



In this paper we are interested in studying the macroeconomic determinants of asset prices given by a multi-input aggregate production technology. We focus exclusively on the producers' first-order conditions that link production variables and state-prices, with sectoral investment playing the key role. We are interested in two sets of questions. First: what properties of investment and production technologies are important for the first and second moments of risk free rates and aggregate equity returns? Second: does a model plausibly calibrated to the U.S. economy have the ability to replicate first and second moments of risk free rates and aggregate equity returns?

The work most closely related to ours is Cochrane's work on production-based asset pricing (1988, 1991). Some of the features that differentiate our work are that we focus explicitly on the equity premium, we use more general functional forms for adjustment cost, and we base our empirical evaluation on the two main sectoral aggregates of U.S. capital investment, namely equipment & software as well as structures. Cochrane (1993) derives a set of asset pricing implications of a production function where the productivity level can be selected in a state-contingent way.

We consider the problem of a representative producer that selects multiple fixed input factors. In order to be able to recover a state-price process, our setup needs to have two related properties. First, markets need to be complete and the producer has to face a full set of state-prices. Second, there needs to be as many predetermined state variables (fixed production factors) as there are states of nature. This assumption of "complete technologies" is necessary in order to be able to recover the full set of state-contingent prices from the production side. In most studies with nontrivial production sectors this property is not satisfied; of course, in a general equilibrium environment it doesn't usually play such an important role.

We calibrate our model to a two-sector representation. We use U.S. data on investment for equipment and software, as well as for structures. This sectoral representation is convenient because these two sectors have natural asymmetries. Indeed, we use the plausible assumption that the capital stock for structures is more difficult to adjust than for equipment and software. As becomes clear below, asymmetries across sectors are crucial for asset prices.

We characterize sectoral asymmetries that ensure that state-prices are positive and that generate a positive equity premium. Specifically, we present a closed form expression for the Sharpe ratio at steady-state as a function of investment volatility and adjustment cost curvature only. We also show that there is an upper bound to the Sharpe ratio more generally in two-state en-  

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(1999), Constantinides (1990); for models with nontrivial production sectors Jermann (1998) and Rouwenhorst (1995); for production asset pricing studies, Cochrane (1988, 1991), Li, Vassalou and Xing (2003), and Gomes, Yaron and Zhang (2002). Other examples of related asset pricing studies with rich production structures are Berk, Green and Naik (1999), Carlson, Fisher and Giammarino (2003), Hugonnier, Morellec and Sundaresan (2005), and Tuzel (2004).

vironments. Our key quantitative findings are the following. For unconditional moments, we can plausibly generate an equity premium of several percentage points with risk free rates having a reasonable mean and volatility. For conditional moments, the expected excess equity return is quite volatile, usually more volatile than the risk free rate. Also concerning excess returns, the correlation between conditional means and volatilities is negative.

The paper is organized as follows. In section 1, we present the model and in section 2 some asset pricing elements. Section 3 introduces functional forms. In section 4 we characterize theoretical links between asset prices and investment. Section 5 contains our calibration and section 6 the quantitative analysis.

## 1 Model

The model represents the producer's choice of capital inputs for a given state price process. Key ingredients are capital adjustment cost and stochastic productivity.

Assume an environment where uncertainty is modelled as the realization of  $s$ , one out of a finite set  $S = (\mathfrak{s}_1, \mathfrak{s}_2, \dots, \mathfrak{s}_N)$ , with  $s_t$  the current period realization and  $s^t \equiv (s_0, s_1, \dots, s_t)$  the history up to and including  $t$ . Probabilities of  $s^t$  are denoted by  $\pi(s^t)$ . Assume an aggregate production function

$$Y(s^t) = F\left(\{K_j(s^{t-1})\}_{j \in J}, s^t, N(s^t)\right),$$

$s^t$  introduces possible stochastic technology in the production of the final good,  $K_j$  is the  $j$ -th capital stock,  $N$  labor. Capital accumulation for capital good of type  $j$  is represented by

$$K_j(s^t) = K_j(s^{t-1})(1 - \delta_j) + Z_j(s^t) I_j(s^t),$$

where  $\delta_j$  is the depreciation rate and  $Z_j(s^t)$  the (stochastic) technology for producing capital goods. We assume  $Z_j(s^t) = Z_j(s^{t-1}) \cdot \lambda^{Z_j}(s_t)$ , with  $\lambda^{Z_j}(s_t)$  following a  $N$ -state Markov process. The total cost of investment in capital good of type  $j$  is given by

$$H_j(K_j(s^{t-1}), I_j(s^t), Z_j(s^t)).$$

This specification will be further specialized below.

The representative firm solves the following problem taking as given state prices  $P(s^t)$

$$\begin{aligned} \max_{\{I, K', N\}} \sum_{t=0}^{\infty} \sum_{s^t} P(s^t) & \left[ F\left(\{K_j(s^{t-1})\}_{j \in J}, s^t, N(s^t)\right) - w(s^t) N(s^t) - \sum_j H_j(K_j(s^{t-1}), I_j(s^t), Z_j(s^t)) \right] \\ \text{s.t. : } [P(s^t) q_j(s^t)] & : K_j(s^{t-1})(1 - \delta_j) + Z_j(s^t) I_j(s^t) - K_j(s^t) \geq 0, \forall s^t, j \end{aligned}$$

with  $s_0$  and  $K_j(s_{-1})$  given and  $P(s_0) = 1$ , without loss of generality. The scaling of the multipliers is chosen so that we get intuitive labels. Indeed,  $q$  represents the marginal value of one unit of installed capital in terms of the numeraire of the same period. In equilibrium, it is also the cost of installing one unit of capital including adjustment cost. Note, for U.S. data on equipment and software,  $Z$  has a strong positive growth trend. Then,  $q$  in this sector will be trending down—reflecting the fact that equipment and software become cheaper over time. Also note that  $q$  is not the ratio of the market value over the book value of capital, that is Tobin’s  $Q$ , but  $qZ$  is. The book value (or replacement cost) of the capital stock is  $K/Z$ , where  $K$  is number of units of the capital good and  $1/Z = p_I$  the price of a unit of capital in terms of the final good.

First-order conditions are summarized by

$$0 = -H_{j,2}(K_j(s^{t-1}), I_j(s^t), Z_j(s^t)) + Z_j(s^t) q_j(s^t),$$

$$q_j(s^t) = \sum_{s_{t+1}} \frac{P(s^t, s_{t+1})}{P(s^t)}$$

$$\cdot \left\{ F_{K_j}(\{K_i(s^t)\}_{i \in J}, s^t, s_{t+1}, N(s^t, s_{t+1})) - H_{j,1}(K_j(s^t), I_j(s^t, s_{t+1}), Z_j(s^t, s_{t+1})) + (1 - \delta_j) q_j(s^t, s_{t+1}) \right\}$$

and for  $N$ ,

$$F_N(\{K_j(s^{t-1})\}, s^t, N(s^t)) - w(s^t) = 0$$

so that substituting out shadow prices we have

$$\sum_{s_{t+1}} P(s_{t+1}|s^t) \left[ \begin{array}{c} F_{K_j}(\{K_j(s^t)\}, s^t, s_{t+1}, N(s^t, s_{t+1})) \\ -H_{j,1}(K_j(s^t), I_j(s^t, s_{t+1}), Z_j(s^t, s_{t+1})) + (1 - \delta_j) \frac{H_{j,2}(K_j(s^t), I_j(s^t, s_{t+1}), Z_j(s^t, s_{t+1}))}{Z_j(s^t, s_{t+1})} \end{array} \right] \cdot \left[ \frac{Z_j(s^t)}{H_{j,2}(K_j(s^{t-1}), I_j(s^t), Z_j(s^t))} \right] = 1,$$

for each  $j$ , where the notation  $P(s_{t+1}|s^t)$  shows the price of the numeraire in  $s_{t+1}$  conditional on  $s^t$  and in units of the numeraire at  $s^t$ . From this condition we define the investment return  $R_j^I(s^t, s_{t+1})$  implicitly through  $\sum_{s_{t+1}} P(s_{t+1}|s^t) R_j^I(s^t, s_{t+1}) = 1$ .  $R_j^I(s^t, s_{t+1})$  is the return we get in  $s_{t+1}$  from adding one (marginal) unit of capital of type  $j$  in state  $s^t$ . The first-order condition shows that in equilibrium adding one marginal unit of a given type of capital produces a change in the profit plan that is worth one unit.<sup>2</sup>

We will specialize the model to have 2 capital inputs and 2 states of nature in each period. In addition to complete markets, that is the producers ability to sell contingent output for each state of nature, we also need to satisfy the requirement of “complete technologies”, that is the ability to move resources independently between all states of nature. This complete technology requirement is needed if we want to be able to recover all state prices from the producers first-order conditions.

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<sup>2</sup>Based on the details given below, strict concavity will be assumed, so that first-order and transversality conditions (given below) are sufficient for a maximum.

Representing the first-order conditions in matrix form we have

$$\begin{bmatrix} R_1^I(s^t, \mathfrak{s}_1) & R_1^I(s^t, \mathfrak{s}_2) \\ R_2^I(s^t, \mathfrak{s}_1) & R_2^I(s^t, \mathfrak{s}_2) \end{bmatrix} \begin{bmatrix} P(\mathfrak{s}_1|s^t) \\ P(\mathfrak{s}_2|s^t) \end{bmatrix} = \mathbf{1}, \text{ or compactly: } R^I(s^t) \cdot p(s^t) = \mathbf{1} \quad (2.1)$$

so that the state price vector is obtained from matrix inversion

$$p(s^t) = (R^I(s^t))^{-1} \mathbf{1}.$$

Clearly, it isn't necessarily the case that this matrix inversion is feasible nor that state prices are necessarily positive for any chosen set of returns. As further discussed below, the requirement for positive state prices will constrain our empirical implementation.

In this environment, the risk free return is given by:

$$1/R^f(s^t) = \mathbf{1}p(s^t) = P(\mathfrak{s}_1|s^t) + P(\mathfrak{s}_2|s^t).$$

It is easy to check the matrix algebra to see that if for one of the investment returns the realized return is not state-contingent  $R_j^I(s^t, s_{t+1}) = R_j^I(s^t)$ , then, as is implied by no-arbitrage, it equals the risk free rate,  $R_j^I(s^t) = R^f(s^t)$ .

Now, consider aggregate equity returns

$$R(s^t, s_{t+1}) = \frac{D(s^t, s_{t+1}) + V(s^t, s_{t+1})}{V(s^t)},$$

where  $D(s^t, s_{t+1}) = F(\{K_j(s^{t-1})\}, s^t, N(s^t)) - w(s^t)N(s^t) - \sum_j H_j(K_j(s^{t-1}), I_j(s^t), Z_j(s^t))$  represents the dividends paid by the firm and  $V(s^t, s_{t+1})$  the ex-dividend value of the firm. Assuming constant returns to scale in  $F(\cdot)$  and  $H_j(\cdot)$ , Hayashi's (1982) result applies, and this return will be equal to a weighted average of the investment returns:

$$R(s^t, s_{t+1}) = \sum_j \frac{q_j(s^t) K_j(s^t)}{\sum_i q_i(s^t) K_i(s^t)} \cdot R_j^I(s^t, s_{t+1}). \quad (2.2)$$

The market price of risk, aka the highest Sharpe ratio, also has a simple expression. Let us, introduce the stochastic discount factor  $m(s_{t+1}|s^t)$  by dividing and multiplying through by the probabilities  $\pi(s_{t+1}|s^t)$ , so that

$$P(s_{t+1}|s^t) = \left( \frac{P(s_{t+1}|s^t)}{\pi(s_{t+1}|s^t)} \right) \pi(s_{t+1}|s^t) = m(s_{t+1}|s^t) \pi(s_{t+1}|s^t).$$

Then, ruling out arbitrage implies  $E_t(m(s_{t+1}|s^t) R^e(s^t, s_{t+1})) = 0$ , for  $\forall R^e(s^t, s_{t+1})$  defined as excess returns. It is then easy to see that

$$\max \frac{E[R^e(s^t, s_{t+1})|s^t]}{Std[R^e(s^t, s_{t+1})|s^t]} = \sqrt{\frac{\sum_{s_{t+1}} P(s_{t+1}|s^t)^2 / \pi(s_{t+1}|s^t)}{[\sum_{s_{t+1}} P(s_{t+1}|s^t)]^2}} - 1.$$

### 3 Functional Forms

In this section, we present the functional forms and the simulation strategies.

