in which we do not impose Assumption T1. In that case, for some parameters, the average pay in small firms also decreases as one moves to the new equilibrium, which increases the firm-size pay premium even further.
The analysis in this section confirms that the simpler static ($\delta = 0$) case considered so far delivers most of the important results. Furthermore, it gives us some additional insights on the role of learning about talent. The only significant change is that we now need to redefine $\Delta$ in order to take into account the value of the option to learn about the ability of a young manager. Consequently, changes to the value of this option can also trigger changes in regime.

For brevity, we present the mathematical steps very briefly, as these are similar to the case of $\delta = 0$. The reader interested only in the intuition for the incremental contribution of this Section may want to skip to Subsection 4.2.

4.1. Equilibrium Existence and Characterization

We start with the case of a no-poaching equilibrium. As we focus on stationary equilibria, we omit the time subscripts.

Let $\pi^o_i(\tau)$ denote the one-period profit of a firm of type $i$ that keeps an old manager with talent $\tau$, and let $\pi^y_i$ denote the one-period profit of a firm of type $i$ that hires a young manager. The firm’s value function $V^o_i(\tau)$ if it retains an old manager is

$$V^o_i(\tau) = \pi^o_i(\tau) + \delta V^y_i;$$

and the value function $V^y_i$ if it hires a young manager is

$$V^y_i = \pi^y_i + \delta F(\tilde{\tau}_i) V^y_i + \delta (1 - F(\tilde{\tau}_i)) E[V^o_i(\tau) \mid \tau \geq \tilde{\tau}_i].$$

An equilibrium requires the existence of values for $V^o_i(\tau)$ and $V^y_i$ that simultaneously solve equations (29) and (30).

A type-$i$ firm retains a manager with talent $\tau$ if

$$V^o_i(\tau) - V^y_i = \pi^o_i(\tau) - \pi^y_i - \delta (1 - F(\tilde{\tau}_i)) (E[V^o_i(\tau) \mid \tau \geq \tilde{\tau}_i] - V^y_i) \geq 0.$$

Stationarity implies that

$$E[V^o_i(\tau) \mid \tau \geq \tilde{\tau}_i] - V^y_i = \frac{\delta (1 - F(\tilde{\tau}_i))}{1 + \delta (1 - F(\tilde{\tau}_i))} (E[\pi^o_i \mid \tau \geq \tilde{\tau}_i] - \pi^y_i) \equiv K_i(\tilde{\tau}_i).$$

Notice that $K_i(\tilde{\tau}_i)$ is the equilibrium option value of learning about the talent of a young manager.
We can then simplify the condition that determines the retention of an old manager to
\[ \pi_i^o (\tau) \geq \pi_i^y + K_i (\tilde{\tau}_i) . \] (33)

In a no-poaching equilibrium, the offer made by a potential h poacher to an i-firm is such that the poacher is indifferent between poaching an old manager or hiring a young manager:
\[ \pi_h^p (\tilde{\tau}_i) = \pi_h^y + K_h (\tilde{\tau}_h) . \] (34)

This condition can be rewritten as follows:
\[ \gamma \theta E[\tau | \tau \geq \tilde{\tau}_i] - \hat{w}_h (\tilde{\tau}_i) = \gamma \theta \mu + K_h (\tilde{\tau}_h) , \] (35)

where \( \hat{w}_h (\tilde{\tau}_i) \) is the wage that a potential h poacher is willing to pay for a manager with ability above the threshold \( \tilde{\tau}_i \). We can similarly define the wage \( \hat{w}_l (\tilde{\tau}_i) \) that a potential l poacher is willing to pay for a manager with ability above the threshold \( \tilde{\tau}_i \) from the following condition:
\[ \gamma E[\tau | \tau \geq \tilde{\tau}_i] - \hat{w}_l (\tilde{\tau}_i) = \gamma \mu + K_l (\tilde{\tau}_l) . \] (36)

We can see that an h-firm has a higher immediate gain from hiring an old manager. However, the option value of an h-firm may be higher than the option value of an l-firm. Therefore, we do not know a priori which of \( \hat{w}_h (\tilde{\tau}_i) \) and \( \hat{w}_l (\tilde{\tau}_i) \) is larger.

The offer made by a firm of type i in order to retain managers with ability \( \tau \geq \tilde{\tau}_i \) is given by
\[ w_i (\tilde{\tau}_i) = \max \{ \hat{w}_l (\tilde{\tau}_i), \hat{w}_h (\tilde{\tau}_i) \} . \] (37)

The thresholds \( \tilde{\tau}_i \) are given by the following equations:
\[ \theta \tilde{\tau}_h - w_h (\tilde{\tau}_h) = \gamma \theta \mu + K_h (\tilde{\tau}_h) , \] (38)
\[ \tilde{\tau}_l - w_l (\tilde{\tau}_l) = \gamma \mu + K_l (\tilde{\tau}_l) . \] (39)

For brevity, here we consider only the case in which \( \hat{w}_h (\tilde{\tau}_i) \geq \hat{w}_l (\tilde{\tau}_i) \) in an equilibrium. The analysis of this case is analogous to the case with \( \delta = 0 \). This case always holds if, for example, \( \delta \) is not too large. We note though that the case in which \( \hat{w}_h (\tilde{\tau}_i) < \hat{w}_l (\tilde{\tau}_i) \) creates no difficulties. This case delivers the same qualitative implications at the transition threshold and is omitted here just for brevity of exposition.

Under the assumption that \( \hat{w}_h (\tilde{\tau}_i) \geq \hat{w}_l (\tilde{\tau}_i) \), we obtain the retention threshold in h-firms
from the following equation:

\[ \tilde{\tau}_h = \gamma E[\tau \mid \tau \geq \tilde{\tau}_h]. \] (40)

We note that the option to learn about the manager’s ability does not affect the retention threshold for an \( h \)-firm. The intuition for this result is that the firm can recoup all of the surplus created by the option of hiring a young manager by decreasing the reward offered to an old manager.

