Firm-to-Firm Relationships and Price Rigidity

Theory and Evidence*

Sebastian Heise
Yale University

December 21, 2015
Job Market Paper
Latest version available here

Abstract
Economists have long suspected that firm-to-firm relationships might increase price rigidity due to the use of explicit or implicit fixed-price contracts. Using confidential, transaction-level import data from the U.S. Census, I study the pass-through of exchange rate changes and show that prices are in fact substantially more responsive to cost shocks in older versus newly formed relationships. Based on additional stylized facts about a relationship’s life cycle and interviews I conducted with purchasing managers, I develop a model in which a buyer-seller pair subject to persistent, stochastic shocks to production costs shares profit risk under limited commitment. Relationships that experience good shocks have lower costs, trade more, and survive longer, which generates the life cycle. Furthermore, since partners in older relationships on average enjoy a greater relationship surplus, alternative matches are less attractive to them, which enables the firms to share profit risk more completely by setting prices that are more responsive to shocks. As qualitatively predicted by the model, pass-through in the data is correlated with trade growth within a relationship and with exporter risk aversion. Once structurally estimated, the model replicates the empirical correlation between relationship age and pass-through. My results suggest that changes to the average length of relationships in the economy – e.g., in a recession, when the share of young relationships declines – can influence price flexibility and thus the effectiveness of monetary policy.

*I am extremely grateful to my advisors, Eduardo Engel, Pinelopi Goldberg, Giuseppe Moscarini, Peter Schott, and Aleh Tsyvinski for their advice and support throughout this project. I am also thankful to Costas Arkolakis, Michael Peters, and Tommaso Porzio for their comments. This material is based upon work supported by the National Science Foundation under Grant Number 1427027. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation or of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.
1 Introduction

This paper examines how relationships between firms affect price flexibility, where I define a relationship as a buyer-seller pair that has been engaged in trade for a certain period of time. Economists have long suspected that relationships might be important for monetary policy, increasing price rigidity due to the use of fixed-price contracts (e.g., Barro (1977), Carlton (1986, 1989)). Such contracts might for example explain why pass-through of exchange rate shocks into prices is low, an important puzzle in international trade.¹ In fact, using U.S. import data I show that long-term relationships – presumably more likely to use either implicit or explicit contracts – display a higher responsiveness of prices to cost shocks than new relationships. My finding implies that an economy’s aggregate price flexibility may vary with the average length of its underlying relationships.

A well-documented fact in the management literature is that long-term relationships account for a large and growing fraction of buyer-seller pairs in the U.S. economy.²³ However, little work has been done to investigate relationships’ aggregate effects, since large-scale datasets mapping the linkages between domestic buyers and sellers are generally unavailable.⁴ To make progress on this issue, I study relationships using trade data from the Longitudinal Firm Trade Transactions Database (LFTTD) of the U.S. Census. These data identify both the U.S. importer and the foreign exporter for each of 130 million arms’ length import transactions conducted by U.S. firms during the past two decades. As in the domestic economy, long-term relationships are common in U.S. imports – in an average quarter, about 53% of U.S. arms’ length imports are sourced within importer-exporter pairs that have been transacting with each other for at least 12 months.

The trade data reveal that prices become more responsive to cost shocks the longer a relationship has lasted. Specifically, within an importer-exporter relationship, the pass-through of exchange rate shocks into import prices is 50% higher when the relationship is four years older. In a new relationship, price movements on average reflect 15% of the exchange rate change since the last transaction, compared to 23% in a four-year relationship. The result is robust to a wide range of specifications, and holds for positive and negative exchange rate shocks. Since pass-through and price flexibility are


²For example, Cannon and Perreault Jr. (1999) survey a sample of more than 400 buyer-supplier pairs from a cross-section of sectors and find that the pairs sampled have on average been transacting with each other for 11 years - even though the buyer has multiple suppliers for the product in 76% of the cases. Kotabe, Martin, and Domoto (2003) find that suppliers in the U.S. automotive industry have on average been transacting with major buyers for 26 years.

³Surveys suggest that long-term relationships have become more common over the last three decades. See e.g. Lyons, Krachenberg, and Henke Jr. (1990), Han, Wilson, and Dant (1993), Helper and Sako (1995), Gaddel and Snebota (2000), Liker and Choi (2004). Case studies of firms are Xerox (David (1993)), John Deere (Golden (1999)), Dell (Jacobs (2003)), Wal-Mart (Hahn (2005)), Boeing, and Lockheed Martin (Avery (2008)).

⁴There exists a large management literature on firm-to-firm relationships. This literature is almost exclusively based on qualitative surveys. See e.g. Noordewier, John, and Nevin (1990), Parkhe (1993), Morgan and Hunt (1994), Cannon and Homburg (2001), Palmaier, Dant, and Grewal (2007). Recent work in economics has examined customers as capital, e.g. Drozd and Nosal (2012), Gourio and Rudanko (2014). However, this literature does not examine how relationships evolve over time and how relationship length affects aggregate outcomes.
strongly correlated (Berger and Vavra (2015)), my findings suggest that long-term relationships have more flexible prices in general.

I document several additional characteristics of relationships, which will form the basis of a model. First, I analyze the dynamics of value traded, price setting, and the break-up probability of buyer-seller pairs, and show that relationships follow a life cycle. New relationships trade small values and have a high likelihood of separation. As the relationship ages and survives, the value traded rises while the transaction price relative to the market and the separation probability fall. Trade declines again near the relationship’s end. This life cycle is quantitatively important: a six-year relationship trades at its peak in year three 21% more than in year one, and exhibits price reductions of about 1.3% on each transaction relative to the market price. Long-term relationships are also valuable to firms. I study break-ups where a foreign supplier is connected to at least three U.S. importers, loses all of them at once, and is never again in the dataset. I interpret these cases as an exporter bankruptcy or a significant strategy change.\(^5\) I show that importers reduce the quantity purchased of affected products by on average 19% and experience a reduction in employment growth by 1.5% in the year after losing a relationship that has lasted at least two years.\(^6\) The size of the effect is smaller for break-ups of shorter relationships. I relate my findings to the survey-based management literature on relationships and show that my results are consistent with that work. Management theory conjectures that relationships evolve through life cycle phases (e.g., Dwyer, Schurr, and Oh (1987)).\(^7\) They begin with an exploration period, followed by relationship growth, increased commitment, and deepening operational linkages. Product obsolescence or a breach of trust eventually lead to decline and termination. Management surveys also suggest that relationships are valuable due to learning about the partner and the accumulation of relationship-specific assets, such as customized equipment (e.g., Palmatier, Dant, and Grewal (2007)). My work is the first to provide quantitative evidence for these features of relationships using large-scale data.

The life cycle findings are also consistent with evidence from 16 interviews I conducted with purchasing managers of mostly large, international firms. These interviews also suggest that relationships enable firms to share risk. Survey evidence from the management literature provides additional evidence on risk sharing in relationships (e.g., Uzzi (1996), Camuffo, Furlan, and Rettore (2007)).

To rationalize the empirical findings, I develop a model of relational contracting in which a buyer and a risk averse seller firm interact repeatedly, and share profit risk using both monetary payments and production quantities, as suggested by interviews and surveys. Both firms have the outside option to leave the relationship to search for an alternate partner. Risk arises due to aggregate shocks to the seller’s production costs, which I will interpret as arising from exchange rate movements. Relationships

\(^5\)I impose a number of other restrictions to ensure that the break-up is not driven by the importer: the importer has to survive for at least two more years, the importer must not account for more than 50% of the exporter’s U.S. sales, and imports of the industry must not be declining in the year of the break-up.

\(^6\)These findings also provide indirect evidence suggesting that the manufacturer ID’s are reliable firm identifiers, supporting the work by Kamal, Krizan, and Monarch (2015).

also experience persistent idiosyncratic shocks to costs reflecting for example the accumulation of specific assets, as suggested in the management literature.\(^8\) I show that stochastic asset accumulation and endogenous separation generate the relationship life cycle. Those relationships that receive good shocks to assets have low marginal costs, trade more, and are less likely to break up. Relationships receiving bad shocks reduce the value traded and separate endogenously when assets are low. Most importantly, under risk sharing, asset accumulation generates increasing pass-through of cost shocks with relationship age. I show that the price, defined as the ratio of monetary payment and quantity ordered, can be decomposed into marginal costs plus an endogenous mark-up term. When neither firm’s outside option binds, the price responds fully to cost shocks, thus stabilizing the risk averse seller’s profits and insuring her perfectly. However, when either firm threatens to leave the relationship, the price response to a cost shock is muted. For example, when a rising price following an adverse cost shock causes the buyer to prefer separation, the mark-up component of the price falls to dampen the price increase, raising the buyer’s profits and providing the right incentives for her to stay in the relationship. As the level of assets increases, the relationship becomes more valuable and the firms’ outside options bind less often, which enables them to smooth the seller’s profits more completely by setting prices that are more responsive to shocks. Since older relationships on average have higher assets, they exhibit higher pass-through. My model generates the positive correlation between pass-through and age via a novel source of mark-up variation that arises endogenously from risk sharing motives. Prior work on incomplete pass-through has usually generated variable mark-ups via assumptions on the market structure (e.g., Krugman (1987), Atkeson and Burstein (2008)).\(^9\)

My baseline assumption that foreign sellers are risk averse is motivated by two main facts. First, the average foreign exporter is smaller than the average U.S. importer (e.g., Bernard, Jensen, and Schott (2009), Di Giovanni, Levchenko, and Rancière (2011)), and firm size has been shown to be negatively correlated with risk aversion (Asanuma and Kikutani (1992)). Second, U.S. stock market capitalization is high relative to other countries (World Bank (2015)), suggesting that a relatively large fraction of firms is publicly listed, and private firms are more likely to be risk averse (Okamuro (2001)). I assume risk neutral buyers to facilitate equilibrium characterization. I examine two alternative models, one with buyer risk aversion and one with risk neutral partners and Nash bargaining, and show that they generate the life cycle but generally do not deliver increasing pass-through with relationship age. I then show that several other implications of the baseline model are in fact supported in the data. First, I find that smaller exporters and exporters with fewer customers conditional on size exhibit significantly higher pass-through for both positive and negative exchange rate shocks, consistent with them being more risk averse.\(^10\) While previous work has documented that small exporters have higher

\(^8\)Parkhe (1993), Dyer (1996), Doney and Cannon (1997), Artz (1999), Palmatier, Dant, and Grewal (2007), among others, find that specific assets are a key source of the value of relationships.


\(^10\)Kawasaki and McMillan (1987), Asanuma and Kikutani (1992), Okamuro (2001), and Camuffo, Furlan, and Rettore
pass-through into import prices (Berman, Martin, and Mayer (2012), Chatterjee, Dix-Carneiro, and Vichyanond (2013)), I also find that pass-through increases more strongly with relationship age for small exporters than for large ones. This novel result is consistent with my model mechanism, since small firms are initially constrained in obtaining risk sharing but converge towards full insurance as the relationship ages, while large firms do not need insurance. Three additional implications of the relationship life cycle also find empirical support. First, pass-through increases with the value and the number of products a relationship trades relative to the first year, in line with pass-through increasing with relationship quality. Second, pass-through is diminished in the year before the relationship is terminated, when specific assets are likely to be low. Third, relationships that have high pass-through in the first year last longer, consistent with such relationships having had a high initial asset draw.

I structurally estimate the model with seller risk aversion by a method of simulated moments and show that, when calibrated to the life cycle, it quantitatively matches the empirical correlation between pass-through and relationship age. This result is a significant success of the model because that moment is not targeted in the estimation. Since the model generates complete pass-through when neither agent is constrained, while in the data pass-through is generally incomplete, I assume that imported inputs or local distribution costs insulate part of the seller’s costs from exchange rates, as documented by Goldberg and Verboven (2001) or Amiti, Itskhoki, and Konings (2014). A model with only buyer risk aversion matches the low level of pass-through through the risk sharing assumption alone, but, as mentioned, implies a counterfactually negative correlation between pass-through and relationship age. The Nash bargaining model misses both pass-through moments. The analysis shows that while risk sharing can generate either the correct level or the correct slope of pass-through with age, an additional assumption is needed to match both moments simultaneously.

Using the estimated model, I run a counterfactual exercise to show that changes in average relationship length affect aggregate price flexibility. Recent work by Berger and Vavra (2015) documents that the pass-through of exchange rate shocks into import prices doubled during the 2008-09 recession. They attribute this increase to time variation in the elasticity of demand. In my setup this effect arises from selection. The number of relationships of age less than one year fell by one fifth in 2008-09. Using the model, I show that the associated increase in the average relationship length explains about 20% of the rise in pass-through. My findings provide a micro foundation of time variation in price flexibility, and suggest that policymakers should take into account the average length of relationships in the economy when making monetary policy choices.

My work exploits the fact that unit values are observable in the trade data to examine how risk sharing under limited commitment affects price setting. Previous work on limited commitment has (2007) show that firm risk aversion is decreasing in firm size. Kawasaki and McMillan (1987) and Asanuma and Kitatani (1992) argue that the supplier’s risk aversion in any particular relationship should decrease as the firm’s customer portfolio becomes less concentrated.
mostly characterized the evolution of wages and consumption.\textsuperscript{11,12} The risk sharing model I develop builds on work by Kocherlakota (1996) and Ligon, Thomas, and Worrall (2002), who examine an endowment economy in which households share income risk via transfer payments. I apply this model to a firm-to-firm setup, extending it in two ways. First, firms share profit risk using both monetary payments and an endogenous production decision. Production is customized to the relationship and hence does not affect the seller’s outside option, in contrast to other work with production or capital such as Marcet and Marimon (1992) or Ligon, Thomas, and Worrall (2000).\textsuperscript{13} In my model, production is a transfer from the seller to the buyer that is used as an additional risk sharing instrument. Second, I assume that relationships experience cost shocks, which affect the relationship’s value and the seller’s outside option, as well as specific asset shocks, which only affect the relationship’s inside value. I show that these asset shocks affect the degree of risk sharing available in the relationship, which in turn determines the responsiveness of prices to shocks.

On the empirical side, my work contributes to the emerging literature on trade relationships. For example, Antràs and Foley (2015) show that customers in longer cross-country relationships obtain more favorable financing terms. Macchiavello and Morjaria (2014) test a model of relational contracting in the Kenyan flower industry, and show that longer relationships can relax limited commitment constraints. Prior work using the LFTTD has mainly focused on the micro-level properties of customer-supplier matches. For example, Eaton, Eslava, Jinkins, Krizan, and Tybout (2014) provide descriptive facts about associations between U.S. importers and Colombian suppliers and estimate a model of exporter learning. Monarch (2015) examines break-ups and switching in relationships with Chinese firms, and Monarch and Schmidt-Eisenlohr (2015) present evidence on multi-product relationships and switching behavior that they argue is consistent with exporter learning. My paper studies how relationships evolve over time and highlights that they can have aggregate effects. My work follows a large literature on the responsiveness of prices to shocks such as Mankiw and Reis (2002) or Nakamura and Steinsson (2008). I suggest a novel mechanism that explains the diminished responsiveness of prices to shocks based on risk sharing.

This paper proceeds as follows. In Section 2, I present the empirical analysis. I first introduce the data, define a relationship, and provide some summary statistics. I then present reduced-form evidence on pass-through and relationship length. Finally, I document additional stylized facts on the evolution of relationships and on their value. These facts form the basis of a model, which I introduce in Section 3. I characterize the model equilibrium, discuss alternative setups, and test model implications in the

\footnotesize

\textsuperscript{12}Macchiavello and Morjaria (2014) consider a two-sided limited commitment model with buyers and sellers. However, they do not assume risk sharing and do not characterize the optimal choice of prices and quantities. Instead, they derive predictions about the value of the relationship and the trade value and test these predictions empirically.

\textsuperscript{13}Introducing capital into a limited commitment model usually produces underinvestment. See also Albuquerque and Hopenhayn (2004), Cooley, Marimon, and Quadrini (2004).
data. In Section 4, I estimate the model and examine its aggregate implications. Section 5 concludes.

2 Firm-to-Firm Relationships: Stylized Facts

2.1 Data

Due to the lack of data mapping customer-supplier linkages in the U.S. domestic economy, I study relationships between U.S. firms and their overseas suppliers using international trade data from the Longitudinal Firm Trade Transactions Database (LFTTD) of the U.S. Census Bureau.\textsuperscript{14} This dataset is based on customs declarations forms collected by U.S. Customs and Border Protection (CBP), and comprises the entire universe of import transactions in goods\textsuperscript{15} made by U.S. firms during the period 1992-2011. The crucial advantage of this dataset is that for each import transaction, it records an identifier of the U.S. importer (called “alpha”) as well as a foreign exporter ID (the “manufacturer ID”). This information on both transaction partners makes the study of relationships possible. Recent work by Monarch (2015) and Kamal, Krizan, and Monarch (2015) suggests that the foreign firm identifiers are reliable over time and in the cross-section. While previous work has usually focused on a subset of the data (e.g., Eaton, Eslava, Jinkins, Krizan, and Tybout (2014)), my work will use the full dataset of about 130 million arms’ length transactions. I will draw on survey evidence throughout the paper to link my findings to results for domestic relationships.

In addition to the firm identifiers, the LFTTD dataset also comprises the 10-digit Harmonized System (HS10) code of the product traded\textsuperscript{16}, the country of the foreign exporter, the value and the quantity shipped (in U.S. dollars), the date of the shipment, and an identifier whether the two transaction parties are related firms.\textsuperscript{17} The U.S. firm identifier can be linked to other Census products such as the Longitudinal Business Database (LBD), which provides annual information at the establishment-level about payroll, number of employees, and NAICS code of the establishment. I aggregate information in the LBD across all of a firm’s establishments in each year, and assign each firm to the industry that is associated with most of its employees.

I focus on arms’ length relationships only and exclude related party transactions, which include for example intra-firm trade, by dropping all transactions in years for which a relationship records at least one related party trade. Associations between related parties are likely to be much deeper than relationships between unrelated firms, due to the substantial equity investments made. I compute (log) prices as unit values by dividing the shipment value by the quantity shipped, as in Monarch (2015).\textsuperscript{18}

\textsuperscript{15}Trade in goods accounted for 83% of all U.S. imports in 2013.
\textsuperscript{16}Examples of HS10 products are “Coconuts, in the inner shell” or “Woven fabrics of cotton, containing 85 percent or more by weight of cotton, weighing no more than 100g/m2, unbleached, of number 43 to 68, printcloth”.
\textsuperscript{17}Based on Section 402(e) of the Tariff Act of 1930, related party trade includes import transactions between parties with various types of relationships including “any person directly or indirectly, owning, controlling, or holding power to vote, 6 percent of the outstanding voting stock or shares of any organization”.

7
and Monarch and Schmidt-Eisenlohr (2015). This is only an imperfect measure of the true price, since it assumes that products are homogeneous within an HS10. In reality, there is likely to be some heterogeneity even within HS10 codes, for example due to quality differences. In my analyses, I treat the same HS10 shipped from different countries as different products, and run regressions using country fixed effects or focus on price changes within the same relationship to alleviate this problem. For all analyses involving prices, I trim the dataset in each quarter by removing transactions whose prices lie below the 1st or above the 99th percentile of the price distribution for the associated product-country pair, and drop price changes larger than four log points (about 400%) within the same relationship. Appendix A discusses the variables and data cleaning operations in more detail.

2.2 Relationships in the Data

I define a relationship as an importer-exporter pair trading at least one, but possibly many, products. One way to measure a relationship’s length would be to assume that the relationship starts at the date of its first transaction and ends with its last transaction observed in the dataset. However, this definition has two shortcomings: first, due to right censoring in 2011, it does not allow me to determine when some relationships end. Second, a number of importer-exporter pairs trade very rarely, which generates zero trade in most years of the association and makes the study of the relationship’s evolution less meaningful. I therefore define active relationships. A relationship is initiated with the first time an importer-exporter pair appears in the data. The relationship has a length of one month at that point. Since many relationships in 1992-1994 are likely to have started before the beginning of the dataset, the data in these years will only be used to initiate relationships, and will be dropped from all analyses. Whenever another transaction between the relationship partners occurs in any good, the relationship age is increased by the number of months passed.\textsuperscript{18} To determine the termination date of a relationship, I first take all importer-exporter-product (HS10 code) observations and record the time passed until the next observation of the same triplet. This provides an idea about how much time typically elapses, for each product, between a relationship’s subsequent transactions of that good. I take the distribution of these gap times for each HS10 product across the entire dataset and determine the 95th percentile of this distribution. I refer to this product-level statistic as the product’s maximum gap time. A relationship is assumed to have ended if for a given importer-exporter pair, first, none of the products previously traded is traded within its maximum gap time, and, second, no new products are traded within that time interval. Based on this definition, a relationship is terminated if no transactions for any product are observed for a significantly longer time period than would be normal. If an importer-exporter pair appears again in the data after the end of a relationship, I treat

\textsuperscript{18} As an alternative to the number of months, I could also define age based on the number of transactions. The results are generally similar.
Table 1: Summary statistics of the dataset

<table>
<thead>
<tr>
<th></th>
<th>Length of Included Relationships</th>
<th>All</th>
<th>&gt;12 months¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arms’ length trade</td>
<td></td>
<td>38%</td>
<td>32%</td>
</tr>
<tr>
<td>Arms’ length trade (always unrelated)</td>
<td></td>
<td>27%</td>
<td>21%</td>
</tr>
<tr>
<td>Arms’ length trade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exporters per importer-HS10, p.a.</td>
<td></td>
<td>2.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Importers per exporter-HS10, p.a.</td>
<td></td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>HS per importer-exporter, p.a.</td>
<td></td>
<td>1.9</td>
<td>3.0</td>
</tr>
<tr>
<td>Average gap time between transactions (months)</td>
<td>0.6</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Average maximum gap time (months)</td>
<td></td>
<td>10.0</td>
<td>—</td>
</tr>
<tr>
<td>Average relationship length (months)</td>
<td></td>
<td>5.7</td>
<td>30.0</td>
</tr>
<tr>
<td>... in Manufacturing</td>
<td></td>
<td>5.9</td>
<td>30.6</td>
</tr>
<tr>
<td>... in Wholesale / Transportation</td>
<td></td>
<td>5.7</td>
<td>30.6</td>
</tr>
<tr>
<td>... in Retail</td>
<td></td>
<td>5.9</td>
<td>28.7</td>
</tr>
</tbody>
</table>

¹ Statistics consider only those relationships that last in total for more than 12 months.

I use this measure of relationship length to take into account that different products trade with different frequency, rather than imposing an arbitrary cut-off for time gaps.

Table 1 provides some summary statistics of the resulting dataset. The first column shows statistics for the entire dataset, while the second column restricts to transactions in relationships that last for more than 12 months in total. The first row shows that 38% of trade takes place in relationships that are arms’ length in the year of the transaction. This figure is computed by taking the trade share of arms’ length relationships in every quarter and averaging across quarters. The second row displays the share of trade in arms’ length relationships restricted to those relationships that are always unrelated. Such relationships account for 27% of trade in an average quarter.

The remaining rows present additional statistics on arms’ length relationships. Rows 3-5 show that each importer has on average more than two arms’ length suppliers per HS10 product, while each exporter tends to have about one U.S. customer. From other datasets, it is well-known that there is significant heterogeneity across importers, with a few large firms having an extremely large number of counterparties. I find that the average time gap between transactions of the same importer-exporter-product triplet is less than one month across all products. The average maximum gap time is about 10 months. The last four rows document average relationship lengths for the entire dataset and by industry of the importer, based on the firm’s NAICS code from the LBD. Figure F.10 in Appendix F.1 shows the distribution of trade by industry of the importer in the average quarter. In terms of

---

19 I keep track of the fact that this is a continued relationship using a dummy variable, which I use in some of the regressions below.

20 See e.g. Blaum, Lelarge, and Peters (2013), for France; Gopinath and Neiman (2014), for Argentina
relationship lengths, I do not find significant differences for these industries.

Figure 1 provides a more detailed distribution of value traded by relationship length. The blue bars in the figure show the length distribution of relationships in an average quarter, based on the current length of that relationship. The figure shows that in an average quarter, 53% of the value traded in arms’ length transactions is accounted for by relationships that have been together for more than 12 months. About 18% is due to pairs that have been together for more than 4 years. However, most matches are actually quite short-lived. The orange bars in Figure 1 display the equally-weighted distribution of buyer-seller associations by length in the average quarter, where the length of the association is calculated at the first transaction of each quarter. Close to 44% of all importer-exporter pairs observed in the average quarter are new matches. Of those associations lasting more than one month, however, many are long-lived, with 7% of them having lasted for more than four years. Comparing the orange and the blue bars, I find that while many associations are very short, such matches account for only 15% of the value traded. Measuring the length of relationships by the number of transactions rather than the number of months yields a similar picture (Figure F.11).

Survey evidence suggests that long-term relationships are also important in the domestic U.S. economy. Table F.14 in Appendix F.2 presents the average length of domestic buyer-seller associations, based on the time passed since the first interaction. The table highlights that the average U.S. relationship is several years old, and relationships with major suppliers can last for decades. This evidence suggests that long-term relationships are not only an international trade phenomenon.

---

21Specifically, I compute the trade value by relationship length in each quarter, and average across quarters. To account for relationship lengths up to 48 months accurately, I drop not only the first three years but the first five years for this analysis, up to and including 1996.
2.3 Reduced-Form Evidence on Pass-Through

I now turn to the main research question of this paper and examine the connection between relationship length and price flexibility. Barro (1977) and Carlton (1986) suggest that long-term relationships could be an important source of price rigidity due to the use of contracts which specify fixed prices for a period of time. To study this question, I examine how relationships of different length affect firms’ price response to an identifiable cost shock. Following Berger and Vavra (2015), I argue that exchange rate shocks can be used as an easily observable source of exogenous variation in the exporter’s costs. Similar to Gopinath and Itskhoki (2010), I examine “medium-run pass-through”, defined here as the share of exchange rate movements since the last transaction that is passed through into prices at the next transaction.\footnote{Gopinath and Itskhoki (2010) define medium-run pass-through based on the next observed price change. Since I use unit values rather than prices, I cannot observe without error the period over which a price is held fixed. I therefore examine pass-through between subsequent transactions.} The exchange rates used are obtained from the OECD’s Monetary and Financial Statistics database and measured in foreign currency units per U.S. dollar. I supplement these data with rates from Datastream for Eurozone countries.\footnote{Euro exchange rates are converted into the implied local rate using the conversion rate at the time of the adoption of the Euro to construct consistent time series for each Eurozone country. In total, I have data for 45 countries, presented in Appendix C.} To focus only on relationships which are market-based throughout their life, I apply a more stringent filter from now on and drop all relationships which are ever related at any point.\footnote{This implies that all relationships that switch status at any point are dropped. In future work, I plan to investigate the link between a relationship’s features and its probability of making a transition into related party status. There is a large theoretical literature on firms’ decisions regarding market-based production vs. integration (see e.g. Grossman and Helpman (2002)). See Carballo (2014) for recent work on this topic using Census export data.} Let \( m \) index an importer, \( x \) the exporter, \( c \) the exporter’s country, and \( h \) the HS10 product. A relationship is indexed by \( mx \).