#### A. The investment cost function

The investment cost function is chosen so as to allow for the ability to have different adjustment cost curvatures across sectors and for balanced growth. We will use the following specification

$$H(K, I, Z) = H(K/Z, I) = H(1, ZI/K) \cdot (K/Z)$$

so that it is homogenous of degree 1 in  $I$  and  $K/Z$  to guarantee balanced growth.<sup>3</sup>

The functional form is assumed to be

$$H(1, ZI/K) \cdot (K/Z) = \left\{ \frac{b}{\nu} (ZI/K)^\nu + c \right\} (K/Z),$$

with  $b, c > 0$ ,  $\nu > 1$  and  $ZI/K \geq 0$ .

As can easily be seen, this function is (1) convex in  $I$  for  $\nu > 1$ . (2) adjustment cost and the direct cost for additional capital goods are separable, trivially so because  $H(1, ZI/K) \cdot (K/Z) = [H(1, ZI/K) - ZI/K + ZI/K] \cdot (K/Z) = [H(1, ZI/K) - ZI/K] \cdot (K/Z) + I \equiv C(1, ZI/K) \cdot (K/Z) + I$ . We impose restrictions on the parameters of  $H(\cdot)$  so that  $C(1, ZI/K) \geq 0$ , that is, the pure adjustment cost is nonnegative.

The different parameters have roughly the following functions.  $\nu$  determines the cost of choosing volatile investment plans. For a given investment process, it determines the volatility of the market price of capital. The parameters  $b$  and  $c$  are used to obtain target values for  $qZ$  (marginal cost) and for the total cost.

With this specification, we have the marginal cost given as

$$H_I(K, I, Z) = H_2(1, ZI/K) = b(ZI/K)^{\nu-1},$$

and we can easily check convexity with respect to investment

$$H_{II}(K, I, Z) = b(\nu - 1)(ZI/K)^{\nu-2}(Z/K) > 0 \text{ for } b > 0 \text{ and } \nu > 1.$$

The case of no-adjustment cost is given by setting  $\nu = b = 1$ ,  $c = 0$ , so that

$$H(1, ZI/K) \cdot (K/Z) = I.$$

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<sup>3</sup>Given the capital accumulation equation, and as we further discuss below,  $IZ$  and  $K$  are cointegrated, and so are  $I$  and  $K/Z$ .

From the first-order condition we obtain a relationship between the investment rate and the marginal cost of capital

$$qZ = H_I(K, I, Z) = b(ZI/K)^{\nu-1}.$$

Normalizing  $Z$  to 1, the elasticity (quantity elasticity of a price) of  $q$  with respect to  $I/K$  is

$$\frac{\partial q}{\partial(I/K)} \frac{I/K}{q} = \nu - 1.$$

## B. Simulation strategy and stationarity of returns

For our quantitative analysis, we will assume that we know the optimal investment process. We then derive the implied investment returns and state-prices. We will assume that investment follows a two-state Markov process. For our simulations, we want returns to be stationary. This imposes some restrictions on technologies and the assumed optimal investment process. In this section we discuss these issues in detail.

Consider the investment return for capital stock  $j$  given our functional forms

$$\begin{aligned} & R_j^I(s^t, s_{t+1}) \\ = & Z_{jt} \cdot \frac{F_{K_j}(\{K_j(s^t)\}, s^t, s_{t+1}, N(s^t, s_{t+1}))}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}} + (Z_{j,t}/Z_{j,t+1}) \cdot \frac{b(1 - \frac{1}{\nu})(Z_{j+1t}I_{j,t+1}/K_{j,t+1})^\nu - c}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}} \\ & + (Z_{j,t}/Z_{j,t+1}) \cdot (1 - \delta_j) \cdot \frac{b(Z_{j,t+1}I_{j,t+1}/K_{j,t+1})^{\nu-1}}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}}, \end{aligned}$$

where we have used a more compact notation for state-dependence. The first term represents the marginal product, the second the adjustment cost reduction next period (growth option) and the third, the leftover capital. A sufficient set of conditions for  $R_j^I(s^t, s_{t+1})$  to be stationary is that each of the 3 different composite variables is stationary. Let us consider each of these terms separately.

First,  $Z_{j,t+1}/Z_{j,t}$ , as seen above, is stationary by assumption. Second, given the specification of the productivity growth rates as finite elements Markov chains, and assuming that sectoral investment growth rates also follow finite element Markov chains, that is,  $I_j(s^t, s_{t+1}) = I_j(s^t) \lambda^{I_j}(s_{t+1})$ , it can easily be shown that for appropriate starting points,  $Z_{jt}I_{j,t}/K_{j,t}$  is bounded. Third, we will make the assumption that the production function is such that

$$Z_{jt}F_{K_j}(\{K_j(s^t)\}, s^t, s_{t+1}, N(s^t, s_{t+1})) \equiv A_j(s_{t+1}).$$

This guarantees stationarity and allows us to focus our analysis on investment dynamics. One implication of this assumption is that to the extent that capital gets cheaper to produce over time,

that is as  $Z_j$  increases, it also becomes less productive at the margin in physical terms, so that in value terms, the marginal product remains constant.<sup>4</sup>

Stationarity of sectoral investment returns is not sufficient for stationarity of aggregate asset returns. Indeed, as shown in equation 2.2, the aggregate return equals a weighted average of the sectoral returns. For stationarity, the weights need to be stationary too. Aggregate returns are given by

$$R(s^t, s_{t+1}) = \sum_j \frac{\frac{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}}{Z_{j,t}} K_{j,t+1}}{\sum_i \frac{b(Z_{it}I_{i,t}/K_{i,t})^{\nu-1}}{Z_{i,t}} K_{i,t+1}} R_j(s^t, s_{t+1}).$$

A sufficient (and necessary) condition for stationarity, given our previous assumptions, is that  $K_{1,t+1}/Z_{1,t}$  and  $K_{2,t+1}/Z_{2,t}$  are cointegrated. Given that the investment capital ratios  $Z_{jt}I_{j,t}/K_{j,t}$  are stationary, this is equivalent to  $I_{1,t}$  and  $I_{2,t}$  being cointegrated. Setting investment expenditure growth rates equal across sectors, that is  $\lambda^{I1}(s_{t+1}) = \lambda^{I2}(s_{t+1})$ , guarantees that  $I_{1,t}$  and  $I_{2,t}$  are cointegrated. To summarize, because individual quantities have stochastic trends, we end up choosing identical investment expenditure growth realizations across sectors to guarantee stationarity of aggregate equity returns. However, we are free to choose the realizations for  $\lambda_t^{Z1}$  and  $\lambda_t^{Z2}$  independently. This is less restrictive than it might appear for several reasons. As seen above, what matters for the investment returns is the behavior of the product  $\lambda_t^{I1}\lambda_t^{Z1}$  and not  $\lambda_t^{I1}$  individually. That is to say that the important element in the calibration is to fit the process of real investment growth rather than the growth in investment expenditure. Moreover, for our empirical counterparts, as shown below, the historical volatilities of  $\lambda^{I1}$  and  $\lambda^{I2}$  are nearly identical, and realizations of the two growth rates are strongly positively correlated. Alternatively, we could introduce additional components for each process that have purely transitory effects and would thus not need to be restricted to ensure balanced growth. However, given the requirement to keep the number of states small, the additional flexibility introduced in this way would be rather limited.

To summarize, we have

$$\begin{aligned} R_j^I(s^t, s_{t+1}) &= \frac{A_j(s_{t+1})}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}} \\ &+ (Z_{j,t}/Z_{j,t+1}) \cdot \frac{b(1 - \frac{1}{\nu})(Z_{j+1t}I_{j,t+1}/K_{j,t+1})^\nu - c}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}} \\ &+ (Z_{j,t}/Z_{j,t+1}) \cdot (1 - \delta_j) \cdot \frac{b(Z_{j,t+1}I_{j,t+1}/K_{j,t+1})^{\nu-1}}{b(Z_{jt}I_{j,t}/K_{j,t})^{\nu-1}}, \end{aligned} \tag{3.1}$$

with,

$$Z_{j,t+1}I_{j,t+1}/K_{j,t+1} = (Z_{jt}I_{j,t}/K_{j,t+1}) \lambda_{t+1}^I \lambda_{t+1}^{Z_j}.$$

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<sup>4</sup>This is related to one of the properties implied by Greenwood, Hercowitz and Krusell's (1997) balanced growth path.

Thus, realized returns are given as functions of the following

$$R_j^I(s^t, s_{t+1}) = R_j^I(Z_j(s^t) I_j(s^t) / K_j(s^{t-1}); \lambda^I(s_{t+1}), \lambda^{Z_j}(s_{t+1}), A_j(s_{t+1})) \text{ for } j = 1, 2.$$

For our simulations, we will generate realizations of all quantities of interest based on a probability matrix describing the law of motion for the exogenous state  $s_{t+1}$ . The law of motion for the rest of the variables follows

$$\begin{aligned} I_j(s^t, s_{t+1}) &= I_j(s^t) \lambda^I(s_{t+1}) \text{ for } j = 1, 2 \\ Z_j(s^t, s_{t+1}) &= Z_j(s^t) \lambda^{Z_j}(s_{t+1}) \text{ for } j = 1, 2 \\ K_j(s^t) &= K_j(s^{t-1}) (1 - \delta_j) + Z_j(s^t) I_j(s^t) \text{ for } j = 1, 2. \end{aligned}$$

Seven variables are a sufficient statistic for the current state of the economy  $s^t$ , namely  $s_t, K_1(s^{t-1}), K_2(s^{t-1}), I_1(s^t), I_2(s^t), Z_1(s^t), Z_2(s^t)$ . Clearly  $K_j(s^t)$  matters too, but it is a function of the state variables. The probability distribution of the shocks is summarized by  $s_t$ , the realization of the return does not depend on  $s_t$ . As initial conditions, we will set  $K_2(s^{-1}) = Z_1(s^0) = Z_2(s^0) = 1$ , and  $K_1(s^{-1})$  is set equal to the historical average of the ratio of the value of capital in this sector relative to the other sector. Initial investment levels are assumed at their implied steady state values.

#### 4 Theoretical analysis

In this section we present a series of theoretical results that help us understand key model mechanisms. First, we show how the equity premium is determined from investment returns, and how investment returns are determined from investment growth. Second, we discuss in detail what constitutes an admissible investment process for a given technology specification. Third, we present the conditions under which we can recover state prices in a model without technology shocks. A key finding of this section is that sectoral differences in the adjustment cost parameters  $\nu_j$  are crucial for allowing us to recover admissible state prices from investment. We present a simple expression for the Sharpe ratio that depends only on the investment cost curvature and the investment volatility. This expression shows that there is a minimum amount of curvature required to have a positive equity premium. We also derive an upper bound for the Sharpe ratio, and discuss to what extent this can explain some of our quantitative findings. Another key finding is that in a model without technology shocks and where interest rates are constant, investment returns have to be constant. Thus, in such an environment, it is not possible to recover state prices from producers first-order conditions.

