USC FBE MACROECONOMICS AND INTERNATIONAL FINANCE WORKSHOP
presented by Dean Corbae
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Credit Scoring and Competitive Pricing of Default Risk

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Legal Environment

- A Ch.7 bankruptcy permanently discharges net debt (liabilities-assets above statewide exemption levels).
- A filer is ineligible for a subsequent Ch. 7 discharge for 6 years (instead forced into Ch. 13 which is a 3-5 year repayment schedule followed by discharge).
- The Fair Credit Reporting Act requires credit bureaus to exclude the filing from credit reports after 10 years (and all other adverse items after 7 years).

Questions

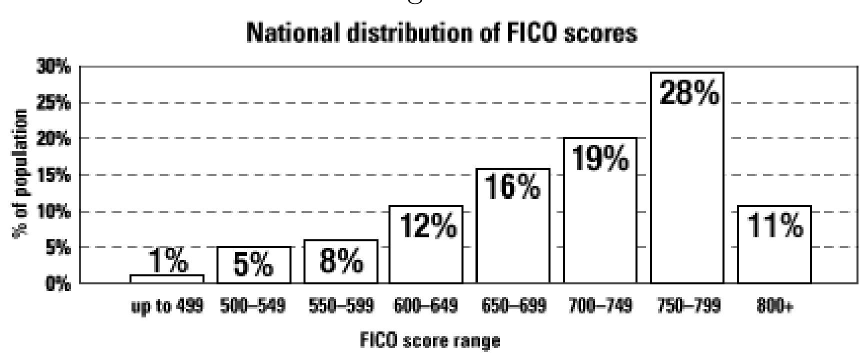
- Why don't we see more lending to people who have filed Ch.7?
- What are the welfare consequences of legal restrictions like FCRA?

Consumption-Smoothing in Practice

- Save or Borrow with the option to default.
- Lenders assess creditworthiness of borrowers.
- FICO credit scores used as an input into the assessment of creditworthiness.
  - FICO scores range between 300 and 850 where the higher the score, the higher the creditworthiness.
  - Over 75% of mortgage lenders and 80% of the largest financial institutions use FICO scores.
- Non-collateralized consumer debt is 10% of aggregate consumption.
- 1 bankruptcy filing per 75 households.

The national distribution of FICO scores

Figure 1



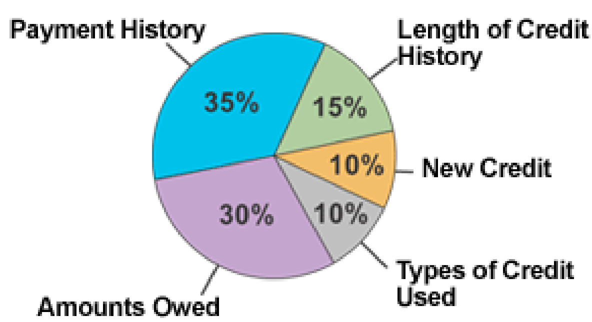
Inputs into FICO Scores

FICO scores are calculated from data in the individual's credit report in five basic catogories:

- Payment history (35%) – includes adverse public records
- Amounts owed (30%)
- Length of credit history (15%)
- Credit limits (10%)
- Types of credit used (10%)

  Ignores:

- Race, color, national origin, sex, and marital status (prohibited by law)
- Age, assets, salary, occupation, and employment history.



Key Properties of FICO Scores

1. Low score raises interest rate
2. Default lowers score, credit access with flag is restricted, removal raises it
3. Increasing indebtedness lowers score

Evidence for Properties

PF1:

FICO Score	Auto Loan	Mortgage
720-850	4.94%	5.55%
700-719	5.67%	5.68%
675-699	7.91%	6.21%
620-674	10.84%	7.36%
560-619	15.14%	8.53%
500-559	18.60%	9.29%

Source: http://www.myfico.com

- PF2: Musto (2004, *Journal of Business*)/Fisher,Filer,Lyons (2004)
  - Removal of Bankruptcy flag substantially raises credit scores
  - Bankruptcy lowers availability of credit
- PF3: On its website Fico advises to keep balances low and pay off debt.

