WHY DIDN'T THE JAPANESE MIRACLE TAKE PLACE BEFORE WORLD WAR II?

by

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1. Introduction

The Japanese miracle, which lifted the Japanese economy from the ashes of the World War II destruction to the present-day prosperity, is well known. Also well known is the "lost decade" of the 1990s. Much less well known, however, is the decades-long stagnation before World War II. Japanese per-capita real GNP remained far below — about 2/3 below — that of the leader country, the U.S., at least since 1885 until the war. The TFP in prewar Japan is only about a half of the TFP level for 1985-88, even after adjusting for the world-wide upward trend in the TFP. Furthermore, there was very little capital deepening, resulting in the low level of per-capita GNP that is also about a half of the detrended postwar level. This paper addresses the question of why the Japanese miracle didn't occur before World War II.

An amazing fact about employment in agriculture in prewar Japan is that it is virtually constant at 14 million persons (more than 60% of working-age population in 1885) throughout the entire prewar period. The constancy strongly suggests that there was a powerful barrier that prevented people from moving out of agriculture. We hypothesize that this sectoral misallocation of resources is responsible for the prewar stagnation. Without this labor barrier, would prewar Japanese GNP been much higher?

We answer this question using a two-sector growth model with agriculture. Our two-sector model builds on the long tradition of modelling the "dual economy" starting with Jorgenson (1961). Its modern renditions are recent papers by Echeverria (1997), Laitner (2000), Gollin-Parente-Rogerson (2002), and others. They feature either non-homothetic preferences to accommodate Engel's Law or a decreasing returns scale in the primary sector's production technology or both. Our model is a marginal extention in its flexible representation of non-homothetic preferences by Frisch demands.

We superimpose the labor barrier on this two-sector model to see to what extent the model can account for the prewar Japanese stagnation. Given observed sectoral TFPs and the labor barrier, the model accounts well for the slow capital deepening. We then lift the labor barrier to predict what would have happened to GNP. We find that per-capita prewar GNP would have been higher by about 60%, less than about 100% that would be needed to put detrended GNP in prewar Japan on a par with its postwar level.

The plan of the paper is as follows. The next section, Section 2, describes those facts about GNP and the TFP in more detail and advances our sectoral misallocation hypothesis. Section 3 presents the two-sector growth model. The asymptotic properties of the model is examined
2. The Basic Idea

A Look at GNP and the TFP

We start out with a look at aggregate output since 1885. Figure 1 shows detrended real percapita GNP (GNP per working-age population) for Japan and the U.S. Detrending is done as follows. We know from the data on the U.S. percapita output that the long-run growth rate is about 2.0% after World War II and about 1.8% prewar. We set the trend level in period \( t \), denoted \( TREN_D_t \), grows at 1.8% in the prewar period, 4% during the war, and 2.0% after the war. Thus

\[
\frac{TREN_D_{t+1}}{TREN_D_t} = \begin{cases} 
1.018 & \text{for } t = 1885, \ldots, 1938, \\
1.040 & \text{for } t = 1939, \ldots, 1944, \\
1.020 & \text{for } t = 1945, \ldots 
\end{cases} \tag{2.1}
\]

We assume the growth rate during the war is as high as 4% because without a high wartime growth rate detrended percapita GNP is higher in the postwar period than in the prewar period. The same trend is used to detrend both Japanese and U.S. GNP. In the figure, detrended Japanese percapita GNP for 1985-1988 is normalized to unity.

There are three features that would catch anyone’s eye. The first is the fabulous growth in the post World War II era, known as the Japanese miracle. There was a five-fold increase in Japan’s GNP per working-age population in 25 years since 1947. The second is the prewar stagnation of several decades: between 1885 and 1940, Japan’s percapita GNP remained about one-third to 40% of the U.S. percapita GNP and about 50% of Japan’s own detrended GNP for 1985-88. The third feature is the stagnation in the 1990s. We have dealt with Japan’s 1990s elsewhere (Hayashi and Prescott (2002)) and won’t be concerned here. The question we address in this paper is why the Japanese miracle didn’t take place until after World War II.

This question should be puzzling to economists because the postwar Japanese miracle is commonly viewed as a convergence to the steady-state path represented by the U.S. economy. For example, a standard intermediate macroeconomics textbook states: “The ‘miracle’ of rapid growth in Japan .... is what the Solow model predicts for countries in which war has greatly reduced the capital stock” (Mankiw (2003, p. 189)). In the neoclassical growth model (be it
the Solow model or the Cass-Koopmans optimal growth model) with technological knowledge available to all countries in the world, if a country grows faster other countries, it must be due to capital deepening.

The popular way to decompose a country's growth rate between capital deepening and technical progress, is to define an index of the economy's overall efficiency, called the TFP (total factor productivity), as

$$ TFP_t = \frac{Y_t}{K_t^\theta L_t^{1-\theta}} = \frac{Y_t}{(K_t L_t)^{\theta}}, \quad (2.2) $$

where $Y_t$ is aggregate output in period $t$, $K_t$ is aggregate capital stock, $L_t$ is labor input (measured in total hours worked), and $\theta$ is a parameter that equals capital's share of aggregate income if the aggregate production function is Cobb-Douglas. This definition of $TFP$ can be rewritten as

$$ \frac{Y_t}{TREND_t} = \frac{TFP_t}{TREND_t^{1-\theta}} \cdot \left( \frac{K_t}{L_t} \right)^{\theta}, \quad (2.3) $$

which means that the trend for the TFP is

$$ TREND_t^{1-\theta}. \quad (2.4) $$

Therefore, if the long-run growth rate of output per capita is 2% and if $\theta = 0.362$, for example, then the growth rate of $TFP$ must be

$$ 1.02^{1-0.362} - 1 = 1.275\%. \quad (2.5) $$

This means that the trend TFP growth rate is about 1.275% postwar and 1.148% ($= 1.018^{1-0.362} - 1$) prewar, if the share parameter $\theta$ equals 0.362.

If the postwar Japanese miracle is entirely due to capital deepening, then the detrended $TFP$, calculated as $TFP_t/TREND_t^{1-\theta}$, should be constant. That this prediction is far off the mark is shown in Figure 2, where the detrended TFP, normalized to unity for 1985-1988, is plotted for years since 1885. The detrended TFP, far from being constant at unity in the postwar period, grew rapidly (i.e., the TFP growth rate was far above 1.275%) in the postwar period until around 1970. Much of the Japanese miracle is due to this rapid increase in the TFP. That the detrended percapita GNP in 1950 was about 30% of its 1985-88 level while the detrended TFP in 1950 was about 45% of its 1985-88 level means capital deepening accounts for only about 20% ($\approx (0.45 - 0.3)/(1 - 0.3)$) of the postwar miracle.

The other striking feature of Figure 2 is the low level of prewar detrended TFP, about 50% of the 1985-88 level. Detrended percapita GNP in the prewar period is also about 50% of
its 1985-88 level, meaning that there was virtually no capital deepening in the prewar period. The detrended TFP for the very early postwar period is even lower than the prewar level, probably because of the aftermath of the war destruction and Japan's lack of access to frontier technology during the war. To understand the prewar stagnation and the postwar miracle, we need to account for the low prewar level and the rapid postwar increase of the TFP.

