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Consumption Risk and the Cost of Equity Capital

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1 Motivation

- Historical average returns vary substantially across different asset classes
- There are significant differences even within equities
- For example, during 1954-2003:
  - Small Growth stocks earned 6.19 % per year, and
  - Small Value stocks earned 17.19 % over the risk free asset.
- In a perfect and informationally efficient market,
  - Realized average returns provide a good measure of what investors expected to earn
  - Assets that are expected to earn higher return would have higher exposure to systematic risk
- In such a world assets with higher average risk premium should have higher exposure to systematic risk

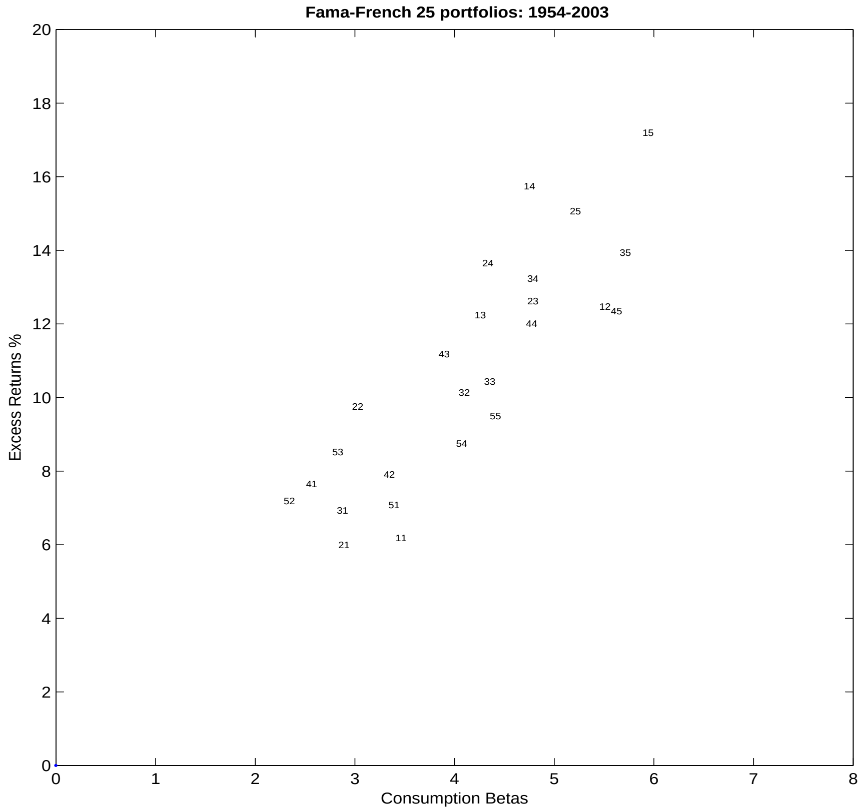
1.1 Measuring Exposure to Systematic Risk

- Standard Economic Theory
  - Covariance between the return on an asset and aggregate per capita consumption growth rate measures exposure to systematic risk
- Consumption CAPM is an attractive model, but previous empirical support is weak.
- Possible Reasons
  - Time aggregation: spot consumption data not available
  - Infrequent adjustment: consumption not continuously adjusted
  - Habit: consumption not fully adjusted

1.2 Our Approach

- We assume
  - There is infrequent adjustment of consumption and investment plans
  - All agents review their decisions at the end of the calendar year, and at possibly random times in between
- Match growth rate in consumption from fourth quarter of one year to the next with the corresponding calendar year return on assets
- Try to minimize the effect due to measurement errors in consumption, errors due to infrequent adjustment of consumption plans, and also reduce the effect of habit
- We find rather strong support for Consumption CAPM

Figure 1
Annual Average Excess Returns and Consumption Betas



1.3 Related Literature

- Breeden 1979, Lucas 1978
  - Provide the theoretical foundations in a perfect markets framework
- Hall 1981, and others
  - Use of quarterly consumption data, and quarterly return
- Hansen and Singleton 1983, and others
  - Use monthly consumption data, and monthly returns
- Brainard, Nelson and Shapiro 1991
  - Recommend the use of longer horizon return for minimizing measurement error in consumption data
- Breeden, Gibbons and Litzenberger 1989
  - Use monthly consumption data, and quarterly return