## A. What determines the equity premium?

We consider here the relationship between investment and asset prices by focusing on the requirements for a positive equity premium. For this issue, a continuous-time representation is more intuitive than our discrete-time model used so far. There are two main findings in this section. First, we show that in order to have a positive equity premium, the investment return with the higher expected return needs to be the more volatile. Second, we show that under some conditions, asymmetries in the investment cost curvature  $\nu$  can generate this property, and we present a simple expression for the Sharpe ratio.

In parallel to our two-state representation in discrete time, assume we have a one-dimensional Brownian motion. Investment returns are given by

$$\frac{dR_j}{R_j} = \mu_j(\cdot) dt + \sigma_j(\cdot) dz, \text{ for } j = 1, 2, \quad (4.1)$$

and assume that the state-price process also follows such a process

$$\frac{d\Lambda}{\Lambda} = -r^f(\cdot) dt + \sigma(\cdot) dz. \quad (4.2)$$

We will assume that the two returns are positively (perfectly) correlated so that  $\text{sign}(\sigma_1) = \text{sign}(\sigma_2)$ . We allow for the drift and diffusion coefficients to change with the state of the economy. For compactness, from now on, our notation will not explicitly acknowledge this.

We want to derive the drift and diffusion terms of the state-price process,  $-r^f$  and  $\sigma$ , from the given return processes, that is from the four values  $\mu_j$  and  $\sigma_j$  for  $j = 1, 2$ . Remember, in this environment, the absence of arbitrage implies that

$$0 = E_t \left( \frac{d\Lambda_t}{\Lambda_t} \right) + E_t \left( \frac{dR_{jt}}{R_{jt}} \right) + E_t \left( \frac{d\Lambda_t}{\Lambda_t} \frac{dR_{jt}}{R_{jt}} \right), \quad (4.3)$$

so that we have

$$0 = -r^f dt + \mu_i dt + \sigma_i \sigma dt,$$

and thus 2 equations and 2 unknowns. The solution of this system is

$$\begin{aligned} r^f &= \frac{\sigma_2 \mu_1 - \sigma_1 \mu_2}{\sigma_2 - \sigma_1} \\ -\sigma &= \frac{\mu_2 - \mu_1}{\sigma_2 - \sigma_1}. \end{aligned}$$

In order to be able to recover the state price process from the two returns, we need  $\sigma_2 - \sigma_1 \neq 0$ . That is, the volatility terms have to be different across sectors; and, implicitly, if we assume continuous loadings of the return processes, their ordering is not allowed to change. This is an invertibility requirement similar to the one for the discrete time case. Compared to the discrete

time case, there is no issue of possibly negative state prices as we discuss below. Indeed, a process such as 4.2 cannot become negative if initially positive.

We can now consider the equity premium. From the pricing equation 4.3, we have the volatility term equal to the Sharpe ratio

$$-\sigma = \frac{\mu_1 - r^f}{\sigma_1} = \frac{\mu_2 - r^f}{\sigma_2},$$

and using the solutions derived above we have

$$\mu_1 - r^f = -\sigma\sigma_1 = \sigma_1 \left[ \frac{\mu_2 - \mu_1}{\sigma_2 - \sigma_1} \right]. \quad (4.4)$$

Clearly, with positively correlated returns, that is  $sign(\sigma_1) = sign(\sigma_2)$ , the sign of both sectoral risk premiums is equal, and thus the sign of the aggregate equity premium, a weighted average of the sectoral premiums, will be the same as for the two sectoral premiums. From equation 4.4 it is easy to see that there is a positive equity premium in the aggregate if, and only if, the return with the higher risk premium is more volatile.<sup>5</sup>

Let us now make the link to the production side of our model. As shown in the appendix, for the continuous-time counterpart of our investment model, without shocks to the marginal product nor the investment specific technology, the realized return to a given capital stock equals

$$\left\{ \frac{A - c}{b \left( \frac{I_t}{K_t} \right)^{\nu-1}} + \left( 1 - \frac{1}{\nu} \right) I_t/K_t - \delta + (\nu - 1) \left[ (\lambda^I - 1) - (I_t/K_t - \delta) + \frac{1}{2} (\nu - 2) \sigma_I^2 \right] \right\} dt + (\nu - 1) \sigma_I dz, \quad (4.5)$$

where  $(\lambda^I - 1)$  and  $\sigma_I$  are drift and diffusion terms of investment. Consider this return when  $(\lambda^I - 1) - (I_t/K_t - \delta) = 0$ . This holds at the deterministic steady state for given  $(\lambda^I - 1)$  and  $\delta$ , assuming  $(\lambda^I - 1) + \delta > 0$ . We can then write the return as

$$\left\{ (\bar{R} - 1) + \frac{1}{2} (\nu - 1) (\nu - 2) \sigma_I^2 \right\} dt + (\nu - 1) \sigma_I dz. \quad (4.6)$$

Where  $\bar{R}$  is the return in the deterministic model counterpart at the steady state.<sup>6</sup> As we discuss further below, it is convenient to calibrate the model by picking a value for  $\bar{R}$ , the steady-state return, independently from other parameters, which implicitly sets  $A$  at a given level.

We can now consider how  $\nu$  and  $\delta$  contribute to the sign and magnitude of the equity premium. For our calibration with two sectors representing equipment and structures, respectively, we will have that structures are harder to adjust and depreciate more slowly than equipment, that is

<sup>5</sup>Indeed, if  $\sigma_1, \sigma_2 > 0$ , this implies that if  $\mu_2 - \mu_1 > 0$ , we need  $\sigma_2 - \sigma_1 > 0$ , and we can see that  $\mu_1 - r^f > 0$ . Alternatively, if  $\sigma_1, \sigma_2 < 0$ , this condition implies that if  $\mu_2 - \mu_1 > 0$  we need  $\sigma_2 - \sigma_1 < 0$ , (sector 2 is more volatile), and then again  $\mu_1 - r^f > 0$ .

<sup>6</sup> $\bar{R} = \frac{A - c}{b(\lambda^I - (1 - \delta))^{\nu-1}} + \left( 1 - \frac{1}{\nu} \right) \lambda^I + \frac{1}{\nu} (1 - \delta)$ .

$\nu_S > \nu_E$ , and  $\delta_S < \delta_E$ . We label equipment as sector 1, and thus  $\nu_2 > \nu_1$ . As is clear from equation (4.6), for a given  $\bar{R}$ , there is no separate role for depreciation rates at steady state. We now show under what conditions we have a positive equity premium at steady state.

Because  $(\nu - 1)$  multiplies  $\sigma_I dz$ , with our sectoral normalization, the volatility term of sector 2 is larger, that is  $\sigma_2 > \sigma_1$ , assuming equal investment volatility  $\sigma_I$  across sectors. To see whether this asymmetry can generate a positive equity premium, we need to check the effect of  $\nu$  on the drift component. It is easy to see that

$$\frac{\partial(\nu - 1)(\nu - 2)}{\partial\nu} = 2\left(\nu - \frac{3}{2}\right) \rightarrow \text{if } \nu > \frac{3}{2} \text{ then } \frac{\partial(\nu - 1)(\nu - 2)}{\partial\nu} > 0.$$

That is, starting from a common curvature parameter  $\nu > \frac{3}{2}$ , and increasing curvature in one sector, the sector with the higher curvature will have a higher drift, everything else equal. Thus, if  $\nu > 1.5$ , and if there are no sectoral asymmetries except the difference in  $\nu$ , we have  $\sigma_2 > \sigma_1$  and  $\mu_2 > \mu_1$ , that is, as shown in equation 4.4 above, the equity premium is positive.

We can directly compute the Sharpe ratio at steady state, assuming  $\bar{R}_j = \bar{R}$  and  $\sigma_{Ij} = \sigma_I$ , as

$$\frac{\mu_j - r^f}{\sigma_j} \Big|_{ss} = \frac{\nu_1 + \nu_2 - 3}{2} \sigma_I. \quad (4.7)$$

Because,  $\sigma_j$  and  $\sigma_I$  have the same sign (given  $\nu_j > 1$ ), a necessary and sufficient condition for a positive equity premium is that  $\nu_1 + \nu_2 > 3$ . Clearly, the equity premium is then increasing in the sum of the curvature parameters.

In our calibration we set  $\bar{R}_2$  and  $\bar{R}_1$  independently from other parameters as a way of implicitly selecting values of  $A_j$ . Given the limited amount of precise direct information about  $A_j$ , we choose this approach. Alternatively, we could take all coefficients determining  $\bar{R}$  as given, and consider the partial derivatives with respect to  $\nu$  and  $\delta$ . A few steps of algebra show that under the standard assumptions that  $\nu > 1$ ,  $\lambda - 1 - \delta > 0$ , and  $(A - c)/b(\lambda - (1 - \delta))^{\nu-1} > 0$ ,  $\frac{\partial\bar{R}}{\partial\nu} > 0$ , and  $\frac{\partial\bar{R}}{\partial\delta} < 0$ . Therefore, given our assumed sectoral asymmetries, both would contribute to  $\mu_2 > \mu_1$ , and with  $\sigma_2 > \sigma_1$ , this would contribute positively to the equity premium.

We can also compute the instantaneous interest rate at steady state, assuming  $\bar{R}_j = \bar{R}$  and  $\sigma_{Ij} = \sigma_I$ , as

$$r^f \Big|_{ss} = (\bar{R} - 1) - (\nu_1 - 1)(\nu_2 - 1) \frac{\sigma_I^2}{2}.$$

This expression shows how investment uncertainty contributes to a lower steady state interest rate by an extent that is determined by the amount of the adjustment cost curvature. This parallels the precautionary saving effect on interest rates in standard consumption based models.

## B. What is an admissible investment process?

In this section we consider the requirements for an investment process to be admissible, in the sense that it represents a solution to the firm's problem for the implied state-price process, and that this price process is itself well behaved. The two key requirements are that the derived state prices have to be positive and that the implied firm value has to be finite. While a large set of investment processes are admissible, these requirements nevertheless impose constraints on our simulations. For this reason, this section also provides the motivations for some of the choices we make in our empirical analysis.

(i) Positive state prices The first key requirement is that state prices have to be positive. It is easy to see from equation 2.1 that the relative state prices in the two-state case are given by

$$\frac{P(s^t, \mathfrak{s}_1)}{P(s^t, \mathfrak{s}_2)} = \frac{R_2^I(s^t, \mathfrak{s}_2) - R_1^I(s^t, \mathfrak{s}_2)}{R_1^I(s^t, \mathfrak{s}_1) - R_2^I(s^t, \mathfrak{s}_1)}. \quad (4.8)$$

As can easily be seen, for nonnegative investment returns, that is  $R > 0$ , state prices are positive if and only if numerator and denominator have the same sign. Economically, this requirement implies that each capital stock has to do (absolutely) better in one state. Indeed, if one type of investment were to generate a higher return in both states, then resources would be moved into this type of capital from the other, meaning that this is not an equilibrium outcome.<sup>7</sup>

To see what properties are needed to satisfy this requirement, consider now a second order Taylor-series approximation of the investment return in equation (4.10, which is the discrete time counterpart of the continuous-time return equation 4.6 discussed above. The second-order Taylor approximation of the investment return is at the steady-state investment capital level around the mean of investment growth. To focus on the quantitatively important channel, we again assume that only investment expenditure growth is allowed to vary, but not the marginal product nor the investment specific technology. Under these assumptions, we can rewrite equation 3.1 for the investment return as

$$R_{t,t+1}^I(s^t, \mathfrak{s}_j) = \frac{A - c}{b \left( \frac{I_t(s^t)}{K_t(s^{t-1})} \right)^{\nu-1}} + \left\{ (1 - \delta) + \left( 1 - \frac{1}{\nu} \right) \left[ \frac{\frac{I_t(s^t)}{K_t(s^{t-1})}}{\frac{I_t(s^t)}{K_t(s^{t-1})} + (1 - \delta)} \right] \lambda(s_{t+1}) \right\} \left( \frac{1}{\frac{I_t(s^t)}{K_t(s^{t-1})} + (1 - \delta)} \lambda(s_{t+1}) \right)^{\nu-1}. \quad (4.9)$$

