Optimal Devaluations*

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Abstract

We analyze optimal fiscal, monetary and exchange rate policy in a simple small open economy model with price setting frictions. We perform our analysis in the tradition of optimal dynamic Ramsey problems. We characterize optimal allocations and the government policies that implement the optimal allocation.

INTRODUCTION

The purpose of this paper is to provide a theoretical framework to characterize optimal fiscal, monetary and exchange rate policy in a small open economy model with varying degrees of price setting restrictions. The contribution of this paper is to carry on the analysis following the dynamic Ramsey literature. Thus, the mapping from policies to allocations is derived from a fully articulated dynamic general equilibrium monetary model with taxes. An important consequence of this approach is that we can jointly study optimal fiscal and monetary policy. In addition, the explicit introduction of preferences provides a natural welfare criteria to evaluate policies.

*PRELIMINARY AND INCOMPLETE
We consider a model in which a fraction of firms is restricted to set prices one period in advance and characterize the optimal cyclical properties of the Ramsey solution. For this economy, we first extend results derived in Correia, Nicolini and Teles (2001a) and show that the set of implementable allocations is independent of the price setting restrictions. Thus, the optimal allocation under sticky prices is the same as the optimal allocation under flexible prices. We then show that the cyclical properties of optimal short run monetary policy depend on the nature of the shock driving the cycle. In summary, if the boom is caused by a shock to the technology of final goods (non-tradables), optimal monetary policy must be procyclical and a devaluation must follow; if it is driven by a technology shock to intermediate goods (tradeable), optimal monetary policy is countercyclical and the exchange rate must decrease. Finally, if the boom is induced by an international terms of trade shock, optimal monetary policy is procyclical and the exchange rate depends on the source of the term of trade shock: if it is driven by a decrease in the price of importable, the exchange rate must be revalued, while if it is driven by an increase in the price of exportables, the exchange rate must be devalued. Another remarkable result is that neither the optimal allocation nor the policy that implements it depend on the degree of price stickiness.

There is an extensive literature that studies optimal monetary and exchange rate policies and characterizes it in terms of its cyclical properties. Obviously, these properties do depend on the mapping from policies to allocations that is derived from the particular model used and on the welfare criterion used. Most of the literature has used reduced form models not explicitly derived from preferences and technologies. Our results differ from most of the literature, sometimes because of the particular model we use, sometimes because of the welfare criteria used.

The model we analyze is very simple. As such, it has at least two weaknesses we want to discuss. First, as most of the modern literature, we impose ad-hoc restric-
tions on the price setting process, instead of modelling the price setting decision and deriving the optimal price setting rules. Thus, we take as a fundamental parameter the fraction of firms that can adjust prices within the period. Therefore, the model is subject to the Lucas critique, and this raises doubts of its usefulness for policy analysis. We do not view this as a significant problem, since we show that both the optimal allocation and the optimal policy are independent of that assumed fundamental parameter. Thus, potential changes in the parameter due to changes in policy will not alter our conclusions regarding optimal policy.

Second, a model as simple as this one is not able to replicate the evidence of open economies, particularly at the business cycle frequency we will be focusing on. Why performing optimal policy exercises in models that are not able to match the data? This is indeed a serious shortcoming, but there does not seem to be obviously better choices available. We went ahead with the analysis, despite this issue, for two reasons: first, we hope that the intuitions we unravel here will prove useful to understand the workings of monetary and fiscal policy in models that can replicate observed patterns for aggregate variables at business cycle frequencies, if these do exhibit price stickiness. Second, we want to explore the implications of price setting restriction for the conduct of optimal fiscal and monetary policy in open economies, above and beyond the empirical relevance of this restrictions in explaining real time dynamics. Since many times policy advice is offered based on the alleged workings of models with sticky prices, clarifying the ways these models work was, for us, a natural question to raise.

The characterization of optimal monetary and exchange rate policies is an old time question. There also seems to be a certain consensus with respect to the way the nominal exchange rate ought to be managed given shocks such as government spending, real exchange rate or productivity shocks. On the other hand, these questions have only very recently started being addressed in general equilibrium dynamic mod-
els. The policy implications derived from the models we analyze are at odds with the conventional wisdom many times. In addition, some of those policy implications do not appear robust to small changes in the environment. Our first, very simple approximation suggest that general equilibrium economics does not seem to support the conventional wisdom in all dimensions.

THE MODEL

We consider a small-open monetary economy model with sticky prices populated by a representative household which derives utility from a non-tradeable final good and leisure. There is a continuum of producers of non-tradeable intermediate goods indexed by \( i \in [0, 1] \), which are used in the production of the final good, and firms that produce a tradeable input. Each firm \( i \) in the non-tradeable intermediate sector is monopolistic competitive and we assume that a fraction \( \alpha \in [0, 1] \) of them is exogenously constrained to set prices in period \( t \) conditional on the information available at most up to period \( t - 1 \) (i.e. they set prices one period in advance). The rest of the firms behave competitively.

Time is discrete and \( s_t \) denotes the state of the economy in period \( t \), which belongs to a finite set of events \( S \). \( s^t = (s_0, s_1, ..., s_t) \) is the history of events up to and including period \( t \) and we denote by \( \pi(s^t) \) the probability as of period 0 of the history \( s^t \) conditional on a given initial event \( s_0 \). Note that this formulation allows for arbitrary stochastic processes \( s_t \).

The government issues a complete set of one period nominal bonds contingent on the histories \( s^{t+1} \). Specifically, a bond indexed by the history \( s^{t+1} \) pays one unit of domestic currency conditional on \( s^{t+1} \) and zero otherwise, which costs \( Q(s^{t+1}|s^t) \) units of domestic currency. We denote by \( Q(s^\tau|s^t) \) the price of one unit of domestic currency in \( s^\tau \) in units of currency at \( s^t \), where \( Q(s^\tau|s^t) \equiv Q(s^\tau|s^{\tau-1}) \, Q(s^{\tau-1}|s^{\tau-2}) \, ... \, Q(s^{t+1}|s^t) \). We also assume that there is an international credit market where one period state
contingent securities denominated and priced in foreign currency are traded at a price $Q^*(s^{t+1}|s^t)$. An analogous definition for $Q^*(s^\tau|s^t)$ holds. Throughout the paper a superscript * is used to denote any foreign variable.