- Grossman, Melino and Shiller 1987, Hall 1988, Kandel and Stambaugh 1990, Singleton 1990
  - Time aggregation bias
- Grossman and Laroque 1990, Lynch 1996, Marshall and Parekh 1999, Gabix and Laibson 2001
  - Infrequent adjustment of consumption
- Constantinides 1990, Sundaresan 1989, Abel 1990, Campbell and Cochrane 1999
  - Time varying risk aversion due to habit
- Bansal and Yaron 2004, Bansal, Dittmar and Lundblad 2004, Daniel and Marshall 1997, Parker and Juliard 2004
  - Long horizon covariation between consumption growth and asset returns

2 A Stylized Model

We assume that there is a representative investor in the economy with time and state separable Von Neumann – Morgenstern utility function for lifetime time consumption from the vantage point of time t given by:

$$E\left[\left(\sum_{s=t}^{\infty}\delta^{s}u(c_s)\right)|F_t\right] \quad (1)$$

We assume that the representative investor reviews her consumption policy and portfolio holdings at periodic intervals in time, for some exogenously given reasons.

Consider an arbitrary point in time, t, where the representative investor reviews her consumption-investment decisions. Such points will occur at times $t = 0, k, 2k, 3k, ...$ i.e., t will be an integral multiple of the decision interval, k.

The investor will choose consumption and investment policies at t, $t = 0, k, 2k, 3k, ...$ so as to maximize expected life time utility, that gives rise to the following relation that must be satisfied by all financial assets:

$$E_t\left[R_{i,t+j}\left(\frac{\delta^j u'(c_{t+j})}{u'(c_t)}\right)\right] = 0, \tag{2}$$

for, $t = 0, k, 2k, ...$ and, $j = 1, 2, ...$

This leads to the following approximate relation:

$$E[R_{i,t+j}] = \lambda_{cj}\beta_{icj}, \tag{3}$$

where, $\beta_{icj} = \frac{Cov(R_{i,t+j}, g_{c,t+j})}{Var(g_{c,t+j})}$, $\lambda_{cj} \simeq \gamma\left[\frac{Var(g_{c,t+j})}{1-\gamma E(g_{c,t+j}-1)}\right]$.

2.1 Data and Econometric Methodology

- We assume that investors make decisions at the end of the fourth quarter of each calendar year
- They set the consumption for the fourth quarter and portfolio holding decisions
- We use per capita real consumption data for nondurables and services for the period 1954-2003
- We use the Two Stage Cross Sectional Regression and GMM to examine the consumption based model

3 Results

Table 1
Annual Average Excess Returns and Consumption Betas

	Average Annual Excess Returns					Consumption Betas				
	Low		book-to-market		High	Low	book-to-market			High
Small	6.19	12.47	12.24	15.75	17.19	3.46	5.51	4.26	4.75	5.94
	5.99	9.76	12.62	13.65	15.07	2.89	3.03	4.79	4.33	5.21
size	6.93	10.14	10.43	13.23	13.94	2.88	4.10	4.35	4.79	5.71
	7.65	7.91	11.18	12.00	12.35	2.57	3.35	3.90	4.77	5.63
Big	7.08	7.19	8.52	8.75	9.50	3.39	2.34	2.83	4.07	4.41

Table 2 Cross Sectional Regression: Excess Returns

$$E[R_{i,t}] = \lambda_0 + \lambda'\beta$$

	const	Δc	R_m	SMB	HML	R^2(adj-R^2)	F(p-value)
estimate	0.14	2.56				0.73	1.71
t-value	(0.05)	(3.89)				0.71	0.10
Shanken-t	(0.02)	(1.98)					
estimate	11.31		-0.56			0.00	2.37
t-value	(2.05)		(-0.09)			-0.04	0.02
Shanken-t	(2.05)		(-0.08)				
estimate	10.43		-3.26	3.12	5.83	0.80	1.59
t-value	(2.66)		(-0.70)	(1.62)	(3.11)	0.77	0.14
Shanken-t	(2.37)		(-0.57)	(1.03)	(2.12)		

Table 2 (cont.)

