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Identifying the New Keynesian Phillips Curve

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Abstract

Phillips curves are central to discussions of inflation dynamics and monetary policy. The hybrid new Keynesian Phillips curve (NKPC) describes how past inflation, expected future inflation, and a measure of real aggregate demand drive the current inflation rate. This paper studies the (potential) weak identification of the NKPC under GMM and traces this syndrome to a lack of persistence in either exogenous variables or shocks. We employ analytic methods to understand the economics of the NKPC identification problem in the canonical three-equation, new Keynesian model. Given U.S., U.K., and Canadian data, we revisit the empirical evidence and construct tests and confidence intervals based on the exact and pivotal Anderson and Rubin (1949) statistic that is robust to weak identification. We also apply the Guggenberger and Smith (2006) LM test to the underlying NKPC pricing parameters. Both tests yield little evidence of forward-looking inflation dynamics.

JEL classification: E31; C32.

Keywords: Phillips curve, Keynesian, identification, inflation.

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1. Introduction

Recent years have witnessed a boom in work on the Phillips curve. For a student of monetary policy and the business cycle steeped in dynamic general equilibrium methods, the revival of Phillips curve research might come as a shock. The shock might be mitigated because the Phillips curve revival features debates on the role of backward- and forward-looking expectations for inflation, on which measure of real aggregate demand most directly influences inflation, on the response of monetary policy to various disturbances, on the costs of disinflation, and on optimal monetary policy. These debates often are framed by the new Keynesian Phillips curve (NKPC) because it appears to provide a tent under which many views of inflation dynamics can exist. However, whether to be inside or outside the Phillips curve revival tent depends on the NKPC being a persuasive description of inflation dynamics.

Variations on the NKPC are just about limitless. The canonical NKPC is driven either by current real marginal cost or today's output gap and is forward-looking in the current expectation of tomorrow's inflation. Galí and Gertler (1999) add lagged inflation to create a 'hybrid NKPC', which they use to address aspects of the debate among Phillips curve revivalists. Specification of the NKPC has important implications for monetary policy, and in particular for how central banks should react to real events while maintaining inflation targets. Although contributions to this research are too numerous to list, besides Galí and Gertler (1999), Fuhrer and Moore (1995), Roberts (1995), and Sbordone (2002) make important empirical contributions. Theory and evidence about the NKPC also are reviewed by Woodford (2003).

The hybrid NKPC is a second-order, linear, expectational difference equation. Hansen and Sargent (1980) and Sargent (1987) study the dynamic and time series properties of this general class of stochastic models. Much empirical work on the NKPC estimates it using instrumental variables (IV) methods, as Galí and Gertler (1999) do. Generally, NKPC parameters prove difficult to pin down even with large instrument sets. This suggests weak identification. Other symptoms of this syndrome include instability of estimates across instrument sets, estimates which may approach those from ordinary least-squares and hence be inconsistent, and Wald tests with size distortions. The goals of this paper are (a) to study the economics underlying weak identification, with a view to drawing lessons and recommendations for applied work and (b) to provide new tests of the NKPC that are robust to weak identification.

Section 2 provides background on the NKPC and begins our study of the economics of weak identification. It shows that identification requires predictability of future marginal cost or the output gap beyond that provided by current marginal cost or current or lagged inflation. This predictability requires higher-order dynamics in marginal cost, or additional instruments beyond lags of the variables in the NKPC.

Section 3 briefly considers a variety of approaches that have been proposed for dealing with the identification problem, while using standard tools of GMM estimation and testing. For example, some researchers have suggested calibrating the discount factor in the Calvo pricing model, focusing on the purely forward-looking NKPC, or indexing with lagged inflation. We note that some of these methods can improve identification, but at the cost of precluding tests of the general, hybrid NKPC.

Section 4 details the GMM identification problem when the hybrid NKPC is set in a typical, three-equation, new Keynesian model. The hybrid NKPC cannot be identified under IV estimation in the baseline version of this model. For the hybrid NKPC to be identified requires that either (a) one of the shocks to the system is persistent or (b) the interest-rate rule involves a lagged interest rate (interest-rate smoothing). Studying the NKPC within the three-equation, new Keynesian model thus does not generally suggest added instruments that could aid identification. The key analytic results are summarized in table 1.

Given these analytical results, section 5 therefore provides tests of the NKPC that are robust to weak identification. We estimate the hybrid NKPC for the U.S., U.K., and Canada, using a range of instruments. We first use the Anderson and Rubin (1949) statistic to test the hybrid NKPC. This test is exact and robust to weak or omitted instruments. Its application yields little evidence of forward-looking inflation dynamics for the U.S. and U.K., but cannot reject the forward-looking NKPC for Canada. We also apply the methods of Guggenberger and Smith (2006), which allow more powerful tests – again robust to weak identification – of the Calvo pricing parameters that underlie the NKPC. These methods too yield little evidence in support of the hybrid NKPC for the U.S., U.K., and Canada.

2. Background

A variety of pricing environments give rise to a hybrid NKPC that describes inflation, π_t:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda x_t, \tag{1}$$

where we use x_t to denote real aggregate demand (either real marginal cost or an output gap). The studies by Rotemberg (1982), Roberts (1997), Fuhrer and Moore (1995), and Galí and Gertler (1999) contain examples of these environments. The underlying pricing behavior can range from smooth adjustment with quadratic costs to a variation of Calvo's contract model (with or without firm-specific capital) in which some price-setters are backward-looking. The hybrid NKPC (1) also may be consistent with the dynamic indexing model suggested by Woodford (2003), assuming it is written in the quasi-difference or change in inflation rather than the level.

An influential example of an environment underlying the NKPC is Calvo's pricing model. There the discount factor is β. A fraction θ of firms are allowed to change prices each period. In Galí and Gertler's hybrid version of the model, meanwhile, a fraction ω are able to change prices but choose not to. Define $\phi = \theta + \omega[1 - \theta(1 - \beta)]$. Then the mapping between these structural paramaters and the reduced-form parameters is:

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}. \tag{2}$$

This mapping is unique when the structural parameters are positive fractions.

It is often convenient to work with the general problem of identifying the parameters γ_f, γ_b, and λ. In this case the environment is linear, so there is no distinction between local and global identification. Throughout the paper we also assume (with one exception) that the roots of relevant difference equations imply stability and uniqueness of solutions, and that the difference equation (1) follows from a pricing model – in which all three parameters are positive – and not an observationally equivalent environment, as in Beyer and Farmer (2004).

The hybrid NKPC (1) is a linear, second-order, stochastic difference equation. Our study draws on tools for formulating these problems under rational expectations developed by Hansen and Sargent (1980) and Sargent (1987). We also draw on studies of estimation in the linear-quadratic model by Gregory, Pagan, and Smith (1993), West and Wilcox (1994), and Fuhrer, Moore, and Schuh (1995).

GMM estimation of the hybrid NKPC (1) uses sample versions of:

$$E[\pi_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0, \tag{3}$$

and instruments z_t. Given moment conditions (3), a necessary condition for identification of $\{\gamma_b, \gamma_f, \lambda\}$ is that there are as many *valid* instruments as parameters (or variables that explain inflation in this linear model). A test based on over-identification requires at least four instruments or four such pieces of information. The instruments must be uncorrelated with the GMM residuals, which are essentially forecast errors. This is the *order* condition. Of course, being dated $t-1$ or earlier is not sufficient for an instrument to be valid: it must possess *incremental* information about π_{t+1}. The matrix of cross-products of the instruments and the right-hand-side variables in the hybrid NKPC cannot be singular. This is the *rank* or 'relevance' condition of IV estimation. We have omitted constant terms, as if the data have been demeaned. Of course, if in applications a constant term is included in the NKPC, a vector of ones can be used as an instrument while adding no net identifying information.

Identification obviously requires that one can predict π_{t+1} with at least one variable other than π_t, π_{t-1}, and x_t. This is a stringent requirement. Stock and Watson (1999) report that few variables have power to forecast U.S. inflation during the great disinflation of the 1980s and 1990s.

But the economic structure can be used to give an alternate perspective on the potential identification problem. The second-order difference equation (1) can be rewritten in present-value form using the methods of Sargent (1987):

$$\pi_t = \delta_1 \pi_{t-1} + \left(\frac{\lambda}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t x_{t+k}, \tag{4}$$

where δ_1 and δ_2 are the stable and unstable roots, respectively, of the characteristic equation:

$$-L^{-1} + \frac{1}{\gamma_f} - \frac{\gamma_b L}{\gamma_f} = 0.$$

We assume that $\{x_t\}$ is of exponential order less than δ_2 so that the infinite sum in (4) is finite, and that the roots yield a unique solution to the difference equation.

Leading the present-value version (4) forward and forecasting gives $E_t \pi_{t+1}$ as:

$$E_t \pi_{t+1} = \delta_1 \pi_t + \left(\frac{\lambda}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t x_{t+1+k}. \tag{5}$$

This restatement of $E_t \pi_{t+1}$ shows that identifying γ_f requires that there be variation in the forecast of the stream $\{x_{t+1}, x_{t+2}, x_{t+3}, ...\}$ that is unrelated to variation in x_t and π_{t-1}. In other words, higher-order dynamics are needed for identification.