The commodities in this economy are a non-tradeable final good, labor, a continuum of non-tradeable intermediate goods and two tradeable intermediate inputs. The domestic economy is able to produce only one of the tradeable inputs and the other must be necessarily imported. Even though both inputs can be imported from abroad, to avoid confusion let us agree to call the former the home input and the latter the foreign input.

Each commodity at period $t$ is indexed by the partial history $s^t$, so the commodity space is the set of contingent sequences whose $t^{th}$ component is a function of $s^t$. As is usually assumed in the optimal taxation literature, the government has to finance an exogenous sequence of expenditures $g(s^t)$ which does not generate utility to the households. The set of policy instruments also belongs to the space of space of contingent sequences. Monetary policy consists of rules for the stock of money $M(s^t)$ and the nominal exchange rate between domestic and foreign currency $\varepsilon(s^t)$. Fiscal policy consists of linear tax rates on labor $\tau^n(s^t)$, taxes on dividends $\tau^d(s^t)$ and linear taxes on the receipts from foreign securities $\tau^*(s^t)$. The introduction of taxes on foreign securities is useful for two reasons: first, it roughly captures the idea of taxing international capital flows and second, they turn out to be important for the implementation of the Ramsey allocation (later on we discuss the consequences of eliminating those taxes). Moreover, since taxation of dividends is equivalent to lump-sum taxes, any optimal policy will have $\tau^d(s^t) = 1$. \footnote{Taxing dividends more than 100\% is not feasible since we assume that firms can produce zero in the periods where $\tau^d(s^t) > 1$ (or set an infinite price) and we focus in equilibria where all goods are produced.} To make the problem interesting, we assume that the proceeds from the dividend taxes are not enough to
finance the sequence of government consumption \( g(s^t) \) and hence, the government
must use distortionary instruments to finance the remaining part of its expenditures.
A government policy is defined as \( \Omega \equiv \{ M, \varepsilon, \tau^n, \tau^* \} \), where we use the compact
notation \( x \) to denote the contingent sequence \( \{ x(s^t) \} \) for any variable \( x \).

Preferences are described by the expected utility function:

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\left( c\left( s^t \right), n\left( s^t \right) \right) \pi\left( s^t \right)
\]  

(1)

where \( 0 < \beta < 1 \), \( c(s^t) \) and \( n(s^t) \) are consumption and labor at \( s^t \) respectively. We
introduce a cash-in-advance constraint for purchases of the final good \( c(s^t) \): of the
total value of final goods traded, we assume, as in Ireland (1996), that a stochastic
fraction \( 1/v(s^t) \) of it has to be paid with currency and the remaining fraction can
be acquired with credit to be cleared at the beginning of the next period. We follow
the timing used in Lucas (1984) where each time period is divided into two non-
overlapping trading sessions. Specifically, at the beginning of period \( t \) every agent
observes the current state \( s_t \) and the government announces the amount of money
that will be introduced into the economy or withdrawn from it during the current
period. The first session consists of trading in securities. At the beginning of the
session all previous debts and trade credits are honored and firms distribute the pre-
vious period’s dividends, if any. The securities that are traded afterwards include:
domestic and international one period nominal state contingent bonds, domestic cur-
currency and shares in the non-tradeable intermediate goods firm’s dividends. In the
second trading session, exchanges of goods, inputs and production activities are car-
ried out. The household splits into a worker-shopper pair. The worker sells labor
to the firms and the shopper acquires the household’s planned consumption for the
current period, paying a fraction \( 1/v(s^t) \) with the currency acquired in the previous
trading session and the remaining part is purchased in exchange for claims to be
honored in the securities market session of the following period. Workers and other
sellers of intermediate goods issue trade credits which are also settled at tomorrow’s
securities trading session. Finally, we assume that the market for foreign currency is
open during the whole period.

The cash in advance constraint faced by the households is

\[
\frac{P_t(s^t)}{v(s^t)} c(s^t) \leq M(s^t)
\]  

(2)

where \(P_t(s^t)\) is the price of the final good in period \(t\). The subindex \(t\) in the price level
is not redundant, since some firms set prices conditional on the information available
at period \(t - 1\). We interpret \(v(s^t)\) as a velocity shock.

Since both the set of domestic bonds and the set of foreign bonds span the set of
states \(S\), we assume, without loss of generality, that households only hold domestic
bonds. At the beginning of the period, after all debts and trade credits are honored
and household observe the current state \(s_t\), the securities trading session opens and
household choose currency \(M(s^t)\) and one period domestic bonds \(B(s^{t+1})\), subject
to the wealth constraint

\[
M(s^t) + \sum_{a_{t+1}} B(s^{t+1}) Q(s^{t+1}, s^t) \leq W(s^t),
\]  

(3)

where \(W(s^t)\) is the amount of nominal wealth after debts and trade credits are hon-
ored.\(^2\) Nominal wealth evolves according to

\[
W(s^{t+1}) = M(s^t) + B(s^{t+1}) + W(s^t) n(s^t) (1 - \tau^n(s^t)) - P_t(s^t) c(s^t).
\]  

(4)

The household’s problem is to maximize (1), by choice of \(c(s^t)\), \(n(s^t)\), \(M(s^t)\) and
\(B(s^{t+1})\) subject to (2), (3), (4) and an arbitrarily large negative lower bound for
\(B(s^{t+1})/P_t(s^t)\). The first order conditions for the household can be summarized in

\(^2\)Since we have assumed that there is full taxation of dividends, we can disregard trade in the
intermediate goods firms’ shares.
the following equations

\[- \frac{u_n(s^t)}{u_c(s^t)} = \frac{W(s^t)}{P_t(s^t)} \left( 1 - \tau^n(s^t) \right) v(s^t) \left( R(s^t) - 1 + v(s^t) \right) \]  \hspace{1cm} (5)

\[\frac{\beta u_c(s^{t+1}) \pi(s^{t+1}|s^t)}{u_c(s^t)} = \frac{P_{t+1}(s^{t+1}) Q(s^{t+1}|s^t) R(s^t) v(s^t) R(s^{t+1}) - 1 + v(s^{t+1})}{R(s^t) - 1 + v(s^t)} \]  \hspace{1cm} (6)

and the cash-in-advance constraint (2), where \( u_c(s^t) \) and \( u_n(s^t) \) denote the marginal utility of consumption and labor in period \( t \), history \( s^t \), and where \( R(s^t) \equiv \left[ \sum_{s^{t+1}} Q(s^{t+1}|s^t) \right]^{-1} \) is the nominal interest rate between periods \( t \) and \( t + 1 \).