The Sectoral Misallocation Hypothesis

The thesis of this paper is that agriculture was a drag on the economy's overall efficiency as measured by the TFP. We were led to this thesis by the following observations. Figure 3 shows that employment in agriculture (here and elsewhere excluding forestry and fishery) was remarkably constant at 14 million persons in prewar Japan. This was taking place when in most other developing and developed countries agricultural employment was steadily declining. The figure also shows that, in sharp contrast to the prewar area, postwar Japan witnessed a steep decline in agricultural employment. As the labor force expands, a constant level of employment means a declining employment share. This is shown in Figure 4, where the employment share of agriculture declined gradually before the war and very sharply postwar. Figure 5 is an XY plot of the two series shown in Figures 2 and 4. It shows a fairly close negative association between the detrended overall TFP and agriculture's employment share.

We hypothesize that there was a barrier in prewar Japan that prevented people from moving out of agriculture. Thanks to this barrier, there was too much labor tied up in the decreasing-returns-to-scale technology called agriculture. For some reason unknown to us at least now, this barrier ceased to operate after World War II. The sectoral misallocation of labor became much less onerous, and this allowed the rapid increase in the overall TFP in postwar Japan. Hansen and Prescott (1999) described the industrial revolution as a switch from a decreasing-returns-to-scale technology (the Malthus technology) to a constant-returns-to-scale technology (the Solow technology). Using their terminology, we can state our sectoral misallocation hypothesis as saying that the transition to Malthus to Solow was inhibited by the barrier to labor mobility.

The rest of this paper is to formalize our sectoral misallocation hypothesis in a two-sector growth model with agriculture and see to what extent the model can account for the prewar stagnation and the postwar miracle. We would be able to declare full and complete victory if the detrended overall TFP implied by the two-sector model without the labor barrier when superimposed on Figure 2, were constant at unity for all years since 1885. To state the paper's conclusion here, our sectoral misallocation hypothesis can account for only a part — albeit a substantial part — of the gap between unity and the actual overall TFP. This is shown in
Figure 6. The reason that our sectoral misallocation hypothesis is not the whole story is that the TFP in the non-agriculture sector grew at a rate far above the 1.275% in the postwar era until around 1970.

3. The Two-Sector Model

In this section we present the two-sector model. The model will be calibrated later in the paper.

Households
There is a stand-in household with $N_t$ working-age members at date $t$. The size of the household evolves over time exogenously. The stand-in household utility function is

$$
\sum_{t=0}^{\infty} \beta^t N_t U(c_{1t}, c_{2t})
$$

(3.1)

where $c_{jt}$ is per-member consumption of good $j$ ($j = 1, 2$).

There are three sources of income for the household. First, measure $E_{jt}$ of the household members work in sector $j$ for $h_{jt}$ hours per week ($j = 1, 2$). If $w_{jt}$ is the wage rate in sector $j$, the household’s total labor income is $w_{1t}h_{1t}E_{1t} + w_{2t}h_{2t}E_{2t}$. Second, if $K_t$ is the capital stock owned by the household, its rental income is $r_tK_t$. Unlike labor, we assume no barrier to capital mobility between sectors, so the rental rate $r_t$ does not depend on which sector the capital is rented out to. Third, there is a rent earned from land, which is an input to sector 1’s production. The period-budget constraint for the household, then, is

$$
Q_t N_t c_{1t} + N_t c_{2t} + K_{t+1} - (1 - \delta)K_t = w_{1t}h_{1t}E_{1t} + w_{2t}h_{2t}E_{2t} + r_tK_t - \tau(r_t - \delta)K_t - \pi_t,
$$

(3.2)

where $\delta$ is the depreciation rate, $Q_t$ is the relative price of good 1 in terms of good 2, $\tau$ is the tax rate on net capital income, and $\pi_t$ is the taxes other than the capital income tax less land rent. The second good is the numeraire, so, for example, $w_{1t}$ is the sector 1 wage rate in terms of good 2. We assume (at least in this version of the paper) that total employment ($E_t \equiv E_{1t} + E_{2t}$) and hours worked ($h_{1t}, h_{2t}$) are exogenous. Therefore the tax on labor income is not distortionary and is included in $\pi_t$.

The household chooses how the exogenously given total employment $E_t$ is divided between two sectors, but there is a barrier to labor mobility requiring that employment in sector 1 be
at least \( \bar{E}_{1t} \):

\[
E_{1t} \geq \bar{E}_{1t} \quad \text{or} \quad E_{2t} \leq \bar{E}_{2t} \equiv E_t - \bar{E}_{1t}.
\] (3.3)

The stand-in household wishes to maximize its utility (3.1) subject to the sequence of period budget constraints (3.2) and the labor barrier (3.3) for \( t = 0, 1, 2, \ldots \). If \( \beta \Lambda_t^{-1} \) is the Lagrange multiplier for the period \( t \) budget constraint (i.e., if \( \Lambda_t \) is the ratio of \( \beta_t \) to the Lagrange multiplier), the first-order conditions are given by

\[
\Lambda_{t+1} = \beta \Lambda_t [1 + (1 - \tau)(r_{t+1} - \delta)],
\] (3.4)

\[
\frac{\partial U(c_{1t}, c_{2t})}{\partial c_{1t}} = \frac{Q_t}{\Lambda_t},
\] (3.5)

\[
\frac{\partial U(c_{1t}, c_{2t})}{\partial c_{2t}} = \frac{1}{\Lambda_t}.
\] (3.6)

For \( E_{2t} \), the household would set \( E_{2t} = 0 \) if \( w_{1t}h_{1t} > w_{2t}h_{2t} \), \( E_{2t} = \bar{E}_{2t} \) if \( w_{1t}h_{1t} < w_{2t}h_{2t} \), and \( E_{2t} \) is indeterminate if \( w_{1t}h_{1t} = w_{2t}h_{2t} \). Since \( \Lambda_t \) is the reciprocal of the Lagrange multiplier for the budget constraint, it measures how wealthy the consumer is. The first-order conditions for consumption, (3.5) and (3.6), can be solved for consumption as

\[
c_{1t} = c_1(Q_t, \Lambda_t) \quad \text{and} \quad c_{2t} = c_2(Q_t, \Lambda_t).
\] (3.7)

This system, relating prices and the (reciprocal of) marginal utility to consumption, is the Frisch demand functions.

