Free for a Fee: The Hidden Cost of Index Fund Investing

Alexi Savov *

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ABSTRACT

I build a rational expectations model consistent with the empirical finding that active funds underperform index funds by as much as their fees. Uninformed households receive privately observed wealth shocks that lead them to rebalance, thereby inducing noise in stock prices. As a result, they fail to attain the buy-and-hold index fund return. The equilibrium net buy-and-hold alpha of informed active funds is negative to make active and index funds equally attractive. I find in the data that high index fund flows forecast low returns and low index fund returns relative to active fund returns. This differential impact can account for most of the buy-and-hold advantage of index funds over active funds.

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I. Introduction

How can you beat the average investor? That depends: before costs – good luck (Fama 1998); after costs – buy index funds (Fama and French 2009; French 2008). Mutual funds hold one quarter of the U.S. stock market.\(^1\) A long literature starting with Jensen (1968) shows that in turn, the average mutual fund holds the market portfolio. In this way, the average investor holds the passive market portfolio but pays active fees to do so. To beat the average investor, just switch to a low-cost index fund. This observation poses a challenge to market efficiency at the portfolio management level: If there are two ways to hold the same portfolio, both should cost the same. This paper addresses that challenge.

I present a rational expectations model consistent with the empirical result that active funds underperform index funds by as much as their fees. In the model, households are subject to wealth shocks causing them to rebalance. The aggregate component of these shocks is not public and so it induces noise in the price system through a wealth effect. As a result, households rebalance in a way that is systematically related to stock mispricing. Buying high and selling low, index fund investors fail to attain the high buy-and-hold returns of index funds. To avoid this problem, households can invest with informed active funds. These funds time the market correctly but charge fees. In equilibrium, to make households indifferent between active and index fund investing, the net active return is below the benchmark return, as in the data.

I develop this idea within the noisy rational expectations framework of Grossman and Stiglitz (1976). The price of a risky asset responds to a cashflow shock and an aggregate shock to nontradable wealth. Informed active managers observe the cashflow shock directly and, through the price, are also able to identify the wealth shock. Households, on the other hand, must filter both shocks from the price, but this cannot be done perfectly. When a positive aggregate wealth shock hits, demand for the risky asset rises and so does its price. Since households do not observe the wealth shock directly, they optimally attribute part of the price rise to cashflow news. As a result, they underestimate the wealth shock and overestimate future cashflows. Now the average household thinks itself wealthier than average and buys more stock. Informed investors, not poorer households, take the other side of the trade. Since households
overestimate future cashflows and underestimate aggregate wealth, they overestimate future returns. In short, the stock becomes overpriced. In subsequent periods the typical household makes a loss at the expense of the active funds.

The story applies conversely to cashflow news. Following a positive shock to cashflows, informed demand bids up prices leading households to overestimate the level of nontradable wealth and underestimate future cashflows. They therefore underestimate future returns and the stock becomes underpriced. In this case, households unwittingly sell to the active managers. This prediction is consistent with the results of Cohen, Gompers, and Vuolteenaho (2002), who find that stocks underreact to cashflow news and that following a positive cashflow shock, institutions buy stocks from individuals.

I interpret the undelegated stock holdings of the uninformed investors in my model as an index fund. A real-world index fund holds a basket of stocks like the constituents of the S&P 500 in proportion to their market capitalization. In the model, there is one asset that can be viewed as an index, and index funds hold shares of that asset. The index funds track the price of this asset well, but the end returns their investors receive are low because they enter and exit at the wrong time.

In addition to index funds, households have access to active funds run by informed managers who time the market correctly in exchange for a competitive per-dollar fee. They deliver a return that benefits from mispricing so that even after accounting for the timing of flows, the gross active fund return has positive timing. This implies that after fees active funds must have negative alpha to make them no more attractive than index funds. Investors in the two types of funds share the cost of information equally.

The model predicts that controlling for the timing of investment should lower index fund returns by as much as the cost of active investing. By contrast, the net returns of active funds should remain roughly the same. Figure 1 offers preliminary evidence consistent with this prediction. The top plot shows the log share of the total stock market held in index and active funds, net of a time trend. The index fund share is more volatile and more pro-cyclical than the active fund share. It tends to peak with the market, suggesting that index fund investors are getting returns that are lower than buy-and-hold returns.

While the top plot of Figure 1 shows the timing of investors into and out of funds, the bottom one shows how active managers themselves are timing the market with the
funds they manage. Specifically, the plot shows the quarter-by-quarter return on a value-weighted portfolio long index funds and short active funds. The average return on this portfolio is about 1% per year, in line with the difference in expenses between the two types of funds. However, the plot also reveals that this return tends to turn negative precisely when index funds are large relative to active funds. To see this, consider the moving average line, which at any given point shows the average return of the long-short portfolio over the following year. As one example, note that near the peak of the stock market in late 1999, index funds held an unusually large share of the market, and, over the next year, underperformed active funds by as much as 2% per quarter. This suggests that during this period index fund investors suffered for two reasons: they increased their holdings at the market peak, and just before a period when active funds did better than index funds. The patterns in Figure 1 are quantified in Table V. In the empirical section of the paper, I check whether they can account for the buy-and-hold advantage of index funds.

To do so, I look at the relationship between flows and subsequent performance at both the aggregate level and the individual fund level. At the aggregate level, as Figure 1 suggests, high index fund investment conditional on prices and other public information is also associated with lower subsequent returns than high active fund investment. At the individual fund level, high flows conditional on past performance lead to low subsequent returns for index funds but not active funds, and this can account for most of the gap in buy-and-hold net alphas. The broader message of both the model and the empirical results is that performance evaluation should consider the end returns earned by investors and not simply buy-and-hold returns.

In the remaining sections of the paper, after discussing related work, I present a simple two-period model and then a dynamic model where wealth shocks inject noise into prices. Both models reproduce the negative net buy-and-hold alpha of active funds. The two-period model, while highly stylized, allows for equilibrium disagreement among the uninformed agents. The dynamic model is more realistic but becomes intractable if the full distribution of beliefs of the uninformed is part of the state. For this reason, in the dynamic model I focus on the case when the idiosyncratic wealth shocks are large enough for households to safely ignore, relying on public signals instead. This model produces the same qualitative results as the two-period model with optimal learning.

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In addition, it allows for computing impulse response functions and time series of prices and returns that motivate the empirical strategy of the paper. Following the presentation of the models, I analyze data on aggregate and individual fund returns and then conclude.

II. Related Literature

The mechanism presented here relies on three ingredients: information costs or else the market would be perfect but uninteresting; multiple shocks to sustain profits that cover these costs; and one of these shocks must be correlated with the investment decision of the uninformed.


In order to obtain closed form linear prices, almost all papers in the information trading literature assume exponential utility and additive reduced-form shocks, suppressing potentially important wealth effects. One exception is Peress (2004) who analyzes wealth effects in the context of the Grossman and Stiglitz (1980) model where the noise is still coming from a reduced-form supply shock. In this paper, it is wealth shocks themselves that act as noise via a wealth effect.

The information structure considered here resembles that in Diamond and Verrecchia (1981). To make the aggregate wealth shock unobservable, I subject households to large idiosyncratic shocks: each household observes its own shocks but that is of little help in deciphering the aggregate. Prices reveal the aggregate shock only partially, and in this way the public information set is poorer than the combined information sets of individual households.

Bernardo and Judd (2000) show how to analyze general equilibrium models with asymmetric information under a general preference specification. In addition, their
technique is easy to adapt to situations with equilibrium disagreement among the
uninformed, as in Diamond and Verrecchia (1981). I rely on their methods to compute
approximate equilibria.

On the empirical side, this paper relates to the mutual fund performance literature.
Starting with Jensen (1968), many studies have found that on a buy-and-hold basis
the average active fund does not cover its fees. Carhart (1997) shows that common
stock factors like value and momentum explain much of the persistence in mutual fund
returns. Wermers (2000) finds that before fees and expenses mutual funds deliver a
positive alpha of 1.3% that turns to negative 1% after fees. This result is consistent
with this paper.

Glode (2009) builds a partial equilibrium model where fund managers optimally
choose to deliver alpha in states with high risk prices. In this way, active funds provide
a form of insurance to investors, allowing them to have unconditionally low net returns.
Glode (2009), as well as Kosowski (2006) and Moskowitz (2000) find that indeed mutual
funds do better in recessions.

As emphasized by Fama (1976) and Roll (1977), any test of market efficiency is also
a test of some particular model of risk, a model that may itself be misspecified. This
observation implies that it is empirically impossible to rule out a risk-based explanation
of any return phenomenon. In the context of mutual funds, it is conceivable that active
funds load positively on some risk factor unknown to the econometrician, whereas all
other equity investors as a group, among them index fund investors, load negatively on
that factor (the sum of all holdings must be the market portfolio). Such a story has to
justify the fees of the active funds; it has to explain why it is costly to have positive
exposure to this risk.

Dichev (2007) shows that the internal rate of return of stocks, a measure that
considers changes in their net supply, is lower than their buy-and-hold return. The
net transfer in Dichev (2007) is from public equity investors to others, whereas in this
paper it is from index to active fund investors.

Edelen (1999) finds that taking into account the liquidity provision of open-end
mutual funds brings their negative net alphas close to zero. Since index funds are also
open-ended, this fact cannot explain their high net buy-and-hold alphas but it does
suggest, as emphasized here, that considering buy-and-hold returns in isolation can be
misleading.

The results of the market-timing literature are mixed. For example, Becker, Ferson, Myers, and Schill (1999) do not find significant evidence of market timing using returns, whereas Jiang, Yao, and Yu (2007) use holdings data and find that the average active mutual fund does have market-timing ability. One possibility is that stock picking dominates market timing as a motive for trade at any given fund, making it difficult to measure market timing ability at the individual fund level. Cremers and Petajisto (2009) show that half or more of active mutual fund trades net out so that as a group the holdings of active mutual funds deviate much less from their passive benchmarks than do individual funds. This observation suggests the possibility that market timing is more important at the aggregate level than at the individual fund level. In addition, the central result in this paper is that active funds need not cover their fees through market timing when index fund investors have negative market timing. Finally, since in the model the benchmark return is a convex combination of the active and index returns, the larger the active fund sector relative to the index fund sector, the smaller the gross market-timing alpha of active funds.

I model active mutual funds as having market-timing rather than stock-picking skill because index funds are immune to stock picking. A rational expectations model where active funds engage in stock picking only, perhaps against some third group of investors, would typically require that active funds have zero net alpha to keep investors from flocking to index funds. Given the significant cost of active management and imposing the equilibrium condition that investors be indifferent between active and index funds, it is the high returns of index funds and not the low returns of active funds that pose a challenge. Unlike stock-picking, market timing can address that challenge.

III. Two-Period Model

In this section I present a simple two-period model that illustrates the basic intuition of the paper. Uninformed investors choose between active and index funds. They trade off poor market-timing against active fees. In equilibrium, the net buy-and-hold active return is lower than the buy-and-hold index return, but on a dollar-weighted basis, and after adjusting for differences in risk, the two funds are equally attractive.
While highly stylized, the two-period model allows for a solution with optimal learning. Since households observe their own wealth shocks which contain partial information about aggregate wealth shocks, and since prices are noisy, some disagreement among the uninformed agents persists in equilibrium (see Diamond and Verrecchia (1981)). In the context of non-CARA utility, this disagreement leads to a pricing function that depends on the full distribution of beliefs, which is itself endogenous. In a dynamic model, this complication heralds intractability. For this reason, in the dynamic model later in the paper I suppress belief heterogeneity among the uninformed by assuming that households do not use the information in their own labor incomes to filter aggregate shocks. This simplification is reasonable when idiosyncratic volatility is much larger than aggregate volatility. The purpose of the two-period model at hand is to show that the intuition of the paper and the qualitative results of the dynamic model obtain in a setting with optimal learning.