	const	Δc	R_m	SMB	HML	log(ME)	log(B/M)	R^2(adj-R^2)
estimate	11.75	1.58	-3.76	3.00	5.75			0.87
t-value	(2.98)	(3.64)	(-0.81)	(1.56)	(3.07)			0.84
Shanken-t	(1.95)	(2.26)	(-0.50)	(0.83)	(1.71)			
estimate	16.20					-0.87	3.46	0.84
t-value	(2.95)					(-1.43)	(3.00)	0.83
estimate	12.19	0.71				-0.71	2.66	0.86
t-value	(2.41)	(1.62)				(-1.23)	(2.12)	0.84
estimate	22.22		-3.80	-0.67	0.96	-1.07	3.04	0.87
t-value	(3.50)		(-0.88)	(-0.23)	(0.37)	(-1.51)	(2.87)	0.84

Figure 2

Realized vs. Fitted Excess Returns: FF25 Portfolios

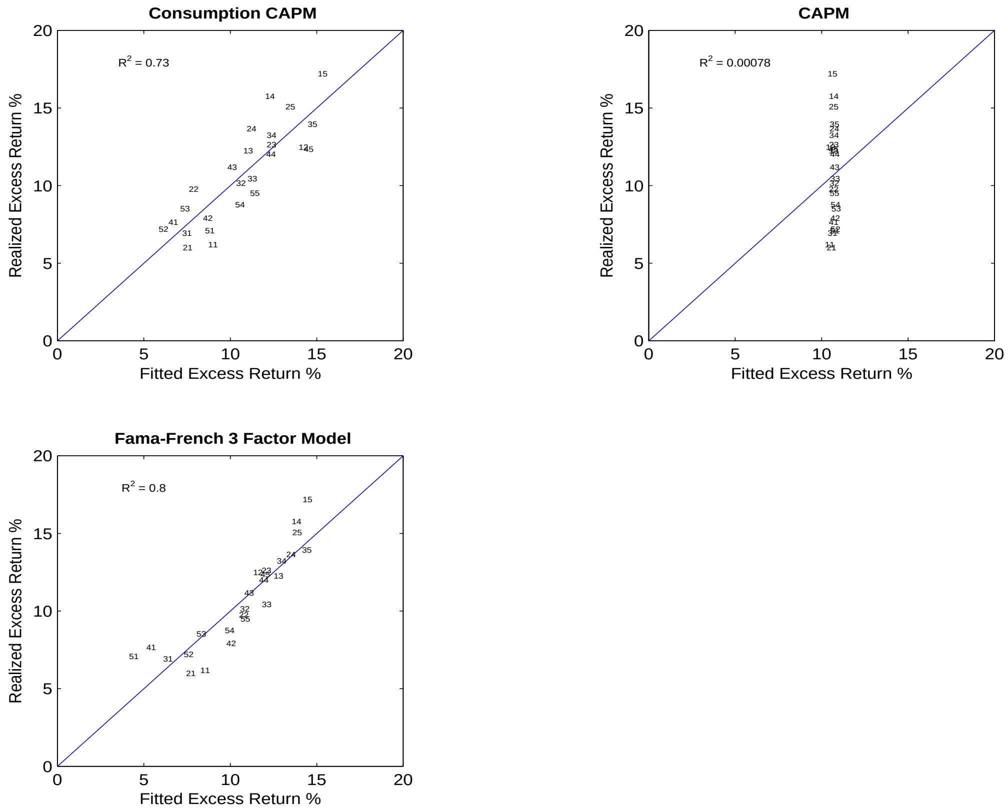


Table 3 Hansen-Jagannathan GMM : Excess Returns

$$E[(1 - b'f)R_{i,t}] = 0$$

	Δc	$HJ - dist$	p-value
estimate	33.01	0.29	0.69
t-value	(25.45)		

	R_m	$HJ - dist$	p-value
estimate	2.10	0.74	0.08
t-value	(6.44)		

	R_m	SMB	HML	$HJ - dist$	p-value
estimate	1.90	0.56	2.61	0.63	0.10
t-value	(4.12)	(0.85)	(5.02)		

Table 4 Optimal GMM : Excess Returns

$$E[(1 - b'f)R_{i,t}] = 0$$

$J \cdot \frac{T-N-K}{N}$ is approximately F(N,T-N-K)

	Δc			$J - statistic$	p-value
estimate	37.21			0.88	0.66
t-value	(33.99)				

	R_m			$J - statistic$	p-value
estimate	3.68			1.39	0.24
t-value	(11.30)				