Firms

The production function for sector 1 is

\[
Y_{1t} = TFP_{1t} K_{1t}^{\theta_1} (h_{1t}E_{1t})^\eta
\]

\[
= A_{1t} K_{1t}^{\theta_1} E_{1t}^\eta \quad \text{with} \quad A_{1t} \equiv TFP_{1t} h_{1t}^{1-\theta_2},
\] (3.8)

where \( TFP_{1t} \) is the total factor productivity and \( K_{1t} \) is the capital input in sector 1. Land is the third input, but since it is constant, its contribution is submerged in the TFP. Because of the existence of the fixed factor of production, we have a decreasing returns to scale in capital and labor:

\[
\theta_1 + \eta < 1.
\] (3.9)

Production in sector 2 does not require land and exhibits constant returns to scale:

\[
Y_{2t} = TFP_{2t} K_{2t}^{\theta_2} (h_{2t}E_{2t})^{1-\theta_2}
\]

\[
= A_{2t} K_{2t}^{\theta_2} E_{2t}^{1-\theta_2} \quad \text{with} \quad A_{2t} \equiv TFP_{2t} h_{2t}^{1-\theta_2}.
\] (3.10)
The marginal productivity conditions for the two sectors are as follows.

\[ r_t = Q_t \theta_1 A_{1t}^\theta_1 K_{1t}^{\theta_1 - 1} E_{1t}^\eta, \quad (3.11) \]
\[ w_{1t} h_{1t} = Q_t \eta A_{1t}^\theta_1 K_{1t}^{\theta_1 - 1} E_{1t}^\eta, \quad (3.12) \]
\[ r_t = \theta_2 A_{2t} \left( \frac{K_{2t}}{E_{2t}} \right)^{\theta_2 - 1}, \quad (3.13) \]
\[ w_{2t} h_{2t} = (1 - \theta_2) A_{2t} \left( \frac{K_{2t}}{E_{2t}} \right)^{\theta_2}. \quad (3.14) \]

**Market Equilibrium**

The second good can be either consumed or invested. We also assume that government expenditure \( G_t \) is on the second good. Thus the market equilibrium conditions are

\[ N_t c_{1t} = Y_{1t}, \quad (3.15) \]
\[ N_t c_{2t} + (K_{t+1} - (1 - \delta)K_t) + G_t = Y_{2t}. \quad (3.16) \]

Three remarks are in order. First, we are assuming that the first good is non-tradable. Second, as in Hayashi and Prescott (2002), we treat claims on foreigners as part of the capital stock, so investment here \((K_{t+1} - (1 - \delta)K_t)\) is the sum of domestic investment and the current account, and \(Q_t Y_{1t} + Y_{2t}\) is GNP (in terms of good 2), not GDP. Third, the government budget constraint is implied by these market equilibrium conditions, the household period-budget constraint above, and the factor exhaustion condition (that payments to factors of production, including land, sum to output).

**Putting All Those Equations Together**

Let \( s_{E_1} \) and \( s_{K_1} \) be the employment and capital share of sector 2:

\[ s_{E_1} = \frac{E_{2t}}{E_{1t} + E_{2t}} = \frac{E_{2t}}{E_t}, \quad s_{K_1} = \frac{K_{2t}}{K_{1t} + K_{2t}} = \frac{K_{2t}}{K_t}. \quad (3.17) \]

Using this definition, recalling that the Frisch demands solve first-order conditions (3.5) and (3.6), and substituting (3.8) into (3.15), we obtain

\[ N_t c_{1t}(Q_t, A_t) = A_{1t} K_{1t}^{\theta_1} E_{1t}^\eta (1 - s_{K_1})^{\theta_1} (1 - s_{E_1})^\eta. \quad (3.18) \]

From the two marginal productivity conditions for capital ((3.11) and (3.13)), we obtain

\[ Q_t = \frac{\theta_2}{\theta_1} A_{2t} A_{1t} K_{1t}^{\theta_2 - \theta_1} E_t^{1 - \eta - \theta_2} \left( \frac{s_{K_1}}{s_{E_1}} \right)^{\theta_2 - 1} \left( 1 - s_{K_1} \right)^{\theta_1 - 1} \left( 1 - s_{E_1} \right)^{-\eta}. \quad (3.19) \]
If the barrier to labor mobility ($E_{1t} \geq \bar{E}_{1t}$ or $E_{2t} \leq \bar{E}_{2t}$) is not binding, then we have the equality of income from two sectors, $w_{1t}h_{1t} = w_{2t}h_{2t}$. Combine this with the two marginal productivity conditions for labor ((3.12) and (3.14)) to obtain

$$Q_t = \frac{1 - \theta_2}{\eta} A_{2t} E_t \left( \frac{K_t}{s_{Kt}} \right)^{\theta_2} \frac{\left( \frac{E_t}{s_{Et}} \right)^{1 - \theta_2}}{(1 - s_{Kt})^{\theta_1} (1 - s_{Et})^{\eta - 1}}. \quad (3.20)$$

Taking the ratio of these two equations, we obtain

$$\frac{\eta}{\theta_1} \frac{1 - s_{Kt}}{1 - s_{Et}} = \frac{1 - \theta_2}{\theta_2} \frac{s_{Kt}}{s_{Et}} \quad \text{or} \quad s_{Kt} = \frac{s_{Et}}{\frac{1 - \theta_2}{\theta_2} \left( \frac{1 - s_{Kt}}{1 - s_{Et}} + s_{Et} \right)}. \quad (3.21)$$

Substituting (3.10) into the market equilibrium condition for good 2 (3.16), we obtain

$$K_{t+1} = (1 - \delta)K_t + (1 - \psi_t)A_{2t} K_t^{\theta_2} E_t^{1 - \theta_2} s_{Kt}^{\theta_1} s_{Et}^{1 - \theta_2} - N_t c_2(Q_t, A_t), \quad (3.22)$$

where $\psi_t$ is the government's share of good 2:

$$\psi_t \equiv \frac{G_t}{Y_{2t}}. \quad (3.23)$$

Substituting (3.13) into the Euler equation (3.4) gives

$$\Lambda_{t+1} = \Lambda_t \beta \left( 1 + (1 - \tau) \left[ \theta_2 A_{2,t+1} \left( \frac{K_{t+1}}{E_{t+1}} \right)^{\theta_2 - 1} \left( \frac{s_{K,t+1}}{s_{E,t+1}} \right)^{\theta_2 - 1} - \delta \right] \right). \quad (3.24)$$

Five equations, (3.18), (3.19), (3.21), (3.22), and (3.24), determine the sequence of endogenous variables ($Q_t, s_{Kt}, s_{Et}, K_t, \Lambda_t$), given the sequence of exogenous variables ($E_t, E_{1t}, N_t, \psi_t, A_{1t}, A_{2t}$). More specifically, for each period $t$, given the exogenous variables and ($K_t, \Lambda_t$), we use (3.18), (3.19), (3.21) to solve for ($Q_t, s_{Kt}, s_{Et}$). If the $s_{Et}$ thus obtained does not satisfy the labor barrier $s_{Et} \leq s_{Et} = \bar{E}_{2t}/E_t$, then we set $s_{Et} = \bar{s}_{Et}$ and use (3.18) and (3.19) to solve for ($Q_t, s_{Kt}$). Substituting the solved-out values for ($Q_t, s_{Kt}, s_{Et}$) into (3.22) and (3.24) yields a system of two nonlinear difference equations in ($K_t, \Lambda_t$). This system will be solved with the initial condition for $K_0$ and an appropriate transversality condition that pins down $\Lambda_0$. The imposition of the transversality condition is done, as in most other studies, by locating the saddle path in the system where variables are suitably detrended.
4. Existence of Asymptotic Steady State

The Detrended System

In solving the model, we will assume that there exists a period $T$ such that,

$$
\text{for } t > T, \quad \frac{E_{t+1}}{E_t} = \frac{N_{t+1}}{N_t} = n > 1, \quad \psi = \psi \in (0, 1), \quad \frac{A_{1,t+1}}{A_{1t}} = g_1 > 1, \quad \frac{A_{2,t+1}}{A_{2t}} = g_2 > 1.