A. Setup

There are three dates and two types of agents, a continuum of households of measure one and informed active managers of measure zero. Households have logarithmic preferences over terminal date consumption. There are three assets in the economy: a riskless bond, a risky asset, and a nontradable claim to future labor income. The bond pays a numeraire gross return $R_f$ and is in zero net supply. The risky asset pays a single dividend payment $D$ at date 2. Let

$$\log D = s + \epsilon,$$  \hspace{1cm} (1)

where $s \sim N(\mu_d, \sigma_s)$, $\epsilon \sim N(0, \sigma_\epsilon)$, and the two shocks are independent. Let $\sigma_d = \sqrt{\sigma_s^2 + \sigma_\epsilon^2}$ and let $h_s = \sigma_s^2 / \sigma_d^2$ measure the precision of the signal $s$. Active fund managers observe $s$ at date 1, whereas uninformed agents have to filter $s$ from the price of the risky asset as best they can.

The third asset is a claim to future labor income, which is subject to both aggregate and idiosyncratic shocks. Specifically, at date 1, households learn their date 2 labor
income $L$. Let

$$
\begin{align*}
  l^1 & \equiv \log L^1 = l + \eta \\
  l^2 & \equiv \log L^2 = l - \eta.
\end{align*}
$$

(2)

Half of all households receive $L^1$ and the other half receive $L^2$. The purpose of this assumption is to simplify the characterization of the distribution of household beliefs. Let $l \sim N(\mu_l, \sigma_l)$ and $\eta \sim N(0, \sigma_\eta)$. Importantly, each household knows its own labor income $l^i$ and whether it is in group 1 or 2, but it does not know the labor income of the other group. In this way, households cannot fully distinguish between aggregate and idiosyncratic labor income shocks. I assume that active fund managers observe $l$. They can infer $\eta$ by observing prices and the cashflow signal.

At date 0, households choose whether to become active or index fund investors. In other words, they commit to purchase their risky asset holdings at date 1 through an active or an index fund. This assumption ensures that active funds absorb some of the wealth shocks. It is not essential that households invest all their risky asset holdings with one type of fund; an alternative where they invest some fraction in each yields the same results as long as that fraction is decided at time 0.

Index funds hold the risky asset and no bonds, and they do not charge any fees. Using their superior information, the active funds choose how much to invest in the risky asset and the riskless bond. The active funds charge a per-dollar proportional fee $f$, which can be thought of as the cost of information since the active fund sector is perfectly competitive.

### B. Portfolio Decisions

Let $P$ be the price of the risky asset. Using lower case letters to denote logs, suppose the active fund managers invest a fraction

$$
x_S = \frac{s - p - rf + \frac{1}{2}\sigma^2}{\sigma^2}
$$

(3)

of assets under management in the risky asset. That is to say, active managers specialize in investing based on the true expected return on the asset, $s - p$. I assume that
active funds do not serve as financial advisors; they do not take into account the labor income of their individual clients in deploying their assets. The households themselves fulfill this task. Beyond this assumption, as pointed out by Admati and Pfleiderer (1990), if active managers invest according to some other transformation of \( s \) measurable with respect to the information set of the households, then households can undo that transformation and so it serves no purpose. For this reason, and to abstract from possible agency frictions between investors and their investment managers, I let the active fund managers follow a simple Merton (1971) portfolio rule.

Let \( R = D/P \) be the gross risky asset return. The active fund return is

\[
R_A = x_S R + (1 - x_S) R_f. \tag{4}
\]

Write the net active return as \( R_{A,\text{net}} = R_A (1 - f) \). In what follows, it will be convenient to work with log returns, which can be written approximately as (see, for example, Campbell and Viceira (2002, p.29))

\[
r_A - r_f = r_f + x_S (r - r_f) + \frac{1}{2} x_S (1 - x_S) \text{Var} (r) \tag{5}
\]

\[
r_{A,\text{net}} - r_f = (r_A - r_f) - f. \tag{6}
\]

Here \( r = d - p \) stands for the log return on the risky asset. At date 1, households fall into one of four groups: active and index fund investors, half of whom have labor income draws in group \( i = 1, 2 \). Index fund investors solve the maximization problem

\[
\max_{x_i^I} E \left[ \log \left( W_i^I + L^I \right) \right] P, L^I, \tag{7}
\]

where

\[
\frac{W_i^I + L^I}{W_0} = x_i^I \left( \frac{D}{P} - R_f \right) + R_f + \frac{L_i^I}{W_0}. \tag{8}
\]

As shown in Appendix A, the optimal portfolio weight is approximately

\[
x_i^I = \frac{E \left[ r - r_f \mid p, L^I \right]}{\text{Var} (r \mid p, L^I)} + \frac{1}{2} \text{Var} (r \mid p, L^I) \left( 1 + e^{L^I - F} - r_f \right), \tag{9}
\]

Since labor income is riskless at date 1, total wealth is equal to financial wealth \( W_0 \) plus
labor income discounted at the risk-free rate. When labor income is large, households tilt their financial wealth towards the stock to balance their overall exposure. This wealth effect is central to the mechanism of the paper because it affects the price of the risky asset. Since households do not know the labor income of all other households, they do not know aggregate total wealth. This makes prices noisy in the sense that uninformed investors do not know the true (as seen by the informed) expected return.

Active fund investors solve

$$\max_{x^i_A} E \left[ \log \left( W^i_A + L^i \right) \right | P, L^i] ,$$

where

$$\frac{W^i_A + L^i}{W_0} = x^i_A (R_{A,net} - R_f) + R_f + \frac{L^i}{W_0}.$$  

The same line of calculations as with the index fund investors gives

$$x^i_A = \frac{E \left[ r_A - r_f \vbar p, l^i \right] - f + \frac{1}{2} Var (r_{A,net} \vbar p, l^i)}{Var (r_{A,net} \vbar p, l^i)} \left( 1 + e^{v - p - r_f} \right).$$

In the case of the active investors, this approximation is less accurate since the active fund return is not normally distributed. In unreported tests, I find that this effect is small, especially when the active fund sector is large, which is the case of interest.

C. Equilibrium

Let $q$ be the fraction of households who decide to become active fund investors at date 0. The market for the risky asset must clear:

$$1 = q \left( x^1_A + x^2_A \right) x^S + \frac{1 - q}{2} \left( x^1_I + x^2_I \right) .$$

Using the rational expectations equilibrium formulation in Grossman (1976) and Grossman and Stiglitz (1976), the resulting pricing function generally depends on the distribution of labor income and has the form

$$p = p (s, l, \eta).$$  

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As the active fund sector becomes large \((q\) is high), prices reveal the private signal \(s\) better. They do not do so perfectly since the demands of the households are also a function of the aggregate labor income. Unfortunately, the nonlinear setup in this paper precludes a proof of the existence or uniqueness of such a pricing function. I rely on numerical techniques to compute a candidate equilibrium, as do Bernardo and Judd (2000).

At date 0, households must be indifferent between becoming active and index fund investors:

\[
E_0[\log (W_A + L)] = E_0[\log (W_I + L)].
\] (15)

Taking the fee \(f\) as given, this equation determines the active share \(q\), and vice versa.

Investors equalize the return on total wealth, adjusting for differences in risk between active and index funds. When active funds are small, they take large positions and this makes them riskier to households. Riskiness aside, since the value of labor income grows at the risk-free rate, equating returns on total wealth amounts to equating returns on financial wealth.

\[D. \text{ Numerical Approach}\]

To solve this problem, I employ the polynomial projection methods described in Judd (1998) and applied in Bernardo and Judd (2000). Specifically, I use Hermitian collocation to calculate both the pricing function and the conditional expectations that figure in the demands of the two types of uninformed investors. The details are in Appendix A. Unlike Bernardo and Judd (2000), there is an additional layer of information asymmetry within the group of uninformed investors. The projection method extends in a straight-forward way to incorporate this complication.

\[E. \text{ Results and Analysis}\]

I present a calibrated example of the two-period model that illustrates its mechanics. The parameter values and convergence diagnostics are listed in Table I. I set the volatilities of both aggregate dividends and aggregate labor income to .25. In addition,
I make labor income 10 times as large as dividends. Both of these assumptions serve to amplify the effects of shocks to labor income as log utility investors are not very risk-averse. Increasing the size of the shocks increases risk premia as would higher risk aversion. I set the volatility of the idiosyncratic component of labor income to 1, making an individual household’s labor income a poor source of information about the aggregate shock. This assumption increases the information asymmetry between informed and uninformed investors. In addition, I set the precision of the private signal to one half. Finally, I set the active fees to 1%, in line with the most recent annualized average fees and expenses of U.S. stock mutual funds.\(^3\) This number potentially understates the cost of active investing as it does not account for trading costs.

The results are in Table II. The equilibrium share of active funds is 25%. In the data, active mutual funds hold close to the same proportion of the U.S. stock market. At this level, active funds push prices very close to the fully revealing price. To show this, I follow Bernardo and Judd (2000) and compute as a measure of price noisiness the percentage absolute deviation of the equilibrium price from the fully revealing price (the price that would obtain under symmetric information). According to this measure, there is 1% noisiness in the equilibrium price system. This low number follows from the assumption of perfect competition in the active fund sector.

The low level of noise limits the profit opportunities of the active funds. To measure these profits, I calculate alphas using a conditional beta model. In each state, the beta of the active fund is the expected active fund risky asset portfolio weight under the uninformed information set. The actual active exposure differs depending on the part of the private signal that is not revealed to the uninformed. This sets up a conditional covariance between the fund return and the risky asset return that shows up as an intercept, or alpha, and measures the market-timing ability of the active funds. I use a similar measure of market timing in the empirical section of the paper.

The equilibrium gross alpha of the active funds given the parameters in Table I is 18 basis points. Fama and French (2009) find an insignificant 10 basis points of gross alpha under the Fama and French (1993) three-factor model, falling to -18 basis points under the CAPM. Wermers (2000) considers hypothetical returns on holdings, that is returns before transaction costs and finds a positive gross alpha of 1.3%. In any case, the gross alpha of active funds in both the model and the data is not large.
The net alpha of active funds in the model is -82 basis points, which is equal to the gross active minus the active fee of 1%. This number matches the -79 basis points in the aggregate results in Table VI under the CAPM and is somewhat higher than the -47 basis points calculated under the three-factor model. In the data, these numbers refer to the difference between the net active fund alpha and index fund alpha. In the model, the net index fund alpha is zero but in the data even index funds charge nonzero fees.

Households are willing to tolerate a negative net active alpha because they lose an equivalent amount due to going in and out of index funds at the wrong time. As Table II shows, even though active funds have negative alpha and index funds do not, households can expect the same return on financial wealth of 3.2% in either type of fund. This is because the active fund delivers a return that is immune to mispricing of the risky asset so the dollar weighted return of active fund investors is close to the active fund return itself. By contrast, the dollar weighted index fund return is low because flows into the index funds have poor timing.

Figure 2 illustrates the profit opportunities that the active funds exploit. It shows the difference between the equilibrium price and the fully revealing price on the left as well as the corresponding difference in the demand of the active funds on the right. The equilibrium price responds much less to the private cashflow signal \( s \) than the fully revealing price. That is to say, prices underreact to cashflow news as in Cohen, Gompers, and Vuolteenaho (2002). This leads the active funds to increase their exposure following a positive cashflow signal. By contrast, the price overreacts to an aggregate labor income shock, \( l \). Following such a shock, active funds reduce their exposure. The intuition behind these results is that households optimally attribute high prices to both cashflow and labor income news. On average they are right, but conditional on high cashflow news they underestimate cashflows and overestimate labor income, and conditional on high labor income they underestimate labor income and overestimate cashflows. This causes the price to underreact to cashflow news and overreact to labor income news.
IV. Dynamic Model

In this section I outline a dynamic model that produces negative market timing for index fund investors and a negative net alpha for active funds. The dynamic model allows for calculating time series of returns and impulse response functions that motivate the empirical section of the paper. However, tractability requires limiting the equilibrium amount of disagreement between agents. Specifically, I do not allow for disagreement within the group of uninformed investors. This assumption is justified in the limit as the idiosyncratic shock to labor income is arbitrarily large so that households can safely ignore their own labor income when forming expectations about aggregate quantities. Without this assumption, equilibrium prices depend on the full distribution of beliefs as in the two-period model of the previous section.

A. Setup

Consider a finite $T$-period economy in continuous time with no intermediate consumption. There are two types of agents, uninformed households of measure one and informed specialists of measure zero. Households have logarithmic preferences over date $T$ consumption, which is the simplest way to introduce wealth effects. These assumptions lead to simple characterizations of portfolio demand, which is convenient for solving the household information filter.