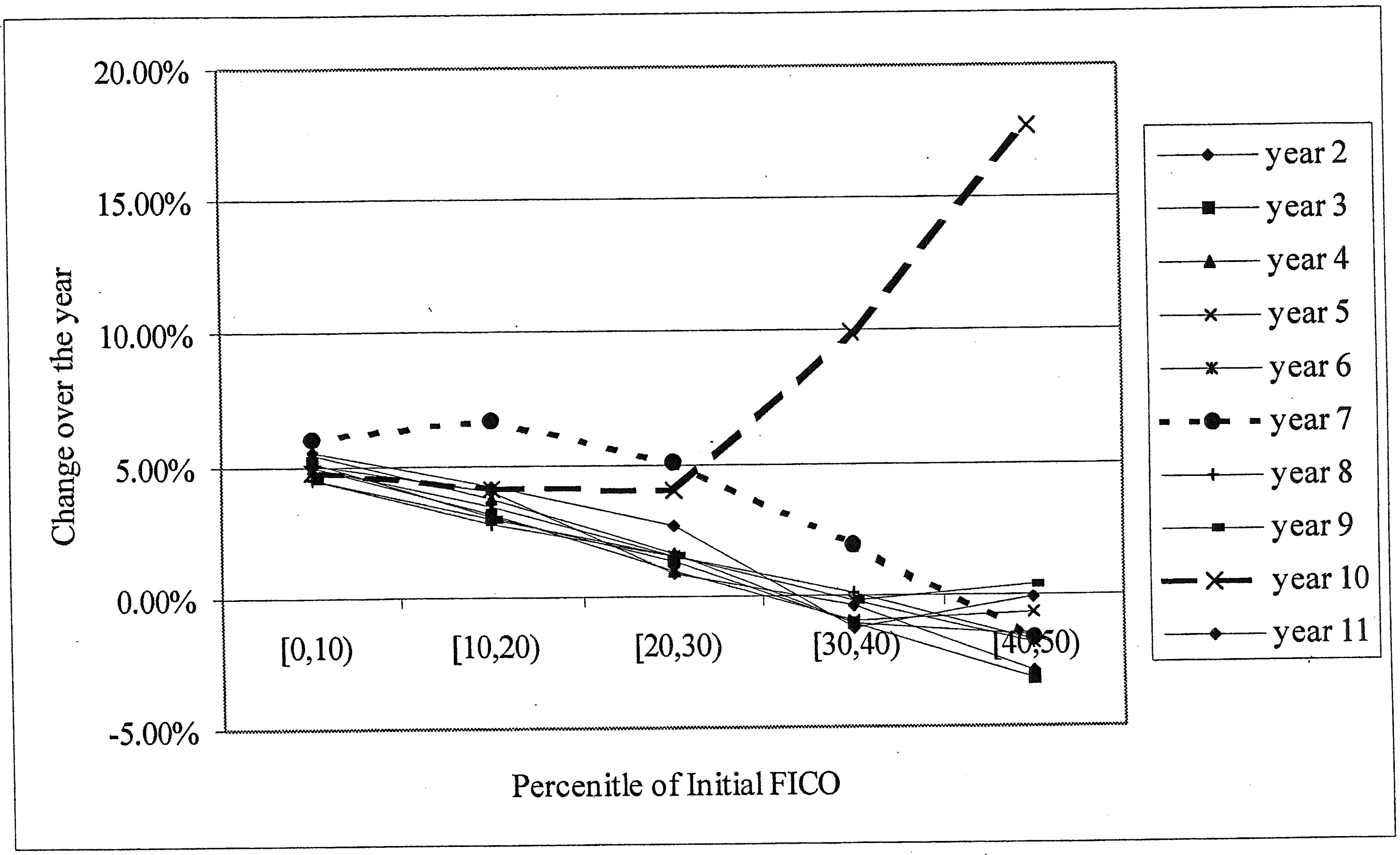


Figure 4. Change in FICO percentile over the n^{th} year, sorted by initial percentile.

Environment

- Model of precautionary savings.
- 2 types of people; type is unobservable and affects preferences.
- Borrowers cannot commit to pay back loans.
- Intermediaries borrow at the world risk-free rate and lend to people.
- Interest rates on loans depend on loan size and on the intermediary's belief that a person is of a given type.
- Beliefs about type are updated according to Bayes' rule.
- Free entry of intermediaries.

Can this Environment deliver PF1-3?

- Crucial inference problem for the intermediary: Is a defaulter or a borrower a low risk type with a low earnings realization or a high risk type?

Relevant Prior Work

- Incomplete Markets:
  - Aiyagari (1994, QJE)
  - Huggett (1993, JEDC)
- Bankruptcy:
  - Athreya (2002, *JME*)
  - Chatterjee, Corbae, Nakajima, and Rios-Rull (2005)
- Reputation:
  - Cole, Dow and English (1995, *IER*)
  - Diamond (1989, *JPE*)
  - Phelan (forthcoming, *JET*)

People

- Unit measure of two types of agents $i_t \in \{g, b\}$ where type affects preferences β_i.
- People switch types with probability $\delta_i > 0$.
- Each period a person's endowment e_t is an i.i.d draw from a distribution η with compact support $E \subset R_{++}$.
- Type, earnings, and cons. are private information, but default decision and asset position are observable.

Intermediaries

- World interest rate r.
- Competitive credit industry accepts deposits $\ell_t > 0$ and makes loans $\ell_t < 0$ to people.
- Price of a loan of size ℓ_{t+1} made to a person with credit history$(\ell_t, h^t(T))$ denoted $q(\ell_{t+1}, \ell_t, h^t(T)) \geq 0$ where $h^t(T) = (d_{t-1}, \ell_{t-1}, d_{t-2}, ..., \ell_{t+1-T}, d_{t-T})$

Recursive Formulation of HH Problem

$$v_i(e, \ell, h; q) = \max_{d \in \{0,1\}} v_i^d(e, \ell, s; q)$$

where

$$v_i^0(e, \ell, h; q) = \max_{c, \ell' \in B(e,\ell,h;q) \neq \varnothing} u_i(c) + \beta_i \, (1 - \delta_i) \int_E v_i \left[e', \ell', h'\right] \eta(de') + \beta^i \, \delta_i \int_E v_{-i} \left[e', \ell', h'\right] \eta(de')$$

with $\quad B(e, \ell, h; q, \Psi) = \{c \geq 0, \ell' \in L \mid c + q(\ell', \ell, h) \cdot \ell' \leq e + \ell\}$

and

$$v_i^1(e, \ell, h; q) = u_i(e) + \beta_i \, (1 - \delta_i) \int_E v_i \left[e', 0, h'\right] \, \eta(de') + \beta_i \, \delta_i \int_E v_{-i} \left[e', 0, h'\right] \, \eta(de')$$

where $h'(T) = \lambda(\ell, d, h; T) = (d, \ell, ..., \ell_{+2-T}, d_{+1-T})$.