Wages for retained managers become

\[ w_i(\tilde{\tau}_i) = \max \{ \gamma \theta (E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu) - K_h(\tilde{\tau}_h), 0 \}. \] (41)

The retention threshold for an \( l \)-firm is given by:

\[ \tilde{\tau}_l - \max \{ \gamma \theta (E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu) - K_h(\tilde{\tau}_h), 0 \} - \gamma \mu - K_l(\tilde{\tau}_i) = 0. \] (42)

We can now state the conditions for the existence of a no-poaching equilibrium:

**Proposition 9 (Existence of no-poaching equilibrium)** A no-poaching equilibrium exists and is unique if and only if \( \Delta^* \leq 1 \), where

\[ \Delta^* \equiv \frac{\theta \gamma (\overline{\tau} - \mu) - \delta \theta (1 - \gamma) \int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau)}{\overline{\tau} - \gamma \mu}. \] (43)

Using the same arguments as in Subsection 3.3, we can easily show that the threshold for the existence of a unique poaching equilibrium is also \( \Delta^* \geq 1 \). We thus omit this proof for brevity. Similarly, it is possible to prove all previous results; the equilibrium values are different, but all results are qualitatively the same.

**4.2. Discussion**

All new results in this section come from the new threshold \( \Delta^* \). The difference between \( \Delta \) and \( \Delta^* \) arises from the term \( \delta \theta (1 - \gamma) \int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau) \), which is the expected present value to an \( h \)-firm of the option to keep a talented old manager (the derivation of this term can be found in the proof of Proposition 9 in the Appendix). A firm only has this option if it hires a young manager at time \( t \) and then learns about the manager’s talent at time \( t + 1 \). To understand the intuition behind this term, first notice that

\[ \theta \int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau) = F(\tilde{\tau}_h) * 0 + \theta (1 - F(\tilde{\tau}_h)) E[\tau \mid \tau \geq \tilde{\tau}_h]. \] (44)
That is, $\theta \int_{\tau_h}^{\tau} \tau dF(\tau)$ is the expected value (as of $t + 1$) of retaining a manager with talent $\tau \geq \tau_h$, if the manager had no bargaining power. To see that this is the case, notice that with probability $F(\tau_h)$ the talent of the manager is below the threshold and the option will expire worthless. With probability $1 - F(\tau_h)$ the option is exercised and the expected profit is $\theta E[\tau | \tau \geq \tau_h]$. However, the firm can only capture a fraction $1 - \gamma$ of this value, because $\gamma \theta \int_{\tau_h}^{\tau} \tau dF(\tau)$ will accrue to the manager in the form of wages. Intuitively, in a no-poaching equilibrium, old managers with talent above the retention threshold expect to receive a fraction $\gamma$ of the surplus that they create. Thus, $1 - \gamma$ can also be seen as the firm’s expected bargaining power with respect to retained managers. The manager’s bargaining power increases as her skills become more general ($\gamma$ increases).

We first note that, as $\delta \to 0$, $\delta (1 - \gamma) \int_{\tau_h}^{\tau} \tau dF(\tau) \to 0$ and thus $\Delta^* \to \Delta$. We also have that $\Delta^* \geq \Delta$, which implies that the no-poaching equilibrium can be sustained for larger values of $\theta$, $\gamma$ and $\eta$. Intuitively, as the option to retain an old manager becomes less valuable, the relative value of poaching a manager increases, which makes poaching more likely to occur in equilibrium.

One of the key results of this Section is that $\Delta^*$ decreases when firms become more impatient, that is, when $\delta$ falls. An alternative interpretation of $\delta$ is that it represents the probability that the firm will be in business in one period from now. Thus, tougher competition in product markets may lead to a decrease in $\delta$. Our results show that increasing product market competition could be yet another reason for an increase in executive labor market mobility.

5. Discussion and Extensions

In Subsection 5.1, we first briefly compare this simple version with an even simpler case: a model in which learning is symmetric. In Subsection 5.2, we introduce effort and incentive pay. In Subsection 5.3, we discuss an extension in which performance is also informative about talent.

5.1. Symmetric Learning

Under symmetric information, the wage function is the same for everyone: $w(\tau)$. An $h$-type firm retains a manager of type $\tau$ if

$$\theta \tau - w(\tau) \geq \theta \gamma \mu, \quad (45)$$
and poaches a manager of type $\tau$ unless

$$\theta \gamma \tau - w(\tau) \leq \theta \gamma \mu. \quad (46)$$

Thus no-poaching wages are given by

$$w(\tau) = \max \{\theta \gamma (\tau - \mu), 0\}. \quad (47)$$

An $l$-firm retains type $\tau$ if

$$\tau - \max \{\theta \gamma (\tau - \mu), 0\} \geq \gamma \mu. \quad (48)$$

Thus, firm $l$ always retains all types $\tau \in [\gamma \mu, \mu]$ and pays them nothing (in all equilibria), and also retains $\tau > \mu$ only if

$$\tau - \theta \gamma (\tau - \mu) \geq \gamma \mu, \quad (49)$$

which holds for sure if $\gamma \theta < 1$, but may also hold for some $\gamma \theta > 1$, in which case the maximum $\tau$ that the firm retains is

$$\tau(\theta) = \frac{\gamma \mu (1 - \theta)}{1 - \theta \gamma}. \quad (50)$$

We have

$$\tau'(\theta) = \gamma \mu \frac{\gamma - 1}{(1 - \theta \gamma)^2} < 0, \quad (51)$$

that is, the maximum type that is retained falls with $\theta$.

An equilibrium in which all types are retained exists only if

$$\tau - \theta \gamma (\tau - \mu) \geq \gamma \mu, \quad (52)$$

which implies the following threshold:

$$\Delta \leq 1. \quad (53)$$

We first note that the condition for the existence of a no-poaching equilibrium is exactly the same as that in the case of asymmetric learning. However, if $\Delta > 1$, the new equilibrium will involve some poaching, but only the most talented managers will be poached. An $l$-firm always retains all $\tau \in [\gamma \mu, \tau(\theta)]$, where $\tau(\theta) > \mu$ for any finite $\theta$. 
Thus, under symmetric information, there is no sharp discontinuity in executive mobility. As \( \Delta \) goes above 1, the fraction of poached managers grows monotonically with \( \theta \) and is given by \( [\tau (\theta), \bar{\tau}] \).