There are three assets in the economy: a risky asset, a riskless storage technology, and nontradable a claim to future labor income. The riskless asset is in zero net supply and yields a numeraire rate of return $r$. The risky asset is a claim to a date $T$ cashflow of $D_T$. Letting $\log D_t$ be the informed expectation of $\log D_T$, write

$$d \log D_t = e^{-\kappa(T-t)} \sigma_t dB.$$  \hspace{1cm} (16)

Since $\log D_t$ is defined as an expectation, it necessarily has no drift. This specification is consistent with an underlying stationary cashflow process that mean-reverts at a rate $\kappa$. The initial value corresponding to $t = -\infty$ is the unconditional average level of dividends.

Informed agents observe $\log D_t$ directly whereas households must filter it from the
stock price. This gives the informed an advantage. They can only profit from this advantage in the presence of “noise” in the price system. I introduce labor income as the source of this noise. I use the term labor income broadly as a stand-in for any asset that (1) is not held by the index funds and (2) is subject to aggregate shocks that are not fully observed at zero cost. I interpret the shocks to the value of this asset as wealth shocks.

By affecting total wealth, labor income impacts the portfolio decisions of households (see for example Heaton and Lucas (1997)). Specifically, a positive shock to labor income tilts total wealth away from stocks. To restore balance, agents respond by tilting their portfolio allocation toward stocks.

To make aggregate labor income unobserved, I add idiosyncratic labor income shocks. Each household knows its own labor income but not the aggregate. Specifically, let \( \log L_T \) be the expectation of date-\( T \) aggregate log-labor income \( \log L_T \) and let \( \log L^h_T \) be the expected log-labor income of household \( h \). Write

\[
d \log L_t = e^{-\kappa(T-t)} \sigma_L' dB \\
d \log L^h_t = e^{-\kappa(T-t)} \left( \sigma_L' dB + \sigma_h dB^h \right).
\]

I assume that \( \sigma^h \gg |\sigma_L| \) to justify excluding individual labor income from the household filtering problem. I also assume that the idiosyncratic shocks \( dB^h \) are independent across households. Since labor income plays the generic role of a nontradable wealth shock here, I do not attempt to capture any particular dynamic of the labor market.

Finally, I introduce an additional public signal that partially reveals the log labor income to dividends ratio \( y = \log L/D \), a key state variable:

\[
de_t = y_t dt + \sigma_e' dB.
\]

Both informed and uninformed agents observe \( e \). The classical economy with no asymmetric information corresponds to the special case \( \sigma_e = 0 \).

The public signal serves two additional purposes. The first is to make the error in the uninformed estimate of \( y \) mean-revert toward zero, which is analogous to endowing households with access to lagged cashflow news. This helps households update their
beliefs about expected returns sooner so active funds can realize their profits before the economy expires. The second is to serve as a reduced-form version of the additional information contained in the cross section of labor income. Diamond and Verrecchia (1981) point out that in a model with multiple private signals equilibrium prices reveal a noisy summary of the set of private signals. In the present model, the distribution of this aggregated signal would depend on the distribution of wealth and beliefs. The public signal $e$ can be viewed as a stand-in for this type of signal.

As in the two-period model, households have a choice between an active and an index fund. I assume that they make that choice at time 0.

B. Portfolio Decisions

The portfolio decision of the active fund managers is particularly simple. Let the price of the risky asset evolve according to the true process

$$\frac{dP_t}{P_t} = \mu_t dt + \sigma_t dB_t. \quad (20)$$

This evolution is seen directly by informed agents only. Suppose the uninformed household estimate of $\mu$ is $\mu^H$ and let informed managers invest a fraction

$$x^S_t = 1 + \frac{\mu_t - \mu^H_t}{\sigma_t \sigma_t^t} \quad (21)$$

of assets under management in the risky asset. As in the two-period model, this formulation assumes that active managers do not take into account the labor income of their clients when deploying their assets. Instead, they specialize in their information advantage, which consists of knowing the true expected return $\mu_t$. Given that, the exact form of the active portfolio weight is unimportant, as discussed in the two-period model.

The key aggregate state variable is the log labor-income to dividends ratio $y = \log L/D$. Since I impose that households agree on aggregate quantities, let $y^H$ be the uninformed estimate of $y$ and let $l^h = L^h/W^h$ be the labor income-to-wealth ratio of household $h$. Suppose household $h$ is an index fund investor. Then household $h$ solves
the Bellman equation

\[ 0 = \max_x E^h \left[ dJ(W^h, l^h, y^H, t) \right], \tag{22} \]

where financial wealth \( W^h \) follows

\[ \frac{dW^h}{W^h} = x \left( \frac{dP}{P} - rdt \right) + rdt. \tag{23} \]

The optimality condition (see Appendix B) is

\[ x = \left( \frac{-J_W}{J_{WW}W^h} \right) \frac{\mu^H - r}{\sigma' \sigma} + \left( \frac{-J_{WL}L^h}{J_{WW}W^h} \right) \frac{(\sigma^H_t)' \sigma}{\sigma' \sigma} + \left( \frac{-J_{Wy}y}{J_{WW}W^h} \right) \frac{(\sigma^H_y)' \sigma}{\sigma' \sigma}. \tag{24} \]

In an incomplete markets setting, there is no known closed-form expression for the value function or by extension for the risky demand \( x \). In the appendix, I show how to use the homogeneity of log utility as in Benzoni, Collin-Dufresne, and Goldstein (2007) to take wealth out of the demand equation. I then apply a linearization around the complete markets case \( l^h = 0 \) similar to Viceira (2001) and Kogan and Uppal (2003) to obtain a simple first-order approximation to the risky asset demand:

\[ x = \frac{\mu^H - r}{\sigma' \sigma} + \left( \frac{\mu^H - r}{\sigma' \sigma} - \frac{(\sigma^H_t)' \sigma}{\sigma' \sigma} \right) l^h. \tag{25} \]

This approach has the added benefit of making household demand linear in labor income \( l^h \). As a result, aggregate demand is linear in aggregate labor income \( l^H \) and does not depend on the full cross-sectional distribution of labor income. Alternative ways of dealing with such dependence, for example the numerical approach of Krusell and Smith (1998) are hard to implement here because writing down the filtering problem of households requires the pricing function as an input. In this paper, idiosyncratic shocks are just a device for making aggregate wealth shocks unobservable and as such are not themselves the main subject. For this reason and for the considerable simplifications it offers, I rely on the linearized demand equation.

Now consider a household that invests in active funds. Let the active fund return
be
\[ dR^A = x^S \left( \frac{dP}{P} - r dt \right) = x^S dR. \] (26)

Since active investing is costly, the wealth evolution in this case is
\[ \frac{dW^h}{W^h} = x (dR^A - f dt) + r dt. \] (28)

Let \( \Delta = y - y^H \) measure the uninformed error in the estimate of \( y \) and let \( \Sigma = E^H [\Delta^2] \).

I show in the filtering section that
\[ E^H [dR^A] \approx \left[ \mu^H + (F \kappa \Delta)^2 \frac{\Sigma}{\sigma' \sigma} \right] dt, \] (29)

where \( F = \log P/D \) is the log price-dividend ratio and \( \kappa_\Delta \) is the persistence in the evolution of \( \Delta \). The second term measures the gross market-timing alpha of the active funds. When households know \( y \), \( \Sigma = 0 \) and the active fund has no alpha. Similarly, when the index fund sector is very small, the error \( \Delta \) has little impact on prices so \( F \Delta = 0 \) and again active funds have no alpha. Let \( \alpha_g \) and \( \alpha_n = \alpha_g - f \) be the gross and net alphas of the active fund. A similar derivation as with the index fund investors gives the portfolio weight
\[ x = \frac{\mu^H + \alpha_n - r}{\sigma' \sigma} + \left( \frac{\mu^H + \alpha_n - r}{\sigma' \sigma} - \frac{(\sigma^H_L)' \sigma}{\sigma' \sigma} \right) l^h. \] (30)

Both active and index fund investors have an optimal policy that tilts their financial wealth towards the risky asset following a positive shock to their labor income. When the shock is at the aggregate level, the price of the risky asset rises.

Finally, households must be indifferent between index and active funds at time zero:
\[ E_0 [\log (W_T + L_T) | Active] = E_0 [\log (W_T + L_T) | Index]. \] (31)

This condition determines the fraction \( q \) of households that choose to become active fund investors. The trade-off is between the fees and the negative market-timing asso-
associated with index fund investing. I solve this condition numerically.

\[ \int_A x^H W^h dh \quad \frac{\mu^H + \alpha_n - r}{\sigma'} \left( \int_A W^h dh \right) \]

\[ = \mu^H + \alpha_n - r \quad \frac{\int_A W^h dh}{W^H} \times \int_A L^h dh \]

\[ + \left( \frac{\mu^H + \alpha_n - r}{\sigma'} - \frac{\sigma_L'}{\sigma'} \right) \left( \int_A L^h dh \right) \]

\[ \int_I x^H W^h dh \quad \frac{\mu^H - r}{\sigma'} \left( \int_I W^h dh \right) \]

\[ = \mu^H - r \quad \frac{\int_I W^h dh}{W^H} + \left( \frac{\mu^H - r}{\sigma'} - \frac{\sigma_L'}{\sigma'} \right) \left( \int_I L^h dh \right). \]

These demands depend on the relative shares of the active and index fund investor base. At the outset, these shares are \( q \) and \( 1 - q \) for the financial wealth shares and \( q \left( L_0^0 / P_0 \right) \) and \( (1 - q) \left( L_0^0 / P_0 \right) \) for the labor income shares. However, over time they may evolve differently depending on the sample path. At this stage, I do not have a way of dealing with this complication. Instead, I solve for prices under the assumption that this distribution remains constant. This holds in expectation when active and index funds are equally risky so that the expected return on wealth is the same for the two groups. I look for parameterizations that satisfy this condition, as did the two-period model. Finally, in a model with more frequent movement between funds, the size of the active fund sector is limited by the skill of the active funds, which would keep it stable. In the present model, there is a mechanical force operating in the same direction: as the active share gets big, active funds fail to cover their fees and so their share declines. With this simplification, let

\[ x^A = \frac{\mu^H + \alpha_n - r}{\sigma'} \quad + \quad \left( \frac{\mu^H + \alpha_n - r}{\sigma'} - \frac{\sigma_L'}{\sigma'} \right) \frac{L^*}{W^H} \]

\[ x^I = \frac{\mu^H - r}{\sigma'} \quad + \quad \left( \frac{\mu^H - r}{\sigma'} - \frac{\sigma_L'}{\sigma'} \right) \frac{L^*}{W^H}. \]
Summing up demand and using \( P = W^H \), the risky asset market clears when

\[
1 = qx^A x^S + (1 - q) x^I. \tag{36}
\]

This equilibrium relation showcases the role of aggregate labor income as noise in the price system. Without it, the only quantity unknown to households would be the true expected return \( \mu \), which shows up in \( x^S \). It would then be possible to filter out \( \mu \) exactly just by observing prices. The presence of labor income, which is also unknown to households, makes this task impossible.

\[D. \text{ Filtering}\]

The market-clearing equation sets up a filtering problem. Households observe the signal \( e \) and prices which depend in part on the actions of the informed specialists. Using these, they update their belief \( y^H \) about the unknown state variable \( y = \log L/D \). Let \( \Delta = y - y^H \) be the error in these beliefs. As shown by Wang (1993), both \( y \) and \( \Delta \) are relevant state variables in this setup.

Let the log price-dividend ratio be given by

\[
\log \frac{P}{D} = F(y, \Delta, t). \tag{37}
\]

Households face a nonlinear filtering problem for \( y \), which is the key to forecasting returns. To solve this filtering problem, I use an extended Kalman-Bucy filter, which essentially linearizes the evolution of prices around the public estimate of the state. The details are in Appendix B.

