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A Model of Crime and the Labor Market*

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Abstract

This paper examines interactions between the labor market and crime. It seems reasonable to believe that changes in policy or technologies that affect the labor market might affect the extent of criminal activities, and vice versa–changes in the crime sector can affect the labor market. To analyze these interactions we construct a search-theoretic model where labor market outcomes and crimes are determined jointly. We establish that a more generous unemployment insurance system raises the total crime rate, although our calibration shows the effect is quantitatively small. A change in preferences toward market work that generates a rise in labor market participation leads to an increase in crime. We also describe the effects that changes in workers' compensation, the availability of crime opportunities, and crime policies have on the labor market.

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1 Introduction

Changes in labor market policies can have unforeseen consequences in terms of criminal activities. As an example, the Job Seeker's Allowance introduced in the United Kingdom in 1996 was aimed at reducing unemployment by shortening the duration of unemployment benefits. According to Machin and Marie (2004), this reform had the unfortunate effect of increasing crime. Similarly, Hoon and Phelps (2003) advocate the use of labor market policies, such as wage subsidies, to reduce the enrollment of low-skilled workers in criminal activities. Changes in preferences or technology can also affect the labor market and crime together. The substantial increase of female participation in the labor force over the last 50 years –which has been attributed to changes in preferences and technology– has been accompanied by a sharp increase in female crime.[1] Witt and Witte (2000) suggest that these two phenomena are related.

The crime sector has also undergone substantial changes. Sentence lengths have been increased in several states, sentencing guidelines have become tougher, and some states have moved to "three-strikes" rules. While it is intuitively plausible that increased deterrence and/or punishment should reduce criminal activity, there is little research on how this might affect job duration, employment and other outcomes of the labor market. Given the pervasiveness and the extent of criminal activity –roughly 7 million persons under correctional supervision in 2003 compared to 9 million people unemployed– it seems worthwhile to develop a model where crime and labor market outcomes are determined jointly, and to use this model to assess qualitatively and quantitatively the effects of various experiments. In this paper we do just that.

We base our model on the canonical description of the labor market proposed by Pissarides (2000) since it has been thoroughly investigated, both qualitatively and quantitatively. According to this description, the labor market is subject to search-matching frictions and the terms of the employment contract are determined through bilateral bargaining. Participation decisions are formalized by assuming that individuals have idiosyncratic preferences for activities while out of the labor force.

We extend this labor market model to account for criminal activity by assuming that all individuals receive random opportunities to commit crimes. As in the standard sequential search

[1] See Greenwood, Seshadri, and Yorukoglu (2005), Fernandez, Fogli, and Olivetti (2004) for explanations of the increase in labor force participation. Over the second half of the last century, the fraction of offenses perpetrated by women has increased by a factor of about five.

framework, individuals follow an optimal stopping rule summarized by a reservation value for crime above which they undertake the crime opportunity. The presence of crime opportunities affects the equilibrium allocation and the welfare of the economy by distorting individuals' and firms' participation decisions.

Assuming that (detected) crimes are punished by periods of imprisonment, employed workers' involvement in criminal activities imposes a negative externality on firms by reducing average job duration. This type of externality, which is well understood in models with on-the-job search (crime can certainly be thought of in a similar way), can lead to inefficient separations if the contract space is restricted to flat wages.[2] We take the approach that employees and employers face no liquidity constraints and can write optimal employment contracts that generate efficient turnover from the point of view of a worker and employer. The optimal contract involves an up-front payment by the worker and a constant wage equal to the worker's productivity. While this contract is not necessarily realistic, it approximates more realistic ones with probationary periods or an upward sloping wage profile.

We prove that equilibrium exists and provide simple conditions for uniqueness.[3] We show that individuals' willingness to engage in criminal activities can be ranked according to their labor force status, unemployed workers being the least choosy in terms of crime opportunities to undertake. To highlight the tractability of the model, we provide a two-dimensional representation of the equilibrium similar in spirit to that in Mortensen and Pissarides (1994). This tractability allows us to study a broad range of experiments. We show analytically that a more generous unemployment insurance system reduces the crime rate of unemployed workers but has ambiguous effect on the crime rate of employed workers. Quantitatively, the total crime rate increases, although the effect is small: A 50% increase in unemployment benefits raises the total crime rate by only 2%. A change in preferences toward market work leads to higher participation in the labor market, but also higher crime by individuals out of the labor force. An increase in the output of a match leads to lower crime by all types of individuals.[4] Quantitatively, the effect is large: a 10% increase in productivity leads to nearly a 20% reduction in the total crime rate. Higher worker's bargaining power leads to higher unemployment but it has ambiguous (and highly nonlinear) effects on

[2]See Burdett and Mortensen (1998), the extensions by Burdett and Coles (2003) and Stevens (2004).

[3]One can consider various extensions of the model that generate multiple steady-state equilibria, however, we find it interesting that a benchmark version of the model predicts a unique equilibrium.

[4]See Freeman (1999) for an extensive review on the relationship between crime and workers' compensation.

the crime rates of employed and unemployed workers. We also study changes in the availability of crime opportunities and crime policies. As an example, if crime opportunities become more readily available for employed workers then the labor market becomes tighter (more firms enter), unemployed workers commit fewer crimes and employed workers become more choosy in terms of the crime opportunities they undertake.

There are few papers with calibrated models of the labor market and crime. Burdett, Lagos, and Wright (2003)– BLW hereafter differs from our paper in several key ways. First, while BLW adopt the wage posting framework of Burdett and Mortensen (1998), we employ the Pissarides model where the terms of the employment contract are determined via bilateral bargaining and where a free-entry condition of firms makes the job finding rate endogenous. This distinction is important because both worker's bargaining strength and the (endogenous) exit rate out-of-unemployment are important determinants of the trade-off that workers face when deciding whether or not to undertake crime opportunities. Also, by extending the Pissarides framework our model is set-up to discuss issues related to the business cycle (which cannot be addressed using the BLW model). Second, in contrast to BLW we consider optimal employment contracts that internalize the effect of workers' crime decisions on the duration of a match. In BLW the employment contract is restricted to a constant wage which leads to a wage distribution and multiple equilibria. Third, our framework endogenizes participation in the labor force which allows us to analyze how changes in preferences towards market work affect crime, and which provides an additional channel through which criminal activities can distort the allocation and lower welfare. In contrast, the distortions introduced by crime in BLW are due solely to the policy that consists of sending criminals to jail. Therefore, it is suboptimal to punish criminals in BLW. Our description of criminal activities also differs from BLW. The value of crime opportunities in our model are random draws from a distribution; this allows us to formalize crime behavior as a standard sequential search problem and to obtain endogenous crime rates for individuals in any state, i.e., employed, unemployed or out of the labor force.

Huang, Liang, and Wang (2004) is also related to our model in that they employ a search-theoretic framework with bilateral bargaining. One key difference is that in their model individuals specialize in criminal activities, while in ours they do not. Their specialization assumption means that employed workers never commit crimes, a result that is at odds with the evidence. Their framework uses an explicit matching process between criminals and victims while we assume stochastic arrivals of random crime opportunities for all individuals. Furthermore, the extent of criminal activity does not affect the duration of jobs in their model so that their wage contract is different from the one we consider. Although they do not formalize participation decisions they do include an endogenous human capital choice.

2 Model

The environment is similar to Pissarides (2000) extended to allow for criminal activity. Time, t, is continuous and goes on forever. The economy is composed of a unit-measure set $\mathscr{K}$ of infinitely-lived individuals indexed by κ and a large measure of firms. There is one final good produced by firms. Each individual is endowed with one indivisible unit of time that has three alternative, mutually exclusive uses: search for a job, work for a firm, or enjoy some utility from staying at home.

The instantaneous utility of a flow of consumption, $c(t)$, is simply $c(t)$ and individuals discount at rate $r > 0$. Individuals are not liquidity constrained and can borrow and lend at rate r. An unemployed worker who is looking for a job enjoys a utility flow b. One can interpret b as the utility from not working or as unemployment benefits paid by the government. Upon entering an employment relationship, a worker pays a hiring fee, ϕ, and receives a constant wage w thereafter. The pair (ϕ, w) will be determined through some bargaining solution. Such contracts will be shown to be optimal. This type of contract can be thought of as an extreme version of a contract with an upward sloping wage profile over time. Another interpretation is that there is an initial probationary period after which wages will be increased. Our contract supposes that there are no liquidity constraints. Although extreme, these assumptions allow us a simple characterization of equilibrium.

Individuals out of the labor force enjoy utility flow κp where $\kappa \geq 0$ is individual-specific and $p \geq 0$ is common across all individuals. Individuals are heterogeneous in terms of their utilities at home. The distribution of the κ's across individuals is given by a continuous distribution, $H(\kappa)$, with density $h(\kappa)$. We interpret the common component p as a proxy for the way society perceives work at home and in the market, or alternatively as the productivity of the technology in the home sector.[5] We assume that κ does not affect the utility of employed or unemployed workers, for

[5] According to Fortin (2005), "...female attitudes towards working women are developed in youth, influenced by parental education and religious affiliation." Or, following Fernandez et al. (2004), p could reflect women's spouses

instance, because of indivisibilities in the use of time.

Firms are composed of a single job, either filled or vacant. Vacant firms are free to enter the labor market. There is a flow cost, $\gamma > 0$, to advertise a vacancy. Vacant firms produce no output while filled jobs produce $y > b$. Firms are risk-neutral and discount future utility at rate $r > 0$.