This expression makes it clear how in this case realized returns are a function of the realization of investment growth rate  $\lambda(s_{t+1})$ , while the only relevant state variable is the current investment-capital ratio  $I_t(s^t)/K_t(s^{t-1})$ . A second-order Taylor approximation is obtained by assuming that the investment-capital ratio is at its steady state  $I_t(s^t)/K_t(s^{t-1}) = \bar{\lambda} - 1 + \delta$ ,

<sup>7</sup>A related requirement is that the matrix  $R$  has to be invertible for us to be able to recover the state prices.

$$R_{t,t+1}^I = \bar{R} + (\nu - 1) \Delta\lambda' + \frac{B}{2} (\Delta\lambda')^2 + o\left((\Delta\lambda')^2\right) \quad (4.10)$$

where  $\Delta\lambda' = \lambda' - E(\lambda')$ . The only difference compared to the continuous-time equation is the second-order term. Here it equals

$$B = \frac{\nu - 1}{\lambda} \left\{ \nu - 1 - \frac{1 - \delta}{\lambda} \right\}^8$$

Assuming IID investment shocks, the size of the up and down deviations from the mean in a two-state setting are identical, that is, we have

$$\Delta\lambda_j(\mathfrak{s}_2) = -\Delta\lambda_j(\mathfrak{s}_1) \equiv \overline{\Delta\lambda_j}, \text{ for each } j \in (1, 2).$$

Moreover, assuming equal investment growth volatility in the two sectors (a reasonable assumption as we discuss later) and positive correlation, we have

$$\overline{\Delta\lambda_1} = \overline{\Delta\lambda_2} = \overline{\Delta\lambda}.$$

Introduce this approximation into the ratio determining relative state prices,

$$\frac{P(\cdot, \mathfrak{s}_1)}{P(\cdot, \mathfrak{s}_2)} = \frac{[\nu_2 - \nu_1] \overline{\Delta\lambda} + \left[ (\bar{R}_2 - \bar{R}_1) + \frac{1}{2} (B_2 - B_1) (\overline{\Delta\lambda})^2 \right] + o\left((\overline{\Delta\lambda})^2\right)}{[\nu_2 - \nu_1] \overline{\Delta\lambda} - \left[ (\bar{R}_2 - \bar{R}_1) + \frac{1}{2} (B_2 - B_1) (\overline{\Delta\lambda})^2 \right] + o\left((\overline{\Delta\lambda})^2\right)} \quad (4.11)$$

As shown by equation 4.11, in order to have positive prices, the first term in the fraction needs to be far away from zero, that is  $[\nu_2 - \nu_1] \overline{\Delta\lambda} \gg 0$  or  $\ll 0$ . This terms needs to be sufficiently far away from 0 so that the remaining terms in the fraction cannot overturn the sign of the numerator and the denominator. Moreover, this first-order term should not be too sensitive to the state of the economy so as to ensure that this property is satisfied everywhere. Thus, clearly, asymmetry in  $\nu$  is needed to generate positive state prices, while the level of  $\delta$  plays no role.

One issue we face in our numerical experiments is that the investment capital ratio can reach very low levels. In this case, the coefficients of a second-order approximation around the current investment-capital ratio can differ substantially from their steady state values. Specifically, as can be seen from equation (4.9), investment returns can get infinitely large as  $I_t(s^t)/K_t(s^{t-1})$  gets close to 0. Indeed, the first term  $(A - c)/b \left( \frac{I_t(s^t)}{K_t(s^{t-1})} \right)^{\nu-1}$  can get infinitely large. Under these conditions, the requirement for positive state prices may be hard to satisfy for all possible paths. What makes this condition hard to satisfy is that it has to hold with probability one. In our simulations we choose to make the marginal product terms,  $A$ , state contingent and to select its value in the low growth state specifically so that state prices are positive for the lowest possible

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<sup>8</sup> With  $(1 - \delta) = \lambda = 1$ , we would have  $B = (\nu - 1)(\nu - 2)$ , which is the term in the continuous time counterpart. Note, we have also made the assumption here that  $E(\lambda') = \bar{\lambda}$ .

$I_t(s^t)/K_t(s^{t-1})$ . As we show below, shocks to  $A$  have only second order effects on asset price implications in general. This is because the level of  $A$  is so small relative to the other terms in the return equation 4.9.

(i).1 Upper bound for Sharpe ratio The requirement for nonnegative state prices has an implication for two-state environments in general, that we present here. Namely, if the two states are equally likely, the Sharpe ratio is bounded by 1. Because we never deviate much from this case in our simulations, we typically are bound by this constraint. Moreover, because the requirement of positive state prices has to hold with probability one, the constraints on the technology and the investment process end up reducing average Sharpe ratios to levels substantially lower than 1.

In a two-state environment, no arbitrage requires that for the excess return on the market  $R^M(s) - R^f$ ,

$$P(\mathfrak{s}_1) \left( R^M(\mathfrak{s}_1) - R^f \right) + P(\mathfrak{s}_2) \left( R^M(\mathfrak{s}_2) - R^f \right) = 0.$$

This implies that, in line with our equation (4.8), the ratio of the state prices is given by

$$\frac{P(\mathfrak{s}_1)}{P(\mathfrak{s}_2)} = \frac{(R^M(\mathfrak{s}_2) - R^f)}{-(R^M(\mathfrak{s}_1) - R^f)} = \frac{[R^M(\mathfrak{s}_2) - ER^M] + [ER^M - R^f]}{-[R^M(\mathfrak{s}_1) - ER^M] - [ER^M - R^f]}.$$

It is easy to see that if both states are equally likely then we have  $[R^M(\mathfrak{s}_2) - ER^M] = -[R^M(\mathfrak{s}_1) - ER^M] = Std(R^M)$ , so that

$$\frac{P(\mathfrak{s}_1)}{P(\mathfrak{s}_2)} = \frac{Std(R^M) + [ER^M - R^f]}{Std(R^M) - [ER^M - R^f]}.$$

Because of the requirement of positive state prices, the denominator has to be positive, and therefore the Sharpe ratio,  $\frac{ER^M - R^f}{Std(R^M)}$ , is bounded,

$$\begin{aligned} Std(R^M) &> [ER^M - R^f] \\ 1 &> \frac{ER^M - R^f}{Std(R^M)}. \end{aligned}$$

Clearly, this result applies to all models without arbitrage in a two-state environment. For instance, it applies to the classic model used by Mehra and Prescott (1985). Based on the history of sectoral investment growth we consider in this study, a reasonable calibration cannot deviate much from the case where up and down movements are equally likely. Thus, the result derived here is relevant. Because the requirement of positive state prices has to hold with probability one, the constraints on the technology and the investment process end up reducing average Sharpe ratios in our simulations to a level substantially below 1. The non-IID case we consider, with first-order

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<sup>9</sup>We have here implicitly normalized the environment so that  $R^M(\mathfrak{s}_2) > R^M(\mathfrak{s}_1)$ . This is without loss of generality.

serial correlation of 0.2, does not substantially affect quantitative findings as shown later in the paper.<sup>10</sup>

(ii) Other requirements The second key requirement is that the value of the firm implied by an investment process and a state price process has to be finite. If not, the equivalence between investment returns and returns to the firm breaks down. A related condition that guarantees optimality of the path satisfying the first-order condition is the transversality condition

$$\lim_{t \rightarrow \infty} \sum_{s^t} \frac{P(s^t)}{P(s_0)} \{A(s^t) + H_I(s^t)(1 - \delta) - H_K(s^t)\} K_t(s^{t-1}) = 0.$$

We check both conditions in our simulations. We also make sure that gross investment returns are nonnegative,  $R \geq 0$ . This limited liability requirement is not necessarily needed. On the other hand, it doesn't impact any quantitative conclusions.

(iii) Models with no technology shocks: Interest rates cannot be constant We consider here the benchmark environment where the only source of uncertainty that firms face are stochastic state prices. That is to say, there are no shocks to the production technology. We have one result: even without technology shocks, investment returns can be optimally state-contingent as long as interest rates are not constant. In an environment where interest rates are constant forever, investment returns are constant too. Thus, with constant interest rates it is not possible to recover the state price process from producers' first-order conditions. The basic economic idea in this section is that if a firm is subject to convex capital adjustment costs, it will not find it optimal to choose a volatile investment plan unless forced by changing prospects in future valuations.

Consider the discrete-time model with no technology shocks. Assuming a general two-state environment where state-prices do not necessarily have a Markov representation. The firm's problem is given by

$$\begin{aligned} & \max_{\{I(s^t), K(s^{t+1})\}} \sum_{t=0}^{T-1} \sum_{s^t} \left[ AK_t(s_{t-1}) - \left\{ \frac{b}{\nu} (I(s^t) / K_t(s^{t-1}))^\nu + c \right\} K_t(s^{t-1}) \right] \times \left( \prod_{j=0}^{t-1} P(s_{j+1}|s^j) \right) \\ & + \sum_{s^T} \Psi K_T(s^{T-1}) \left( \prod_{j=1}^{T-1} P(s_{j+1}|s^j) \right), \end{aligned}$$

subject to

$$0 = (1 - \delta) K_t(s^{t-1}) + I(s^t) - K_{t+1}(s^t),$$

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<sup>10</sup>For the case with unequal probabilities, through a similar argument, we can obtain a bound

$$\left( \frac{1 - \pi}{\pi} \right)^{0.5} > \frac{[ER^M - R^f]}{std(R^M)},$$

where  $\pi$  is the probability of the state with the low return realization. Thus, with postively skewed returns the bound is tighter, with negatively skewed returns it is looser.

with  $K(s_0)$  given,  $P(s_0|s^0) = 1$ , and  $\Psi > 0$  a parameter; assuming that  $s_t \in (\mathfrak{s}_1, \mathfrak{s}_2)$ .

The solution of this problem for  $T \rightarrow \infty$  is equivalent to the solution of the general version of our problem with enough regularity so that firm value is finite. However, it is easy to see in this model why interest rate volatility is needed. Indeed, from  $T - 1$  to  $T$ , without technology shocks, the return to capital equals the risk free rate. For the second-to-last return-period, that is, from  $T - 2$  to  $T - 1$ , it can be checked that the return is given by

$$R_{T-2,T-1}(s^{T-2}, \mathfrak{s}_j) = \frac{\alpha \left( \frac{\Psi}{R_{T-1,T}^f(s^{T-2}, \mathfrak{s}_j)} \right)}{\alpha \left( \frac{\Psi}{R_{T-1,T}^f(s^{T-2}, \mathfrak{s}_1)} \right) P(\mathfrak{s}_1|s^{T-2}) + \alpha \left( \frac{\Psi}{R_{T-1,T}^f(s^{T-2}, \mathfrak{s}_2)} \right) P(\mathfrak{s}_2|s^{T-2})}, \text{ for } j = 1, 2.$$

with  $\alpha(x) = (A - c) + (1 - \delta)x + (1 - \frac{1}{\nu}) \left(\frac{1}{b}\right)^{\frac{1}{\nu-1}} x^{\frac{\nu}{\nu-1}}$ . Clearly, if the interest rate  $R_{T-1,T}^f(s^{T-2}, \mathfrak{s}_j)$  is constant, that is if it does not depend on  $\mathfrak{s}_j$ , then,  $R_{T-2,T-1}(s^{T-2}, \mathfrak{s}_j) = R_{T-2,T-1}(s^{T-2}) = R_{T-2,T-1}^f(s^{T-2})$ . However, to the extent that one-period interest rates are state-contingent at  $T - 1$ , the return to the firm from  $T - 2$  to  $T - 1$  will be state-contingent, and it will depend on the technology of the firm, in particular, the parameters of the adjustment cost function. Going backwards in time, this same argument can be made if all future one-period interest rates are constant. We summarize these derivations in the following proposition.