(4.1)
$$

That is, after $T$ periods, the growth rates of those exogenous variables become constant. We will assume for those constant growth rates that

$$
g_1 g_2^{\frac{\epsilon_1}{1-\eta}} n^{\eta_2 + \eta_1 - 1} > 1,

(4.2)
$$

which (as we will see below) guarantees sector 1 output per working-age population to grow (rather than shrink) in the long run.

In standard one-sector growth models, with a constant-returns-to-scale technology and CRRA (constant relative risk aversion) preferences, there exists a steady state growth path in that the capital stock converges to the unique constant-growth path regardless of its initial value and remains on that path once on the path (this latter property distinguishes a steady state from an asymptotic steady state). In our two-sector growth model with the first sector being agricultural sector, we wish to respect Engel’s Law (that the food share of expenditure keeps decreasing as the household gets wealthier) in consumption of good 1 and allow a decreasing returns to scale in production. All the while we want the model to have an asymptotic steady state in that $K_t$ converges to a unique constant-growth path and both $s_{Kt}$ and $s_{Et}$ converge to the same constants that do not depend on their initial values. We won’t require the model to have a steady state, rather than an asymptotic steady state, because we wish to allow the food share to be ever-decreasing.

To see under what conditions the model has such an asymptotic steady state, define:

$$
X_{Kt} \equiv A_{2t}^{\frac{1}{1-s_2}} E_t, \quad k_t \equiv \frac{K_t}{X_{Kt}}, \quad X_{Qt} \equiv A_{1t}^{\frac{1}{1-s_1}} A_{2t}^{\frac{1}{s_2}} E_t^{1-s_1 - \eta}, \quad q_t \equiv \frac{Q_t}{X_{Qt}}, \quad X_{At} \equiv \frac{X_{Kt}}{N_t}, \quad \lambda_t \equiv \frac{A_{1t}}{X_{At}}.

(4.3)
$$

To anticipate the discussion below, $X_{Kt}$, $X_{Qt}$, and $X_{At}$ turn out to be trends for $K_t$, $Q_t$, and
\( \Lambda_t \), respectively. Equations (3.18), (3.19), (3.22), (3.24) can be rewritten as

\[
\frac{c_1(q_t X_{Qt}, \lambda_t X_{At})}{X_{At}/X_{Qt}} = k_t^{\theta_1} (1 - s_{Kt})^{\theta_1} (1 - s_{Et})^n, \tag{4.4}
\]

\[
q_t = \frac{\theta_2}{\theta_1} k_t^{\theta_2 - \theta_1} \left( \frac{s_{Kt}}{s_{Et}} \right)^{\theta_2 - 1} (1 - s_{Et})^{\theta_1 - 1} (1 - s_{Et})^n, \tag{4.5}
\]

\[
\frac{X_{K,t+1}}{X_{K,t}} k_{t+1} = (1 - \delta)k_t + (1 - \psi_t) k_t^{\theta_2} s_{K,t}^{\theta_1} s_{Et}^{\theta_1 - \theta_2} - \frac{c_2(q_t X_{Qt}, \lambda_t X_{At})}{X_{At}}, \tag{4.6}
\]

\[
\frac{X_{\Lambda,t+1}}{X_{\Lambda,t}} \lambda_{t+1} = \lambda_t \beta \left\{ 1 + (1 - \tau) \left[ \theta_2 k_{t+1}^{\theta_2 - 1} \left( \frac{s_{K,t+1}}{s_{Et,t+1}} \right)^{\theta_2 - 1} - \delta \right] \right\}. \tag{4.7}
\]

In the long run, with total employment \( E_t \) growing without limit, the labor barrier \( E_{1t} \geq E_{1t} \) or \( s_{Et} \leq s_{Et} \equiv \frac{E_{1t} - E_{1t}}{E_{1t}} \) ceases to be binding. Therefor, for \( t \) large enough, the endogenous variables \( (q_t, s_{Et}, s_{Et}, k_t, \lambda_t) \) satisfy (3.21) as well as the above four equations. With (4.4), (4.5), and (3.21) defining \( (q_t, s_{Et}, s_{Et}) \) implicitly as a function of \( (k_t, \lambda_t) \), this five-equation system is a system of two nonlinear difference equations in \( (k_t, \lambda_t) \).

In this detrended system of two nonlinear difference equations, the terms \( \frac{X_{K,t+1}}{X_{K,t}} \) and \( \frac{X_{\Lambda,t+1}}{X_{\Lambda,t}} \) are constant for \( t > T \), but because of the terms involving \( c_1 \) in (4.4) and \( c_2 \) in (4.6), the system is not autonomous even for \( t \) sufficiently large. However, the saddle path for this detrended system converges to an equilibrium (that is, the original system before detrending converges to an asymptotic steady state) if there exist functions \( \chi_1(q, \lambda) \) and \( \chi_2(q, \lambda) \) such that

\[
\left| \frac{c_1(q X_{Qt}, \lambda X_{At})}{X_{At}/X_{Qt}} - \chi_1(q, \lambda) \right| \to 0, \quad \left| \frac{c_2(q X_{Qt}, \lambda X_{At})}{X_{At}} - \chi_2(q, \lambda) \right| \to 0 \quad \text{as} \quad t \to \infty. \tag{4.8}
\]

Under this “asymptotic independence” condition, with a suitable initial value for \( \lambda_0 \), the pair \( (k_t, \lambda_t) \) converges to the steady state \( (k^*, \lambda^*) \). So indeed \( X_{Kt} \) is the trend for \( K_t \) and \( X_{At} \) is the trend for \( A_t \). It is easy to show the following:

- \( q_t \)'s trend is \( X_{Qt} \), because \( q_t \), being a function of \( (k_t, \lambda_t) \), converge to a constant.

- The trend for sector 2 output is the same as that for \( K_t \), which is \( X_{Kt} \). So the trend for sector 2 output per working-age population equals \( X_{At} \).

- The trend for sector 1 output per working-age population is \( X_{At}/X_{Qt} \propto A_{1t} A_{2t}^{1/2} E_{1t}^{\theta_1 n^{-1}} \).

By (4.2), this trend is positive.

- \( q_t c_1(\lambda_t, A_t) \) and \( c_2(q_t, A_t) \) share the same trend of \( X_{At} \), implying that the value share of sector 1 will be constant in the long run.