The labor market is subject to search-matching frictions. The flow of hirings is given by the aggregate matching function $m(U,V)$ where U is the measure of unemployed workers actively looking for jobs and V is the measure of vacant jobs. The matching function $m(\cdot,\cdot)$ is strictly increasing and strictly concave with respect to each of its arguments and it exhibits constant returns to scale. Furthermore, $m(0,\cdot) = m(\cdot,0) = 0$ and $m(\infty,\cdot) = m(\cdot,\infty) = \infty$. Following Pissarides' terminology, we define $\theta \equiv V/U$ as labor market tightness. Each vacancy is filled according to a Poisson process with arrival rate $\frac{m(U,V)}{V} \equiv q(\theta)$. Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate $\frac{m(U,V)}{U} = \theta q(\theta)$. Filled jobs receive negative idiosyncratic productivity shocks, with a Poisson arrival rate s, that render matches unprofitable.[6]

Individuals in the economy receive an opportunity to commit a crime according to a Poisson process with arrival rate λ_i, where i indicates the individual's state: $i = u$ if unemployed, $i = e$ if employed and $i = o$ if out of the labor force. So, the availability of crime opportunities depends on one's labor force status. The value of a crime is ε, where ε is a random draw from a distribution $G(\varepsilon)$ with support $[0,\bar{\varepsilon}]$. A worker who commits a crime is caught and sent to jail with probability π. When in jail an individual cannot make any productive use of time but receive a flow of utility x (which can be negative). A prisoner exits jail according to a Poisson process with arrival rate δ. We assume that the average time spent in jail is independent of the value ε of the crime.[7]

The expected instantaneous loss from being victimized is equal to τ and is independent of one's labor force status. Also, we let individuals in jail be victimized which simply means that they have some property that can be stolen just like other individuals. Also, we assume that firms do not suffer directly from criminal activities. We impose a "balanced budget" requirement according to

attitudes toward working women.

[6]One could adopt a more general description of the idiosyncratic shocks received by firms and endogenize s. See Mortensen and Pissarides (1994).

[7]According to the U.S. Sentencing Commission Guidlines Manual the length of incarceration has more to do with the violent nature of the crime and the number of past offenses than the value of the crime. For a larceny less than \$10,000 (75% of thefts are under \$10,000) and if the criminal has not been convicted more than once, the Sentencing Commission Guidlines suggests a period of incarceration ranging from 0 to 6 months. If it is the criminals second or third offense then the suggested penalty is 4-10 months. If the theft is violent, such as a robbery, and the crime is still less than \$10,000, the guidelines suggest incarceration for 33-41 months.

which the aggregate amount stolen by criminals is equal to the loss incurred by victims.

3 Bellman equations

This paper focuses on steady state equilibria where the distribution of individuals across states and the measure of vacancies are constant over time. As a consequence, market tightness and matching probabilities are time invariant. In this section we write down the flow Bellman equations for individuals and firms and characterize the employment contract.

3.1 Individuals

An individual is in one of the following four states: Out of the labor force (o), unemployed (u), employed (e), or in prison (p). The value of being an individual in state $i \in \{o,u,e,p\}$ is denoted $\mathcal{V}_i$. The flow Bellman equations for individuals' value functions are

$$r\mathcal{V}_u = b-\tau+\theta q(\theta)\left(\mathcal{V}_e-\mathcal{V}_u-\phi\right)+\lambda_u\int\left[\varepsilon+\pi(\mathcal{V}_p-\mathcal{V}_u)\right]^+dG(\varepsilon), \quad (1)$$

$$r\mathcal{V}_e = w-\tau+s\left(\mathcal{V}_u-\mathcal{V}_e\right)+\lambda_e\int\left[\varepsilon+\pi\left(\mathcal{V}_p-\mathcal{V}_e\right)\right]^+dG(\varepsilon), \quad (2)$$

$$r\mathcal{V}_o = \kappa p-\tau+\lambda_o\int\left[\varepsilon+\pi\left(\mathcal{V}_p-\mathcal{V}_o\right)\right]^+dG(\varepsilon), \quad (3)$$

$$r\mathcal{V}_p = x-\tau+\delta\left[\max\left(\mathcal{V}_u,\mathcal{V}_o\right)-\mathcal{V}_p\right]. \quad (4)$$

where $[x]^+=\max(x,0)$. When there is no ambiguity we omit the dependence of the value functions on κ. Equation (1) has the following interpretation. An unemployed worker enjoys a utility flow of $b-\tau$ where b is the income of unemployed workers and τ is the (expected) cost of being victimized. A job is found with an instantaneous probability $\theta q(\theta)$. Upon taking a job an individual pays a hiring fee, ϕ (or receives an up-front payment if $\phi<0$), and enjoys the capital gain $\mathcal{V}_e-\mathcal{V}_u$. When unemployed the individual receives an opportunity to commit a crime with instantaneous probability λ_u. The value of the crime opportunity is drawn from the cumulative distribution $G(\varepsilon)$. If a worker chooses to commit a crime she enjoys utility ε but is at risk of being caught and sent to jail with probability π, in which case she suffers a capital loss, $\mathcal{V}_p-\mathcal{V}_u$. From (2), an employed worker receives a wage w, loses the job with an instantaneous probability s and has the opportunity to commit a crime with an instantaneous probability λ_e. As can be seen in (3), an individual out of the labor-force enjoys the utility κp and receives an opportunity to commit crime with an instantaneous probability λ_o. According to (4), an imprisoned worker receives consumption flow

x, suffers the loss τ, and exits jail with an instantaneous probability δ. After release a decision has to be made whether to participate in the labor force as an unemployed worker or to be out of the labor force. In steady-state a prisoner who was previously in the labor force returns to the labor force upon release from jail. Similarly, a prisoner who was previously out of the labor-force returns to home production activities after exiting jail.

From (1), (2) and (3), an individual κ in state i chooses to commit a crime whenever $\varepsilon \geq \varepsilon_i$ where

$$\varepsilon_u = \pi(\mathscr{V}_u - \mathscr{V}_p), \quad (5)$$

$$\varepsilon_e = \pi(\mathscr{V}_e - \mathscr{V}_p), \quad (6)$$

$$\varepsilon_o(\kappa) = \pi[\mathscr{V}_o(\kappa) - \mathscr{V}_p(\kappa)]. \quad (7)$$

From (5)-(7) the value of the marginal crime, that makes an individual in a given state indifferent between undertaking the crime or not, ε_i, is the expected cost of punishment, $\pi(\mathscr{V}_i - \mathscr{V}_p)$.

An individual chooses to stay at home if $\mathscr{V}_o(\kappa) \geq \mathscr{V}_u$. From (3), the utility from staying at home is increasing with κ. Hence, there exists a threshold κ_u such that an individual chooses not to participate in the labor force if $\kappa \geq \kappa_u$, where κ_u satisfies $\mathscr{V}_o(\kappa_u) = \mathscr{V}_u$. From (5) and (7), $\varepsilon_o(\kappa_u) = \varepsilon_u$; the marginal worker out of the labor force has the same reservation value for crime opportunities as an unemployed worker. Therefore, from (1) and (3), and using the fact that $\int_{\varepsilon_i}^{\bar{\varepsilon}}(\varepsilon - \varepsilon_i)dG(\varepsilon) = \int_{\varepsilon_i}^{\bar{\varepsilon}}[1 - G(\varepsilon)]d\varepsilon$ from integration by parts,

$$\kappa_u p = b + \theta q(\theta)(\mathscr{V}_e - \mathscr{V}_u - \phi) + (\lambda_u - \lambda_o)\int_{\varepsilon_u}^{\bar{\varepsilon}}[1 - G(\varepsilon)]d\varepsilon. \quad (8)$$

According to (8), the reservation utility, κ_u, below which individuals choose to participate in the labor market, is such that the flow utility from staying at home, the left-hand side of (8), is equal to the the sum of the income flow received when unemployed, the expected surplus from finding a job and the difference of the returns from criminal activities for unemployed individuals and individuals out of the labor force, the right-hand side of (8).

3.2 Firms

Firms participating in the market can be in either of two states: they can hold a vacant job (v) or a filled job (f). Firms' flow Bellman equations are

$$r\mathscr{V}_v = -\gamma + q(\theta)\left(\phi + \mathscr{V}_f - \mathscr{V}_v\right), \quad (9)$$

$$r\mathscr{V}_f = y - w - s\left(\mathscr{V}_f - \mathscr{V}_v\right) - \lambda_e \pi \left[1 - G(\varepsilon_e)\right](\mathscr{V}_f - \mathscr{V}_v). \quad (10)$$

According to (9), a vacancy incurs an advertising cost γ; finds an unemployed worker with an instantaneous probability q in which case it enjoys the capital gain $\phi + \mathscr{V}_f - \mathscr{V}_v$. According to (10), a filled job enjoys a flow profit $y - w$ and is destroyed if a negative idiosyncratic productivity shock occurs, with an instantaneous probability s, or if the worker commits a crime and is caught, an event occurring with an instantaneous probability $\lambda_e \pi[1 - G(\varepsilon_e)]$. Free-entry of firms implies $\mathscr{V}_v = 0$ and therefore, from (9),

$$\mathscr{V}_f + \phi = \frac{\gamma}{q(\theta)}. \quad (11)$$

From (11), the firms' surplus from a match, the sum of the value of a filled job and the hiring fee, is equal to the average recruiting cost incurred by the firm.

3.3 Employment contract

In establishing the employment contract we take as given the existence of a commitment technology for the firm. That is, once the workers pays the hiring fee, the firm will not renege on the agreed future compensation.