Final goods at period \( t \) are produced from the continuum of non-tradeable intermediate goods with the constant returns to scale technology

\[ y(s^t) = \left[ \int_0^1 y(i,s^t) \frac{\theta^1}{\theta^t} \right]^{1/\theta} ; \quad \theta > 1 \]  \hspace{1cm} (7)

where \( y(s^t) \) denotes the final good, \( y(i,s^t) \) the intermediate input of variety \( i \) and \( \theta \) is the elasticity of substitution between intermediate goods. Since intermediate good producers are monopolistic competitive firms, existence of an equilibrium requires \( \theta > 1 \).

Final goods producers choose inputs \( y(i,s^t) \) and output \( y(s^t) \) to maximize

\[ P_t(s^t) y(s^t) - \int_0^1 p_t(i,s^t) y(i,s^t) \, di \]

subject to the production function (7), where \( p_t(i,s^t) \) denotes the price of the intermediate good of type \( i \) in period \( t \). This problem gives the demand functions

\[ y^d(i,s^t) = \left[ \frac{P(s^t)}{p_t(i,s^t)} \right]^\theta y(s^t) \]  \hspace{1cm} (8)

The zero profit condition determines the equilibrium price \( P_t(s^t) \) as the following function of \( p_t(i,s^t) \):

\[ P_t(s^t) = \left[ \int_0^1 p_t(i,s^t)^{1-\theta} \, di \right]^{1/(1-\theta)} \]
The technology to produce the intermediate good \( i \in [0, 1] \) is the constant returns to scale production function

\[
y (i, s^t) = F \left( n_y (i, s^t), x (i, s^t), h (i, s^t), z_y (s^t) \right)
\]  

(9)

where \( n_y (i, s^t) \) is labor, \( x (i, s^t) \) denotes the home input, \( h (i, s^t) \) the foreign input and \( z_y (s^t) \) is an aggregate productivity shock common across varieties \( i \in [0, 1] \) satisfying \( \partial F / \partial z_y > 0 \). The introduction of the two types of tradeable intermediate good is done in order to analyze how the optimal policy responds to terms of trade shocks defined as \( d^* (s^t) \equiv p^*_x (s^t) / p^*_h (s^t) \) where \( p^*_x (s^t) \) and \( p^*_h (s^t) \) denote the price in foreign currency of the home-produced tradeable intermediate good and foreign intermediate good respectively. To determine a nominal exchange rate, we assume that all international exchanges of goods are carried out in foreign currency. The domestic price in local currency of each tradeable goods are denoted by \( p_x (s^t) \) and \( p_h (s^t) \) respectively. We find it convenient to idealize the existence of intermediaries carrying out all the international exchanges of tradeable intermediate goods. The assumption that those intermediaries demand foreign currency to import goods, or receive foreign currency when they sell exports abroad does not have other effect than determining a nominal exchange rate. Since the intermediary activity consists mainly of transporting the intermediate inputs from the dock to the firms or vice-versa, absence of arbitrage opportunities implies the purchasing power parity (PPP) conditions

\[
p_x (s^t) = \varepsilon (s^t) p^*_x (s^t)
\]

\[
p_h (s^t) = \varepsilon (s^t) p^*_h (s^t).
\]

(10)

Output of the home-produced tradeable intermediate good is given by the linear production function

\[
X (s^t) = z_x (s^t) n_x (s^t)
\]
where $z_x(s^t)$ is a technology shock and $n_x(s^t)$ is labor. Competitive pricing ensures that the value of the marginal product equals the nominal wage rate, $W(s^t)$,

$$p_x(s^t)z_x(s^t) = W(s^t). \quad (11)$$

In what follows, it is convenient to characterize the unit cost of production of any non-tradeable intermediate good firm $i$. This cost minimization problem is given by

$$\min_{n_y,x,h} Wn_y + p_xx + phh$$

subject to $F(n_y, x, h, z_y) = 1$. Since the technology is constant returns to scale, the unit cost function is increasing, homogeneous of degree one and concave in the input prices, and decreasing in $z_y$. The homogeneity of degree one of the unit cost function implies that it can be written as

$$W \phi \left( \frac{p_x}{W}, \frac{px}{W}, \frac{ph}{W}, z_y \right)$$

However, using the equilibrium condition (11) and that the PPP conditions (10) imply $p_x(s^t)/p_h(s^t) = p_x^*(s^t)/p_h^*(s^t) = d^*(s^t)$, the unit cost function becomes

$$W(s^t) \phi (z_x(s^t), z_y(s^t), d^*(s^t))$$

where $\phi(\cdot)$, defined as

$$\phi (z_x, z_y, d^*) \equiv \varphi \left( 1, \frac{1}{z_x}, \frac{1}{z_xd^*}, z_y \right),$$

depends only on the exogenous shocks and satisfies $\partial \phi / \partial z_x < 0$, $\partial \phi / \partial z_y < 0$, $\partial \phi / \partial d^* < 0$ and $\partial (z_x \phi) / \partial z_x > 0$. We will find it convenient to use the shorter notation $\phi (s^t)$ instead of $\phi (z_x(s^t), z_y(s^t), d^*(s^t))$.

Moreover, the constant returns to scale assumption implies that the input ratios are the same across all non-tradeable intermediate goods firms, thus for all $i \in [0,1]$

$$\frac{x(i, s^t)}{n_y(i, s^t)} = \kappa_x(s^t) \quad \text{and} \quad \frac{h(i, s^t)}{n_y(i, s^t)} = \kappa_h(s^t). \quad (12)$$
It follows, then, that

\[ y(i, s^t) = n_y(i, s^t) \tilde{F}(s^t), \]  

(13)

where

\[ \tilde{F}(s^t) \equiv F(1, \kappa_x(s^t), \kappa_h(s^t), z_y(s^t)) \]

is the same for all types \( i \in [0, 1] \).

The industry structure in the non-tradeable intermediate goods sector is monopolistic competition as in Dixit and Stiglitz (1977). As mentioned above, we introduce price stickiness by assuming that the firms in the interval \( i \in [0, \alpha] \) are constrained to set prices at period \( t \) conditional on the information available up to period \( t - 1 \). Equivalently, we can think of those firms, called sticky firms, as setting prices one period in advance. The remaining fraction of firms, called flexible firms, is allowed to set prices at \( t \) conditional on all the information available at that period.