Error in the public estimate of the underlying state leads to abnormal flows into index funds. The active fund managers profit by taking the opposite side. In the empirical section of the paper, I use a vector auto-regression that includes cashflow news and aggregate fund flows. In the model, these two variables jointly span the aggregate labor income and dividend shocks. In analyzing the data, I apply a Kalman filter to construct public estimates of these shocks as reflected in cashflow news and flows, given prices and other public sources of information. With these estimates, I construct dollar-weighted portfolios that reflect abnormal fluctuations into and out of
mutual funds.

E. Numerical Approach

I solve the dynamic model numerically using similar projection methods as in the two-period model. I conjecture a pricing function of the form

$$\log \frac{P_t}{D_t} = -r(T-t) + \left( e^{-2\kappa(T-t)} - 1 \right) F(y, \Delta, t).$$

(38)

The complete markets case $y = -\infty$ is solved by $F = \sigma_D^2 / (4\kappa)$. I approximate $F$ to satisfy the market-clearing equation (36) using the Chebyshev collocation method described in Judd (1998) and Judd (1992). I use Chebyshev rather than the Hermite polynomials from the two-period model because the extended Kalman filter gives the conditional distributions of the state variables. The details are in Appendix B. Once I estimate the pricing function, I clear the information market by simulating 100,000 samples of length 100 and comparing the expected utility of active and index fund investing.

F. Results and Analysis

I present a calibrated example to illustrate the mechanics of the model. The parameter values are in Table III. I interpret the risky asset as a claim to one share of the market portfolio at the end of each year. Unlike in the two-period model, the solution to the portfolio problem here requires a lower labor income share. I set labor income to half the size of the risky asset. To preserve the strong effect of labor income on expected returns, I increase the volatilities of both labor income and cashflow shocks to 0.5. While high, these volatilities lead to risk premia that would obtain in an economy with higher risk aversion and less risk.

I set the cost of information, the active fund fee, to 2% of assets under management, which is higher than the most recent fees and expenses of active funds. In the model, the fees stand for the full cost of information, that is they also include trading costs. Wermers (2000) finds a gap between holdings-based and net returns-based mutual fund alphas of 2%.
Table IV summarizes the results. The equilibrium share of active funds is 40%, suggesting that active managers possess valuable information. They achieve a gross alpha of 150 basis points and a net alpha of -50 basis points. In the data, this number is -79 basis points under the CAPM and -47 basis points under the three-factor model of Fama and French (1993), as shown in Table VI.

As Table IV shows, even though active funds have a negative net alpha, expected financial wealth is the same for both active and index fund investors. This is because index fund investors exhibit negative market timing. The noise in the price system leads them to make occasional mistakes that informed active managers can exploit.

The volatility of financial wealth is also very similar across the two types of funds, suggesting that active funds are not particularly risky. This leads investors to essentially equalize dollar-weighted returns between the two types of funds. This observation motivates the empirical tests based on dollar-weighted returns in the next section.

Figures 3 and 4 show the dynamics of the positive and negative market timing of informed and uninformed agents. In Figure 3, a positive shock to expected labor income raises the price of the risky asset. From the price, households infer that both labor income and cashflows are likely to be higher. In this way, they overestimate future cashflows and underestimate labor income. This makes the risky asset overpriced (the dotted line shows the price in the same economy under symmetric information). Since high cashflows and low labor income are associated with high expected returns, households overestimate future expected returns. This causes the index fund share of the market to increase as the active fund share falls. In subsequent periods, the price of the risky asset grows less slowly than households expect, leading them to revise their beliefs. On a risk-adjusted basis, the active funds make profits at the expense of households.

Figure 4 shows the effect of a dividend shock. This effect is documented empirically by Cohen, Gompers, and Vuolteenaho (2002) who find that institutions tend to buy from individuals following positive cashflow news. In the figure, higher expected dividends cause active managers to increase their demand for the stock, pushing up the price. Households attribute part of the price increase to higher labor income, underestimating cashflows and overestimating labor income. This causes the stock to become underpriced. In response, active managers increase their holdings and households de-
crease them. In subsequent periods the stock price rises faster than households expect, once again leading to higher risk-adjusted profits for the active funds.

An important feature of the model made clear in Figures 3 and 4 is that the mistimed flows by active fund investors into active funds do not negate the market timing profits of the active funds. Consider the active fund investor portfolio weights in the two figures. These weights are the effective exposure of active fund investors to the risky asset, that is they represent the quantity $x^A x^S$ in the model (see equations (21),(35), and (36)). In both figures, this quantity diverges from neutral in the same direction as the active manager’s portfolio weight $x^S$. While the direction is the same, the magnitude is lower, reflecting the fact that active fund investors time their funds negatively just like index fund investors. Nevertheless, even on a dollar-weighted basis, active fund investors are profiting by taking contrary positions to index fund investors. This has to be the case in order for the sum of the holdings of active and index fund investors to equal the fixed supply of the risky asset. It is the fees, not the flows, that turn the gross active fund alpha into a negative net alpha to make active and index funds equally attractive.

Finally, note that labor income shocks lead to more price distortion than dividend shocks. This is because under log utility dividends have a large first-order impact on price while labor income matters only through the relatively low risk premium. Uninformed investors therefore attribute most price changes to cashflow news. As a result, they generally fail to detect labor income shocks, which creates large profit opportunities for the active funds.

V. Data and Empirical Results

In this section I test the prediction that once the timing of investment is taken into account, index funds no longer outperform active funds. Starting with reduced-form predictability regressions, I find that the share of the stock market invested in index funds forecasts low subsequent market returns. Next, using dollar-weighted portfolios that capture the return on financial wealth of investors, I find evidence of negative market timing by index fund investors and small positive market timing by active managers that help explain most of the buy-and-hold advantage of index funds over
active funds. I conclude the empirical section with a look at individual fund returns that corroborates the aggregate evidence.

I use the Survivor-Bias-Free U.S. Mutual Fund Data from the Center for Research in Security Prices. Following Fama and French (2009) and Cremers and Petajisto (2009), I consider only U.S. equity funds, excluding sector funds.

For the aggregate tests, I use quarterly data from January 1986 through June 2009. I choose this frequency to correspond with the other macroeconomic time series. Elton, Gruber, and Blake (2001) discuss biases in the CRSP data prior to 1984. In light of their concerns, and since there are few index funds before 1986, I begin the sample in 1986. The individual fund tests do not require macroeconomic aggregates, allowing me to use monthly data. I begin in January 1991 since monthly total net assets are unavailable earlier. I identify 214 index funds from the Lipper S&P objective code and the CRSP index fund flag.6

A. Return Predictability

The model predicts that following large inflows into index and active funds, average returns are low as the risky asset becomes overpriced. Moreover, active funds should outperform index funds following high inflows as the active managers reduce their holdings of the risky asset in anticipation of low returns. In this section, I test these propositions in reduced form by running predictability regressions. Specifically, I consider specifications of the form

\[
R_{t+1,t+12} = a + b(\text{Index fund share})_t + c (p - d)_t + \epsilon_{t+12} \tag{39}
\]

\[
R_{t+1,t+12} = a + b(\text{Active fund share})_t + c (p - d)_t + \epsilon_{t+12}. \tag{40}
\]

The results of the predictability regressions are in Table V. The left-most panel shows that the total share of the stock market held in index funds is a strong negative
predictor of future market returns at a one-year horizon. The $R^2$ from this regression is 16%, whereas the $R^2$ using the price-dividend ratio is 12%. In a joint specification, the index fund share drives out the price-dividend ratio in predicting returns. The index fund share also predicts negatively the return on a strategy long index funds and short active funds. That is to say, following periods of growth of index funds, index funds tend to underperform active funds. This finding was also apparent in Figure 1, especially around the burst of the bubble in 2000.

Table V also shows that the active fund share of the market is not a good predictor of future market returns and is a positive predictor of the difference between index and active fund returns. In the model, the index and active shares of the market (recall that these are AUM divided by market cap) are perfectly correlated, so the model cannot explain the difference in behavior between these two variables. A possible explanation consistent with the intuition of the paper is that active fund investors are more likely to rely on financial advisors who are largely compensated by fund managers and give timing advice.

Turning to the cross-section of assets, Table V shows that the index fund share predicts positively the return on the momentum factor, a curious finding that, given the point estimate of a small negative index fund momentum loading, is another reason index fund investors tend to suffer low returns following high inflows. The active fund share is a strong negative predictor of the size and value premia. This finding suggests that SMB and HML tend to perform poorly following strong growth of mutual funds relative to other forms of investment, as active funds account for the large majority of mutual fund assets.

The fact that the index fund share of the market predicts low index fund returns relative to active fund returns is the key feature of the data consistent with the intuition of the paper. In the remainder of the empirical section, I check whether this relationship is large enough to account for the buy-and-hold advantage of index funds over active funds.
B. Aggregated Funds

I construct an aggregate active fund and an aggregate index fund as value-weighted portfolios of the individual funds in the CRSP data. Fama and French (2009) also use this approach to examine an aggregate mutual fund without making the active/passive distinction. For each of the two funds, I calculate a time series of buy-and-hold returns. From these, I obtain buy-and-hold net alphas by running time-series regressions:

\[
R_{I,t} = \alpha_I + \beta_I Mkt_t + s_I SMB_t + h_I HML_t + \epsilon_{I,t} \quad (41)
\]

\[
R_{A,t} = \alpha_A + \beta_A Mkt_t + s_A SMB_t + h_A HML_t + \epsilon_{A,t}. \quad (42)
\]

The three benchmark returns are as in Fama and French (1993), namely the market return and two long-short returns that capture the size (SMB) and value (HML) effects. I also consider specifications that use the momentum factor (MOM) as in Carhart (1997). I find that index funds outperform active funds by 47 basis points per year under the three-factor model and 79 basis points under the CAPM (Table VI). These differences are not statistically significant, but the negative alpha of active funds is marginally significant. The weak significance of the initial results that are the focus of the paper reduces the power of the tests. I get around this problem in part by looking at differences in differences; at whether index fund returns drop by more than active fund returns when the timing of investors is taken into account. In addition, others like Fama and French (2009) use longer samples and achieve higher precision, suggesting that the initial results are robust.

In the model, the timing of fund flows is the result of shocks to nontradable wealth. Since these shocks are unobservable, I rely on flows themselves to identify them. The drawback is that other sources of aggregate fluctuation like changes in risk aversion can also create aggregate flows. In its most general form the intuition of this paper can apply to some of these shocks as well, as long as they have a publicly unobserved aggregate component.

To see how flows reveal the aggregate wealth shock, consider the impulse response functions in Figures 3 and 4. A positive shock to wealth leads to inflows that forecast low returns as the stock becomes overpriced. Positive cashflow news lead to outflows...
that forecast high returns due to underpricing. These flows are conditional on public observables like the price of the asset and the public signal, which is why I condition on the price-dividend ratio and other public information in the tests that follow.

Using data on the total net assets of active and index funds, I construct empirical analogs to the timed portfolios of the model. Total net assets correspond to the quantities $x^I WH$ and $x^A WH$ (see equations (21) and (35)). To isolate the portfolio weights $x^I$ and $x^A$, I divide by total financial wealth from the Federal Reserve Flow of Funds.\footnote{Using data on the total net assets of active and index funds, I construct empirical analogs to the timed portfolios of the model. Total net assets correspond to the quantities $x^I WH$ and $x^A WH$ (see equations (21) and (35)). To isolate the portfolio weights $x^I$ and $x^A$, I divide by total financial wealth from the Federal Reserve Flow of Funds.\footnote{I use innovations in these exposures as a stand-in for the wealth shocks together with various measures of cashflow news in a vector auto-regression. These quantities span the information set of the informed agents. Uninformed investors must filter them using public information like prices and other signals. I mimic this process with a Kalman filter. The exact state-space formulation is in Appendix C and the cashflow proxies are discussed below. I use the filtered values to construct timed portfolios. Specifically, let $E^H [x^I]$ and $E^H [x^A]$ represent the filtered values of $x^I$ and $x^A$ and define

$$R_{I,t+1}^* = \frac{x^I_t}{E^H_t [x^I_t]} R_{I,t+1} = \frac{x^I_t}{E^H_t [x^I_t]} R_{A,t+1}. \quad (43)$$

These dollar-weighted returns capture the tendency of investors to time their funds as well as the tendency of active fund managers to time the market. This timing is reflected in time-series intercepts:

$$R_{I,t}^* = \alpha_I^* + \beta_I^* Mkt_t + s_I^* SMB_t + h_I^* HML_t + \epsilon_{I,t}^* \quad (44)$$

$$R_{A,t}^* = \alpha_A^* + \beta_A^* Mkt_t + s_A^* SMB_t + h_A^* HML_t + \epsilon_{A,t}^*. \quad (45)$$

The main hypothesis is that $0 = \alpha_I > \alpha_A$, whereas $0 > \alpha_I^* = \alpha_A^*$. Adjusting for flows, active and index funds should perform equally and both should underperform the benchmark. Note that by construction the dollar-weighted portfolios have time-varying betas but using those betas on the right would be a tautology. The goal is to measure market timing, that is the covariance between changes in market beta and returns. This covariance shows up in the intercept, which I call market-timing alpha.