Intermediary's Problem

- Can borrow or lend at rate $r \geq 0$.
- $\alpha(\ell', \ell, h) \geq 0$ is the measure of (ℓ', ℓ, h) type contracts sold by an intermediary
- Expected profits from this sale is $\pi(\ell', \ell, h;\, q, p) \cdot \alpha(\ell', \ell, h)$, where

$$\pi(\ell', \ell, h;\, q, p) = \begin{cases} \frac{[1-p(\ell',\ell,h;\,q)]}{1+r}(-\ell') - q(\ell', \ell, h)(-\ell') & \text{if } \ell' < 0 \\ q(\ell', \ell, h)\ell' - \frac{\ell'}{1+r} & \text{if } \ell' \geq 0 \end{cases}$$

where $p(\ell', \ell, h)$ is the fraction of people with with history (ℓ, h) expected to default on a loan of size ℓ' tomorrow.

- An intermediary solves the linear problem

$$\max_{\alpha(\ell',\ell,h)\geq 0} \sum_{\ell',(\ell,h)} \pi(\ell', \ell, h;\, q) \cdot \alpha(\ell', \ell, h)$$

Equilibrium Conditions

An equilibrium is (q^*, p^*, μ^*) where μ is the distn of agents over $\{g, b\} \times L \times H$ where H is the set of all possible h satisfying:

1. Given q^*, $D_i(\ell, h; q^*)= \{e \mid d_i(e, \ell, h; q) = 1\}$ and $E_i(\ell', \ell, h; q^*)e \mid \ell'_i(e, \ell, h; q) = \ell'\}$ are consistent with hh optimization.

2. q^* is consistent with intermediary optimization.

$$q^*(\ell', \ell, h) = \begin{cases} (1+r)^{-1}[1 - p^*(\ell', \ell, h)] & \ell' < 0 \\ (1+r)^{-1} & \ell' \geq 0 \end{cases} \tag{1}$$

where

$$p^*(\ell', \ell, h) = [1 - \eta(D_g(\ell', h'; q))] \cdot \Psi(\ell', 0, \ell, h)$$

$$+[1 - \eta(D_b(\ell', h'; q))] \cdot (1 - \Psi(\ell', 0, \ell, h))$$

and $\Psi(\ell', d, \ell, h)$ is the post-type shock probability that an individual who chooses (ℓ', d) in history (ℓ, h) is of type g at the beginning of the following period required to satisfy Bayes' rule wherever possible

3. μ^* is a fixed point of

$$\begin{aligned}
&\mu'(i',\ell',h') \\
=\;& \sum_{i,\ell,h}\int_e \delta(i'|i)\mathbf{1}_{\{\ell'_i(e,\ell,h;q^*)=\ell',\lambda(\ell,d_i(e,\ell,h;q^*),h;T)=h'\}}d\eta(e)\mu(i,\ell,h).
\end{aligned}$$

The Formula for Updating Type Scoring

- Intermediaries update assessments of agent type as

$$\Pr(g|\ell', d, \ell, h) = \frac{\Pr(\ell', d, \ell, h|g) \Pr(g)}{\Pr(\ell', d, \ell, h)}$$

$$= \frac{\Pr(\ell', d|g, \ell, h) \Pr(g|\ell, h)}{\Pr(\ell', d|g, \ell, h) \Pr(g|\ell, h) + \Pr(\ell', d|b, \ell, h) \Pr(b|\ell, h)}.$$

- Two sorts of observable and mutually exclusive events:
  - Person of type i with history (ℓ, h) defaults on a loan of size ℓ :

$$\Pr(0, 1|i, \ell, h) = \eta(D_i(\ell, h; q))$$

  - Person of type i with history (ℓ, h) does not default and chooses ℓ' :

$$\Pr(\ell', 0|i, \ell, h) = \eta(E_i(\ell', \ell, h; q))$$

- Current type score given by

$$\Pr(i|\ell, h) = \frac{\mu^*(i, \ell, h)}{\sum_{j \in \{g,b\}} \mu^*(j, \ell, h)}$$

- Then, given that type can change at the beginning of the next period

$$\Psi(\ell', d, \ell, h) = (1-\delta_g) \Pr(g|\ell', d, \ell, h) + \delta_b [1 - \Pr(g|\ell', d, \ell, h)]$$

- With $T = \infty$, if for any two distinct histories (ℓ, h) and $(\ell, \widehat{h})$ such that $\Pr(g|\ell, h) = \Pr(g|\ell, \widehat{h})$ we have $\Psi(\ell', d, \ell, h) = \Psi(\ell', d, \ell, \widehat{h})$ for all ℓ', d then we can replace the infinite-dimensional state variable h by the scalar $s \in [0, 1] = \Pr(g|\ell, h)$.

When is a Type Score a *Credit* Score?

1. For $\sigma_1 > \sigma_2$ and $\ell' < 0$

$$q^*(\ell', \sigma_1) \geq q^*(\ell', \sigma_2).$$

2. For $\ell < 0$

$$\Psi^*(0, 1, \ell, s) < s \ \forall s.$$

3. For $\ell' < 0$ and $\ell' < \ell$

$$\Psi^*(\ell', 0, \ell, s) < s \ \forall s.$$

How Can We Get This?