Define $\mathscr{S} \equiv \mathscr{V}_e - \mathscr{V}_u + \mathscr{V}_f$ as the total surplus of a match (Recall that $\mathscr{V}_v = 0$). From (2) and (10),

$$r\mathscr{S} = y - \tau - r\mathscr{V}_u - s\mathscr{S} + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} \left[\varepsilon - \pi\mathscr{S} - \pi(\mathscr{V}_u - \mathscr{V}_p)\right] dG(\varepsilon). \quad (12)$$

Equation (12) has the following interpretation. A match generates a flow surplus, $y - \tau - r\mathscr{V}_u$, composed of the output of the job minus the loss due to victimization and the permanent income of an unemployed person, $r\mathscr{V}_u$. The match is destroyed if an exogenous shock occurs, with an instantaneous probability s, or if the worker commits a crime and is caught. In this case, the value $\mathscr{S}$ of the match is lost and the worker goes to jail which generates an additional capital loss $\mathscr{V}_u - \mathscr{V}_p$. The value of the match also incorporates the crime opportunities undertaken by the employed worker.

Suppose a worker and a firm could *jointly* determine the crime opportunities undertaken by the worker. It can be seen from (12), that the surplus of the match is maximized if

$$\varepsilon_e = \pi(\mathscr{S} + \mathscr{V}_u - \mathscr{V}_p) = \pi\left(\mathscr{V}_e + \mathscr{V}_f - \mathscr{V}_p\right). \tag{13}$$

Comparison of (6) and (13) reveals that if $\mathscr{V}_f > 0$ then the total surplus of the match is not maximized. Employed workers commit too much crime because they do not internalize the negative externality they impose on the firm if they commit a crime and are sent to jail.

In the following, we will allow the firm to charge an up-front fee, ϕ, so that the worker and the firm reach an efficient contract.[8] The employment contract (ϕ, w) is determined by the generalized Nash solution where the worker's bargaining power is $\beta \in [0,1]$. The contract satisfies

$$(\phi, w) = \arg\max\left(\mathscr{V}_e - \mathscr{V}_u - \phi\right)^{\beta}\left(\mathscr{V}_f + \phi\right)^{1-\beta}. \tag{14}$$

Lemma 1 *The employment contract solution to (14) is such that*

$$w = y, \tag{15}$$

$$\phi = (1-\beta)\left(\mathscr{V}_e - \mathscr{V}_u\right). \tag{16}$$

According to Lemma 1, the wage is set to be equal to the worker's productivity. Since the worker gets the entire output generated by the match this wage setting guarantees that the worker internalizes the effect of their crime decision on the total surplus of the match. The up-front payment is used to split the surplus of the match according to each agent's bargaining power.[9]

4 Equilibrium

In this section we will establish that the model has a simple recursive structure that can be exploited to study the properties of equilibrium. In particular, we will show that we can reduce the model

[8] In the wage-posting model of Burdett et al. (2003), firms are restricted to post contracts promising a constant wage. This restriction generates an inefficient turnover of workers and, for some parameter values, a nondegenerate distribution of wages. For a similar result in a search-on-the-job model, see Stevens (2004). If the employment contract is restricted to a constant wage, the bargaining set may not be convex in which case the Nash solution cannot be used. For an elaboration of this point in a related context, see Shimer (2005).

[9] Alternatively, the optimal contract could take the form of a constant wage w and a payment from the worker to the firm if the worker is caught committing a crime. This transfer would exactly compensate the firm for its lost surplus. In addition, note that any flat wage contract is dominated by a contract with an upward sloping wage profile. So even in the presence of liquidity constraints, the optimal contract would require the wage to increase with tenure. A loose, but somewhat more realistic interpretation, is to think of ϕ in terms of a probation period. For a related discussion, see Chapter 5 in Mortensen's book on wage dispersion

to two equations and two unknowns, market tightness (θ) and the reservation value for crime opportunities (ε_u).

First, we can use the free-entry condition of firms to express the worker's and firm's surpluses from a match as functions of market tightness. From (11), $\mathscr{V}_f = 0$ implies

$$\phi = \frac{\gamma}{q(\theta)}. \tag{17}$$

The gain from filling a vacancy is equal to the up-front payment, ϕ, which equals the average recruiting cost incurred by the firm to fill a vacancy. The expected surplus received by an unemployed worker who finds a job is

$$-\phi + \mathscr{V}_e - \mathscr{V}_u = \frac{\beta\gamma}{(1-\beta)q(\theta)}. \tag{18}$$

The worker's surplus from a match is $\frac{\beta}{1-\beta}$ times the expected recruiting costs incurred by firms.

Using the Bellman equations (1), (2) and (4), as well as the expression for the worker's surplus, (18), the crime decisions (5)-(7) can be rewritten as follows:

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_u = b - x + \frac{\beta}{1-\beta}\theta\gamma + \lambda_u \int_{\varepsilon_u}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon, \tag{19}$$

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_e = y - x + \frac{(\delta-s)\gamma}{q(\theta)(1-\beta)} + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon. \tag{20}$$

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_o(\kappa) = \kappa p - x + \lambda_o \int_{\varepsilon_o(\kappa)}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon. \tag{21}$$

Given θ, (19)-(21) determine a unique list $[\varepsilon_u, \varepsilon_e, \varepsilon_o(\kappa)]$ of threshold values for crime decisions. Notice that (19)-(21) correspond to standard optimal stopping rules where the left-hand side represents the gain from stopping (expressed in flow terms and adjusted for the probability of being caught) and the right-hand side is the flow gain from continuing to search for opportunities. Also, (19) gives us our first relationship between ε_u and θ.

Let us turn to the determination of market tightness. Substitute (18) into (1) and integrate the integral term in (1) by parts, the permanent income of an unemployed worker obeys

$$r\mathscr{V}_u = b - \tau + \frac{\beta}{1-\beta}\theta\gamma + \lambda_u \int_{\varepsilon_u}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon. \tag{22}$$

From (2) and (22) and using the fact that $\mathscr{V}_e - \mathscr{V}_u = \gamma/[(1-\beta)q(\theta)]$, market tightness satisfies

$$\frac{(r+s)\gamma}{(1-\beta)q(\theta)} = y - b - \frac{\beta}{(1-\beta)}\theta\gamma - \lambda_u \int_{\varepsilon_u}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon. \tag{23}$$

Given the thresholds ε_u and ε_e, (23) determines a unique θ. Up to the last two terms on the right-hand side, (23) is identical to the equilibrium condition in the Pissarides model. If crime activities are more valuable for unemployed workers than for employed ones, i.e., the sum of the last two terms is negative, then the presence of crime opportunities tends to reduce market tightness. This will be the case if the arrival rates of crime opportunities are the same for employed and unemployed workers, $\lambda_e = \lambda_u$, since $\varepsilon_e > \varepsilon_u$. Using (6)

$$\varepsilon_e = \varepsilon_u + \frac{\pi\gamma}{(1-\beta)q(\theta)}. \tag{24}$$

Substituting ε_e by its expression given by (24) into (23) we obtain a relationship between ε_u and θ,

$$\begin{aligned} \frac{(r+s)\gamma}{(1-\beta)q(\theta)} &= y - b - \frac{\beta}{(1-\beta)}\theta\gamma - \lambda_u \int_{\varepsilon_u}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon \\ &\quad + \lambda_e \int_{\varepsilon_u + \frac{\pi\gamma}{(1-\beta)q(\theta)}}^{\bar{\varepsilon}} [1-G(\varepsilon)]\, d\varepsilon. \end{aligned} \tag{25}$$

Equation (25) gives us our second relationship between ε_u and θ. According to (25), if $\lambda_u[1-G(\varepsilon_u)] > \lambda_e[1-G(\varepsilon_e)]$ then θ increases with ε_u. This condition is satisfied, for instance, if $\lambda_u = \lambda_e$.

From (8), the reservation utility at home, below which individuals participate in the labor force satisfies

$$\kappa_u p = b + \frac{\beta}{1-\beta}\theta\gamma + (\lambda_u - \lambda_o)\int_{\varepsilon_u}^{\bar{\varepsilon}} [1-G(\varepsilon)] d\varepsilon. \tag{26}$$

Notice that τ, the cost of being victimized, does not appear in the equilibrium conditions (19), (24), (25) and (26). Therefore, the requirement that the losses incurred by victims are the gains of the criminals is irrelevant for the equilibrium.

Finally, we characterize the steady-state distribution of individuals across states. The state of an individual is comprised of her labor force status and utility at home. Denote $n_i(\kappa)$ the density measure of individuals in state $i \in \{e,u,o,p\}$ with utility at home κp. More precisely, $\int_E n_i(\kappa) d\kappa$ is the measure of individuals in state i whose utility at home is $\kappa \in E \subseteq \mathscr{K}$. Consider individuals who do not participate in the labor force, $\kappa \geq \kappa_u$. The condition that the flows in and out of each state are equal implies

$$n_o(\kappa)\lambda_o\pi[1-G(\varepsilon_o(\kappa))] = \delta n_p(\kappa), \tag{27}$$

$$n_o(\kappa) + n_p(\kappa) = h(\kappa). \tag{28}$$

According to (27) the flow of individuals from out-of-the-labor-force to jail, $n_o(\kappa)\lambda_o\pi[1-G(\varepsilon_o(\kappa))]$, has to be equal to the flow of individuals from jail to out-of-the-labor-force, $\delta n_p(\kappa)$. Equation (28) states that individuals with utility at home greater than κ_u are either out of the labor force or in jail.