Constructing the filtered variables requires a measure of fundamentals. As there
is no obvious candidate, I use three different measures. The first is a value-weighted average of the expected long-term earnings growth from the Institutional Brokers’ Estimate System (henceforth IBES) from Thompson Reuters. I use this measure because it alone picks up the dramatic rise in the price-dividend ratio in the late 1990s, a central feature of the sample. The second measure is the one-year expected earnings growth from the Survey of Professional Forecasts at the Philadelphia Federal Reserve. The third measure is the VIX index from the Chicago Board Options Exchange. While VIX does not measure expected cashflows, it reflects expected volatility which can also be thought of as a measure of “fundamentals”.

These measures are publicly available, typically with a lag, but they require sophistication on the part of investors. Constructing the IBES measure requires manipulation and a large subscription fee. For the purposes of this paper, it is enough that uninformed investors do not use these measures. I include them with the public signals with one lag.

Figure 5 shows the results of the Kalman filter under the base specification with IBES. The estimated parameters are reported in Table VI. The most striking feature in Figure 5 is the run-up in IBES long-term expected earnings growth. This measure tracks the rise in the price-dividend ratio well, so that the Kalman filter does not attribute the entire run-up in prices during the late 1990s to a wealth shock.

The results of the main performance regressions are in Table VI. The buy-and-hold advantage of index funds is greatly cut or eliminated once timing is considered. Looking at the dollar-weighted returns, under the CAPM the index fund advantage drops from 79 to 20 basis points and under the three-factor model it drops from 47 to -19 basis points. Dollar-weighting affects index funds by 60 basis points more than active funds, and this difference comes in with a t-statistic of 1.5 to 1.6.

Both strategies have big negative alphas but at least some of that is due to time-varying risk premia as both strategies tend to go long at market peaks. In unreported tests, I use the price-dividend ratio to instrument the betas of the dollar-weighted portfolios and the results are unaffected, largely because the portfolio construction already conditions on prices. I report an additional way of dealing with time-varying risk premia below.

To check robustness, I consider alternative state and signal specifications. The
results are reported in the Tables VII-XI. In Table VII, I add the momentum factor. The results are very close to the CAPM specification. In Table VIII, I use the Survey of Professional Forecasters (SPF) instead of the IBES forecast as a proxy for expected cashflow growth. Substituting this proxy does not impact the relative performance of active and index funds.

In Table IX, I use the VIX index as an alternative measure of fundamentals as it moves strongly with prices. This weakens the results under the CAPM where most of the advantage of index funds remains. As for the three-factor model, not much changes.

Table X applies $x^I$, the exposure to index funds, to both the index and active fund returns in constructing the dollar-weighted portfolio, dropping $x^A$. The idea is to control for potential differences between index fund flows and active fund flows and focus on the timing ability of the active managers. Applying this technique reduces the results in magnitude by about half although it increases their statistical significance since applying the same flows to both the index and active fund return makes estimating their difference more precise.

Finally, Table XI uses shares of total market capitalization rather than shares of financial wealth to construct the dollar-weighted portfolios. This measure of leverage is less pro-cyclical as it scales by the price level and in this way deals with the time-varying expected returns issue. The dollar-weighted intercepts of both the active and index funds are less negative. As predicted, only the index fund intercept drops when time-varying leverage is applied. Under the CAPM, the advantage of index funds falls in half, whereas under the three-factor model, it once again turns slightly negative.

\section*{C. Individual Funds}

In this section, I report results for dollar-weighted returns of individual funds. This level of analysis is closer to the decision of the individual investor. I rely on reduced-form regressions as I have no information on the financial wealth of investors in any given fund.
I calculate monthly flows into fund $i$ at time $t$ as

$$ f_{i,t+1} = \log \left( \frac{TNA_{i,t+1}}{TNA_{i,t}} \right) - \log (R_{i,t+1}), \quad (46) $$

where $TNA$ stands for total net assets and $R$ is the monthly return. To isolate the part of fund flows due to unexpected wealth shocks, I consider a forecasting model. For example, flows tend to respond positively to past returns in the case of active funds. They also tend to be persistent, or to exhibit time trends related to the lifecycle of the fund. To control for these effects, write

$$ f_{i,t+1} = a_i + b_i f_{i,t} + c_i \log(R_{i,t+1}) + d_i \log(R_{i,t-6\rightarrow t}) + g_i t + h_i t^2 + e_{i,t+1}. \quad (47) $$

Here $\log(R_{i,t-6\rightarrow t})$ is the fund return over the past six months, and it is meant to capture the return-chasing behavior of investors. I consider five nested versions of this model.

The time-varying exposure of the individual fund investor represents a managed portfolio whose average risk-adjusted return should be lower than the benchmark buy-and-hold return. I call this a dollar-weighted return:

$$ R_{i,t+1}^{*} = \frac{x_{h} E_t [x_{h}^t]}{E_t [x_{h}^t]} R_{t+1}. \quad (48) $$

The conditional expectation in the denominator is obtained by exponentiating the predicted value from the flow predictability regression (47):

$$ R_{i,t+1}^{*} = \exp \left( e_{i,t+1} - \frac{\sigma_{e}^2}{2} \right) R_{t+1}. \quad (49) $$

The variance term is simply a Jensen correction. The model predicts that the buy-and-hold returns of index funds are higher than those of active funds by as much as the fees that active funds charge, whereas the dollar-weighted returns of index and active funds should be equal.

To check this hypothesis, I regress the time series of buy-and-hold and dollar-weighted returns of all funds on the market return, the three Fama French factors (Fama and French 1993), and momentum (Carhart 1997). For the buy-and-hold returns, the
time series intercept measures the timing ability of the fund less fees and expenses. For the dollar-weighted returns, the intercept also includes the timing of investors into and out of the fund. The model predicts that the buy-and-hold returns of active funds should be lower than those of index funds, while their dollar-weighted returns should be equal.

The results of this test are in Tables XII and XIII. The top panel of Table XII shows five different specifications for predicting fund flows. Column (5) includes all controls in regression (47), while column (1) has time trends only. I run the predictability model separately for each fund and report average coefficients across all funds. Both past flows and returns are associated with future flows. The $R^2$ estimates indicate that much of the variation in flows is unexplained and that there is a large idiosyncratic component of flows.

The next three panels of Table XII show the differential impact of dollar-weighting on active and index funds. The main hypothesis is that regressing the difference between dollar-weighted and buy-and-hold alphas on an index fund dummy should produce a negative coefficient. This is indeed the case across 14 of the 15 specifications. The results are strongest in the columns (4) and (5), which include all the controls for predicting flows. In the full specifications, index fund returns drop by between 50 and 80 basis points relative to active fund returns. In fact, active fund returns are largely unaffected by dollar-weighting, as indicated by the intercepts, which are close to zero. In the full specification, the coefficient on the index fund dummy is significant under the three- and four-factor models (the t-statistics are 2.10 and 1.98), and marginally significant for the CAPM (t-statistic of 1.70).

Controlling for past flows and past returns appears to be important for these results. One possibility is a return-chasing story that lies outside this model (see Chevalier and Ellison (1997) and Sirri and Tufano (1998)). An active fund that performs well will see new inflows as investors update their beliefs about the manager’s skill. If skill exhibits decreasing returns to scale (Berk and Green 2004), inflows will predict lower future net alpha.

Whereas Table XII shows the difference in differences between dollar-weighted and buy-and-hold alphas for active and index funds, Table XIII shows the underlying levels of alpha. Looking at buy-and-hold returns, index funds outperform active funds by
about 60 basis points per year under the CAPM or 80 basis points under the FF3F. This result is consistent with the findings in Fama and French (2009) among others, and with the model of the paper.

Looking at dollar-weighted intercepts, the apparent advantage of index funds falls or disappears entirely, depending on the specification for predicting flows. In columns (4) and (5), which include all controls for predicting flows, index funds appear just as (un-) attractive as active funds. This result can be seen in the zero coefficient on the index fund dummy once dollar-weighting is applied.

Overall, the results of the empirical tests suggest that after taking flows into account, both types of funds have net dollar-weighted returns that are lower than the benchmark returns by an amount in line with the average level of active fees.

VI. Conclusion

If you get a raise, should you buy stocks? Only if someone else gets a pay cut. Aggregate shocks do not affect the optimal allocation of risk. The premise of this paper is that households might not know if any given shock they receive is an aggregate one or not. This uncertainty creates noise in the prices that sustains information rents. Households trade off the benefits of risk sharing against private losses that Hirshleifer (1971) called “the distributive aspect of access to superior information”. These losses are incurred through poor market timing and so they affect well-diversified index fund investors. Active funds offer a costly solution: In equilibrium, net active alpha is negative to make active and index funds equally attractive.

In the data, I find evidence that abnormal flows into index funds are associated with low subsequent index fund returns. As in the model, this is not the case with active funds. The share of the stock market held in index funds tends to peak with prices and in subsequent periods active funds do better than index funds. Most of the buy-and-hold advantage of index funds over active funds disappears after taking into account the timing decisions of investors.

As Berk and Green (2004) point out, no alpha does not imply no skill. In this paper, I show conditions under which even negative (net) alpha does not imply no skill. Since the cost of information is a net cost to society (French 2008), and since investors must
be indifferent between alternative forms of investment, all investors must bear that cost equally, either directly or indirectly.
A Two-Period Model Appendix

This section presents the calculations behind the solution of the two-period model.

A. Household Portfolio Decisions

For the index fund investors, using the approximation techniques described in Campbell and Viceira (2002, p.29),

\[
\log\left(\frac{W_i + L^i}{W_0}\right) = \log\left(x^i_I (R - R_f) + R_f + \frac{L^i}{W_0}\right) \tag{50}
\]

\[
= r_f + \log\left(x^i_I (e^{r - r_f} - 1) + 1 + e^{r - w_0 - r_f}\right) \tag{51}
\]

\[
\approx r_f + \log\left(1 + e^{r - w_0 - r_f}\right) + \frac{x^i_I}{1 + e^{r - w_0 - r_f}} (r - r_f) \tag{52}
\]

\[+ \frac{1}{2}\frac{x^i_I}{1 + e^{r - w_0 - r_f}} \left(1 - \frac{x^i_I}{1 + e^{r - w_0 - r_f}}\right) \text{Var}(r|p,l) \tag{53}\]

The optimal portfolio weight is

\[
x^i_I = E\left[r - r_f|p,l\right] + \frac{1}{2} \text{Var}(r|p,l) \left(1 + e^{r - p - r_f}\right), \tag{54}\]

I have replaced \(W_0\) with \(P\) since the risky asset is assumed to be distributed symmetrically across households. That is, all households have equal financial wealth at date 1.