My specifications extend the standard pass-through regressions run for example in Campa and Goldberg (2005), Gopinath and Itskhoki (2010), or Berger and Vavra (2015) by taking into account the identity of the importer-exporter pair. Specifically, I compute pass-through between subsequent transactions of the same relationship, and include as regressor the relationship length in addition to the standard country- and product-level effects. I first examine pass-through in the cross-section of relationships. Each transaction in the dataset is allocated to buckets based on the length of the \( mx \) relationship at the point of the transaction. I aggregate the transaction-level data at the quarterly level in order to smooth out noise in the unit values, and compute exchange rate shocks for each transaction as the cumulative change in the exchange rate since the last transaction of the same relationship-product triplet. I then run

\[
\Delta \ln(p_{mxchi}(t)) = \beta_0 + \beta_1 \Delta \ln(e_{ct}) + \gamma_c + \xi_h + \omega_t + \epsilon_{mxchi}(t),
\]

for each of relationship buckets of length \((T - 1, T]\), for \(T = 1, ... , 7\) years, respectively. Here, \( \Delta \ln(p_{mxchi}(t)) \) is the log nominal price change between transaction \(i\) and \(i-1\) for relationship \(mx\).
trading product \( h \) in quarter \( t \), \( \Delta \ln(e_{ct}) \) is the cumulative exchange rate change since the last transaction of the relationship for that product, and \( \gamma_c, \xi_h, \) and \( \omega_t \) are country, product, and quarter fixed effects. Coefficient \( \beta_1 \) measures the response of prices to the cumulative change in the exchange rate since the last transaction. The coefficients and 95% confidence intervals for this regression are depicted in Figure 2. The results reveal that pass-through actually increases with relationship age. New relationships exhibit pass-through of about .17, which increases to .24 for relationships of age five years. Overall, the average level of the coefficient is comparable in magnitude to the aggregate pass-through for all U.S. imports of around 0.2 documented by Gopinath and Itskhoki (2010).

To account for potential selection issues, I examine a more stringent specification that focuses on the evolution of pass-through within the same relationship as that relationship ages. I run

\[
\Delta \ln(p_{mxi(t)}) = \beta_0 + \beta_1 \Delta \ln(e_{ct}) + \beta_2 \text{Months}_{mxi(t)} + \beta_3 \Delta \ln(e_{ct}) \cdot \text{Months}_{mxi(t)} + \gamma_{mxi} + \omega_t + \epsilon_{mxi(t)},
\]

where \( \text{Months}_{mxi(t)} \) measures the length of a given importer-exporter relationship in months at the time of the last transaction taking place in quarter \( t \), and \( \gamma_{mxi} \) are relationship-product fixed effects. These fixed effects also control for the country of the exporter. The coefficient of interest is \( \beta_3 \), which measures the response to an exchange rate innovation for each additional month a specific relationship has lasted. The first column in Table 2 presents coefficient \( \beta_1 \) in the standard regression of price changes on exchange rate changes. As before, this coefficient is about 0.2. Column 2 presents my main specification. It displays coefficients \( \beta_1 \) and \( \beta_3 \) of regression (2). Similar to before, pass-through is increasing in the length of the relationship. For each additional month a relationship has lasted, the responsiveness of prices to exchange rate shocks rises by 0.0015. Thus, pass-through in a relationship that is four years old is about 7.2 percentage points higher than pass-through at the point when the
relationship was new (a 47% increase). This is a substantial effect, and highlights that pass-through increases even within the same relationship. The finding suggests that long-term relationships, which presumably are the most likely to rely on contracts, do not have more rigid prices than new relationships as has been claimed in the literature. In fact, prices become more flexible with relationship length, at least in response to exchange rate shocks. My result aligns well with previous work showing that trade between related partners exhibits higher exchange rate pass-through and less sticky prices than trade between arms’ length partners (Neiman (2010)), since related partners can be thought of as having a very close relationship.

The third column of Table 2 shows the same regression but using dummies for relationship length: \( d_{med} = 1 \) for relationships that are between one and four years old, and \( d_{long} = 1 \) for relationships that are older than four years. Pass-through in old relationships is almost 80% higher than in new ones. Finally, the fourth and fifth column present the pass-through regressions only for positive and negative exchange rate shocks, respectively. The positive coefficients in both regressions indicate that pass-through goes in the direction of the shock in both cases. As before, long-term relationships have higher pass-through for both types of shocks. Thus, buyers in new relationships participate less in cost increases when the exchange rate appreciates, but also get smaller price reductions when the exchange rate declines. I also find that pass-through of foreign currency appreciations (increasing costs) is higher than pass-through following depreciations. This asymmetry of the price response to cost shocks is consistent with evidence from domestic studies of price setting such as Peltzman (2000) or Fabiani and Druant (2005).

Since relationships do not necessarily trade in every quarter, the price changes I observe could be based on a selected sample. This problem biases the fixed effects estimator if the selection is correlated with the errors in (2). To examine the impact of this issue, as a first pass I restrict the dataset to only those importer-exporter-product triplets that trade in every quarter for the duration of their association, and re-estimate the within-relationship regression. The results are very similar.
to before (column 1 in Table F.15). More formally, I implement a selection model to correct for the selection bias. Given the panel nature of my dataset, pass-through and selection are likely to depend on unobservable heterogeneity at the level of the relationship-product triplet. I therefore apply the selection correction for panel data proposed in Wooldridge (1995) to my problem, which approximates the fixed effects using leads and lags of observable variables. I discuss the correction procedure in Appendix B. Column 2 in Table F.15 shows the pass-through results from this exercise. The results are similar to the baseline regression, even though the selection term $\lambda$ is statistically significant.

I conduct a number of additional robustness checks. First, Gopinath and Itskhoki (2010) document that pass-through into import prices is significantly higher when pricing occurs in the foreign currency, rather than in U.S. dollars. Unfortunately, the currency of invoice is not observed in the LFTTD. To get a sense of whether the pass-through results differ based on currency choice, I construct a group of countries with a relatively high share of local currency pricing, using the countries listed in Table 1 in Gopinath and Itskhoki (2010). I then run regression (2) for this group of countries and for its complement (columns 3-4 in Table F.15). I find that while the level of pass-through is higher in the group of countries with high local currency pricing, pass-through is increasing in relationship length for both groups. I conduct a similar exercise by sorting transactions into three groups based on the share of local currency pricing within two-digit HS code product categories, using the pricing information in Table 4 of Gopinath and Itskhoki (2010).25 The results for the three groups again show higher pass-through for long-term relationships (columns 5-7 in Table F.15).

I next examine whether pass-through increases with relationship age even when the total length of the relationship is fixed. Column 1 of Table F.16 presents the results from running regression (2) for only the subset of relationships that last for in total between 24 and 35 months. Columns 2 and 3 redo the regression for two different total length groups. My results suggest that there is a time effect on pass-through even when controlling for how long the relationship lasts in total.

Third, I aggregate the data at annual or monthly frequency, respectively, to study how aggregation affects the regression results. Columns 4 and 5 of Table F.16 show that the results are similar. As a final check, I examine whether prices and exchange rates are cointegrated. Burstein and Gopinath (2014) reject the null hypothesis that the U.S. import price index, the nominal exchange rate, and the foreign producer price index are not cointegrated. If cointegration were present, the regression in differences would still yield the correct results for the short-run adjustment, but would miss long-run dynamics. To test for cointegration, I examine whether prices and exchange rates are unit roots. I define a lagged observation as the previous transaction, regardless of how much time has passed since then, since a given cumulative exchange rate movement should have the same effect on prices regardless of the time gap. I drop all relationship-product triplets with fewer than 20 transactions to have sufficiently long

---

25The “high” group contains all product groups in which at least 20% of goods are priced in foreign currency, the “medium” group all product groups with foreign currency pricing for 10-19% of the goods, and the “low” group contains the remainder.
time series for each panel. Since the panels contain a heterogeneous number of transactions, I test for unit roots using the test in Im, Pesaran, and Shin (2003). The test strongly rejects the null that all panels contain a unit root for exchange rates ($p < .0001$) (Table F.17). The exchange rate is only observed when the relationship transacts, and therefore the series is not the standard exchange rate process. The null cannot be rejected for prices. Since both series need to be unit roots for cointegration to be present, differencing seems to be a valid approach.

2.4 Further Properties of Relationships

I document a number of additional stylized facts, which will guide me in the development of a theory linking relationships and pass-through. The facts I study are motivated by a large, mostly survey-based management literature on relationships. This literature has suggested that relationships, defined as buyer-seller pairs interacting repeatedly, follow a life cycle and are valuable for firms. I first discuss these findings. I then take these results to the trade data and provide quantitative evidence supporting them for U.S. import transactions. Finally, I will build on this additional evidence to specify my model of a relationship, and use the previous evidence of pass-through dynamics to validate the model.

Management research suggests that relationships follow a life cycle (Dwyer, Schurr, and Oh (1987), Ring and van de Ven (1994)). While the details differ across papers, at its core this work proposes that relationships begin with an exploration stage, in which buyers search for partners and run trials by placing small purchase orders with possible suppliers (Egan and Mody (1992)). The relationship is still very loose at that point, and the focus is on selection based on satisfaction with performance and bargaining. In the build-up and maturity stage, the benefits of being in the relationship gradually increase as it accumulates specific assets and trust. Commitment to the relationship increases and the partners derive value through lower costs, market access, and information sharing. Informal agreements increasingly replace formal contracts, and the partners work together to solve their problems jointly. In the final decline phase, the relationship unravels, for example because of changing product requirements, increased transaction costs, or a breach of trust. Jap and Anderson (2007) test the life cycle theory based on a qualitative survey of 1,540 customers of a U.S. chemicals manufacturer, and confirm its main implications for these relationships.

Management research also suggests that relationships are valuable for firms. One source of value are investments into relationship-specific assets, such as customized equipment. These investments signal commitment to the relationship, thus making break-ups less likely (Anderson and Weitz (1992), Parkhe (1993)). Higher assets have been shown to increase product quality, lead times, and firm profitability (Dyer (1996), Palmatier, Dant, and Grewal (2007)). Relationships are also valuable due to learning about the partner (Egan and Mody (1992)). As the quality of the partner becomes clear, commitment to the relationship and trust increase (Ganesan (1994), Leuthesser (1997)). These effects can improve

26 Several authors have since extended their work, e.g. Wilson (1995), Jap and Anderson (2007)
relationship performance due to lower transaction costs (Dyer and Singh (1998)) and faster adaptation to market conditions (Palmatier, Dant, and Grewal (2007)).

The trade data provide evidence supporting the survey-based results. First, I study the path of value traded, relationship prices, and the hazard rate of relationship break-ups to provide quantitative evidence of a life cycle. Second, I examine the effect of relationship separations that are plausibly exogenous from the perspective of the importer to show that relationships are valuable. I will develop a model of relationships based on these facts.

Value and number of products traded

I begin by examining the link between value traded and relationship age. I first sort relationships into groups, based on whether they last for three years but less than four years, four years but less than five years, and so on. I examine the evolution of trade values separately for these groups to ensure that my results are not driven by composition effects. For each relationship, I compute the total value traded within its first 12 months, months 13-24, etc., up to the maximum full number of years for which the relationship is alive, and regress this on dummies indicating the relationship length. Since many relationships do not trade in every year, I apply a smoothing procedure to fill in years with zero trade. Otherwise, since by definition each relationship trades a positive quantity in the first year, I would find a sharp drop in trade from year one to year two. I therefore assume that the value purchased is equally distributed across subsequent years with zero trade.27 I distribute the last trade of the relationship linearly over a time period corresponding to the average time gap between transactions for that relationship. Letting \( \tau \) be the age of the relationship in years, and \( \tau^* \) be the total number of full years the relationship exists, I then run the following regression for \( \tau^* = 3, 4, 5, 6, 7 \):

\[
\ln(y_{mx\tau}) = \beta_0 + \sum_{i=2}^{\tau^*} \beta_i d_i + \gamma_{mx} + \epsilon_{mx\tau}. \tag{3}
\]

Here, \( \ln(y_{mx\tau}) \) is the (log of) the total trade value of relationship \( mx \) in year \( \tau \), \( d_i \) are relationship age dummies, and \( \gamma_{mx} \) are relationship fixed effects. Figure 3 plots the \( \beta_i \) coefficients and 95% confidence bands from these regressions, with year one normalized to zero.

The figure is consistent with the life cycle theory of relationships. For all relationships lasting at least four years, the value traded follows a hump-shaped pattern, with the value traded increasing over the first few years and then stabilizing and declining gradually. The effect is quantitatively important: for example, for relationships lasting six years, the value traded in year three is 21% higher than in year one. Trade values in the last year are below the initial starting point, consistent with problems and abandonment of the relationship. There is a very clear ordering based on how long the relationship eventually lasts: relationships that last six years have a stronger increase in trade than relationships lasting only five years, and so on. The patterns could be consistent with two explanations: on the

27This is consistent with a linear inventory policy with repurchase once the inventory level hits zero.
one hand, there could be selection based on persistent shocks to the relationship, such as demand fluctuations. On the other hand, pairs that start out better could actively invest more into their relationship, which therefore survives longer (see e.g., Ganesan (1994)).

Figures F.12a-F.12b in Appendix F.1 show that the number of products traded and the number of transactions per year follow a similar pattern as the total value. My findings are consistent with recent results by Fitzgerald, Haller, and Yedid-Levi (2015), who examine the growth patterns of exporters selling to a new destination country. They show that for exporters selling to a new market, the initial years after market entry are characterized by steep growth in revenues and quantities, while the years before market exit are characterized by decline. My results highlight that the pattern also holds at the level of the individual relationship.

Relationship prices

I next examine the path of prices over the duration of a relationship. I focus on importer-exporter-product triplets because overall relationships may trade several products. For each transaction \( i \) in quarter \( t \), I compute the relative log price \( \ln(p_{mxchi}(t)) \) by taking the log transaction price and subtracting the log average price for that product-country combination in that quarter. This removes product- or country-specific price trends. I discard all product-country-quarter cells that do not contain at least five observations.\(^{28}\) I then regress the relative price on dummies measuring how often the triplet has transacted. Let \( d_6, d_{11}, d_{16}, d_{21}, \) and \( d_{41} \) be dummies for whether the triplet has conducted 6-
10, 11-15, 16-20, 21-40, or 41-60 transactions, respectively (observations beyond transaction 60 are dropped). I include two additional control variables: first, I include the pair’s overall relationship length in months at transaction \(i\), \(\text{Months}_{mxi}(t)\). This variable captures whether prices are affected by overall relationship length, across all products. Second, I control for whether the relationship has previously been broken up and is now continuing, \(\text{Cont}_{mxi}(t)\). This variable picks up whether price setting is different in continued relationships compared to ones that are genuinely new. Thus, I run:

\[
\ln(\tilde{p}_{mxi}(t)) = \beta_0 + \sum_j \rho_j d_j + \beta_1 \cdot \text{Months}_{mxi}(t) + \beta_2 \cdot \text{Cont}_{mxi}(t) + \gamma_{mxh} + \epsilon_{mxi}(t),
\]

where \(\gamma_{mxh}\) are fixed effects for the triplet.\(^{29}\)

The regression shows that the relative price obtained in a relationship declines monotonically up to about 1.3% per purchase by transaction 41-60, with an additional reduction by .03% per relationship month (column 1 in Table 3). The management literature has similarly found evidence that long-term relationships provide price discounts to the buyer (Kalwani and Narayandas (1995), Cannon and Homburg (2001), Claycomb and Frankwick (2005)). This literature suggests that price declines are the result of a direct effect due to (possibly required) productivity improvements and learning curve effects (Lyons, Krachenberg, and Henke Jr. (1990), Kalwani and Narayandas (1995), Ulaga (2003)), and an indirect effect due to quantity discounts as order volumes rise (Cannon and Homburg (2001), Claycomb and Frankwick (2005)). I will incorporate both of these effects in my model below.

To test whether quantity discounts might be present, I first re-run regression (4), using the log deviation of quantity ordered from the market average for the product as dependent variable (column 2 in Table 3). The results show that the quantity ordered increases with the number of transactions, which may increase the price discount the buyer receives. To investigate this effect, I need to specify an assumption how prices are set. In my model, I will assume that buyers face a concave revenue function in quantity ordered and choose quantities to maximize profits. Longer relationships enable the buyer-seller pair to reduce marginal costs, for example due to productivity improvements, which lowers the price charged by the seller. In this setup, a regression of price on quantity will suffer from endogeneity bias since quantities depend on price. I therefore need to find an exogenous demand shifter to separate supply curve shifts due to productivity improvements from movements along the supply curve caused by higher quantities ordered.

My demand instrument is the weighted average gross output of the downstream industries of the imported good, where the weights are constructed via the “Use” table of the 2002 input-output table of the BEA.\(^{30}\) The identifying assumption behind this instrument is that when downstream industries’ output is high, their demand for inputs is large, and hence importers selling to these industries increase their imported inputs. Since prices are computed relative to the market average, the effect of industry-

\(^{29}\)As for the pass-through regressions, I do not need country dummies since these are a linear combination of triplet fixed effects.

\(^{30}\)I use the most detailed input-output matrix containing 417 industries.
wide price trends on demand is stripped out. The industry gross output figures are obtained for the period 1997-2011 from the BEA, and matched with the industries recorded in the IO table. Since detailed industry outputs are only available at annual frequency, I also use U.S. GDP as a second instrument to introduce quarterly variation. I detrend both variables using an HP filter.

The first-stage regression is run for each of 18 broad product categories presented in Appendix C. I regress log quantity on \( ProdDown_{1t} - ProdDown_{18t} \) and \( GDP_{1t} - GDP_{18t} \), where the transaction’s cyclical downstream demand component, \( ProdDown_{yt} \), and the cyclical GDP component, \( GDP_{yt} \), are set to their value if the transaction falls into product category \( y \), and are set to 0 otherwise. This specification allows for different cyclical responses by product category. The results from running (4) using the actually observed transaction quantity are shown in column 3 of Table 3, and the results using the instruments are shown in column 4. These results show that prices decline due to a direct effect alone. On average, relative price falls by about 0.7% by transaction 41-60. In Appendix F.2, Table F.20 repeats column 1 of Table 3 for different product categories. Price declines tend to be strongest for differentiated products such as chemicals, metal products, and machinery, and weakest for minerals, leather products, or textiles. While I cannot adjust prices to account for changing quality, these results provide suggestive evidence that a main driver behind the price declines is customization and associated productivity improvements, which cannot be generated for more standardized products.
Hazard rate of breaking up a relationship

I finally analyze the hazard rate of breaking up a relationship at a given age, conditional on having reached that age. Let $\tau$ be a relationship’s age in months, and $I\{\tau_{mxt} = \tau\}$ be an indicator that is equal to 1 if relationship $mxt$ with age equal to $\tau$ breaks up in month $t$. I define $\omega_{mxt}$ as the relationship’s value traded during the past twelve months, which will be used as a weight for the relationship’s importance. Recall that a relationship ends only when the maximum gap time has elapsed for all its products, and hence a relationship does not need to trade at $t$ to be ongoing. The weighted hazard rate at $t$ is defined as a weighted average over all relationships having that length at $t$:

$$\{\bar{I}_{mxt}|\tau_{mxt} = \tau\} = \frac{\sum_{mxt} \omega_{mxt} I\{\tau_{mxt} = \tau\}}{\sum_{mxt} \omega_{mxt}}.$$ (5)

The hazard rate for the sample is computed by taking a simple average over the hazard rates in all months. Figure 4 shows that this break up hazard declines very rapidly: from 55% in the first month to 6% in month 12.\(^{31}\) This result is related to findings by Eaton, Eslava, Jinkins, Krizan, and Tybout (2014) and Monarch (2015) who report a high rate of attrition in the first year of a relationship.\(^{32}\) I find that relationships are actually very likely to break up within the first months (usually after the first transaction), but once they clear that hurdle they are likely to continue. The findings align well with the presence of an “exploration phase” of the relationship life cycle. The results also mirror the negative association between job tenure and separation in worker-firm relationships (e.g., Mincer and

---

\(^{31}\) For this analysis, in order to account for relationship lengths up to 48 months accurately, I drop not only the first three years but the first five years, up to 1997.

\(^{32}\) For relationships with Colombian and Chinese suppliers, respectively.
Jovanovic (1979)). Pries (2004) estimates that the separation hazard for workers is more than 60% in the first year, and falls to less than 10% by year five, somewhat similar to my findings for firms.

**Value of relationships**

I now turn to the second claim made by the management surveys and examine whether relationships are valuable to firms. To this purpose, I analyze whether importers reduce the quantity purchased and experience lower employment growth following a plausibly exogenous break-up of the relationship with one of their suppliers. An adverse effect would suggest that relationships have value. In my main specification, I study the importer’s quantity purchased before and after a break-up, since the LFTTD data do not contain information on profits or sales. I assume that quantities are correlated with these variables.

Since I do not have additional data on the exporters, I make the identifying assumption that a break-up is plausibly exogenous from the importer’s perspective if the exporter involved had at least three active relationships at the time of the break-up, loses all of these simultaneously, and is never again seen in the dataset. My definition seeks to capture for example a bankruptcy or significant strategy change of the exporter, which require the importer to suddenly replace an established relationship. The fact that the exporter is still in three active relationships suggests that the break-up is sudden. I impose two additional conditions to ensure that separations are not caused by the importer. First, I use only importers that survive in the LBD for at least for two more calendar years after the break-up takes place. Second, I compute the share of the exporter’s U.S. sales accounted for by each importer during the year before the break-up, and consider break-ups as exogenous only if the importer accounts for less than 50% of the exporter’s U.S. sales in that year. I impose these conditions to eliminate cases where problems originate at the importer but spill over to the exporter due to the importer’s importance. To rule out that the declines in quantity are driven by industry-wide forces, I consider break-ups only for products whose total U.S. imports are increasing in the year of the break-up. Hence, quantity declines after a break-up would run counter the industry-wide trend.

I run a regression of the total quantity imported of product \( h \) by importer \( m \) in year \( t \) on a dummy for whether the importer experienced a relationship break-up impacting that product. To track the time path of quantities around the time of a break-up, I run separate regressions with dummies for whether a break-up happens in the following calendar year, in the current year, in the previous year, two years ago, and three years ago. These dummies are denoted \( d_{mh,i}^{b} \), with \( i \in \{ t + 1, t, t - 1, t - 2, t - 3 \} \). Since importers often have many marginal suppliers, I consider only relationships that are important from the perspective of the importer, defined as cases where the relationship supplied at least 50% of

---

33 “Simultaneously” means that the maximum gap time has not yet elapsed for these relationships. Break-ups in 2011 are not counted due to right-censoring.

34 I require the broad HS6 industry to be increasing to capture wider industry trends. The results are similar if HS10 industries are used.
importer $m$’s purchases of product $h$ over the past year.\footnote{Note that this regression uses the entire dataset, from 1995. All restrictions discussed so far only affect whether the break-up dummy is set equal to 1.} Thus, I run:

$$
\ln(q_{mht}) = \beta_0 + d_{mh,i}^b + \gamma_{mh} + \xi_t + \epsilon_{mht}, \tag{6}
$$

where $\gamma_{mh}$ are importer-product fixed effects and $\xi_t$ are year fixed effects. If relationships are valuable, the quantity ordered should decline sharply in the year of a break-up, and then recover gradually as the lost relationship is replaced. I drop break-ups where the importer has not recovered the pre-break-up level of purchases by the third year after the separation to eliminate cases where the reduction in quantity is permanent.

Figure 5 traces out the quantity patterns of these regressions for broken up relationships that have lasted at least 24 months, 12-24 months, and less than 12 months, respectively.\footnote{I am currently in the process of disclosing the coefficient for year $-2$ as well.} I normalize the coefficient in the year before the break-up to zero. The figure shows that losing an important long-term relationship is significantly more costly for importers than losing a relatively new relationship. For relationships that have lasted at least 24 months, the quantity imported in the calendar year after the break-up is about 19 percentage points below the quantity imported in the year before the break-up, and recovers only gradually. The drop is significantly smaller for relationships of age 12-24 months. Columns 1-5 of Table 4 present the coefficients of the regression for relationships that have lasted at least 24 months before the break-up. These coefficients can be interpreted as deviations from the average quantity traded in the relationship.