If type characteristics are such that

$$D_g(\ell, s; q^*) \subseteq D_b(\ell, s; q^*) \forall \ell, s,$$

Might get (1), (2) and (3).

Special Case Assumptions

- $0 = \beta_b < \beta_g$ (Type b are completely myopic)
- $\delta_i \in (0,1)$ and $1 - \delta_g > \delta_b$.
- $L = \{-x, 0, x\}$.
- Off-the-equilibrium path beliefs:
  - If neither type defaults,
    $\Psi(0, 1, -x, s) = (1 - \delta_g)s + \delta_b(1 - s)$
  - If neither type chooses ℓ',

    $\Psi(\ell', 1, \ell, s) = (1 - \delta_g)s + \delta_b(1 - s)$.

(Weak) Characterization of Type Scores

Lemma 0. For $\sigma > 0$, if some type g do not default (i.e. $D_g(-x, \sigma;\, q^*) \neq E$), then $q^*(-x, \sigma) > 0$.

- Follows from intermediary optimization that

$$q^*(\ell, \sigma) = (1 + r)^{-1} \times [1 - \eta(D_g(-x, \sigma;\, q^*))] \cdot \sigma$$

Characterization - (cont.)

If an equilibrium with $q^*(-x, \sigma) > 0$ for all type scores $\sigma > 0$ exists....

Proposition 1. Type b with debt default and type b without debt borrow.

(i) For all ℓ, s, no type b save.

- Saving means consuming less today and you don't care about your future score.

(ii) For all s, all type b default.

- $(d_b = 0, \ell'_b = 0)$ is strictly dominated by $(d_b = 1, \ell'_b = 0)$ since $\ell = -x$ is wiped clean.
- $(d_b = 0, \ell'_b = -x)$ is strictly dominated by $(d_b = 1, \ell'_b = 0)$ since $q^*(-x, \sigma) \leq 1/(1 + r) < 1$ for any σ.

(iii) For $\ell \in \{0, x\}$ all type b borrow.

- Borrowing maximizes current consumption provided loan prices are strictly positive.
- If loan prices are zero, then agents are endogenously borrowing constrained.

Characterization - (cont)

Propositions 2&3. If a person doesn't borrow, he is type g.

- (2) If some type g save (i.e. $\eta(E_g(x, \ell, s; q^*)) > 0$, an assumption necessary to avoid $\frac{0}{0}$ in Bayesian updating formula), then result follows from Prop1(i), saver isn't type b.
- (3) If some type g choose $\ell' = 0$, then result follows from Prop1(iii), agents choosing $\ell' = 0$ aren't type b.

Characterization - (cont)

Proposition 4. Type scores decrease following default (i.e. $\Pr(g|0, 1, -x, s) \leq s$).

- By Prop1(ii), all type b default. Then if some type g do not default, default increases the probability (via Bayes' rule) the person is of type b.

Propositions 5&6. Type scores: (i) decrease if person borrows (i.e. if $\ell \in \{0, x\}, \Pr(g| - x, 0, \ell, s) \leq s$); (ii) increase if person pays off debt (i.e. if $\ell' \in \{0, x\}, \Pr(\ell', 0, -x, s) = 1 > s$); and (iii) increase if person maintains debt (i.e. $\Pr(-x, 0, -x, s) = 1 > s$)

- (i). By Prop1(iii), all type b borrow if $\ell \in \{0, x\}$. Then if some type g do not borrow, borrowing increases the probability (via Bayes' rule) the person is of type b.
- (ii). By Prop1(iii) and Bayes' rule, saving signals agent is type g.
- (iii) Prop 1(ii) and Bayes' rule, not defaulting with debt signals agent is type g.

Characterization - (cont)

- Props 4-6 refer to $\Pr(g|\ell', d, \ell, h)$ and not Ψ.

- The fact that type changes induces "mean reversion" in Ψ. Let $\varphi \equiv \Pr(g|\ell', d, \ell, h)$, then:

  $$\varphi < \frac{\delta_b}{\delta_g + \delta_b} \Longrightarrow \Psi > \varphi \text{ and } \varphi > \frac{\delta_b}{\delta_g + \delta_b} \Longrightarrow \Psi < \varphi.$$

- This means that if the person's current period score is low it is possible for his next period score to rise after default.

- Musto documents mean reversion in scores.

Numerical Results

- Parameter Values:
  - $\beta_g = 0.75$
  - $r = (0.8/0.75) - 1$
  - $\delta_g = 0.1$, $\delta_b = 0.5$
  - $x = 0.5$
  - Uniform distribution over a 61 element grid of earnings given by $\{0.35, 0.7, 1.05, \ldots, 21.0\}$.

Numerical Results (cont)
Equilibrium $\Psi^*(\ell', d, \ell, s)$

- $\ell = -x$

1. $\Psi^*(0, 1, -x, s) < s$ for $s > 0$ 0.69 (mean rev)
2. $\Psi^*(-x, 0, -x, s) =(1 - \delta_g)s + \delta_b(1 - s)$ (o-e-p)
3. $\Psi^*(0, 0, -x, s) =(1 - \delta_g)s + \delta_b(1 - s)$ (o-e-p)
4. $\Psi^*(x, 0, -x, s) =(1 - \delta_g)$

For every s, some type g default and some save

- $\ell = 0$

1. $\Psi^*(-x, 0, 0, s) < s$ for $s > 0.51$ (mean rev)
2. $\Psi^*(0, 0, 0, s) = 1 - \delta_g$
3. $\Psi^*(x, 0, 0, s) = 1 - \delta_g$