Consider next workers who participate in the labor market ($\kappa < \kappa_u$). The distribution $[n_u(\kappa), n_e(\kappa), n_p(\kappa)]$ is determined by the following steady-state conditions:

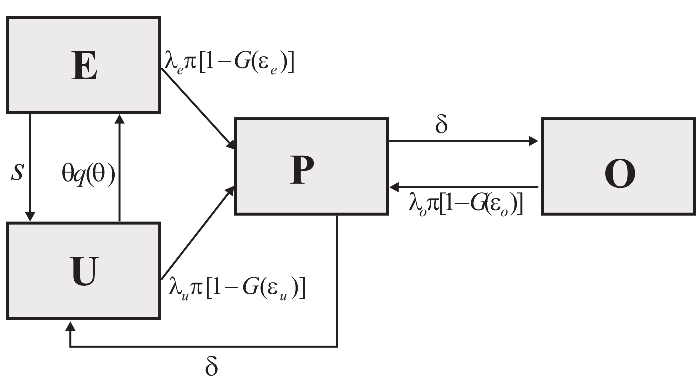
$$sn_e(\kappa) + n_p(\kappa)\delta = \{\theta q(\theta) + \lambda_u \pi[1 - G(\varepsilon_u)]\}\, n_u(\kappa), \tag{29}$$

$$\theta q(\theta) n_u(\kappa) = \{s + \lambda_e \pi[1 - G(\varepsilon_e)]\}\, n_e(\kappa), \tag{30}$$

$$n_e(\kappa) + n_u(\kappa) + n_p(\kappa) = h(\kappa). \tag{31}$$

According to (29) the flows in and out of unemployment must be equal. The measure of individuals (with flow of utility at home κ) entering unemployment is the sum of the employed workers who lose their jobs, $sn_e(\kappa)$, and the criminals who exit jail, $n_p(\kappa)\delta$. The flow of individuals exiting unemployment corresponds to individuals finding jobs, $\theta q(\theta)n_u(\kappa)$, or unemployed individuals committing crimes and sent to jail, $\lambda_u\pi[1-G(\varepsilon_u)]n_u(\kappa)$. Similarly, (30) prescribes that the flows in and out of employment must be equal in steady state. According to (31), individuals with utility at home less than κ_u are either employed, unemployed, or in jail. Figure 1 diagrams the above-mentioned flows.

Figure 1: Worker Flows



The equilibrium unemployment rate u is defined as the fraction of individuals in the labor force who are unemployed,

$$u = \frac{\int_0^{\kappa_u} n_u(\kappa)d\kappa}{\int_0^{\kappa_u} n_e(\kappa)d\kappa + \int_0^{\kappa_u} n_u(\kappa)d\kappa}. \tag{32}$$

From (30), it satisfies

$$u = \frac{s+\lambda_e\pi\left[1-G(\varepsilon_e)\right]}{\theta q(\theta)+s+\lambda_e\pi\left[1-G(\varepsilon_e)\right]}. \tag{33}$$

As in Mortensen and Pissarides (1994), the unemployment rate decreases with market tightness and increases with the job destruction rate which, in our model, depends on ε_e. The rate at which jobs are destroyed is endogenous, it depends on employed workers' decision to commit crimes. The participation rate is computed as the fraction of individuals who are not in jail who choose to participate in the labor market. It satisfies

$$\mathscr{P} = \frac{\int_0^{\kappa_u} n_e(\kappa)d\kappa + \int_0^{\kappa_u} n_u(\kappa)d\kappa}{1-\int_0^{\bar{\kappa}} n_p(\kappa)d\kappa}. \tag{34}$$

Assuming that the instantaneous losses that individuals suffer from criminal activities are equal to the crime opportunities undertaken in the economy, we find that

$$\tau = \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) \int_0^{\kappa_u} n_e(\kappa)d\kappa + \lambda_u \int_{\varepsilon_u}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) \int_0^{\kappa_u} n_u(\kappa)d\kappa + \lambda_o \int_{\kappa_u}^{\infty} \int_{\varepsilon_o(\kappa)}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) n_o(\kappa)d\kappa. \tag{35}$$

We are now ready to define an equilibrium for the model.

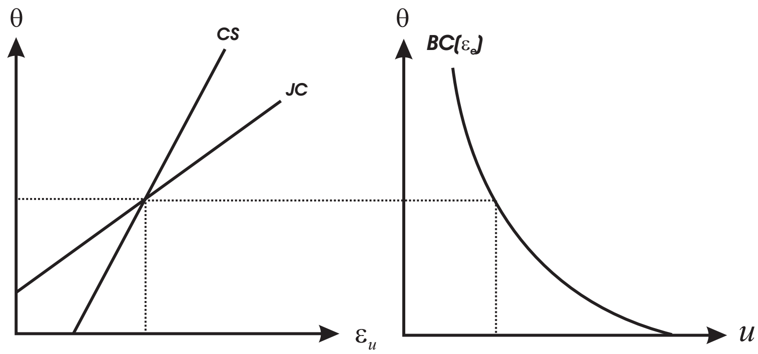
Definition 2 *A steady-state equilibrium is a list* $\{\theta,\ \kappa_u,\ \varepsilon_u,\ \varepsilon_e,\ \varepsilon_o(\kappa),\ n_e(\kappa),\ n_u(\kappa),\ n_o(\kappa),\ n_p(\kappa)\}$ *such that:* θ *satisfies (25);* κ_u *satisfies (26);* $\{\varepsilon_u,\ \varepsilon_e,\ \varepsilon_o(\kappa)\}$ *satisfies (19), (24), (21);* $\{n_e(\kappa),\ n_u(\kappa),\ n_o(\kappa),\ n_p(\kappa)\}$ *satisfies (27)-(31) and* τ *that satisfies (35).*

As indicated above, the model is recursively solvable. First, the crime decisions of individuals out of the labor force are determined independently of other endogenous variables by (21). Second, the pair (θ, ε_u), is determined jointly from (19) and (25). Third, knowing (θ, ε_u), one can use (24) and (26) to find ε_e and κ_u. Finally, knowing $\{\theta, \kappa_u, \varepsilon_u, \varepsilon_e, \varepsilon_o(\kappa)\}$ the steady-state distribution $\{n_e(\kappa), n_u(\kappa), n_o(\kappa), n_p(\kappa)\}$ is obtained from (27)-(31).

Figure 2 represents the determination of the pair (θ, ε_u). We denote *CS* (crime schedule) the curve representing (19) and *JC* (job creation) the curve representing (25). Recall that *CS* always slopes upward while *JC* can slope upward or downward depending on the the values of λ_e and λ_u. In the case where $\lambda_u = \lambda_e$ the two curves slope upward. Along *CS*, as the number of vacancies per unemployed increases, unemployed workers are less likely to commit crimes. Along *JC*, as the frequency of crime by the unemployed falls, the number of jobs in the market increases. The Beveridge curve (33) is denoted $BC(\varepsilon_e)$. It shifts with the reservation value ε_e which from (24) is uniquely determined from θ and ε_u. In Figure 2, the curves *CS* and *JC* intersect once. The following lemma establishes that this result holds in general.

Lemma 3 *In the space (ε_u, θ) the curve JC intersects the curve CS from above.*

Figure 2: Equilibrium



Interestingly, the determination of equilibrium is reminiscent of the one in Mortensen and Pissarides (1994) where labor market tightness and the job destruction rate are determined jointly. The *CS* curve in our model is analogous to the job destruction curve in the Mortensen-Pissarides model in that workers' crime decisions affect the duration of a job. Denote ε_u^0 the value of ε_u that solves (19) when $\theta = 0$.

Proposition 4 *There exists a unique equilibrium such that $\theta > 0$ if*

$$y - b + (\lambda_e - \lambda_u) \int_{\varepsilon_u^0}^{\bar{\varepsilon}} [1 - G(\varepsilon)] d\varepsilon > 0. \tag{36}$$

Proposition 4 shows that equilibrium exists and is unique. So despite the possibility of strategic complementarities between individuals' crime decisions and firms' entry decisions, there is no multiple steady-state equilibria in this model. The condition (36) for firms entering the market requires that the rate at which unemployed workers receive crime opportunities is not too high compared to the arrival rate of crime opportunities for employed workers; obviously, it is satisfied if $\lambda_e = \lambda_u$.

Proposition 5 *In any equilibrium where $\theta > 0$, $\varepsilon_e > \varepsilon_u$ and $\varepsilon_o(\kappa) \geq \varepsilon_u$ for all $\kappa \geq \kappa_u$.*

Proposition 5 shows that unemployed workers are less picky than other individuals when choosing which crime opportunities to accept. To see this, note that employed workers are paid their marginal product which is larger than the income they receive when unemployed. Therefore, the opportunity cost of being caught and sent to jail is higher for employed workers. Also, individuals who choose not to participate in the labor force have a higher expected utility than individuals who are unemployed. Consequently, those individuals suffer a higher cost of being sent to jail than unemployed workers. In the particular case where $\lambda_u = \lambda_e = \lambda_o$ Proposition 5 implies that the crime rate of unemployed workers is larger than the crime rates of employed workers and individuals out of the labor force.

The following Proposition provides a condition under which the equilibrium is characterized by no criminal activities. Denote $\hat{\theta}$ the value of market tightness that solves

$$\frac{(r+s)\,\gamma}{q(\hat{\theta})} = (1-\beta)\,(y-b) - \beta\hat{\theta}\gamma. \tag{37}$$

This is the market tightness that would prevail in an economy without crime.

Proposition 6 *If*

$$\frac{(r+\delta)}{\pi}\bar{\varepsilon} \leq b - x + \frac{\beta}{1-\beta}\hat{\theta}\gamma \tag{38}$$

then the equilibrium is such that $\theta = \hat{\theta}$ and no crime occurs.

According to Proposition 6, there is no crime in equilibrium provided that the probability of being caught is sufficiently high and the time spent in jail is sufficiently long.

5 Welfare

We assume that the discount rate is close to 0 and measure society's welfare, $\mathscr{W}$, as the sum of all agents' utility flows in steady state,

$$\mathscr{W} = \int [n_u(\kappa)\,(b-\theta\gamma) + n_e(\kappa)y + n_o(\kappa)\kappa p + n_p(\kappa)x]\,d\kappa. \tag{39}$$

From (39) welfare is the sum of unemployed workers' consumption (b), the output y produced in each match, the utility flows of workers out of the labor force (κp) and in jail (x) minus the recruiting expenses incurred by firms to find unemployed workers (γ).[10] The planner is subject to the same matching frictions, summarized by $m(U,V)$, as individuals and takes π, the technology to catch criminals, and δ, the jail sentence, as given. So the planner can only choose agents' participation (including entry of vacancies) and crime decisions.