To examine how replacing a lost relationship affects the importer’s quantity purchased, I re-run regression (6) for relationships lasting at least 24 months, and interact the break-up indicator with a
Table 4: Break-up regressions, using quantity purchased, relationships 24 months or older

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: $\ln(q_{mht})$</td>
<td>$t + 1$</td>
<td>$t$</td>
<td>$t - 1$</td>
<td>$t - 2$</td>
<td>$t - 3$</td>
<td>$t - 1$</td>
<td>$t - 2$</td>
</tr>
<tr>
<td>$d_{mb,i}$</td>
<td>1.166***</td>
<td>0.0105</td>
<td>-0.0736**</td>
<td>-0.0517</td>
<td>0.1451***</td>
<td>-0.0812**</td>
<td>-0.0750*</td>
</tr>
<tr>
<td>(0.0293)</td>
<td>(0.0250)</td>
<td>(0.0321)</td>
<td>(0.0336)</td>
<td>(0.0379)</td>
<td>(0.0400)</td>
<td>(0.0440)</td>
<td></td>
</tr>
<tr>
<td>$d_{mb,i} \cdot d_{new}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0215</td>
<td>0.0559</td>
</tr>
<tr>
<td>(0.0669)</td>
<td>(0.0682)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed effects</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
</tr>
<tr>
<td>Observations</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
</tr>
</tbody>
</table>

dummy $d_{mb,t-1}^{new}$. This dummy is equal to one if a new relationship is formed in the same country for the same product in the year after the break-up. I focus on the same country only to avoid picking up new relationships that trade a different variety or a different quality level of the original relationship’s product. I then run

$$\ln(q_{mht}) = \beta_0 + d_{mb,t-1}^b + d_{mb,t-1}^{b} \cdot d_{mb,t-1}^{new} + \gamma_{mh} + \xi_t + \epsilon_{mht}. \quad (7)$$

Column 6 of Table 4 shows that creating a new relationship in the year after the break-up reduces the drop in quantities slightly, if at all. This suggests that a long-term relationship is valuable and cannot be immediately replaced with a new one. Column 7 redoes the regression with an interaction term measuring whether a new relationship has been formed in the two years since the break-up.

To estimate the real losses of relationship destruction, I use the LBD to examine the employment growth of firms affected by exogenous break-up. I calculate the growth as the log change in employment across all the firm’s plants from one year to the next. Since firms are likely to also have many domestic relationships, the effect is expected to be quite small. Columns 1 in Table 5 shows that for relationships that have lasted at least 24 months, employment growth is 1.5% below average in the year after a break-up. The remaining two columns show that the employment effects are smaller and statistically insignificant for shorter relationships.

As a robustness check, I re-run regressions 6 and 7 without imposing any restrictions on break-ups other than that the exporter must have had at least three customers and lose all of them, and that the exporter account for at least half of the importer’s purchases. The results for relationships that have lasted at least 24 months are presented in Table F.18 in Appendix F.2, for both quantities (columns 1-7) and employment growth (column 8). The results are strengthened compared to the baseline case.

Overall, my findings provide strong evidence that relationships generate rents. Related work by Barrot and Sauvagnat (2014) and Carvalho, Nirei, and Saito (2014) shows that firms whose suppliers are disabled due to natural disasters experience a significant reduction in sales growth. My work suggests that losses in fact vary with relationship length, and are not present for new relationships. The
Table 5: Break-up regressions, using employment growth

<table>
<thead>
<tr>
<th>$\Delta \ln(e_{mh,t})$</th>
<th>$\geq 24$</th>
<th>$12 - 24$</th>
<th>$&lt; 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$d_{mh,t-1}$</td>
<td>$-0.0149^*$</td>
<td>$-0.0126$</td>
<td>$-0.0020$</td>
</tr>
<tr>
<td></td>
<td>$(0.0086)$</td>
<td>$(0.0078)$</td>
<td>$(0.0036)$</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
<td>$mh,t$</td>
</tr>
<tr>
<td>Observations</td>
<td>9,542,000</td>
<td>9,542,000</td>
<td>9,542,000</td>
</tr>
</tbody>
</table>

Table 6: Statistics of exogenous break-ups, for relationships lasting at least 24 months

<table>
<thead>
<tr>
<th>$\geq 24$ months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. months until new supplier found</td>
</tr>
<tr>
<td>Avg months until new supplier for rel $\geq 24$ months found</td>
</tr>
<tr>
<td>Avg. number of suppliers tried before rel $\geq 24$ months</td>
</tr>
<tr>
<td>Excess gap time between transactions</td>
</tr>
</tbody>
</table>

results also mirror similar findings for firm-worker relationships. For example, Jacobson, LaLonde, and Sullivan (1993) find that when high-tenure workers lose their job due to mass layoffs, they experience a significant and prolonged wage drop even when they find a new job at a similar firm.

Table 6 provides some additional statistics for break-ups of relationships that have lasted at least 24 months. After an exogenous break-up, it takes U.S. importers on average 17 months to find a new supplier of the same good. Finding a new supplier with whom the relationship will last more than 24 months takes even longer, on average 20 months, and on average importers unsuccessfully try out 0.9 suppliers before forming that long-term relationship. The fourth row shows that the time needed to find a new supplier for a good exceeds the average time gap time of that good by on average 11 months. Thus, locating a supplier to replace a lost relationship takes a significant amount of time.

2.5 Interview Evidence

I conducted 16 interviews with purchasing managers and executives of 15 companies in Germany, the United States, and Chile to obtain additional qualitative evidence about firms’ relationships with their suppliers. More than half of the companies interviewed are well-known, leading players in their industry. Interview partners were found via personal connections, LinkedIn, and via the Yale Career Network. In total, 9 of the respondents are manufacturing firms, 2 are supermarkets, 2 are apparel retailers, and 2 are grocery wholesalers. The interviews were mostly conducted over the phone, and lasted between 20 and 30 minutes. The detailed interview transcripts are available on request.

\[37\] I thank Prof. Eduardo Engel for conducting one of the interviews in Chile and for making the other one possible.

\[38\] The firms are based in the following industries: Electrical and optical equipment (4), steel production (1), car manufacturing (1), tobacco (1), pharmaceuticals (1), and cardboard manufacturing (1).
Table 7: Summary of Interview Responses

<table>
<thead>
<tr>
<th>Fact</th>
<th>Sample: n=16</th>
<th>Sample</th>
<th>Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most of our transactions are done via long-run relationships</td>
<td></td>
<td>10</td>
<td>100%</td>
</tr>
<tr>
<td>Over time, we order more from the same supplier</td>
<td></td>
<td>15</td>
<td>87%</td>
</tr>
<tr>
<td>Over time, we customize and develop products together</td>
<td></td>
<td>13</td>
<td>92%</td>
</tr>
<tr>
<td>2 We get better prices in relationships due to quantity discounts</td>
<td></td>
<td>6</td>
<td>67%</td>
</tr>
<tr>
<td>Prices may improve due to efficiency gains</td>
<td></td>
<td>5</td>
<td>40%</td>
</tr>
<tr>
<td>3 We run a small pilot to find out about new suppliers</td>
<td></td>
<td>12</td>
<td>100%</td>
</tr>
<tr>
<td>We learn in less than 3 months if we want to work with a new supplier</td>
<td></td>
<td>8</td>
<td>75%</td>
</tr>
</tbody>
</table>

presents a summary of my findings. The “Sample” column indicates with how many interview partners I spoke about this topic. The “Agree” column shows what fraction of respondents agreed with the statement.

As expected, I find that long-run relationships with suppliers are important. The majority of respondents reported that long-term relationships account for 80% to almost all of their supplier relationships. Most firms are aware of a lot of potential suppliers, which they usually get to know via trade fairs, trade magazines, or the internet. Once a supplier has been selected, firms often conduct an initial test run by ordering a small quantity. This aligns well with the exploration phase of the life cycle theory. Learning about the supplier’s overall quality is usually quick, and takes only one or at most a few transactions. However, entering into a legal agreement, transferring technology, or setting up the logistics of a relationship may take a long time. Especially manufacturing firms and apparel retailers often conduct a lengthy evaluation involving formal audits and negotiations before starting a long-term relationship with a supplier. This suggests that the main friction is not so much about finding a supplier in general, but rather finding a supplier meeting the company’s criteria.

Once the relationship with a supplier has been established, relationships deepen over time. This includes ordering larger quantities from the supplier by allocating him a higher share of production or by growing quantities in line with increasing sales of the final output. Manufacturing firms often also ask for greater customization of the product. If the supplier is good, smaller existing connections may be replaced by this supplier and additional products may be ordered. Several respondents stated that they are able to obtain price reductions from long-term relationships because they order higher quantities. In the manufacturing sector, respondents also mentioned efficiency improvements in the production process. All these facts accord well with the build-up of a relationship. Most firms try to have at least two suppliers for a given input to be able to compare prices and to keep a check on the supplier’s bargaining power. While firms try to remain aware of alternatives, they rarely switch suppliers once a successful long-term relationship is in place. Interview respondents cited the high costs of building up a new relationship as a key reason not to switch unless there is no other option. One executive in the apparel industry mentioned that following a break-up she lost an entire year of sales of a product due to the time it took to audit a new supplier and to ensure its quality.
An important advantage of long-term relationships is that they enable firms to share risk. An interview partner at a Chilean metal-mechanic company stated that a long-term supplier will provide reliable shipments and not cancel the delivery of a product when market conditions are tight. Long-term relationships also provide the assurance that prices will not be changed arbitrarily. For example, an apparel industry manager mentioned that long-term relationship partners are willing to work together on price and help each other as demand fluctuates. Surveys from the management literature further corroborate these findings. Uzzi (1996) documents for the New York garment industry that long-term relationship partners exchange favors, for example by placing an order with a long-term supplier in slack times to help him make profits. Normann (2008) describes contracts implemented in the technology sector to share the risk of demand and price fluctuations. Camuffo, Furlan, and Rettore (2007) provide evidence from the Italian air conditioning sector that cost fluctuations are absorbed by both relationship partners, and Hennessy and Lawrence (1999) discuss risk sharing in agricultural contracts. This evidence suggests that risk sharing is common in long-term relationships.

3 Model

To rationalize the empirical findings, I develop a model of relational contracting in which a buyer and a seller firm interact repeatedly, and share profit risk using both monetary payments and production quantities, as suggested by interviews and surveys.\footnote{While in principle firms could also protect themselves against risk using financial derivatives, in practice only a small share of currency exposures is hedged. Allayannis and Ofek (2001) find that firms in the S&P500 in 1993 hedge on average 14.5% of their foreign currency exposure. Lel (2012) finds for a sample of listed international firms that about 20% of their currency exposures are hedged. Small firms are likely to hedge even less.} I introduce firm risk aversion as a parsimonious way to capture firms’ desire for reliable inputs, predictable prices, and flexible adjustments of contractual terms. Risk arises due to shocks to the seller’s production costs resulting from exchange rate movements. Production costs are also affected by persistent, idiosyncratic shocks which reflect for example the accumulation of specific assets, such as customized equipment. I show that asset accumulation changes the value of the relationship and generates the life cycle facts discussed in the previous section. Furthermore, I show that it also explains the increasing pass-through profile observed in the data by affecting the extent to which the firms can share risk.

In my baseline model, I assume that foreign exporters are more risk averse than U.S. importers, based on three empirical facts. First, the average foreign exporter is significantly smaller than the average U.S. importer (Table F.19 in Appendix F.2). For example, Bernard, Jensen, and Schott (2009) document that the average U.S. importer in 2000 had 319 employees. For the average French exporter, the figure is about 70 (Di Giovanni, Levchenko, and Rancière (2011)). Since a number of management studies find that small firms are more risk averse than large ones (e.g., Kawasaki and McMillan (1987), Asanuma and Kikutani (1992)), risk aversion should be higher for foreign exporters than for U.S.
importers. Second, a relatively large share of the U.S. economy is listed on the stock market. U.S. stock market capitalization was 129.6% of GDP in 2005, compared to a world average of 93.6% and an average of the Euro area of 60.5% (World Bank (2015)). Okamuro (2001) shows that unlisted firms are more risk averse than listed ones, since owners of private firms may often be personally liable for the firm’s debts. Third, most foreign countries have less liquid financial markets than the U.S., making hedging more difficult for foreign firms (Beck, Demirgüç-Kunt, and Levine (2009)). These facts point towards higher risk aversion of foreign exporters compared to U.S. importers. To simplify the characterization of my model, I assume in the baseline setup that importers are risk neutral. In Section 3.3, I present results for two-sided risk aversion and for a model with only risk averse buyers, and show that the latter model generates a counterfactually negative correlation between pass-through and age under plausible assumptions. A model with risk neutral firms and Nash bargaining also cannot match the empirical pass-through patterns. Section 3.4 shows that several pass-through moments in the data are in fact consistent with seller risk aversion, and provides evidence supporting the life cycle.

3.1 Model Setup

Let time be discrete and indexed by \( t \). A buyer firm purchases a quantity \( q_t \) from a seller in each period \( t \) until either firm chooses to leave the relationship. The buyer produces output \( y_t = f(q_t) \) and faces a demand function for his final output given by \( p^F = p^F(y) \). Thus, the buyer’s revenues as a function of inputs are

\[
R(q_t) = p^F(f(q_t))f(q_t). \tag{8}
\]

I assume that the buyer’s revenues are bounded, so that \( R'(q) = 0 \) for \( q > \bar{q} \). On the interval \( q \in [0, \bar{q}] \), I assume \( R'(q) > 0 \), \( R''(q) < 0 \), and \( R(0) = 0 \). This revenue function can be generated for example by a downward sloping demand function and linear production. Let \( \lim_{q \to 0} R'(q) = \infty \), which will guarantee that the overall profits of the relationship are positive if the order size is sufficiently small.

The seller produces the buyer’s input \( q_t \) according to a linear production function

\[
q_t = a_t l_t, \tag{9}
\]

where \( l_t \) is labor input and \( a_t \) is a productivity shifter affecting the seller’s marginal costs. I will interpret this variable as relationship-specific assets, in line with the management evidence discussed in Section 2.4, but more broadly an increase in \( a_t \) can reflect any process that reduces costs or that raises the amount of quality produced per unit of input. Labor input is purchased at exogenous cost \( w_x \) drawn each period from a finite set of states \( x \in \{1, ..., X\} \) with transition probabilities \( Q_{x' \mid x} \) following

\footnote{For example production could require a factor that is in limited supply, such as land, which is combined with the purchased inputs in a Leontief fashion. In my simulations, I will set \( \bar{q} \) very large.}
a Markov chain. Below, I will associate changes in production costs with fluctuations in exchange rates.

At the beginning of a relationship, relationship-specific assets $a_0$ are drawn from a continuous distribution $G(a)$ with support on $(0, \infty)$. This feature reflects the fact that matches can be of different quality, and captures the “exploration” aspect of the life cycle theory. I assume that the value of $a_0$ is unknown before the first transaction of the relationship, and becomes perfectly known once the first transaction has occurred. This modeling choice is in line with the interview evidence showing that buyers learn the quality of their supplier quickly. The high separation rate of relationships after the first transaction also suggests that learning and selection are most important at that point. Once the value of specific assets is known, it evolves according to

$$a_{t+1} = (1 - \delta)a_t + \epsilon_{t+1}. \tag{10}$$

This process\textsuperscript{41} reflects on the one hand depreciation of specific assets, such as physical wear and tear or staff turnover that breaks personal bonds. Second, assets receive random shocks $\epsilon_t \sim N(0, \sigma^2)$ that capture investments or disagreements between the partners.

In each period, seller firms deliver a quantity $q_t \geq 0$ to the buyer, and in return receive a monetary payment $T_t \geq 0$. I define price as the unit value, $p_t = T_t/q_t$. Both partners can leave the relationship at any time. Let the value of the buyer’s outside option be given by a fixed constant $V \geq 0$. This reflects the buyer’s ability to search for a new seller and start a new relationship. In the quantitative exercise, I will endogenize $V$ by estimating the probability of finding a new match. Sellers have an outside option value of $U(w_x) \geq 0$ satisfying $U'(w_x) < 0$. I assume that it is more difficult for sellers with higher costs to find a new match, which reduces the value of their outside option. As will become apparent when I characterize the model’s solution, this assumption is necessary to ensure that the seller’s profits are sometimes increasing under the optimal contract. Note that neither outside option depends on $a$, reflecting the assumption that the assets are fully specific to the relationship.

Sellers are risk averse with utility function $u$ over profits $\Pi$ which satisfies $u'(\Pi) > 0$ and $u''(\Pi) < 0$. Let $h_t = ((a_0, w_0), ..., (a_t, w_t))$ denote the history of play and states up to date $t$. A contract $\Xi$ is a sequence of functions $(q(h_t), T(h_t))_{t=0}^{\infty}$ specifying a quantity purchased $q(h_t)$ and a transfer payment $T(h_t)$ for every such history. The value of this contract to the buyer is given by

$$J(\Xi; h_t) = R(q(h_t)) - T(h_t) + E \sum_{o=t+1}^{\infty} \beta^{o-t} \{ R(q(h_o)) - T(h_o) \}, \tag{11}$$

where $\beta$ is the discount factor, and the expectation is taken with respect to $a$ and $w_x$ conditional on the initial state. In each period, buyers make profits calculated as revenues minus the payment to the

---

\textsuperscript{41}I assume an exogenous asset accumulation process to focus on the implications of risk sharing on prices. It would be interesting to extend the model to allow for investment in specific assets, and to study the implications for risk sharing in that setup.
seller. I allow for the possibility that the relationship is terminated. After such histories, \( q(h_o) = 0 \) and \( T(h_o) = 0 \) forever.

The seller’s value of the relationship is similarly given by

\[
W(\Xi; h_t) = u(T(h_t) - \frac{w_t}{a_t} q(h_t)) + E \sum_{o=t+1}^{\infty} \beta^{o-t} \left\{ u(T(h_o) - \frac{w_o}{a_o} q(h_o)) \right\} .
\]

Since the agents cannot commit, they only stay in the relationship in history \( h_t \) if the value of the contract in that history exceeds their outside options. I call a contract \( \Xi \) self-enforcing if for all histories in which there exist sequences \( (q(h_o), T(h_o))_{o=t}^{\infty} \) such that \( J(\Xi; h_t) \geq V \) and \( W(\Xi; h_t) \geq U(w_t) \), one of these sequences is implemented. Separation is constrained efficient under this contract since agents only break up in those histories in which no allocation of quantities and transfers exists that would incentivise both agents to stay.

As in Thomas and Worrall (1988) and Kocherlakota (1996), I write down a recursive formulation of the problem which involves an additional state variable \( W \) that captures the value promised to the seller in a given state. To simplify notation, I index states by \( s = (a, w_x) \). Let \( J(s_0, W) \) be the buyer’s value of being in a relationship with specific assets and costs \( s_0 = (a_0, w_0) \) and promised value to the seller \( W \) before the current period’s state \( s = (a, w_x) \) is revealed. The buyer’s problem is to choose a quantity traded \( q_s \), transfer \( T_s \), and continuation value \( W_s \) for each state \( s \) such that the seller receives in expectation the value that was promised to him and the participation constraints hold. Thus, the contracting problem before the current period’s state is revealed is

\[
J(s_0, W) = \max_{\{q_s, T_s, W_s\}} \left\{ E \left[ R(q_s) - T_s + \beta J(s, W_s) \right] \right\}
\]

subject to

\[
W = E \left[ u(T_s - \frac{w_x}{a} q_s) + \beta W_s \right]
\]

\[
R(q_s) - T_s + \beta J(s, W_s) \geq V
\]

\[
u(T_s - \frac{w_x}{a} q_s) + \beta W_s \geq U(w_x).
\]

The problem shows that the buyer maximizes his current period profits \( R(q_s) - T_s \) plus the continuation value of \( J(s, W_s) \), subject to the promise-keeping constraint (14). Equations (15) and (16) are the participation constraints of the buyer and the seller. Let \( \Gamma \) be the set of states \( (s_0, W) \) for which the contracting problem has a solution. For \( (s_0, W) \notin \Gamma \), a self-enforcing contract does not exist and the relationship separates endogenously. In that case, the buyer receives value \( V \) and the seller receives \( U(w_x) \).

My model shows that risk sharing has implications for price setting, extending existing work which has usually studied consumption or wage dynamics (e.g., Thomas and Worrall (1988), Ligon, Thomas,
and Worrall (2002)). The setup differs from standard risk sharing models in which households share income risk, such as Kocherlakota (1996), because, first, buyers and sellers share risk using two instruments, monetary transfers $T_s$ and quantity purchased $q_s$. Since production is assumed to be relationship-specific and therefore does not affect the outside option, payments and quantities both act as transfers that can be used for risk sharing. Second, the model contains two stochastic processes. A higher level of costs $w_x$ reduces the overall size of the surplus that can be split, and lowers the value of the seller’s outside option of leaving the relationship. These shocks are analogous to endowment shocks for one agent as in Attanasio and Ríos-Rull (2000), Kocherlakota (1996), or Ligon, Thomas, and Worrall (2002). On the other hand, shocks to specific assets $a$ affect only the value of being inside the relationship (the inside option) while leaving the separation value unchanged. I show below that a higher level of assets improves the scope for risk sharing, similar to a setup in which agents can save using a public technology (Ábrahám and Laczó (2014)). Better risk sharing will lead to prices that are more responsive to shocks.

3.2 Characterization

I show that the evolution of profits in my model resembles previous results, such as Kocherlakota (1996). Based on this, I then derive a set of new results regarding the evolution of prices, and show that the model generates the life cycle.

**Dynamic Evolution of Profits**

The solution to the problem specifies quantities $q_s$, transfers $T_s$, and promised values $W_s$ for every state $s$, given an initial $(s_0, W)$. Using similar arguments as Thomas and Worrall (1988), the set of self-enforcing contracts in every state is convex, the continuation values lie on a compact interval $[W_s, \bar{W}_s]$, and the Pareto frontier defined by $J(s_0, W)$ is concave (see Appendix D.1). Define $f(s|s_0)$ as the implied transition density between states $s$. Let $\lambda$ be the Lagrange multiplier on the promise-keeping constraint (14), and let $f(s|s_0)\nu_s$ and $f(s|s_0)\mu_s$ be the Lagrange multipliers on the participation constraints (15) and (16), respectively, for state $s$. These satisfy $\nu_s \geq 0$, $\mu_s \geq 0$. The first-order conditions of the problem with respect to $q_s$, $T_s$, and $W_s$ are

\begin{align}
\frac{w_x}{a}(\lambda + \mu_s)u'(T_s - \frac{w_x}{a}q_s) &= R'(q_s)(1 + \nu_s) \quad \forall (s_0, W) \in \Gamma \\
(\lambda + \mu_s)u'(T_s - \frac{w_x}{a}q_s) &= (1 + \nu_s) \quad \forall (s_0, W) \in \Gamma \\
(\lambda + \mu_s) + (1 + \nu_s)J_W(s, W_s) &= 0 \quad \forall (s_0, W) \in \Gamma.
\end{align}
Combining equations (17) and (18) and cancelling terms yields

\[ R'(q_s) = \frac{w_x}{a}, \quad (20) \]

which pins down a unique \( q \in [0, \bar{q}] \) by strict concavity of \( R \). This equation shows that under the optimal contract, the quantity ordered is always first-best, corresponding to the choice of a vertically integrated firm. The reason for this is that while a change in \( T_s \) has equal and offsetting effect on both agents’ profits, a one unit increase in quantity purchased lowers the seller’s profits by \( \frac{w_x}{a} \) but increases the buyer’s profits by \( R'(q_s) \). Varying \( q_s \) therefore affects the overall surplus that can be split between the agents. Agents thus choose a level of \( q_s \) to maximize the surplus regardless of their history of interactions or outside options, and use the transfer \( T_s \) to split this surplus. The result implies that when studying prices, \( p_s = T_s/q_s \), the denominator will not be affected by whether agents are constrained or not. The problem therefore becomes similar to one in which quantities are exogenously given and only transfers are chosen endogenously. Since \( R \) is strictly concave, quantities ordered are decreasing in costs and increasing in relationship-specific assets. Therefore, relationships experiencing good shocks to assets trade more, while a reduction in assets lowers the trade volume. This feature will allow me to match the relationship life cycle. The equation also shows that the responsiveness of quantity to shocks depends on the curvature of the revenue function. If \( R''(\cdot) \) is large in absolute value, the quantity response to cost shocks is small, as suggested in empirical evidence on import elasticities (Hooper, Johnson, and Marquez (2000)).

Seller profits are \( \Pi_s = T_s - \frac{w_x}{a} q_s \). Combining equations (18) and (19) relates the seller’s profits to promised continuation values:

\[ u'(\Pi_s) = -\frac{1}{J_W(s, W_s)} \quad \forall (s_0, W) \in \Gamma. \quad (21) \]

The equation shows that under the optimal contract, seller profits and future promises to the seller are positively correlated. Thus, incentives to the seller are provided both by higher profits today and by promising him higher profits in the future. Since the continuation values \( W_s \) lie in a compact interval, the equation implies that the set of efficient seller profits in each state \( s \) must also be a compact interval \([\Pi_s, \bar{\Pi}_s]\).

Using the envelope condition \( J_W(s_0, W_{s_0}) = -\lambda \) together with equation (19) yields

\[ (1 + \nu_s)J_W(s, W_s) + \mu_s = J_W(s_0, W). \quad (22) \]

This equation characterizes the dynamic evolution of the contract. By equation (21), the marginal values define a unique profit level \( \Pi_s \) of the seller for each state. Given the efficient profit level, these profits are implemented by a unique transfer \( T_s \) and the optimal quantity choice \( q_s \) from equation (20). The equation highlights that the two-dimensional problem of choosing sequences of \((q_s, T_s)\) collapses
to a single dimensional problem of determining the evolution of the seller’s profits. Profits follow a
similar process as in Ligon, Thomas, and Worrall (2002), where I denote by $\Pi_{s_0}$ the seller’s profits in
the past state and by $\Pi_s$ her profits in the new state:

**Proposition 1.** Under the optimal contract, for every state $s$, seller profits evolve according to

$$
\Pi_s = \begin{cases} 
\Pi_s & \text{if } \Pi_{s_0} > \Pi_s \geq \Pi_s \\
\Pi_{s_0} & \text{if } \Pi_{s_0} \in [\Pi_s, \bar{\Pi}_s] \\
\Pi_s & \text{if } \Pi_{s_0} < \Pi_s \leq \bar{\Pi}_s.
\end{cases}
$$

(23)

If $\Pi_s > \bar{\Pi}_s$, separation occurs and both agents receive their outside option.

**Proof.** See Appendix D.2.

Since the seller is risk averse with respect to profits, the optimal contract reduces his profit volatility
as much as possible. Seller profits are constant when neither outside option binds. In such states, the
buyer absorbs all shocks to costs and assets via his payments and quantity choices. When the buyer’s
outside option binds, the seller’s profits are reduced to incentivise the buyer to stay in the contract.
This reduction is as small as possible, to the upper bound of the seller’s profit interval associated with
a self-enforcing contract. Similarly, when the seller’s outside option binds, his profits are increased to
the lower bound of the profit interval. The relationship is terminated when the interval is empty.