We now consider the problem of the intermediate goods firms. Strictly speaking, full taxation of dividends imply that the pricing and production decisions of the firms are indeterminate. We find it convenient, instead, to think of each firm maximizing after-tax dividends for \( \tau^d_j < 1 \), and then considering the limiting economy as \( \lim_{j \to \infty} \tau^d_j = 1 \) for all \( s^t \).

Flexible firms face the static optimization problem of maximizing nominal dividends

\[ \max_{p_t(i, s^t)} \left[ p_t(i, s^t) - W(s^t) \phi(s^t) \right] y^d(i, s^t) \]

subject to the demand function (8). The optimal pricing rule determines the price as a constant mark-up over the marginal cost,

\[ p_t^{fl}(s^t) \equiv p_t(i, s^t) = \frac{\theta}{\theta - 1} W(s^t) \phi(s^t) \text{ for } i \in (\alpha, 1]. \]  

(14)

Sticky firms set prices at period \( t \) conditional on information available up to period \( t - 1 \). Since \( p_t(i, s^{t-1}) \) does not depend on the new information available at period \( t \), it could be the case that along the sequence \( \tau^d_j < 1 \) for some \( j \), the firm runs negative
dividends if forced to satisfy all the demand, so it will be optimal to produce zero. On the other hand, if the price is higher than the marginal cost, demand determines production. For any finite price \( p_t (i, s_{t-1}) \), there will be positive demand of that variety, hence existence of equilibrium with finite prices requires positive production of each type. Throughout we assume that shocks are sufficiently small so that sticky firms produce positive quantities.

A sticky prices firm chooses the period \( t \) price conditional on information at \( t - 1 \). Its problem is

\[
\max_{p_t(i, s_{t-1})} \sum_{s_t} Q \left( s^t | s_{t-1} \right) \left[ p_t (i, s_{t-1}) - W \left( s^t \right) \phi \left( s^t \right) \right] y^d (i, s^t)
\]

where \( y^d (i, s^t) = p_t (i, s_{t-1})^\theta P_t (s^t)^{-\theta} y (s^t) \). The pricing rule is

\[
p_{st} (s_{t-1}) \equiv p_t (i, s_{t-1}) = \frac{\theta}{\theta - 1} \sum_{s_t} \psi (s^t) W (s^t) \phi (s^t)
\]

where

\[
\psi (s^t) \equiv \frac{Q \left( s^t | s_{t-1} \right) P_t^\theta (s^t) y (s^t)}{\sum_{s_t} Q \left( s^t | s_{t-1} \right) P_t^\theta (s^t) y (s^t)}
\]

Instead of setting a constant mark-up over the marginal cost as the flexible firms do, sticky firms set a constant mark-up over a weighted average of the marginal costs across all states.

We now describe the aggregate constraint of the economy with the rest of the world. The trade balance measured in units of the home-produced intermediate good is defined as

\[
TB (s^t) = X (s^t) - \int_0^1 x \left( i, s^t \right) di - \frac{p_{h} (s^t)}{p_x (s^t)} \int_0^1 h \left( i, s^t \right) di,
\]

where \( X (s^t) - \int_0^1 x \left( i, s^t \right) di \) are the net exports of the home produced intermediate good and \( \frac{p_{h} (s^t)}{p_x (s^t)} \int_0^1 h \left( i, s^t \right) di \) denotes the imports of foreign intermediate inputs measured in units of the home-produced intermediate good.
The equation that determines the evolution of the country’s net foreign assets at period \( t \), in foreign currency, is given by

\[
\sum_{s^{t+1}} B^* (s^{t+1}) Q^* (s^{t+1} | s^t) = B^* (s^t) + p_x^* (s^t) TB (s^t)
\]

where \( B^* (s^t) \) denotes the net holdings of foreign securities of the economy as a whole.

Solving the previous equation starting from period 0 forward and focusing on bounded real allocations, we obtain the economy foreign sector feasibility constraint,

\[
0 = B^*_0 + \sum_{t=0}^{\infty} \sum_{s^t} p_x^* (s^t) TB (s^t) Q^* (s^t | s^0)
\]

where \( B^*_0 \) is the initial holding of foreign assets.

Assuming that foreign securities are priced by risk neutral investors we obtain

\[
Q^* (s^{t+1} | s^t) = \beta \frac{p_x^* (s^t)}{p_x^* (s^{t+1})} \pi (s^{t+1} | s^t)
\]

where it was assumed that foreign investors have the same discount factor \( \beta \) as domestic households. Thus, the foreign sector constraint becomes

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t TB (s^t) \pi (s^t) = -\frac{B^*_0}{p^*_0} \tag{17}
\]

Absence of arbitrage opportunities between domestic and foreign bonds imply

\[
Q (s^{t+1} | s^t) = \frac{Q^* (s^{t+1} | s^t)}{(1 - \tau^+ (s^{t+1}))} \frac{\epsilon(s^t)}{\epsilon(s^{t+1})} \tag{18}
\]

Final goods market-clearing is given by

\[
c (s^t) + g (s^t) = y (s^t) \tag{19}
\]

and market clearing in the labor market requires

\[
n (s^t) = n_x (s^t) + \int_0^1 n_y (i, s^t) \, di. \tag{20}
\]
An allocation $\mathcal{R}$ and a price system $\mathcal{P}$ are contingent sequences $\mathcal{R} = \{c, n, M, B, B^*, n_x, n_y(i), x(i), h(i), X, y^d(i)\}$ and $\mathcal{P} = \{Q, Q^*, p_x, p_m, p(i), W\}$ for $i \in [0, 1]$. Given a government policy $\Omega$, an allocation $\mathcal{R}$ and a price system $\mathcal{P}$ are an equilibrium if (i) households solve their utility maximization problem; (ii) the price of the home input satisfies the competitive pricing equation (11); (iii) final goods producers solve their maximization problem; (iv) intermediate goods producers act optimally, that is, (12) hold for all $i \in [0, 1]$ and they follow the pricing rules (14) if $i \in (\alpha, 1]$ and (15) if $i \in [0, \alpha]$; (v) the final goods and input market clearing conditions (19), (20) are satisfied; (vi) the economy-wide feasibility constraint (17) holds; and (vii) the no-arbitrage conditions (18) and (10) hold.\footnote{By Walras' Law, the budget constraint of the government, implicit in the previous equations, is also satisfied.}