Recall that when a present-value (5) is projected on current information the number of lags is one less than the lag length in the underlying forecasting model. Thus, for example, if x_t follows an autonomous, Jth-order autoregression, then $J \geq 2$ is necessary for identification and $J \geq 3$ is necessary for overidentification with GMM. Marginal cost x_t must be predictable with at least one variable other than x_{t-1} and π_{t-1}. In particular, if x_t follows a first-order Markov process then the parameters of the second-order difference equation in inflation (1) cannot be identified by GMM. Pesaran (1987, Propositions 6.1 and 6.2) derived similar results. He observed that identifying information is available when the lag length in the process for x_t is longer than that in the difference equation.

Including the lagged, endogenous variable in predicting x may partly capture the additional information used by price-setters in forecasting. Campbell and Shiller

(1987) and Boileau and Normandin (2002) develop this approach. Suppose then that the investigator predicts x with once-lagged or twice-lagged inflation in the hope of providing over-identification. Again, the lag length in the projection of the present value on current information is one less than the lag length in the forecasting equation. Recall that π_{t-1} is already in the estimating equation, so for π_{t-2} to be available as an instrument requires that x_t be predictable with π_{t-3}.

A number of researchers have used only lagged instruments in estimating (3). For example, Galí and Gertler (1999) used up to four lags of six instruments. Denote this instrument set by z_{t-1}. Using lagged instruments moves the forecasting platform back in time, but does not alter the result that higher-order dynamics or additional, valid variables other than x and π are needed for identification.

Ma (2002) was the first to raise the issue of the potential weak identification of the NKPC with GMM. He studied the U.S. Phillips curve using the S-sets derived by Stock and Wright (2000). Mavroeidis (2004) provides a good discussion of the econometrics of weak identification and traces it to the properties of exogenous variables. He suggested using the concentration parameter as a measure of weakness of identification, but did not provide formal tests. Mavroeidis (2005) shows by simulation that standard GMM tools may be unreliable when applied to the NKPC. Our main contributions are twofold: (a) we trace the potential weak identification of the NKPC to the *economic* properties of the underlying new Keynesian model and (b) we provide identification-robust tests of the forward-looking component of the NKPC for the U.S., U.K., and Canada for the reduced-form parameter γ_f and for the underlying parameters of the hybrid NKPC. Dufour, Khalaf, and Kichian (2006) simultaneously with this paper proposed and applied identification-robust tests based on Anderson and Rubin (1949) to U.S. NKPCs measured with an output gap. Perhaps the study most similar to ours is Cochrane (2006). He studies the identification of Taylor rules in new Keyenesian economic environments.

Our focus excludes other potential sources of identification, such as structural breaks, time-varying coefficients, stochastic volatility, or the use of survey data on inflation expectations. We also do not focus on the system estimator – where additional identifying information is available from cross-equation and covariance restrictions. Systems estimation has been studied by, among others, Fuhrer and Moore (1995), Sbordone (2002, 2005), Kurmann (2005), Lindé (2005), Jondeau and Le Bihan (2003), and Fuhrer and Olivei (2004).

3. GMM Approaches

The purpose of this section is to review several plausible suggestions for avoiding weak identification or non-identification and for transforming the moment condition or instruments while continuing to apply GMM , and to see whether they provide solutions. Several methods are practical, but they generally rely on specific, extra information or preclude testing the hybrid NKPC. All the paper's analytical results are contained in table 1.

First, in some circumstances, the investigator may know the value of λ, either from theory or from some auxiliary statistical work. For example, if $x_t \sim I(1)$ then x_t and π_t will be cointegrated with parameter λ, which could be estimated from a static regression, as originally proposed by Granger and Engle (1987). This common, stochastic trend restriction can potentially aid identification of the remaining parameters, γ_f and γ_b.

Result 1. If a consistent estimate $\hat{\lambda}$ is available, then an additional instrument is available in z_t but not in z_{t-1}.§

When λ is known or estimated from auxiliary information then x_t becomes a valid instrument for π_{t+1}; the instruments x_t and π_{t-1} can be used to identify γ_f and γ_b. But with instruments z_{t-1} three variables in (3) remain to be forecasted, $\{\pi_{t+1}, \pi_t, x_t\}$, even given an estimate $\hat{\lambda}$. Thus, a two-step procedure cannot identify the two other parameters unless other instruments are already available.

The last part of Result 1 is a generalization of an example found in Pagan, Gregory, and Smith (1993), who modelled x_t as a random walk. According to Pagan, Gregory, and Smith, lagged instruments could not identify the parameters of the difference equation without higher-order dynamics in the x-process. Result 1 also is relevant to price-setting rules that are written in terms of the level of prices, rather than the inflation rate, because the price level is more likely to be nonstationary yet cointegrated with the fundamental.

Result 2. Restricting $\gamma_b = 0$, or $\gamma_b = 1 - \gamma_f$, or calibrating a discount factor β in an underlying pricing model may aid identification but precludes testing of the hybrid NKPC. §

If the investigator imposes $\gamma_b = 0$, so that the NKPC is purely forward-looking then the variable π_{t-1} is now free to play the role of an instrument for π_{t+1}. Mavroeidis (2004, 2005) provides a detailed discussion of this case.

A number of authors suggest imposing the restriction $\gamma_f + \gamma_b = 1$. This restriction means that there is no long-run tradeoff between inflation and real activity in levels. Rudd and Whelan (2006) provide methods for testing this restricted model in systems estimation. Given this restriction, the NKPC becomes:

$$\Delta\pi_t = \tilde{\gamma}_f E_t \Delta\pi_{t+1} + \tilde{\lambda} x_t, \tag{6}$$

where $\tilde{\gamma}_f = (1 - \gamma_b)/\gamma_b$ and $\tilde{\lambda} = \lambda/\gamma_b$. Woodford (2003) shows that this revised Phillips curve (6) is implied by a staggered pricing mechanism in which firms cannot commit to a new price but instead set their price at date t by adding lagged, aggregate inflation to fully index their previous period's price. The rewriting (6) again shows that π_{t-1} again is now eligible as an instrument.

However, an odd implication of the second approach in result 2 is that is predicts that $\gamma_b > 0.5$. Notice that $\tilde{\gamma}_f$ acts as the discount factor in revised Phillips curve difference equation (6). One thus would normally expect $\tilde{\gamma}_f < 1$ which implies $\gamma_b > 0.5$. Woodford (2003) notes that several studies that impose $\gamma_f + \gamma_b = 1$ obtain estimates in which backward-looking dynamics dominate forward-looking ones.

The NKPC sometimes is viewed as stemming from an underlying Calvo pricing model, with three deep parameters: β, a discount factor, the fraction of firms able to change price, and the fraction able to change price that do not. Pre-setting β amounts to setting γ_f conditional on γ_b and λ, and so it may allow identification.

An important corollary of result 2, however, is that naturally none of the restrictions in result 2 can be tested if the two remaining parameters are just identified. For example, one cannot test the hybrid NKPC against the purely forward-looking one.

As an interesting way to provide evidence on the hybrid NKPC, Rudd and Whelan (2001), Galí, Gertler, and López-Salido (2005), and Guay, Luger, and Zhu (2002) solve the hybrid NKPC difference equation forward, but truncate after K leads. Rudd and Whelan (2005) motivate instrumenting the present discounted value of x instead of π_{t+1} by the possibility of specification error. This leads them to estimate by instrumental variables:

$$E_{t-1}(\pi_t - \delta_1 \pi_{t-1} - \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{K} {\delta_2}^{-k} x_{t+k}). \tag{7}$$

Result 3. Solving forward and truncating provides no additional information to aid identification (or improve efficiency).§

The difference equation – solved forward and truncated – still involves the three parameters $\{\gamma_f, \gamma_b, \lambda\}$. Were there valid instruments for each future x_{t+k} in (4), these parameters would be overidentified because (7) contains more variables than parameters when $K \geq 1$. Nonetheless, the number of relevant instruments is unchanged, so the conditions for identification are unchanged.

In GMM estimation it is sometimes useful to use lagged residuals as instruments. For example, they have helpful scaling properties, for they are of the same order of magnitude as the residuals that are being minimized in estimation. The next result shows that this device is unavailable for the NKPC.

Result 4. Whether z_t or z_{t-1} is adopted, the GMM residual is a MA(1) process, so any instrument set must exclude once-lagged GMM residuals. §

The GMM residual includes a two-step forecast error, which naturally follows a first-order moving average. Thus the residual is correlated with the lagged residual, which violates the orthogonality condition. However, this moving average can be accounted for in constructing the weighting matrix in GMM estimation.

In summary, several methods suggested to ward off weak identification are practical, but they generally rule out testing of the hybrid NKPC. Thus identification and testing requires higher-order dynamics or additional variables to predict inflation. A natural question concerns the economic interpretation of these features. The next section looks at their availability in a new Keynesian economic model.