**Payments and Prices**

I now characterize the evolution of prices associated with the optimal risk sharing contract. I show
that prices can be written as marginal costs plus a mark-up term. When either agent is constrained
following a cost shock, variation in mark-ups causes the price response to the shock to be dampened.
Since agents with a lower level of assets are more frequently constrained, pass-through rises with assets.

The first step of the analysis involves examining the response of the seller’s profit intervals $\Pi_s$
and $\bar{\Pi}_s$ to a shock. This characterization determines under what circumstances either agent can be
constrained. I show that given my assumptions, both the upper and the lower bound of the profit
interval are strictly decreasing in costs $w_x$, and the profit intervals expand with the level of assets.

**Proposition 2.** The profit intervals $[\Pi_s, \bar{\Pi}_s]$ associated with a self-enforcing contract have the following
properties:

1. $\Pi_s$ is decreasing in both $w_x$ and $a$.
2. $\bar{\Pi}_s$ is decreasing in $w_x$ and increasing in $a$.

**Proof.** See Appendix D.3.
The intuition for the comparative statics with respect to costs is that since $U'(w_x) < 0$, higher costs worsen the seller’s outside option. This lowers the minimum profits required by the seller to stay in the contract. The upper bound of the profit intervals is declining in $w_x$ because higher costs shrink the joint profits that can be split, which reduces the highest profit level that the seller can obtain while delivering to the buyer her fixed outside option value. The comparative statics with respect to assets highlight that the seller accepts a lower level of minimum profits if assets are higher. When assets are high, the expected joint surplus in future periods increases, which improves the scope for risk sharing and makes the relationship more valuable to the seller. Similarly, more assets make the relationship more valuable to the buyer, and increase the maximum amount of profits the seller can be allocated under the efficient contract.

Conditional on assets, the seller’s outside option may only bind after a shock that reduces costs, since such a shock may lead her minimum profit level to exceed her current profits. On the other hand, the buyer’s outside option may only bind after a shock that raises costs because the buyer claims the residual value of the relationship. These results show how the assumption $U'(w_x) < 0$ affects my results. If the seller’s outside option were constant, then a shock that reduces costs would not cause the seller’s outside option to bind. As I show below, in that case the model cannot generate diminished pass-through in response to negative cost shocks, since neither agent’s outside option binds after such a shock. When $U'(w_x) < 0$, pass-through may be diminished for both positive and negative shocks.

Since the seller is fully insured against profit fluctuations when neither constraint binds, risk sharing improves with the level of assets, since a higher asset level expands the profit intervals associated with a self-enforcing contract. In particular, if conditional on assets there exists a non-empty intersection of the profit intervals associated with all cost levels, the seller is fully insured against cost shocks once profits fall into that interval.

I next derive expressions for the payments $T_s$ made from the buyer to the seller. From equations (18) and (23), I obtain

$$T_s = \begin{cases} 
\frac{w_x}{a} q_s + (u')^{-1} \left( \frac{1 + \nu_s}{\lambda} \right) & \text{if } \Pi_{s0} > \Pi_s \geq \bar{\Pi}_s \\
\frac{w_x}{a} q_s + (u')^{-1} \left( \frac{1}{\lambda} \right) & \text{if } \Pi_{s0} \in [\Pi_s, \bar{\Pi}_s] \\
\frac{w_x}{a} q_s + (u')^{-1} \left( \frac{1}{\lambda + \mu_s} \right) & \text{if } \Pi_{s0} < \Pi_s \leq \bar{\Pi}_s.
\end{cases}$$

Equation (24) shows that if the buyer’s constraint binds ($\nu_s > 0$), the transfer payment $T_s$ is reduced relative to the unconstrained case, since $(u')^{-1}$ is strictly decreasing. Intuitively, since the buyer’s outside option is binding, the payment made by the buyer is reduced in order to incentivise him to stay in the contract. On the other hand, if the seller’s outside option is binding ($\mu_s > 0$), transfer payments are increased to raise the seller’s profits.

The response of payments to cost shocks is ambiguous when neither constraint binds. This ambi-
guity arises because the seller’s total production costs, $C_s = \frac{w_x}{a} q_s = \frac{w_x}{a} R^{-1}(\frac{w_x}{a})$, may either increase or decrease following a cost shock. On the one hand, higher marginal costs make production more expensive, which increases $C_s$ and therefore the payment required to stabilize the seller’s profits. On the other hand, higher marginal costs also reduce quantities ordered $q_s$, which reduces $C_s$. The overall effect depends on which of the two effects dominates. However, the effect of shocks on prices is clear when neither agent is constrained. Combining the equations in (24), I obtain:

$$p_s = \frac{T_s}{q_s} = \frac{w_x}{a} + \frac{1}{q_s} \left[ (u')^{-1} \left( \frac{1 + \nu_s}{\lambda + \mu_s} \right) \right]. \quad (25)$$

Prices consist of two parts: first, sellers receive their marginal cost. The second term is a mark-up to marginal costs, which depends itself on two forces: the quantity ordered and the outside options. Through the quantity effect, buyers ordering higher quantities pay a lower mark-up, assuming the Lagrange multipliers remain fixed. This matches the empirical evidence from the management literature discussed in section 5.2. suggesting that sellers offer quantity discounts (e.g., Cannon and Homburg (2001)). The effect arises because when the quantity ordered is high, less needs to be paid per unit in addition to costs to deliver the required utility level to the seller. The second source of mark-up variation arises from the Lagrange multipliers. When the buyer’s constraint binds, $\nu_s > 0$ holds, and therefore the mark-up is reduced since offering the buyer a lower price incentivises him to stay in the contract. Conversely, when $\mu_s > 0$ the mark-up is increased, incentivising the seller to stay in the contract. Prices satisfy $\partial p_s / \partial a < 0$ when neither constraint binds, which generates the life cycle fact that prices are decreasing with the quality of the relationship.42

When neither constraint binds, $\partial p_s / \partial w_x > 0$, and therefore prices move in the same direction as costs. For example, an adverse cost shock raises marginal costs and lowers quantities, which increases the mark-up. Under full risk sharing, the buyer pays a price that exactly offsets the change in the per-unit production cost $\frac{w_x}{a}$, plus an amount that offsets the fall in the seller’s profits due to the lower quantity ordered. As shown below, the second effect implies that under full risk sharing with seller risk aversion pass-through exceeds one.43

Equation (25) does not make clear whether a binding constraint may overturn the positive correlation between cost shocks and price, and for example cause prices to fall when costs increase. To examine this effect, I make the additional assumption that the buyer’s revenue function $R(q)$ is isoelastic. This revenue function has the following properties:

**Lemma 1.** Assume that the buyer’s revenue function $R(q)$ is isoelastic, with $R(q) = kq^r$, where $k > 0$

---

42 If shocks to $a$ represent improvements in product quality rather than reductions in marginal costs, the result holds if prices are defined per unit of quality.

43 The fact that mark-ups vary with quantity choice is not needed for the main story of this paper. The pass-through results below still go through even if quantity is assumed fixed, $m_s = \bar{m}$. In that case, pass-through will be bounded between zero and one.
and \( 0 < r < 1 \) by concavity of revenues. Then:

1. The seller’s production cost \( C_s \) is strictly decreasing in \( w_x \) and strictly increasing in \( a \).

2. Conditional on assets, the transfer payments \( T_s \) are decreasing in \( w_x \) for any value of the Lagrange multipliers. Conditional on costs, the transfer payments always increase with \( a \).

Proof. See Appendix D.4.

When the buyer’s revenue function is isoelastic and concave, an increase in costs lowers the quantity ordered relatively more than proportionally, and the seller’s total production costs fall. Stabilizing the seller’s profits therefore requires a reduction in the transfer payment. If the buyer’s outside option binds in response to the cost increase, the drop in payments is amplified to keep the buyer in the relationship. Conversely, a reduction in costs increases the transfer payment, and a binding seller constraint strengthens this increase. Thus, with isoelastic revenues, binding Lagrange multipliers always amplify the response of payments to a cost shock. Payments therefore unambiguously decline in costs. Payments always increase in the level of assets because quantity ordered rises more than proportionally with assets, and therefore the total production costs go up.

Using the isoelastic revenue function, I can derive conditions under which binding Lagrange multipliers mute the price response to a cost shock but do not change the direction of the response. In the discussion below, I focus on small cost shocks, defined as cases where taking the derivative with respect to \( w \) is approximately valid. The proof of the proposition in the appendix also discusses arbitrarily large shocks to costs.

**Proposition 3.** Fix \( a \) and consider two states \( s_0 = (a, w_0) \) and \( s = (a, w_x) \). Then:

1. If \( w_x > w_0 \) and \( \nu_s > 0 \), then for small i.i.d. cost shocks \( p_x > p_0 \) for all \( p_0 \geq \frac{w_0}{a} \). For small persistent shocks, \( p_x > p_0 \) unless \( \Delta \Pi^B > 0 \), where \( \Delta \Pi^B \) is the change in the buyer’s profits under the efficient contract, and at the same time the price level satisfies \( p_0 < \bar{p}(s_0, s) \).

2. If \( w_x < w_0 \) and \( \mu_s > 0 \), then prices satisfy \( p_x < p_0 \) if and only if \( U(w_x) - U(w_0) < K(s_0, s) \), where \( K(s_0, s) \) is a constant.

Proof. See Appendix D.5.

When either agent is constrained, mark-ups move to offset some of the change in costs. The first part of the proposition shows that for small cost shocks prices rise when the buyer’s participation constraint binds, except when her profits actually increase after a cost shock.\(^{44}\) This case occurs when the relationship is close to termination, since incentives to the buyer cannot be provided via future profits in that case, and therefore the buyer’s profits in the current period may be raised despite the

\(^{44}\)For shocks that are not small, there exists a constant \( M_0 > 0 \) such that prices rise if \( \Delta \Pi^B > -M_0 \) and prices lie in an interval \( p_0 \in \left[ \frac{w_0}{a}, \bar{p}(s_0, s) \right) \). See the proof for details.
fact that the overall relationship surplus falls. Such a profit increase must be implemented via falling prices if the current price level is low, while for high prices the reduction in quantity ordered alone may be enough to raise profits. The second part of the proposition shows that the price response to a cost decrease depends on the slope of the seller’s outside option. If a negative cost shock causes the seller’s outside option to increase by a sufficiently large amount, the price must go up to keep her in the contract.

With my interpretation that changes in $w_x$ are due to observable exchange rate movements, I can derive an equation for exchange rate pass-through using a log-linear approximation. I define exchange rates in units of foreign currency. Thus, an appreciation of the foreign currency translates into an increase in costs for the foreign producer.

**Lemma 2.** Fix a value of $a$. For small changes of $w_x$ pass-through into prices satisfies

$$\hat{p}_s = \xi_1\hat{w}_x + \xi_2\hat{\nu} + \xi_3\hat{\mu},$$

(26)

where hats denote log deviations from the current state and $\xi_1 > 1$, $\xi_2 < 0$, and $\xi_3 > 0$. Furthermore, $\xi_1$ satisfies $\frac{\partial \xi_1}{\partial a} < 0$, $\frac{\partial \xi_1}{\partial k} < 0$, and $\lim_{r \to 0} \xi_1 = 1$.

**Proof.** See Appendix D.6. □

Equation (26) highlights that when no constraint binds ($\hat{\nu} = 0$ and $\hat{\mu} = 0$), pass-through exceeds one. As discussed above, in response to a cost shock the buyer pays the seller an amount that offsets the change in total production cost $\frac{w_x}{a}q$, plus an amount that offsets the fall in the seller’s profits due to the lower quantity ordered. This second effect pushes pass-through above one. Pass-through is reduced when either agent’s constraint binds. For example, when costs fall causing the seller to hit her outside option ($\hat{\mu} > 0$), then since $\xi_3 > 0$ there is an offsetting effect causing prices to fall less than in the unconstrained case. The diminished pass-through result holds even if the conditions in Proposition 3 are not satisfied. In that case, pass-through is reduced by such an extent that it actually becomes negative.

When the constraints do not bind, pass-through is decreasing in the level of assets. As the level of assets increases, quantity ordered rises, lowering the mark-up over marginal costs. This diminishes the importance of the change in mark-ups relative to the change in marginal costs following a cost shock. As assets go to infinity, the mark-up goes to zero and pass-through approaches one, since only the marginal cost effect matters. In my quantitative estimation, I show that given the life cycle observed in the data assets do not vary much in absolute terms, and therefore pass-through is basically constant outside of the constrained region. If the buyer’s revenue function is very concave ($r \to 0$), the quantity effect on mark-ups vanishes and pass-through approaches exactly one when neither agent is constrained.
My limited commitment model is in the tradition of models that generate imperfect pass-through via varying mark-ups, as in Krugman (1987), Goldberg and Verboven (2001), or Atkeson and Burstein (2008). My setup introduces a novel source of mark-up variation based on risk sharing, which generates an endogenous evolution of mark-ups. Prior work has usually generated mark-ups by assuming a specific market structure. The model does not generate stable prices in long-term relationships, as predicted by e.g. Barro (1977), because seller’s are risk averse with respect to profits, not prices. Sellers are therefore not harmed by high price volatility, and in fact prefer it if volatile prices lead to a smooth profit stream.

**Accumulation of Specific Assets**

As shown above, the model generates increasing quantities and falling prices as more assets are accumulated. By Lemma 1, if the revenue function is isoelastic, value traded increases as well. To match the life cycle, it remains to show that relationships terminate if the level of specific assets is sufficiently low.

**Lemma 3.** There exist threshold levels \(a^*(w_x)\) for each level of costs \(w_x\) such that if \(a < a^*(w_x)\) the relationship is endogenously terminated. This separation is efficient. Furthermore, for two values of costs \(w_0 < w_x\), the thresholds satisfy \(a^*(w_0) < a^*(w_x)\) if \(J((a^*(w_0), w_x), U(w_x)) < V\).

**Proof.** See Appendix D.7.

For a low enough value of assets, there exists no combination of quantities, payments, and promises such that both relationship partners prefer to stay in the relationship. Separation is therefore efficient. The second part of the lemma shows that if the value of the relationship falls more strongly with costs than the outside option value, the minimum level of assets needed to sustain the relationship is increasing in costs. Since assets and costs follow different stochastic processes, knowing only the ratio \(w_x/a\) is not sufficient to determine whether a relationship should be terminated.

In the first period of the relationship, costs \(w_x\) are observable but the initial asset level is unknown by the firms. I therefore assume that in the first period buyers place an order based on their expectation of assets to maximize profits. Once the order arrives, the initial value of \(a\) is observed. Buyer and seller then agree on a payment that splits their current period profits giving a fraction \(\phi_0\) to the buyer, and determine whether the relationship should be terminated. If the relationship is continued, the agents set an initial promised value for the seller \(W\) that under the optimal contract would give the buyer a share \(\phi_0\) of profits if state \(s_0\) occurred again. In this setup, the buyer’s initial order size \(m_0\) satisfies

\[
R'(q_0) = w E \left[ \frac{1}{a} \right]
\]  

and the initial price \(p_0\) satisfies

\[
p_0 = (1 - \phi_0) \frac{R(q_0)}{q_0} + \phi_0 \frac{w_x}{a}.
\]
Given my assumptions on the revenue function, positive relationship profits can always be guaranteed by placing an order that is sufficiently small. Consequently, there is no termination before the assets become known. Since the initial price is set based on the exogenous parameter $\phi_0$, any pass-through may be generated in the first period by selecting $\phi_0$ appropriately. In the model estimation below, I will set the initial price to match the average price decline in the first year, which is around 1%.

My assumptions imply the following value functions in the first period before the level of assets is revealed, conditional on costs:

\[
J^{new}(w_0) = \max_q \left\{ \int_0^\infty [R(q) - T(q; w_0, u)] g(u)du + \beta E \left[ \max \{J(s, W), V\} \right] \right\}
\]

(29)

and

\[
W^{new}(w_0) = \int_0^\infty [T(q; w_0, u) - \frac{w_0}{w} q] g(u)du + \beta E \left[ \max \{W, U(w_x)\} \right],
\]

(30)

where $T(q; w_0, a)$ is the transfer payment that is implemented once the level of assets becomes known.

The life cycle is generated through the stochastic evolution of specific assets. The relationship is terminated in the first period if the initial level of assets is revealed to be too low. This feature generates the high initial probability of relationship termination. Relationships that last for many periods on average receive many positive shocks, which move their assets away from the endogenous termination bound. This raises the value traded, lowers price, and decreases the probability that the relationship will be terminated in the next period. Relationships that terminate must have received a number of bad shocks that has brought assets back down to the termination bound.

Given my production structure and the stochastic asset evolution, any model with efficient separations can generate the life cycle. To illustrate this point, in Appendix E I solve a simple Nash bargaining model with free entry on the buyer side and show that it can generate the life cycle moments. However, this model fails to match the empirical pass-through patterns. First, since the match surplus is always split in fixed proportions under Nash bargaining, this model cannot generate the increasing pass-through pattern obtained under limited commitment. I show that with an isoelastic revenue function, pass-through is actually decreasing in the level of specific assets, for similar reasons as in the limited commitment setup. Second, pass-through is always greater than one under Nash bargaining with free entry, regardless of the bargaining weights. This result holds because a cost shock not only raises production costs but also lowers the quantity ordered, which acts as an additional force raising price. Therefore, while the Nash bargaining model is capable of matching the life cycle, it delivers counterfactual results with regard to pass-through. I confirm this quantitatively in Section 4.
3.3 Alternative Assumptions for Risk Aversion

The model with seller risk aversion generates pass-through that is greater than one when neither agent is constrained. Since pass-through in the data is considerably lower than one, I explore different assumptions of risk aversion to test whether the limited commitment model is also able to generate the correct level of pass-through. When buyers are risk averse, they seek to stabilize their own profits, which causes them to respond less than fully to any cost shocks. I can derive pass-through equations similar to (26) given different assumptions of risk aversion.

**Proposition 4.** Fix \( a \), and consider the effect of a small change of \( w_x \) on prices, where the buyer is allowed to be risk averse. Then, pass-through is

\[
\hat{p}_s = \varsigma_1 \hat{w}_x + \varsigma_2 \hat{\nu} + \varsigma_3 \hat{\mu},
\]

(31)

where

1. If both agents are risk averse with the same utility function of the CRRA class, then \( \varsigma_1 = 1, \varsigma_2 < 0, \) and \( \varsigma_3 > 0 \).

2. If the buyer is risk averse and the seller is risk neutral, then \( \varsigma_1 < 1, \varsigma_2 < 0, \varsigma_3 > 0, \) and \( \frac{\delta K}{\delta a} > 0 \).

3. If the buyer is risk averse and the seller is risk neutral, then the price response to a positive cost shock is amplified if the seller’s outside option binds. Such a case can never arise if for a fixed asset level, \( U(w_x) - U(w_0) < K(s_0, s) < 0 \) for all \( s_0 = (a, w_0) \) and \( s = (a, w_x) \) with \( w_x > w_0 \).

**Proof.** See Appendix D.8.

The proposition shows that pass-through declines as the buyer becomes more risk averse relative to the seller. If both firms are equally risk averse, then pass-through is one when neither is constrained, resulting from the sum of the effect due to declining marginal costs and the reduction in quantity ordered. Binding constraints now no longer necessarily mute the price response to shocks, since the seller’s profits are no longer constant in response to shocks and her outside option may therefore also bind in response to a cost increase.

When only the buyer is risk averse, pass-through is lower than one when neither agent is constrained, and may even be negative. The seller bears the risk of cost shocks, and prices are only adjusted to offset any changes in quantities when no constraint binds. The effect of binding outside options depends on the shape of the seller’s outside option \( U(w_x) \). A cost increase reduces both her relationship value and her outside option. Consequently, it depends on the relative slopes of the two whether a shock causes the seller’s outside option to bind. If the outside option value declines more strongly than the relationship value in all cost states, the seller’s outside option never binds after a cost increase if assets are held fixed. On the other hand, if \( U(w_x) \) is relatively constant, the outside option may bind
following a cost increase. In that case, since $\varsigma_3 > 0$, a binding seller constraint amplifies the price response to positive cost shocks compared to the unconstrained case.

Pass-through with risk averse buyers is increasing in the level of specific assets because prices are now given by the buyer’s per-unit revenue minus a “mark-down” term that depends on the quantity ordered. By a mirror argument to the one in Section 3.2, as assets converge to infinity pass-through approaches one, but this time from below. When assets change only relatively little, as revealed in the quantitative exercise, this effect is negligible.

### 3.4 Empirical Tests of Model Implications

I now present additional evidence supporting the baseline setup with seller risk aversion. First, I consider two proxies for higher seller risk aversion and examine their impact on pass-through. Second, I examine several implications of the life cycle for pass-through.

**Risk Sharing and Pass-Through**

Existing work on risk sharing suggests that smaller firms are more risk averse than large ones (e.g., Kawasaki and McMillan (1987)). One reason for this result is that smaller firms are more likely to be owner-operated and have a smaller number of projects, making them less able to diversify risk (see e.g. Greenwald and Stiglitz (1990), Engel, Fischer, and Galetovic (1998)). Additionally, Kawasaki and McMillan (1987) and Asanuma and Kikutani (1992) suggest that firms with a higher concentration on a particular customer should be more risk averse with respect to that relationship, since fluctuations in the business with an important customer are harder to smooth out. Through the lens of my model, small firms and firms with a smaller network of customers should therefore have higher pass-through than large firms if these are approximately risk neutral. This finding should hold for both positive and negative exchange rate shocks.

To examine the effect of size on pass-through, I compute the exporter’s size as the sum of its total real exports in the dataset. I then run the following specification:

$$\Delta \ln(p_{mxchi(t)}) = \beta_0 + \beta_1 \Delta \ln(e_{ct}) + \beta_2 \ln(\text{Size}_x) + \beta_3 \Delta \ln(e_{ct}) \cdot \ln(\text{Size}_x) + \beta_4 \ln(\text{Size}_m) + \gamma_c + \xi_h + \omega_t + \epsilon_{mxchi(t)},$$

(32)

where the variables are defined as in Section 2.3, $\text{Size}_x$ is the size of the exporter, and $\text{Size}_m$ is the size of the importer based on its total real imports. The regression compares pass-through controlling for country, product, quarter, and the size of the importer across exporters of different size. Coefficient $\beta_3$ is the coefficient of interest.

As predicted, I find that pass-through is declining with the size of the exporter (Column 1 in Panel a of Table 8). An increase in firm size by one log point is associated with a reduction in pass-through.
by .027. Columns 2-3 of Table 8 highlight that small firms have a higher responsiveness of prices for both positive and negative exchange rate shocks. Thus, small firms obtain larger price increases following a rise in costs and larger price declines following cost decreases. This supports my risk sharing mechanism. In particular, the fact that smaller firms also obtain larger price increases than large firms cannot be explained by an alternative story in which large importers simply benefit from higher bargaining power when dealing with small suppliers.

While the size effect on pass-through is consistent with previous work by Berman, Martin, and Mayer (2012) and Chatterjee, Dix-Carneiro, and Vichyanond (2013), risk sharing also implies that pass-through should increase more strongly with relationship length for small firms than for large firms. Small firms in new relationships are initially not able to share risk well, but obtain better insurance as the relationship ages due to the relaxation of enforcement constraints. This effect should be less important for large firms. To test this implication, I run the specification with triple interaction terms:

\[
\Delta \ln(p_{mchi(t)}) = \beta_0 + \beta_1 \Delta \ln(e_{chlt}) + \beta_2 \ln(\text{Size}_x) + \beta_3 \text{Months}_{mxi(t)}
\]

\[
+ \beta_4 \Delta \ln(e_{chlt}) \cdot \ln(\text{Size}_x) \cdot \text{Months}_{mxi(t)} + \kappa \chi_{chlt} + \gamma_c + \xi_h + \omega_t + \epsilon_{mchi(t)}, \tag{33}
\]
where $x_{cht}$ is a vector containing all three combinations of interactions between $\Delta \ln(e_{ct})$, $\ln(\text{Size}_x)$, and $\text{Months}_{mx_i}(t)$. Risk sharing suggests that $\beta_4$ is negative. Column 4 in Table 8 shows that this is the case. Small firms not only start a relationship with a higher level of pass-through, but also increase pass-through more rapidly as the relationship ages, consistent with better risk sharing. The results hold separately for both positive and negative exchange rate shocks (Columns 5-6). These new results add significant support to my proposed mechanism.

The second variable correlated with risk aversion is the size of the exporter’s customer network. I construct this variable by counting the number of U.S. firms an exporter sells to in each year, across all products. I then re-run regression (32) for the log number of customers instead of size, but control for $\ln(\text{Size}_x)$ separately to avoid picking up the size effect already documented. The regression therefore measures the effect of an additional customer conditional on firm size. As expected, conditional on size, average pass-through falls as the number of customers increases (Column 1 of Panel b in Table 8). An increase in the customer network by 1% lowers pass-through by .032. The result holds for positive and negative exchange rate shocks (columns 2-3). The result is again consistent with risk sharing. Exporters with more customers are able to diversify more easily across buyers, and therefore require less risk sharing and less responsive prices. Columns 4-6 run a regression with triple interaction terms. The simple interaction effects are similar to before, but the relationship length effect on pass-through is statistically indistinguishable across exporters with a different number of customers. Overall, these results show that smaller exporters and exporters with a smaller number of customers set prices that are more responsive to exchange rate shocks, supporting the risk sharing motive.