Let us summarize what our assumption of asymmetric learning is contributing to the analysis. First, from a purely methodological viewpoint, this assumption allows us to compare the two equilibria while keeping all other parameters constant. This is convenient because it allows us to analyze the differences in the characteristics of executive labor markets that are explained only by the nature of the equilibrium, and not by any of the other parameters in the economy. This is obviously not possible in the symmetric learning case, because the progression from one equilibrium to another is smooth.

Second, with asymmetric learning, drastic changes in the characteristics of executive labor markets may occur even when key parameters, such as firm heterogeneity (\( \theta \)) and the importance of firm-specific skills (\( \gamma \)), change slowly and continuously. That is, in our model, asymmetric learning is a natural way of generating a discontinuity in an otherwise continuous environment.\(^{16}\)

Third, the asymmetric learning case allows us to compare the relative efficiency of the two types of equilibria. Under symmetric information, turnover is always efficient. Firms replace their managers either when firms expect outside managers to be better than incumbents or when incumbents are poached. In the latter case, better managers move to more productive firms, which improves allocational efficiency. Asymmetric learning generates two types of turnover, one good and one bad. The good (efficient) turnover is the same as that in the symmetric information case. Bad turnover happens when firms fire some managers, who have talent above the average, to avoid lowering their pay and inviting poaching. Thus, the new economy equilibrium is more efficient than the old economy equilibrium, because in the new economy there is both less bad turnover and more good turnover (poaching).

Fourth, there is too much poaching in the asymmetric learning case. Intuitively, the existence of a few very talented individuals creates an externality: firms have incentives to poach managers from other firms hoping to recruit a superstar. If firms know the identity of the superstars, no externality would occur, as firms would only try to poach the superstars. Asymmetric information implies that firms poach too many managers hoping to find a superstar.

Finally, we note that the asymmetric learning model has different empirical implications.\(^{16}\)

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\(^{16}\)We note however that asymmetric learning is not sufficient for such discontinuity. It is possible to make the transition smooth again, for example, by allowing for a continuum of firm types. We make only the weaker claim that asymmetric learning makes discontinuities more likely to occur.
For example, in the asymmetric learning case, we can meaningfully speak of a firm size-pay premium, in the sense that, controlling for all other observable executive characteristics, executive pay is a function of firm size. In the symmetric learning case, all executives with the same general talent are paid the same, regardless of which firms they work for.

5.2. Effort and Incentive Pay

Our model is silent about the composition of executive pay. In this subsection, we introduce a simple moral hazard problem that allows us to analyze the composition of pay and compare them across the two equilibria.

Let us now assume that managerial effort is another productive input. Effort is measured by \( e \in (0, 1) \), which is private information to the manager. As before, managers are risk-neutral, have no initial wealth, and are protected by limited liability. We assume that effort imposes a nonpecuniary cost to the manager according to a quadratic cost function \( c(e) = \frac{e^2}{2} \).

We modify the technology to incorporate effort:

\[
R_{jt}(i_j, s_{jt}, \tau_{jt}, e_{jt}) = \begin{cases} 1, & \text{with probability } i_j s_{jt} \tau_{jt} e_{jt} \\ 0 & \text{with probability } 1 - i_j s_{jt} \tau_{jt} e_{jt} \end{cases} \tag{54}
\]

We keep the same restrictions on the these variables as before.

Each firm now offers their managers a contract \((w, b)\), where \( w \geq 0 \) is a fixed wage and \( b \in [0, 1] \) is the fraction of the output that is given to the manager. Given the binomial nature of the technology, we do not need to consider more complicated contracts.

We need to impose a different restriction on the derivative of \( E[\tau^2 \mid \tau \geq \tilde{\tau}] \). We first define the following function:

\[
\varepsilon(\tilde{\tau}) \equiv \frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \frac{\tilde{\tau}}{E[\tau^2 \mid \tau \geq \tilde{\tau}]} \tag{55}
\]

Notice that \( \varepsilon(\tilde{\tau}) \) is the elasticity of the second non-centered moment of the truncated distribution \( F(\tau \mid \tau \geq \tilde{\tau}) \) with respect to the truncation point \( \tilde{\tau} \). We now assume:

**Assumption T1’** \( \varepsilon(\tilde{\tau}) \leq 2 \) for all \( \tilde{\tau} \in [\underline{\tau}, \bar{\tau}] \).

Assumption T1’ is not very restrictive. For example, we have that:

**Lemma 1** If \( f \) is log-concave, then \( \varepsilon(\tilde{\tau}) \leq 2 \) holds for all \( \tilde{\tau} \in [\underline{\tau}, \bar{\tau}] \).
5.2.1. Monopoly

To fix ideas, we briefly develop here a benchmark case in which there is no competition.

Suppose initially that there is only one firm. The firm knows the talent of its incumbent manager, $\tau$. Because there are no other firms, if the firm wishes to replace its manager, it must hire a young manager.

Let $\pi_i$ denote the profit of a firm of type $i$. Suppose first that the firm decides to retain its manager. The program of a type-$i$ firm is to:

$$\max_{b,w} E[\pi_i] = (1 - b)i\tau e - w, \quad (56)$$

subject to

$$\begin{cases}
    bid - e = 0 & (IC) \\
    bid - \frac{\sigma^2}{2} + w \geq 0 & (IR) \\
    w \geq 0 & (LL)
\end{cases} \quad (57)$$

The first constraint is the incentive compatibility (IC) constraint for the manager, the second one is her participation or individual rationality (IR) constraint, and the last one is the limited liability (LL) constraint. In an optimal contract, LL binds and $w$ is always zero. This is true in all the cases that we consider. Thus, to simplify the exposition we ignore $w$ from now on. Notice that IC implies that IR is not binding. The optimal bonus is $b^M = \frac{1}{2}$, where the superscript $M$ refers to the optimal solution in the monopolistic scenario. This bonus is independent of the type of the firm. The (expected) profit from retaining the incumbent manager is given by $\pi^*_i = \frac{i^2 \gamma^2}{4}$.