4. Economic Sources of Weak Identification

Up to this point, we have discovered (or rediscovered) that identifying the hybrid NKPC depends on the multi-step predictability of the x-process. However, real marginal cost or the output gap is endogenous in a dynamic, stochastic, general-equilibrium model. We study identification in a more complete model in this section. It seems natural to work with a typical, new Keynesian, trinity (*i.e.*, three-equation) model (NKTM) consisting of an NKPC, a linearized, dynamic IS schedule, and a Taylor rule. Let y be the output gap and R be the central bank's discount rate (the nominal federal funds rate in the U.S.). The system is:

$$\begin{aligned} \pi_t &= \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda y_t + \epsilon_{\pi t} \\ y_t &= \beta_f E_t y_{t+1} + \beta_b y_{t-1} - \beta_R (R_t - E_t \pi_{t+1}) + \epsilon_{yt} \\ R_t &= \omega_\pi \pi_t + \omega_y y_t + \epsilon_{Rt}. \end{aligned} \tag{8}$$

Our interest is in estimating the hybrid NKPC by replacing $E_t\pi_{t+1}$. We derive the forecasting implications of the NKTM (8) to do this. Using the policy rule to replace the interest rate in the equations for inflation and the output gap gives:

$$\begin{aligned} \pi_t &= \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda y_t + \epsilon_{\pi t} \\ y_t &= \beta_f \varphi E_t y_{t+1} + \beta_R \varphi E_t \pi_{t+1} + \beta_b \varphi y_{t-1} - \beta_R \omega_\pi \varphi \pi_t + \varphi (\epsilon_{yt} - \beta_r \epsilon_{Rt}) \end{aligned} \tag{9}$$

where

$$\varphi \equiv (1 + \beta_R \omega_y)^{-1}. \tag{10}$$

Let us stack: $w_t = (\pi_t \;\; y_t)'$, which allows us to write the system (9) as:

$$w_t = cE_t w_{t+1} + dw_{t-1} + fw_t + \epsilon_t, \tag{11}$$

where the 2×2 matrices of the system of second-order difference equations (11) are:

$$c = \begin{pmatrix} \gamma_f & 0 \\ -\beta_R \varphi & \beta_f \varphi \end{pmatrix}, \qquad d = \begin{pmatrix} \gamma_b & 0 \\ 0 & \beta_b \varphi \end{pmatrix},$$

and f has zeros on the diagonal:

$$f = \begin{pmatrix} 0 & \lambda \\ -\beta_R \omega_\pi & 0 \end{pmatrix}.$$

The vector shock is given by: $\epsilon_t = (\epsilon_{\pi t} \;\; \varphi(\epsilon_{yt} - \beta_r \epsilon_{Rt}))'$, which implies that it is not possible to identify innovations to y_t separately from innovations to R_t. The bivariate system (11) can be written:

$$w_t = [I - f]^{-1} cE_t w_{t+1} + [I - f]^{-1} dw_{t-1} + [I - f]^{-1} \epsilon_t. \tag{12}$$

This system is in exactly the same form as our original hybrid NKPC (1), except that π and x have been replaced by w and ϵ. Thus, the persistence and covariance properties of the shock vector ϵ_t will be important, just as the x_t properties were important earlier. Given that elements of f are non-zero, so that current values appear in the system, we require that the elements of ϵ_t be uncorrelated with each other. However, the rescaled shocks $[I - f]^{-1}\epsilon_t$ will be cross-correlated.

We assume uniqueness and stability, and specifically that $\omega_\pi > 1$. This restriction on monetary policy satisfies the well-known Taylor principle. Under this restriction to aggressive policies, only fundamental shocks, ϵ_t, drive inflation and the output gap. The unique solution takes a first-order form:

$$w_t = aw_{t-1} + b\epsilon_t, \tag{13}$$

where a and b are 2×2 matrices. The solution (13) is the equilibrium vector process of the new Keynesian economy (8). Solving for a and b by guess-and-verify methods leads to a system of polynomials in the lag operator. Factoring a multivariate spectral density matrix usually requires numerical methods; a and b cannot be found analytically in general. For discussion and examples, see Hansen and Sargent (1981) and Sayed and Kailath (2001). Nonetheless, the form of the solution (13) tells us much about the necessary conditions for identification.

Result 5. In the new Keynesian, three-equation model, the hybrid NKPC cannot be identified by single-equation GMM.§

The result follows from the first-order Markov nature of w_t. With y_t and π_{t-1} already entering the hybrid NKPC, there are no further variables available to instrument for π_{t+1} in GMM estimation.

There will be higher-order dynamics in the *univariate* time series process for y_t implied by the NKTM. Marginalizing the VAR gives:

$$y_t = a_{22} y_{t-1} + a_{21} a_{12} \sum_{j=0}^{\infty} a_{11}^{j} y_{t-2-j}. \tag{14}$$

But there is no additional information in the lagged values of y beyond that contained in π_{t-1}. Thus, finding higher-order dynamics in y is necessary, but not sufficient for identification in GMM. Although the NKTM can produce higher-order output dynamics, as in (14), these do not yield relevant instruments. Lagged inflation already enters the hybrid NKPC. Result 5 implies that identifying the NKPC must rely on cross-equation restrictions in this system.

Persistent shocks are another potential source of of identifying information. Suppose the shock vector follows a Jth-order autoregression:

$$\epsilon_t [I - \xi(L)] = \vartheta_t, \tag{15}$$

where ϑ_t is a vector of innovations. Pass $[I - \xi(L)]$ through the first-order solution (13) and substitute using the VAR of (15) to produce:

$$w_t [I - aL][I - \xi(L)] = b\vartheta_t. \tag{16}$$

The system (16) entails a VAR$(J+1)$ in inflation and the output gap.

Result 6. One of the shocks to inflation, to the output gap, or to the interest rate must be persistent for the hybrid NKPC to be identified by GMM in the NKTM (8).§

Identifying the second-order difference equation in inflation in GMM requires at least second-order dynamics. A necessary condition for these dynamics to arise is that the intrinsic, first-order dynamics of the NKTM (8) be augmented with first-order dynamics in at least one shock. Given there are no zero elements in $[I - f]^{-1}$, all three shocks from the NKTM (8) affect π_t. Thus, persistence in at least one shock is sufficient for identification. Shock persistence also translates into serial correlation in inflation and the output gap. This finding may help to explain the long lags in estimated NKTM inflation and output gap equations reported, for example, by Lindé (2005) and Jondeau and Le Bihan (2003). Likewise, Ireland (2004) found that an autocorrelated cost shock in the NKPC was necessary for empirical success of the NKTM.

There is an analogous result when the NKTM (8) possesses multiple equilibria. Lubik and Schorfheide (2004) study a NKTM that associates the indeterminacy with passive monetary policy, $\omega_\pi < 1$, and sunspot (*i.e.* extrinsic) shocks. Under $\omega_\pi < 1$, they show that the rational expectations forecast of π_t and y_t is a first-order VAR with forecast innovations a function of the fundamental shocks ϵ_t and the rational expectation forecast errors, ϕ_t:

$$[I - \tau_w L]E_t w_{t+1} = \tau_\vartheta \vartheta_t + \tau_\phi \phi_t, \tag{17}$$

where $\phi_{t+1} = [y_{t+1} - E_t y_{t+1} \quad \pi_{t+1} - E_t \pi_{t+1}]'$ and the τ matrices are functions of the parameters of the NKTM. Given the linear NKTM (8), this class of passive monetary policies also permits ϕ_t to be a linear function of ϵ_t and a vector of sunspot shocks, ψ_t. It follows from these facts – $E_t w_{t+1}$ is the VAR(1) of (17) and ϕ_t depends on ψ_t, besides fundamental shocks – that w_t becomes a (restricted) bivariate ARMA process rather than a pure bivariate autoregression:

$$[I - \mu L]w_t = \kappa_\vartheta[I - \mu\theta_\vartheta L]\vartheta_t + \kappa_\psi[I - \mu\theta_\psi L]\psi_t, \tag{18}$$

where μ denotes the stable eigenvalue of (17) and the κ and θ matrices are functions of the NKTM parameters. Note that the first-order moving average of the bivariate

ARMA process (18) are functions of the fundamental and sunspot shocks. The econometrician focuses on the sunspot to connect the observed data to one of the multiple equilibria. This motivates Lubik and Schorfheide to argue that the sunspot shock interpretation of indeterminacy (created by $\omega_\pi < 1$) explains serially correlated inflation and output gap data.

Result 7. When the new Keynesian, three-equation model (8) possesses multiple equilibria and the rational expectations forecast errors are a (linear) function of the fundamental and extrinsic shocks, the GMM estimator of the hybrid NKPC is not identified.§

The key to Result 7 is that the lack of restrictions on the rational expectations forecast errors under indeterminacy provides no additional identification information. Although fundamental and sunspot shocks are news for an econometrician attempting to estimate the NKTM (8), these shocks do not help forecast π_{t+1}. However, this approach to identifying the NKPC within a larger model imposes persistence and cross-equation restrictions on the forecast innovation of the bivariate ARMA process (18) of y_t and π_t, which can yield additional information for identification in the system.

The NKTM is a monetary model, in which the central bank's policy tool is its discount rate, R_t. Although our analysis of the NKPC with the NKTM uses the Taylor rule to substitute for the discount rate in the dynamic IS schedule, it seems reasonable to use R_t as an instrument.

Result 8. With the Taylor rule in the NKTM (8), the current, nominal, policy interest rate, R_t is not a valid instrument in the NKPC.§

The nominal interest rate is a natural predictor of π_{t+1} and so might seem to be a natural instrument. It is invalid because under the Taylor rule R_t is set as a proportion ω_π of the current inflation rate π_t which in turn is the dependent variable in the hybrid NKPC. The correlation between R_t and $\epsilon_{\pi t}$ violates the order condition.

Result 9. Lagged policy interest rates are valid but inefficient instruments in the NKTM.§

Recall that the solution (13) describes the optimal forecast of π_{t+1} in the NKTM based on lags of inflation and the output gap. Meanwhile, inspection of the lagged Taylor rule shows that the nominal interest rate contains information on the lagged output gap and inflation but (a) with an error ϵ_R and (b) with Taylor-rule coefficients on the lagged values of inflation and the output gap that will not correspond to the elements of the optimal coefficient matrix a given in the solution (13).