For every s, some type g choose each asset position

- $\ell = x$

  1. $\Psi^*(-x, 0, x, s) = \delta_b$
  2. $\Psi^*(0, 0, x, s) = 1 - \delta_g$
  3. $\Psi^*(x, 0, x, s) = 1 - \delta_g$

For every s, some type g dissave and some save

Numerical Results (cont)

- Figure 2 - Default prob. of type g.
- Figure 3 - Default induces drop in score for $s > 0.7$
- Figure 4 - Price rises linearly

$$q^*(-x, \sigma) = (1 + r)^{-1} \times [1 - \eta(D_g(-x, \sigma;\ q^*)] \cdot \sigma$$

- Figure 5 & 6 - Prices and Score updates associated with borrowing - running down assets is costly.
- Figure 7 - Decline in default prob. by type g reflects drop in the "value of maintaining a reputation" measured by

$$q(-x, \Psi^*(x, 0, -x, s)) - q(-x, \Psi^*(0, 1, -x, s)).$$

Figure 2: Default Probability of Type g

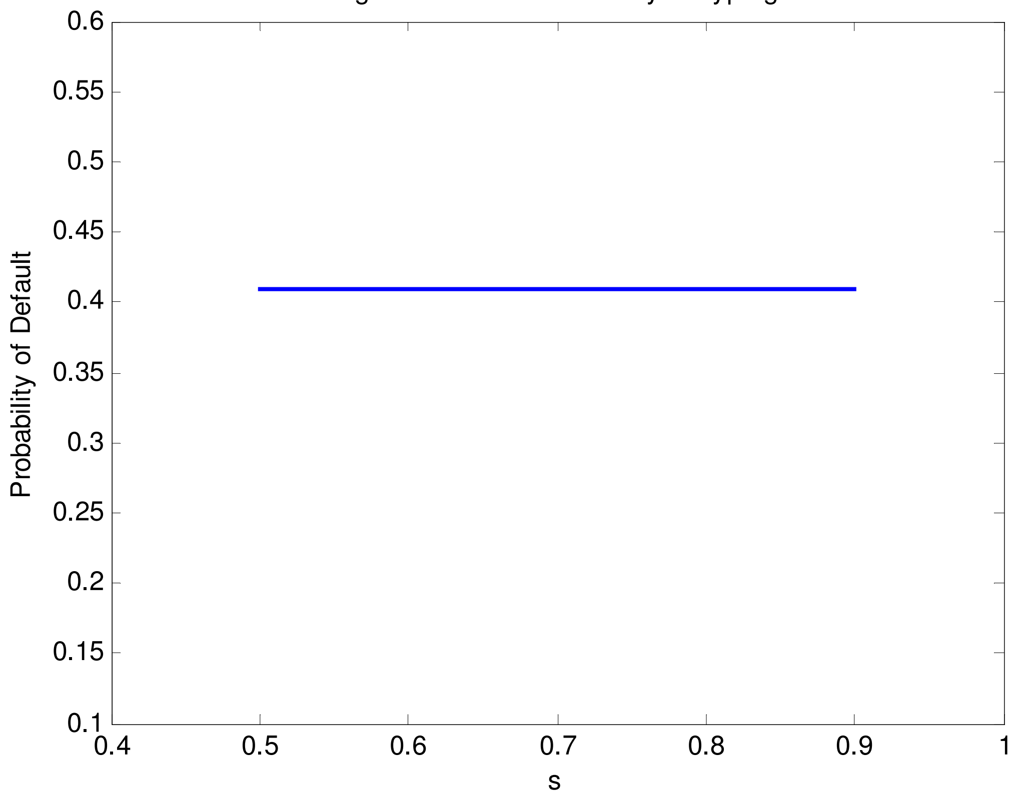


Figure 3: Posterior for Defaulters