Suppose now that the firm decides to fire its manager and replace her with a young manager. Using the fact that $w = 0$, the program of a type-$i$ firm is now:

$$\max_b E[\pi_i] = E [(1 - b)i\gamma e]$$

subject to

$$\begin{cases}
    b\gamma i\tau - e = 0 & (IC) \\
    b\gamma i\tau e - \frac{\sigma^2}{2} \geq 0 & (IR)
\end{cases} \quad (59)$$

The optimal bonus is still $b^M = \frac{1}{2}$. The expected profit from hiring a young manager

$$\pi^y_i = \frac{i^2 \gamma^2 E[\tau^2]}{4} = \frac{i^2 \gamma^2 (\sigma^2 + \mu^2)}{4}. \quad (60)$$
The firm retains its incumbent manager if and only if \( \tau_i^o \geq \pi_i^y \), or
\[
\tau \geq \hat{\tau}^M \equiv \gamma \sqrt{\sigma^2 + \mu^2}.
\]  
(61)

5.2.2. Existence of a No-Poaching Equilibrium

We now describe the derivation of a no-poaching equilibrium.

If a firm chooses to hire a young manger, it faces no competition. Thus, the optimal bonus is the same as that in the monopolistic case: \( b_i^o = \frac{1}{2}, i = l, h \). Thus, in what follows we only need to derive the equilibrium values for \((b_i, \tilde{\tau}_i), i = l, h\).

A firm of type \( i \) that retains a manager with talent \( \tau_i \) and pays a bonus \( b_i \) has an expected profit of \( b_i(1 - b_i)\tau_i^2 \). Under no poaching, firm \( i \) retains the manager if
\[
b_i(1 - b_i)\tau_i^2 \geq \frac{i^2\gamma^2 E[\tau^2]}{4}.
\]  
(62)

This rule implies a threshold decision (given \( b_i \)) to retain the manager only if \( \tau_i \geq \tilde{\tau}_i \), where
\[
\tilde{\tau}_i = \sqrt{\frac{\gamma^2 E[\tau^2]}{b_i(1 - b_i)4}}, \quad i = l, h.
\]  
(63)

This condition links the equilibrium thresholds to the offers that are made to the incumbent managers. Note that \( \tilde{\tau}_i \) is an increasing function of \( b_i \in \left[ \frac{1}{2}, 1 \right] \).

We now derive the no-poaching condition for an \( h \)-firm. The minimum offer that a manager working for firm \( i \) would accept is \( \frac{b_i}{\gamma} \). If \( \theta \gamma > 1 \), a manager from a type-\( l \) firm would accept to work for a type-\( h \) firm even if the bonus was lower than \( b_l \). Thus the minimum offer that a type-\( h \) firm makes to a type-\( i \) incumbent manager is \( b_i' = \max \left\{ \frac{b_i}{\gamma}, \frac{1}{2} \right\} \), because it is never optimal to offer a bonus that is lower than \( \frac{1}{2} \) (the optimal bonus in the monopolistic case).

Assuming that the minimum offer is made, if the poaching attempt is successful, firm \( h \)'s profit is
\[
b_i' [1 - b_i'] \theta^2 \gamma^2 E[\tau^2] | \tau \geq \tilde{\tau}_i.
\]  
(64)

This profit is decreasing in \( b_i' \in \left[ \frac{1}{2}, 1 \right] \).

In a no-poaching equilibrium, this profit must satisfy two conditions. First, \( b_i' = \frac{b_i}{\gamma} \), otherwise a type-\( h \) firm could offer a bonus of \( \frac{1}{2} \) to poach a manager from a type-\( l \) firm. Second, this profit must not be larger than the one obtained by hiring a young manager.
These two conditions imply the following one:

\[
\frac{b_i}{\theta_i} \left[ 1 - \frac{b_i}{\theta_i} \right] E[\tau^2 \mid \tau \geq \tilde{\tau}_i] \leq \frac{E[\tau^2]}{4}.
\]  

(65)

Unless \( b_i = \frac{1}{2} \), the condition above cannot hold with slack in equilibrium, because if it did, firm \( i \) could reduce \( b_i \) and still avoid the poaching of its manager. Thus, in a no-poaching equilibrium, either condition (65) must hold with equality or \( b_i = \frac{1}{2} \). We have that:

\[
b_l = \max \left\{ \frac{\theta_i}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 \mid \tau \geq \tilde{\tau}_l]}} \right) \frac{1}{2} \right\}, \text{ and}
\]

(66)

\[
b_h = \max \left\{ \frac{\gamma}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 \mid \tau \geq \tilde{\tau}_h]}} \right) \frac{1}{2} \right\},
\]

(67)

which, together with condition (63) determine the equilibrium pairs \((b_l, \tilde{\tau}_l)\) and \((b_h, \tilde{\tau}_h)\), if an equilibrium exists.

The following proposition establishes the necessary and sufficient condition for the existence of a unique no-poaching equilibrium.

**Proposition 10 (Existence of no-poaching equilibrium)** A no-poaching equilibrium exists and is unique if and only if \( \Delta^{**} \leq 1 \), where

\[
\Delta^{**} \equiv \frac{\tau^* + \sqrt{\tau^{*2} - \gamma^2 E[\tau^2]}}{\theta_i \left( \tau^* + \sqrt{\tau^{*2} - E[\tau^2 \mid \tau \geq \tau^*]} \right)},
\]

(68)

and

\[
\tau^* = \arg \max_{\tilde{\tau}_l \in [\underline{\tau}]} (1 - b_l) b_l \tilde{\tau}_l^2.
\]

(69)

**5.2.3. Existence of a Poaching Equilibrium**

We now describe the derivation of a poaching equilibrium. As before, \( b_i^p = \frac{1}{2} \), \( i = l, h \).