Result 10. Interest-rate smoothing in monetary policy may provide an alternate source of identification.§

There is much debate about whether the discount rate can be partly explained by its own lag due to persistent shocks or to an interest rate smoothing policy. Suppose the policy rule is:

$$R_t = (1 - \upsilon)(\omega_\pi \pi_t + \omega_y y_t) + \upsilon R_{t-1} + \epsilon_{Rt} \tag{19}$$

with $0 < \upsilon < 1$. The current interest rate thus reflects information on the entire history of inflation, the output gap, and policy shocks ϵ_{Rt}. The output gap inherits this memory because R_t enters the equation for the output gap in the NKTM (8). Thus, additional instruments become available in the same way that Result 6 adds them using shock persistence.

This section has focused on the bivariate VAR in $\{w_t\}$ implied by the NKTM (8) because of our interest in instrumenting π_{t+1} in the hybrid NKPC. Thus, we have not studied the complete reduced form, or addressed the identification of other NKTM parameters. The main result of this section is that a persistent shock or an interest rate smoothing policy is necessary for the hybrid NKPC to be identified by GMM within the NKTM. Table 1 summarizes the results of this section.

5. Revisiting the Evidence: Robust Tests

The theory in section 4 suggests that candidate instruments may not be available for GMM estimation of the NKPC, unless there are autocorrelated shocks or interest-rate smoothing. Identifying the NKPC under GMM may be challenging. We thus turn to tests of the NKPC that are valid even under weak identification. We draw inference without using a system by using simple statistics from Anderson and Rubin (1949) plus some recent developments. We next apply our results to the estimation and testing of hybrid NKPCs for the U.S., U.K., and Canada. The data consists of GDP inflation and measures of real marginal cost. The appendix describes the data sources.

5.1 Statistics

First, we study the time-series properties of x_t. We estimate univariate autoregressions for x_t, and test the lag length from $J = 1$ to $J = 6$ lags using a likelihood ratio statistic, the AIC, and the SIC. Recall from section 2 that – if there are no instruments other than lags of x – then $J \geq 2$ is necessary for identification in GMM. We next include lagged values of inflation and report the results of a pre-test of the null hypothesis that $\{\pi_t\}$ does not Granger-cause $\{x_t\}$. Section 2 also noted that finding a role for lagged inflation in forecasting x suggests that further instruments may be available. These could include lags of inflation beyond the first two or other variables that lead to Granger-causality because of the superior information of price-setters.

Second, our main interest is in instrumental-variables estimation, so we estimate:

$$E[\pi_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0 \tag{20}$$

by GMM and report point estimates and standard errors as well as the J-test statistic of over-identifying restrictions and its p-value. Following Result 4, GMM estimators will allow for a first-order moving average in the GMM residual. The weighting matrix will be the continuous-updating version introduced by Hansen, Heaton, and Yaron (1996), which has good finite-sample properties and is invariant to the normalization of the hybrid NKPC (1). Of course, these standard estimates and inferences may be suspect due to weak identification but the idea is to show how conclusions may differ between these methods and those that are robust to weak identification.

Third, we calculate Anderson-Rubin (1949) statistics to test several hypotheses, and find the implied confidence intervals. Excellent surveys of inference under weak identification are provided by Dufour (2003) and Andrews and Stock (2005). For the just-identified case, the Anderson-Rubin test is preferred, according to these authors.

The statistics from GMM estimation (20) depend on nuisance parameters under weak identification. In contrast, the AR statistics are pivotal in finite samples. To test $H_0: \ \gamma_f = \gamma_{f0}$ one projects as follows:

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t, \qquad (21)$$

with auxiliary variables u_t, then constructs the Anderson-Rubin (AR) F-statistic for $H_0': \ \alpha_3 = 0$. The idea is that there should be no further role for u_t at the true value for γ_f. In our case, γ_f is a scalar. This yields a $F(k+2, T-k)$ statistic, where $k+2$ is the total number of exogenous variables and instruments. The Anderson-Rubin (AR) statistic provides an exact test, which is robust to (a) weak instruments and (b) omitted instruments. We do not need all the u-elements necessarily, but power is lower if irrelevant instruments are included. The test statistic also is robust to misspecification of the forecasting rule for π_{t+1} (*i.e.*, its size is not affected, though again its power may be).

The distributional assumption underlying the statistic's being pivotal in finite samples is normality of the GMM residuals. In the literature, the main drawbacks to this approach arise when the structural equation is non-linear, or when there is more than one endogenous, explanatory variable and we want to study subsets of their coefficients. But here the hybrid NKPC is linear, and γ_f is a scalar.

The AR statistics also can be used to construct confidence intervals. A confidence set is:

$$C(\alpha) = \{\gamma_{f0} : AR(\gamma_{f0}) \leq F_\alpha(k, T-k-2)\}. \qquad (22)$$

Since γ_f is a scalar, there is a quadratic solution, given by Zivot, Startz, and Nelson (1998). The coefficients of the quadratic equation are functions of the data and the $F-$statistic at significance level α and degrees of freedom $\dim(u)$ and $T-2-k$. With over-identification this confidence set can be empty. Without identification, it can be unbounded. More generally, Andrews and Stock (2005, p 18) describe how this confidence interval may be conservative (too wide) if there are other endogenous right-hand-side variables.

Fourth, one criticism of the Anderson-Rubin tests is that they lack power when there is overidentification. We find these tests reject for the U.S. and U.K. data. Thus, a lack of power is not a a central issue in this study. A variety of approaches have been proposed recently to correct this potential shortfall. We apply the LM test of Guggenberger and Smith (2006), which is robust to weak instruments and suffers no power loss as over-identification rises. Guggenberger and Smith (GS) also present Monte Carlo work in which their LM statistic has good sampling properties. For example, its power properties are comparable to the test of Kleibergen (2005), and sometimes superior when there are many instruments. Thus our evidence complements Dufour, Khalaf, and Kichian (2006) who apply Kleibergen's test to the U.S. NKPC.

A further advantage of the GS LM statistic is that it allows tests of parameters from nonlinear moment condition models. Recall that the NKPC interprets the parameters $\{\gamma_f, \gamma_b, \lambda\}$ as arising from an underlying structure with parameters $\{\beta, \theta, \omega\}$ given the mapping (2). We employ the GS LM statistic to test hypotheses on several values of θ and ω. In this approach we assume that parameters not under test are strongly identified. In test statistics, these parameters are replaced by consistent estimates. For methods to test subsets of parameters under weak identification, also see Dufour and Tamouti (2003), Guggenberger and Smith (2006, section 2.3), and section 7.8 of Andrews and Stock (2005).

5.2 United States

Table 2 presents evidence on real marginal cost dynamics for a U.S. sample of $1949Q1 - 2001Q4$. We fail to reject the null that inflation does not Granger-cause real marginal cost according to the first two rows of table 2. In addition, the AIC and LR statistic choose a lag length of 3, while the SIC selects a lag length of 1 for the x-autoregression. It is not surprising then that the second-order coefficient in this regression is insignificantly different from zero. The implication of these pre-tests is that finding relevant instruments may be a challenge in the U.S. data. Although U.S. real marginal cost is persistent (the half life of a shock is about seven quarters), there is not strong evidence of higher-order dynamics in U.S. real marginal cost. Campbell and Shiller (1987) and Boileau and Normandin (2002) also show that the presence of other predictors of x_t should lead to a role for lagged inflation, yet we find none here. Thus, the quest for other instruments may not be fruitful.

Table 3 contains single-equation GMM estimates. Most of the work is done by the instruments $\{\pi_{t-1}, x_t, x_{t-2}\}$, as is suggested by the pre-test evidence that only x_t and x_{t-2} help forecast x_{t+1}. Adding further instruments increases the precision slightly but does not lead to significant changes in the estimates. The J-test clearly does not reject the over-identifying restrictions.

The estimated weight attributed to backward-looking inflationary expectations, $\hat{\gamma}_b$, ranges from 0.28 to 0.42, depending on the instrument set. The GMM estimates show these expectations are dominated by forward-looking expectations because $\hat{\gamma}_f$ ranges from 0.52 to 0.70. The response of π_t to x_t, denoted $\hat{\lambda}$, also takes plausible values, between 0.1 and 0.9 percent, but is not statistically significant (for a five percent test). Our results are comparable to those of Galí and Gertler (1999, table 2), but we obtain smaller and insignificant estimates of λ using smaller instrument sets.

Table 3 includes estimates of $\{\theta, \omega, \beta\}$ on the U.S. data. The estimates imply that about 90 percent of firms have the opportunity to change price each quarter and of those 30-55 percent choose not to do so. We also find reasonable estimates for the discount factor β.

Next, we provide tests for the forward-looking component in the U.S. NKPC that are robust to weak identification. Table 4 presents AR F-statistics and their associated p-values based on equation (21) and a grid of potentially 'true' $\gamma_f = \gamma_{f0}$. We set γ_{f0} to [0.0, 0.2, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99]. The AR statistics in the first row reveal little evidence against the null of $\gamma_f = \gamma_{f0}$, for any of these values of γ_{f0} given $u_t = x_{t-2}$. When we add instruments though – in the next two rows – we can reject any of the null hypotheses at standard significance levels. Thus, lags of real marginal cost besides x_{t-2} matter for predicting the quasi-difference of π_t and π_{t+1}. The test is correctly sized even if these added instruments are weak, which gives us a formal rejection of the forward looking model.