- Sector 1's output relative to sector 2's is asymptotically zero if and only if \( q_t \)'s trend is positive (i.e., if and only if \( g_1 < g_2^{1/2} n^{-1/2} \)).
Frisch Demands

Since the first good is food, we wish to incorporate Engel’s Law in preferences, while imposing asymptotic independence (4.8). The Frisch demand provides a convenient parameterization for that purpose. Recall that a system of Frisch demands is derived from a “profit function”.¹ That is, let \( p \) be the price vector and \( \Lambda \) the reciprocal of the marginal utility. Then the Frisch demand for good \( i \) can be obtained by partial differentiation:

\[
- \frac{\partial \pi(p, \Lambda)}{\partial p_i} = c_i(p, \Lambda) \quad (i = 1, 2, \ldots, n),
\]

where \( n \) in this paragraph is the number of goods. One of the parameterizations of the profit function considered by Browning, Deaton, and Irish (1985) is

\[
\pi(p, \Lambda) = -\alpha_0 \Lambda - d(p) - \Lambda \sum_{i=1}^{n} \mu_i \log \left( \frac{p_i}{\Lambda} \right), \quad d(p) = \sum_{i=1}^{n} d_i p_i + \sum_{i \neq j} d_{ij} \sqrt{p_i} \sqrt{p_j}.
\]

For the present case of two goods, we have \( n = 2, Q = p_1/p_2, \) and \( p_2 = 1 \). The Frish demands become

\[
c_1(Q, \Lambda) = d_1 + d_{12} \sqrt{Q} + \mu_1 \frac{\Lambda}{Q} \quad \text{and} \quad c_2(Q, \Lambda) = d_2 + \frac{d_{12}}{\sqrt{Q}} + \mu_2 \Lambda.
\]

Provided (4.1) and (4.2), the asymptotic independence condition (4.8) holds if

\[
either \quad d_{12} = 0 \quad \text{or} \quad \frac{X_M}{(X_{Q_1})^{3/2}} \rightarrow \infty, \quad \sqrt{X_{Q_1}} X_M \rightarrow \infty \quad \text{as} \quad t \rightarrow \infty.
\]

Preferences under the former condition \( d_{12} = 0 \) has the “direct” representation

\[
U(c_1, c_2) = \mu_1 \log(c_1 - d_1) + \mu_2 \log(c_2 - d_2),
\]

which is linear-logarithmic version of the Stone-Gary utility function. Under either condition, the asymptotic Frisch demands, \( \chi_1(q, \lambda) \) and \( \chi_2(q, \lambda) \), are given by

\[
\chi_1(q, \lambda) = \mu_1 \frac{\lambda}{q} \quad \text{and} \quad \chi_2(q, \lambda) = \mu_2 \lambda,
\]

which is none other than the Cobb-Douglas (linear-logarithmic) preferences.

The steady-state values of the endogenous variables under this system (4.11) of Frisch demands are derived in the Appendix 1.

¹See, e.g., Browning, Deaton, and Irish (1985)
5. Calibration

Without loss of generality, we can set \( \mu_2 = 1 \) (if \( \mu_2 \neq 1 \), redefine \( \Lambda \) to be \( \mu_2 \Lambda \)). In this version of the paper, we set \( d_2 = 0, d_{12} = 0 \). The following is the list of remaining model parameters and how they are calibrated. Calibrated values are in Table 1.

\( \theta_1, \eta \) (the share parameters for sector 1): These parameters are taken from Hayami (1975) (Table 2-5). \( \theta_1 = 0.2, \eta = 0.5 \) (so land rent’s share is 0.3).

\( \theta_2 \) (capital’s income share in sector 2): Hayashi and Prescott (2002) report that the capital share for 1984-89 is 0.362. The share is fairly unchanged throughout the postwar period. For the prewar period, there is a capital share estimate for private nonagricultural sector by Minami and Ono (1978). It rises from 39.4% for 1896 to 54.2% for 1940. Rather than letting \( \theta \) change over time, we set \( \theta = 0.362 \) even for both the postwar and prewar periods.

\( \delta \) (depreciation rate): its calibrated value is calculated as the average of the ratio of depreciation to the capital stock.

\( \tau \) (tax rate on capital income): the postwar value is taken from Hayashi and Prescott (2002). For the prewar period, the average of the ratio of our estimate of taxes on capital income to the estimate of capital income from Minami and Ono (1978) is about 0.17. Presuming that the Minami-Ono capital income estimate might be overstated, we set \( \tau = 0.2 \).

\( \beta \) (discounting factor): With \( d_2 = 0, d_{12} = 0 \) and \( \mu_2 = 1 \), we have \( c_{2t} = \Lambda_t \) (see (4.11)). Substituting this and (3.13) into the Euler equation (3.4),

\[
\frac{c_{2,t+1}}{c_{2t}} = \beta \left[ 1 + (1 - \tau) \left( \theta_2 \frac{Y_{2t}}{K_{2t}} - \delta \right) \right].
\]

We take the sample average of both sides and solve for \( \beta \). The prewar calibrated value is smaller than the postwar value, mainly because the capital-output ratio for sector 2 is substantially lower before the war.

\( d_1 \) (food subsistence level): In this version, we set it equal to 0.4 times the 1885 value of sector 1 output per working age population.

\( \mu_1 \) (asymptotic expenditure share of food): The Engel coefficient is about 0.15 in recent years for Japan. Thus \( \mu_1 / (\mu_1 + \mu_2) \approx 0.15 \). With \( \mu_2 \) set equal to unity, \( \mu_1 \) should be about 0.15/(1-0.15).
6. Findings

We have calibrated the two-sector model to the Japanese economy for the perwar and postwar periods. We now wish to use this calibrated model to answer two questions: (1) with the labor barrier in place, to what extent does the model track historical data on the TFP and capital accumulation? (2) what would have happened to the TFP and capital accumulation had there been no labor barrier? To this end we run four simulations — one with the labor barrier and one without for the prewar and postwar periods separately.

Initial Conditions and Exogenous Variables

In the two prewar simulations, the actual capital stock in 1885 is taken as the initial capital stock, while in the two postwar simulations, which starts from 1951, the initial capital stock is the actual capital stock in 1951. In all simulations, the exogenous variables are:

\[ A_{1t} (= TFP_{1t} h_{1t}^{\gamma}) \text{ and } A_{2t} (= TFP_{2t} h_{2t}^{1-\beta}) \], where \( TFP_{jt} \) is the TFP in sector \( j \), \( h_{jt} \) is average hours worked (\( j = 1, 2 \)),

\( E_t \) (aggregate employment),

\( \bar{E}_{1t} \) (lower bound for sector 1 employment \( E_{1t} \)),

\( \psi_t \) (share of government expenditure in sector 2's output),

\( N_t \) (working-age population).

For all these variables except for the lower bound \( \bar{E}_{1t} \), we use their actual values for the sample period (1885-1940 for prewar simulations and 1951-1989 for postwar simulations). Figure 7 shows the detrended TFP for two sectors (detrending is by deflation by \( TREND_t^{1-\beta} \), where the trend function \( TREND_t \) is defined in (2.1)). Regarding the lower bound \( \bar{E}_{1t} \), we set it equal to the observed employment \( E_{1t} \) (so \( \bar{E}_{1t} = E_{1t} \)) for the prewar period. For the postwar period, we set \( \bar{E}_{1t} = E_{1t} \) until 1970 and \( \bar{E}_{1t} = 0 \) thereafter, thus assuming that the labor barrier ceased to exist after 1970.