For brevity, we consider only Case 1. If all managers from \( l \)-firms are poached, \( h \)-firms compete for managers, driving the poaching bonus to

\[
\hat{b}_h = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 \mid \tau \geq \tilde{\tau}_l]}} \right).
\]

(70)

Because both \( h \) and \( l \) firms only avoid poaching by firms of their own type, we have \( \tilde{\tau}_l^p = \)
\[ \hat{\tau}_h^p = \hat{\tau}^p \] and
\[ b_i^p = b_n^p = \max \left\{ \frac{\gamma^p}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}^p]}} \right), \frac{1}{2} \right\}. \quad (71) \]

For a no poaching equilibrium to exist, we need an \( l \)-firm to be unwilling to fight for a manager of type \( \tau^* \), which implies the following proposition

**Proposition 11 (Existence of poaching equilibrium)** A poaching equilibrium exists and is unique if and only if \( \Delta^{**} > 1 \).

5.2.4. Comparing the Two Equilibria

Most of the results proved in the case of no effort also hold (with the appropriate modifications) in the current case. For brevity, here we only focus on the new results that are related to \( b \), which is our measure of pay-performance sensitivity. Again, here we consider the case of \( \Delta^{**} = 1 \). The fact that \( \hat{b}_h > \hat{b}_n^p = b_h \) implies the following result.

**Remark 7** In large firms, the pay-performance sensitivity in the new economy is larger than the pay-performance sensitivity in the old economy.

This result follows from the fact that more intense competition for managers in the poaching equilibrium bids up the pay for the poached managers. Limited liability then implies that total pay and performance-based pay are positively linked. This result also implies that, in the new economy, the pay-performance sensitivity is larger for experienced managers hired from the outside than for experienced inside managers. Intuitively, because outside managers lack firm-specific skills, their performance-based bonuses must be larger so that outside managers can expect the same pay as managers retained by large firms.

5.3. Learning from Performance

To be done.

6. Conclusions

To be done.
7. Appendix

7.1. Discussion of Assumption T1

Although we can prove most of the results without T1 (at the cost of more complexity), we also is note that T1 is not too restrictive. For example, any uniform distribution with $\tau \geq \frac{1}{2}$ trivially satisfies Assumption T1.

Assumption T1 is similar to (but more restrictive than) assuming log-concavity of $F$, which implies $\frac{\partial E[\tau \mid \tau \geq \hat{\tau}]}{\partial \tau} \leq 1$. In fact, most of our results can be proven with log-concavity alone. Log-concavity is not very restrictive. Many families of distributions, including the Uniform distribution and (truncated) Normal distributions, are log-concave.

We also do not need T1 to hold for all $\hat{\tau} \in [\tau, \bar{\tau}]$; most of our results can be proven for any differentiable $F$ as long as $\tau \geq \frac{1}{2}$, for a large set of parameters:

**Lemma 2** For any twice-differentiable $F$ such that $\bar{\tau} \geq \frac{1}{2}$, there exists a $\tau_F$ such that condition T1 holds for all $\hat{\tau} \in [\tau_F, \bar{\tau}]$.

Proof.

\[
\frac{\partial E[\tau \mid \tau \geq \hat{\tau}]}{\partial \hat{\tau}} = \frac{f(\hat{\tau})}{1 - F(\hat{\tau})} \{ E[\tau \mid \tau \geq \hat{\tau}] - \hat{\tau} \} \quad (72)
\]

\[
= \frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \left\{ \int_{\hat{\tau}}^\tau \tau f(\tau) d\tau - \hat{\tau} [1 - F(\hat{\tau})] \right\} \quad (73)
\]

\[
= \frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \left\{ \int_{\hat{\tau}}^\tau (\tau - \hat{\tau}) f(\tau) d\tau \right\} \quad (74)
\]

Integrating by parts yields

\[
\frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \left\{ (\tau - \hat{\tau}) F(\tau) |_{\hat{\tau}}^\tau - \int_{\hat{\tau}}^\tau F(\tau) d\tau \right\} =
\]

\[
= \frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \left\{ \tau F(\hat{\tau}) - \hat{\tau} F(\hat{\tau}) - \int_{\hat{\tau}}^\tau F(\tau) d\tau \right\} \quad (75)
\]

\[
= \frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \left\{ \int_{\hat{\tau}}^\tau [1 - F(\tau)] d\tau \right\}
\]

Now we take the limit as $\hat{\tau} \to \tau$

\[
\lim_{\hat{\tau} \to \tau} \frac{dE[\tau \mid \tau \geq \hat{\tau}]}{d\hat{\tau}} = \lim_{\hat{\tau} \to \tau} \frac{f(\hat{\tau})}{[1 - F(\hat{\tau})]^2} \int_{\hat{\tau}}^\tau [1 - F(\tau)] d\tau \quad (75)
\]
Using L’Hôpital’s rule twice,

\[
\lim_{\tilde{\tau} \to \tilde{\tau}} \frac{f'(\tilde{\tau}) \int_{\tilde{\tau}}^{\infty} [1 - F(\tau)] d\tau - [1 - F(\tilde{\tau})] f(\tilde{\tau})}{-2 [1 - F(\tilde{\tau})] f(\tilde{\tau})} = \lim_{\tilde{\tau} \to \tilde{\tau}} \frac{f'(\tilde{\tau}) \int_{\tilde{\tau}}^{\infty} [1 - F(\tau)] d\tau + \frac{1}{2}}{-2 [1 - F(\tilde{\tau})] f(\tilde{\tau}) + \frac{1}{2}} = \frac{1}{2}
\]

which implies (by continuity) that \( \frac{\partial E[\tau \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq \tau \) in a neighborhood of \( \tau \). ■

7.2. Proofs

Proposition 1.

Proof. If. The threshold for an \( h \)-firm is given by

\[
\tilde{\tau}_h - \gamma E[\tau \mid \tau \geq \tilde{\tau}_h] = 0.
\]

For \( \tilde{\tau}_h = 0 \), the left-hand side of (79) becomes \(- \gamma \mu < 0\). For \( \tilde{\tau}_h = \tau \), the left-hand side of (79) becomes \( \tau - \gamma \tilde{\tau} > 0 \). Therefore a solution exists. The solution is unique if \( 1 - \gamma \frac{\partial E[\tau \mid \tau \geq \tilde{\tau}_h]}{\partial \tilde{\tau}_h} > 0 \), which is implied by T1.