For the active fund investors, the same approach gives

\[
x^i_I = E\left[r_{A,net} - r_f|p,l\right] + \frac{1}{2} \text{Var}(r_{A,net}|p,l) \left(1 + e^{r - p - r_f}\right), \tag{55}\]

Using the relation \(r_{A,net} - r_f = r_A - r_f - f\),

\[
x^i_I = E\left[r_A - r_f|p,l\right] - f + \frac{1}{2} \text{Var}(r_A|p,l) \left(1 + e^{r - p - r_f}\right), \tag{56}\]

B. Numerical Solution

I use Hermite polynomials on a Gauss-Hermite grid to take advantage of the normality of the shocks. Specifically, let \(h_n(x)\) denote the \(n\)th order Hermite polynomial.
evaluated at a standardized $x$ (subtract mean, divide by standard deviation) and write

$$
\hat{p}_N (s, l, \eta; a) = \sum_{n_1+n_2+n_3=0}^N (a_{n_1} a_{n_2} a_{n_3}) h_{n_1} (s) h_{n_2} (l) h_{n_3} (\eta). \quad (57)
$$

for some maximum polynomial degree $N$ and vector of constants $a$. The vector $a$ is chosen to make deviations from market-clearing orthogonal to the span of the zero through $N$th order Hermite polynomials of the relevant state variables. Specifically, $a$ solves

$$
E \left[ \left( 1 - \left[ \frac{q}{2} (x_1^A + x_2^A) x^S + \frac{1-q}{2} (x_1^I + x_2^I) \right] \right) \otimes (h_{n_1} (s) h_{n_2} (l) h_{n_3} (\eta)) \right] = 0, \quad (58)
$$

$$
n_1 + n_2 + n_3 = 0, \ldots, N. \quad (59)
$$

In addition to the price, the other unknown functions are the conditional expectations and conditional variances that appear in the four portfolio demands. I estimate these using the same method. For example, consider the conditional expectation

$$
f (p, l^I) = E [s | p, l^I]. \quad (60)
$$

For a given pricing function parameterized by $a$, the function $f$ can be approximated by

$$
\hat{f}_N (p, l^I; b) = \sum_{n_1+n_2=0}^N (b_{n_1} b_{n_2}) h_{n_1} (p) h_{n_2} (l^I). \quad (61)
$$

The vector of constants $b$ is set to minimize the error in the set of equations

$$
E \left[ \left( s - \hat{f}_N (p, l^I; b) \right) \otimes (h_{n_1} (p) h_{n_2} (l^I)) \right] = 0 \quad (62)
$$

$$
n_1 + n_2 = 0, \ldots, N. \quad (63)
$$

These orthogonality conditions amount to a non-linear filter. As $N$ becomes large, the conditional expectation function $\hat{f}$ becomes exact. The market-clearing and conditional expectation orthogonality conditions set up a just-identified system of equations.
B Dynamic Model Appendix

This section details the mathematical steps in analyzing the model.

A. Household Portfolio Decisions

Substituting the wealth dynamics and applying Ito’s Lemma to the Bellman equation,

\[
0 = \max \left( -J_T + J_W W^h \left[ x \left( \mu^H - r \right) + r \right] + \frac{1}{2} J_W W \left( W^h \right)^2 x^2 \hat{\sigma}^2 \right) + \frac{1}{2} J_{LL} \left( L^h \right)^2 \left( \sigma^2 + \left( \sigma^H \right)^2 \right) + J_W L^h x \sigma \sigma^H
\]

\[
+ \frac{1}{2} J_{yy} \left( \sigma^H \right)' \sigma_y + J_W y^h x \sigma \sigma^H + J_L y^h \sigma^H \sigma_y. \quad (64)
\]

Following Benzoni, Collin-Dufresne, and Goldstein (2007), the homogeneity of log utility implies that

\[
J_W \left( W^h, L^h, y^H, \tau \right) = J_W \left( \frac{1}{2}, L^h, y^H, \tau \right) W^h. \quad (67)
\]

Thus define the function

\[
g \left( l^h, y^H, \tilde{\tau}, \tau \right) = [J_W \left( 1, l^h, y^H, \tau \right)]^{-1} = \left( W^h J_W \left( W^h, L^h, y^H, \tau \right) \right)^{-1}. \]

In a model with intertemporal consumption, \( g \) is the consumption-to-wealth ratio. Substituting into the expression for \( x \):

\[
x = \left( \frac{g}{g - l^h \bar{g}^h} \right) \mu^H - r - \left( \frac{g l^h}{g - l^h \bar{g}^h} \right) \frac{\sigma^H}{\sigma} - \left( \frac{g \bar{g}^h}{g - l^h \bar{g}^h} \right) \frac{\sigma^H}{\sigma} \quad (68)
\]

\[
= \frac{\mu^H - r}{\sigma^2} + \left( \frac{g l^h}{g - l^h \bar{g}^h} \right) \left( \frac{\mu^H - r}{\sigma^2} - \frac{\sigma^H}{\sigma} \right) - \left( \frac{g \bar{g}^h}{g - l^h \bar{g}^h} \right) \frac{\sigma^H}{\sigma}. \quad (69)
\]

The first term is the familiar Merton (1971) portfolio weight. The second term adjusts for the level and stock correlation of labor income. The last term adjusts for changes in the investment opportunity set and is likely small given the assumption of log utility.

Labor income introduces non-linearity in the demand for the risky asset. As a result, through the market clearing condition, prices generally depend on the full distribution of labor income across households, vastly complicating the solution of the problem.
Krusell and Smith (1998) come up with an approximate numerical solution to this problem but that would make it hard to write down the household filtering problem. Instead, following Viceira (2001) and Kogan and Uppal (2003), I linearize around a known solution to obtain a linear demand function. This in turn leads to prices that only depend on the aggregate level of labor income.

As the equation above shows, \( l^h = 0 \) corresponds to the standard Merton problem where all assets are tradeable. The log-utility investor in that case invests

\[
\begin{align*}
  x|_{l^h=0} &= \frac{\mu - r}{\sigma^2}. \\
  \tag{70}
\end{align*}
\]

Therefore when labor income is small relative to financial wealth we can write

\[
\begin{align*}
  x &\approx \frac{\mu - r}{\sigma^2} + \frac{\partial}{\partial l^h} \left( \frac{gl^h}{g - l^h g} \right) \bigg|_{l^h=0} \left( \frac{\mu - r}{\sigma^2} - \frac{\sigma_H}{\sigma} \right) l^h \\
  &\approx \frac{\mu - r}{\sigma^2} + \left( \frac{\mu - r}{\sigma^2} - \frac{\sigma_H}{\sigma} \right) l^h. \\
  \tag{71}
\end{align*}
\]

Assuming that \( l^h \) is small is perhaps unrealistic. However, in reality a large part of labor income is publicly observed so the part that is only privately observed can be small.

\[38\]

B. Filtering

Let the log price dividend ratio be given by

\[
\log \frac{P}{D} = F(y, \Delta, t). \\
\tag{73}
\]

Since \( P_T = D_T \), \( F \) must satisfy the boundary condition \( F(y, \Delta, 0) = 0 \). The two public sources of information, prices and the public signal \( e \) evolve according to

\[
\begin{align*}
  d \log P &= d \log D + dF(y, \Delta, t) \\
  de &= ydt + e^{-\kappa(T-t)} \sigma_e dB. \\
  \tag{74}
\end{align*}
\]
Households observe prices, which follow

\[ d \log P = d \log D + F_t dt + F_y dy + F_\Delta d \Delta + \frac{1}{2} F_{yy} dy^2 + F_{y\Delta} dy d \Delta + \frac{1}{2} F_{\Delta \Delta} d \Delta^2 \]  \hspace{1cm} (76) \]

Suppose that \( d \Delta = -\kappa \Delta dt + e^{-\kappa (T-t)} \sigma'_\Delta dB \) as will be verified shortly and recall the dynamics of \( y \):

\[ dy = e^{-\kappa (T-t)} (\sigma_L - \sigma_D)' dB \]  \hspace{1cm} (77) 
\[ = e^{-\kappa (T-t)} \sigma'_y dB. \]  \hspace{1cm} (78) 

Substituting into the price dynamics,

\[ d \log P = \left[ F_t - F_\Delta \kappa \Delta + \frac{1}{2} F_{yy} e^{-2\kappa (T-t)} \sigma'_y \sigma_y + F_{y\Delta} e^{-2\kappa (T-t)} \sigma'_y \sigma_\Delta \right] dt + \left[ \frac{1}{2} F_{\Delta \Delta} e^{-2\kappa (T-t)} \sigma'_\Delta \sigma_\Delta \right] dt \]
\[ + e^{-\kappa (T-t)} (\sigma_D + F_y \sigma_y + F_\Delta \sigma_\Delta)' dB \]  \hspace{1cm} (79) 
\[ = \mu_p (y, \Delta, t) + e^{-\kappa (T-t)} \sigma_P (y, \Delta, t)' dB. \]  \hspace{1cm} (80)

To solve this nonlinear filtering problem, I use an extended Kalman-Bucy filter, which essentially linearizes \( \mu_p \) and \( \sigma_P \) around \( (y^H, 0, t) \). This filter is not efficient in a statistical sense but is nevertheless standard owing to its versatility and robustness. Thus write

\[ \mu_p (y, \Delta, t) \approx \mu_p (y^H, 0, t) + \frac{\partial \mu_p}{\partial y} (y^H, 0, t) (y - y^H) + \frac{\partial \mu_p}{\partial \Delta} (y^H, 0, t) \Delta \]  \hspace{1cm} (83) 
\[ \hat{\mu}_p (y, \Delta, t) \approx \mu_p (y^H, 0, t). \]  \hspace{1cm} (84) 

Similarly,

\[ \hat{\sigma}_P (y, \Delta, t) \approx \sigma_P (y^H, 0, t). \]  \hspace{1cm} (85)
Therefore, the innovation in prices from the point of view of the uninformed households is

\[
d \log P - d \log \hat{P} = \left( \mu_p(y, \Delta, t) - \mu_p(y^H, 0, t) \right) dt + \sigma_p(y, \Delta, t)' dB \tag{86}
\]

\[
\approx \left( \frac{\partial \mu_p}{\partial y} (y - y^H) + \frac{\partial \mu_p}{\partial \Delta} \right) dt + \sigma_p' dB \tag{87}
\]

\[
\approx -F_{\Delta \kappa \Delta} \Delta dt + e^{-\kappa(T-t)} \left( \sigma_D + F_y \sigma_y + F_{\Delta \sigma \Delta} \right)' dB. \tag{88}
\]

From the standpoint of the informed, households tend to be surprised at a rate proportional to their estimation error \( \Delta \). They update their beliefs about \( y \) via

\[
dy^H = \beta \left[ \frac{d \log P - E^H [d \log P]}{de - E^H [de]} \right] \tag{89}
\]

\[
= \beta \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right) \tag{90}
\]

The resulting dynamics of \( \Delta \) are

\[
d\Delta = dy - dy^H \tag{91}
\]

\[
= e^{-\kappa(T-t)} \sigma_y' dB - \beta \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right) \tag{92}
\]

\[
= -\beta \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_y - \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix} \beta' \end{bmatrix}' dB. \tag{93}
\]

It follows that

\[
\kappa_{\Delta} = \frac{\beta_2}{1 + \beta_1 F_{\Delta}} \tag{94}
\]

\[
\sigma_{\Delta} = \sigma_y - \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix} \beta' \tag{95}
\]

\[
= \sigma_y - \begin{bmatrix} \sigma_D + F_y \sigma_y + F_{\Delta \sigma \Delta} & \sigma_e \end{bmatrix} \beta' \tag{96}
\]

\[
= \frac{\sigma_y - \begin{bmatrix} \sigma_D + F_y \sigma_y & \sigma_e \end{bmatrix} \beta'}{1 + F_{\Delta} \beta_1}. \tag{97}
\]
To get $\beta$, write

$$\beta = E^H \begin{pmatrix} (y + dy - y^H - E^H [dy]) \\
\cdot \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right) \right)^t \end{pmatrix} (98)$$

$$E^H \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right)^t (99)$$

$$E^H \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right)^{-1} (100)$$

Let $\Sigma = E^H [\Delta^2]$. The numerator is

$$E^H \left( \Delta + e^{-\kappa(T-t)} \sigma_y dB \right) \left( \begin{bmatrix} -F_{\Delta \kappa \Delta} \\ 1 \end{bmatrix} \Delta dt + e^{-\kappa(T-t)} \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dB \right)^t = \left( -F_{\Delta \kappa \Delta} \begin{bmatrix} -F_{\Delta \kappa \Delta} & 1 \end{bmatrix} \right) \Sigma dt + e^{-2\kappa(T-t)} \sigma_y \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dt (102)$$

The denominator is

$$\left( -F_{\Delta \kappa \Delta} \begin{bmatrix} -F_{\Delta \kappa \Delta} & 1 \end{bmatrix} \right) \Sigma dt + e^{-2\kappa(T-t)} \sigma_y \begin{bmatrix} \sigma_P & \sigma_e \end{bmatrix}' dt (103)$$

To get $\Sigma$, write

$$d\Sigma = E^H [d\Delta^2] (104)$$

The steady state is

$$\Sigma = e^{-2\kappa(T-t)} \left( \frac{\sigma_y \sigma_{\Delta \sigma}}{2\kappa_{\Delta}} \right) (108)$$
These formulas give a system of equations describing the evolution of the filtered variables.