**Proposition 1** *If one-period interest rates are constant in every period, without technology shocks, the returns to the firm (and the investment returns) are equal to the one-period interest rate.*

A consequence of this result is that, without technology shocks, if investment returns (representing optimal choice) for one firm are state-contingent, then one-period interest rates cannot be constant. The importance of this result is that we cannot work with a “nice” benchmark environment with constant interest rates, in general.<sup>11</sup> On the other hand, as we show in the quantitative applications below, interest rate volatility doesn’t have to be excessively high, even though investment returns can be quite volatile.

## 5 Calibration

Parameter values are assigned based on 3 types of criteria. First, a set of parameter values are picked to match direct empirical counterparts. Second, some parameters are chosen to yield the best implications for key asset pricing moments. Third, some parameters are chosen to make sure the derived state-prices are admissible. We first present a short summary of our baseline

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<sup>11</sup>Note, setting  $\nu = 1 = b$ , and  $c = 0$  for one of the firms in our analysis would seem to imply constant interest rates. However, this is not an admissible specification, because the first-order conditions do not describe optimal firm behavior in general, as this problem is linear.

calibration. The details and the specification with shocks to the investment technology are given thereafter.

### A. Summary

The table lists the parameters to calibrate and the chosen baseline values:

$$\begin{aligned}
\rho &= (0, 0.2); & fr &= 1 \\
\begin{bmatrix} \lambda^I(\mathfrak{s}_1) \\ \lambda^I(\mathfrak{s}_2) \end{bmatrix} &= \begin{bmatrix} 0.9712 \\ 1.0954 \end{bmatrix} \\
\delta_e, \delta_s &= [0.112, 0.031] \\
\overline{(K_e/Z_e)/(K_s/Z_s)} &= 0.6 \\
\nu_e &= 2.5, \nu_s = 5; & b_e, b_s, c_e, c_s : \overline{qZ} &= 1.5 \\
A_e, A_s &: \text{ so that } \bar{R} = 1.03 \\
A_j(\mathfrak{s}_1) &= A_j(1-x); & A_j(\mathfrak{s}_2) = A_j(1+x) : x &= .236.
\end{aligned}$$

$\rho$  is the first-order serial correlation,  $fr$  is the relative frequency of the low growth state. A set of parameters is chosen based on direct empirical counterparts; namely,  $\rho$ ,  $fr$ ,  $(\delta_e, \delta_s)$ , and  $\overline{(K_e/Z_e)/(K_s/Z_s)}$ . The investment realizations are chosen to mimic first and second moments of postwar US investment growth, with the additional restriction that the lowest investment-capital ratio is above 0. In order to replicate steady-state values for  $qZ$ , we pick  $(b_e, b_s)$ ;  $(c_e, c_s)$  is then determined to have the lowest possible total cost. For the curvature parameters, based on casual observation, we assume:  $\nu_e < \nu_s$ ; with the exact values picked to maximize the model's fit.  $\bar{R}$ , and thus  $A_e$  and  $A_s$ , are picked to minimize the impact of hard to measure quantities and to help the model. Finally,  $x$  is chosen so that state-prices and gross investment returns are always positive.

### B. Details of different parts of calibration

(i) Investment and productivity processes We consider the empirical counterparts of three quantities

$I \cdot Z \equiv$  Investment ( addition to capital stock in units of capital good)

$I \equiv$  Investment expenditure in units of numeraire final good (consumption)

$Z \equiv$  Investment-specific technological change.

In the model,  $1/Z = P_I/P_C \equiv p_I$ , where  $P_I$  and  $P_C$  are the prices of the investment good and the consumption good, respectively. Therefore,  $p_I = 1/Z$  is the replacment cost for capital (not including adjustment cost), or the bookvalue of capital.

We will use the following time series: the quantity index of investment in each sector to give us  $I_Z$ , the deflator of investment goods and the deflator of nondurables and services to jointly give us  $Z$  for each sector. We use annual data for these series, for equipment & software and structures, covering 1947-2003.

The calibration of the investment growth process is in two steps. First, the probability matrix is determined to match the serial correlation and the frequency of low and high growth states. These two moments do not depend on the shock values themselves but only on the probabilities. Specifically, the two diagonal elements of the probability matrix are given as

$$\pi_{11} = \frac{\rho + fr}{1 + fr}; \quad \pi_{22} = \frac{1 + fr \cdot \rho}{1 + fr},$$

where  $\rho \equiv$  autocorrelation and  $fr \equiv p_1/p_2$ , that is the relative frequency of state 1 (the recession state). For the relative frequency we have counted the number of realizations of investment growth below its mean: there are 26/56 for equipment and 27/56 for structures. Given that this is very close to 50% we choose a frequency ratio of 1. As shown in the table below, the first-order serial correlations of the growth rates of investment are 0.13 and 0.28, respectively, and .08 and .27 for investment expenditure. The common  $\rho$  is set at the average for investment of 0.2. We also consider the case  $\rho = 0$ .

$\lambda^{Ie} - 1 = 3.81\%$	$\sigma^{Ie} = 6.98\%$	$\rho^{Ie} = .08$
$\lambda^{Is} - 1 = 2.85\%$	$\sigma^{Is} = 7.94\%$	$\rho^{Is} = .27$
$\lambda^{IZe} - 1 = 5.71\%$	$\sigma^{IZe} = 7.81\%$	$\rho^{IZe} = .13$
$\lambda^{IZs} - 1 = 2.29\%$	$\sigma^{IZs} = 6.86\%$	$\rho^{IZs} = .28$
$\lambda^{Ze} - 1 = 1.82\%$	$\sigma^{Ze} = 2.56\%$	$\rho^{IZe} = .66$
$\lambda^{Zs} - 1 = -.44\%$	$\sigma^{Zs} = 2.35\%$	$\rho^{IZe} = .31$

For the baseline calibration, we set the mean of  $\lambda^I - 1$  at the common 3.33% per year, the average of the historical investment growth rates across the two sectors. The implied standard deviation is 6.21%. This is 20% lower than the historic average across the two sectors. With this reduction in volatility the investment-capital ratio for structures at the steady state corresponding to the low growth state is at  $\lambda^I(\mathbf{s}_1) - 1 + \delta_s = 0.9712 - 1 + 0.031 = 0.0022$ , which makes the investment-capital ratio strictly positive with probability one.<sup>12</sup> An alternative approach to achieving this objective would be to specify a richer investment process, but this would come at

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<sup>12</sup>This is not an issue for equipment & software because the depreciation rate is higher.

the cost of a loss in transparency. The perfect positive correlation in the model is not that far from the historical reality. Indeed, the historical sample correlations for investment across the two sectors is 0.61 and 0.64, for investment and investment expenditure, respectively.

We also present results for the case where the investment specific technology  $Z_j$  is allowed to vary in both sectors. For this calibration, we select the 6 values for the realized growth rates of investment expenditure and the sector specific technological progress, in order to match as closely as possible the 8 means and the standard deviations for  $IZ_e$ ,  $IZ_s$ ,  $Z_e$  and  $Z_s$ , where each standard deviation is reduced by 30 for the reason explained in the previous paragraph. This objective can be achieved quite well. The empirical correlations of sectoral investment with its specific technological growth are 0.43 and  $-0.32$ , while the correlations of the technological growth across sectors is 0.3. Clearly, due the limited degrees of freedom in our two state process we cannot match all these correlations. As we show below, for most quantities of interest, this consideration is quantitatively second order.

(ii) Depreciation rates We need the depreciation rates for equipment and software as well as for structures:  $(\delta_e, \delta_s)$ . We can directly compute depreciation rates from the Fixed Assets tables, and take the sample average. We get 13.06% and 2.7% for 1947-2002. Because NIPA's depreciation includes physical wear as well as economic obsolescence, we adjust the data to take into account that depreciation in the model covers only physical depreciation. To do this we add the price increase in the capital good. So that

$$\delta_t = \frac{D_t}{K_t} + p_{I,t}/p_{I,t-1} - 1 = \frac{D_t}{K_t} + (Z_{t-1}/Z_t - 1),$$

with  $D_t$  depreciation according to NIPA. This adjustment decreases depreciation by 1.82% for equipment and -0.44% for structures, so that we have  $(\delta_e, \delta_s) = (.112, .031)$ .

(iii) Relative size of capital stocks The ratio of the capital stocks,  $(K_{e,t}/Z_{e,t}) / (K_{s,t}/Z_{s,t})$ , is needed only for computing aggregate returns, which, as we have seen, are a function of the price weighted sum of the two capital stocks. In the model, the ratio of the physical capital stocks  $K_{e,t}/K_{s,t}$  is trending, while the ratio of the book values of the capital stocks (in terms of the consumption good)  $(K_{e,t}/Z_{e,t}) / (K_{s,t}/Z_{s,t})$  is not trending.

We use Current-Cost Net Stocks of Fixed Assets from the BEA. With this data, for the period 1947-2002,  $mean((K_{e,t}/Z_{e,t}) / (K_{s,t}/Z_{s,t})) = 0.6$ . Based on this we set the steady-state value of equipment at 0.6 of the value of structures.

(iv) Adjustment cost and marginal product To parameterize the adjustment cost function, we choose the following procedure:

1) Pick  $\nu$  to get good results for asset prices under the restriction that  $\nu_e < \nu_s$ . Specifically, the selected values, roughly, generate the highest equity premium with the lowest (reasonable) return volatility, by also guaranteeing positive state prices. More formal searches have selected very similar parameter combinations.

2) Pick  $b$  so that  $qZ$  is consistent with average values reported in the literature.

3)  $c$  is then picked to minimize the overall amount of output lost due to adjustment cost.

In addition to casual empiricism there is also other evidence that suggests that adjustment cost curvature is larger for structures than for equipment and software. In particular, the fact that the serial correlation of the growth rates is somewhat higher for structures than for equipment can be interpreted as an expression of the desire to smooth investment over time due to the relatively high adjustment cost.

There are many examples of studies that estimate  $qZ$ . Lindenberg and Ross (1981) report averages for two-digit sectors for the period 1960-77 between .85 and 3.08. Lewellen and Badrinath (1997) report an average of 1.4 across all sectors for the period 1975-91. Gomes (1999) reports an average of 1.56. Based on this, we will use a steady-state target value for  $qZ$ ,  $\overline{qZ}$ , of 1.5 for both sectors. One problem with using empirical studies to infer the required heterogeneity in the sectoral costs is that most studies consider adjustment costs by sector of activity. For our analysis, we would need information about the adjustment cost by type of capital.

For our baseline calibration, the mean average adjustment cost (obtained in simulations) is 0.07 and 0.09 for equipment & software and structures, respectively. These values depend primarily on the target value for  $qZ$ , which itself does not affect much the model's asset pricing implications.

The marginal product coefficients  $A_e$  and  $A_s$  are set implicitly so as to have the steady-state return  $\bar{R}_j$  equalized in the two sectors, to replicate the mean risk free rate, and to make sure the firm value is finite and the transversality conditions are satisfied. The implied values for  $(A_1, A_2) = (0.1492, 0.0612)$ .

Finally, the volatility of the marginal production terms  $x$  is set so that for all paths the implied state-prices are positive and the gross returns are positive. While this parameter is useful in insuring that implied state-prices are admissible, it has only second-order effects on key asset pricing moments. This is because the marginal product component  $A$  represents a small part of the return. Of course, it is reasonable to assume that there are productivity shocks that are positively correlated with investment.

## 6 Quantitative findings

The key asset pricing moments we are interested in are first and second unconditional moments for equity and risk free returns. We also consider time-varying means and volatilities.