	R_m	SMB	HML	$J - statistic$	p-value
estimate	1.70	2.61	3.63	0.87	0.75
t-value	(3.75)	(4.61)	(7.84)		

Table 5

Cross Sectional Regression: FF25 and R_f ($E[R_f] = 1.69\%$)

	$const$	Δc	R_m	SMB	HML	R^2(adj-R^2)
estimate	1.32	2.57				0.81
t-value	(0.71)	(4.47)				0.81
Shanken-t	(0.36)	(2.26)				
estimate	5.48		6.43			0.19
t-value	(2.81)		(1.78)			0.16
Shanken-t	(2.64)		(1.40)			
estimate	2.86		5.99	3.15	6.38	0.82
t-value	(5.13)		(2.23)	(1.58)	(3.27)	0.80
Shanken-t	(4.26)		(1.47)	(0.97)	(2.14)	

Table 6
Factor Correlation Coefficients

	Δc	R_m	SMB	HML
Δc	1.00			
R_m	0.27	1.00		
SMB	0.08	0.27	1.00	
HML	0.20	-0.14	-0.02	1.00

Table 7

Regress SDF on Return Space (FF25 and R_f)

	CCAPM					Fama-French 3 Factor				
	Low	book-to-market			High	Low	book-to-marke			High
Small	0.00	-0.32	-0.07	0.23	0.12	-0.02	0.04	-0.26	0.11	0.16
	-0.05	0.46	-0.12	0.17	-0.12	-0.03	0.15	0.01	-0.07	0.04
size	0.06	-0.30	-0.22	0.27	-0.06	-0.11	-0.04	-0.03	0.09	-0.11
	0.13	-0.29	0.29	0.06	-0.12	0.09	-0.14	0.03	0.07	0.02
Big	-0.01	0.08	0.05	-0.26	0.02	0.07	0.02	0.16	-0.21	-0.13
		R_f	1.00				R_f	1.09		

$$SDF_{CCAPM} = 2.67 - 0.69 * \Delta c$$

$$SDF_{FF3} = 1.43 - 0.03 * (R_m - R_f) - 0.01 * SMB - 0.04 * HML$$

Coefficients are estimated by HJ-GMM

Table 8
SDF Mimicking Portfolios

	Mean	Std	Sharpe Ratio
M_{CCAPM}	3.49	3.62	0.49
M_{FF3}	2.53	2.76	0.29
M*	0.32	1.27	-0.79