**EQUILIBRIUM ALLOCATIONS: NECESSARY CONDITIONS**

An allocation rule $\mathcal{R}(\Omega)$ is the equilibrium mapping from the set of policies $\Omega$ into allocations (which of course, can be empty). The Ramsey problem consists choosing a policy $\Omega$ such that the resulting allocation rule $\mathcal{R}(\Omega)$ maximizes the household’s utility among the set of equilibrium allocations induced by the policy $\Omega$. The strategy we follow below consists in deriving a set of necessary conditions that an equilibrium allocation $\mathcal{R}(\Omega)$ has to satisfy. We then proceed to maximize the utility of the household subject to the set of allocations derived from the necessary conditions. Since those conditions need not be sufficient for an equilibrium, the obtained allocation solves a relaxed problem that may not be implementable as an equilibrium. We then show that the allocation is an equilibrium in our environment and hence, it is indeed the optimal Ramsey allocation. For the rest of the paper, we assume that $B_0 = 0$ and $B^*_0 = 0$, so that there is no initial wealth to tax.
The next proposition takes derives the necessary conditions that an equilibrium allocation has to satisfy:

**Proposition 1:** If an equilibrium exists for a given policy \( \Omega \), then the equilibrium allocation \( \alpha(\Omega) \) satisfies the following three conditions:

i) **Feasibility,**

\[
c(s^t) + g(s^t) = \tilde{F}(s^t) \left[ \alpha n_y^{st}(s^t) \frac{\theta - 1}{\pi} + (1 - \alpha) n_y^{fl}(s^t) \frac{\theta - 1}{\pi} \right] \frac{\theta}{\pi} \tag{21}
\]

where \( \tilde{F}_t(s^t) \) was defined in (13).

ii) **Current account sustainability,**

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t TB(s^t) \pi(s^t) = 0 \tag{22}
\]

where

\[
TB(s^t) = z_x(s^t) \left[ n(s^t) - \left[ \alpha n_y^{st}(s^t) + (1 - \alpha) n_y^{fl}(s^t) \right] \tilde{F}(s^t) \phi(s^t) \right]
\]

and iii)

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left[ u_c(s^t) c(s^t) + u_n(s^t) n(s^t) \right] \pi(s^t) = 0 \tag{23}
\]

**Proof:** In the appendix.

Condition iii) follows from the household’s intertemporal budget constraint and it includes the optimization problem of all the agents in the economy. Interestingly, note that neither i), ii) nor iii) include the velocity shocks \( v(s^t) \). This implies that the (relaxed) optimal allocation will be independent of it, in particular, monetary policy will respond to accommodate any change in \( v(s^t) \). Correia, Nicolini and Teles (2004) go one step further and prove, in a closed economy version of this model, that in fact, the previous conditions are sufficient to characterize all allocations that any planner with preferences increasing in \( c \) and decreasing in \( n \) would choose.
Remark: Using standard arguments (see for example, Chari, Christiano and Kehoe 199X) it can be shown that with the available policy instruments, i), ii) and iii) above are necessary and sufficient conditions for an equilibrium allocation in a flexible prices economy. In other words, an equilibrium with flexible prices satisfy those conditions and conversely, if an allocation satisfies i), ii) and iii) above, then there are prices and government policies which implement that allocation as an equilibrium with flexible prices.

THE RAMSEY PROBLEM

We assume that there is a commitment technology through which the government binds itself forever to a particular policy $\Omega$ chosen at period 0. Also, the government is able to make rational forecasts about future actions and prices, in particular, the government knows the allocation rule $N(\Omega)$ mapping policies to allocations.

The relaxed Ramsey problem is to maximize (1), by choice of $\{c, n, n_x, n_y^{st}, n_y^{fl}\}$, subject to the necessary conditions (21), (22) and (23). The first order conditions of this problem imply, after some algebra,

\begin{equation}
uc(s^t) \left[ 1 + \lambda \frac{u_{nc}(s^t)c(s^t)}{u_c(s^t)} \right] + \lambda u_{nc}(s^t) n(s^t) = \eta z_x(s^t) \phi(s^t) \tag{24}
\end{equation}

\begin{equation}
un(s^t) \left[ 1 + \lambda \frac{u_{nn}(s^t)n(s^t)}{u_n(s^t)} \right] + \lambda u_{cn}(s^t) c(s^t) = \eta z_x(s^t) \tag{25}
\end{equation}

\[n_y^{st}(s^t) = n_y^{fl}(s^t)\]

where $\lambda$ is the Lagrange multiplier on (23) and $\eta$ is the multiplier on (22).

Since $n_y^{st}(s^t) = n_y^{fl}(s^t) = n_y(s^t)$, feasibility can be written as

\[c(s^t) + g(s^t) = n_y(s^t) \tilde{F}(s^t). \tag{26}\]

Equations (24), (25) and (26) together with constraints (22) and (23) completely
characterize the relaxed Ramsey allocation, where the trade balance becomes

\[ TB\left(s^t\right) = z_x\left(s^t\right) \left[n\left(s^t\right) - n_y\left(s^t\right) \tilde{F}\left(s^t\right) \phi\left(s^t\right)\right]. \quad (27) \]

Specifically, equations (24) and (25) can be used to solve for consumption and labor as a function of the multipliers \(\lambda\) and \(\eta\). Then we can use (26) and the labor market clearing condition to solve for \(n_y\left(s^t\right)\) and \(n_x\left(s^t\right)\) as a function of \(\lambda\) and \(\eta\). At this point the allocation is expressed as a function of the multipliers. The present value constraints (22) and (23) can be used to find the values for \(\eta\) and \(\lambda\).

Since the set of equilibrium allocations is included in the set of allocations that satisfy conditions i), ii) and iii) of Proposition 1, it follows that if the outcome of the relaxed Ramsey problem can be implemented as an equilibrium allocation, then it is the optimal allocation. We then have,

**Proposition 2:** The relaxed Ramsey allocation can be implemented as an equilibrium with sticky prices and hence it is, indeed, the Ramsey allocation.