To conduct our welfare analysis, we normalize individuals' utility flow in jail to $x = 0$ and assume that $b > 0$ and $\kappa > 0$ so that prisoners get the lowest utility. As long as $\pi > 0$ the planner would always want to have no crime. To see this notice that crime opportunities have no value for society since they are pure transfers from the victim to the criminal. However, the punishment associated with criminal activities generates a cost to society since individuals in jail are not productive. By setting $\varepsilon_o = \varepsilon_u = \varepsilon_e = \bar{\varepsilon}$ the planner guarantees that all workers out of the labor force enjoy their utility at home instead of the lower utility flow in jail, and it maximizes the number of individuals who are either employed or unemployed.

From (33) $u = \frac{s}{\theta q(\theta)+s}$ and assuming $\varepsilon_u = \varepsilon_e = \varepsilon_o = \bar{\varepsilon}$ the expression for social welfare can be simplified to

$$\mathscr{W} = H(\kappa_u)\left[\frac{s}{s+\theta q(\theta)}(b-\theta\gamma) + \frac{\theta q(\theta)}{s+\theta q(\theta)}y\right] + p\int_{\kappa_u}^{\infty} \kappa dH(\kappa). \tag{40}$$

According to (40) there is a measure $H(\kappa_u)$ of workers who participate in the labor force; a fraction $\frac{s}{s+\theta q(\theta)}$ are unemployed and receive b while the remaining fraction are employed and generate output y. The total number of vacancies is $H(\kappa_u)\frac{s\theta}{s+\theta q(\theta)}$ and each vacancy incurs a flow cost γ. The last term on the right-hand side of (40) captures the sum of utility flows of individuals who are not participating. Maximizing welfare with respect to θ and κ_u leads to the following two necessary conditions:

$$\frac{s\gamma}{[1-\eta(\theta)]\,q(\theta)} = y - b - \frac{\eta(\theta)}{1-\eta(\theta)}\gamma\theta, \tag{41}$$

$$\kappa_u p = b + \frac{\eta(\theta)}{1-\eta(\theta)}\gamma\theta, \tag{42}$$

where $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$ is the elasticity of the matching function. The allocation prescribed by (41) and (42) is independent of the policy (π,δ). Therefore, the maximum social welfare is the same for all crime policies.

[10]The number of vacancies in equilibrium is equal to the product of market tightness, θ, and the measure of unemployed workers, $\int n_u(\kappa)d\kappa$.

Let us next turn to the comparison of the equilibrium allocation and the optimal one. Provided that condition (38) is not satisfied some crimes occur in equilibrium. As a consequence, there is a positive measure of agents who are in jail which creates a welfare loss. This loss can be removed by setting $\pi = 0$.[11] However, in our model the presence of crime opportunities has other undesirable effects by distorting individuals' and firms' decisions to enter the market. To see this, suppose that $\pi = 0$. Then, agents take all crime opportunities, $\varepsilon_e = \varepsilon_u = \varepsilon_o = 0$. The equilibrium conditions (25) and (26), assuming that $r \approx 0$, become

$$\frac{(r+s)\gamma}{(1-\beta)q(\theta)} = y - b - \frac{\beta}{(1-\beta)}\theta\gamma + (\lambda_e - \lambda_u)\int_0^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon, \tag{43}$$

$$\kappa_u p = b + \frac{\beta}{1-\beta}\theta\gamma + (\lambda_u - \lambda_o)\int_0^{\bar{\varepsilon}}[1-G(\varepsilon)]d\varepsilon. \tag{44}$$

Compare (41) and (42) with the equilibrium conditions (43) and (40). If all individuals receive crime opportunities at the same rate, $\lambda_e = \lambda_u = \lambda_o$, the planner's choices coincide with the equilibrium allocation if and only if $\beta = \eta(\theta)$, the Hosios condition holds.

If the arrival rates of crime opportunities are not equal for individuals with different labor status, then crime does affect participation decisions. For instance, if $\lambda_u > \lambda_e$ too few firms open vacancies (assuming the Hosios condition holds). Similarly, if $\lambda_o > \lambda_u$ then participation in the labor force tends to be too low.

6 Calibration

The model is calibrated to the U.S. labor market. The unit of time corresponds to one year and the rate of time preference is set to $r = 0.048$. The output from a match is normalized to $y = 1$ following Shimer (2005). The flow of utility when unemployed is $b = 0.4$[12]

The matching function is assumed to be Cobb-Douglas, $m(U,V) = AU^{\eta}V^{1-\eta}$ with constant returns to scale and we set $\eta = 0.5$. Although Shimer (2005) chose 0.72 we use a value of 0.5 which is in the middle of the range of estimates found in the literature (see Petrongolo and Pissarides (2001)). We set the bargaining power of the worker, $\beta = 0.5$, which means the Hosios (1990) condition holds. The distribution of utilities in the home sector is exponential, $g(\kappa) = e^{-\kappa}$. We

[11] In the model by Burdett, Lagos and Wright (2003, 2005), crime imposes a cost on society only when individuals are sent to jail. The optimal policy in their model is therefore not to punish criminals.

[12] The choice of this parameter is controversial, here we follow Shimer (2005).

calibrate p so the model's labor force participation rate matches that of the U.S. in 2003, which was 66%.

To calibrate the job finding rate, we use the strategy found in Shimer (2005). Let u_t^s denote the number of workers unemployed for less than one month in month t, and u_t be the total number of unemployed in month t. The job finding rate is defined as

$$f_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}. \tag{45}$$

For the years 1951-2003 $f_t = 0.45$ per month, implying the annualized expected number of job offers, $\theta q(\theta)$, is 5.40. The parameters A and γ are chosen to match the average job finding rate and the average $v-u$ ratio. In the model the vacancy to unemployment ratio, θ, is arbitrary and normalized to one. Therefore, we set $A = 5.40$ and $\gamma = 0.513$.[13]

We infer the job separation rate using the two unemployment series given above. In the data, when a worker is separated from her job, she has on average half a month to find a new job before she is recorded as unemployed. Therefore, letting e_t be the number employed in month t we calculate the separation rate as

$$s_t = \frac{u_{t+1}^s}{e_t(1-\frac{1}{2}f_t)}, \tag{46}$$

which is 0.034. This implies an annualized rate of 0.408, i.e., jobs last, on average, about 2 years.

The crimes considered are Type I property crimes as defined by the FBI, which includes larceny, burglary, and motor vehicle theft. We exclude violent and drug-related crimes because they are not necessarily driven by economic incentives.[14] Finally, the FBI defines Forgery, Fraud, and Embezzlement as a Type II offense and does not collect the number of these types of crimes.

The crime rate (crimes per 1000 persons) is taken from the Uniform Crime Reports for the population sixteen and over. The probability of being caught is derived from the number sent to prison divided by the number of crimes, implying $\pi = 0.019$. We exclude those sentenced to probation when calculating the probability of being caught because individuals on probation or parole may not be forced out of employment and/or home production. The mean length of incarceration for those convicted of a property crime was 16 months in 2002, so that $\delta = 0.75$. The average per capita loss from crime is calculated by taking the dollar value stolen divided by

[13] These numbers differ from that of Shimer (2005) due to the fact that we use annual rather than quarterly data and use 0.5 rather than 0.72 for the elasticity of the matching function.

[14] See Cozzi (2005) for an analysis on the link between drugs and crime.

the number of individuals and normalized by the wage, implying $\tau = 0.002$.[15] Since we do not have much information on the utility or disutility from being in jail, we let $x = 0$.[16]

We assume that the distribution of the value of crime opportunities $G(\mu_g, \sigma_g)$ is log normal.[17] We choose the mean of the log of ε, μ_g and the standard deviation, σ_g, to target the average amount stolen, the remaining parameters $\lambda_e = \lambda_u = \lambda_o$ target the overall crime rate. The average amount stolen in the data is approximately $1243, calculated as the ratio of the dollar value stolen divided by the number of crimes. The crime rates for employed, unemployed, and non-participants, (which correspond to $\lambda_i[1 - G(\varepsilon_i)]$ in the model) are 4.5%, 27.5%, and 2.5%, respectively. The crime rate when in a particular labor force state is computed as the product of the number of crimes and the percent incarcerated when in the particular state, divided by the number in that state. Unfortunately, there is little direct evidence to assist in choosing σ_g. The benchmark sets $\sigma_g = 1$, and weighting the three expectations of the dollar value stolen for each state by the proportion of crime committed in each state gives the result $\mu_g = -5.951$.[18]

Table 1 provides a summary of the parameters used in the calibration.

7 Comparative statics and quantitative findings

In this section we examine qualitatively and quantitatively how changes in some relevant variables affect crime and labor market outcomes.

7.1 Workers' compensation

In accordance with the seminal work of Becker (1968), it has been well documented that workers' compensation is an important determinant of crime (Gould, Weinberg, and Mustard (2002)). Workers' compensation has two components in our model: the hiring fee at the start of the employment

[15] The total number of property crimes is reported in the Uniform Crime Reports, 2004, Table 1. The total dollar amount lost from crime is published in the Uniform Crime Reports 2004, Table 24. The population is non-institutional as defined and calculated by the Bureau of Labor Statistics.

[16] We have tested different values for x and have verified that the calibration is basically unaffected. The threshold values ε_i fall as x rises, which decreases our target for μ_g. The effects on the arrival rates of crime are found to be quite small.