**Pass-Through and the Relationship Life Cycle**

I test three additional implications of the life cycle model in the data, and show that they are not rejected. First, the model predicts that pass-through is higher for those relationships which have a higher level of relationship-specific assets. Since assets are correlated in my model with the observed trade value, I can test this implication. I run a regression of the form given in (2), where I replace relationship length with value traded. To eliminate cross-sectional effects, I normalize the value traded in the first year of a relationship to one, and examine whether pass-through is positively correlated with trade relative to year one. Define $d_{med}^{med}(mx,i(t))$ as a dummy that is equal to one if the relationship trades 25%-50% more in the year associated with transaction $i$ than in year one, and $d_{high}^{med}(mx,i(t))$ be a dummy that is one if the relationship trades more than 50% more than in the first year. I then run

$$\Delta \ln(p_{mxchi}(t)) = \beta_0 + \beta_1 \Delta \ln(e_{ct}) + \beta_2 d_{med}^{med}(mx,i(t)) + \beta_3 d_{med}^{med}(mx,i(t)) + \beta_4 \Delta \ln(e_{ct}) \cdot d_{med}^{med}(mx,i(t)) + \beta_5 \Delta \ln(e_{ct}) \cdot d_{high}^{high}(mx,i(t)) + \gamma_{mxh} + \omega_t + \epsilon_{mxchi}(t).$$

(34)

Column 1 of Table 9 shows that pass-through is increasing in value traded. Relationships trading more than 50% more than in the first year exhibit pass-through that is about 3.1 percentage points higher.
Table 9: Pass-through model implications

<table>
<thead>
<tr>
<th>$\Delta \ln(p_{mcxht})$</th>
<th>Value</th>
<th>Products</th>
<th>Last year</th>
<th>Quadratic</th>
<th>Total length</th>
<th>Total length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>$\Delta \ln(e_{cht})$</td>
<td>.1904 ***</td>
<td>.1968 ***</td>
<td>.2188 ***</td>
<td>.1407 ***</td>
<td>.1149 ***</td>
<td>.1060 ***</td>
</tr>
<tr>
<td></td>
<td>(.0053)</td>
<td>(.0051)</td>
<td>(.0056)</td>
<td>(.0085)</td>
<td>(.0107)</td>
<td>(.0110)</td>
</tr>
<tr>
<td>$d_{med}$</td>
<td>.0266 *</td>
<td>0.0003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0150)</td>
<td>(.0172)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{high}$</td>
<td>.0310 ***</td>
<td>.0183 *</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0093)</td>
<td>(.0101)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{first}$</td>
<td>- .0621 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0094)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{last}$</td>
<td>- .0176 *</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0093)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Months$</td>
<td>.0022 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Months^2$</td>
<td>- .0000 **</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totmonths</td>
<td>.0005 †</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed effects</td>
<td>$mzh,t$</td>
<td>$mzh,t$</td>
<td>$mzh,t$</td>
<td>$mzh,t$</td>
<td>$mzh,t$</td>
<td>$mzh,t$</td>
</tr>
<tr>
<td>Observations</td>
<td>16,902,000</td>
<td>16,902,000</td>
<td>16,902,000</td>
<td>16,902,000</td>
<td>16,902,000</td>
<td>16,902,000</td>
</tr>
</tbody>
</table>

Significance levels: *** : 99% level, ** : 95% level, * : 90% level, † : 89% level

than in that year. The effect is smaller but significant for relationships with a moderate increase in trade value.

As an alternative specification, I run the same regression using the number of products traded instead of value. While in my model all relationships trade only one product, it seems plausible that relationships that increase the number of products traded are good relationships that have increased specific assets. Column 2 of Table 9 documents that a higher number of products compared to the first year is associated with higher pass-through, although the effect is small.

The second prediction I examine is that relationships close to separation have a low level of specific assets. Such relationships should therefore have diminished pass-through. To test this implication, I run the pass-through regression (2) with dummies for whether the relationship is in its first or in its last year. The regression coefficients are only identified for relationships lasting at least three years. Column 3 of Table 9 shows that pass-through is diminished in the last year of a relationship, as predicted. Pass-through is about 1.8% lower in the last relationship year than in the years in the middle of the relationship. Since value traded increases at a diminishing rate in my model and in the data (see Figure 3), another test of the model fit is to introduce a quadratic term into the baseline
pass-through regression and to examine its significance. I find that indeed the increase in pass-through with relationship length is a quadratic (Column 4).

A third implication of the model is that relationships that start out with a higher level of specific assets last longer, since they are further away from the termination threshold. Such relationships should be characterized by high pass-through. To test this implication, I calculate for each relationship the total number of months it exists. I then examine whether a given importer exhibits higher pass-through in the first year for those of his relationships that last longer by running

\[
\Delta \ln(p_{mxchi(t)}) = \beta_0 + \beta_1 \Delta \ln(e_{ct}) + \beta_2 \text{Totmonths}_{mx} + \beta_3 \Delta \ln(e_{ct}) \cdot \text{Totmonths}_{mx}
\]

\[+ \ln(\text{AvgSize}_x) + \gamma_c + \xi_{mh} + \omega_t + \epsilon_{mxchi(t)},\]

(35)

where \(\text{Totmonths}_{mx}\) is the total length of the relationship in months, and \(\text{AvgSize}_x\) controls for the average size of the exporter in the relationship by calculating the sum of his total export value divided by the number of quarters the exporter is in the data. Furthermore, \(\xi_{mh}\) are importer-product fixed effects. I use average size rather than total size of the exporter because an exporter with longer relationships has by definition a higher export value. I therefore normalize this figure by the number of quarters the exporter is active. I run the regression only for transactions in the first year of a relationship, and consider only pass-through based on price changes that occur between subsequent quarters, since the asset level could have changed significantly over a longer time horizon. The coefficient of interest is identified by comparing cases where the same importer buys the same product in the same quarter from two exporters of the same size located in the same country, where the relationships with the two last for a different number of months. Columns 5 in Table 9 highlights that higher pass-through in the first year is indeed associated with a longer relationship. A relationship lasting one month longer has pass-through in the first year that is about .05% higher. Column 6 repeats the regression using dummies for relationships that last in total one to four years and more than four years. These findings lend additional support to the relationship life cycle theory.

4 Quantitative Analysis

I now structurally estimate the model. This step has two purposes: first, I show that the model with seller risk aversion, when calibrated to the relationship life cycle, matches the increase in pass-through with relationship age observed in the data. Since this moment is not targeted in the estimation, matching it is a significant success of my model. I compare the baseline model with seller risk aversion to alternative models and show that only the baseline model generates a positive correlation between pass-through and age. Second, I use the calibrated model to study how changes in the distribution of relationships affect aggregate pass-through. During the 2008-09 recession, the number of relationships
of age less than one year fell by one fifth. I show that the associated increase in the average length of relationships can explain about 20% of the increase in pass-through observed during that period. My paper is the first to show that the distribution of relationships in the economy may have aggregate effects.

4.1 Functional forms

Let a time period correspond to one quarter. I assume that there is a unit mass of buyers indexed by \( j \), and a continuum of sellers indexed by \( k \). A buyer’s production function is given by \( f(q) = q^\alpha \), where \( \alpha \in (0,1) \). Buyers are monopolistic competitors for their product, and face a demand in their downstream market given by \( p_F(y) = by^{-1/\rho} \), where \( \rho > 1 \) measures the elasticity of demand and \( b \) is a constant determining the level of demand. With these functional forms, the buyer’s revenue is \( R(q) = bq^\alpha(\rho - 1)/\rho \), which is isoelastic with \( \rho = \alpha(\rho - 1)/\rho \). The constant \( b \) could be endogenized by assuming that buyers sell to a final goods firm that aggregates all products based on a Dixit-Stiglitz aggregator. I assume that the seller’s utility function is of the form \( u(\Pi_s) = \ln(\Pi_s) \). Unmatched buyers have a probability \( \pi_j \) to meet a new seller in the next period. Therefore, a buyer’s outside option is given by

\[
V = \beta[(1 - \pi_j)V + \pi_j E F_{new}(w_{x'})],
\]

where the expectation is with respect to the state of \( w_x \) in the next period. I assume that a seller is able to shift his production to a new customer immediately, so that his outside option depends on his current level of costs, as assumed previously. The seller’s probability of meeting an unmatched buyer is denoted by \( \pi_k \). Hence, the seller’s outside option is given by

\[
U(w_x) = \pi_k W_{new}(w_x) + (1 - \pi_k)\beta EU(w_{x'}),
\]

where the value of being unmatched is normalized to zero. Initial assets are drawn from a lornormal distribution with parameters \((\mu_a, \sigma_a)\), and I let costs follow a random walk in logs given by

\[
\ln(w_{x'}) = \ln(w_x) + \xi
\]

with \( \xi \sim N(0, \sigma_w^2) \).

To generate the correct level of pass-through in the baseline model, I assume that exchange rate shocks affect only a fraction of the seller’s costs, for example due to imported inputs or local distribution costs (e.g., Goldberg and Verboven (2001), Amiti, Itskhoki, and Konings (2014)). I therefore model costs as

\[
w = w_F(e)^\omega w_L^{1-\omega},
\]

45
where \( w_F(e) \) are costs affected by exchange rate movements, \( w_L \) are local costs, and \( \omega \) governs the relative shares of each. Assuming that local costs are constant, I have that

\[
\Delta \log(w) = \omega \Delta \log(w_F(e)).
\]

I will estimate \( \omega \) to match the level of pass-through observed in the data for the baseline model.

4.2 Estimation and Identification

Estimating the Parameter Values

I set several parameters of the model exogenously. First, I normalize the demand parameter \( b = 1 \). I choose an elasticity of substitution of \( \rho = 4 \) as in Nakamura and Steinsson (2008), which delivers a mark-up of buyers for their final goods close to estimates by Berry, Levinsohn, and Pakes (1995). I impose curvature in the production function to dampen the quantity response to cost shocks and set \( \alpha = .6 \). The quarterly discount factor is set to \( \beta = 0.988 \). One difference between the model and the data is that a firm cannot produce in the model once it loses its (unique) supplier, while in the data firms often have multiple suppliers, and can shift between them. To obtain a cleaner estimate of the buyer’s matching probability, I estimate \( \pi_j \) using exogenous break-ups, since after such break-ups buyers seek to replace their lost supplier as quickly as possible. As shown in Section 2.4, firms losing a supplier exogenously suffer a loss of output in the interim. The time required to find the next supplier in this case should therefore reflect more cleanly how fast firms are able to find a new match. I use the excess gap time from Table 6 to estimate the quarterly probability of finding a new match. For the seller’s matching probability \( \pi_k \), I use the average time passed between the first transaction with a new buyer to the next time the seller meets a new customer, which in the data is about 14 months. While a higher \( \pi_k \) implies a better outside option for the seller, determining the matching probability accurately is not crucial because the distribution of initial assets is chosen in the estimation step to generate the correct separation rate. When the seller’s probability of finding a buyer increases, the distribution shifts to the left so that separation stays the same as before.

I set \( \sigma_W \) by calculating in the data the quarterly standard deviation of exchange rate shocks relative to the previous exchange rate, across all currencies used. I find this relative standard deviation to be 6.8%, and therefore set \( \sigma_W = .068 \). Finally, I normalize the initial value of \( w_x \). While dynamically both \( w_x \) and \( a \) affect the problem separately due to their different evolution equations, I find in my simulation that this effect is small and that I can match the model for arbitrary levels of initial \( w_x \) by choosing the appropriate initial asset level. I therefore assume that initial costs have a mean of \( \bar{w}_x = .23 \) and are uniformly distributed around that wage level. These parameters are summarized in the top part of Table 10.

Six parameters remain to be estimated: the mean and standard deviation of the initial distribution
of specific assets, $\mu_a$ and $\sigma_a$, the depreciation rate of specific assets $\delta$, the standard deviation of the shocks to these assets $\sigma_e$, the initial split of the relationship surplus $\phi_0$, and the share of traded costs $\omega$. I estimate these parameters to match the cross-sectional distribution of relationships by age from Section 2.2, the relationship life cycle moments discussed in Section 2.4, and the probability of finding a new match after break-up from Table 6. Furthermore, I target the average pass-through level from Section 2.3. In total, I target 16 moments.

First, I choose eight data points from the life cycle of values traded that reflect the overall shape of the relationship life cycle. I target the value traded in the second year of three-year relationships, in the second and third year of four- and five-year relationships, and in the second, third, and fourth year of six-year relationships. Value traded in the model corresponds to $T_s = p_s q_s$. Second, I seek to match the average price decline during the first four quarters. Since an average relationship trades twice per month, I target a price decline in the first year of about 1% based on the results in Table 3. Third, I choose three data points from the quarterly hazard rate of relationship destruction. I target the break-up probability in the first and second quarter to match the high initial rate of destruction, and after 16 quarters to capture the break-up probability of long-term relationships. Fourth, I target the cross-sectional share of value traded by relationships that are one and two quarters old to match the share of young relationships in the data, and the share of relationships that are more than 16 quarters old to match the share of old relationships. Finally, based on the pass-through regressions I target an average pass-through level of 0.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand parameter ($b$)</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Elasticity of substitution ($\rho$)</td>
<td>4</td>
<td>Mean markup in Berry, Levinsohn, and Pakes (1995)</td>
</tr>
<tr>
<td>Curvature of production function ($\alpha$)</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Discount factor ($\beta$)</td>
<td>0.988</td>
<td></td>
</tr>
<tr>
<td>Buyer matching probability ($\pi_j$)</td>
<td>0.28</td>
<td>Based on Table 6</td>
</tr>
<tr>
<td>Seller matching probability ($\pi_k$)</td>
<td>0.33</td>
<td>Time gap between subsequent new buyers</td>
</tr>
<tr>
<td>Standard dev of exchange rate shocks ($\sigma_W$)</td>
<td>0.068</td>
<td>Std. of quarterly exchange rate changes</td>
</tr>
<tr>
<td>Mean of costs ($\bar{w}_x$)</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Mean of initial distribution ($\mu_a$)</td>
<td>$-0.35$</td>
<td></td>
</tr>
<tr>
<td>Standard dev of initial distribution ($\sigma_a$)</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Depreciation rate ($\delta$)</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Standard dev of shocks ($\sigma_e$)</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Buyer initial bargaining share ($\phi_0$)</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Share of traded costs ($\omega$)</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Simulation Parameters
Identification

I now describe how I use these data moments to identify the parameters. I choose the parameters of the initial asset distribution \((\mu_a, \sigma_a)\) to match the initial probability of break-ups, the increase in value traded in the second year compared to the first year, and the price decline in the first year. The parameter \(\mu_a\) is the main determinant of the probability of break-up in the first quarter of the match. If \(\mu_a\) is low, many relationships have low initial profits, which increases their initial separation probability. The parameter \(\sigma_a\) affects the value traded and the price decline relative to the first transaction. If the standard deviation of initial assets is high, relationships that survive after the initial value of \(a\) is revealed on average received larger positive shocks. They therefore exhibit a higher increase in value traded and steeper price declines compared to the first transaction.

The depreciation rate of assets \(\delta\) and the standard deviation of the shocks to assets \(\sigma_\epsilon\) are identified from the entire life cycle profile of values, the cross-sectional age distribution, and the hazard rate of break-ups. A higher value of \(\delta\) implies a flatter profile of values traded, since any positive shock has a smaller impact. Higher depreciation also means that the cross-sectional distribution is more concentrated at younger ages, and the break-up hazard is higher. A larger \(\sigma_\epsilon\) steepens the quantity and price profile, since those relationships that survive on average received larger positive shocks. On the other hand, it also makes the break-up hazard flatter, since even relationships that start out with a high level of initial assets may be destroyed quickly. The value of \(\sigma_\epsilon\) is identified because it affects all years of the relationship life cycle, while the initial distribution parameter \(\sigma_a\) affects only the first year.

The initial split of the surplus \(\phi_0\) is chosen to match the price decline in the first year. A higher initial bargaining weight for the buyer leads to a lower price at the first transaction, which in turn results in on average smaller price declines or even price increases during the first year of the relationship. The level of \(\phi_0\) also affects the probability of separation in the first period, since a higher buyer share raises the buyer’s outside option of a new relationship while decreasing the seller’s. Finally, the parameter \(\omega\) is chosen to generate the correct level of pass-through.

Let the true values of the parameters in the data be \(\Theta\), and denote the estimated parameters by \(\hat{\Theta}\). Denote the vector of data moments and model moments by \(G(\Theta)\) and \(G(\hat{\Theta})\), respectively. For a given \(\hat{\Theta}\), I simulate the model with 10,000 firms for 1,000 periods, and discard the first 200 periods as burn-in.\(^{45}\) I then solve

\[
J = \min_{\hat{\Theta}} E \left[ (G(\Theta) - G(\hat{\Theta}))' (G(\Theta) - G(\hat{\Theta})) \right].
\]

The bottom part of Table 10 shows the estimated parameter values in the baseline setup. The average level of initial assets is about 0.71, with a standard deviation of about 0.05. Thus, initial assets are relatively concentrated.

\(^{45}\)The estimation of the limited commitment model is done similarly to e.g. Attanasio and Ríos-Rull (2000) or Morten (2013).
4.3 Quantitative Performance

The baseline model generates an increase in pass-through with relationship age that matches the data. The first and second column of Table 11 compare the data moments to the moments generated by the limited commitment model with seller risk aversion (LC1). The fourth-last row of the table provides the value of the objective function $J$, where a value closer to zero indicates a better model fit. The limited commitment model matches the life cycle moments very well. To illustrate this more clearly, Figures F.13-F.15 graphically compare the life cycle of values, the break-up hazard, and the cross-sectional distribution between the model and the data. The life cycle is clearly visible, and the break-up hazard and cross-sectional distribution are well-matched. The only shortcoming of the model is that the life cycle is compressed. The setup overstates the increase in value traded for short relationships, and understates the increase for long relationships. For example, in the data the value traded in 4-year relationships increases by 0.9% between years one and three, and by 20.7% for 6-year relationships. In the model, the corresponding values are 9.3% and 12.7%, respectively. The reason the model has trouble matching this fact is that it generates a higher dispersion across cohorts only via a larger variance of asset shocks $\sigma_\epsilon$. However, a higher shock variance also increases the fraction of relationships that get destroyed early, raising the fraction of young relationships and flattening the
break-up hazard, which are matched relatively well. One possibility to generate a more dispersed profile would be to introduce heterogeneity in $\delta$ or $\sigma_\epsilon$ for different levels of specific assets, as this would weaken the link between break-ups and the dispersion of the life-cycle.

In Figure 6 I plot pass-through as a function of relationship-specific assets, for $w_x = .23$. This figure highlights the main mechanism of the model: for low levels of initial assets pass-through is severely diminished, since in this region the participation constraints of the two agents bind frequently. As assets are accumulated, the relationship moves out of the constrained region and pass-through increases. The blue dashed line depicts the distribution of initial assets. It shows that many relationships start out in the initially constrained region, where pass-through is below the average level of 0.2. The average pass-through level in the model is recorded in the fifth-last row of Table 11, and matches the data by design.

The solid black line in Figure 7a presents pass-through in the cross-section of firms in my model economy, for different relationship age buckets. Pass-through is clearly higher for older relationships. This result arises because young relationships have a relatively low level of specific assets, set by the initial asset distribution, and are therefore more frequently constrained. Surviving relationships accumulate assets, thus increasing pass-through. While some older relationships are also near termination, other relationships have significantly increased their amount of assets, more than offsetting the effect from relationships near termination and raising the average responsiveness of prices relative to new relationships. The model fits the empirical slope of pass-through in the cross-section very well (red line in the figure, with 95% confidence intervals). In both the model and the data pass-through increases strongly during the first years and is relatively flat after that, matching the model intuition that pass-through is the same once the constraints are no longer binding. Pass-through in the empirical
cross-section is 43% higher in relationships that have lasted five years compared to a new relationship. My model predicts an increase in pass-through by 45% (reported in the last row of Table 11). Matching this fact is a significant success of the model, as it depends on non-trivial objects such as the value of Lagrange multipliers and on how important young relationships are relative to old ones in the cross-section. Furthermore, this moment was not part of the estimation.

The baseline model requires an assumption about local costs to match the correct level of pass-through. To examine whether risk sharing on its own can generate the correct level of pass-through, I consider a model with only buyer risk aversion, which based on my analysis above generates the lowest level of pass-through. The third column of Table 11 presents the estimated moments in model (LC2), where I have again assumed log utility. As before, the life cycle moments are well-matched. Furthermore, the average level of pass-through is now about 0.23, which is near the level observed in the data. Pass-through cannot be further reduced because assuming that only the buyer is risk averse is the limiting case of the model. The main problem with this setup is that it implies a negative correlation between pass-through and relationship age for young relationships (Figure 7b). The solid black line in the figure shows that pass-through in the first year of an average relationship is about 0.28, and then drops sharply to about 0.18 in the second year. As discussed in Section 3.3, this result arises because in the model with buyer risk aversion binding constraints amplify the response of prices to shocks if the outside option is relatively stable. This pattern of pass-through is at odds with the data. This setup therefore cannot explain why pass-through increases with relationship age.

The third model I estimate is the Nash bargaining model described in Appendix E, modified slightly by imposing the same outside options as in the limited commitment model rather than free entry. This modification can lower pass-through below one. I do not impose the local costs assumption. The fourth column of Table 11 shows that this model also matches the relationship life cycle. However,
while value traded, the break-up hazard, and cross-sectional distribution are well-matched, the model
does not generate the right magnitude of the price decline. The reason for this outcome is that in the
limited commitment models the parameter \( \phi_0 \) governs the price only in the first period, with prices
being set based on the risk sharing contract after that. The parameter can therefore be chosen freely to
generate a price decline of the right size. In the Nash bargaining model on the other hand, the shares
of the buyer and the seller are fixed throughout the life of the relationship. Reducing the buyer’s share
in the first period thus affects all prices and does not generate a significant change in the price decline
in the first period.

The Nash bargaining model is also unable to match any of the pass-through facts. As expected,
the level of pass-through is close to one.\(^{46}\) Furthermore, the slope of pass-through with age is virtually
zero, since the split of the surplus is always constant (red dashed line in Figure 7b). Overall, the
estimation shows that if one only seeks to match the relationship life cycle, a simple Nash bargaining
model does just as well as the limited commitment model. However, if one also desires to match the
pass-through facts, the limited commitment model with seller risk aversion is preferable.

4.4 Aggregate Implications

I use the calibrated model to show that changes in the average length of relationships can affect
aggregate pass-through. In recent work, Berger and Vavra (2015) document using import price data
from the Bureau of Labor Statistics that pass-through during recessions is significantly higher than in
expansions. In particular, they find that pass-through doubled in 2008-09, from about 0.2 to 0.4. They
attribute this result to time variation in the elasticity of demand. My framework generates increasing
pass-through through a selection mechanism. The number of U.S. import relationships of age less
than one year fell by about 20% during this period, compared to a significantly smaller drop in the
number of older relationships (Figure 8). Using the model, I can estimate the effect of the associated
increase in the average age of relationships on pass-through. Since it becomes harder for sellers to
find a relationship partner during a downturn, I worsen the seller’s outside option and assume that
sellers never prefer to separate. I then calibrate a new termination bound such that I match the overall
decline in the number of relationships in 2008-09. This exercise removes those relationships with the
lowest level of assets, which are mostly young. The exercise provides a first pass to understand how
relationship length affects the responsiveness of prices. I assume that the downturn lasts for four
quarters.

Figure 9 shows that the recession shock in my model causes an increase in pass-through by about 4
percentage points, from about 0.2 to about 0.24. The change in the distribution of relationships on its
own can therefore explain about 20% of the overall effect documented in Berger and Vavra (2015). The

\(^{46}\)As discussed in Appendix E, if I use the outside options discussed above rather than free entry, pass-through may
be below one.
result highlights that shocks which alter the distribution of relationships in the economy may effect aggregate price flexibility.

5 Conclusion

This paper has presented evidence that long-term buyer-seller relationships affect aggregate price flexibility. Examining pass-through of exchange rate changes, I have shown that a relationship’s prices become more responsive to cost shocks as the relationship ages. This effect is large: a four-year relationship exhibits pass-through that is about 50% higher than a new relationship. To shed light on how relationships generate this result, I have documented a set of stylized facts about the dynamic evolution of relationships. I have shown that relationships follow a life cycle: they begin by trading little and initially have a high break-up probability. As the relationship ages, the trade volume rises and prices relative to the market fall. Trade starts to decline again at some point and the relationship eventually terminates. I have also documented that importers that are separated from a long-term relationship supplier for plausibly exogenous reasons experience a decline in quantity imported and reduced employment growth in the year after the separation, consistent with relationships being valuable. The break-up losses depend on the length of the relationship and are not present for new relationships.

Motivated by survey evidence and own interviews, I have developed a model of risk sharing in which relationships accumulate specific assets. This model can explain the increased responsiveness of prices to shocks in older relationships. Older relationships on average have a higher level of assets, which makes the relationship more valuable and enables better profit risk sharing via prices that are more responsive to shocks. I have shown that my model matches a number of additional facts in the
Figure 9: Pass-through response to recession shock

data, and is quantitatively able to generate the life cycle and the correct size of the correlation between pass-through and relationship age. The framework suggests a new mechanism to explain time variation in the responsiveness of prices to shocks, due to changes in the relationship length distribution. My findings suggest that when evaluating the effectiveness of monetary policy, policymakers should take into account the distribution of relationships.

This work forms the beginning of a broader research agenda aimed at understanding how long-term relationships affect the U.S. economy. In follow-up work, I investigate the cyclical properties of relationship formation and destruction (Heise (2015)). This work has already shown that in both the 2001 and the 2009 recessions, trade declined mainly due to a reduction in the formation of new relationships, while relationship destruction remained virtually constant. The project seeks to link the dynamic properties of relationships to the recent work on the network properties of the U.S. economy, for example Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Kelly, Lustig, and Van Nieuwerburgh (2013), and Di Giovanni, Levchenko, and Mejean (2014), which takes the network structure of the economy as given. I plan to also examine how relationship length and price flexibility are connected in the cross-section, and how the relative bargaining power of the buyer and the seller affects price setting.
References


Appendix

A Construction of the Dataset

This section describes in detail the preparation of the dataset. The first task is to ensure consistency of the importer identifiers. The alpha variable in the LFTTD identifies the U.S. importer at the firm level, and is analogous to the firm ID in other Census datasets, such as the LBD. However, in 2002, the Census Bureau changed the firm identification codes for single unit firms, making these identifiers inconsistent over time. For single unit firms, I therefore map the alphas in the LFTTD to the Census File Numbers (CFNs) in the LBD, and use these to obtain time-consistent firm identifiers from the LBD. For multi-unit firms, I retain the original identifiers. As a robustness check of these identifiers, I use the Employer Identification Numbers (EINs), which are also reported in the LFTTD, as an alternative identifier. These are tax IDs defined at the level of a tax unit. Consequently, a given firm may have several EINs, and an EIN may comprise several plants. Using this variable yields nearly identical results to my analyses using the firm ID variable. The main difference is that relationships based on the EIN are shorter.