**Proof:** In the appendix.

In the optimal allocation \(y^{st}\left(s^t\right) = y^{fl}\left(s^t\right)\), then it follows from (8) that \(p^{st}_t\left(s^{t-1}\right) = p^{fl}_t\left(s^t\right) = P_t\left(s^t\right)\) for all \(s_t\). In other words, the government undoes the price stickiness by manipulating policy instruments to make the price level fully predictable one period in advance. This result is reminiscent to the Diamond and Mirrless (1971) prescription according to which it is not optimal to tax intermediate goods. In fact, the different prices of the varieties \(i \in [0,1]\) due to price stickiness is isomorphic to an economy with flexible prices but in which a tax on the firms \(i \in [0,\alpha]\) is included. Undoing the price stickiness is equivalent to eliminating that tax rate (see Correia, Nicolini and Teles (2004) for a further development of this idea).

Note that velocity shocks \(v\left(s^t\right)\) and the fraction of sticky firms \(\alpha\) in the economy are irrelevant for the Ramsey allocation. It also follows from (24), (25), (26) and (27)
that the Ramsey allocation \( c(s^t) \), \( n_x(s^t) \), \( n_y(s^t) \), \( n(s^t) \) and \( TB(s^t) \) at any \( s^t \) only depends on the realization of the stochastic processes dated at period \( t \), as government purchases \( g(s^t) \), productivity shocks \( z_x(s^t) \) and \( z_y(s^t) \), and the terms of trade shock \( d^*(s^t) \) and not on the history of realizations.

Further, government expenditure shocks do not affect \( c(s^t) \) nor \( n(s^t) \). The proof follows directly from the fact that \( c(s^t) \) and \( n(s^t) \) can be solved as a function of the shocks \( z_x(s^t) \), \( z_y(s^t) \) and \( d^*(s^t) \), and of the two multipliers \( \lambda \) and \( \eta \), which do not depend on the particular realization of \( g(s^t) \). Assume, for example, that government expenditures increases: equation (26) shows that in order to keep consumption constant, the amount of labor allocated to the production of final goods has to increase to meet the higher government expenditures. Equation (27) shows that the trade balance decreases. In other words, the availability of international credit allows the planner to insulate all government expenditure shocks through borrowing and lending. Of course, more final goods have to be produced, so more labor is allocated to the intermediate goods sector. Hence the trade balance decreases for two reasons: first, intermediate goods firms demand more tradeable inputs of both types and second, at the same time the country reduces the production of the home inputs to allocate more labor to the production of intermediate goods and hence, to final goods.

In what follows we find it convenient to assume that utility is separable between consumption and labor,

\[
    u(c, n) = U(c) - V(n)
\]

where \( U'(c) > 0 \), \( U''(c) < 0 \), \( V'(n) > 0 \) and \( V''(n) > 0 \). The separability of consumption and labor turns out to be a convenient assumption. In this case the first order conditions with respect to \( c(s^t) \) and \( n(s^t) \) become

\[
    U'(c(s^t)) \left[ 1 + \lambda \left( 1 + \frac{U''(c(s^t)) c(s^t)}{U'(c(s^t))} \right) \right] = \eta z_x(s^t) \phi(s^t)
\]

(28)

\[
    V'(n(s^t)) \left[ 1 + \lambda \left( 1 + \frac{V''(n(s^t)) n(s^t)}{V'(n(s^t))} \right) \right] = \eta z_x(s^t)
\]

(29)
We further assume that the left side of equation (28) is decreasing in \( c(s') \) and the left side of (29) is increasing in \( n(s') \).\(^4\)

We now study how the optimal allocation changes with the different shocks. Start with a positive shock to the intermediate goods technology \( z^y(s') \). It follows from equations (28) and (29), and the fact that \( \phi(s') \) is decreasing in \( z_y(s') \), that \( c(s') \) increases and \( n(s') \) remains constant. The exact reallocation of labor between sectors and the change in the trade balance is indeterminate. A positive shock to the terms of trade \( d^* (s') = p^*_x (s') / p^*_h (s') \), has qualitatively the same effects as a productivity shock to the final goods technology. Indeed, both work through a reduction in the marginal cost of production in the final goods sector.

A positive shock to the home input technology \( z_x(s') \) has a different effect. Equation (28) implies that \( n(s') \) increases, and given that \( z_x(s') \phi(s') \) is increasing in \( z_x (s') \), \( c(s') \) decreases. Not only labor is reallocated from the intermediate goods sector to the tradeable input sector but it is also optimal to increase aggregate labor as a whole. The trade balance increases for two reasons: intermediate goods firms reduce the demand of tradeable inputs and total production of the home input increases. Intuitively, the shock to the tradeable sector determines when it is good times to export and when it is not. When the country’s productivity in the tradeable sector increases, it is optimal to allocate more labor to it, to reduce labor allocated to the intermediate goods sector and hence, to reduce consumption. Further, since consumption and leisure are both normal, leisure also decreases. The next table summarizes the results.

\(^4\)If \( U'' (C) C / U' (C) \) and \( V'' (N) N / V' (N) \) are roughly constant, this requirement is satisfied.
We now study the policy implications of the optimal allocation. Questions such as under what circumstances flexible exchange rates are superior to fixed exchange rates or vice-versa follows from our analysis.

As we showed in the proof of proposition 2, there is an indeterminacy in the implementation of the Ramsey allocation. In particular, there is an equilibrium which implements the Ramsey allocation for any sequence of interest rates \( R(s^t) \). (It is straightforward to eliminate this indeterminacy: by extending the model to incorporate a cash-credit good setting, as in Lucas and Stokey (1983), the indeterminacy disappears.) In our case it is convenient to focus on the equilibrium where \( R(s^t) = 1 \) (i.e. the Friedman rule), since it is the only equilibrium where the labor tax rate and the price level of the final good \( P_t(s^{t-1}) \) does not depend on the velocity shock \( v(s^t) \). At the Friedman rule households are indifferent about what level of money to choose and the cash-in-advance ceases to bind. We focus on the equilibrium where, in fact, the cash in advance constraint holds at equality.