[17] Exponential and uniform distributions were also tried. The results from the exponential distribution were not remarkably different. In contrast, the uniform distribution resulted in the calibrated values for the arrival rates of crime opportunities to be very low, nearly two hundred times lower than under the log normal.

[18] Choosing $\sigma_g = 1.25$ leads to arrival rates that are large, for the unemployed it is nearly 150 opportunities per year. While choosing $\sigma_g = .75$ leads to fewer than 2 opportunities per year for the unemployed.

Table 1: Parameters

r	0.048	real interest rate
b	0.400	unemployed utility flow
β	0.500	bargaining power of workers
η	0.500	elasticity of matching function
γ	0.513	recruiting cost
s	0.408	job destruction rate
A	5.400	efficiency of matching technology
p	0.844	preferences for non-market activity
x	0.000	utility flow when in jail
π	0.019	apprehension probability
δ	0.750	rate of exit from jail
$\lambda_e = \lambda_u = \lambda_o$	4.343	flow of crime opportunities
μ_g	−5.951	mean of log normal crime distribution
σ_g	1.000	s.d. of log normal crime distribution

relationship that depends on workers' bargaining power, and a wage equal to labor productivity. We consider both components in turn.

Proposition 7 *An increase in β: reduces θ; raises κ_u if $\beta < \eta(\theta)$ and decreases it if $\beta > \eta(\theta)$; increases ε_u if $\beta < \eta(\theta)$ and decreases it if $\beta > \eta(\theta)$; increases ε_e if $\delta > s$ and $\beta > \eta(\theta)$ or $\delta < s$ and $\beta < \eta(\theta)$, and increases it otherwise.*

The previous Proposition shows that an increase in β has two effects on unemployed worker's utility. On the one hand, workers enjoy a larger share of the match surplus which tends to make them better-off (they pay a lower hiring fee). On the other hand, a higher worker's bargaining power reduces firms' incentives to open vacancies, and therefore also reduces the job finding rate of workers. The former effect dominates if $\beta < \eta$. In this case, ε_u increases so that the unemployed workers are less likely to engage in crime, and more agents participate in the labor force. If $\beta > \eta$ then the opposite happens.

Suppose that the value of being unemployed, $\mathcal{V}_u$, increases. The effect of β on the crime rate of employed workers depends also on the average jail sentence and job duration since employed workers and individuals in jail will ultimately end up in the pool of unemployed. The transition from employment to unemployment occurs at rate s while the transition from jail to unemployment occurs at rate δ. If $\delta > s$ then the value of being in jail tends to increase relatively more, raising the incentive to commit crimes. In contrast, if $\delta < s$ then employed workers commit fewer crimes.

So, if $\delta > s$, the crime behavior of employed workers varies in the opposite direction as the crime behavior of unemployed workers.

In order to summarize a worker's employment contract in one number we define an "equivalent constant-wage" as the constant wage that generates the same discounted sum of payments as (ϕ, w). More precisely,

$$\bar{w} = y - \{r + s + \lambda_e [1 - G(\varepsilon_e)]\}\phi. \tag{47}$$

In Table 2, when β increases from a low value to 0.5 (the Hosios condition in our calibration), it raises the value of being unemployed and therefore reduces the likelihood of committing crime by unemployed workers and workers out of the labor force. On the other hand, since $\delta > s$ employed workers commit more crime. The overall effect on crime is ambiguous and it depends on the composition of the population across labor force status. For our calibration, the positive effect of an increase in β on the crime rate of the employed dominates and the overall crime rate increases. Furthermore, participation increases, unemployment decreases and employment increases.

The effects on crime of an increase in β above 0.5 are qualitatively symmetric to those described above. The value of being unemployed falls and therefore both unemployed workers and workers out of the labor force commit more crimes. Since $\delta > s$ employed workers commit fewer crimes. In the labor market, an increase in β reduces tightness and increases unemployment. Since participation falls, employment is reduced unambiguously.

Quantitatively, the relationship between the total crime rate and β is non-monotonic and highly non-linear. Reducing workers' bargaining power from 0.5 to 0.01, corresponding to a reduction of workers' compensation of about 30%, generates a reduction in the total crime rate of about 8%. On the other hand, raising workers' bargaining power from 0.5 to 0.99, which corresponds to an increase in workers' compensation of 5%, increases total crime by 66%.

We now consider the second component of compensation, a change in the output from a match.

Proposition 8 *An increase in y raises θ, κ_u and ε_i for $i = o, u, e$.*

As workers become more productive a larger measure of firms enter the market. Graphically, the *JC* curve shifts upward and both θ and ε_u increase. The fact that the labor market becomes tighter implies the cost to an unemployed worker of being caught committing a crime increases. As a consequence, unemployed workers commit fewer crimes. Similarly, the wage, which is equal to productivity, increases, raising employed workers' cost of being caught committing a crime.

Table 2: Changes in Bargaining Power, (β)

β	0.01	0.05	0.10	0.50	0.90	0.95	0.99
Equivalent wage	0.660	0.813	0.864	0.952	0.985	0.990	0.997
Labor Force							
Employed	55%	61%	62%	61%	52%	45%	27%
Unemployed	1%	1%	2%	5%	12%	17%	28%
NILF	45%	38%	36%	34%	36%	38%	45%
Crime							
Pr(Commit Crime | e)	0.033	0.042	0.045	0.050	0.045	0.042	0.033
Pr(Commit Crime | u)	0.148	0.096	0.085	0.074	0.085	0.096	0.148
Pr(Commit Crime | o)	0.045	0.031	0.028	0.025	0.028	0.031	0.045
Total Crime Rate	38.93	38.53	39.78	42.42	43.98	46.80	70.26

So the crime rate of employed workers falls. Finally, since productivity in the market increases, participation increases as well.

Quantitatively, increasing productivity by 10% decreases the overall crime rate and the probability of committing crime for each labor force status by roughly 25%. Labor force participation rises by four percentage points and unemployment is essentially unaffected. In carrying out this experiment we have kept the distribution of crime values, $G(\varepsilon)$, fixed. More generally, there may be a relationship between productivity, y, and $G(\varepsilon)$. That is, as the output from a match increases it may be that the value of crime opportunities also increases.

Table 3: Effects of Changing Productivity (y)

	y		
	0.9	1.0	1.1
Labor Force			
Employed	57%	61%	65%
Unemployed	5%	5%	5%
NILF	38%	34%	30%
Crime			
Pr(Commit Crime | e)	0.065	0.050	0.039
Pr(Commit Crime | u)	0.095	0.074	0.058
Pr(Commit Crime | o)	0.031	0.025	0.020
Total Crime Rate	53.65	42.42	33.93

7.2 Unemployment benefits

Over the last decade several countries have reduced the generosity of their unemployment insurance systems in order to increase the incentives of the unemployed to accept jobs and to reduce the pressure on wages, for example The Job Seekers Allowance in the U.K. To illustrate the effects of unemployment benefits in our model, we consider an increase in the income flow, b, received by unemployed workers.[19]

Proposition 9 *An increase in b: reduces θ; raises κ_u; raises ε_u; decreases ε_e if $\delta > s$ and increases it if $\delta < s$.*

In Figure 2, for given θ, an increase in b provides unemployed workers with lower incentives to commit crimes: The curve CS shifts to the right. For given ε_u, an increase in b raises the threat point of workers when bargaining so that fewer firms enter the market: The curve JC shifts downward. Although the overall effect seems ambiguous, Proposition 9 establishes that the measure of vacancies per unemployed falls as well as unemployed workers' incentives to commit crimes. Furthermore, a larger fraction of individuals participate in the labor force.

The effect of an increase in b on the crime rate of the employed depends on the relative sizes of δ and s. [20] Quantitatively, δ is almost twice s, therefore the crime rate increases for those employed when b rises, though only slightly, from 0.050 to 0.051 as shown in Table 4. In addition, the findings also suggest that, in contrast to the previous studies that focus on the partial equilibrium effect of unemployment benefits on the crime rate of unemployed workers, overall crime increases with the level of unemployment benefits, although the change is quite small, from 42.42 to 43.32.[21]

7.3 Crime opportunities

In the following propositions we consider how the availability of crime opportunities affect individuals' crime behavior and the labor market.

Proposition 10 *An increase in λ_e: raises θ, κ_u and ε_u; the effect on ε_e is ambiguous.*

[19] Unemployment insurance benefits, in practice, require certain eligibility conditions and are usually terminated after a fixed number of periods. We abstract from these in the model and calibration. For a more detailed treatment, see Holmlund (1998).

[20] A related result can be found in Burdett, Lagos and Wright (2003).

[21] We have also considered an increase in b financed using a tax on filled jobs. The findings are similar to those in Table 4, although the increase in the crime rate is somewhat larger. This occurs due to the fact that after-tax income is now lower when b is increased, so that more employed people commit crime.

Table 4: Effects of Changing Unemployment Benefits (b)

	b		
	0.2	0.4	0.6
Labor Force			
Employed	61%	61%	61%
Unemployed	4%	5%	6%
NILF	34%	34%	33%
Crime			
Pr(Commit Crime | e)	0.048	0.050	0.051
Pr(Commit Crime | u)	0.077	0.074	0.070
Pr(Commit Crime | o)	0.026	0.025	0.024
Total Crime Rate	41.73	42.42	43.32

An increase in the arrival rate of crime opportunities for employed workers moves *JC* in Figure 2 upward since the value of being employed increases. Consequently, both θ and ε_u increase: The market becomes tighter and unemployed workers commit fewer crimes. Since crime opportunities arrive at a higher frequency, employed workers become more choosy in terms of the crime opportunities they undertake. Therefore, the effect on the crime rate of employed workers, as well as the overall effect on crime, is ambiguous. Finally, more individuals participate in the market.