For periods after the end of the sample period, the projected values of those exogenous variables are set as follows:

hours worked (\( h_{1t}, h_{2t} \): their projected values are set equal to the values at the end of the sample period.
TFP: their projected growth rates are 1.148% for both sectors for the prewar simulation, while for the postwar simulations the projected growth rate is 1.275% for sector 2 and 0 for sector 1 (agriculture).

aggregate employment ($E_t$) and the working-age population ($N_t$): for both prewar and postwar, their projected growth rates are set at 1%.

government share of sector 2 output ($\psi_t$): for both prewar and postwar periods, its value is set at 0.12.

the lower bound for sector 1 employment ($E_{1l}$): For the postwar period, we set it to 0; for the prewar period, we set it to its 1940 value.

Therefore, we are assuming that in the prewar period agents did not anticipate the actual development of the exogenous variables in the postwar period, let alone the war.

Simulation Results

Given the initial conditions and the sequences of exogenous variables, we can solve the model and calculate the sequence of endogenous variables ($Q_t, s_{Kt}, s_{Et}, K_t, \Lambda_t$). Figure 8a reports the sequence of $s_{Et}$ (sector 2’s employment share) from the two prewar simulations, one with the labor barrier and one without. The thick line is the sequence without the labor barrier, showing that without the barrier a far higher fraction of the labor force would have been employed in sector 2. The red line is the employment share with the barrier. Because the constraint setting the upper bound on sector 2 employment, it is always equal to the actual share. In this prewar simulation with the barrier, the constraint is binding for more than 100 years. Figure 8b shows the capital share for sector 2. The capital share with the labor barrier is larger than that without the barrier. This is to be expected. Figure 9 shows that capital accumulation would have been much larger in the prewar period were it not for the barrier. The figure also shows that our model with the labor barrier can account for the virtual absence of prewar capital deepening.

Prewar real per capita GNP shown in Figure 1 and used in calculating the TFP shown in Figure 2 is in 1934-36 prices. Thus, if $Y_{1t}$ and $Y_{2t}$ are outputs from the two sectors, real GNP is calculated as $Y_{1t} + Y_{2t}$ for all $t = (1885, ..., 1940)$. If $\bar{Y}_{jt}$ ($j = 1, 2$) is output for sector $j$ from the simulation in question, it would be wrong to compare the simple sum $\bar{Y}_{1t} + \bar{Y}_{2t}$ with actual real GNP $Y_{1t} + Y_{2t}$, because the relative price is different between the two economies. Nevertheless, in Figure 6, we report the detrended TFP implied by the simple sum $\bar{Y}_{1t} + \bar{Y}_{2t}$ from
the prewar unconstrained simulation.\textsuperscript{2} Looking at Figure 6, the economy's overall efficiency measured by the overall TFP is about 20% to 30% higher without the labor barrier. The efficiency gain, of course, is greater in the initial years because the barrier gradually gets less onerous as the agriculture’s employment share gradually declines. The much faster capital accumulation (shown in Figure 9) and the higher efficiency (shown in Figure 6) combined mean a substantially higher output level. This is shown in Figure 10 (for a moment, ignore postwar simulations starting 1951, also shown in the figure). Without the labor barrier, the model predicts that detrended prewar GNP would have been 70 to 80% — rather than 40 to 50% as in the historical data — of the 1985-88 level.

Turning to the postwar period, the constrained model does much less well. Unlike the prewar simulations, the constrained model (with the labor barrier for 1951-69) fits the data poorly, particularly for the period 1970-89. One possible reason is that our assumption that sector 1 good is a nontraded good is unsuitable for those years. For food, the import dependency ratio (the ratio of imports to the sum of domestic production and net imports), which is less than 2.2% until 1970 (see Table 1-19, volume 14 of LTES), sharply increased afterwards. Our model chooses the agricultural employment share of about 12% for 1973-89, which is less than the actual share for those years.

\textsuperscript{2}We also used a chain index number implied by the assumed utility function, with roughly similar results.
Appendix 1: Details on Model Solution

In this appendix, we describe our solution method in detail.

Five Equations

Our model, in detrended form, consists of five equations — (4.4), (4.5), (3.21), (4.6), and (4.7) — with the Frisch demands given by (4.11). We can assume without loss of generality that \( \mu_2 = 1 \) (we can do this because \( \mu_1, \mu_2 \) enter the system only through \( \mu_1 / \mu_2 \)). Thus the five-equation system can be written as

\[
\frac{d_1}{X_{\Delta t}/X_{qt}} + \frac{d_{12} \sqrt{q_t}}{X_{\Delta t}/(X_{qt})^{3/2}} + \mu_1 \lambda \frac{1}{q_t} = k_1^{\theta_1} (1 - s_{Kt})^{\theta_1} (1 - s_{Et})^\eta, \tag{A.11}
\]

\[
q_t = \frac{\theta_2}{\theta_1} k_1^{\theta_2 - \theta_1} \left( \frac{s_{Kt}}{s_{Et}} \right)^{\theta_2 - 1} \left( 1 - s_{Kt} \right)^{\theta_1 - 1} \left( 1 - s_{Et} \right)^{-\eta}, \tag{A.12}
\]

\[
s_{Kt} = \frac{s_{Et}}{\left( 1 - \theta_2 \right) \theta_1 (1 - s_{Et}) + s_{Et}}, \tag{A.13}
\]

\[
\frac{X_{K,t+1}}{X_{Kt}} k_{t+1} = (1 - \delta) k_t + (1 - \psi_t) k_t^{\theta_2} s_{Kt} s_{Et}^{1 - \theta_2} - \left( \frac{d_2}{X_{\Delta t}} + \frac{d_{12}}{\sqrt{q_t} X_{\Delta t} \sqrt{X_{qt}} + \lambda} \right), \tag{A.14}
\]

\[
\frac{X_{\Delta t+1}}{X_{\Delta t}} \lambda_{t+1} = \lambda_t \beta \left( 1 + (1 - \tau) \left[ \frac{\theta_2}{\theta_1} k_{t+1}^{\theta_2 - 1} \left( \frac{s_{K,t+1}}{s_{Et,t+1}} \right)^{\theta_2 - 1} - \delta \right] \right). \tag{A.15}
\]

In this system, the endogenous variables are \((q_t, s_{Kt}, s_{Et}, k_t, \lambda_t)\), while the exogenous variables are the trends \((X_{Kt}, X_{qt}, X_{\Delta t})\), defined in (4.3), and \( \psi_t \) (the government's share of sector 2 output, defined in (3.23)).