C. Numerical Solution

In this section I briefly describe the numerical approach I use to solve the model. The goal is to obtain a numerical approximation to the pricing function that satisfies the market-clearing equation (36) in conjunction with the Kalman filter.

The state variables are the log labor income-dividends ratio \( y = \log L/D \) and the error in its public estimate, \( \Delta = y - y^H \). I conjecture a pricing function of the form

\[
\log P_t D_t = -r(T-t) + \left( e^{-2\kappa(T-t)} - 1 \right) F(y, \Delta, t) .
\] (109)

This specification is motivated by the boundary conditions

\[
\lim_{t \to T} \log P/D = 0 \quad (110)
\]

\[
\lim_{y \to -\infty} \log P/D = -r(T-t) + \frac{\sigma_D\sigma_D}{4\kappa} \left( e^{-2\kappa T} - 1 \right) . \quad (111)
\]

Intuitively, as the economy approaches its terminal date \( T \), no-arbitrage requires that the price of the asset converges to its imminent cashflow. As labor income becomes arbitrarily small compared to dividends, the economy is close to the complete markets case and the closed-form solution (111) is readily available.

I form a grid using the roots of the Chebyshev polynomials and later check the robustness of the solution on an evenly-spaced grid. I use a fifth-order approximation to minimize approximation error. As suggested by Judd and Gaspar (1997) for multivariate settings, I rely on complete rather than tensor polynomials. Finally, since one of the state variables is endogenous, I use a least squares approach as in Judd (1992).

C Data and Empirical Results Appendix

I consider a state vector consisting of index and active fund exposure and fundamentals. The signal is the price-dividend ratio and the lagged fundamentals. For example, in
the specification with IBES long-term EPS growth, let

\[
x_t = \begin{bmatrix} \log x^I_t \\ \log x^A_t \\ IBES_t \\ 1 \end{bmatrix}, \quad y_t = \begin{bmatrix} (p - d)_t \\ IBES_{t-1} \end{bmatrix}, \quad z_t = \begin{bmatrix} (p - d)_{t-1} \end{bmatrix}. \tag{112}
\]

Here \( x \) and \( y \) are the state and signal vectors, and \( x^I \) and \( x^A \) are index and active fund total net assets scaled by financial wealth. Since the price-dividend ratio is highly persistent, I use the lagged price-dividend ratio as an additional control \( z \) in the signal equation, as suggested by Hamilton (1994, p.372). This system forms a vector auto-regression. The state transition and signal equations are

\[
x_{t+1} = Fx_t + Q\epsilon_{t+1} \tag{113}
\]
\[
y_t = Az_t + Gx_t + R\epsilon_t. \tag{114}
\]

The shocks in the two equations are the same, allowing for correlation between the state and the signal. I impose a priori block-exogeneity restrictions on \( F, A, \) and \( G \). Specifically,

\[
F = \begin{bmatrix} f_{11} & 0 & 0 & f_{14} \\ 0 & f_{22} & 0 & f_{24} \\ 0 & 0 & f_{33} & f_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} \\ 0 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ 0 & 0 & g_{23} & g_{24} \end{bmatrix}. \tag{115}
\]
Table I. Two-period model parameter values
This table shows the parameter values used in the numerical solution of the two-period model. The solution is based on the projection methods of Judd (1998). The mean squared error is from the orthogonality conditions on the market-clearing condition; it is given as a fraction of the price of the risky asset.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>Active fee (per dollar of AUM)</td>
<td>0.01</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Log interest rate</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Total log-dividend volatility</td>
<td>0.25</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Private signal precision</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Volatility of aggregate log-labor income shocks</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>Volatility of idiosyncratic log-labor income shocks</td>
<td>1</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>Average log-dividend</td>
<td>$\log(1)$</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>Average log-labor income</td>
<td>$\log(10) - 0.5\sigma^2_\eta$</td>
</tr>
<tr>
<td><strong>Numerical approximation parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Polynomial degree</td>
<td>7</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean-squared error</td>
<td>$4.385 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
Table II. Two-period model results
This table presents the results from the calibrated example of the two-period model. The parameter values are in Table I. The table shows the equilibrium fraction of wealth invested in each type of fund, expected utility, expected financial wealth, gross and net alphas, and the noisiness of the equilibrium price. The alphas are risk-adjusted under the information set of the uninformed. The noisiness is measured as the absolute deviation of the equilibrium price from the fully revealing price as a percentage of the equilibrium price (see Bernardo and Judd (2000)).

<table>
<thead>
<tr>
<th></th>
<th>Index</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion (q)</td>
<td>0.75</td>
<td>0.25</td>
</tr>
<tr>
<td>(E[\log(W+L)])</td>
<td>2.027</td>
<td>2.027</td>
</tr>
<tr>
<td>(E[W])</td>
<td>1.032</td>
<td>1.032</td>
</tr>
<tr>
<td>Gross alpha</td>
<td>0</td>
<td>0.18</td>
</tr>
<tr>
<td>Net alpha</td>
<td>0</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

Noisiness: 1%
Table III. Dynamic model parameter values

This table shows the parameter values used in the numerical solution of the model. The solution method is Chebyshev collocation of equation (36). The log price-dividend ratio is of the form

$$\log P_t/D_t = -r (T - t) + \left( e^{-2\kappa(T-t)} - 1 \right) F(y, \Delta, t),$$

where \( y = \log \frac{L}{D} \) and \( \Delta = y - y^H \) (see text for details). The function \( F \) is approximated using complete Chebyshev polynomials of degree 0 through \( N \). The solution uses a tensor grid of the roots of the Chebyshev polynomials of order \( N \). The mean squared error is for the equally-weighted state-by-state excess demand; it is given as a fraction of total financial wealth.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>Active fund per-dollar fee</td>
<td>0.02</td>
</tr>
<tr>
<td>( T )</td>
<td>Number of periods</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma_D )</td>
<td>Dividend volatility at ( T )</td>
<td>\begin{bmatrix} 0.5 &amp; 0 &amp; 0 \end{bmatrix}^T</td>
</tr>
<tr>
<td>( \sigma_L )</td>
<td>Labor income volatility at ( T )</td>
<td>\begin{bmatrix} 0.5 &amp; 0 &amp; 0 \end{bmatrix}^T</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>Signal volatility at ( T )</td>
<td>\begin{bmatrix} 0 &amp; 0 &amp; 0.5 \end{bmatrix}^T</td>
</tr>
<tr>
<td>( \sigma_h )</td>
<td>Labor income dispersion at ( T )</td>
<td>1</td>
</tr>
<tr>
<td>( \mu_D )</td>
<td>Average log-dividend</td>
<td>\log(1)</td>
</tr>
<tr>
<td>( \mu_L )</td>
<td>Average log-labor income</td>
<td>\log(0.5) - 0.5\sigma_h^2</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Rate of mean reversion</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Chebyshev collocation parameters

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Polynomial degree</td>
<td>5</td>
</tr>
<tr>
<td>( K )</td>
<td>Grid width in standard deviations</td>
<td>6</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean-squared error</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Table IV. Dynamic model results
This table presents the results from the calibrated example of the dynamic model. The parameter values are in Table III. The table shows the equilibrium fraction of wealth invested in each type of fund, expected utility, expected financial wealth and its volatility, gross and net alphas, and the mean and volatility of returns.

<table>
<thead>
<tr>
<th></th>
<th>Index</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion $(q)$</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>$E_0 [\log (W_T + L_T)]$</td>
<td>0.453</td>
<td>0.453</td>
</tr>
<tr>
<td>$E_0 [W_T]$</td>
<td>1.133</td>
<td>1.133</td>
</tr>
<tr>
<td>St. Dev $(W_T)$</td>
<td>0.633</td>
<td>0.637</td>
</tr>
<tr>
<td>Gross alpha</td>
<td>0.000</td>
<td>1.508</td>
</tr>
<tr>
<td>Net alpha</td>
<td>0.000</td>
<td>-0.492</td>
</tr>
<tr>
<td>Expected return</td>
<td></td>
<td>17.533%</td>
</tr>
<tr>
<td>Volatility of returns</td>
<td></td>
<td>48.595%</td>
</tr>
</tbody>
</table>
**Table V. Return predictability**

Predicting returns using the log of assets under management of index and active funds as a percentage of total stock market capitalization ("Index share" and "Active share"), and the market price-dividend ratio. Estimates are from the regressions

\[ R_{t+1,t+12} = a + b(\text{Index share or active share})_t + c(p - d)_t + \epsilon_{t+12}. \]

The returns are the market excess return, the returns on the Fama French factors SMB and HML, the momentum factor MOM, and the return on a portfolio long index funds and short active funds. The coefficients use the persistent predictor correction of Stambaugh (1999). The t-statistics use 12-lag Newey-West standard errors. Monthly data, January 1991 through June 2009.

<table>
<thead>
<tr>
<th>MRF</th>
<th>SMB</th>
<th>HML</th>
<th>MOM</th>
<th>Index - Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index share</td>
<td>-0.78</td>
<td>-0.66</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.81)</td>
<td>(2.40)</td>
<td>(0.62)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>P/D</td>
<td>-0.20</td>
<td>-0.14</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.38)</td>
<td>(1.66)</td>
<td>(0.96)</td>
<td>(1.53)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>16%</td>
<td>12%</td>
<td>22%</td>
<td>1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MRF</th>
<th>SMB</th>
<th>HML</th>
<th>MOM</th>
<th>Index - Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active share</td>
<td>-0.20</td>
<td>-0.30</td>
<td>-0.89</td>
<td>-0.88</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.40)</td>
<td>(0.67)</td>
<td>(3.95)</td>
<td>(4.04)</td>
</tr>
<tr>
<td>P/D</td>
<td>-0.22</td>
<td>-0.21</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.59)</td>
<td>(2.45)</td>
<td>(1.19)</td>
<td>(1.88)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0%</td>
<td>12%</td>
<td>13%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Table VI. Aggregate active and index fund returns

Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. The top panel shows CAPM and three-factor (FF3F) alphas. The dollar-weighted strategy is given by equations (44) and (45) and has time-varying exposure equal to the difference between actual and publicly estimated aggregate household exposure. The bottom panel shows the estimated transition dynamics. \( x^I \) and \( x^A \) are the fractions of financial wealth held in index and active funds. IBES is the market-level long-term earnings growth forecast from I/B/E/S. The sample covers January 1986 through June 2009.

<table>
<thead>
<tr>
<th>Fund performance</th>
<th>( \text{CAPM} )</th>
<th>( \text{FF3F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH Alpha ( (t) )</td>
<td>0.04 (0.10)</td>
<td>-0.75 (2.27)</td>
</tr>
<tr>
<td>DW Alpha ( (t) )</td>
<td>-1.43 (2.57)</td>
<td>-1.63 (3.61)</td>
</tr>
<tr>
<td>BH–DW ( (t) )</td>
<td>1.47 (3.09)</td>
<td>0.88 (2.41)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kalman filter</th>
<th>( (p - d)_{t-1} )</th>
<th>( x^I_t )</th>
<th>( x^A_t )</th>
<th>IBES ( t )</th>
<th>1</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>( x^I_t )</td>
<td>0.89</td>
<td>( x^A_t )</td>
<td>0.92</td>
<td>IBES ( t+1 )</td>
<td>0.98</td>
</tr>
<tr>
<td>t-statistics</td>
<td>IBES ( t+1 )</td>
<td>(22.69)</td>
<td>(24.79)</td>
<td>(38.40)</td>
<td>(2.72)</td>
<td>(2.14)</td>
</tr>
<tr>
<td>State transition</td>
<td>( (p - d)_t )</td>
<td>0.93</td>
<td>0.05</td>
<td>0.75</td>
<td>0.65</td>
<td>99%</td>
</tr>
<tr>
<td>IBES ( t-1 )</td>
<td>0.98</td>
<td>0.00</td>
<td>94%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal equation</td>
<td>( (p - d)_t )</td>
<td>57.09</td>
<td>1.42</td>
<td>(2.49)</td>
<td>(3.84)</td>
<td></td>
</tr>
<tr>
<td>IBES ( t-1 )</td>
<td>(38.74)</td>
<td>(1.75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

49
Table VII. Aggregate active and index fund returns, specification with momentum Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. CAPM and four-factor (FF3F+MOM) alphas. See Table VI for details. January 1986 through June 2009.