Correlation Coefficients

	M_{CCAPM}	M_{FF3}	M*
M_{CCAPM}	1.00		
M_{FF3}	0.57	1.00	
M*	0.13	0.26	1.00

Figure 3

Fama-French 25 Portfolios, Risk Free Asset and Efficient Frontier

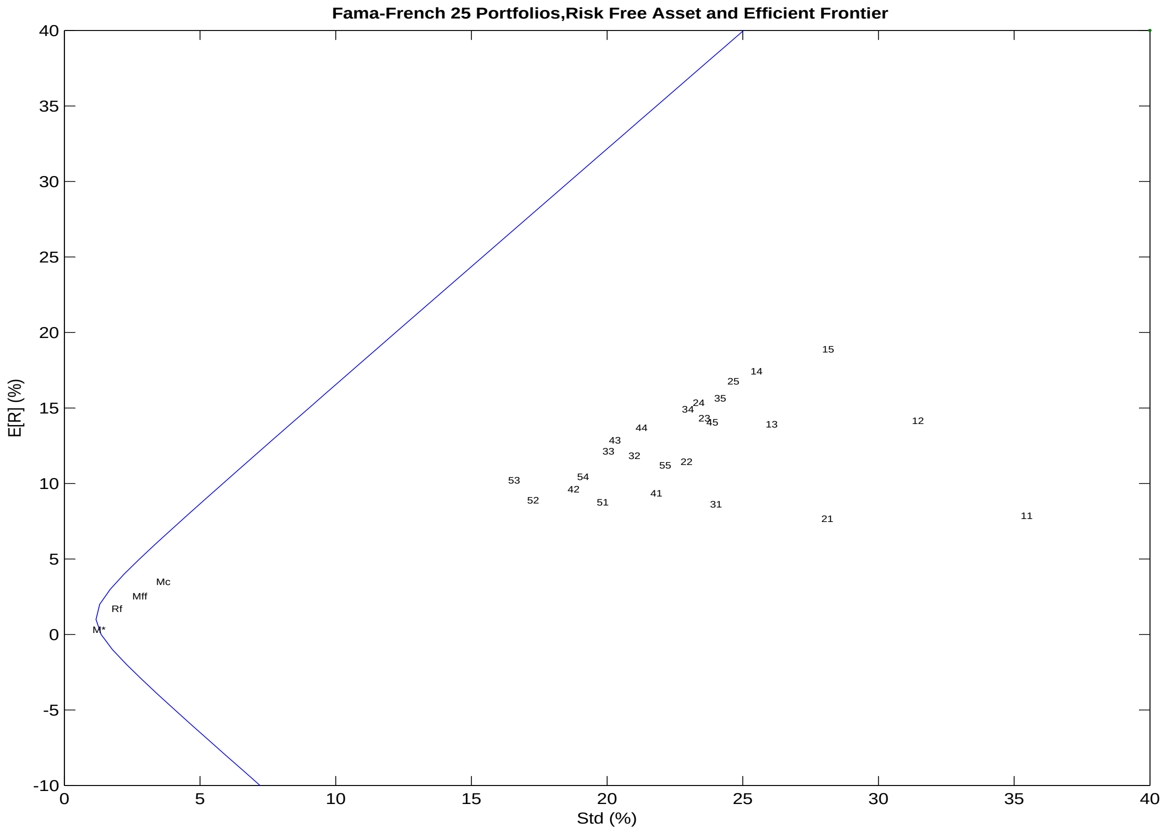


Table 9

Gibbons, Ross and Shanken Test: FF25 Excess Returns

Time Series Regression: $R_{i,t} = \alpha_i + \beta_i f_t + \varepsilon_{i,t}$

$$GRS = \frac{T-N-K}{N}[1 + E_T(f)'\widehat{\Omega}^{-1}E_T(f)]^{-1}\widehat{\alpha}'\widehat{\Sigma}^{-1}\widehat{\alpha} \sim F_{N,T-N-K}$$

	CCAPM*	CAPM	FF 3 Factor
GRS	1.13	2.07	1.65
p-value	0.38	0.04	0.12

* CCAPM using consumption mimicking portfolio

Table 10: Different Consumption Data

Nondurable + Services - Shoes -Clothing

(Lettau and Ludvigson's data: 1954-2002)

	$const$	Δc	R^2(adj-R^2)
	All Quarters: $Q_t - Q_{t+4}$		
estimate	5.10	1.53	0.31
t-value	(3.67)	(5.11)	0.28
Shanken-t	(2.77)	(3.67)	
	$Q4 - Q4$		
estimate	-0.23	2.29	0.66
t-value	(-0.07)	(3.52)	0.65
Shanken-t	(-0.04)	(1.88)	

Table 10: Different Consumption Data (cont.)

Nondurable + Services: Separate Price Index

	$const$	Δc	R^2(adj-R^2)
All Quarters: $Q_t - Q_{t+4}$			
estimate	4.81	1.36	0.32
t-value	(3.52)	(4.77)	0.29
Shanken-t	(2.76)	(3.55)	

$Q4 - Q4$

estimate	-0.70	2.28	0.65
t-value	(-0.21)	(3.56)	0.63
Shanken-t	(-0.12)	(1.89)	

Table 10: Different Consumption Data (cont.)

Nondurable + Services - Housing: Separate Price Index

	$const$	Δc	R^2(adj-R^2)
All Quarters: $Q_t - Q_{t+4}$			
estimate	3.44	1.63	0.39
t-value	(2.38)	(5.11)	0.36
Shanken-t	(1.83)	(3.73)	

$Q4 - Q4$			
estimate	-1.98	2.47	0.72
t-value	(-0.57)	(3.65)	0.71
Shanken-t	(-0.32)	(1.99)	

Table 10: Different Consumption Data (cont.)