The Asymptotic Steady State

As assumed in the text, we have \( X_{\Delta t}/X_{qt} \to \infty, \psi_t \to \psi \) as \( t \to \infty \). Under the asymptotic independence assumption (4.8), this five-equation system is asymptotically autonomous. Let \((q, s_K, s_E, k, \lambda)\) be the equilibrium of this system. It satisfies

\[
\frac{\mu_1}{q} = k_1^{\theta_1} (1 - s_{K})^{\theta_1} (1 - s_{E})^\eta, \tag{A.16}
\]

\[
q = \frac{\theta_2}{\theta_1} k_1^{\theta_2 - \theta_1} \left( \frac{s_K}{s_E} \right)^{\theta_2 - 1} (1 - s_K)^{\theta_1 - 1} (1 - s_E)^{-\eta}, \tag{A.17}
\]

\[
s_K = \frac{s_E}{\left( 1 - \theta_2 \right) \theta_1 (1 - s_E) + s_E}, \tag{A.18}
\]

\[
\gamma n k = (1 - \delta) k + (1 - \psi) k_1^{\theta_2} s_K^{\theta_2} s_E^{1 - \theta_2} - \lambda, \tag{A.19}
\]

\[
\gamma = \beta \left( 1 + (1 - \tau) \left[ \frac{\theta_2}{\theta_1} k_{t+1}^{\theta_2 - 1} \left( \frac{s_K}{s_E} \right)^{\theta_2 - 1} - \delta \right] \right), \tag{A.10}
\]

where, with \( g_2 \) and \( n \) as defined in the text,

\[
\gamma \equiv g_2 \gamma(n). \tag{A.11}
\]

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This can be solved uniquely for \((q, s_K, s_E, k, \lambda)\) with
\[
s_K = \frac{\frac{1}{2} s_1}{1 - \psi + \frac{s_2}{s_1 \mu_1}}.
\]
(A1.12)

\[
s_E = \frac{s_k}{(1 - s_2) \theta_1} + \left(1 - \frac{s_2}{(1 - s_2) \theta_1}\right) s_K.
\]
(A1.13)

\[
k = \frac{\frac{3 - 1 + \delta}{\theta_2}}{\left(\frac{3 - 1 + \delta}{\theta_2}\right) \frac{s_1}{s_2} - 1}.
\]
(A1.14)

\[
\lambda = \frac{k \theta_2}{\mu_1 \theta_1} \left(1 - s_K\right).
\]
(A1.15)

The Solution Method

To illustrate our solution method, assume for simplicity that
\[
d_{12} = 0, d_2 = 0.
\]
(A1.16)

We eliminate \(q_t\) from (A1.1) and (A1.2) to obtain
\[
Z_{1t} (1 - s_{Kt})^{\theta_1} + \frac{\theta_1}{\theta_2} Z_{2t} (1 - s_{Et})^{\theta_2} \left(\frac{s_{Kt}}{s_{Et}}\right)^{\theta_1} = (1 - s_{Kt})(1 - s_{Et})^{\eta},
\]
(A1.17)

where
\[
Z_{1t} \equiv \frac{d_1}{(X_{At} / Y_{At})}, \quad Z_{2t} \equiv \frac{\mu \lambda}{k^{\eta}}.
\]
(A1.18)

This equation implicitly defines \(s_{Kt}\) as a function of \((s_{Et}, k_t, \lambda_t, X_{At}/X_{At})\). Conditional on \((k_t, \lambda_t)\) and \(X_{At}/X_{At}\), the graph of this function, viewed as a function of \(s_{Et}\), is as drawn in Appendix Figure 1. If the labor barrier \(s_{Et} \leq \bar{s}_{Et}\) is not binding, the intersection of this graph with the upward-sloping curve representing (A1.3), also drawn in the figure, is the equilibrium sectoral allocation of capital and labor given \((k_t, \lambda_t, X_{At}/X_{At})\). To calculate such \((s_{Kt}, s_{Et})\), substitute (A1.3) into (A1.18) to obtain
\[
\frac{\theta_1}{\theta_2} Z_{2t} = x(s_{Et})^{\theta_2 - 1}\left[1 - s_K(s_{Et}) - Z_{1t} (1 - s_K(s_{Et}))^{\theta_1} \left(\frac{1 - \theta_2}{\theta_2} \eta\right) x(s_{Et})^\eta\right]
\]
(A1.19)

where
\[
x(s_{Et}) \equiv \frac{1}{s_{Et} + \frac{(1 - \theta_2) \eta}{\theta_2 \eta} (1 - s_{Et})}, \quad s_K(s_{Et}) \equiv \frac{s_{Et}}{s_{Et} + \frac{(1 - \theta_2) \eta}{\theta_2 \eta}(1 - s_{Et})}.
\]
(A1.20)

The \(s_{Et}\) that solves this equation is the equilibrium labor allocation if the barrier is not binding. If the labor barrier is binding, then the equilibrium allocation is on the former curve with \(s_{Et} = \bar{s}_{Et}\).

With \((s_{Kt}, s_{Et})\) thus determined as functions of \((k_t, \lambda_t)\), the two equations, (A1.4) and (A1.5), form a system of nonlinear difference equations in \((k_t, \lambda_t)\). We use the shooting algorithm to solve for the saddle path.
Appendix 2: Incorporating Intermediate Inputs

The model in the text (and Appendix 1) ignores intermediate inputs. This is a standard simplification in one-sector models, but in multi-sector models the omission may alter conclusions. In this appendix, we explicitly allow intermediate inputs for Sector 1.

The Model
The production function for sector 1 is

\[ Y_{1t} = TFP_{1t} K_{1t}^{\theta_1} (h_{1t} E_{1t})^\eta M_t^\alpha \]
\[ = A_{1t} K_{1t}^{\theta_1} E_{1t}^\eta M_t^\alpha \quad \text{with} \quad A_{1t} \equiv TFP_{1t} h_{1t}^\eta, \quad (A2.1) \]

The contribution of land is implicit in this production function, so we have a decreasing returns to scale in capital, labor, and intermediate inputs:

\[ \theta_1 + \eta + \alpha < 1. \quad (A2.2) \]

The marginal productivity conditions for sector 1 are

\[ r_t = Q_t \theta_1 A_{1t} K_{1t}^{\theta_1-1} E_{1t}^\eta M_t^\alpha, \quad (A2.3) \]
\[ w_{1t} h_{1t} = Q_t \eta A_{1t} K_{1t}^{\theta_1} E_{1t}^{\eta-1} M_t^\alpha, \quad (A2.4) \]
\[ 1 = Q_t \alpha A_{1t} K_{1t}^{\theta_1} E_{1t}^{\eta-1} M_t^{\alpha-1}. \quad (A2.5) \]

With the production function for sector 2 being the same as in the text, the marginal productivity conditions for sector 2 are (3.13) and (3.14).