The threshold for an \( l \)-firm is given by

\[
\tilde{\tau}_l - \theta \gamma E[\tau \mid \tau \geq \tilde{\tau}_l] + \gamma \mu (\theta - 1) = 0.
\]

For \( \tilde{\tau}_l = \tau \), the left-hand side of (80) becomes \( \tau - \gamma \mu < 0 \). For \( \tilde{\tau}_l = \tau \), the left-hand side of (80) becomes

\[
\tau - \theta \gamma (\tau - \mu) > \gamma \mu,
\]

which is equivalent to \( \theta \gamma < \frac{\tau - \mu}{\tau - \gamma \mu} \). Therefore a solution exists, provided this condition holds.

To prove the uniqueness of the equilibrium it is sufficient to show that the left-hand side of (80) increases with \( \tilde{\tau}_l \):

\[
1 - \theta \gamma \frac{\partial E[\tau \mid \tau \geq \tilde{\tau}_l]}{\partial \tilde{\tau}_l} > 1 - \tau \frac{\partial E[\tau \mid \tau \geq \tilde{\tau}_l]}{\partial \tilde{\tau}_l} > 0,
\]

due to Assumption T1. Thus, for \( \frac{\theta \gamma (\tau - \mu)}{\tau - \gamma \mu} < 1 \) there is a unique equilibrium \( \tilde{\tau}_l \).\(^{17}\)

\(^{17}\)Uniqueness here is in a generic sense. The equilibrium is not unique if parameters are such that \( \Delta = 1 \).
Only if. If \( \frac{\theta_s(\tau - \mu)}{\tau - \gamma \mu} > 1 \), equation (80) has no solution. ■

**Proposition 2.**

**Proof.** 1. This follows from equations (79) and (80) and the fact that the left-hand side of (80) is increasing in \( \tilde{\tau}_l \).

2. This follows from \( \tilde{\tau}_l > \tilde{\tau}_h \) and the stationarity condition \( \beta_i = \frac{1}{2 - \overline{F}(\tilde{\tau}_i)}, \ i = l, h \).

3. \( w(\tilde{\tau}_i) = \theta \gamma(\overline{E}[\tau \mid \tau \geq \tilde{\tau}_i] - \mu) \) is increasing in \( \tilde{\tau}_i \), and \( \tilde{\tau}_l > \tilde{\tau}_h \). ■

**Proposition 3.**

**Proof.** It follows immediately from the inspection of conditions (12), (79) and (80). ■

**Proposition 4.**

**Proof.** It follows immediately from the inspection of conditions (12), (79) and (80). ■

**Proposition 5.**

**Proof.** *Only if.* The maximum that an \( l \)-firm is willing to pay in order to retain \( \tau \) is:

\[
\tau - w_i^{\text{max}}(\tau) = \gamma \mu \Rightarrow w_i^{\text{max}}(\tau) = \tau - \gamma \mu. \tag{83}
\]

The maximum that an \( h \)-firm is willing to pay in order to poach a manager with talent \( \tau \) is:

\[
\gamma \theta \tau - \hat{w}_h(\tau) = \gamma \theta \mu \Rightarrow \hat{w}_h(\tau) = \gamma \theta \tau - \gamma \theta \mu. \tag{84}
\]

Poaching can only occur if an \( l \)-firm is not willing to match an offer made by an \( h \)-firm to the best possible manager, i.e. when \( \hat{w}_h(\tau) \geq w_i^{\text{max}}(\tau) \). Hence a necessary condition for the existence of a poaching equilibrium is:

\[
\theta \geq \frac{\tau - \gamma \mu}{\gamma(\tau - \mu)}. \tag{85}
\]

*If.* Assuming that an equilibrium exists, we characterized one set of quantities for each possible (mutually exclusive) case. Condition \( \Delta \geq 1 \) implies that an \( l \)-firm does not want to deviate and fight a poaching attempt because, if it cannot retain the best possible manager, it cannot win a fight for any manager. ■

**Proposition 6.**

**Proof. Case 1.** 1. \( \hat{\tau}_h^p = \hat{\tau}_l^p \) (immediate from inspection of (18) and (22)).

2. Turnover in \( l \)-firms is 100%.

3. Offered wage is \( w_l^p < w_h^p \) (see (19) and (23)) and effective wage paid is zero in \( l \)-firms.
Case 2. 1. $\hat{\tau}^p_h = 1 > \hat{\tau}^p_i$, because all $h$-firms are poachers.
2. Turnover in $h$-firms is 100%.
3. Wage is $\hat{w}_h = w_i^p$ for all old managers. ■

**Proposition 7.**

**Proof.** Inspection of (18) and (19), (21), and (23) reveals that $\theta$ has no impact on the threshold or the wages paid by type-$l$ firms. Equation (21) implies that $\hat{w}_h$ (and thus $w_i^p$) increases with $\theta$ in Case 1. In Case 2, $\hat{w}_h = w_i^p$, which is independent of $\theta$. ■

**Proposition 8.**

**Proof.** (19) implies that $w_i^p$ increases with $\gamma$ in both cases. In Case 1, $\hat{w}_h = w_i^p$, and they both increase with $\gamma$ (see (21)). In Case 2, $\hat{w}_h = w_i^p$, and they both increase with $\gamma$ (see (19)). ■

**Lemma 1.**

**Proof.** If $f$ is a log-concave density, then $V'[\tau \mid \tau \geq \tilde{\tau}] \leq 0$ and $E'[\tau \mid \tau \geq \tilde{\tau}] \leq 1$ (see e.g. An (1988)). We have

$$\frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} - 2E[\tau \mid \tau \geq \tilde{\tau}] \frac{\partial E[\tau \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq 0,$$

therefore

$$\frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq 2E[\tau \mid \tau \geq \tilde{\tau}].$$