In the proof of proposition 2 we provide the decentralization for any sequence of interest rates \( R(s^t) \). Here we write down the relevant equations when the government follows the rule \( R(s^t) = 1 \) for all \( s^t \). The equations that determine the decentralization

<table>
<thead>
<tr>
<th>( \uparrow z_x(s^t) )</th>
<th>( c(s^t) )</th>
<th>( n(s^t) )</th>
<th>( n_x(s^t) )</th>
<th>( n_y(s^t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \uparrow z_y(s^t), d^x(s^t) )</td>
<td>+</td>
<td>=</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( \uparrow g(s^t) )</td>
<td>=</td>
<td>=</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>( \uparrow v(s^t) )</td>
<td>=</td>
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</table>

**DECENTRALIZATION**
of the Ramsey allocation if $\alpha > 0$ are given by, $P_0 (s^{-1}) = p_0^{st}$,

$$P_{t+1} (s^t) = P_t (s^{t-1}) \sum_{s_{t+1}} \beta \frac{U' (c (s^{t+1})) \pi (s^{t+1} | s^t)}{U' (c (s^t))} \tag{30}$$

$$M (s^t) = \frac{P_t (s^{t-1}) c (s^t)}{v (s^t)} \tag{31}$$

$$\frac{V' (n (s^t))}{U' (c (s^t))} = \left( \frac{\theta - 1}{\theta} \right) \frac{1 - \tau^n (s^t)}{\phi (s^t)} \tag{32}$$

$$W (s^t) = \frac{\theta - 1}{\theta \phi (s^t)} P_t (s^{t-1}) \tag{33}$$

The decentralization of the equilibrium is obtained recursively as follows: Given the initial price $P_0 (s^{-1}) = p_0^{st}$, the first equation is the condition that justifies the interest rate $R (s^t) = 1$ and determines recursively the price of the final good. The cash in advance condition (31) determines the money supply at period $t$ that justifies the price $P_t (s^{t-1})$ given the allocation and the velocity shock. Then (32) pins down the labor tax rate, and (33) determines the equilibrium wage rate. The price of the home input follows from the pricing equation (11). Finally, the exchange rate and the domestic price of the foreign input follow from the PPP conditions (10). Note that even though the final good price is sticky, the rest of the prices (i.e. wage rate, exchange rate and prices of inputs) do depend on the current realization of the state. Given the previous equilibrium prices and policies, $Q (s^{t+1} | s^t)$ follows from (6) and the tax on foreign bonds is determined from (18).

Since velocity shocks are irrelevant in the Ramsey allocation, the government accommodates the money supply to eliminate any fluctuation in $v (s^t)$, as can be seen in equation (31). Note, further, that the rest of the policies and nominal prices are also independent of the velocity shocks; in particular, the exchange rate $\varepsilon (s^t)$ does not vary with $v (s^t)$. If we interpret velocity shocks as monetary shocks (or LM shocks), as in Lahiri and Vegh (2003), the previous result is consistent with the view that the exchange rate should not respond to monetary shocks.
We consider now a positive shocks to the intermediate goods technology \( z_y(s^t) \). The Ramsey allocation prescribes increasing consumption and letting the aggregate amount of labor constant. Since the final good price \( P_t(s^{t-1}) \) is given, it follows from (31) that the stock of money \( M(s^t) \) increases. Further, since \( \phi(s^t) \) decreases with \( z_y(s^t) \), it follows from (33) that the nominal wage increases and hence, from (11) and (10) the input prices \( p_x(s^t) \) and \( p_h(s^t) \) increase and the nominal exchange rate \( \varepsilon(s^t) \) also increases.

Now assume that there is an increase in the terms of trade \( d^*(s^t) = p^*_x(s^t) / p^*_h(s^t) \). The qualitative effect on the optimal allocation is, as shown in the previous section, identical to a positive shock to the final goods technology \( z_y(s^t) \), namely, \( c(s^t) \) increases and \( n(s^t) \) remains constant. As in the previous case, the increase in \( c(s^t) \) is attained through an increase in the stock of money \( M(s^t) \), the nominal wage increases and the domestic price of the home input also increases. The behavior of the exchange rate, however, depends on how the increase in the terms of trade is made: through an increase in \( p^*_x(s^t) \), a decline in \( p^*_h(s^t) \), or some other change such that the \( d^*(s^t) \) increases. According to the PPP equation (10) for good \( x \), if the increase in the terms of trade is driven by an increase in \( p^*_x(s^t) \), the movement in the exchange rate and the domestic price of the home input is indeterminate. If, however, the increase in the terms of trade is driven by a decline in the \( p^*_h(s^t) \), equation (10) for the good \( h \) and the fact that \( p_x(s^t) \) increases imply that \( \varepsilon(s^t) \) must go up. The domestic price of the foreign input could either increase or decrease.

Finally consider a positive shock to the home input technology \( z_x(s^t) \). The solution to the Ramsey problem implies that aggregate labor increases, and consumption decreases. Since the price level remains constant, (31) implies that the government withdraws money from the economy. Since \( \phi(s^t) \) decreases with \( z_x(s^t) \), (33) implies that the nominal wage rate increases. Also, the domestic price of the home input
decreases. To see this, note that from (33) and (11) we obtain

\[ P(s^{t-1}) = \frac{\theta}{\theta - 1} p_x(s^t) z_x(s^t) \phi(s^t), \]

hence, since \( z_x(s^t) \phi(s^t) \) increases with \( z_x(s^t) \), \( p_x(s^t) \) has to decrease. Further, from the PPP conditions (10) imply that both, the nominal exchange rate \( \varepsilon(s^t) \) and the domestic price of the foreign input decline.

As can be seen from the previous analysis, the exchange rate implied by the optimal allocation does change in the presence of the real shocks \( z_y, z_x, p_x^* \) and \( p_h^* \). This conclusion is also consistent with the traditional view that the exchange rate should vary when real shocks hit the economy.

The following table summarizes the findings of this section.

<table>
<thead>
<tr>
<th>( M(s^t) )</th>
<th>( \varepsilon(s^t) )</th>
<th>( \tau^n(s^t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \uparrow z_x(s^t) )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \uparrow z_y(s^t), d^*(s^t) )</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \uparrow g(s^t) )</td>
<td>=</td>
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<tr>
<td>( \uparrow v(s^t) )</td>
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</table>
Appendix

Proof of proposition 1:

We now prove that $i)$, $ii)$ and $iii)$ have to be satisfied in any equilibrium. Condition $i)$ follows by using (7) and (13) into the necessary condition (19). Condition $ii)$ follows from the definition of equilibrium, and the equality for the trade balance follows from (12), (20) and by noting that the cost minimization problem of the intermediate good firms implies

$$z_x (s^t) \tilde{F} (s^t) \phi (s^t) = \left[ z_x (s^t) + \kappa_x (s^t) + \frac{\kappa_h (s^t)}{d^x (s^t)} \right]$$

We now show that $iii)$ is necessary. Use (3) and (4) to obtain the household’s intertemporal budget constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} \frac{Q (s^t | s^0)}{R (s^t)} \left[ P (s^t) c (s^t) + M (s^t) [R (s^t) - 1] - W (s^t) (1 - \tau_n (s^t)) n (s^t) \right] = 0$$

Using the cash in advance constraint (2) at equality and rearranging we find

$$0 = \sum_{t=0}^{\infty} \sum_{s^t} \frac{Q (s^t | s^0)}{R (s^t)} \left\{ P (s^t) c (s^t) \frac{R (s^t) - 1 + v (s^t)}{v (s^t)} - W (s^t) (1 - \tau_n (s^t)) n (s^t) \right\}$$

Introducing (5) and (6) into the previous equation we obtain (23) QED.