The foreign manufacturer ID (or “exporter ID”) combines the name, the address, and the city of the foreign supplier.\textsuperscript{47} Monarch (2015) and Kamal, Krizan, and Monarch (2015) conduct a variety of robustness checks of this variable, and find that it is a reliable identifier of firms both over time and in the cross-section. Importantly, importers are explicitly warned by the U.S. CBP to ensure that the manufacturer ID reflects the true producer of the good, and is not an intermediary or processing firm. For the HS10 codes, I use the concordance by Pierce and Schott (2012) to ensure the consistency of the codes, since some of them change over time. With regard to the date, I use the date of the shipment from the foreign country as the date of the transaction, rather than the arrival date in the U.S.. The export date is the date at which the foreign supplier completed the transaction, and based on which the transaction terms should be set. I aggregate all transactions between the same partners in the same HS10 code on the same day into one by summing over the values and quantities of that day. Further aggregation is done on a monthly or quarterly basis when needed.

Several additional data cleaning operations are performed. First, I remove all transactions that do not include an importer ID, exporter ID, or HS code. I also remove all observations for which the recorded date is erroneous, and drop observations for which the exporter ID does not start with a letter (since it should start with the country name) or has fewer than three characters. I also remove observations which are missing a value or a quantity. Note that due to the cleaning operations and the removal of related party transactions, the aggregate value of trade based on my sample is significantly lower than the total value of trade recorded in official publications. In order to remove the general effect of inflation, I deflate the transaction values using the quarterly GDP deflator from FRED. I keep only imports used for consumption by dropping warehouse entries.

\textsuperscript{47}Specifically, it contains the ISO2 code for the country’s name, the first three letters of the producer’s city, six characters taken from the producer’s name and up to four numeric characters taken from its address. See Monarch (2014) for details.
B Correcting the Pass-Through Regressions for Selection

I re-write regression specification (2) as

$$\Delta \ln(p_{\text{mxcht}}) = z_{\text{mxcht}}^1 \beta + \gamma_{\text{mxh}} + \omega_t + \tilde{\epsilon}_{\text{mxcht}},$$

(41)

where $z_{\text{mxcht}}^1$ is a 1xK vector of regressors used in the pass-through regression and includes unity, $\beta$ is a 1xK vector of parameters, $\gamma_{\text{mxh}}$ accounts for relationship-product specific unobserved heterogeneity, $\omega_t$ captures unobserved time-varying effects, and $\tilde{\epsilon}_{\text{mxcht}}$ is an error term. I drop the explicit reference to the transaction number $i$ and use only the time index $t$ to denote the transaction. The selection equation is specified as

$$s_{\text{mxcht}} = 1 \left[ z_{\text{mxcht}} \delta + \xi_{\text{mxh}} + \varrho_t + \tilde{a}_{\text{mxcht}} > 0 \right],$$

(42)

where $s_{\text{mxcht}}$ is a selection indicator, $z_{\text{mxcht}} = [z_{\text{mxcht}}^1, z_{\text{mxcht}}^2]$ is a vector of regressors, $\xi_{\text{mxh}}$ is relationship-product specific unobserved heterogeneity, $\varrho_t$ is time-dependent unobserved heterogeneity, and $\tilde{a}_{\text{mxcht}}$ is a normally distributed error term.

If firms choose not to trade for unobservable reasons, then $E[\tilde{\epsilon}_{\text{mxcht}}|z_{\text{mxcht}}, \gamma_{\text{mxh}}, \omega_t, s_{\text{mxcht}} = 1] \neq 0$, and the standard fixed effects estimator produces inconsistent estimates. While differencing equation (41) could remove the triplet-fixed effect and eliminate the selection problem, this approach only works if $E[\Delta \tilde{\epsilon}_{\text{mxcht}}|z_{\text{mxcht}}^1, z_{\text{mxcht}}^1, \omega_t, \omega_t, s_{\text{mxcht}} = s_{\text{mxcht}} - 1 = 1] = 0$.

This equation does not hold if for example selection is time-varying. In such cases, the estimation needs to take the selection process into account. A standard approach in the literature to estimate a selection model in panel data is based on Wooldridge (1995). This approach parametrizes the conditional expectations of the unobservables via a linear combination of observed covariates.

To simplify, I assume that the time-varying unobservables depend linearly on U.S. GDP according to

$$\omega_t = GDP_t \varphi_1 + e_1$$

(43)

and

$$\varrho_t = GDP_t \varphi_2 + e_2.$$  

(44)

I define $\epsilon_{\text{mxcht}} = \tilde{\epsilon}_{\text{mxcht}} + e_1$ and $a_{\text{mxcht}} = \tilde{a}_{\text{mxcht}} + e_2$. Then, the problem can be written as

$$\Delta \ln(p_{\text{mxcht}}) = z_{\text{mxcht}}^1 \beta + GDP_t \varphi_1 + \gamma_{\text{mxh}} + \epsilon_{\text{mxcht}},$$

(45)

with

$$s_{\text{mxcht}} = 1 [z_{\text{mxcht}} \delta + GDP_t \varphi_2 + \xi_{\text{mxh}} + a_{\text{mxcht}} > 0].$$

(46)

I now apply the approach of Wooldridge (1995) to my problem. The method is based on four main assumptions. I follow the discussion in Dustmann and Rochina-Barrachina (2007), and let bold letters indicate vectors or matrices that include all periods.

**Assumption 1.** The conditional expectation of $\xi_{\text{mxh}}$ given $(z_{\text{mxch1}}, ..., z_{\text{mxcht}})$ is linear.
Based on this assumption, the selection equation (46) can be written as
\[ s_{mxcht} = 1 \left[ \psi_0 + z_{mxch1}\psi_1 + ... + z_{mxchT}\psi_T + GDP_t\phi_2 + v_{mxcht} > 0 \right], \] (47)
where \( v_{mxcht} \) is a random variable. Thus, selection is assumed to depend linearly on all leads and lags of the explanatory variables.

**Assumption 2.** The error term \( v_{mxcht} \) is independent of the entire matrix of observables \([z_{mxch} \quad GDP]\) and is distributed \( v_{mxcht} \sim N(0,1) \).

**Assumption 3.** The conditional expectation of \( \gamma_{mxh} \) given \( z_{mxch} \) and \( v_{mxcht} \) is linear.
Under this assumption,
\[ E[\gamma_{mxh}|z_{mxch}, v_{mxcht}] = \pi_0 + z_{mxch1}\pi_1 + ... + z_{mxch1}\pi_T + \phi_t v_{mxcht}. \] (48)
While the Wooldridge approach allows \( \phi_t \) to be time-varying, I make the assumption that it is constant.

**Assumption 4.** The error term in the main equation satisfies
\[ E[\epsilon_{mxcht}|z_{mxch}, GDP, v_{mxcht}] = E[\epsilon_{mxcht}|v_{mxcht}] = \rho v_{mxcht}. \] (49)
I additionally apply the simplification by Mundlak (1978) and assume that \( \gamma_{mxh} \) and \( \xi_{mxh} \) depend only on the time averages of the observables \( \bar{z}_{mxch} \), rather than on the entire lead and lag structure. Dustmann and Rochina-Barrachina (2007) also use this assumption in their application. The assumption is necessary here since the dataset is extremely large, and therefore estimating the coefficients on all leads and lags is computationally infeasible. Under these assumptions, I can re-write the main equation as
\[ \Delta \ln(p_{mxcht}) = z_{mxcht}^1\beta + \bar{z}_{mxch}\pi + GDP_t\phi_1 + \mu\lambda[z_{mxcht}\rho + \bar{z}_{mxch}\eta + GDP_t\phi_2] + \epsilon_{mxcht}, \] (50)
where \( \lambda(\cdot) \) denotes the inverse Mill’s ratio. The selection equation is given by
\[ s_{mxcht} = 1[z_{mxcht}\rho + \bar{z}_{mxch}\eta + GDP_t\phi_2 + v_{mxcht} > 0]. \] (51)
While it would be desirable to estimate the equation on a fully squared dataset that records a missing observation in every quarter between 1995 and 2011 in which a relationship-product triplet does not trade, such a dataset would be considerably too large for estimation, in particular since many relationships trade only a few times. To operationalize the estimation, I therefore assume that new relationships are randomly formed. This assumption is supported by the high hazard rate of separation after the first transaction observed in the data. More strongly, I assume that there is no selection problem regarding the start of a relationship-product triplet, which allows me to exclude all missing trades before the start of a triplet from the selection problem. Furthermore, I retain missing trades after the last transaction of a relationship-product triplet for only four quarters, and interpret this as relationship partners “forgetting” their transaction partner for that product after that time. While these assumptions are obviously stylized, they allow me to reduce the dataset to a manageable size. Given these assumptions, for each triplet the time averages \( \bar{z}_{mxch} \) are only taken over the relevant period.
As in the main text, $z_{mxcht}$ contains the cumulative exchange rate change $\Delta \ln(e_{ct})$, the length of the relationship in months $\text{Months}_{mxt}$, and the interaction of the two. I add several variables that should predict selection. I include the level of the exchange rate, the log real value traded at the last transaction, the time gap in quarters since the last transaction of the triplet, and the average time gap since the last transaction across all U.S. importers. A higher value traded at the last transaction should diminish the probability to transact again, while this probability should increase with the time gap since the last transaction. On the other hand, a larger average time gap across all exporters implies that this is a product that is less frequently traded, which should reduce the probability of trade in a given quarter. My exclusion restriction is that the average time gap at the product level is unrelated to pass-through, and therefore does not enter the main equation (50). Thus, $z_{mxcht}^1$ includes all regressors except this variable. If I impose the strong assumption that $\epsilon_{mxcht}$ is normally distributed, I can estimate the system via Maximum Likelihood in the same way as a Heckman selection model.

C Product and Country Categories

Table 12: List of product categories

<table>
<thead>
<tr>
<th>Product category</th>
<th>HS 2 code</th>
<th>Product category</th>
<th>HS 2 code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animal products</td>
<td>01 - 05</td>
<td>Textiles</td>
<td>50 - 63</td>
</tr>
<tr>
<td>Vegetables</td>
<td>06 - 14</td>
<td>Footwear</td>
<td>64 - 67</td>
</tr>
<tr>
<td>Fats</td>
<td>15</td>
<td>Stones and ceramics</td>
<td>68 - 70</td>
</tr>
<tr>
<td>Food</td>
<td>16 - 24</td>
<td>Jewelry</td>
<td>71</td>
</tr>
<tr>
<td>Mineral products</td>
<td>25 - 27</td>
<td>Metals and metal products</td>
<td>72 - 83</td>
</tr>
<tr>
<td>Chemicals</td>
<td>28 - 38</td>
<td>Machinery</td>
<td>84 - 85</td>
</tr>
<tr>
<td>Plastics</td>
<td>39 - 40</td>
<td>Transportation</td>
<td>86 - 89</td>
</tr>
<tr>
<td>Leather products</td>
<td>41 - 43</td>
<td>Optical products</td>
<td>90 - 92</td>
</tr>
<tr>
<td>Wood products</td>
<td>44 - 49</td>
<td>Arms</td>
<td>93</td>
</tr>
</tbody>
</table>

Table 13: List of countries

<table>
<thead>
<tr>
<th>Australia</th>
<th>Czech Republic</th>
<th>India</th>
<th>Mexico</th>
<th>South Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>Denmark</td>
<td>Indonesia</td>
<td>Netherlands</td>
<td>South Korea</td>
</tr>
<tr>
<td>Belgium</td>
<td>Estonia</td>
<td>Ireland</td>
<td>New Zealand</td>
<td>Spain</td>
</tr>
<tr>
<td>Brazil</td>
<td>Finland</td>
<td>Israel</td>
<td>Norway</td>
<td>Sweden</td>
</tr>
<tr>
<td>Canada</td>
<td>France</td>
<td>Italy</td>
<td>Poland</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Chile</td>
<td>Germany</td>
<td>Japan</td>
<td>Portugal</td>
<td>Taiwan</td>
</tr>
<tr>
<td>China</td>
<td>Greece</td>
<td>Latvia</td>
<td>Russia</td>
<td>Thailand</td>
</tr>
<tr>
<td>Colombia</td>
<td>Hungary</td>
<td>Lithuania</td>
<td>Slovak Republic</td>
<td>Turkey</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>Iceland</td>
<td>Luxemburg</td>
<td>Slovenia</td>
<td>United Kingdom</td>
</tr>
</tbody>
</table>
D Proofs

D.1 Properties of the contracting problem

This proof follows closely Thomas and Worrall (1988). I show: (i) The set of self-enforcing contracts is convex, (ii) for every state \( s \) in which a self-enforcing contract exists, the values \( W_s \) that are associated with self-enforcable contracts lie in a compact interval \([W_\text{a}, W_\text{b}]\), (iii) The Pareto frontier \( J(s_0, W) \) is decreasing, strictly concave, and differentiable in \( W \) on \((W, \bar{W})\), (iv) For each value of \( W^t \in [\bar{W}, \bar{W}] \) there exists a unique continuation value of the contract \( \Xi \) at time \( t \) in which \( W(\Xi; (h_{t-1}, s)) = W^t_s \) and \( J(\Xi; (h_{t-1}, s)) = J(s, W^t_s) \).

(i) Consider any two self-enforcing contracts \( \Xi \) and \( \Xi' \). Define the convex combination of the two contracts as \( \Xi^0 = (q^0(h_t), T^0(h_t)) \in [0, \tilde{q}] \), where \( q^0(h_t) = \partial q(h_t) + (1 - \vartheta)q'(h_t) \) and \( T^0(h_t) = \partial T(h_t) + (1 - \vartheta)T'(h_t) \) for all \( h_t \) and any \( \vartheta \in (0, 1) \). By the concavity of \( R(\cdot) \), \( J(\Xi^0; h_t) \geq \partial J(\Xi; h_t) + (1 - \vartheta)J(\Xi'; h_t) \geq V \) for all \( h_t \). Likewise, by concavity of \( u \), \( W(\Xi^0; h_t) \geq U(w_t) \) for all \( h_t \). Thus \( \Xi^0 \) is self-enforcing, and hence the set of self-enforcing contracts is convex.

(ii) I first show that the set of \( W_s \) associated with self-enforcing contracts is bounded. From equation (12), a high value of \( W_s \) is associated with high transfer payments \( P_s \) and low quantities \( m_s \), which by equation (11) reduces the buyer’s value. Therefore, for \( W_s \) sufficiently high, the buyer’s participation constraint must eventually bind. Similarly, for \( W_s \) sufficiently low, the seller’s participation constraint must eventually bind. This defines two bounds \( W_{a} \) and \( \bar{W}_{b} \) associated with self-enforcing contract values. To show that the interval \( I_s \) between these values is closed, consider a sequence \( W^s_{\nu} \in I_s \) such that \( \lim_{\nu \to \infty} W^s_{\nu} = W_s \). Take a sequence of contracts \( \Xi^\nu \) associated with these values. Since quantities are by assumption contained in the interval \([0, \bar{q}]\) and the buyer’s outside option is finite, transfers must be contained in an interval \([0, \bar{T}]\). If payments were higher than \( \bar{T} \), the buyer’s outside option would be violated even if she made zero payments in all future states and received \( \bar{q} \). The set of quantities and payments is therefore compact in the product topology state-by-state. Then, there exists a sub-sequence of contracts that converges state-by-state to a limiting contract, \( \Xi^\infty \). Since \( u \) and \( R \) are continuous, the sequences \( R(q^\nu) \) and \( u(T^\nu - \frac{w}{a} q^\nu) \) converge pointwise as well. Then, using (11) and (12), since \( \beta < 1 \), by the Dominated Convergence Theorem the expectation terms also converge. Therefore, after any history the limit of the value of the seller equals the value of the limiting contract. The same holds for the buyer. Since each \( \Xi^\nu \) is self-enforcing, \( \Xi^\infty \) is as well, and it gives the seller a gain of \( W_s \), the limit of the sequence of net gains. Hence, \( I_s \) is closed.

(iii) The function \( J(s_0, W) \) is decreasing in \( W \) because a higher promised utility to the seller is associated with a higher payment to the seller for a given quantity, or a lower quantity for a given transfer. This reduces the buyer’s value from (11). Strict concavity of \( J(s_0, W) \) follows since \( R \) and \( u \) are strictly concave and the set of self-enforcing contracts is convex. If \( W \) is increased, either \( T_s \) must rise or \( q_s \) must fall. By concavity either of these operations reduces \( J(s_0, W) \) by more than in a linear case. Differentiability follows by the same argument as in Thomas and Worrall (1988).

(iv) Existence follows from the fact that \( I_s \) is compact. Since the set of self-enforcing contracts is convex and \( u \) is strictly concave, the continuation values must be unique.
D.2 Proof of Proposition 1

From equation (21), \( \Pi_s = L(J_W(s, W_s)) \), where \( L \) is a continuous and strictly decreasing function. It is decreasing because when \( W_s \) rises, \( J_W(s, W_s) \) falls, by strict concavity, and profits and future promises co-vary positively, as shown in the main text. Replacing the \( J_W(s, W_s) \) in equation (22) by \( L^{-1}(\Pi_s) \) yields:

\[
L^{-1}(\Pi_s) = (1 + \nu)L^{-1}(\Pi_s) + \mu_s
\]  

(52)

There are four possible cases:

(i) If \( \Pi_{s0} > \Pi_s \geq \Pi_{s2} \), then since the new profits \( \Pi_s \) under a self-enforcing contract needs to lie in \([\Pi_s, \Pi_{s2}]\), it follows that \( \Pi_s < \Pi_{s0} \). Since \( L \) is decreasing, \( L^{-1}(\Pi_{s0}) < L^{-1}(\Pi_s) \). From equation (21), \( L^{-1}(\Pi_{s0}) = J_W(s, W_s) \) is negative, and therefore it must be the case that \( \nu_s > 0 \) for equation (52) to hold. Thus, the firm’s self-enforcement constraint is binding, and therefore \( \Pi_s = \bar{\Pi}_s \).

(ii) If \( \bar{\Pi}_s \geq \Pi_s \geq \Pi_{s0} \), then \( \nu_s = \mu_s = 0 \): if we had \( \nu_s > 0 \), then by definition from equation (15) the seller’s profits must be at the top of the interval in the following period, \( \Pi_s = \bar{\Pi}_s \). Furthermore, it must be the case that \( \mu_s = 0 \), since both constraints cannot bind simultaneously, or else the contract would not be self-enforcing in this state. Since \( \bar{\Pi}_s \geq \Pi_{s0} \), if \( \nu_s > 0 \) then \( L^{-1}(\Pi_{s0}) \geq L^{-1}(\bar{\Pi}_s) > (1 + \nu_s)L^{-1}(\Pi_s) \). This contradicts equation (52), since \( \mu_s = 0 \) and so this cannot hold with equality. Similarly, if \( \mu_s > 0 \), then \( \nu_s = 0 \) and \( \Pi_s = \Pi_{s0} \). Then \( L^{-1}(\Pi_{s0}) \leq L^{-1}(\Pi_{s0}) < L^{-1}(\bar{\Pi}_s) + \mu_s \), which again contradicts equation (52). Hence, \( \nu_s = \mu_s = 0 \), and from (52), \( \Pi_s = \Pi_{s0} \).

(iii) If \( \Pi_{s0} < \Pi_s \leq \bar{\Pi}_s \), then \( \mu_s > 0 \), using the same logic as in (i). Hence, \( \Pi_s = \bar{\Pi}_s \).

(iv) If \( \Pi_{s2} > \bar{\Pi}_s \), then the interval of profits that are consistent with a self-enforcing contract is empty. In this case, the relationship is terminated.

D.3 Proof of Proposition 2

Part 1: (i) Lower interval bound is decreasing in \( w_x \): Fix \( a \) and consider a level of costs \( w_0 \) such that the seller’s outside option exactly binds. Call this state \( s_0 = (a, w_0) \). From the seller’s participation constraint (16), it must be the case that \( u(\Pi_{s0}) + \beta W = U(w_0) \). Consider a shock to \( w_0 \) that changes costs to \( w_x < w_0 \), while assets remain fixed at \( a \). Call the new state \( s = (a, w_x) \). Since \( U'(w) < 0 \), the participation constraint is now violated, with \( u(\Pi_{s0}) + \beta W < U(w_x) \). Therefore, either the seller’s profit allocation, his promised value, or both must be increased. Since the seller’s constraint is binding, we have \( \mu_s > 0 \), while if the relationship is not terminated the buyer’s constraint cannot bind and therefore \( \nu_s = 0 \). From equation (22), it then follows that

\[
J_W(s, W_s) + \mu_s = J_W(s_0, W).
\]

From equation (21), this equation can be re-written in terms of utility as

\[
\frac{1}{w'(\Pi_s)} - \mu_s = \frac{1}{w'(\Pi_{s0})}.
\]
Therefore, $\Pi_s > \Pi_{s_0}$, and hence the lower bound of the profit intervals is decreasing in $w_x$. This result holds for any Markov process of shocks.

(ii) Lower interval bound is decreasing in $a$: Fix $w_0$ and take a level of assets $a$ such that the seller exactly receives his outside option. Call this initial state $s_0 = (w_0, a)$. Thus, $u(\Pi_{s_0}) + \beta W = U(w_0)$. Consider a change of $a$ to $a' < a$. Call the new state $s = (w_0, a')$, and note that the value of the outside option is unchanged since $U(w_0)$ does not depend on $a$. By equation (21), the adjustment of $\Pi_{s_0}$ depends on the behavior of $J_W(s_0, W)$ in response to the shock. By the envelope condition, $J_W(s_0, W) = -\lambda_0$. If assets were i.i.d., then the Lagrange multiplier would not depend on the current value of $a$, since the promise-keeping constraint is only required to hold in expectation. In that case, given the fixed outside option value, $\Pi_{s_0} = \Pi_s$ and $W = W_s$. However, with a persistent asset process the slope of $J$ varies with current assets. Consider two relationships with sellers that have been promised the same value $W$, distinguished only by different current asset levels given by $a$ in relationship one and $a' < a$ in relationship two. Assume that the buyers would like to provide an additional $\epsilon$ of value to the sellers by raising $W$. Since assets are persistent and a lower level of assets reduces the joint surplus that can be split between the agents, the buyer in relationship two is more likely to be constrained in the future. By definition, those states in which the buyer is constrained cannot be used to provide additional utility to the seller. Consequently, the buyer in relationship two has to provide more utility to the seller in the remaining states, to counterbalance the effect that he will more likely be constrained. Given concave utility, this is costly. Therefore, in the original problem, lower assets in state $s$ imply $J_W(s, W) < J_W(s_0, W)$, i.e., the slope of the buyer’s value function at $W$ is steeper (more negative) when assets are lower. From equation (21), this implies at $s = (w_0, a')$:

$$u'(\Pi_{s_0}) > -\frac{1}{J_W(s, W)}.$$  

Since the outside option of the seller is unchanged and therefore the new profits and promises must satisfy $u(\Pi_s) + \beta W_s = U(w_0)$, this implies that the seller is now constrained, and $\Pi_s > \Pi_{s_0}$ and $W_s < W$.

Part 2. (i) Upper interval bound is decreasing in $w_x$: Similar to before, consider state $s_0 = (a, w_0)$ and let the buyer’s outside option exactly bind. Define $\Upsilon_{s_0} = R(q_0) - \frac{w_0}{a}q_0$ as the static surplus of the match. From the buyer’s participation constraint (15), $\Upsilon_{s_0} - \Pi_{s_0} + \beta J(s_0, W) = V$. Consider a shock to $w_0$ that changes costs to $w_x > w_0$ while assets remain fixed at $a$. Call the new state $s = (a, w_x)$. The total surplus that can be split between the buyer and the seller is decreasing in $w_x$, by concavity of $R(q)$ and the optimal choice of $q$ from $R'(q) = \frac{w}{a}$. Therefore, $\Upsilon_s < \Upsilon_{s_0}$. Furthermore, since costs are persistent, $J(s, W) < J(s_0, W)$. As a consequence, the buyer’s participation constraint is now violated, $\Upsilon_s - \Pi_{s_0} + \beta J(s, W) < V$. Using the same steps as in the derivation of the lower bound,

$$\left(1 + \nu_x\right)\frac{1}{u'(\Pi_s)} = \frac{1}{u'(\Pi_{s_0})}.$$  

Therefore, $\Pi_s < \Pi_{s_0}$, and the upper bound of the profit intervals is decreasing in $w_x$.

(ii) Upper interval bound is increasing in $a$: The proof proceeds in the same way as in 2(i). Fix $w_x$ and consider two states $s_0 = (a, w_x)$ and $s = (a', w_x)$ with $a' < a$. Assume that in $s_0$ the buyer’s outside option exactly binds so that $\Upsilon_{s_0} - \Pi_{s_0} + \beta J(s_0, W_{s_0}) = V$. A shock that decreases assets has the same effect as an increase in $w_x$, and therefore by the same arguments as before, $\Upsilon_s - \Pi_{s_0} + \beta J(s, W) < V$, where $\Upsilon_s$ are the joint profits under $a'$. Hence, the buyer’s outside option binds, and as before $\Pi_s < \Pi_{s_0}$.
D.4 Proof of Lemma 1

Part 1: When $F(m)$ is isoelastic, from equation (20) quantity purchased satisfies

$$q_s = (kr)^{1-r} \left( \frac{a}{w_x} \right)^{\frac{1-r}{r}}.$$  

(53)

Therefore, total production costs are given by

$$C_s = \frac{w_x}{a} q_s = (kr)^{1-r} \left( \frac{a}{w_x} \right)^{\frac{1-r}{r}}.$$  

(54)

Since by concavity $0 < r < 1$, the expression is increasing in $a$ and decreasing in $w_x$.