The asymptotic 95 percent confidence interval $C(\alpha = 0.05)$ of γ_f, given in (22), reflects the evidence of table 4. The solution yields $C(0.05) = \{-6.75, 0.05\}$, $\{0.60, 0.84\}$, and $\{-0.10, 1.24\}$ for $u_t = x_{t-2}$, $\{x_{t-1}, x_{t-2}\}$, and $\{x_{t-1}, \ldots, x_{t-4}\}$, respectively. The smallest (just-identified) and largest (overidentified) instrument sets yield asymptotic 95 percent confidence intervals that contain zero. Only the information vector with the first two lags of x produces a confidence interval with reasonable values of γ_f.

These wide confidence intervals yield the same, negative conclusion as do the large S-sets found by Ma (2002). It is interesting to note the contrast with some of the findings of Dufour, Khalaf, and Kichian (2006) who use a real-time output gap measure in the U.S. NKPC and cannot reject a significant, forward-looking component in U.S. inflation using robust methods. It remains an open question how sensitive test results are to the choice between using marginal cost or the output gap in the NKPC or, indeed, to different ways of measuring these variables.

Table 5 presents the GS test statistics for various values of θ and, separately, of ω. The grids for these underlying parameters are suggested by the GMM estimates in table 3. Across a grid of values, the test rejects each value of θ with the exception of $\theta = 0.9$ on the just-identified instrument set, which yields an estimate of β greater than one. When further instruments are added, this value too is rejected. With larger instrument sets, the test also rejects each value of ω. For those ω that do not reject the GS test, the estimates of θ are between 0.93 and 1.03. Joint tests (not shown but available on request) lead to similar conclusions.

5.3 United Kingdom

The estimation sample for the U.K. is $1961Q1 - 2000Q4$. Table 2 shows that the Granger causality pre-test provides strong evidence of predictability in both directions. This result implies that lagged values of inflation (beyond the first two lags) may be available as instruments. The second set of pre-tests indicate a lag length in the x-autoregression of $J = 5$ using the LR test and SIC. This places more of the history of x in the instrument vector z_t than in the U.S. case.

Table 6 contains estimates of the U.K. hybrid NKPC. The GMM estimates depend on instrument choice. Once lags up to x_{t-4} are included, the coefficients accord with theory and are estimated with some precision. However, the over-identifying restrictions are rejected when x_t is an instrument. When x_t is not an instrument, the estimates of γ_f, γ_b, and λ are significant at the ten percent level or better. Neiss and Nelson (2002) obtain statistically significant estimates of λ, but use dummy variables to control for a variety of price shocks. Table 6 also shows that the Calvo parameters are quite sensitive to the instrument set, a potential sign of weak identification.

In contrast with the standard methods in table 6, however, table 7 gives evidence against the null of $\gamma_f = \gamma_{f0}$ for the U.K. hybrid NKPC. The significance levels of the AR statistics average 0.03 in table 7, for the projection (21), on the same grid of values of γ_{f0} used for table 4. Only two of the 16 AR statistics have p-values that exceed ten percent, which are associated with the instrument x_{t-1}, and γ_{f0} near unity.

The AR 95 percent asymptotic confidence interval (22) of γ_f is $C(0.05) = \{-0.87, -0.01\}$ when $u_t = x_{t-1}$, and $C(0.05) = \{0.02, 0.97\}$ when $u_t = \{x_{t-1}, ..., x_{t-4}\}$. Thus the confidence interval of γ_f has the wrong sign with the smaller information set. The confidence interval takes the correct sign using the larger information set and matches values set for γ_{f0} in table 7. Nonetheless, with equal probability γ_f runs from economically meaningless values to values that reveal an important role for forward-looking inflationary expectations.

Table 8 presents the GS test statistics. The results are even more negative than in the U.S. case shown in table 5. The test rejects each value of θ or ω that we investigate. Overall, then, the methods that are robust to weak identification provide a different impression of the U.K. NKPC than do standard methods.

5.4 Canada

Estimating and testing the Canadian NKPC uses data from $1963Q1$ to $2000Q4$. Table 2 shows that Canadian inflation Granger-causes real marginal cost. This table also shows real marginal cost fails to Granger-cause inflation – in contrast to results for the U.K. and U.S. data. The pre-tests for lag length reveal a persistence pattern similar to that in U.S. real marginal cost, according to the LR test, the AIC, and the SIC. In the time series for $\{x_t\}$, once-lagged costs play a large predictive role and thrice-lagged costs play an additional role, that is statistically significant. However, a half-life of 8.5 quarters with respect to a shock to its third-order, autoregressive process shows that Canadian real marginal cost is more persistent than it is in the U.K. and the U.S. data.

Table 9 contains estimates of the hybrid NKPC parameters γ_f, γ_b, and λ for Canada. They suggest that the hybrid NKPC is poorly identified. For example, the point estimates $\hat{\gamma}_f$ and $\hat{\gamma}_b$ are sensitive to the instrument set. When we include π_{t-2} as an instrument, these two coefficients are similar to those found in the U.S. data, with a large role for future inflation. Guay, Luger, and Zhu (2003) estimate the hybrid NKPC using a wider range of instruments. They increase precision and reject the over-identifying restrictions. However, we reproduce their finding that $\hat{\lambda}$ is insignificant. This indicates little role for real marginal cost in Canadian inflation dynamics. Table 9 also shows that either we cannot find an economically plausible value for the discount factor β or we obtain a wide 95 percent asymptotic confidence interval for β that runs from 0.71 to 1.24 (using the largest instrument vector).

Table 10 includes AR statistics that favor forward-looking inflation dynamics for Canada, which is the opposite found in the U.S. and the U.K. data. None of the hypothesized values of γ_f can be rejected at the five percent level in table 10. Thus, the AR 95 percent asymptotic confidence intervals (22) for γ_f, $C(0.05)$, should cover plausible values. For instrument vectors $u_t = x_{t-2}$, $\{x_{t-1}, x_{t-2}\}$, and $\{x_{t-1}, \ldots, x_{t-4}\}$, $C(0.05) = \{-0.00,\ 0.97\}$, $\{-0.00,\ 0.78\}$, and $\{-0.00,\ 0.97\}$, respectively.

The non-rejection in table 10 may reflect a lack of test power. Table 11 presents the GS test statistics for Canada. Recall that this test has greater power in over-identified cases. With relatively small instrument sets, there is some support for the selected values of θ or ω, but at these values the estimates of γ_f are less than 0.1. But with larger instrument sets, all the values of θ and ω we condider are rejected.

Thus, the GS LM tests limit support for the hybrid NKPC in Canadian data.

6. Conclusion

This paper is about identification problems in the hybrid new-Keynesian Phillips curve (NKPC). We show that estimation of the hybrid NKPC faces a fundamental source of non-identification: weak, higher-order dynamics. By setting the hybrid NKPC in a new Keynesian, three-equation model, we find the hybrid NKPC cannot be identified by GMM. In this setting, the current nominal interest rate also is ineligible as an instrument, as long as a Taylor rule applies. One solution to the identification problem is to posit persistent shocks either to real aggregate demand, inflation, or monetary policy, as is often implicitly done in the literature. Table 1 collects and summarizes our analytical results.

We draw on the Anderson-Rubin statistic to provide a new set of tests of the forward-looking inflation model. These test statistics are exact, pivotal, and robust to either weak or omitted instruments. The tests reveal little evidence of forward-looking expectations driving U.S. or U.K. inflation. When we add to test power by using the tests derived by Guggenberger and Smith (2005), we also reject the model of inflation for Canada. It is noteworthy that in all three cases the conventional J-test statistic does not reject the over-identifying restrictions.

Rejecting a necessary condition is sufficient for rejecting a model. As usual, then, an advantage of the limited-information approach is that it may alert us to an empirical difficulty without requiring a complete model. But whether the test rejections here are due to misspecification of the economic model of inflation, or to measurement problems, or to the assumptions of rational expectations is worthy of further exploration.

Appendix: Data Sources

United States

The price level P_t is the GDP implicit price deflator. The GDP deflator is available in chain weight form and in implicit form (all the U.S. results are based on the implicit GDP deflator).

Nominal unit labor cost (ULC) is the ratio of the index of hourly compensation in the non-farm business sector, labelled COMPNFB, to output per hour of all persons in the non-farm business sector, labelled OPHNFB. COMPNFB is an index of the nominal wage. OPHNFB is an index of the average product of labor. These can be found in the Federal Reserve Bank of St. Louis' FRED databank. Thus, ULC is a measure of labor's share.

Real ULC equals nominal ULC deflated by P_t. Inflation is $100 \ln(P_t/P_{t-1})$ and real ULC is $100(1 + a)\ln(\text{COMPNFB}_t/\text{OPHNFB}_t) - 100 \ln P_t$, where a is a function of the steady-state markup and labor's share parameter in the firm's production function. This adjustment renders real ULC stationary and $a = 1.08$.

The entire sample runs from $1947Q1$ to $2002Q4$, but the estimation sample period is $1949Q1 - 2002Q4$, $T = 212$.

United Kingdom

The inflation rate is measured with the GDP deflator, and x is a measure of the log of real marginal cost. Data sources are given by Katharine Neiss and Edward Nelson (2002), who kindly provided the data. The estimation period is $1961Q1$ to $2000Q4$, so $T = 168$.