<table>
<thead>
<tr>
<th>Fund performance</th>
<th>CAPM</th>
<th>FF3F+MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Index</td>
<td>Active</td>
</tr>
<tr>
<td>BH Alpha</td>
<td>0.04</td>
<td>-0.75</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.10)</td>
<td>(2.27)</td>
</tr>
<tr>
<td>DW Alpha</td>
<td>-1.43</td>
<td>-1.63</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.57)</td>
<td>(3.61)</td>
</tr>
<tr>
<td>BH–DW</td>
<td>1.47</td>
<td>0.88</td>
</tr>
<tr>
<td>(t)</td>
<td>(3.09)</td>
<td>(2.41)</td>
</tr>
</tbody>
</table>
Table VIII. Aggregate active and index fund returns, SPF specification
Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. The top panel shows CAPM and three-factor (FF3F) alphas. The dollar-weighted strategy is given by equations (44) and (45) and has time-varying exposure equal to the difference between actual and publicly estimated aggregate household exposure. The bottom panel shows the estimated transition dynamics. SPF is the expected one-year earnings of the corporate sector from the Survey of Professional Forecasters at the Philadelphia Federal Reserve, scaled by consumption. The sample covers January 1986 through June 2009.

<table>
<thead>
<tr>
<th>Fund performance</th>
<th>Fund performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPM</td>
</tr>
<tr>
<td>BH Alpha</td>
<td>0.04</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>DW Alpha</td>
<td>-1.17</td>
</tr>
<tr>
<td>(t)</td>
<td>(1.95)</td>
</tr>
<tr>
<td>BH−DW</td>
<td>1.21</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.41)</td>
</tr>
</tbody>
</table>

Kalman filter

\[
(p - d)_{t-1} \quad x^I_t \quad x^A_t \quad SPF_{t+1} \quad 1 \quad R^2
\]

State transition

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^I_{t+1} )</td>
<td>0.89</td>
</tr>
<tr>
<td>( x^A_{t+1} )</td>
<td>0.92</td>
</tr>
<tr>
<td>SPF_{t+1}</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Signal equation

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (p - d)_t )</td>
<td>0.96</td>
</tr>
<tr>
<td>SPF_{t-1}</td>
<td>0.98</td>
</tr>
</tbody>
</table>

\[ SPF_t \]

51
Table IX. Aggregate active and index fund returns, VIX specification

Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. The top panel shows CAPM and three-factor (FF3F) alphas. The dollar-weighted strategy is given by equations (44) and (45) and has time-varying exposure equal to the difference between actual and publicly estimated aggregate household exposure. The bottom panel shows the estimated transition dynamics. $x^I$ and $x^A$ are the fractions of financial wealth held in index and active funds. VIX is the CBOE implied volatility index, averaged over the last month of the quarter. The sample covers January 1986 through June 2009.

<table>
<thead>
<tr>
<th>Fund performance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPM</strong></td>
</tr>
<tr>
<td>BH Alpha</td>
</tr>
<tr>
<td>(t)</td>
</tr>
<tr>
<td>(t)</td>
</tr>
<tr>
<td>DW Alpha</td>
</tr>
<tr>
<td>(t)</td>
</tr>
<tr>
<td>BH−DW</td>
</tr>
<tr>
<td>(t)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kalman filter</th>
</tr>
</thead>
</table>
| $\begin{align*}
(p - d)_{t-1} & \quad x_t^I \quad x_t^A \quad VIX_t \quad 1 \quad R^2 \\
\text{Coefficients} & \quad -0.67 \quad 85\% \\
\text{t-statistics} & \quad 22.69 \quad 24.79 \quad (10.08) \quad (3.52) \\
\text{State transition} & \quad 0.89 \quad 0.92 \quad 0.73 \quad 0.06 \quad 53\% \\
\text{Signal equation} & \quad 0.97 \quad 0.15 \quad -0.03 \quad -0.29 \quad 1.00 \quad 99\% \\
\text{Coefficients} & \quad 0.73 \quad 0.06 \quad 53\% \\
\text{t-statistics} & \quad 103.54 \quad 5.10 \quad (7.50) \quad (7.60) \\
\end{align*}$ |
Table X. Aggregate active and index fund returns, index fund exposure only
Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. The top panel shows CAPM and three-factor (FF3F) alphas. The dollar-weighted strategy is given by equation (44) and has time-varying exposure equal to the difference between actual and publicly estimated aggregate household exposure. The bottom panel shows the estimated transition dynamics. $x^I$ is the fractions of financial wealth held in index and active funds funds. IBES is the market-level long-term earnings growth forecast from I/B/E/S. The sample covers January 1986 through June 2009.

<table>
<thead>
<tr>
<th>Fund performance</th>
<th>CAPM</th>
<th>FF3F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Index</td>
<td>Active</td>
</tr>
<tr>
<td>BH Alpha</td>
<td>0.04</td>
<td>-0.75</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.10)</td>
<td>(2.27)</td>
</tr>
<tr>
<td>DW Alpha</td>
<td>-1.40</td>
<td>-1.97</td>
</tr>
<tr>
<td>(t)</td>
<td>(2.52)</td>
<td>(3.47)</td>
</tr>
<tr>
<td>BH−DW</td>
<td>1.44</td>
<td>1.22</td>
</tr>
<tr>
<td>(t)</td>
<td>(3.04)</td>
<td>(2.68)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kalman filter</th>
<th>((p - d)_{t-1})</th>
<th>(x^I_t)</th>
<th>IBEST_t</th>
<th>1</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x^I_{t+1})</td>
<td>0.89</td>
<td>-0.67</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(IBEST_{t+1})</td>
<td>0.98</td>
<td>0.00</td>
<td>94%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-statistics</td>
<td>(22.69)</td>
<td>(2.72)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(IBEST_{t+1})</td>
<td>(38.40)</td>
<td>(0.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Signal equation</th>
<th>((p - d)_t)</th>
<th>(IBEST_{t-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(IBEST_{t-1})</td>
<td>0.96</td>
<td>0.01</td>
</tr>
<tr>
<td>t-statistics</td>
<td>(38.74)</td>
<td>(1.75)</td>
</tr>
</tbody>
</table>
Table XI. Aggregate active and index fund returns, shares of market cap
Buy-and-hold (BH) and dollar-weighted (DW) alphas of the aggregated active and index funds. The
top panel shows CAPM and three-factor (FF3F) alphas. The dollar-weighted strategy is in (44)
and (45) and has time-varying exposure equal to the difference between actual household exposure
and its public estimate. The bottom panel shows the estimated transition dynamics. \(x^{IW}/P\)
and \(x^{AW}/P\) are the fractions of total market cap held in index and active funds. IBES is the

<table>
<thead>
<tr>
<th>Fund performance</th>
<th>CAPM</th>
<th>FF3F</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH Alpha (t)</td>
<td>0.04</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(2.27)</td>
</tr>
<tr>
<td>DW Alpha (t)</td>
<td>-0.39</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>(0.72)</td>
<td>(2.31)</td>
</tr>
<tr>
<td>BH–DW (t)</td>
<td>0.43</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td>(0.11)</td>
</tr>
</tbody>
</table>

Kalman filter

\[
(p - d)_{t-1} \quad \left( x^{I W}/P \right)_t \quad \left( x^{A W}/P \right)_t \quad IBES_t \quad 1 \quad R^2
\]

<table>
<thead>
<tr>
<th>State transition</th>
<th>Coefficients</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^{I W}/P)_{t+1}</td>
<td>0.95</td>
<td>-0.27</td>
</tr>
<tr>
<td>(x^{A W}/P)_{t+1}</td>
<td>0.96</td>
<td>-0.08</td>
</tr>
<tr>
<td>IBES_{t+1}</td>
<td>0.98</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Signal equation</th>
<th>Coefficients</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p - d)_t</td>
<td>0.93</td>
<td>0.02</td>
</tr>
<tr>
<td>IBES_{t-1}</td>
<td>0.98</td>
<td>0.00</td>
</tr>
</tbody>
</table>

IBES \(t-1\)
Table XII. Individual fund alphas, differences in difference
Each column uses a separate specification for predicting flows. The conditioning variables are the lagged flow, the lagged one and six-month returns, and a linear and quadratic time trends. I regress the difference between a fund’s buy-and-hold (BH) and dollar-weighted (DW) alphas on an index fund dummy. The buy-and-hold (BH) alpha is from a regression of the fund’s return on the factors. The dollar-weighted (DW) alphas are for the returns given by equation (49). The sample includes domestic equity fund returns from January 1991 through June 2009.

<table>
<thead>
<tr>
<th>Flow predictability</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>3.57</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>3.06</td>
</tr>
<tr>
<td>Flow_{i,t}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_{t+1}</td>
<td></td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_{t-6-t}</td>
<td></td>
<td>0.15</td>
<td></td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td></td>
<td></td>
<td>Yes</td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>R^2</td>
<td></td>
<td>22%</td>
<td>21%</td>
<td>10%</td>
<td>27%</td>
</tr>
</tbody>
</table>

\[
Flow_{i,t+1} = a_i + b_i(Flow_{i,t}) + c_i(R_{i,t+1}) + d_i(R_{i,t-6-t}) + g_i t + h_i t^2 + \epsilon_{i,t+1}
\]

\[
(Alpha \ DW)_i - (Alpha \ BH)_i = a + b(Index \ Fund)_i + \epsilon_i
\]

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<td>0.01%</td>
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<td>0.09%</td>
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Table XIII. Individual fund alphas, levels
Each column presents different specification for predicting flows (see Table XII). Buy-and hold (BH) and dollar-weighted (DW) alphas. The dollar-weighted alphas are for the returns given by equation (49). The data covers domestic equity funds from January 1991 through June 2009.

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(Alpha BH)$_i = a + b$(Index Fund)$_i + \epsilon_i$

(Alpha DW)$_i = a + b$(Index Fund)$_i + \epsilon_i$
Figure 1. Index and active fund market shares

The top plot shows the de-trended log shares of the market held in index and active funds. The shares are defined as the log of assets under management divided by total stock market capitalization. The bottom plot shows the aggregate index fund return minus the aggregate active fund return, together with a one-year forward-looking moving average. The sample covers January 1986 through June 2009.
Figure 2. Two-period model: Prices and informed demand
The left panel shows the difference between the equilibrium price and the fully revealing price. The right panel shows the difference between the equilibrium demand of the active fund and the same demand when prices are fully revealing. The plots set the idiosyncratic shock $\eta$ to zero. The parameter values are in Table I.
Figure 3. Impulse response: Wealth shock

Filtered estimates, prices, leverage, and returns following a one standard deviation shock to labor income. The parameter values are in Table III. Dotted lines represent the corresponding quantities in an economy with symmetric information ($|\sigma_e| = 0$). The active investor risky asset exposure is the effective exposure, taking into account the portfolio choice of the active fund manager.
Figure 4. Impulse response: Dividend shock
Filtered estimates, prices, leverage, and returns following a one standard deviation shock to dividends. The parameter values are in Table III. Dotted lines the corresponding quantities in an economy with symmetric information (|σ_e| = 0). The active investor risky asset exposure is the effective exposure, taking into account the portfolio choice of the active fund manager.
Figure 5. Kalman filter estimates
This figure shows the filtered state variables against their true values. These are the index and active fund shares of wealth ($x^I$ and $x^A$) and the IBES long-term EPS growth forecast. The signals are the price-dividend ratio and the lagged values of the IBES variable. The sample covers January 1986 through June 2009.
References