Part 2: Combining the equations in (24) into one, transfers satisfy

$$T_s = (u')^{-1} \left( \frac{1 + v_s}{\lambda + \mu_s} \right) + (kr)^{1-r} \left( \frac{a}{w_x} \right)^{\frac{1-r}{r}}.$$  

(55)

This expression is clearly decreasing in $w_x$ and increasing in $a$ when neither constraint binds. By Proposition 2, conditional on assets the buyer’s constraint can only bind after a shock increasing costs, and so $\frac{\partial \nu_s}{\partial w_x} \geq 0$. Since $(u')^{-1} \cdot \cdot \cdot \cdot < 0$. Hence, in the case of a binding buyer constraint

$$\frac{\partial T_s}{\partial w_x} = \left. \frac{\partial (u')^{-1} \left( \frac{1 + v_s}{\lambda + \mu_s} \right)}{\partial v_s} \right|_{<0} \frac{\partial v_s}{\partial w_x} - \left. \frac{r}{1-r} (kr)^{\frac{1-r}{r}} a^{r/(1-r)} w_x^{1/(r-1)} \right|_{<0} < 0.$$  

A binding buyer constraint amplifies the effect of a shock that raises costs and transfers fall even more.

Similarly, the seller’s constraint can only bind after a reduction in costs, and thus $\frac{\partial \mu_s}{\partial w_x} \leq 0$. Furthermore, $\frac{\partial \mu_s}{\partial w_x} (u')^{-1} \cdot \cdot \cdot \cdot > 0$. Therefore, in the case of a binding seller’s constraint

$$\frac{\partial T_s}{\partial w_x} = \left. \frac{\partial (u')^{-1} \left( \frac{1}{\lambda + \mu_s} \right)}{\partial \mu_s} \right|_{<0} \frac{\partial \mu_s}{\partial w_x} - \left. \frac{r}{1-r} (kr)^{\frac{1-r}{r}} a^{r/(1-r)} w_x^{1/(r-1)} \right|_{<0} < 0.$$  

A binding seller constraint amplifies the effect of a shock that lowers costs and transfers rise even more.

Finally, by Proposition 2, an increase in assets expands the interval of profit levels associated with a self-enforcing contract and therefore the constraints can never bind in this case. Consequently, the derivative of the first part of (55) with respect to assets is zero in that case and payments increase when assets go up. On the other hand, when assets fall, either constraint could bind and hence the effect on transfers cannot be determined unambiguously.

D.5 Proof of Proposition 3

Part 1: Positive cost shocks: Fix the level of assets $a$, and consider a buyer facing costs $w_0$ such that his outside option holds with equality. Call this state $s_0 = (a, w_0)$. Then, $\Upsilon s_0 = \bar{\Pi}s_0 + \beta J(s_0, \bar{W}) = V$, where
\[ \Upsilon_{s_0} = R(q_0) - \frac{w_a}{a} q_0. \] Consider an arbitrary shock that raises costs to \( w_x > w_0 \). Call the new state \( s = (a, w_x) \). Since \( \Upsilon_s < \Upsilon_{s_0} \) and, by persistence of the shock process, \( J(s, W) < J(s_0, W) \), I have \( \Upsilon_s - \Pi_{s_0} + \beta J(s, W) < V \). Using equation (21), then \( \Pi_s < \Pi_{s_0} \) and \( \bar{W}_s < \bar{W}. \) The new seller profits and the promised value are defined by \( \Upsilon_s - \Pi_s + \beta J(s, \bar{W}_s) = V. \) The buyer’s profits in state \( s \) therefore satisfy \( \Upsilon_s - \Pi_s \leq \Upsilon_{s_0} - \Pi_{s_0} \) if \( J(s, \bar{W}_s) \geq J(s_0, W) \), and \( \Upsilon_s - \Pi_s > \Upsilon_{s_0} - \Pi_{s_0} \) if the opposite holds. In principle, both are possible. However, if the cost process is i.i.d., then \( J(s_0, W) = J(s, \bar{W}) \), and since \( \bar{W}_s < \bar{W} \) it follows that \( J(s, \bar{W}_s) > J(s_0, W) \). In that case, the buyer’s profits must fall.

I show first that if the drop in the buyer’s current profits between the two states is large enough, then the price must necessarily increase. Using the expression for quantities from Lemma 1, the buyer’s profits before the shock are

\[
\Upsilon_{s_0} - \Pi_{s_0} = k \frac{1}{1-r} \left[ \frac{r}{1-r} \left( \frac{a}{w_0} \right)^{1-r} - p_0 r \frac{1}{1-r} \left( \frac{a}{w_0} \right)^{1-r} \right].
\]

(56)

If the price is held fixed at its initial level \( p_0 \) in response to the cost shock, then the buyer’s profits change by

\[
\Delta \Pi^B = \Upsilon_s - \Pi_s - (\Upsilon_{s_0} - \Pi_{s_0}) = k \frac{1}{1-r} \left( \frac{a}{w_0} \right)^{1-r} \frac{r}{1-r} \left[ p_0 r \left( \frac{a}{w_0} \right) \left[ 1 - \left( \frac{w_0}{w_x} \right)^{1-r} \right] - \left[ 1 - \left( \frac{w_0}{w_x} \right)^{1-r} \right] \right].
\]

(57)

This expression is increasing in \( p_0 \), since by assumption \( w_x > w_0 \). Therefore, at fixed prices, the buyer’s profits increase the least in response to the shock if the initial price is at its lowest possible level, \( p_0 = 0 \). Plugging this price into equation (57) defines a profit change \( \Delta \Pi^B \) such that if under the optimal contract \( \Delta \Pi^B < M_0 \), the price paid by the buyer must increase to \( p_s > p_0 \), since a constant price level does not lower her profits sufficiently.

Consider next the case of \( \Delta \Pi^B = M(w_x - w_0) \), where \( M \) is a constant that satisfies \( M(w_x - w_0) \geq M_0 \). Assume that prices are held fixed at \( p_0 \) in response to the shock. From equation (56), I obtain

\[
\Delta \Pi^B \geq M(w_x - w_0) \quad \Leftrightarrow \quad p_0 \geq \frac{w_0}{a r} \left[ 1 - \left( \frac{w_x}{w_0} \right)^{r/(1-r)} \right] + M(w_x - w_0) \left( \frac{w_0}{k r a} \right)^{1-r} \left[ 1 - \left( \frac{w_0}{w_x} \right)^{1/(1-r)} \right]^{-1}.
\]

(58)

Define the right-hand side of the inequality as \( \tilde{p}(s_0, s) \). The inequality states that for profits to change by at least the required amount while keeping prices fixed, it must be the case that \( p_0 \geq \tilde{p}(s_0, s) \). If \( p_0 < \tilde{p}(s_0, s) \), then the profit change due to the quantity effect alone is too small and therefore price must fall to raise the buyer’s profits further. In this case, \( p_s < p_0 \). If the cost shock is small so that \( w_x \approx w_0 \), then applying L’Hôpital’s rule yields the simplified condition

\[
p_0 \geq \frac{w_0}{a} + (1-r) \left( \frac{w_0}{k r a} \right)^{1-r} w_0 M.
\]

If the buyer’s profits are required to fall, \( M < 0 \), then for small shocks \( p_x > p_0 \) holds for all \( p_0 \geq \frac{w_x}{a} \), and hence in all cases in which the seller makes non-negative profits. Since profits always decrease for i.i.d. shocks, \( p_x > p_0 \) always holds in that case.

The previous steps considered the case where the buyer is at her constraint in state \( s_0 \). To conclude the proof, consider the case where in state \( s_0 \) the buyer is unconstrained, with initial promise to the seller given by \( W < \bar{W} \), and seller profits \( \Pi_0 \). By equation (21), \( \Upsilon_{s_0} - \Pi_{s_0} > \Upsilon_{s_0} - \Pi_{s_0} \). Since quantity ordered is the same regardless of whether the buyer is constrained, it must be that the price paid under \( W \) is less than the
price paid under \( \bar{W} \). Let the price paid in the initially constrained case be given by \( \bar{p}_0 \), and in the initially unconstrained case by \( p_0 \). If the cost change to \( w_x > w_0 \) causes the buyer’s constrained to bind, then as before \( \Upsilon_s - \bar{\Pi}_s + \beta J(s, \bar{W}_s) = V \). The price paid in that state, \( p_s \), is independent of whether the constraint was binding previously or not. If \( p_s > \bar{p}_0 \) in the case where the buyer was initially constrained, then clearly also \( p_s > p_0 \). Hence, if a cost increase is accompanied by a price increase if the buyer was initially constrained, it also leads to a price increase if the buyer was initially unconstrained.

**Part 2:** Negative cost shocks: Fix a level of assets \( a \), and consider a seller with costs \( w_0 \) such that his outside option holds with equality. Call this state \( s_0 = (a, w_0) \). Then, \( \Pi_{s_0} + \beta W = U(w_0) \). Using the expression for quantities based on Lemma 1, the seller’s profits are given by

\[
\Pi(p_0, w_0) = (p_0 - \frac{w_0}{a}) (kr)^{1/(1-r)} \left( \frac{a}{w_0} \right)^{1/(1-r)}.
\]  

(59)

Consider a shock that reduces costs to \( w_x < w_0 \). Call this new state \( s = (a, w_x) \). By Proposition 2, \( \Pi_s > \Pi_{s_0} \). If the price is left constant at \( p_0 \), then the seller’s post-shock profits are given by \( \Pi(p_0, w_x) \), which satisfies \( \Pi(p_0, w_x) > \Pi(p_0, w_0) \) since the derivative of equation (59) with respect to \( w_0 \) is negative. Since under complete pass-through the seller’s profits are constant and the price is decreased, this implies that any level of seller profits in the interval \( [\Pi_{s_0}, \Pi(p_0, w_x)] \) can be implemented with a price change that goes in the same direction as the cost shock. From equation (21) and using the fact that the Pareto frontier \( J(s, W_s) \) is strictly concave in \( W_s \), I can define an implicit function that relates \( \Pi_s \) to \( W_s \):

\[
W_s = J_W^{-1}(s, -\frac{1}{u^0(\Pi_s)}) \equiv G_s(\Pi_s).
\]

Since the seller’s profits are adjusted to the shock such that she is exactly at the constraint, her new profit level must satisfy \( u(\Pi_s) + G_s(\Pi_s) = U(w_x) \). Therefore, prices change in the same direction as costs if

\[
U(w_x) - U(w_0) < u(\Pi(p_0, w_x)) + G_s(\Pi(p_0, w_x)) - u(\Pi_{s_0}) - G_s(\Pi_{s_0}) \equiv K(s_0, s).
\]

If \( U(w_x) - U(w_0) \geq K(s_0, s) \), then the seller’s post-shock profit level must at least be equal to \( \Pi(p_0, w_x) \) and therefore price cannot fall.

Consider now the case where the seller is initially unconstrained, with profits \( \Pi_{s_0} \) and promised value \( \bar{W} \) such that \( \Pi_{s_0} + \beta \bar{W} > U(w_0) \). Let the seller become constrained after the shock lowering \( w_0 \) to \( w_x \). Since \( m_0 \) depends only on \( w_0 \) and not on the promised value \( \bar{W} \), it must be the case that the initial price satisfies \( \bar{p}_0 > p_0 \) (using again the fact that promised values and profits are positively correlated by equation (21)). Therefore, if \( U(w_x) - U(w_0) < K(s_0, s) \) holds, it must also be true that \( p_s < \bar{p}_0 \), since \( p_s < p_0 < \bar{p}_0 \). Thus, a negative cost shock also reduces prices if the seller is initially unconstrained. In this case, the price falls even if \( U(w_x) - U(w_0) \) is slightly larger than \( K(s_0, s) \).
D.6 Proof of Lemma 2

If \( F(m) \) is isoelastic, the price is

\[
p_s = \frac{w_x}{a} + \left( \frac{w_x}{k r a} \right)^{\frac{1}{1-r}} (u')^{-1} \left[ \frac{1 + \nu_s}{\lambda + \mu_s} \right].
\]  

Taking a log-linear approximation around the current state, assuming \( a \) is fixed, yields:

\[
\hat{p}_s = \xi_1 \hat{w}_x + \xi_2 \hat{\nu}_s + \xi_3 \hat{\mu}_s,
\]  

where hats indicate log deviations from the current state, bars indicate the current state, and

\[
\xi_1 = \frac{\hat{w}_x}{\bar{a}} + \left( \frac{\bar{w}_x}{k r \bar{a}} \right)^{\frac{1}{1-r}} (u')^{-1} \left[ \frac{1 + \bar{\nu}_s}{\lambda + \bar{\mu}_s} \right],
\]

\[
\xi_2 = \frac{\left( \frac{\bar{w}_x}{k r \bar{a}} \right)^{\frac{1}{1-r}} \frac{1}{\lambda + \bar{\mu}_s} [\bar{u}'(\cdot)^{-1}]'}{\frac{\bar{w}_x}{\bar{a}} + \left( \frac{\bar{w}_x}{k r \bar{a}} \right)^{\frac{1}{1-r}} (u')^{-1} \left[ \frac{1 + \bar{\nu}_s}{\lambda + \bar{\mu}_s} \right]},
\]

and

\[
\xi_3 = -\frac{\left( \frac{\bar{w}_x}{k r \bar{a}} \right)^{\frac{1}{1-r}} \frac{1 + \bar{\nu}_s}{(\lambda + \bar{\mu}_s)^2} [\bar{u}'(\cdot)^{-1}]'}{\frac{\bar{w}_x}{\bar{a}} + \left( \frac{\bar{w}_x}{k r \bar{a}} \right)^{\frac{1}{1-r}} (u')^{-1} \left[ \frac{1 + \bar{\nu}_s}{\lambda + \bar{\mu}_s} \right]}.
\]

Since \( r < 1 \) and \((u')^{-1}(\cdot) > 0\) since \( u \) is an increasing function, it follows that \( \xi_1 > 1 \). Therefore, pass-through is greater than 1 when neither agent is constrained. Since \( u \) is increasing and concave, \((u')^{-1}(\cdot)\) is decreasing, and therefore \([\bar{u}'(\cdot)^{-1}]'(\cdot) < 0\). Consequently, \( \xi_2 < 0 \) and \( \xi_3 > 0 \).

To see the second part of the lemma, multiply both the numerator and the denominator of equation (62) by \( \bar{a} \), and re-write the expression as

\[
\hat{\xi}_1 = \frac{A + \left( \frac{1}{1-r} \right) B(\bar{a})}{A + B(\bar{a})},
\]

where \( A \) is a constant independent of \( \bar{a} \), and \( B(\bar{a}) \) is decreasing in \( \bar{a} \). Taking the derivative with respect to \( \bar{a} \) yields:

\[
\frac{\partial \hat{\xi}_1}{\partial \bar{a}} = \frac{\left( \frac{r}{1-r} \right) B'(\bar{a}) A}{[A + B(\bar{a})]^2} < 0.
\]

Therefore, \( \hat{\xi}_1 \) is declining in \( \bar{a} \). By a similar argument, pass-through is declining in \( k \). When \( r \to 0 \), equation (62) becomes exactly equal to one.
D.7 Proof of Lemma 3

From equations (11) and (12), the total value of the relationship to the buyer and the seller at history $h_t$ is given by

$$S(\Xi; h_t) = R(q(h_t)) - T(h_t) + u(T(h_t) - \frac{w_t}{a_t} q(h_t)) + \sum_{o=t+1}^{\infty} \beta^{o-t} \left\{ R(q(h_o)) - T(h_o) + u(T(h_o) - \frac{w_o}{a_o} q(h_o)) \right\}. \quad (63)$$

Consider two histories of states $h_t$ and $h_t'$ under the same contract $\Xi$, where the only difference between the two histories is that given assets $a_{t-1}$, the asset shock in period $t$ led to different asset levels in that period satisfying $a_t' < a_t''$. By concavity of $R(q(h_t))$ and since at the optimum $R'(q(h_t)) = \frac{w_t}{q_t}$, the total surplus to be split between the two partners in period $t$, $\Upsilon(a_t) = R(q(h_t)) - \frac{w_t}{a_t} q(h_t)$, satisfies $\Upsilon(a_t') < \Upsilon(a_t'')$. Since $a_t$ is persistent, it must be that $S(\Xi; h_t(a_t')) < S(\Xi; h_t(a_t''))$, where $h_t(a_t')$ denotes the history under $a_t'$, and similarly for $h_t(a_t'')$. Since the outside options are unchanged between the two cases, $S(\Xi; h_t(a_t')) - V - U(w_t) < S(\Xi; h_t(a_t'')) - V - U(w_t)$. By continuity of the surplus function in assets, there exists a value $a^*(w_t)$ such that $S(\Xi; h_t(a^*(w_t))) - V - U(w_t) = 0$. If $a_t < a^*(w_t)$, both outside options must bind, and therefore the relationship must be terminated. If $a_t \geq a^*(w_t)$, there exists a sequence of $(q(h_o), T(h_o))_{o=t}^{\infty}$ such that the agents’ outside options are not violated. Since this makes both agents at least as well off as termination, the relationship is not ended and separation is efficient.

For the second part of the proof assume the seller is promised his outside option value, $W = U(w_0)$. Following similar arguments as before, for costs $w_0$ the threshold asset level is defined by $a^*(w_0)$ such that under these assets $J((a^*(w_0), w_0), U(w_0)) = V$. Consider an increase in costs to $w_x > w_0$. As before, higher costs lower the period surplus and the overall value of the relationship. They also lower the seller’s outside option, $U(w_x) < U(w_0)$. Then, if $J((a^*(w_0), w_x), U(w_x)) < V$, $a^*(w_x) > a^*(w_0)$, since the buyer’s value is increasing in assets.

D.8 Proof of Proposition 4

Part 1: Consider the problem described in equations (13)-(16), but assume now that both agents are risk averse. I denote the buyer’s utility function by $u^b(\cdot)$, and the seller’s utility function by $u^s(\cdot)$. Using the same steps as in the main text, the optimal order size is still given by equation (20). However, taking the first-order condition with respect to $T_s$ and re-arranging now yields

$$\frac{(u^b)'(T_s - \frac{w_{x}}{a} q_s)}{(u^b)'(R(q_s) - T_s)} = \frac{1 + \nu_s}{\lambda + \mu_s}. \quad (64)$$

Thus, under the optimal contract the ratio of marginal utilities is stabilized. Equation (64) states that when the buyer is constrained ($\nu_s > 0$), he must be allocated a higher share of the profits. Since $m_s$ is always chosen first-best, the result implies that the payments made by the buyer, $P_s$, must be reduced relative to the unconstrained case, as in the main model. Similarly, when the seller is unconstrained ($\mu_s > 0$), the payments are increased.

If both utility functions are from the CRRA family, $u(\Pi) = \frac{(\Pi)^{1-\gamma} - 1}{1-\gamma}$, with $0 < \gamma \leq 1$, then the expression becomes

$$\frac{(R(q_s) - T_s)^{\gamma'}}{(T_s - \frac{w_{x}}{a} q_s)^{\gamma'}} = \frac{1 + \nu_s}{\lambda + \mu_s}. \quad (65)$$
If the two agents have the same utility function, then $\gamma^i = \gamma^j$ and I can derive a closed-form expression for prices $p_s = P_s/m_s$:

$$p_s = \frac{1}{1 + \left(\frac{1 + \nu_s}{\lambda + \mu_s}\right)^{1/\gamma}} \frac{R(q_s)}{q_s} + \frac{\left(\frac{1 + \nu_s}{\lambda + \mu_s}\right)^{1/\gamma} w_x}{1 + \left(\frac{1 + \nu_s}{\lambda + \mu_s}\right)^{1/\gamma} a}.$$  

(66)

This equation shows that prices can be written as a convex combination of per-unit revenues and marginal costs, with the weights dependent on the Lagrange multipliers. Call these weights $\phi$ and $1 - \phi$, respectively. As can be seen in Appendix E, this formulation is very similar to the pricing equation in a Nash bargaining setup. Assuming that the revenue function is isoelastic with $R(q) = kq^r$, prices are given by

$$p_s = \phi \frac{w_x}{ra} + (1 - \phi) \frac{w_x}{a}.$$  

(67)

Assuming that $a$ is fixed, a log-linear approximation of this equation yields

$$\hat{p}_s = \hat{w}_x + \varsigma_2 \hat{\nu}_s + \varsigma_3 \hat{\mu}_s,$$  

(68)

where

$$\varsigma_2 = \frac{1}{\hat{p}_s} \left[ \frac{1}{\gamma} \frac{1 + \nu_s}{\lambda + \mu_s} \left(\frac{1 + \nu_s}{\lambda + \mu_s}\right)^{1/\gamma - 1} \right] \left(\frac{\hat{w}_x}{\hat{a}} - \frac{\hat{w}_x}{ra}\right) < 0$$

and

$$\varsigma_3 = \frac{1}{\hat{p}_s} \left[ \frac{1 + \nu_s}{\gamma} \frac{1 + \nu_s}{\lambda + \mu_s} \left(\frac{1 + \nu_s}{\lambda + \mu_s}\right)^{1/\gamma - 1} \right] \left(\frac{\hat{w}_x}{ra} - \frac{\hat{w}_x}{a}\right) > 0$$

as required.

**Part 2:** If the buyer is risk averse and the seller is risk neutral, then the analogue to equation (64) is

$$u'(R(q_s) - T_s) = \frac{1 + \mu_s}{\lambda + \nu_s},$$  

(69)

where $u(\cdot)$ is now the buyer’s utility function, $\lambda$ is the Lagrange multiplier on the promise-keeping constraint for the promise made from the seller to the buyer, and $\nu_s$ and $\mu_s$ are the Lagrange multipliers on the buyer’s and the seller’s participation constraint, respectively. The equation for prices is given by

$$p_s = \frac{R(q)}{q} - \frac{1}{q} \left(\frac{u'}{1 - u'}\right)^{-1} \left[\frac{1 + \mu_s}{\lambda + \nu_s}\right].$$  

(70)

I assume that the seller’s outside option is sufficiently large so that she at least breaks even, and thus $p_s \geq \frac{w_x}{a} > 0$. With isoelastic revenues, the expression becomes

$$p_s = \left(\frac{w_x}{ra}\right) - \left(\frac{w_x}{kra}\right)^{1/(1-r)} \left(\frac{u'}{1 - u'}\right)^{-1} \left[\frac{1 + \mu_s}{\lambda + \nu_s}\right].$$  

(71)

As before, when the buyer is constrained ($\nu_s > 0$) prices are reduced, and when the seller is constrained ($\mu_s > 0$)
prices are increased. The log-linear approximation is given by

\[ \hat{p}_s = \sigma_1 \hat{w}_x + \sigma_2 \hat{u}_s + \sigma_3 \hat{\mu}_s, \quad (72) \]

with

\[
\begin{align*}
\sigma_1 &= \frac{\bar{w}_x}{\bar{r}^a} - \left( \frac{1}{1-r} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} (u')^{-1} \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] < 1, \\
\sigma_2 &= \left( \frac{1 + \mu_x}{\lambda + \nu_x} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} [(u')^{-1}]' \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] < 0, \\
\sigma_3 &= -\left( \frac{1}{\lambda + \nu_x} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} [(u')^{-1}]' \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] > 0,
\end{align*}
\]

which may be negative. The other coefficients are

\[
\begin{align*}
\sigma_1 &= \frac{\bar{w}_x}{\bar{r}^a} - \left( \frac{1}{1-r} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} (u')^{-1} \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] < 1, \\
\sigma_2 &= \left( \frac{1 + \mu_x}{\lambda + \nu_x} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} [(u')^{-1}]' \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] < 0, \\
\sigma_3 &= -\left( \frac{1}{\lambda + \nu_x} \right) \left( \frac{\bar{w}_x}{k \bar{r}^a} \right)^{1/(1-r)} [(u')^{-1}]' \left[ \frac{1 + \mu_x}{\lambda + \nu_x} \right] > 0,
\end{align*}
\]

where the inequalities follow because \([(u')^{-1}]' (\cdot) < 0.

To show that pass-through is increasing in the level of assets, note that I can multiply equation (69) by \(\hat{a}\) and re-write it as

\[ \sigma_1 = A - \frac{1}{1-r} B(\hat{a}) > 0, \quad (74) \]

where \(B'(\hat{a}) < 0\). Taking the derivative with respect to \(\hat{a}\) then yields

\[ \frac{\partial \sigma_1}{\partial \hat{a}} = -\left( \frac{r}{1-r} \right) B'(\hat{a}) A > 0. \quad (75) \]

Part 3: Fix \(a\) and consider a seller with costs \(w_0\) such that his outside option exactly binds. Call this state \(s_0 = (a, w_0)\). Using a similar notation as in the baseline model, in this state I have \(\Upsilon_{s_0} = \bar{\Pi}_{s_0}^B + \beta W(s_0, \bar{J}) = U(w_0)\), where \(\Upsilon_{s_0} = F(m_0) - \frac{w_0}{a} m_0\) are the joint profits, \(\bar{\Pi}_{s_0}^B\) are the buyer’s profits in state \(s_0\) under the optimal contract when the seller’s constraint exactly binds, and \(W\) is the seller’s value as a function of the state and the promise made to the buyer, \(\bar{J}\). Consider a shock increasing costs to \(w_x > w_0\). Call this new state \(s = (a, w_x)\). Assume that the seller’s outside option in this new state were not binding. Then, given the optimal choice of \(m_x = F^{-1}(\frac{w_x}{a})\), joint profits must fall and since the buyer’s profits are held fixed at \(\bar{\Pi}_{s_0}^B\), the seller’s profits must decrease. Therefore, the seller’s new value of the relationship is now \(\Upsilon_s - \bar{\Pi}_{s_0}^B + \beta W(s, J_s)\), which may be less than the new outside option value \(U(w_x)\), which is decreasing in \(w_x\). If \(w_x\) were i.i.d., then if the seller’s outside option does not bind the continuation value is unchanged, \(W(s_0, \bar{J}) = W(s, J_s)\). With persistent shocks, however, the cost shock in state \(s\) makes high cost states in the future more likely. Since the buyer receives \(\bar{\Pi}_{s_0}^B\) in all states in which no outside option binds, the seller’s expected future profits fall across those states. Furthermore, the seller cannot be better off under \(w_x\) than under \(w_0\) due to the now higher likelihood of moving to high-cost states in which his own outside option binds, since his outside option falls with costs and therefore such states have a low value relative to state \(s\). Similarly, low cost states in which his outside option might bind are becoming less likely. Therefore, \(W(s, J_s) < W(s_0, \bar{J})\). Using the analogue of condition (21) and strict concavity of the seller’s value function, I can express the value promised to the buyer as a function of current

77
The seller’s outside option does not bind at \( s \) if
\[
\check{K}(s_0, s) \equiv (\Upsilon_s - \Upsilon_{s_0}) + \left( W(s, H(s, \Pi^B_{s_0})) - W(s_0, H(s_0, \Pi^B_{s_0})) \right) > U(w_x) - U(w_0).
\] (76)

Since the left-hand side is negative, this inequality cannot hold if \( U(w_x) \) is constant across states. From equation (71), if the seller’s outside option binds, \( \mu_s > 0 \), then price increases by more than in the unconstrained case. Therefore, the price response to the cost shock is amplified.