Canada

The inflation rate is measured with the GDP deflator, while x is the log of the labour share in the non-farm, business sector. Data sources are given by Guay, Luger, and Zhu (2003), who kindly provided the data. The estimation period is $1963Q1$ to $2000Q4$.

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Table 1
Summary of New Keynesian Phillips Curve Identification Results

Result 1. If a consistent estimate $\hat{\lambda}$ is available, then an additional instrument is available in z_t but not in z_{t-1}.

Result 2. Restricting $\gamma_b = 0$, or $\gamma_b = 1 - \gamma_f$, or calibrating a discount factor β in an underlying pricing model may aid identification but precludes testing of the hybrid NKPC.

Result 3. Solving forward and truncating provides no additional information to aid identification (or improve efficiency).

Result 4. Whether z_t or z_{t-1} is adopted, the GMM residual is a MA(1) process, so any instrument set must exclude once-lagged GMM residuals.

Result 5. In the NKTM, the hybrid NKPC cannot be identified by single-equation GMM.

Result 6. Either the shock to inflation, the output gap, or the interest rate must be persistent for the NKPC to be identified by GMM in the NKTM.

Result 7. When the NKTM possesses multiple equilibria and the rational expectations forecast errors are a (linear) function of the fundamental and extrinsic shocks, the GMM estimator of the hybird-NKPC is not identified.

Result 8. With the Taylor rule in the NKTM, the current nominal interest rate, R_t, is not a valid instrument in the NKPC.

Result 9. Lagged policy interest rates are valid but inefficient instruments under the NKTM.

Result 10. Interest-rate smoothing in monetary policy may provide an alternate source of identification.

Table 2
Granger Non-Causality Tests

Country	Lag length (d.f.)	$p \quad \pi \not\rightarrow x$	$p \quad x \not\rightarrow \pi$
U.S.	3	0.18	0.05
U.S.	4	0.24	0.08
U.K.	4	0.01	0.00
U.K.	5	0.01	0.00
Canada	3	0.00	0.73
Canada	4	0.00	0.63

Notes: The lag lengths, $\hat{J}$, are the same as those selected by information criteria. Entries are p-values for the null hypothesis that the first variable does not Granger cause the second variable. Data sources and sample sizes are given in the data appendix.

Table 3
U.S. New Keynesian Phillips Curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0$$

1949Q1 - 2002Q4 $T = 212$

Instruments	$\hat{\gamma}_f$ (se)	$\hat{\gamma}_b$ (se)	$\hat{\lambda}$ (se)	$\hat{\omega}$ (se)	$\hat{\theta}$ (se)	$\hat{\beta}$ (se)	$\chi^2(df)$ (p)
π_{t-1}, x_t, x_{t-2}	0.685 (0.357)	0.300 (0.247)	0.001 (0.007)	0.406 (0.465)	0.961 (0.163)	0.965 (0.306)	—
$\pi_{t-1}, x_t, ...x_{t-2}$	0.527 (0.298)	0.415 (0.205)	0.009 (0.005)	0.566 (0.321)	0.902 (0.094)	0.797 (0.480)	2.11(1) (0.34)
$\pi_{t-1}, x_t, ...x_{t-4}$	0.706 (0.223)	0.275 (0.158)	0.008 (0.006)	0.333 (0.263)	0.892 (0.064)	0.961 (0.169)	3.47(3) (0.48)
$\pi_{t-1}, \pi_{t-2}, x_t, ..., x_{t-4}$	0.701 (0.188)	0.278 (0.141)	0.009 (0.005)	0.338 (0.234)	0.893 (0.054)	0.956 (0.135)	3.48(4) (0.63)

Note: The estimation sample runs from 1949Q1 to 2002Q4, based on the complete 1947Q1−2001Q2 sample. Tests of the over-identifying restrictions use the J-statistic.

Table 4
U.S. NKPC: Tests of $H_0: \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

1949Q1 - 2002Q4 $\qquad T = 212$

γ_{f0} =	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
u_t =								
x_{t-2}	2.15 (0.14)	1.31 (0.25)	0.21 (0.65)	0.04 (0.83)	0.00 (0.97)	0.07 (0.80)	0.21 (0.64)	0.39 (0.53)
x_{t-1}, x_{t-2}	4.43 (0.01)	5.17 (0.01)	5.85 (0.00)	5.83 (0.00)	5.68 (0.00)	5.45 (0.00)	5.17 (0.01)	4.90 (0.01)
$x_{t-1}, ..., x_{t-4}$	2.47 (0.05)	2.92 (0.02)	3.34 (0.01)	3.33 (0.01)	3.24 (0.01)	3.10 (0.02)	2.93 (0.02)	2.77 (0.03)

Notes: The Anderson-Rubin statistics are based on equation (21). Dufour (2003) contains details of the Anderson-Rubin statistic and test.

Table 5
U.S. NKPC: Tests of $H_0: \ \theta = \theta_0$ or $H_0: \ \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(1)$

1949Q1 – 2002Q4 $T = 212$

$\theta_0 \ =$	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
$z_t \ =$							
$\{\pi_{t-1}, x_t, x_{t-2}\}$	41.32 (0.00)	68.00 (0.00)	43.30 (0.00)	29.88 (0.00)	22.44 (0.00)	11.29 (0.00)	2.36 (0.12)
$\{\pi_{t-1}, x_t, ..., x_{t-2}\}$	52.72 (0.00)	51.46 (0.00)	47.71 (0.00)	35.06 (0.00)	30.21 (0.00)	26.86 (0.00)	17.27 (0.00)
$\{\pi_{t-1}, \pi_{t-2}, x_t, ..., x_{t-2}\}$	50.19 (0.00)	46.59 (0.00)	30.57 (0.00)	32.00 (0.00)	23.31 (0.00)	15.17 (0.00)	18.60 (0.00)
$\omega_0 \ =$	0.075 (p)	0.15 (p)	0.25 (p)	0.35 (p)	0.45 (p)	0.50 (p)	0.60 (p)
$\{\pi_{t-1}, x_t, x_{t-2}\}$	6.96 (0.01)	5.01 (0.03)	1.18 (0.28)	0.26 (0.61)	0.01 (0.90)	0.16 (0.69)	1.34 (0.25)
$\{\pi_{t-1}, x_t, ..., x_{t-2}\}$	24.80 (0.00)	35.10 (0.00)	15.71 (0.00)	30.27 (0.00)	28.84 (0.00)	28.04 (0.00)	28.87 (0.00)
$\{\pi_{t-1}, \pi_{t-2}, x_t, ..., x_{t-2}\}$	30.18 (0.00)	26.93 (0.00)	19.94 (0.00)	23.69 (0.00)	23.83 (0.00)	19.41 (0.00)	18.20 (0.00)

Notes: The Guggenberger-Smith *LM* statistic tests the null that θ=θ_0 or ω=ω_0. See Guggenberger and Smith (2006) for details.

Table 6
U.K. New Keynesian Phillips Curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t]$$

$1961Q1$ – $2000Q4$ $\quad T = 168$

Instruments	$\hat{\gamma}_f$ (se)	$\hat{\gamma}_b$ (se)	$\hat{\lambda}$ (se)	$\hat{\omega}$ (se)	$\hat{\theta}$ (se)	$\hat{\beta}$ (se)	$\chi^2(df)$ (p)
π_{t-1}, x_t, x_{t-1}	-2.699 (4.782)	2.396 (3.047)	0.924 (1.531)	0.702 (0.113)	0.492 (0.337)	-1.608 (1.412)	—
$\pi_{t-1}, x_{t-1}, ..., x_{t-4}$	0.935 (0.266)	0.019 (0.192)	0.334 (0.152)	0.011 (0.114)	0.570 (0.065)	0.953 (0.113)	4.40(2) (0.22)
$\pi_{t-1}, x_t, ..., x_{t-4}$	0.234 (0.200)	0.535 (0.120)	0.062 (0.133)	0.562 (0.139)	0.801 (0.303)	0.307 (0.287)	9.82(3) (0.04)
$\pi_{t-1}, \pi_{t-2}, x_t, ..., x_{t-4}$	0.233 (0.153)	0.621 (0.107)	-0.045 (0.089)	0.842 (0.224)	1.569 (2.120)	0.201 (0.242)	15.94(4) (0.01)

Notes: The estimation sample runs from $1961Q1$ to $2000Q4$, based on the complete $1959Q3-2001Q2$ sample. Otherwise, see the notes to table 3.

Table 7
U.K. NKPC: Tests of $H_0: \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

1961Q1 - 2000Q4 $T = 168$

γ_{f0} =	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
u_t =								
x_{t-1}	6.84 (0.01)	6.53 (0.01)	5.00 (0.03)	4.32 (0.04)	3.63 (0.06)	2.98 (0.09)	2.40 (0.12)	1.94 (0.17)
$x_{t-1}, ..., x_{t-4}$	4.52 (0.00)	4.58 (0.00)	4.53 (0.00)	4.47 (0.00)	4.40 (0.00)	4.32 (0.00)	4.24 (0.00)	4.18 (0.00)

Notes: See the notes to tables 4 and 6.