Seasonally Unadjusted Consumption Data

	$const$	Δc	R^2(adj-R^2)
All Quarters: $Q_t - Q_{t+4}$			
estimate	2.40	2.35	0.45
t-value	(1.62)	(6.43)	0.43
Shanken-t	(1.02)	(3.95)	

$Q4 - Q4$

estimate	0.88	2.82	0.76
t-value	(0.25)	(3.91)	0.75
Shanken-t	(0.15)	(2.19)	

Table 11: Different Test Portfolios

	$const$	Δc	R^2(adj-R^2)
	18 Size Portfolios		
estimate	-0.44	2.60	0.81
t-value	(-0.09)	(1.68)	0.80
	18 Book/Market Portfolios		
estimate	2.62	1.79	0.80
t-value	(0.97)	(2.94)	0.79
	19 Earning/Price Portfolios		
estimate	1.94	2.09	0.53
t-value	(0.93)	(3.85)	0.50
	19 Cashflow/Price Portfolios		
estimate	2.81	1.72	0.59
t-value	(1.19)	(3.46)	0.56

Table 12

Different Frequency Data: 1960-2003

	Monthly Consumption Data			Quarterly Consumption Data			Annual Consumption Data		
	$const$	Δc	$R^2(\overline{R^2})$	$const$	Δc	$R^2(\overline{R^2})$	$const$	Δc	$R^2(\overline{R^2})$
Monthly	7.70	0.02	0.00						
Return	(2.61)	(0.17)	-0.04						
Quarterly	8.34	0.03	0.00	4.52	0.33	0.22			
Return	(2.80)	(0.15)	-0.04	(1.83)	(1.59)	0.18			
Annual	-1.83	2.01	0.41	-1.19	2.68	0.69	10.12	1.32	0.21
Return	(-0.51)	(2.33)	0.38	(-0.37)	(3.49)	0.68	(3.70)	(1.61)	0.18

Table 13
Using Other Quarterly Consumption Data

	$const$	Δc	R^2(adj-R^2)
		Q1-Q1	
estimate	5.10	1.18	0.27
t-value	(2.00)	(2.39)	0.24
		Q2-Q2	
estimate	7.70	0.88	0.18
t-value	(3.05)	(1.68)	0.14
		Q3-Q3	
estimate	8.64	1.38	0.30
t-value	(2.98)	(2.71)	0.27

Table 14

Regress Consumption Growth on Macro News

	Regression R-square				Correlation Coefficient			
	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4
unemployment rate*	0.47	0.41	0.29	0.28	-0.69	-0.64	-0.54	-0.53
term spread*	0.14	0.13	0.09	0.10	-0.38	-0.37	-0.30	-0.32
default spread*	0.26	0.22	0.13	0.19	-0.51	-0.46	-0.36	-0.43
30 year bond yield*	0.01	0.01	0.02	0.05	-0.10	-0.09	-0.12	-0.22
10 year bond yield*	0.01	0.00	0.03	0.06	-0.09	-0.01	-0.17	-0.25
1 year bond yield*	0.03	0.01	0.04	0.11	-0.16	-0.08	-0.21	-0.34
market return**	0.04	0.01	0.01	0.07	0.20	0.12	0.09	0.26

* 4 quarters change, **12 months return

Table 15

Consumption Beta in Contractions and Expansions

	$const$	Contraction	Expansion	R^2(adj-R^2)
estimate	0.86	0.98	0.23	0.65
t-value	(0.50)	(6.11)	(0.67)	0.62
estimate	0.84	1.06		0.65
t-value	(0.50)	(7.51)		0.62
estimate	6.10		1.40	0.33
t-value	(4.71)		(4.78)	0.26

$$E_t[R_{t+4}] = \alpha_1 I_t + \alpha_2(1 - I_t) + \beta_1 \Delta c_{t+4} I_t + \beta_2 \Delta c_{t+4}(1 - I_t)$$

Table 15 (cont.)
Consumption Beta in Contractions and Expansions

Number of quarters of contractions: 43

Number of quarters of expansions: 157

$$E(R|Contraction) = Beta(Contraction) * RiskPrem(Contraction)$$

$$E(R|Expansion) = Beta(Expansion) * RiskPrem(Expansion)$$

Suppose $Beta(Contraction) = scale * Beta(Expansion)$

$=> E(R) = Beta(Contraction) * WeightedRiskPrem$

Assumption: Beta Expansion is not measured well using aggregate data.

4 Conclusion

- When matching the CCAPM model with data it is necessary to choose the time interval over which consumption growth and returns are measured so as to minimize the effect of
  - measurement errors in consumption data
  - well documented within year calendar seasonal patterns in returns
  - infrequent review of consumption-investment decisions by investors
  - time variations in risk aversion that may arise due to habit etc
- We find rather strong support for the CCAPM in the data