Table 1 presents the model implications for the baseline calibration as well as empirical counterparts for a set of moments. Model results are based on sample moments of very long simulated time series. For unconditional moments, the key finding is that the model is able to generate an equity premium of several percentage points with reasonable volatility for the equity return as well as for the risk free rate. The model's mean Sharpe ratio is about one third of the one that is implied in the historic equity premium. Consistent with our analysis in subsection A., given the higher adjustment cost curvature for structures relative to equipment and software, structures have a higher return volatility and a higher risk premium than equipment and software.

The model is able to generate considerable time-variation in conditional risk premiums. Indeed, the standard deviation of the one-period ahead conditional equity premium is 5.2%, which is considerably higher than the standard deviation of the risk free rate at 2.24%. There is a variety of empirical studies measuring return predictability. For example, Campbell and Cochrane (1999) report  $R^2$ 's of 0.18 and 0.04 for regressions of excess returns on lagged price-dividend ratios at a one-year horizon for the periods 1947 – 95 and 1871 – 1993, respectively. Combining the  $R^2$  with the volatility of the excess returns,  $\sqrt{R^2}std(R - R^f)$  provides an estimate of the volatility of the conditional equity premium. Setting  $R^2 = 0.1$  this would be  $\sqrt{0.1} \times 0.17 = 5.27\%$ . Thus, the model's value of 5.2% is close.

What is driving expected excess returns? In general, assuming the absence of arbitrage, we have that

$$E_t \left( R_{t+1} - R_t^f \right) = - \frac{\sigma_t(m_{t+1})}{E_t m_{t+1}} \sigma_t(R_{t+1}) \rho_t(m_{t+1}, R_{t+1}).$$

Possibly, return volatility  $\sigma_t(R_{t+1})$  can drive risk premiums, but this doesn't seem empirically relevant. According to Lettau and Ludvigson (2004) this is not the case for the U.S. postwar period. Indeed, they find strong negative correlations between conditional means and volatilities. Our model is consistent with this fact. Indeed, for the baseline calibration the correlation between conditional means and volatilities is  $-0.56$ . This negative correlation seems very robust to parameter changes.

Most standard models cannot replicate this finding of a negative correlation between conditional means and volatilities. With CRRA utility and lognormal consumption, expected returns

are given by

$$E_t \left( R_{t+1} - R_t^f \right) \cong -\gamma \cdot \sigma_t (\ln C' / C) \cdot \sigma_t (R_{t+1}) \cdot \rho_t (m_{t+1}, R_{t+1}).^{13}$$

In the Mehra-Prescott setup, all terms in the equation are roughly constant, with the correlation,  $\rho_t (m_{t+1}, R_{t+1})$ , roughly equal to one. In Campbell and Cochrane's model,  $\frac{\sigma_t(m_{t+1})}{E_t m_{t+1}}$  displays considerable variation. However, as is clear from their Figures 4 and 5, conditional means and volatilities are positively correlated.

What is driving the negative correlation between the conditional mean and volatility in our model? It can be shown under fairly general assumptions that this correlation is actually positive for individual (sectoral) investment returns at steady state levels of investment/capital ratios. And it is positive for sectoral returns in all simulations. The negative correlation displayed for the aggregate returns is generated by movements in the sectoral weights. For instance, when investment capital ratios are low in both sectors, the value of the more volatile sector declines by relatively more.

Let us focus now directly on the Sharpe ratio

$$\frac{E_t \left( R_{t+1} - R_t^f \right)}{\sigma_t (R_{t+1})} = -\frac{\sigma_t (m_{t+1})}{E_t m_{t+1}} \rho_t (m_{t+1}, R_{t+1}). \quad (6.1)$$

Given the volatile conditional means and the negative correlation between conditional means and volatilities, Sharpe ratios are very volatile. According to Lettau and Ludvigson (2004), for quarterly data, market implied Sharpe ratios have a mean of 0.39 and a standard deviation of 0.448, which implies a coefficient of variation of  $0.448/0.39 = 1.15$ . In our model, for the baseline calibration, this ratio equals,  $0.26/0.18 = 1.44$ . That is, considering that our model generates average Sharpe ratios of roughly 1/3 of the ones implied by the aggregate market, it nevertheless has the ability to generate considerable volatility in Sharpe ratios

What drives the volatility of the Sharpe ratio? Both parts on the right hand side of equation 6.1 contribute. As shown in Table 1, the market price of risk is moving, but its mean and standard deviation differ from those of the market's Sharpe ratio. The mean of the market price of risk is (obviously) larger, while the volatility is lower. Remember the model structure, given that we have a two-state setup, the conditional correlation  $\rho_t (m_{t+1}, R_{t+1})$  can only be 1 or -1. Clearly, therefore,  $\rho_t (m_{t+1}, R_{t+1})$  is switching between values of 1 and -1 as a function of the state of the economy. To further investigate this regime shifting property, note that if we make the shock process IID the mean and standard deviation of the Sharpe ratio and the market price of risk are much closer to each other than in the baseline case with serial correlation (and asymmetric

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<sup>13</sup>The approximation comes from replacing  $\sqrt{\exp [\gamma^2 \text{var}_t (\ln C' / C)] - 1}$  by  $\gamma \cdot \sigma_t (\ln C' / C)$

states) as seen in Table 2. That is to say that the occurrence of a positive (conditional) correlation between the market return and the stochastic discount factor, and thus a negative Sharpe ratio, is less common in the IID case.

Table 3 and 4 show results for the calibrations with investment specific technology shocks  $Z$ . In Table 3 the correlation of  $Z$  with same sector investment growth equals, 1, in Table 4, it is -1. While there are some quantitative differences compared to the baseline case and between the two cases considered here, none of our main conclusions are affected. Note, for maximum comparability, we only recalibrated  $\bar{R}$  and  $x$  to make sure implied state-prices are admissible.

To further illustrate model properties, we consider the implications from feeding through the investment realizations for the U.S. for the period 1947-2003. Given that investment growth in our model has only two values, the fit of the driving process is not perfect. Nevertheless, as shown in Figure 1, the fit is very good, with correlations between the model and the data of 0.78 and 0.71 for equipment and structures respectively. Figure 2 shows that the model's generated returns are indeed related to actually realized stock returns, with a correlation of 0.48. Figure 3 shows conditional moments. The two panels on the left show that conditional volatility is more persistent (and thus history dependent) than expected returns. The right hand side panel shows the market price of risk and the market's Sharpe ratio. Considering the 1990s, through the series of 8 high realizations in investment growth, expected returns, and Sharpe ratios are declining over time. The figure also shows that with a low investment growth realization, the market Sharpe ratio becomes negative, and thus the conditional correlation  $\rho_t(m_{t+1}, R_{t+1})$  becomes positive. It is interesting here to consider again the calibration with IID investment growth to further highlight the persistent component driving risk premiums. Figure 3b, presents the realized conditional moments corresponding to the IID case we presented in Table 2. Remember, the relevant state of the economy is summarized by the two investment-capital ratios,  $(I_j(s^t)/K_j(s^{t-1}))_{j=1,2}$ . The sequence of positive investment growth realization in the 90s, pushes up these ratios, leading to lower Sharpe ratios. Here, the conditional mean becomes even more persistent than the volatility. Only three times in the postwar period does the market Sharpe ratio become negative. In the 1990s, it is at the 6<sup>th</sup> realization of a high investment rate that the market Sharpe ratio becomes negative.

#### A. Discussion and sensitivity

We provide here some further discussion about the factors driving quantitative results.

Let us reconsider equation (4.7) of the Sharpe ratio at steady state in the continuous-time

model

$$\frac{\mu_j - r^f}{\sigma_j} \Big|_{ss} = \frac{\nu_1 + \nu_2 - 3}{2} \sigma_I. \quad (6.2)$$

Using the values from our baseline calibration,  $(\nu_1, \nu_2) = (2.5, 5)$  and  $\sigma_I = 6.21\%$ , the Sharpe ratio computed from (6.2) is 0.14. As shown in Table 1 and 2, average Sharpe ratios obtained from the simulations in the discrete-time model are at 0.18 for the baseline case. Thus, the continuous-time approximation at steady state gets slightly less than 80% of the simulated average Sharpe ratios. To get a sense of how much of the difference is due to the approximation and how much to the off steady-state realizations, note that in the baseline model, the Sharpe ratio is at 0.148 at the steady-state. Thus, the somewhat higher Sharpe ratios reported from our simulations are mainly a product of the off steady-state behavior. Of course, the baseline model also is subject to stochastic marginal products, that is, shocks to  $A$ . But not surprisingly, this has only minor effects. Table 5 reports simulation results that further confirm this point. Here the shocks to marginal products in the equipment sectors are turned off, while they are maintained for structures so as to assure that implied state prices are always positive. The main effect of this is to reduce the volatility for returns on equipment by about 2.5 percentage points. As a consequence, the Sharpe ratio gets to 0.16, compared to 0.18 in the benchmark case. As suggested by equation (6.2), turning off  $A$  shocks in the equipment sector has an effect on the Sharpe ratio primarily because of the asymmetry introduced across the sectors, rather than because of the volatility in  $A$  itself. Shocks to  $A$  have no second-order effects, and thus they do not directly affect drift terms.

## 7 Conclusions

We have studied the implications of producers' first-order conditions for asset prices in a model where convex adjustment cost play a major role. One lesson of this analysis is that some asymmetries across sectors are crucial. One reason for this is that the considered technology does not allow a firm to make state-contingent investment decisions for each capital stock individually. State-contingent investment decisions are possible through the combination of the two stocks. If both sectoral technologies were identical, optimal decision would generate identical returns, which would make it impossible to recover the state price process.

Our analysis demonstrates the ability of a simple investment cost representation to explain aggregate asset prices. Investment cost curvature and investment volatility are the main ingredients, not only to explain return volatility but also risk premiums. We show in detail how investment curvature and volatility determine key asset pricing features.

The quantitative asset pricing implications from a basic representation of the production side are encouraging. With reasonable assumptions on the quantities, risk premiums and interest rates

come close to explaining observed empirical counterparts. Despite being very stylized, the model has rich implications for time-varying moments. Specifically, the negative correlation between means and volatilities is noteworthy.

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## Appendix: Continuous-time model

This appendix presents a continuous-time investment model that replicates the setup of our discrete-time environment. The technology side of the model follows Abel and Eberly (1994) but without shocks. The main difference is that here the firm faces stochastic state prices, while in their case pricing is risk neutral. We also present the steps needed to derive the return equation (4.5).

The capital stock evolves as  $dK_t = (I_t - \delta K_t) dt$ , and the investment cost is given by

$$H(I_t, K_t) = \left\{ \frac{b}{\nu} (I_t/K_t)^\nu + c \right\} K_t,$$

which is homogenous of degree one in  $I$  and  $K$ . The gross profit is given as

$$AK_t.$$

Assume that the state-price process is given as

$$d\Lambda_t = -\Lambda_t r(x_t) dt + \Lambda_t \sigma(x_t) dz_t,$$

where  $dz_t$  is a one-dimensional Brownian motion, and

$$dx_t = \mu_x(x_t)dt + \sigma_x(x_t)dz_t.$$

We assume that the functions  $\mu_x(x_t)$ ,  $\sigma_x(x_t)$ ,  $r(x_t)$  and  $\sigma(x_t)$  satisfy the regular conditions such that there are solutions for the above two SDEs.