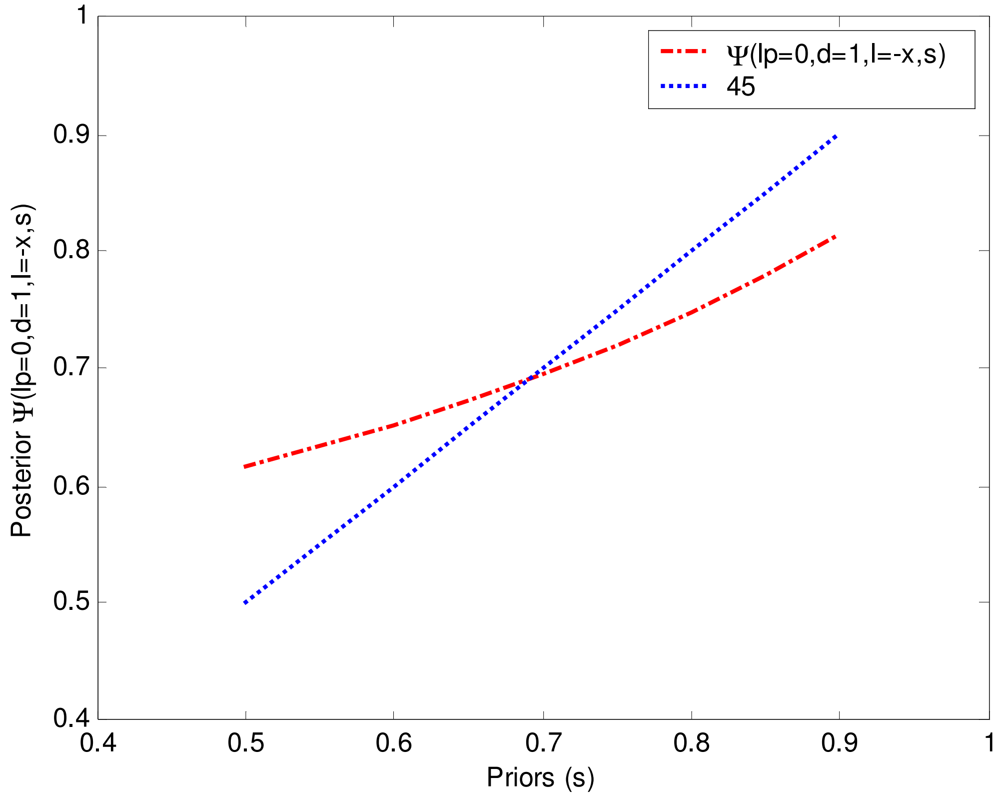


Figure 4: Equilibrium q Function

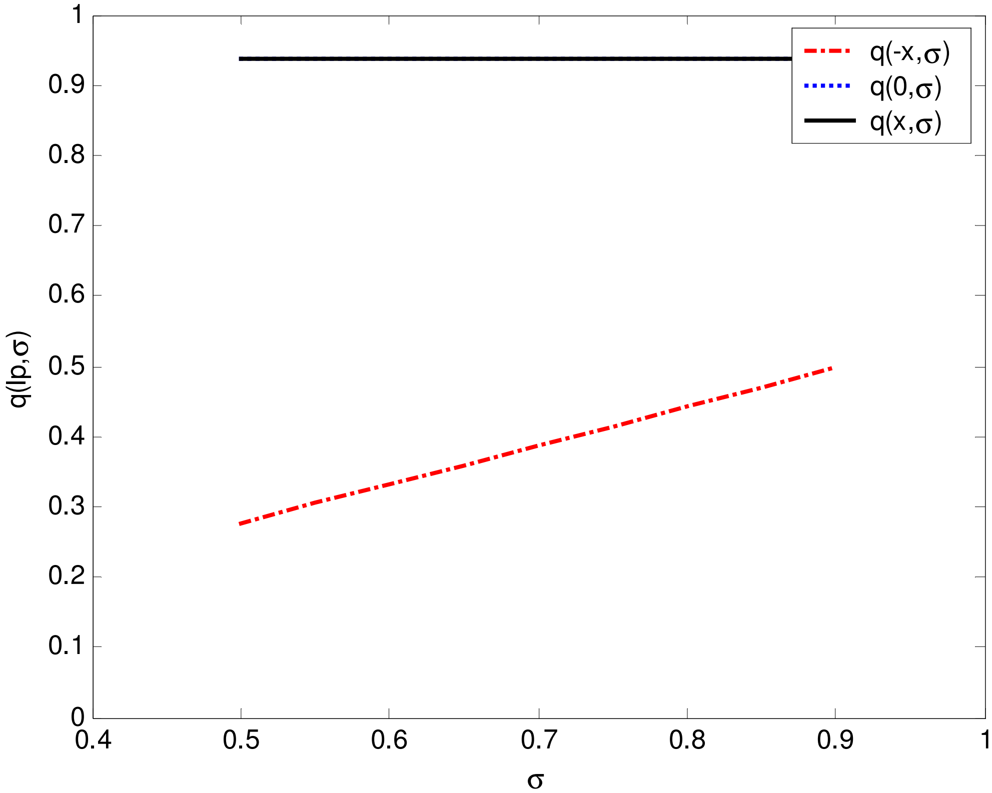


Figure 5: Equilibrium q for Different Initial Assets

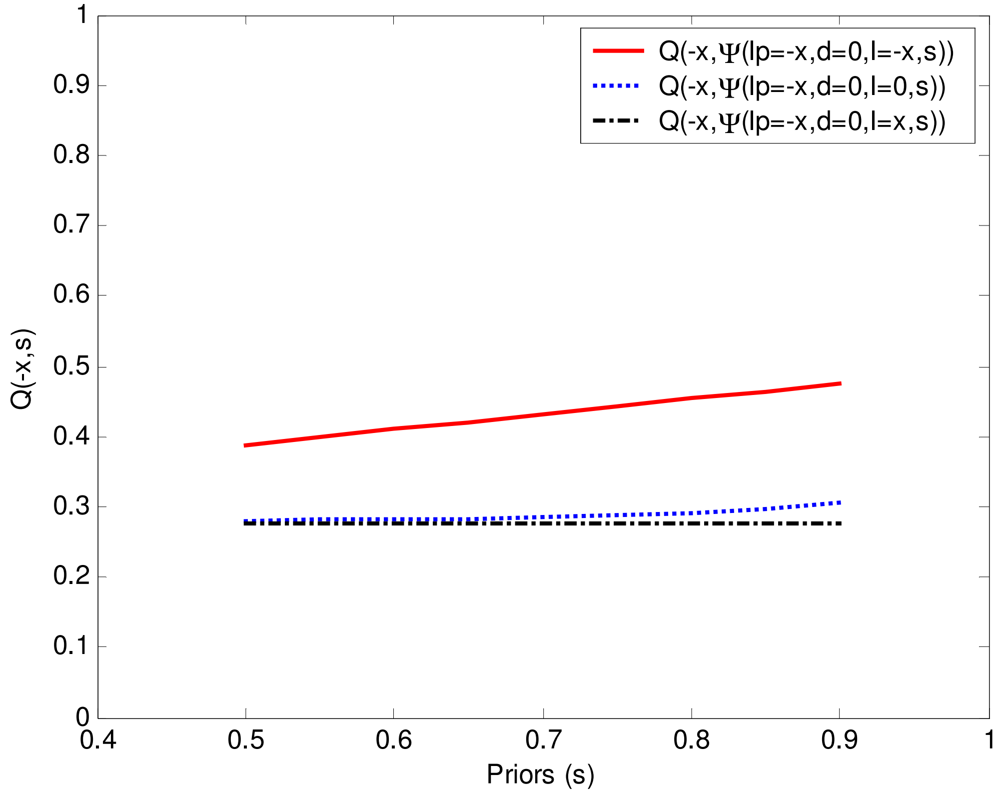


Figure 6: Posterior for Borrowers for Different Initial Assets

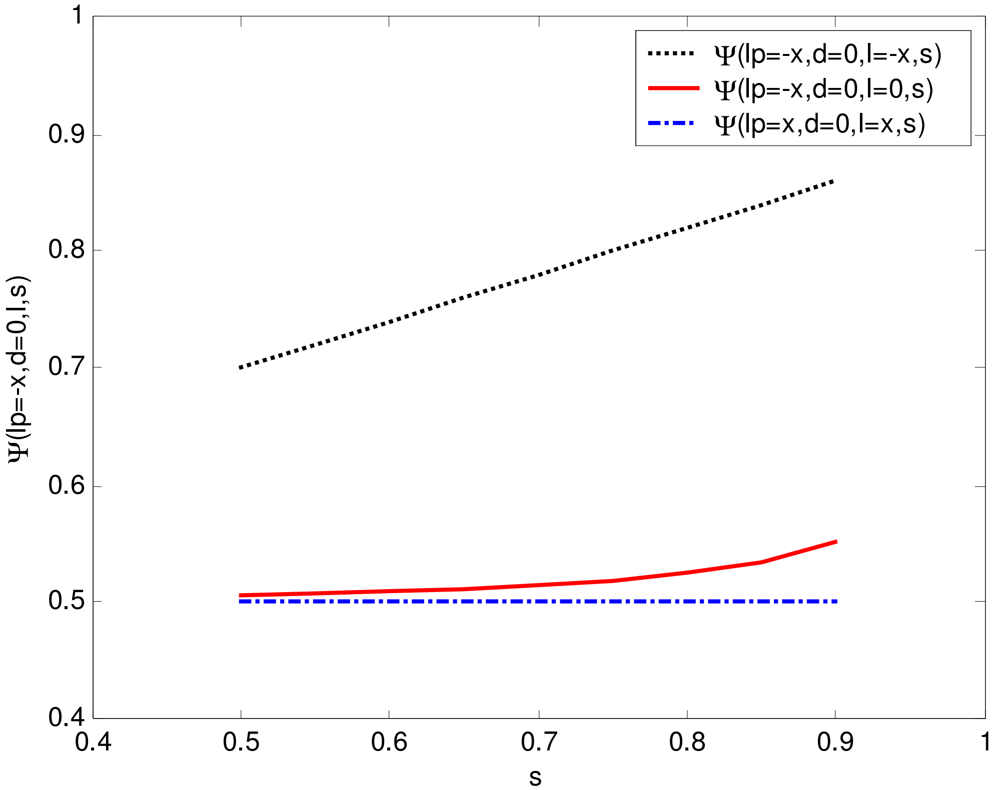


Figure 7: Partial Value of a Reputation

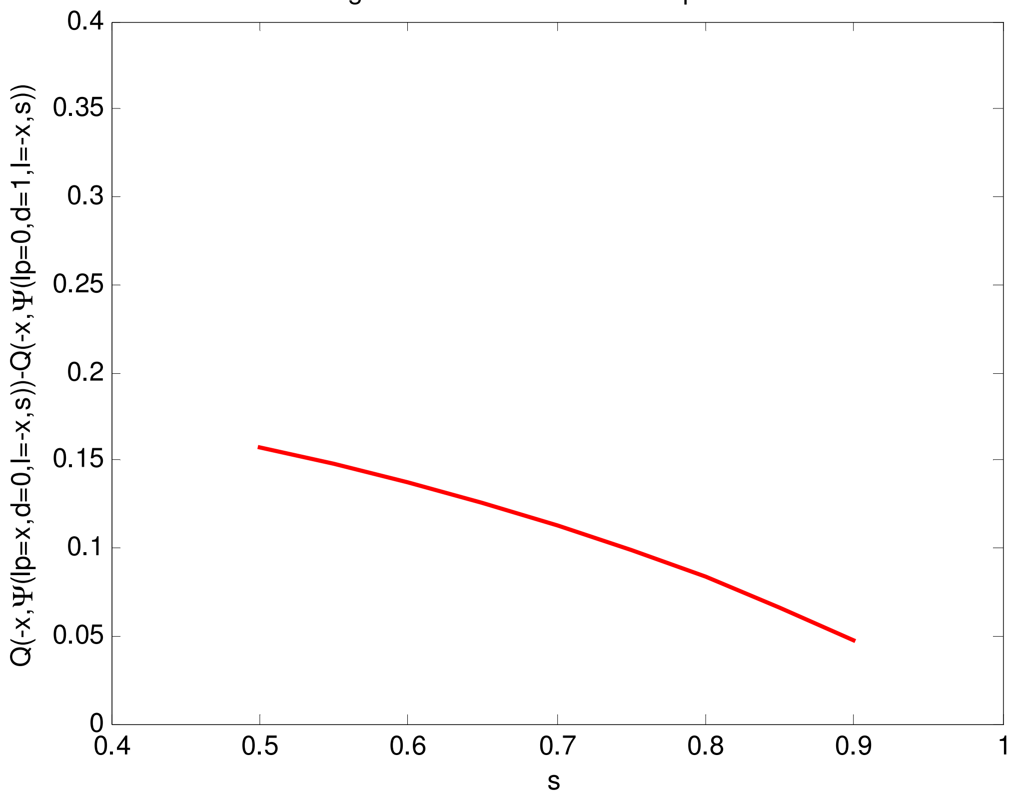
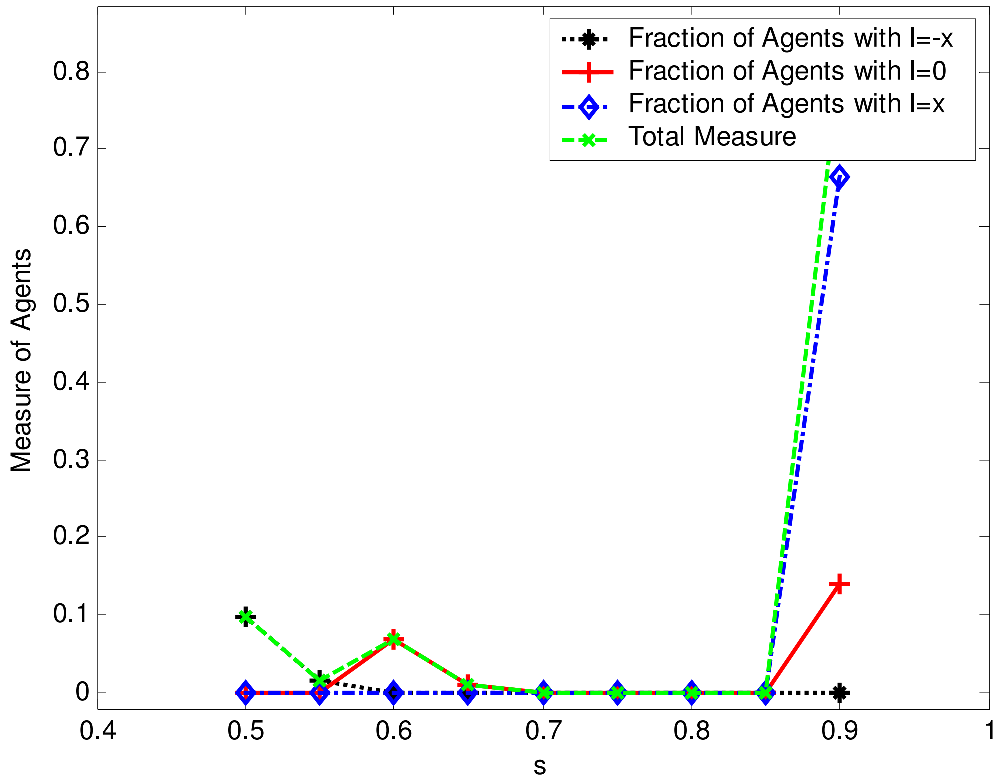


Figure 8: Fraction of Agents Across type Scores at the Stationary Distribution



- Figure 8 - Invariant Distribution
  - 12% are borrowers and have low to medium type scores.
  - Most people with high scores either hold positive assets (66%) or no assets (14%).
  - There are some agents with no assets but medium type scores (8%).

Is a Credit Score a Type Score?

Yes...

- PF1: Negative relationship between interest rates and type score.
- PF2: Default reduces type score.