Table 8
U.K. NKPC: Tests of $H_0: \ \theta = \theta_0$ or $H_0: \ \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(1)$

$1961Q1$ - $2000Q4$ $\qquad T = 168$

θ_0 =	0.40 (p)	0.50 (p)	0.55 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)
z_t =							
$\{\pi_{t-1}, x_t, x_{t-1}\}$	24.12 (0.00)	20.41 (0.00)	14.58 (0.00)	11.90 (0.00)	9.82 (0.00)	9.75 (0.00)	9.51 (0.00)
$\{\pi_{t-1}, x_t, ..., x_{t-4}\}$	30.83 (0.00)	28.33 (0.00)	28.37 (0.00)	27.66 (0.00)	26.98 (0.00)	22.14 (0.00)	19.43 (0.00)
$\{\pi_{t-1}, x_{t-1}, ..., x_{t-4}\}$	25.24 (0.00)	21.89 (0.00)	19.34 (0.00)	15.87 (0.00)	14.16 (0.00)	13.29 (0.00)	13.24 (0.00)
ω_0 =	0.05 (p)	0.25 (p)	0.40 (p)	0.50 (p)	0.55 (p)	0.65 (p)	0.70 (p)
$\{\pi_{t-1}, x_t, x_{t-1}\}$	9.33 (0.00)	8.77 (0.00)	9.51 (0.00)	9.61 (0.00)	9.35 (0.00)	9.02 (0.00)	9.08 (0.00)
$\{\pi_{t-1}, x_t, ..., x_{t-4}\}$	39.73 (0.00)	42.22 (0.00)	36.12 (0.00)	26.15 (0.00)	39.04 (0.00)	37.12 (0.00)	38.75 (0.00)
$\{\pi_{t-1}, x_{t-1}, ..., x_{t-4}\}$	18.44 (0.00)	19.99 (0.00)	17.25 (0.00)	34.97 (0.00)	36.07 (0.00)	34.21 (0.00)	37.81 (0.00)

Notes: See the notes to tables 5 and 6.

Table 9
Canadian New Keynesian Phillips Curve

$$E[\pi_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} - \lambda x_t | z_t] = 0$$

$1963Q1$ – $2000Q4$ $\qquad T = 152$

Instruments	$\hat{\gamma}_f$ (se)	$\hat{\gamma}_b$ (se)	$\hat{\lambda}$ (se)	$\hat{\omega}$ (se)	$\hat{\theta}$ (se)	$\hat{\beta}$ (se)	$\chi^2(df)$ (p)
π_{t-1}, x_t, x_{t-2}	-0.197	0.868	0.039	0.736	0.892	-0.188	—
	(2.085)	(1.374)	(0.074)	(0.186)	(0.040)	(1.726)	
$\pi_{t-1}, x_t, ..., x_{t-2}$	0.277	0.562	0.021	0.663	0.891	0.366	0.29(1)
	(0.768)	(0.514)	(0.027)	(0.287)	(0.033	(1.201)	(0.86)
$\pi_{t-1}, x_{t-1}, ..., x_{t-4}$	-1.052	1.466	0.061	0.785	0.902	-0.625	1.25(3)
	(1.274)	(0.876)	(0.049)	(0.061)	(0.039)	(0.405)	(0.87)
$\pi_{t-1}, \pi_{t-2}, x_{t-1}, ..., x_{t-4}$	0.716	0.274	0.005	0.341	0.911	0.979	2.48(4)
	(0.167)	(0.121)	(0.009)	(0.191)	(0.053)	(0.133)	(0.78)

Notes: The estimation sample is $1963Q1-2000Q4$ with leads and lags taken from a $1961Q1-2001Q1$ sample. Otherwise, see the notes to table 3.

Table 10
Canadian NKPC: Tests of $H_0: \ \gamma_f = \gamma_{f0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

1963Q1 - 2000Q4 $T = 152$

γ_{f0} =	0.00 (p)	0.20 (p)	0.50 (p)	0.60 (p)	0.70 (p)	0.80 (p)	0.90 (p)	0.99 (p)
u_t =								
$\{x_{t-2}\}$	0.01 (0.91)	0.07 (0.80)	0.22 (0.64)	0.28 (0.60)	0.33 (0.57)	0.37 (0.54)	0.41 (0.52)	0.43 (0.51)
$\{x_{t-1}, x_{t-2}\}$	0.40 (0.67)	0.31 (0.74)	0.20 (0.82)	0.18 (0.84)	0.18 (0.84)	0.19 (0.83)	0.20 (0.82)	0.22 (0.80)
$\{x_{t-1}, ..., x_{t-4}\}$	0.69 (0.60)	0.82 (0.52)	0.95 (0.44)	0.96 (0.43)	0.96 (0.43)	0.94 (0.44)	0.91 (0.46)	0.87 (0.48)

Notes: See the notes to tables 9 and 4.

Table 11
Canadian NKPC: Tests of $H_0: \ \theta = \theta_0$ or $H_0: \ \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1 - \theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(1)$

$1963Q1$ - $2000Q4$ $\qquad T = 152$

θ_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
z_t =							
$\{\pi_{t-1}, x_t, x_{t-2}\}$	2.14 (0.14)	4.63 (0.03)	11.81 (0.00)	26.09 (0.00)	28.53 (0.00)	8.49 (0.00)	0.06 (0.81)
$\{\pi_{t-1}, x_t, ..., x_{t-2}\}$	7.33 (0.01)	7.41 (0.01)	7.90 (0.00)	4.49 (0.03)	14.08 (0.00)	19.25 (0.00)	2.51 (0.11)
$\{\pi_{t-1}, x_t, x_{t-1}, ..., x_{t-4}\}$	48.53 (0.00)	34.22 (0.00)	33.63 (0.00)	28.57 (0.00)	19.86 (0.00)	12.00 (0.00)	9.64 (0.00)
ω_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.65 (p)	0.70 (p)	0.75 (p)	0.80 (p)
$\{\pi_{t-1}, x_t, x_{t-2}\}$	8.40 (0.00)	6.86 (0.01)	3.46 (0.06)	1.57 (0.21)	0.33 (0.56)	0.03 (0.86)	1.32 (0.25)
$\{\pi_{t-1}, x_t, ..., x_{t-2}\}$	6.54 (0.01)	4.75 (0.03)	2.88 (0.09)	2.41 (0.12)	3.16 (0.08)	6.54 (0.01)	14.46 (0.00)
$\{\pi_{t-1}, x_t, x_{t-1}, ..., x_{t-4}\}$	21.29 (0.00)	20.68 (0.00)	19.96 (0.00)	19.15 (0.00)	18.47 (0.00)	15.77 (0.00)	10.90 (0.00)

Notes: See the notes to tables 5 and 9.

Auxiliary Tables

Joint Tests

Not for Publication

Table 4b
U.S. NKPC: Tests of $H_0: \ \gamma_f = \gamma_{f0}, \ \gamma_b = \gamma_{b0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} - \gamma_{b0}\pi_{t-1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

$1949Q1 - 2002Q4 \qquad T = 212$

γ_{f0} =	0.00 (p)	0.25 (p)	0.50 (p)	0.68 (p)
$u_t = \{x_{t-2}\}$				
$\gamma_{b0} = 0.00$	4.49 (0.04)	3.35 (0.07)	1.68 (0.20)	0.52 (0.47)
$\gamma_{b0} = 0.30$	4.38 (0.04)	2.68 (0.10)	0.63 (0.43)	0.00 (0.98)
$\gamma_{b0} = 0.45$	4.00 (0.05)	1.97 (0.17)	0.16 (0.69)	0.13 (0.72)
$u_t = \{x_{t-1}, x_{t-2}\}$				
$\gamma_{b0} = 0.00$	2.77 (0.06)	2.86 (0.06)	3.19 (0.04)	3.62 (0.03)
$\gamma_{b0} = 0.30$	3.48 (0.03)	4.13 (0.02)	5.17 (0.01)	5.71 (0.00)
$\gamma_{b0} = 0.45$	3.92 (0.02)	4.87 (0.01)	5.94 (0.00)	6.15 (0.00)
$u_t = \{x_{t-1}, ..., x_{t-4}\}$				
$\gamma_{b0} = 0.00$	2.15 (0.08)	2.10 (0.08)	2.11 (0.08)	2.19 (0.93)
$\gamma_{b0} = 0.30$	2.29 (0.06)	2.51 (0.04)	2.95 (0.02)	3.25 (0.01)
$\gamma_{b0} = 0.45$	2.38 (0.05)	2.79 (0.03)	3.40 (0.01)	3.64 (0.01)

Notes: The Anderson-Rubin statistics are based on the null that γ_{f0}=0 and γ_{b0}=0. Otherwise, see the notes to tables 3 and 4.