Proof of proposition 2:

We now prove that there is an equilibrium which is consistent with the allocation of the relaxed Ramsey problem. Below we find a price system $P$ and a government policy $\Omega$ such that the proposed allocation satisfies conditions $i)$ to $vii)$ in the definition of equilibrium.

Assume first that $\alpha > 0$ and pick an arbitrary sequence $R (s^t)$. The relaxed Ramsey allocation implies that the price level is fully predictable and $P_t (s^{t-1}) = p^{lt}_t (s^{t-1}) = p^{lt}_t (s^{t-1})$. Given $P_t (s^{t-1})$, equation (14) determines the nominal wage

$$W (s^t) = \frac{\theta - 1}{\theta \phi (s^t)} P_t (s^{t-1}) \quad (A1)$$
then (11) determines \( p^* (s^t) \) and the PPP conditions (10) pin down the nominal exchange rate \( \varepsilon (s^t) \) and the price \( p^h (s^t) \). Introducing (A1) into (5) we obtain the labor tax rate \( \tau^* (s^t) \) consistent with the household’s intratemporal condition. Given any sequence of prices \( P_t (s^{t-1}) \) and the arbitrary sequence \( R (s^t) \), the household’s condition (6) can be used to find the prices \( Q (s^{t+1}|s^t) \), which are given by

\[
Q (s^{t+1}|s^t) = \frac{\beta \pi (s^{t+1}|s^t) u_c (s^{t+1}) R (s^{t+1}) v (s^{t+1})}{P_{t+1} (s^{t+1}) [R (s^{t+1}) - 1 + v (s^{t+1})]} \frac{P_t (s^t) [R (s^t) - 1 + v (s^t)]}{u_c (s^t) R (s^t) v (s^t)} \tag{A2}
\]

We have the additional restriction \( \sum_{s_{t+1}} Q (s^{t+1}|s^t) = 1/R (s^t) \) that has to be satisfied. Summing the previous equation over all \( s_{t+1} \) and rearranging we obtain

\[
\frac{u_c (s^t) v (s^t)}{P_t (s^t) [R (s^t) - 1 + v (s^t)]} = \sum_{s_{t+1}} \frac{\beta \pi (s^{t+1}|s^t) u_c (s^{t+1}) R (s^{t+1}) v (s^{t+1})}{P_{t+1} (s^{t+1}) [R (s^{t+1}) - 1 + v (s^{t+1})]} \tag{A3}
\]

When \( \alpha > 0 \), the price at \( t \) depends on \( s^{t-1} \), hence the previous equation can be written as

\[
P_{t+1} (s^t) = P_t (s^{t-1}) \frac{R (s^t) - 1 + v (s^t)}{u_c (s^t) v (s^t)} \sum_{s_{t+1}} \frac{\beta \pi (s^{t+1}|s^t) u_c (s^{t+1}) R (s^{t+1}) v (s^{t+1})}{[R (s^{t+1}) - 1 + v (s^{t+1})]} \tag{A4}
\]

Given \( P_0 (s^{-1}) = p_0^s \), the allocation and the arbitrary sequence \( R (s^t) \), (A4) determines the whole sequence of prices \( P_t (s^{t-1}) \) which justifies the sequence \( R (s^t) \). Furthermore, the price levels \( P_t (s^{t-1}) \) can be justified by setting \( M (s^t) \) according to

\[
M (s^t) = \frac{c (s^t)}{v (s^t)} P_t (s^{t-1}) \tag{A5}
\]

Given the nominal exchange rate \( \varepsilon (s^t) \) and the price \( Q (s^{t+1}|s^t) \) obtained above, the arbitrage condition (18) pins down the tax on foreign bonds \( \tau^* (s^t) \).

Note that by construction, flexible and sticky prices firms will find it optimal to set \( p_t^s (s^{t-1}) = p_t^f (s^{t-1}) = P_t (s^{t-1}) \). Given \( c \) and \( g \), use (26) to obtain \( n_g (s^t) \), the demand for inputs \( x (i, s^t) \) and \( h (i, s^t) \) follow from (12) and \( n_x (s^t) \) follows from the labor market clearing condition (20). The holdings of domestic bonds follows from
solving the household’s budget constraint at period $t$ forward:

$$B(s^t) = \sum_{j=t}^{\infty} \sum_{s^j|s^t} \frac{Q(s^j|s^t)}{R(s^j)} \left[ P_j(s^{j-1}) c(s^j) + M(s^j) (R(s^j) - 1) - W(s^j) n(s^j) (1 - \tau^n(s^j)) \right]$$

and the holdings of foreign bonds follows from solving the economy-wide constraint starting from period $t$ forward:

$$B^*(s^t) = -\sum_{j=t}^{\infty} \sum_{s^j|s^t} p^*_x(s^j) z_x(s^j) \left[ n(s^j) - n_y(s^j) \hat{F}(s^j) \phi(s^j) \right] Q^*(s^j|s^t)$$

Since all the conditions of an equilibrium are satisfied, the relaxed Ramsey allocation is implementable and therefore it is, indeed, the optimal allocation.

If $\alpha = 0$ (i.e. in a flexible prices economy), prices need not be fully predictable. Besides the indeterminacy in the nominal interest rate $R(s^t)$, there is indeterminacy in the price level, since any price sequence $P_t(s^t)$ satisfying (A3) is implementable as an equilibrium, in particular the one derived for the case $\alpha > 0$.

An easy way to eliminate the indeterminacy of the nominal interest rate $R(s^t)$, is by adding a credit good. The marginal rate of substitution between cash and credit goods pins down the interest rate, as shown in Lucas and Stokey (1983). QED.
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