Proposition 11 *An increase in λ_u: reduces θ; increases κ_u; raises ε_u; decreases ε_e if $\delta - s > 0$ and increases it if $\delta - s < 0$.*

Following an increase in λ_u, the *CS* curve in Figure 2 moves to the right since unemployed workers can afford to be more selective in terms of their crime projects when crime opportunities are more readily available. The curve *JC* moves downward since the fact that workers can commit more crimes raises their disagreement point in the bargaining. Proposition 11 establishes that ε_u increases and θ falls. The effect on the crime rate of employed workers is ambiguous and depends on the sign of $\delta - s$. The intuition is similar to the one for an increase in β.

Propositions 10 and 11 show that an increase in λ_e has very different effects on the labor market and crime compared to an increase in λ_u. If it becomes easier to commit crimes when employed, labor market tightness increases whereas labor market tightness decreases if unemployed workers have a greater access to crime opportunities. In both cases, labor force participation increases.

Table 5 shows the quantitative effects of changes in λ_e and λ_u on labor force participation are negligible. Although the effect on the crime rate is ambiguous in theory, the total crime rate rises for both experiments in our calibration. The crime rate rises from 42.42 to 56.42 when the employed receive two more crime opportunites per period, from 4.34 to 6.34. The crime rate also rises with an increase in crime opportunities of the unemployed, although the effect is slightly smaller, from 42.42 to 44.00. The main reason that the change in λ_u has a much smaller effect on the crime rate is that it affects a much smaller share of the population.

Table 5: Changes in λ_e and λ_u

	λ_e			λ_u		
	2.34	4.34	6.34	2.34	4.34	6.34
			Labor Force			
Employed	61%	61%	61%	61%	61%	61%
Unemployed	5%	5%	5%	5%	5%	5%
NILF	34%	34%	34%	34%	34%	34%
			Crime			
Pr(Commit Crime | e)	0.027	0.050	0.073	0.050	0.050	0.050
Pr(Commit Crime | u)	0.074	0.074	0.074	0.040	0.074	0.108
Pr(Commit Crime | o)	0.025	0.025	0.025	0.025	0.025	0.025
Total Crime Rate	28.39	42.42	56.42	40.85	42.42	44.00

7.4 Crime Policies

Imposing harsher punishments on criminals or increasing apprehension probabilities are obvious ways to reduce crime.[22] However, such changes will also affect the labor market. While our model can deliver quantitative effects for crime rates and for the labor market for given changes in policies, it can also be used to examine the welfare consequences due to those changes.

The following proposition considers the effects of a change in two particular policies, an increase in the length of jail sentences and an improvement in the technology to catch criminals.

Proposition 12 *Assume $\lambda_e = \lambda_u = \lambda_o$. An increase in δ or a decrease in π: decreases θ; decreases ε_i for $i = o, e, u$; decreases κ_u.*

[22] Levitt (2004) argues that crime has fallen in the 90's because of an increase in police surveillance. Bedard and Helland (2000) find sizeable deterrence effects of custody rate and punitiveness (as measured by the distance between prisons and cities) changes on female crime. They find that a 10% rise in the custody rate for women reduces female violent crime by approximately 5%. Increasing the average within state prison distance by 40 miles reduces the female violent crime rate by approximately 7%.

An increase in δ, the Poisson rate at which an individual exits jail, moves the *CS* curve to the left. Since the punishment for committing crimes is weaker, both unemployed and employed workers commit more crimes and firms open fewer vacancies. If the arrival rate of crime opportunities is the same for all individuals then fewer individuals wish to participate in the market. The effects of an increase in π, the probability of being caught, are opposite to those of an increase in δ.

Table 6: Changes in Crime Policy

	δ			π		
	0.65	0.75	0.85	0.016	0.019	0.022
				Labor Force		
Employed	61%	61%	61%	61%	61%	61%
Unemployed	5%	5%	5%	5%	5%	5%
NILF	34%	34%	34%	34%	34%	34%
				Crime		
Pr(Commit Crime | e)	0.037	0.050	0.064	0.077	0.05	0.034
Pr(Commit Crime | u)	0.052	0.074	0.098	0.111	0.074	0.051
Pr(Commit Crime | o)	0.017	0.025	0.034	0.039	0.025	0.017
Total Crime Rate	30.69	42.42	55.67	65.72	42.42	28.60
Welfare	0.998	0.9979	0.9979	0.9978	0.9979	0.998

The quantitative findings with respect to δ and π are substantial as seen in Table 6. Increasing the probability of being caught committing a crime by about 10% cuts the total crime rate by about one third. Also, if the average duration spent in jail rose by 2 months, we would see a drop in total crime by a factor of one quarter. Note that the labor is unaffected, suggesting that one can likely ignore the effects of crime policies on the labor market.

Welfare increases with the length of time prisoners must spend in jail and when the probability of being caught rises.

7.5 Women and preferences towards market work

We conclude this section by recalibrating the model for women to examine whether the changes in preferences toward market activities, as captured by the parameter p, that generates increased participation of women in the labor market can also generate higher involvement in criminal activity.

We recalculate the job finding and separation rates for women only, using total female unemployment as well as female short term unemployment, i.e., those unemployed less than 5 weeks.[23]

The female property crime rate is calculated as the product of the total number of property crimes and the percent of female arrests.[24] The annual number of females sent to prison is the product of the total number of convictions, the percentage of the total who were female, and the percent of those females who were incarcerated.[25] In the tables that follow we calculate the total crime rate as if the population were only females. It differs from the statistics reported in the Uniform Crime Reports in that the population in our model is only composed of females.

Proposition 13 *A decrease in p raises κ_u and decreases ε_o. It does not affect ε_e or ε_u.*

According to Proposition 13, as the utility of nonmarket activities falls, participation increases since the benefits of staying at home are smaller. Also, agents out of the labor force have higher incentives to commit crime, ε_o decreases, because the potential costs of doing so have fallen.

We again use the basic strategy outlined above but now use only females in the parameterization. Table 7 provides the new parameterization. In the tables that follow we calculate the total crime rate as if the population were only females. It differs from the statistics reported in the Uniform Crime Reports in that the population in our model is only composed of females. To calculate the crime rate for the entire population (as reported in the Uniform Crime Reports) multiply the total crime rate numbers in the tables below by the fraction of females in the population, 0.512.

To quantify the increase in the female labor force participation rate witnessed over the last 40 years, rising from 0.36 to 0.59, we assume the average gains from staying at home have fallen relative to market participation. The lower portion of Table 8 reports the steady-state outcome for crime corresponding to these different values of p. The change in preferences that is responsible for the increased participation in the labor market generates a 44% increase in crime. So, a change in preferences towards market work can explain a quantitatively significant increase in female crime.

[23]The female data series which began in 1976 can be found at http://www.bls.gov/cps. Implicitly, our assumption that the job finding rate is gender specific implies that the labor market is segmented by gender. This may reflect the fact that males and females are specialized by occupations.

[24]The percent of female arrests is located in Table 42 of the 2004 Uniform Crime Reports. The percent of females convicted to jail or prison can be found in "State Court Sentencing of Convicted Felons, 2002," Table 2.4 and the average time spent in jail or prison is located in Table 2.6. The percent of females incarcerated when in labor force status $i = \{e, u, o\}$ is taken from the most recent Survey of Federal and State Correctional Facilities, 1997. The number of females in $i = \{e, u, o\}$ is taken from the Bureau of Labor Statistics for 2003. See Heimer (2000) for further information on the calculation of the female crime rate.

[25]The values are taken from the most recent survey "State Court Sentencing of Convicted Felons, 2002," Tables 1.1, 2.1, and 2.4, respectively. These numbers represent both state and federal convictions. The Bureau of Criminal Justice Statistics collects data at the state level and then estimates that another 6% are convicted at the federal level.

Table 7: Parameters

r	0.048	real interest rate
b	0.400	unemployed utility flow
β	0.5	bargaining power of workers
η	0.5	elasticity of matching function
γ	0.503	recruiting cost
s	0.456	job destruction rate
A	5.208	efficiency of matching technology
p	1.011	preferences for non-market activity
x	0.00	utility flow when in jail
π	0.015	apprehension probability
δ	0.706	rate of exit from jail
$\lambda_e = \lambda_u = \lambda_o$	1.649	flow of crime opportunities
μ_g	-5.955	mean of log normal crime distribution
σ_g	1.00	s.d. of log normal crime distribution

Table 8: Labor Force Participation and Crime

year	1960	2000
p	2	1.011
Employed	33%	54%
Unemployed	3%	5%
NILF	64%	41%
Crime		
Pr(Commit Crime | e)	0.031	0.031
Pr(Commit Crime | u)	0.044	0.044
Pr(Commit Crime | o)	0.009	0.014
Total Crime Rate	16.95	24.40

The rise in female crime can be separated into two effects. The first effect arises from the fact that a woman's time is relatively less valuable at home. Therefore, the cost of being caught committing a crime is smaller, implying women who stay at home have a higher probability of committing crime. The second effect is due to the change in the composition of the labor force. More women are either employed or unemployed, and both types of women commit crimes at a higher frequency than those not in the labor force, as can be seen in the crime portion of Table 8. The change in the composition alone explains about two-thirds of the total increase in crime while a change in the crime rates alone explain the remaining one third.

8 Conclusion

A search-theoretic model was constructed and calibrated in which labor market outcomes and crimes are determined jointly. Our description of the labor market follows the canonical model of Pissarides (2000) extended to include a participation decision. Criminal activities are described as the result of rational decisions to undertake crime opportunities that occur randomly. The tractability of the model allows us to show how various parameters (unemployment benefits, workers' bargaining power, availability of crime opportunities) affect the equilibrium. The model also generates crime rates that differ according to labor force status - the unemployed have the highest propensity to commit crime compared to being employed or out of the labor force - a feature that is present in the data. We also calibrate the model using data for women to investigate the effect changes in labor force participation of women have on crimes committed by women.