The firm maximize its value

$$V = \max_{\{I_{t+s}\}} E_t \left\{ \int_0^\infty [AK_{t+s} - H(I_{t+s}, K_{t+s})] \frac{\Lambda_{t+s}}{\Lambda_t} ds \right\}.$$

Given the dynamics of  $\Lambda_t$ , it is obvious that the firm's value function  $V$  is independent of  $\Lambda_t$ . Following from the Markov property of the state variable  $x_t$ , the firm's value function would be a function of  $(K_t, x_t)$ . The HJB equation is

$$rV = \max_{\{I_t\}} \left\{ [AK_t - H(I_t, K_t)] + (I_t - \delta K_t) V_K + \mu_x V_x + \frac{1}{2} \sigma_x^2 V_{xx} + \sigma \sigma_x V_x \right\}.$$

The first-order condition is

$$H_I(I_t, K_t) = V_K \equiv q_t$$

That is,

$$\begin{aligned} V_K &= b(I_t/K_t)^{\nu-1} \\ I_t &= \left( \frac{V_K}{b} \right)^{\frac{1}{\nu-1}} K_t \end{aligned}$$

Because we have constant returns to scale in  $K_t$ , following Hayashi, it is easy to see that  $V(K_t, x_t) = K_t V_K(x_t)$ . Thus, it is clear that optimal investment follows an Ito process,  $dI_t/I_t = \mu_I(K_t, x_t) dt + \sigma_I(K_t, x_t) dz_t$ .

Define realized returns to the firm as

$$\frac{AK_t - H(I_t, K_t)}{V_t} dt + \frac{dV_t}{V_t}.$$

Given the Hayashi result and the first-order conditions

$$\frac{AK_t - H(I_t, K_t)}{V_t} dt + \frac{dV_t}{V_t} = \frac{AK_t - H(I_t, K_t)}{q_t K_t} dt + \frac{dK_t}{K_t} + \frac{dq_t}{q_t}.$$

Using  $q_t = b(I_t/K_t)^{\nu-1}$  and Ito's lemma, we can derive the return equation 4.5 given in the main text.

Table 1  
 Asset Pricing Implications: Baseline calibration

	$R^M$	$R^M - R^f$	$R^f$	Market Price of Risk	Sharpe Market	$R^{E\&S}$	$R^{E\&S} - R^f$	$R^S$	$R^S - R^f$
Mean		3.59%	1.43%	0.26	0.18		2.14%		5.09%
Std	20.52%		2.24%	0.19	0.26	12.06%		27.23%	
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Std[E( $R^M - R^f$  t)]	5.20%		Corr(Ep , StdR)	-0.56					
Std[Std( $R^M - R^f$  t)]	0.51%								
		Corr( IKZ , E( $R^M - R^f$  t) )	Corr( IKZ , $R^f$ )	Corr( IKZ , MPR )					
E&S		-0.03	-0.68	-0.58					
S		-0.34	-0.32	-0.82					
		Corr( $\lambda^{I\&S}$ , R)							
E&S, S		0.98							
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Real returns 1947-2003	$R^M$	$R^M - R^f$	$R^f$	Sharpe Market					
Mean		8.35%	1.09%	0.49					
Std	17.24%		2.07%						
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( $v_1=2.5$ , $v_2=5$ , $R=1.03$ , $x=0.23643$ , reduction $\sigma_{\Delta I} =20\%$ )									

Table 2

Asset Pricing Implications: IID case; (no serial correlation)

	$R^M$	$R^M - R^f$	$R^f$	Market Price of Risk	Sharpe Market	$R^{E\&S}$	$R^{E\&S} - R^f$	$R^S$	$R^S - R^f$
Mean		3.52%	1.43%	0.20	0.18		2.13%		4.94%
Std	20.75%		1.91%	0.13	0.15	12.21%		27.09%	
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Std[E( $R^M - R^f t$ )]	3.04%		Corr( $E_p$ , StdR)	-0.95					
Std[Std( $R^M - R^f t$ )]	0.45%								
	Corr( IKZ , E( $R^M - R^f t$ ) )		Corr( IKZ , $R^f$ )		Corr( IKZ , MPR )				
E&S	-0.62		-0.70		-0.54				
S	-0.89		-0.35		-0.78				
	Corr( $\lambda^{I\&S}$ , R )								
E&S, S	0.99								
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Real returns 1947-2003	$R^M$	$R^M - R^f$	$R^f$		Sharpe Market				
Mean		8.35%	1.09%		0.49				
Std	17.24%		2.07%						
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( $v_1=2.5$ , $v_2=5$ , $R=1.03$ , $x=0.23643$ , reduction $\sigma_{\Delta I} = 20\%$ )									

Table 3

Asset Pricing Implications: with shocks to investment technology, positive correlation  $\lambda^1$  and  $\lambda^2$ 

	$R^M$	$R^M - R^f$	$R^f$	Market Price of Risk	Sharpe Market	$R^{E\&S}$	$R^{E\&S} - R^f$	$R^S$	$R^S - R^f$
Mean		2.84%	2.01%	0.29	0.17		1.73%		4.04%
Std	17.14%		2.95%	0.20	0.30	10.53%		22.59%	
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Std[ $E(R^M - R^f t)$ ]	5.01%		Corr( $E_p$ , StdR)	-0.70					
Std[Std( $R^M - R^f t$ )]	0.29%								
	Corr( IKZ , $E(R^M - R^f t)$ )		Corr( IKZ , $R^f$ )		Corr( IKZ , MPR )				
E&S	0.11		-0.74		0.14				
S	-0.33		-0.29		-0.16				
	Corr( $\lambda^2$ , R)								
E&S, S	0.98								
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Real returns 1947-2003	$R^M$	$R^M - R^f$	$R^f$		Sharpe				
Mean		8.35%	1.09%		0.49				
Std	17.24%		2.07%						
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( $v_1=2.5$ , $v_2=5$ , $R=1.033$ , $x=0.28245$ , reduction $\sigma_{\Delta I} = 30\%$ )									

Table 4

Asset Pricing Implications: with shocks to investment technology, negative correlation  $\lambda_I$  and  $\lambda_Z$ 

	$R^M$	$R^M - R^f$	$R^f$	Market Price of Risk	Sharpe Market	$R^{E\&S}$	$R^{E\&S} - R^f$	$R^S$	$R^S - R^f$
Mean		4.13%	1.19%	0.30	0.20		2.77%		5.49%
Std	20.97%		3.17%	0.21	0.30	14.06%		26.45%	
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Std[ $E(R^M - R^f t)$ ]	6.07%		Corr( $E_p$ , StdR)	-0.66					
Std[Std( $R^M - R^f t$ )]	0.29%								
	Corr( IKZ , $E(R^M - R^f t)$ )		Corr( IKZ , $R^f$ )		Corr( IKZ , MPR )				
E&S	0.11		-0.57		0.15				
S	-0.32		-0.08		-0.16				
	Corr( $\lambda^Z$ , R )								
E&S, S	0.98								
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Real returns 1947-2003	$R^M$	$R^M - R^f$	$R^f$		Sharpe				
Mean		8.35%	1.09%		0.49				
Std	17.24%		2.07%						
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( $v_1=2.5$ , $v_2=5$ , $R=1.033$ , $x=.30846$ , reduction $\sigma_{\Delta I} = 30\%$ )									

Table 5

Asset Pricing Implications: Baseline calibration; No A shocks in sector 1 (Equipment)

	$R^M$	$R^M - R^f$	$R^f$	Market Price of Risk	Sharpe Market	$R^{E\&S}$	$R^{E\&S} - R^f$	$R^S$	$R^S - R^f$
Mean		2.89%	2.19%	0.24	0.16		1.45%		4.35%
Std	19.47%		2.39%	0.17	0.25	9.69%		27.24%	
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Std[E( $R^M - R^f$  t)]	4.65%		Corr(Ep , StdR)	-0.33					
Std[Std( $R^M - R^f$  t)]	1.01%								
		Corr( IKZ , E( $R^M - R^f$  t) )	Corr( IKZ , $R^f$ )	Corr( IKZ , MPR )					
E&S		0.06	-0.87	-0.58					
S		-0.25	-0.60	-0.83					
		Corr( $\lambda^{I\&S}$ , R)							
E&S, S		0.98							
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Real returns 1947-2003	$R^M$	$R^M - R^f$	$R^f$	Sharpe Market					
Mean		8.35%	1.09%	0.49					
Std	17.24%		2.07%						
<hr/>									
( $v_1=2.5$ , $v_2=5$ , $R=1.03$ , $x=0.23643$ , reduction $\sigma_{\Delta I} =20\%$ )									