- PF3:Increasing indebtedness reduces type score.

We've verified these properties continue to hold with $\beta_b >0$.

Legal Restrictions on History Length (FCRA)

- Now let $T = 1$.
- 4 possible $(\ell_t, h^t(1)) = (\ell_t, d_{t-1})$ histories at any date t.
- Same equilibrium behavior (discreteness ????).
- Compute the changes in average score following removal of the bankruptcy flag - compare to Figure 1 in Musto.
- Credit score with default is given by $\Pr(g|0,1) = 0.62$.
- Average score after the default leaves the credit record (next period for $T = 1$) is given by $\sum_{i,(\ell',0)\supset(0,1)} \eta(E_i(\ell',0,0,1))\mathbf{\Psi}^*(\ell',\mathbf{0},\mathbf{0},\mathbf{1}) = \mathbf{0.75}$.
- Jump in score is 21%. Musto found that for individuals in the highest pre-default quintile of credit scores, they jumped ahead of 19% of households after the score left their record.
- Rough example where ex-ante welfare is identical for the $T = 1$ and $T = \infty$ equilibria despite large movements in credit scores associated with the legally imposed removal of adverse events from individual credit histories.0.

Still Lots of Questions

- How robust is our theory of credit scores to a richer asset space? Important implications for signalling.
- Will type scores behave like credit scores for other type differences - such as attitude to risk or earnings risk?
- Do high credit scores encourage the "good risks" (the patient types in this model) to default more frequently in the data? Implication is integral to a reputation-based theory of credit scores.
- When does s fail to be a "sufficient statistic" in the updating function? Important since Musto documents that a person's score rises when information about a past bankruptcy leaves a person's credit history.