Replacing in the elasticity we need the following condition to hold:

$$\frac{2E[\tau \mid \tau \geq \tilde{\tau}]\tilde{\tau}}{E[\tau^2 \mid \tau \geq \tilde{\tau}]} \leq 2 \tag{88}$$

which can be rewritten as:

$$\tilde{\tau} \int_{\tilde{\tau}}^{\tau} \frac{\tau f(\tau)}{1 - F(\tilde{\tau})} \leq \int_{\tilde{\tau}}^{\tau} \frac{\tau^2 f(\tau)}{1 - F(\tilde{\tau})}, \tag{89}$$

which always holds. ■

**Proposition 9.**

**Proof.** We first determine the equilibrium values for $K_h(\tilde{\tau}_h)$ and $K_i(\tilde{\tau}_i)$. Define $H(\tilde{\tau}_i) = \frac{\delta(1-F(\tilde{\tau}_i))}{1+\delta(1-F(\tilde{\tau}_i))}$. The equilibrium wage in an $h$-firm is

$$w_h(\tilde{\tau}) = \gamma \theta E[\tau \mid \tau \geq \tilde{\tau}] - \gamma \theta \mu - H(\tilde{\tau}_i) (\theta E[\tau \mid \tau \geq \tilde{\tau}_h] - w_h(\tilde{\tau}_h) - \theta \gamma \mu) \tag{90}$$
or
\[
\omega_h(\tau_h) = \theta \frac{\gamma - H(\tau_h)}{1 - H(\tau_h)} E[\tau | \tau \geq \tau_h] - \theta \gamma \mu, 
\] (91)
which implies
\[
K_h(\tau_h) = H(\tau_h) \theta \left( E[\tau | \tau \geq \tau_h] - \frac{\gamma - H(\tau_h)}{1 - H(\tau_h)} E[\tau | \tau \geq \tau_h] + \gamma \mu - \gamma \mu \right) 
\] (92)
which simplifies to
\[
K_h(\tau_h) = \theta \delta (1 - \gamma) \int_{\tau_h}^{\tau} \tau dF(\tau), 
\] (93)
and we then have
\[
K_l(\tau_l) = H(\tau_l) (1 - \theta \gamma) E[\tau | \tau \geq \tau_l] + (\theta - 1) \gamma \mu + K_h(\tau_h). 
\] (94)

\[
K_l(\tau_l) = H(\tau_l) [1 - \theta \gamma] E[\tau | \tau \geq \tau_l] + (\theta - 1) \gamma \mu + K_h(\tau_h). 
\] (95)

\textbf{If.} The existence and uniqueness of the threshold for an \( h \)-firm is proven in Proposition 1. The threshold for an \( l \)-firm is given by:
\[
\tau_l = \max \{ \gamma \theta (E[\tau | \tau \geq \tau_l] - \mu) - K_h(\tau_h), 0 \} - \gamma \mu - K_l(\tau_l) = 0. 
\] (96)
For \( \tau_l = \tau \), we have \( \tau - \gamma \mu - K_l(\tau_l) < 0 \).
\[
K_l(\tau) = \frac{\delta}{1 + \delta} [(1 - \theta \gamma) \mu + (\theta - 1) \gamma \mu + K_h(\tau_h)] = \frac{\delta}{1 + \delta} [(1 - \gamma) \mu + K_h(\tau_h)] 
\] (97)
For \( \tau_l = \tau \), \( K_l(\tau) = 0 \) and we have \( \tau_l - \gamma \theta (\tau - \mu) - \gamma \mu + K_h(\tau_h) > 0 \), which is equivalent to \( \theta \gamma < \frac{\tau - \gamma \mu + K_h(\tau_h)}{\tau - \mu} \). Therefore a solution exists, provided this condition holds.

To prove the uniqueness of the equilibrium we need to prove that the left-hand side of (80) increases with \( \tau_l \):
\[
1 - \theta \gamma \frac{\partial E[\tau | \tau \geq \tau_l]}{\partial \tau_l} - \frac{\partial K_l(\tau_l)}{\partial \tau_l} > 0 
\] (98)
We have that
\[
\frac{\partial K_l(\tau_l)}{\partial \tau_l} = \frac{\partial H(\tau_l)}{\tau_l} [1 - \theta \gamma] E[\tau | \tau \geq \tau_l] + (\theta - 1) \gamma \mu + K_h(\tau_h) + \frac{\partial E[\tau | \tau \geq \tau_l]}{\partial \tau_l} H(\tau_l), 
\] (99)
and
\[
(1 - \theta \gamma) \frac{\partial E[\tau | \tau \geq \tau_l]}{\partial \tau_l} H(\tau_l), 
\] (100)
Thus we have

\[
1 - \theta \gamma \frac{\partial E[\tau | \tau \geq \tilde{\tau}_l]}{\partial \tilde{\tau}_l} - \frac{\partial K_l(\tilde{\tau}_l)}{\partial \tilde{\tau}_l} = (1 - \theta \gamma)[1 - H(\tilde{\tau}_l)] \frac{\partial E[\tau | \tau \geq \tilde{\tau}_l]}{\partial \tilde{\tau}_l} - \frac{\partial H(\tilde{\tau}_l) K_l(\tilde{\tau}_l)}{\partial \tilde{\tau}_l} H(\tilde{\tau}_l) > 0
\]  

(101)

(102)
due to Assumption T2 and the fact that \( H(\tilde{\tau}_l) < 1 \) and \( \frac{\partial H(\tilde{\tau}_l)}{\partial \tilde{\tau}_l} \leq 0 \).