If the seller’s outside option was initially not binding at \( w_0 \), then the seller’s initial profits must have been higher than in the case where he starts out constrained. Calling the buyer’s profits in that case \( \hat{\Pi}^B_{s_0} \), we have that \( \Upsilon_{s_0} - \hat{\Pi}^B_{s_0} \geq \Upsilon_{s_0} - \Pi^B_{s_0} \). Using the analogue of equation (21), it must then also be the case that \( W(s_0, \check{J}) \geq W(s_0, \bar{J}) \). Following the same argument as above, then
\[
\Upsilon_s - \Pi^B_{s_0} + \beta W(s, \check{J}_s) \geq \Upsilon_s - \Pi^B_{s_0} + \beta W(s, J_s).
\]
Hence, if the seller’s outside option does not bind in the case where he starts out constrained, it also does not bind if he started out with a higher profit level. Therefore, if condition (76) holds, then the seller cannot be constrained in state \( s \) and there is no amplification.

## E Nash Bargaining Setup

I describe a simple Nash bargaining setup with free entry and show that it is able to generate the relationship life cycle. Assume there is a unit mass of buyers \( j \in [0, 1] \) and a continuum of sellers indexed by \( k \). I make the same assumptions about production functions, specific assets, and costs as before. Buyer and seller firms meet in a frictional market. Let \( x_b \) be the mass of unmatched buyer firms and \( x_s \) be the mass of unmatched sellers, and define \( \theta = x_b / x_s \) as market tightness. I assume a standard CES matching function with elasticity \( \iota \), which generates the matching probabilities \( q(\theta) \) for the buyer and \( f(\theta) \) for the seller given by
\[
\pi_j(\theta) = (1 + \theta^i)^{-\frac{1}{\iota}} \quad \text{(77)}
\]
\[
\pi_k(\theta) = \theta(1 + \theta^i)^{-\frac{1}{\iota}}. \quad \text{(78)}
\]
Buyers pay a per-period cost \( c \) to search for matches, while sellers make zero profits when unmatched.

The firms use Nash bargaining to choose quantities \( q \) and transfers \( T \). Let the buyer’s bargaining weight be \( \phi \). As in the limited commitment model, quantities in the first period are chosen to maximize the buyer’s profits in expectation, taking the realization of costs \( w_x \) as given. The payments then implement a split of profits using \( \phi \) once the initial value of \( a \) has been revealed. Let \( V, J^{new}(w_x) \), and \( J(s) \) be the value of an unmatched buyer, the buyer’s first-period value of a relationship with cost realization \( w_x \), and the buyer’s value of an established relationship, respectively. Similarly, let \( U, W^{new}(w_x) \), and \( W(s) \) be the value of an unmatched seller, a seller in a new relationship, and a seller in an established relationship. Then, the value of an unmatched buyer is given
by:

\[ V = -c + \beta E [\pi_j(\theta) J^{\text{new}}(w_x) + (1 - \pi_j(\theta)) V], \]  

where the expectation is taken with respect to costs. The value of a new match with cost \( w_x \) is

\[ J^{\text{new}}(w_x) = \max_q \left\{ \int_0^\infty \left[ R(q) - T(q; w_x, u) \right] g(u) du + \beta E \left[ \max \{ J(s), V \} \right] \right\}, \]  

where \( T(q; w_x, a) \) are the payment made when ordering \( q \), given costs \( w_x \) and after assets \( a \) are revealed, determined by Nash bargaining. The equation states that buyers choose the order quantity in new relationships to maximize expected static profits, taking as given \( w_x \) and applying the expectation to assets.

Once the value of assets becomes known, buyers decide whether to stay in the relationship. I impose free entry of buyers, so that at all times \( V = 0 \), which implies that

\[ E [J^{\text{new}}(w_x)] = \frac{c}{\beta q(\theta)}. \]  

An unmatched seller has the value function

\[ U = \beta [\theta \pi_k(\theta) W^{\text{new}}(w_x) + (1 - \theta \pi_k(\theta)) U], \]  

where

\[ W^{\text{new}}(w_x) = \int_0^\infty \left[ T(q; w_x, u) - \frac{w_x}{a} q \right] g(u) du + \beta E \left[ \max \{ W(s), U \} \right]. \]  

Once the value of specific assets is known, the buyer’s value function becomes

\[ J(s) = R(q) - T(q) + \beta E \left[ \max \{ J(s'), V \} \right], \]  

and the seller’s value function is

\[ W(s) = T(q) - \frac{w_x}{a} q + \beta E \left[ \max \{ W(s'), U \} \right], \]  

where continuation value depends on the evolution of costs and specific assets.

**Payments and quantities**

Given weight \( \phi \) on the buyer, the optimal payment satisfies

\[ T(q) = \arg\max (J(s) - V)^\phi (W(s) - U)^{1-\phi} . \]  

Taking the first-order condition with respect to \( P \) and re-arranging gives:

\[ \phi (W(s) - U) = (1 - \phi) (J(s) - V). \]
From equations (79)-(82), I have that

\[
0 = (1 - \phi) (J(s) - V) - \phi (W(s) - U) \\
= (1 - \phi) R(q) - (1 - \phi) T(q) + (1 - \phi) \beta E \left[ \max \{J(s'), V\} \right] \\
+ (1 - \phi) c - (1 - \phi) \beta E \left[ \pi_b(\theta) J_{\text{new}}(w_x) + (1 - \pi_b(\theta)) V \right] - \phi T(q) + \phi \frac{w_x}{a} q \\
- \phi \beta E \left[ \max \{W(s'), U\} \right] + \phi \beta \left[ \theta \pi_b(\theta) W_{\text{new}}(w_x) + (1 - \theta \pi_b(\theta)) U \right].
\]

(88)

I can use the fact that condition (87) has to hold at each point in time to simplify and obtain:

\[
T(q) = (1 - \phi) \left[ R(q) + c \right] + \phi \left[ \frac{w_x}{a} q \right] \\
+ \phi \theta \pi_b(\theta) \beta E \left[ W_{\text{new}}(w_x) - U \right] - (1 - \phi) \pi_b(\theta) \beta E \left[ J_{\text{new}}(w_x) - V \right].
\]

(89)

By equation (87), this becomes

\[
T(q) = (1 - \phi) \left[ R(q) + c \right] + \phi \left[ \frac{w_x}{a} q \right] + (1 - \phi) \beta \pi_b(\theta) (\theta - 1) E \left[ J_{\text{new}}(w_x) - V \right].
\]

(90)

Using the free entry condition (81) and re-arranging yields

\[
T(q) = (1 - \phi) \left[ R(q) + \theta c \right] + \phi \left[ \frac{w_x}{a} q \right].
\]

(91)

Plugging equation (91) back into (84) and solving for the order quantity yields

\[
q = (R')^{-1} \left( \frac{w_x}{a} \right).
\]

(92)

Thus, the quantity ordered in the Nash bargaining model is the one that maximizes the size of the static joint profits, as in the limited commitment model. Hence, quantity is increasing in the level of specific assets. Similarly, the quantity ordered in the first period is

\[
q_0 = (R')^{-1} \left( \frac{w_x}{a} \right).
\]

(93)

Furthermore, if \( R(q) \) is isoeelastic and concave, then by the same reasoning as before \( \frac{w_x}{a} q \) is increasing in \( a \). From equation (91), the value traded is therefore increasing in assets.

**Relationship life cycle**

I can show that separation is efficient in the Nash bargaining model. Adding up (84) and (85), and deducting (82), I obtain a total match surplus over the outside value of

\[
\tilde{S}(s) = R(q) - \frac{w_x}{a} q + \beta E \left[ \max \{\tilde{S}(s'), 0\} \right] - \beta \theta \pi_b(\theta) (1 - \phi) ES_{\text{new}}(w_x')
\]

(94)

Using the free entry condition (81), this becomes

\[
\tilde{S}(s) = R(q) - \frac{w_x}{a} q + \beta E \left[ \max \{S(s'), 0\} \right] - \frac{1 - \phi}{\phi} \theta c \theta.
\]

(95)
By concavity of $R(q)$ and since at the optimum $R'(q) = \frac{w_x}{a}$, the static per period surplus that can be divided between the two parties $R(q) - \frac{w_x}{a}q$ is increasing in $a$. Therefore, the total surplus is rising in $a$. By continuity, there exist threshold levels $a^*(w_x)$ such that the relationship is terminated if $a < a^*(w_x)$. By the same arguments as before, separation is efficient.

This generates the relationship life cycle by the same arguments as before. As relationship-specific assets increase, value traded increases and the separation probability falls. Relationships that separate must on average have received bad shocks to assets that have brought these back to the termination bound. This generates the hump-shaped pattern of value traded.

**Pricing and implications for pass-through**

Prices satisfy $p(q) = T(q)/q$, and hence

$$p(q) = \phi \frac{w_x}{a} + (1 - \phi) \frac{R(q) + \theta c}{q}. \quad (96)$$

As in the limited commitment model, there is both a direct and an indirect effect. However, here the mark-up only varies because of changes in quantities. Since the split of the relationship surplus is always the same, there are no Lagrange multipliers as in the limited commitment model that would mute the price change. If $R(q)$ is isoelastic and concave, with $R(q) = kq^r$, $k > 0$, $0 < r < 1$, then solving for quantities and plugging into equation (96) yields:

$$p(q) = \phi \frac{w_x}{a} + (1 - \phi) \frac{w_x}{ra} + [(1 - \phi)\theta c] \left( \frac{w_x}{kr a} \right)^{\frac{1}{1-r}}. \quad (97)$$

The equation highlights that prices are declining in the level of specific assets, thus matching this feature of relationships. However, pass-through is actually decreasing in $a$. Fix $a$ at a given level $\bar{a}$ and take a log linear approximation of equation (97) around the current state. This yields:

$$\hat{p} = \phi \frac{\bar{w}_x}{\bar{a}} + (1 - \phi) \frac{\bar{w}_x}{\bar{r} \bar{a}} + [(1 - \phi)\theta c] \left( \frac{\bar{w}_x}{k\bar{r} \bar{a}} \right)^{\frac{1}{1-r}} \frac{1}{1-r} \hat{w}, \quad (98)$$

where hats denote log deviations and bars indicate steady state values. This equation shows two things. First, for small changes in $w_x$, pass-through of cost shocks into prices is greater than one in the Nash bargaining model with free entry, since $r < 1$. Second, this constant is actually declining in $\bar{a}$ due to of the last term in the numerator, which is multiplied by $1/(1-r)$. The proof of this result is analogous to the proof in Appendix D.6. Thus, the Nash bargaining model does not deliver increasing pass-through with relationship quality. The effect goes in the opposite direction.
Estimation

I find that the model with free entry generally does not match the life cycle moments very well. In particular, it severely understates the separation probability in the first period. The reason for this is that as the value of a relationship is reduced to raise the probability of separation, the value of the outside options also falls, since it depends on the value of future relationships. Furthermore, buyers exit the market due to free entry, which makes it more difficult for sellers to find a match and also reduces their outside option. These effects reduce both the value of the relationship and the outside option, and prevent me from making the separation probability sufficiently high while generating the correct matching probability $q$ observed in the data. This problem could be addressed in two ways: first, I could introduce a cost of maintaining the relationship, for example to maintain the specific assets. This cost would lower the value of being in a relationship and increase separation, allowing me to match all moments. The second option is to remove the free entry assumption and to impose outside options (36) and (37), with exogenous matching probabilities. Since this approach brings the model in line with the limited commitment setup, I will follow this route. The price in this model is given by

$$p(q) = \phi \frac{w_x}{a} + (1 - \phi) \frac{w_x}{ra} + q \left\{ \phi \left[ W^{new}(w_x) - \beta E[U(w_x')] \right] - (1 - \phi) \beta \left[ E[J^{new}(w_x)] - V \right] \right\} \left( \frac{w_x}{kra} \right)^{1/r}.$$

(99)

Pass-through now depends on the term in curly brackets. If this term is positive, pass-through is greater than one. If it is negative, pass-through is below one.
F Additional Figures and Tables

F.1 Additional Figures

Figure F.10: Trade distribution by industry

Figure F.11: Average share of trade by relationship length (in number of transactions)
Figure F.12: Relationship life cycle

(a) Number of products traded

(b) Number of transactions

Figure F.13: Total value traded

<table>
<thead>
<tr>
<th>Data</th>
<th>Simulated (LC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[Diagram of data and simulated values]
Figure F.14: Hazard rate of breaking up a relationship (quarterly)

<table>
<thead>
<tr>
<th>Data</th>
<th>Simulated (LC)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Data" /></td>
<td><img src="image2" alt="Simulated (LC)" /></td>
</tr>
</tbody>
</table>

Figure F.15: Average share of trade by length (in quarters)

<table>
<thead>
<tr>
<th>Data</th>
<th>Simulated (LC)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Data" /></td>
<td><img src="image4" alt="Simulated (LC)" /></td>
</tr>
</tbody>
</table>
### F.2 Additional Tables

#### Table F.14: Domestic relationships

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>Type of relationship</th>
<th>Average length (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ganesan (1994)</td>
<td>5 department store chains, 52 matched vendors</td>
<td>Random</td>
<td>2.9 (retailer) / 4.2 (vendor)</td>
</tr>
<tr>
<td>Doney and Cannon (1997)</td>
<td>209 manufacturing firms from SIC 33-37</td>
<td>1st or 2nd choice in recent purchasing decision</td>
<td>11</td>
</tr>
<tr>
<td>Artz (1999)</td>
<td>393 manufacturers from SIC 35-38</td>
<td>Major supplier, at least 3 years</td>
<td>8.8</td>
</tr>
<tr>
<td>Cannon and Perreault (1999)</td>
<td>426 firms, mainly manufacturing and distributors</td>
<td>Main supplier of last purchasing decision</td>
<td>11</td>
</tr>
<tr>
<td>Kotabe, Martin, and Domoto (2003)</td>
<td>97 automotive component suppliers</td>
<td>Major buyer</td>
<td>26.3</td>
</tr>
<tr>
<td>Ulaga (2003)</td>
<td>9 manufacturers from SIC 34-38</td>
<td>Close relationship for an important component</td>
<td>2-25</td>
</tr>
<tr>
<td>Claycomb and Frankwick (2005)</td>
<td>174 manufacturers in SIC 30 and 34-38</td>
<td>Key supplier, mature relationship</td>
<td>7.5</td>
</tr>
<tr>
<td>Jap and Anderson (2007)</td>
<td>1,540 customers of an agricultural chemical manufacturer</td>
<td>Random</td>
<td>17</td>
</tr>
<tr>
<td>Krause, Handfield, Tylor (2007)</td>
<td>373 automotive and electronics manufacturers, 75 matched suppliers</td>
<td>Firms have recently worked to improve performance</td>
<td>12.4</td>
</tr>
</tbody>
</table>

#### Table F.15: Pass-through robustness I

<table>
<thead>
<tr>
<th>$\Delta \ln(p_{mcxht})$</th>
<th>Countries</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No breaks</td>
<td>Selection</td>
</tr>
<tr>
<td>$\Delta \ln(e_{cht})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(.0105)</td>
<td>(.0051)</td>
</tr>
<tr>
<td>(2)</td>
<td>(.0011)</td>
<td>(.0012)</td>
</tr>
<tr>
<td>$\Delta \ln(e_{cht}) \cdot Months$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(.0003)</td>
<td>(.0001)</td>
</tr>
<tr>
<td>(2)</td>
<td>(.0071)</td>
<td>(.0011)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FE</th>
<th>mzx,t</th>
<th>mzx,t</th>
<th>mzx,t</th>
<th>mzx,t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs</td>
<td>8,256,000</td>
<td>13,967,000</td>
<td>5,151,000</td>
<td>11,751,000</td>
</tr>
</tbody>
</table>

86
Table F.16: Pass-through robustness II

<table>
<thead>
<tr>
<th>$\Delta \ln(p_{mxht})$</th>
<th>Fixed length (in months)</th>
<th>Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\geq 24, &lt; 36$</td>
<td>$\geq 36, &lt; 48$</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\Delta \ln(e_{cht})$</td>
<td>.1106***</td>
<td>.1860***</td>
</tr>
<tr>
<td></td>
<td>(.0282)</td>
<td>(.0252)</td>
</tr>
<tr>
<td>$\Delta \ln(e_{cht}) \cdot Months$</td>
<td>.0049**</td>
<td>.0001</td>
</tr>
<tr>
<td></td>
<td>(.0021)</td>
<td>(.0013)</td>
</tr>
<tr>
<td>FE</td>
<td>$mxh,t$</td>
<td>$mxh,t$</td>
</tr>
<tr>
<td>Obs</td>
<td>3,431,000</td>
<td>2,711,000</td>
</tr>
</tbody>
</table>

Table F.17: Im-Pasher-Shin test for unit roots

<table>
<thead>
<tr>
<th>$e_{mxht}$</th>
<th>$p_{mxht}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\bar{\tilde{Z}}$</td>
<td>266.3661</td>
</tr>
<tr>
<td>$p$-value</td>
<td>1</td>
</tr>
<tr>
<td>Panels</td>
<td>65,100</td>
</tr>
<tr>
<td>Observations</td>
<td>1,676,000</td>
</tr>
</tbody>
</table>

Table F.18: Break-up regressions, using quantity purchased, relationships 24 months or older (robustness specification)

<table>
<thead>
<tr>
<th>$d^h_{mh,i}$</th>
<th>$d^h_{mh,i} \cdot d_{new}^{mh,i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t + 1$</td>
<td>$t$</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>.1846***</td>
<td>.0405***</td>
</tr>
<tr>
<td>(.0141)</td>
<td>(.0123)</td>
</tr>
<tr>
<td>$d^h_{mh,i} \cdot d_{new}^{mh,i}$</td>
<td>.0827**</td>
</tr>
<tr>
<td></td>
<td>(.0334)</td>
</tr>
<tr>
<td>FE</td>
<td>$mh,t$</td>
</tr>
<tr>
<td>Obs</td>
<td>9,542,000</td>
</tr>
</tbody>
</table>
Table F.19: Firm size of exporters and importers

<table>
<thead>
<tr>
<th>Country</th>
<th>Employees (Mean)</th>
<th>Shipment value (Mean, $ '000)</th>
<th>Year</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>70</td>
<td>2,006</td>
<td>2000</td>
<td>Di Giovanni, Levchenko, and Mejean (2014)</td>
</tr>
<tr>
<td>Belgium</td>
<td>6,250.3</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Cambodia</td>
<td>2,061.7</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Cambodia</td>
<td>1,087.3</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>2,630.0</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Mexico</td>
<td>4,233.7</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1,245.6</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Norway</td>
<td>1,416.5</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Peru</td>
<td>1,628.8</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Portugal</td>
<td>1,613.8</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Senegal</td>
<td>624.3</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Sweden</td>
<td>2,479.7</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
<tr>
<td>Uganda</td>
<td>1,425.9</td>
<td></td>
<td>2000</td>
<td>World Bank Exporter Dynamics Database</td>
</tr>
</tbody>
</table>
Table F.20: Price regression by industry - overall effect

<table>
<thead>
<tr>
<th></th>
<th>Animal</th>
<th>Vegetables</th>
<th>Fats</th>
<th>Food</th>
<th>Minerals</th>
<th>Chemicals</th>
<th>Plastics</th>
<th>Leather</th>
<th>Wood</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_6$</td>
<td>$-0.0016^{**}$</td>
<td>$-0.0035^{***}$</td>
<td>$-0.0007$</td>
<td>$-0.0040^{**}$</td>
<td>$-0.0028$</td>
<td>$-0.0032^*$</td>
<td>$-0.0063^{**}$</td>
<td>$-0.0025^{**}$</td>
<td>$-0.0008$</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.0007)</td>
<td>(0.0039)</td>
<td>(0.0008)</td>
<td>(0.0028)</td>
<td>(0.0017)</td>
<td>(0.0018)</td>
<td>(0.0011)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>$d_{11}$</td>
<td>$-0.0012$</td>
<td>$-0.0053^{***}$</td>
<td>$0.0013$</td>
<td>$-0.0035^{***}$</td>
<td>$0.0018$</td>
<td>$-0.0038^*$</td>
<td>$-0.0061^{***}$</td>
<td>$-0.0010$</td>
<td>$-0.0041^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.0051)</td>
<td>(0.0010)</td>
<td>(0.0034)</td>
<td>(0.0022)</td>
<td>(0.0022)</td>
<td>(0.0015)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>$d_{16}$</td>
<td>$-0.0011$</td>
<td>$-0.0073^{***}$</td>
<td>$-0.0015$</td>
<td>$-0.0033^{**}$</td>
<td>$0.0040$</td>
<td>$-0.0058^{**}$</td>
<td>$-0.0063^{**}$</td>
<td>$-0.0007$</td>
<td>$-0.0057^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0009)</td>
<td>(0.0061)</td>
<td>(0.0011)</td>
<td>(0.0039)</td>
<td>(0.0026)</td>
<td>(0.0026)</td>
<td>(0.0017)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>$d_{21}$</td>
<td>$-0.0015^*$</td>
<td>$-0.0122^{***}$</td>
<td>$-0.0085$</td>
<td>$-0.0067^{***}$</td>
<td>$0.0012$</td>
<td>$-0.0088^{***}$</td>
<td>$-0.0084^{***}$</td>
<td>$-0.0019$</td>
<td>$-0.0063^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.0062)</td>
<td>(0.0011)</td>
<td>(0.0034)</td>
<td>(0.0025)</td>
<td>(0.0024)</td>
<td>(0.0016)</td>
<td>(0.0019)</td>
</tr>
<tr>
<td>$d_{41}$</td>
<td>$-0.0029^{***}$</td>
<td>$-0.0095^{***}$</td>
<td>$-0.0047$</td>
<td>$-0.0073^{**}$</td>
<td>$0.0081^*$</td>
<td>$-0.0121^{***}$</td>
<td>$-0.0048$</td>
<td>$-0.0040^*$</td>
<td>$-0.0123^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0010)</td>
<td>(0.0083)</td>
<td>(0.0014)</td>
<td>(0.0043)</td>
<td>(0.0034)</td>
<td>(0.0031)</td>
<td>(0.0021)</td>
<td>(0.0024)</td>
</tr>
</tbody>
</table>

Instruments: No No No No No No No No No
Fixed effects: mzh mzh mzh mzh mzh mzh mzh mzh mzh
Observations: 1,766,000 2,761,000 87,000 2,688,000 238,000 1,518,000 2,561,000 2,798,000 2,574,000

<table>
<thead>
<tr>
<th></th>
<th>Textiles</th>
<th>Footwear</th>
<th>Ceramics</th>
<th>Jewelry</th>
<th>Metal prods</th>
<th>Machinery</th>
<th>Transport</th>
<th>Optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_6$</td>
<td>$0.0008^{***}$</td>
<td>$0.0002$</td>
<td>$-0.0046^{***}$</td>
<td>$-0.0051$</td>
<td>$-0.0066^{***}$</td>
<td>$-0.0107^{***}$</td>
<td>$-0.0073^{***}$</td>
<td>$-0.0029^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0006)</td>
<td>(0.0013)</td>
<td>(0.0008)</td>
<td>(0.0011)</td>
<td>(0.0017)</td>
<td>(0.0015)</td>
<td></td>
</tr>
<tr>
<td>$d_{11}$</td>
<td>$0.0007^*$</td>
<td>$-0.0011$</td>
<td>$-0.0069^{***}$</td>
<td>$-0.0145^{***}$</td>
<td>$-0.0090^{***}$</td>
<td>$-0.0136^{***}$</td>
<td>$-0.0086^{***}$</td>
<td>$-0.0028$</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0007)</td>
<td>(0.0017)</td>
<td>(0.0048)</td>
<td>(0.0010)</td>
<td>(0.0014)</td>
<td>(0.0020)</td>
<td>(0.0019)</td>
</tr>
<tr>
<td>$d_{16}$</td>
<td>$0.0004$</td>
<td>$-0.0011$</td>
<td>$-0.0065^{***}$</td>
<td>$-0.0180^{***}$</td>
<td>$-0.0123^{***}$</td>
<td>$-0.0182^{***}$</td>
<td>$-0.0095^{***}$</td>
<td>$-0.0098^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0009)</td>
<td>(0.0020)</td>
<td>(0.0055)</td>
<td>(0.0012)</td>
<td>(0.0016)</td>
<td>(0.0023)</td>
<td>(0.0022)</td>
</tr>
<tr>
<td>$d_{21}$</td>
<td>$-0.0010^{**}$</td>
<td>$-0.0035^{***}$</td>
<td>$-0.0086^{***}$</td>
<td>$-0.0171^{***}$</td>
<td>$-0.0146^{***}$</td>
<td>$-0.0239^{***}$</td>
<td>$-0.0078^{***}$</td>
<td>$-0.0094^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0008)</td>
<td>(0.0019)</td>
<td>(0.0052)</td>
<td>(0.0011)</td>
<td>(0.0015)</td>
<td>(0.0020)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>$d_{41}$</td>
<td>$-0.0024^{***}$</td>
<td>$-0.0089^{***}$</td>
<td>$-0.0098^{***}$</td>
<td>$-0.0202^{***}$</td>
<td>$-0.0170^{***}$</td>
<td>$-0.0307^{***}$</td>
<td>$-0.0096^{***}$</td>
<td>$-0.0136^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0010)</td>
<td>(0.0026)</td>
<td>(0.0067)</td>
<td>(0.0014)</td>
<td>(0.0019)</td>
<td>(0.0024)</td>
<td>(0.0027)</td>
</tr>
</tbody>
</table>

Instruments: No No No No No No No No No
Fixed effects: mzh mzh mzh mzh mzh mzh mzh mzh mzh
Observations: 20,890,000 3,526,000 2,612,000 543,000 5,553,000 8,680,000 2,347,000 2,387,000