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9 Appendix

Proof of Lemma 1 According to Nash's axioms, (ϕ, w) must be pairwise Pareto-efficient. Since the up-front payment ϕ allows the worker and the firm to transfer utility perfectly, the wage, w, must be chosen to maximize the total surplus of the match. The comparison of (6) and (13) shows that the match surplus is maximized iff $\mathscr{V}_f = 0$. From (10), $\mathscr{V}_f = 0$ requires $w = y$. Finally, the first-order condition of (14) with respect to ϕ yields (16).

Proof of Lemma 3 The slope of CS in the (ε_u, θ) space is

$$\left.\frac{d\theta}{d\varepsilon_u}\right|_{CS} = (1-\beta)\frac{r+\delta+\lambda_u\pi[1-G(\varepsilon_u)]}{\pi\beta\gamma}.$$

The slope of JC in the (ε_u, θ) space is

$$\left.\frac{d\theta}{d\varepsilon_u}\right|_{JC} = (1-\beta)\frac{\lambda_u[1-G(\varepsilon_u)]-\lambda_e[1-G(\varepsilon_e)]}{\beta\gamma-\{(r+s)\gamma+\lambda_e\pi\gamma[1-G(\varepsilon_e)]\}\frac{q'(\theta)}{[q(\theta)]^2}}.$$

Observing that

$$\frac{r+\delta}{\pi}+\lambda_u[1-G(\varepsilon_u)] > \lambda_u[1-G(\varepsilon_u)]-\lambda_e[1-G(\varepsilon_e)]$$

and

$$\beta\gamma \le \{(r+s)\gamma+\lambda_e\pi\gamma[1-G(\varepsilon_e)]\}\frac{-q'(\theta)}{[q(\theta)]^2}+\beta\gamma,$$

it is easy to see that

$$\left.\frac{d\theta}{d\varepsilon_u}\right|_{JC} < \left.\frac{d\theta}{d\varepsilon_u}\right|_{CS}.$$

Proof of Proposition 4 Summing (19) and (25) one obtains

$$\frac{(r+s)\gamma}{(1-\beta)q(\theta)}+\left(\frac{r+\delta}{\pi}\right)\varepsilon_u = y-x+\lambda_e\int_{\varepsilon_u+\frac{\pi\gamma}{(1-\beta)q(\theta)}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon. \tag{48}$$

From (48), it can be checked that θ is a strictly decreasing function of ε_u. So if a solution to (19) and (48) exists then it is unique. Denote $\varepsilon_u(\theta)$ the solution ε_u to the equation (19). Since $b-x-\tau>0$ then $\varepsilon_u(\theta)>0$. Furthermore, $\varepsilon_u(\theta)$ is non-decreasing in θ. Define $\Gamma(\theta)$ as

$$\Gamma(\theta) = y-x+\lambda_e\int_{\varepsilon_u(\theta)+\frac{\pi\gamma}{(1-\beta)q(\theta)}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon-\frac{(r+s)\gamma}{(1-\beta)q(\theta)}-\left(\frac{r+\delta}{\pi}\right)\varepsilon_u(\theta).$$

An equilibrium is then a θ that solves $\Gamma(\theta)=0$. Using the expression for $\left(\frac{r+\delta}{\pi}\right)\varepsilon_u(\theta)$ given by (19), we have

$$\Gamma(0)=y-b+(\lambda_e-\lambda_u)\int_{\varepsilon_u^0}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon.$$

So if (36) holds then $\Gamma(0)>0$. Furthermore, $\Gamma(\infty)=-\infty$. Therefore, a solution exists and it is such that $\theta>0$.

Proof of Proposition 5 The result according to which $\varepsilon_e>\varepsilon_u$ comes from (24). From (21), $\varepsilon_o(\kappa)$ is nondecreasing in κ. Since $\varepsilon_o(\kappa_u)=\varepsilon_u$ we have $\varepsilon_o(\kappa)\geq\varepsilon_u$ for all $\kappa\geq\kappa_u$.

Proof of Proposition 6 From Proposition 5, no crime occurs in equilibrium iff $\varepsilon_u\geq\bar{\varepsilon}$. From (23) if $\varepsilon_u\geq\bar{\varepsilon}$ then $\theta=\hat{\theta}$. From (19) the condition $\varepsilon_u\geq\bar{\varepsilon}$ requires (38).

Proof of Proposition 9 The pair (ε_u,θ) is uniquely determined by (19) and (48). Differentiating these two equations, it is straightforward to show that $d\varepsilon_u/db>0$ and $d\theta/db<0$. From (20) the sign of $d\varepsilon_e/db$ is the same as $s-\delta$. To establish that $d\kappa_u/db>0$ we first show that $d\mathscr{V}_u/db>0$. To see this, consider an individual such that $\kappa<\kappa_u$. From (4)

$$(r+\delta)\mathscr{V}_p=x-\tau+\delta\mathscr{V}_u.$$

Rearranging (5) and using the previous equation,

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_u=r\mathscr{V}_u-x+\tau.$$

Since $d\varepsilon_u/db>0$ then $d\mathscr{V}_u/db>0$. From (3) and (4),

$$r\mathscr{V}_o=\kappa p-\tau+\lambda_o\int\left[\varepsilon+\pi\left(\frac{x-r\mathscr{V}_o}{r+\delta}\right)\right]^+dG(\varepsilon),$$

and $d\mathscr{V}_o/db=0$. Since $\mathscr{V}_u=\mathscr{V}_o(\kappa_u)$ then $d\kappa_u/db>0$.

Proof of Proposition 7 The pair (ε_u,θ) is determined by (19) and (48). Differentiating these two equations one can establish that $d\theta/d\beta<0$. In order to determine the effects on ε_u we adopt the following change of variable: $\tilde{\gamma}=\gamma/[(1-\beta)q(\theta)]$. Equations (19) and (48) can now be rewritten as

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_u=b-x+\frac{\beta}{1-\beta}q^{-1}\left[\frac{\gamma}{(1-\beta)\tilde{\gamma}}\right]\gamma+\lambda_u\int_{\varepsilon_u}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon, \tag{49}$$

$$(r+s)\,\tilde{\gamma}+\left(\frac{r+\delta}{\pi}\right)\varepsilon_u=y-x+\lambda_e\int_{\varepsilon_u+\pi\tilde{\gamma}}^{\bar{\varepsilon}}[1-G(\varepsilon)]\,d\varepsilon. \tag{50}$$

Equations (49) and (50) determine ε_u and $\tilde{\gamma}$. The term $\frac{\beta}{1-\beta}q^{-1}\left[\frac{\gamma}{(1-\beta)\tilde{\gamma}}\right]$ on the RHS of (49) increases in β if $\beta < \eta(\theta)$. Differentiating (49) and (50) one can show that $d\varepsilon_u/d\beta > 0$ if $\beta < \eta(\theta)$ and $d\varepsilon_u/d\beta < 0$ if $\beta > \eta(\theta)$. To determine the effect of an increase in β on ε_e we use (20) which can be reexpressed as

$$\left(\frac{r+\delta}{\pi}\right)\varepsilon_e = y - x + (\delta - s)\tilde{\gamma} + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon. \tag{51}$$

From (50) there is a negative relationship between ε_u and $\tilde{\gamma}$. Therefore, sign$(d\varepsilon_e/d\beta)$ =sign$[(s-\delta)d\varepsilon_u/d\beta]$. Following the proof of Proposition 9 we can show that κ_u increases with ε_u.

Proof of Proposition 8 Equation (19) is independent of y or s. Therefore, it is easy to show from (19) and (48) that both θ and ε_u increase following an increase in y or a decrease in s. From (24) one can show that

$$\frac{d\varepsilon_e}{dy} = \frac{d\varepsilon_u}{dy} + \frac{\pi\gamma}{(1-\beta)}\left(\frac{-q'}{q^2}\right)\frac{d\theta}{dy} > 0.$$

Similarly, $\frac{d\varepsilon_e}{ds} < 0$. Following the proof in Proposition 9 one can establish that $\mathscr{V}_u$ and κ_u increase with y or $1/s$.

Proof of Proposition 12 The pair (ε_u, θ) is determined jointly by (19) and (25) where (25) is independent of δ and π. It is straightforward to show that $d\varepsilon_u/d\delta < 0$, $d\theta/d\delta < 0$ and $d\varepsilon_u/d\pi > 0$, $d\theta/d\pi > 0$. From (24), $d\varepsilon_e/d\delta < 0$ and $d\varepsilon_e/d\pi > 0$. From (26) if $\lambda_u = \lambda_o$ then $d\kappa_u/d\delta$ has the same sign as $d\theta/d\delta$, and $d\kappa_u/d\pi$ has the same sign as $d\theta/d\pi$.

Proof of Proposition 10 Differentiating (19) and (25) one can establish that $d\theta/d\lambda_e > 0$ and $d\varepsilon_u/d\lambda_e > 0$. From (24), $d\varepsilon_e/d\lambda_e > 0$. Following the proof in Proposition 9 we can show that $d\mathscr{V}_u/d\lambda_e > 0$ and $d\kappa_u/d\lambda_e > 0$.

Proof of Proposition 11 Differentiating (48) and (19), one can establish that $d\varepsilon_u/d\lambda_u > 0$ and $d\theta/d\lambda_u < 0$. According to (20), the sign of $d\varepsilon_e/d\lambda_u$ is the same as the sign of $s - \delta$. Following the proof in Proposition 9 we can show that $d\mathscr{V}_u/d\lambda_u > 0$ and $d\kappa_u/d\lambda_u > 0$.