\textit{Only if.} The same as in Proposition 1. \( \blacksquare \)

\textbf{Proposition 10}

\textbf{Proof.} Existence: The equilibrium values \( \tilde{\tau}_i \) and \( b_i \) for a type-l firm are given by:

\[
\begin{align*}
\left\{ 
\begin{array}{l}
b_i = \max \left\{ \frac{1}{2}, \frac{\theta \gamma}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \tilde{\tau}_l]}} \right) \right\} \\
(1 - b_i)b_i\tilde{\tau}_l^2 = \frac{\gamma^2 E[\tau^2]}{4}
\end{array}
\right.
\]

(103)

Let

\[
\tau^* = \arg \max (1 - b_i)b_i\tilde{\tau}_l^2,
\]

(104)

which for now we assume is uniquely defined. For \( \tilde{\tau}_l = \tau^* \):

\[
(1 - b_i)b_i\tau^* \geq \frac{\gamma^2 E[\tau^2]}{4} \iff b_i \leq \frac{1}{2} \left( 1 + \sqrt{1 - \frac{\gamma^2 E[\tau^2]}{\tau^*}} \right).
\]

(105)

The maximum offer that an \( h \)-firm would make to a manager given \( \tilde{\tau}_l = \tau \) is:

\[
b'_i = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \tau^*]}} \right).
\]

(106)

Therefore if \( b'_i \theta \gamma \leq \frac{1}{2} \left( 1 + \sqrt{1 - \frac{\gamma^2 E[\tau^2]}{\tau^*}} \right) \), then \( (1 - b_i)b_i\tau^* \geq \frac{\gamma^2 E[\tau^2]}{4} \). The former always holds if:

\[
\frac{\theta \gamma}{2} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \tau^*]}} \right) \leq \frac{1}{2} \left( 1 + \sqrt{1 - \frac{\gamma^2 E[\tau^2]}{\tau^*}} \right)
\]

(107)

If the condition given by equation (107) holds, then an equilibrium exists.

Uniqueness: We only need to show that \( \tau^* \) is uniquely defined. For \( b_i = \frac{1}{2} \), the function \( (1 - b_i)b_i\tilde{\tau}_l^2 \) is strictly increasing and the proof is trivial.
For \( b_t = \frac{\partial}{\partial \tau_i} \left( 1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]}} \right) \), let \((1 - b_t)b_t \hat{\tau}_i^2 \equiv B\), then

\[
\frac{\partial B}{\partial \tau_i} = 2 \hat{\tau}_i b_t \gamma \theta (1 - b_t \gamma \theta) - \frac{\hat{\tau}_i^2 \gamma \theta (2 b_t \gamma \theta - 1) \partial b_t}{\partial \tau_i}
= 2 \hat{\tau}_i \gamma \theta \left( b_t (1 - b_t \gamma \theta) - \frac{\hat{\tau}_i^2 (2 b_t \gamma \theta - 1) \partial b_t}{2} \right)
\tag{108}
\]

Therefore \( \frac{\partial b_t}{\partial \tau_i} \) becomes:

\[
\frac{\partial B}{\partial \tau_i} = 2 \hat{\tau}_i \gamma \theta b_t (1 - b_t \gamma \theta) - \frac{(2 b_t \gamma \theta - 1) \hat{\tau}_i}{8(2 b_t - 1)} \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\frac{\partial E[\tau^2]}{\partial \tau_i}}{\partial \tau_i}
\tag{109}
\]

Hence, \( \frac{\partial B}{\partial \tau_i} \) becomes:

\[
\frac{\partial B}{\partial \tau_i} = 2 \hat{\tau}_i \gamma \theta \left( b_t (1 - b_t \gamma \theta) - \frac{(2 b_t \gamma \theta - 1) \hat{\tau}_i}{8(2 b_t - 1)} \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\partial E[\tau^2 | \tau \geq \hat{\tau}_i]}{\partial \tau_i} \right)
\tag{110}
\]

Since \( b_t (1 - b_t) = \frac{E[\tau^2]}{4E_G[\tau^2 | \tau \geq \hat{\tau}_i]} \) and \( b_t (1 - \gamma b_t) \geq \hat{b}(1 - \hat{b}) \) for \( \gamma \theta \leq 1 \):

\[
\frac{\partial B}{\partial \tau_i} \geq 2 \hat{\tau}_i \gamma \theta \left( \frac{E[\tau^2]}{4E_G[\tau^2 | \tau \geq \hat{\tau}_i]} - \frac{(2 b_t \gamma \theta - 1) \hat{\tau}_i}{8(2 b_t - 1)} \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\partial E[\tau^2 | \tau \geq \hat{\tau}_i]}{\partial \tau_i} \right)
\geq \frac{E[\tau^2]}{2E_G[\tau^2 | \tau \geq \hat{\tau}_i]} \left( 1 - \frac{(2 b_t \gamma \theta - 1) \hat{\tau}_i}{2(2 b_t - 1)} \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\partial E[\tau^2 | \tau \geq \hat{\tau}_i]}{\partial \tau_i} \right)
\tag{111}
\]

T1' implies

\[
\left( 1 - \frac{1}{2} \frac{\hat{\tau}_i}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\partial E[\tau^2 | \tau \geq \hat{\tau}_i]}{\partial \tau_i} \right) \geq 0,
\]

Therefore \( \frac{\partial B}{\partial \tau_i} \geq 0 \) and a unique maximum exists at \( \tau^* = \tau \). If \( \gamma \theta > 1 \), we have either \( \frac{\partial B}{\partial \tau_i} \geq 0 \) or \( \frac{\partial B}{\partial \tau_i} < 0 \). Suppose \( \frac{\partial B}{\partial \tau_i} < 0 \). Define

\[
Z(\theta \gamma) = b_t (1 - b_t \gamma \theta) - \frac{(2 b_t \gamma \theta - 1) \hat{\tau}_i}{8(2 b_t - 1)} \frac{E[\tau^2]}{E[\tau^2 | \tau \geq \hat{\tau}_i]} \frac{\partial E[\tau^2 | \tau \geq \hat{\tau}_i]}{\partial \tau_i}
\tag{112}
\]

Then

\[
\frac{\partial B}{\partial \tau_i} \frac{\partial \theta \gamma}{\partial \tau_i} = 2 \hat{\tau}_i \gamma \theta \frac{\partial Z(\theta \gamma)}{\partial \gamma \theta} < 0,
\tag{113}
\]

which means that once \( \frac{\partial B}{\partial \tau_i} \) becomes negative, then it can only get more negative as \( \theta \gamma \) increases, if \( \theta \gamma > 1 \). Thus \( \tau^* \) is either a unique interior maximizer or \( \tau^* = \tau \).□

**Proposition 11.**

**Proof.** It follows from the same arguments as in the proof of Proposition 10. □
References


Murphy, K. J., and J. Zabojnik. 2006. Managerial Capital and the Market for CEOs. Queen’s University, working paper.