Solving (A2.5) for \( M_t \) and substituting it into (A2.1), (A2.3), and (A2.4), we obtain

\[ Y_{1t} = \alpha \gamma_{1t} Q_t^{1-\alpha} \tilde{A}_{1t} K_{1t}^{\bar{\theta}_1} E_{1t}^\bar{\eta}, \quad (A2.6) \]
\[ r_t = \bar{\theta}_1 \gamma_{1t} Q_t^{1-\alpha} \tilde{A}_{1t} K_{1t}^{\bar{\theta}_1-1} E_{1t}^\bar{\eta}, \quad (A2.7) \]
\[ w_{1t} h_{1t} = \bar{\eta} \gamma_{1t} Q_t^{1-\alpha} \tilde{A}_{1t} K_{1t}^{\bar{\theta}_1} E_{1t}^{\bar{\eta}-1}, \quad (A2.8) \]

where

\[ \tilde{A}_{1t} = A_{1t}^{1-\alpha}, \quad \bar{\theta}_1 = \frac{\theta_1}{1-\alpha}, \quad \bar{\eta} = \frac{\eta}{1-\alpha}. \quad (A2.9) \]

It is then easy to see that the model can be reduced to (3.21) and the following four equations:

\[ Nc_1(Q_t, \Lambda_t) = \alpha \gamma_{1t} Q_t^{1-\alpha} \tilde{A}_{1t} K_{1t}^{\bar{\theta}_1} E_{1t}^\bar{\eta} (1 - s_{Kt})^{\bar{\theta}_1} (1 - s_{Et})^\bar{\eta}, \quad (A2.10) \]

\[ Q_t^{1-\alpha} = \frac{\bar{\theta}_1}{\theta_1} \frac{A_{1t}}{A_{1t}} K_{1t}^{\theta_1-\bar{\theta}_1} E_{1t}^{1-\theta_1-\bar{\eta}} \left( \frac{s_{Kt}}{s_{Et}} \right)^{\bar{\theta}_1-1} (1 - s_{Kt})^{\bar{\theta}_1} (1 - s_{Et})^\bar{\eta}, \quad (A2.11) \]

\[ K_{t+1} = (1 - \delta) K_t + (1 - \psi_t) A_{2t} K_{1t}^{\theta_2} E_{1t}^{1-\theta_2} s_{Kt}^{1-\theta_2} \]
\[ - \alpha \gamma_{1t} Q_t^{1-\alpha} \tilde{A}_{1t} K_{1t}^{\bar{\theta}_1} E_{1t}^{\eta} (1 - s_{Et})^{\bar{\eta}} - Nc_2 (Q_t, \Lambda_t), \quad (A2.12) \]

\[ \Lambda_{t+1} = \Lambda_t \beta \left\{ 1 + (1 - \tau) \left[ \bar{\theta}_2 A_{2,t+1} \left( \frac{K_{t+1}}{E_{t+1}} \right)^{\theta_2-1} \left( \frac{s_{K,t+1}}{s_{E,t+1}} \right)^{\theta_2-1} - \delta \right] \right\}. \quad (A2.13) \]
Detrending

Define

\[ X_{Kt} \equiv A_{2t}^{1-\delta_2} E_t, \quad k_t \equiv \frac{K_t}{X_{Kt}}, \quad X_{Qt} \equiv \left( \frac{\hat{A}_{1t}^{-1} A_{2t}^{1-\delta_2} E_t^{1-\delta_1-\delta} \alpha^{-\tau_2}}{\tau_2} \right)^{1-\alpha}, \quad q_t \equiv \frac{Q_t}{X_{Qt}}, \quad \] (A2.14)

\[ X_{Lt} \equiv \frac{X_{Kt}}{N_t}, \quad \lambda_t \equiv \frac{L_t}{X_{Lt}}. \]

Then these four equations become

\[ \frac{c_1(X_{Qt}q_t, X_{Lt}\lambda_t)}{X_{Lt}/X_{Qt}} = q_t^{\tau_2} k_t^{-\delta_1} (1 - s_{Kt})^{\delta_1} (1 - s_{Et})\gamma, \] (A2.15)

\[ q_t^{\gamma/\gamma - \alpha} = \theta_2 k_t^{\beta_2 - \beta_1} \frac{(s_{Kt}/s_{Et})^{\theta_2 - 1}}{(1 - s_{Kt})^{\delta_1 - 1}(1 - s_{Et})\gamma}, \] (A2.16)

\[ \frac{X_{K,t+1}}{X_{Kt}} k_{t+1} = (1 - \delta)k_t + (1 - \psi_2)k_t^{\beta_2} s_{Kt}^{1-\theta_2} \]
\[ - \alpha q_t^{\gamma/\gamma - \alpha} k_t^{\delta_1} (1 - s_{Kt})^{\delta_1} (1 - s_{Et})\gamma - \frac{c_2(X_{Qt}q_t, X_{Lt}\lambda_t)}{X_{Lt}}, \] (A2.17)

\[ \frac{X_{Lt,t+1}}{X_{Lt}} \lambda_{t+1} = \lambda_t \beta \left\{ 1 + (1 - \tau) \left[ \theta_2 k_t^{\beta_2 - 1} \left( \frac{s_{K,t+1}}{s_{Et,t+1}} \right)^{\theta_2 - 1} - \delta \right] \right\}. \] (A2.18)
References


Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prewar</th>
<th>Postwar</th>
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<tbody>
<tr>
<td>$d_1$ (minimum subsistence level for good 1)</td>
<td>40% of sector 1 output in 1885</td>
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</tr>
<tr>
<td>$d_{12}$ (interaction term in consumption)</td>
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<td></td>
</tr>
<tr>
<td>$d_2$ (minimum subsistence level for good 2)</td>
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<td></td>
</tr>
<tr>
<td>$\mu_1$ (asymptotic consumption share of good 1)</td>
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</tr>
<tr>
<td>$\mu_2$ (asymptotic consumption share of good 2)</td>
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<td>$\theta_1$ (capital share in sector 1)</td>
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<tr>
<td>$\eta$ (labor share in sector 1)</td>
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<tr>
<td>$\theta_2$ (capital share in sector 2)</td>
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<td>$\delta$ (depreciation rate)</td>
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<td>$\beta$ (discounting factor)</td>
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<td>0.966</td>
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<tr>
<td>$\tau$ (tax rate on capital income)</td>
<td>0.2</td>
<td>0.5</td>
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</tbody>
</table>
Figure 1: GNP Per Working-Age Population relative to Trend
(1.8% Prewar, 2.0% Postwar, Japan's 1985-1988 = 1.0)
Figure 2: Japan's TFP relative to Trend (1.148% prewar, 1.275% postwar)
1985-1988 = 1.0
Figure 3: Employment in Agriculture, 1885-2001
Figure 5: Agriculture's Employment Share and Overall TFP Relative to Trend, 1885-1940, 1950-2000
Figure 6: Japan's TFP relative to Trend (1.148% prewar, 1.275% postwar)
1985-1988 = 1.0
Figure 7: TFP in Two Sectors Relative to Trend (1.148% Prewar, 1.275% Postwar)
1885=1.0
Figure 8a: Employment Share of non-Agriculture, 1885-1940
Figure 8b: Capital Share of non-Agriculture, 1885-1940
Figure 9: Capital Stock per Working-age Population, 1885-1940
1885=1.0

- Japan
- Japan, with barrier
- Japan, without barrier
Figure 10: GNP per Working-age Population relative to Trend, 1885-1940
Japan's 1985-88=1.0

[Graph showing trends in GNP per working-age population with various simulation scenarios from 1885 to 2000]