Table 5b
U.S. NKPC: Tests of $H_0: \ \theta = \theta_0, \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(2)$

$1949Q1$ – $2002Q4$ $\qquad T = 212$

θ_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
$z_t = \{\pi_{t-1}, x_t, x_{t-2}\}$							
$\omega_0 = 0.075$	53.51 (0.00)	39.29 (0.00)	63.91 (0.00)	34.67 (0.00)	19.52 (0.00)	6.06 (0.05)	5.68 (0.06)
$\omega_0 = 0.25$	42.44 (0.00)	56.23 (0.00)	56.39 (0.00)	33.11 (0.00)	24.79 (0.00)	8.57 (0.01)	1.87 (0.39)
$\omega_0 = 0.50$	48.98 (0.00)	49.01 (0.00)	43.74 (0.00)	30.60 (0.00)	22.11 (0.00)	13.43 (0.00)	6.23 (0.04)
$z_t = \{\pi_{t-1}, x_t, ..., x_{t-2}\}$							
$\omega_0 = 0.075$	73.81 (0.00)	72.06 (0.00)	62.90 (0.00)	42.37 (0.00)	27.93 (0.00)	22.67 (0.00)	25.15 (0.00)
$\omega_0 = 0.25$	72.46 (0.00)	63.53 (0.00)	55.95 (0.00)	32.33 (0.00)	22.76 (0.00)	16.67 (0.00)	15.99 (0.00)
$\omega_0 = 0.50$	49.15 (0.00)	49.32 (0.00)	48.64 (0.00)	30.08 (0.00)	20.80 (0.00)	13.96 (0.00)	13.04 (0.00)
$z_t = \{\pi_{t-1}, \pi_{t-2}, x_t, ..., x_{t-2}\}$							
$\omega_0 = 0.075$	73.99 (0.00)	71.63 (0.00)	63.57 (0.00)	40.27 (0.00)	31.77 (0.00)	21.72 (0.00)	21.84 (0.00)
$\omega_0 = 0.25$	68.88 (0.00)	65.91 (0.00)	56.60 (0.00)	29.97 (0.00)	20.47 (0.00)	16.29 (0.00)	22.65 (0.00)
$\omega_0 = 0.50$	64.65 (0.00)	52.19 (0.00)	52.95 (0.00)	35.88 (0.00)	22.80 (0.00)	16.66 (0.00)	20.36 (0.00)

Notes: See table 5 for details.

Table 7b
U.K. NKPC: Tests of $H_0: \ \gamma_f = \gamma_{f0}, \ \gamma_b = \gamma_{b0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} - \gamma_{b0}\pi_{t-1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

$1961Q1$ - $2000Q4$ $\qquad T = 168$

γ_{f0} =	0.00 (p)	0.25 (p)	0.50 (p)	0.68 (p)
$u_t = \{x_{t-1}\}$				
$\gamma_{b0} = 0.00$	3.57 (0.06)	3.84 (0.05)	4.06 (0.05)	3.98 (0.05)
$\gamma_{b0} = 0.30$	5.12 (0.02)	5.57 (0.02)	5.85 (0.02)	5.00 (0.03)
$\gamma_{b0} = 0.60$	6.54 (0.01)	6.83 (0.01)	6.69 (0.01)	4.47 (0.04)
$u_t = \{x_{t-1}, ..., x_{t-4}\}$				
$\gamma_{b0} = 0.00$	1.83 (0.13)	2.00 (0.10)	2.25 (0.07)	3.18 (0.02)
$\gamma_{b0} = 0.30$	2.57 (0.04)	2.91 (0.02)	3.34 (0.01)	4.36 (0.00)
$\gamma_{b0} = 0.60$	3.79 (0.01)	4.26 (0.00)	4.69 (0.00)	4.89 (0.00)

Notes: See the notes to tables 4b, 6, and 7.

Table 8b
U.K. NKPC: Tests of $H_0: \ \theta = \theta_0, \ \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1 - \theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(2)$

$1961Q1$ – $2000Q4$ $\qquad T = 168$

θ_0 =	0.40 (p)	0.50 (p)	0.55 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)
$z_t = \{\pi_{t-1}, x_t, x_{t-1}\}$							
$\omega_0 = 0.50$	13.05 (0.00)	12.12 (0.00)	13.66 (0.00)	15.93 (0.00)	25.88 (0.00)	31.20 (0.00)	29.96 (0.00)
$\omega_0 = 0.60$	12.40 (0.00)	16.35 (0.00)	19.00 (0.00)	22.23 (0.00)	33.00 (0.00)	35.05 (0.00)	33.99 (0.00)
$\omega_0 = 0.70$	17.60 (0.00)	24.03 (0.00)	25.65 (0.00)	27.05 (0.00)	33.37 (0.00)	36.88 (0.00)	36.08 (0.00)
$z_t = \{\pi_{t-1}, x_t, ..., x_{t-4}\}$							
$\omega_0 = 0.50$	21.50 (0.00)	29.55 (0.00)	30.57 (0.00)	31.17 (0.00)	21.23 (0.00)	21.72 (0.00)	21.74 (0.00)
$\omega_0 = 0.60$	29.15 (0.00)	39.17 (0.00)	33.53 (0.00)	23.03 (0.00)	22.64 (0.00)	40.43 (0.00)	48.32 (0.00)
$\omega_0 = 0.70$	25.77 (0.00)	26.75 (0.00)	41.89 (0.00)	27.44 (0.00)	38.61 (0.00)	22.64 (0.00)	38.16 (0.00)
$z_t = \{\pi_{t-1}, x_{t-1}, ..., x_{t-4}\}$							
$\omega_0 = 0.50$	29.99 (0.00)	20.27 (0.00)	20.17 (0.00)	19.69 (0.00)	16.79 (0.00)	15.45 (0.00)	14.72 (0.00)
$\omega_0 = 0.60$	37.32 (0.00)	26.52 (0.00)	27.77 (0.00)	27.05 (0.00)	23.38 (0.00)	25.09 (0.00)	22.88 (0.00)
$\omega_0 = 0.70$	37.68 (0.00)	38.20 (0.00)	38.42 (0.00)	38.63 (0.00)	36.09 (0.00)	29.56 (0.00)	32.16 (0.00)

Notes: See table 8 for details.

Table 10b
Canadian NKPC: Tests of $H_0: \gamma_f = \gamma_{f0}, \gamma_b = \gamma_{b0}$

$$\pi_t - \gamma_{f0}\pi_{t+1} - \gamma_{b0}\pi_{t-1} = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 x_t + \alpha_3 u_t$$

Anderson-Rubin Statistic

$1963Q1 - 2000Q4$ $T = 152$

γ_{f0} =	-0.20 (p)	0.20 (p)	0.35 (p)	0.50 (p)
$u_t = \{x_{t-2}\}$				
$\gamma_{b0} = 0.00$	0.29 (0.59)	0.09 (0.76)	0.03 (0.86)	0.00 (1.00)
$\gamma_{b0} = 0.50$	0.10 (0.75)	0.02 (0.89)	0.14 (0.71)	0.34 (0.56)
$\gamma_{b0} = 0.85$	0.00 (0.98)	0.25 (0.61)	0.46 (0.50)	0.64 (0.42)
$u_t = \{x_{t-1}, x_{t-2}\}$				
$\gamma_{b0} = 0.00$	0.18 (0.84)	0.06 (0.94)	0.02 (0.98)	0.00 (1.00)
$\gamma_{b0} = 0.50$	0.26 (0.77)	0.22 (0.80)	0.24 (0.78)	0.29 (0.75)
$\gamma_{b0} = 0.85$	0.46 (0.63)	0.51 (0.60)	0.52 (0.69)	0.53 (0.69)
$u_t = \{x_{t-1}, ..., x_{t-4}\}$				
$\gamma_{b0} = 0.00$	0.35 (0.84)	0.46 (0.77)	0.54 (0.71)	0.64 (0.63)
$\gamma_{b0} = 0.50$	0.39 (0.81)	0.74 (0.57)	0.91 (0.46)	1.03 (0.40)
$\gamma_{b0} = 0.85$	0.57 (0.68)	0.97 (0.42)	1.05 (0.38)	1.05 (0.38)

Notes: See tables 10, 4b, and 9 for details.

Table 11b
Canadian NKPC: Tests of $H_0: \ \theta = \theta_0, \omega = \omega_0$

$$E[\gamma_f \pi_{t+1} - \pi_t + \gamma_b \pi_{t-1} + \lambda x_t | z_t] = 0,$$

$$\gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi}, \quad \lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\phi}, \quad \phi = \theta + \omega[1-\theta(1-\beta)]$$

Guggenberger-Smith LM Statistic $\sim \chi^2(2)$

$1963Q1$ – $2000Q4$ $\qquad T = 152$

θ_0 =	0.40 (p)	0.50 (p)	0.60 (p)	0.75 (p)	0.80 (p)	0.85 (p)	0.90 (p)
$z_t = \{\pi_{t-1}, x_t, x_{t-2}\}$							
$\omega_0 = 0.40$	46.88	40.23	40.26	37.63	30.20	18.46	5.85
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.05)
$\omega_0 = 0.65$	38.52	37.87	44.36	40.49	25.31	5.72	1.56
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.05)	(0.46)
$\omega_0 = 0.75$	49.61	52.12	32.55	39.45	34.85	8.33	0.06
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.97)
$z_t = \{\pi_{t-1}, x_t, ..., x_{t-2}\}$							
$\omega_0 = 0.40$	46.38	45.80	45.69	39.69	37.74	19.29	5.68
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)
$\omega_0 = 0.65$	43.54	42.02	52.89	40.05	29.58	8.88	2.21
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.33)
$\omega_0 = 0.75$	53.68	54.97	46.61	44.10	54.04	7.65	2.35
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.31)
$z_t = \{\pi_{t-1}, x_t, x_{t-1}, ..., x_{t-4}\}$							
$\omega_0 = 0.40$	31.54	36.99	43.64	34.15	24.62	21.07	19.74
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$\omega_0 = 0.65$	31.38	40.38	40.36	30.28	18.90	14.59	14.93
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$\omega_0 = 0.75$	37.11	30.64	44.75	46.79	21.11	8.57	6.59
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)

Notes: See table 11 for details.