Figure 1

### Realized investment growth 1948-2003

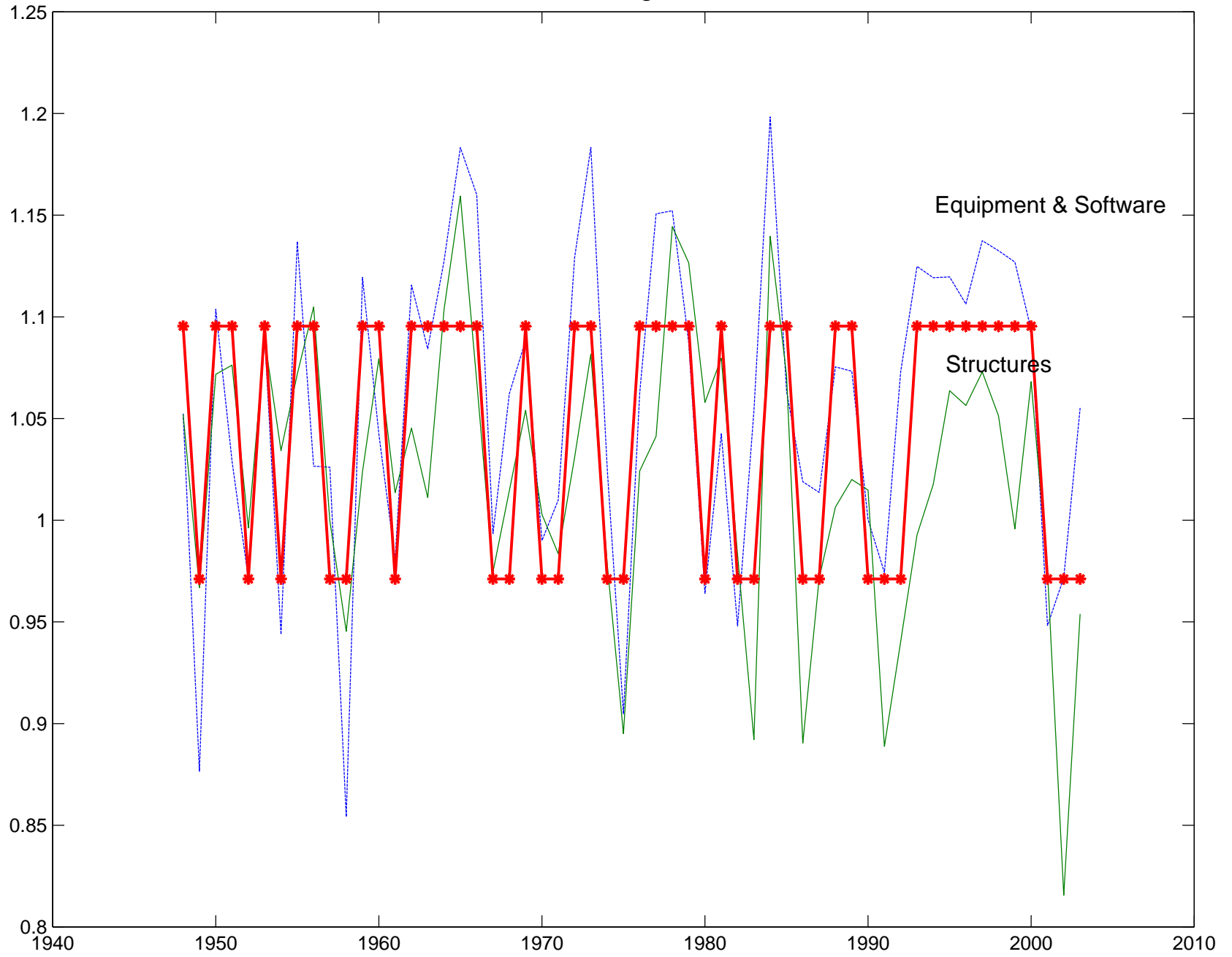


Figure 2

Realized market returns 1948-2002

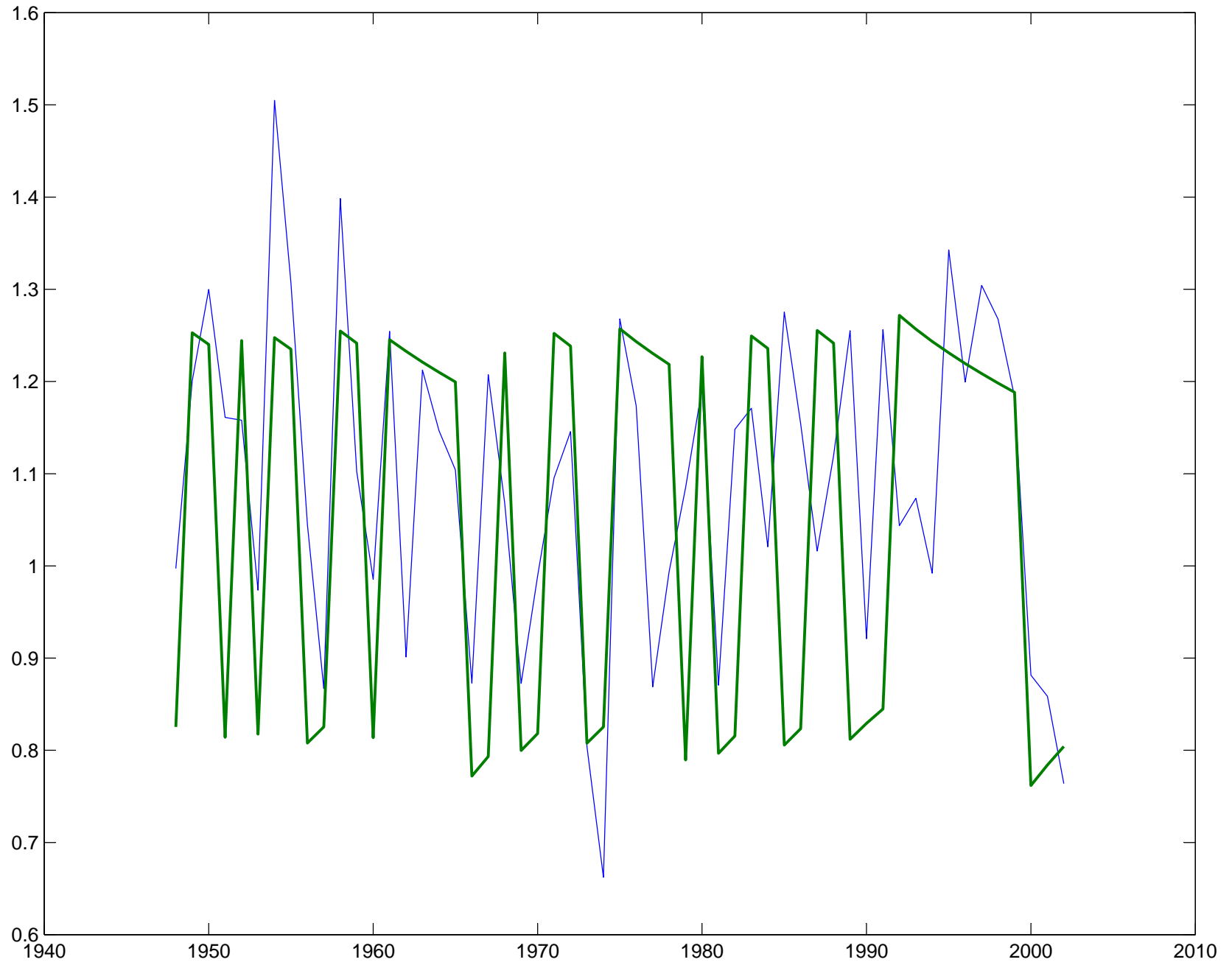
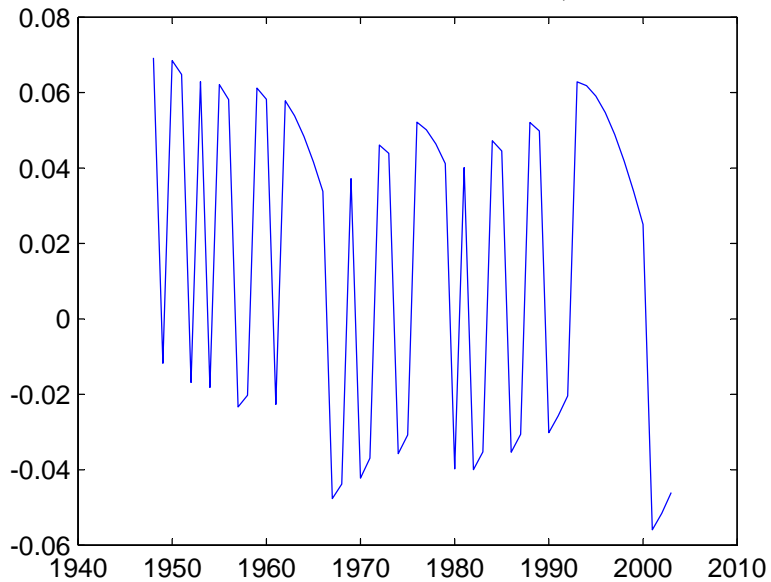


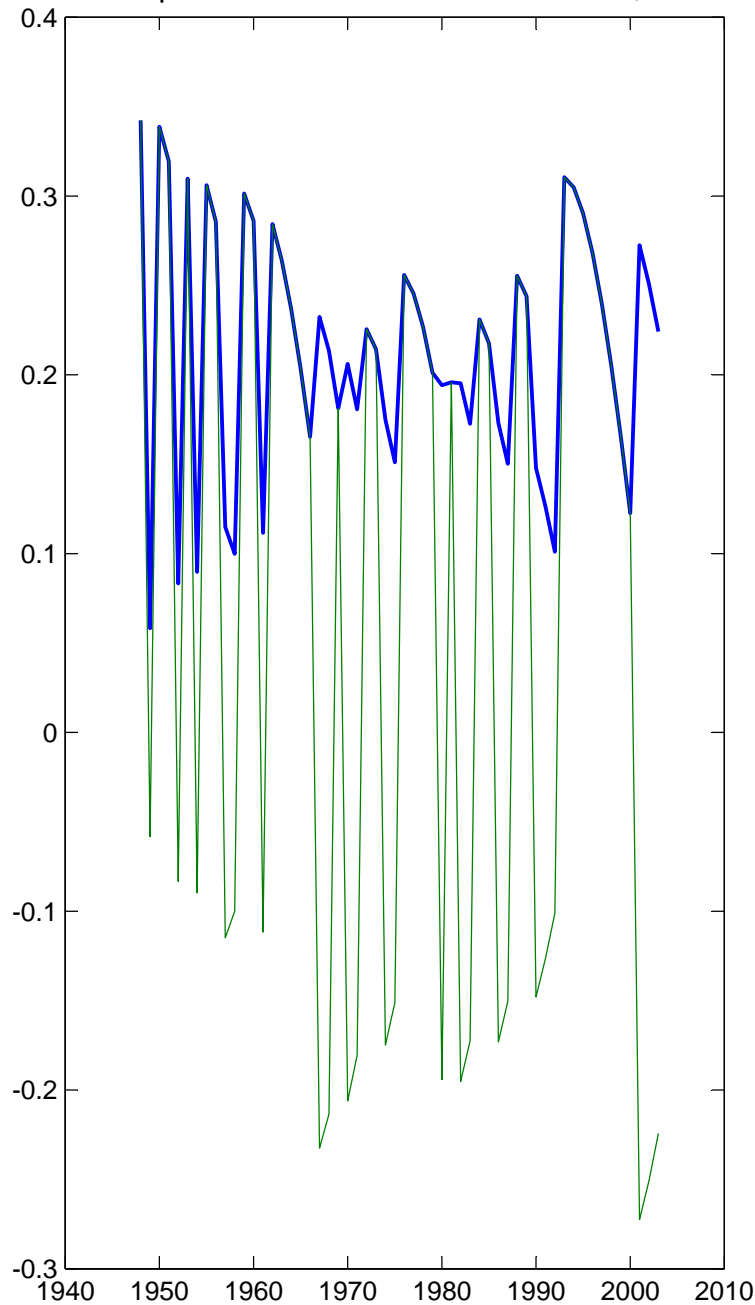
Figure 3a

Excess returns: Conditional mean, 1948-2003

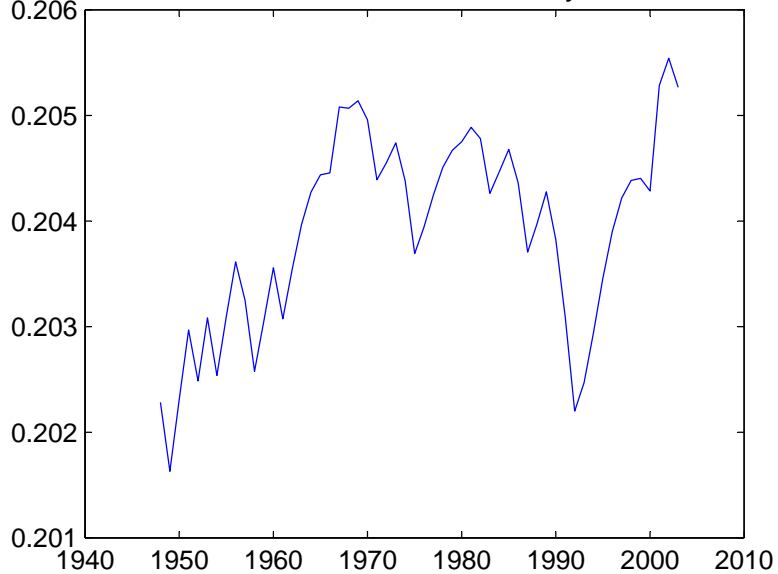


### Baseline Calibration

Market Sharpe Ratio and Market Price of Risk, 1948-2003

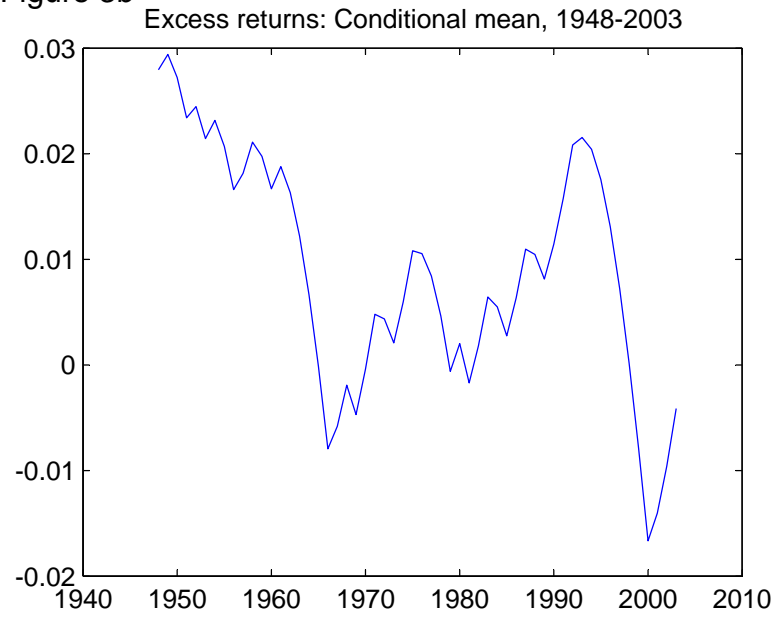


Excess returns: Conditional volatility, 1948-2003



# IID Calibration

Figure 3b



Market Sharpe Ratio and Market Price of Risk, 1948-2003

