Governing Adaptation

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Abstract

We consider an organization in which two activities need to be coordinated with each other (standardization), and yet also need to respond to local conditions (customization). Control of the activities can be allocated among two privately informed but biased local agents (employees) and an uninformed but unbiased central agent (manager). We analyze how the allocation of decisions rights within the organization affects strategic communication among the participants and the decisions that the organization makes. By modeling the participants as strategic actors, we endogenize both the biases in equilibrium decisions, conditional on the information available to the decision-maker(s), and imperfections in the information itself, with communication between the local agents and the decision-maker(s) modeled as a cheap-talk game. The results provide a qualified formalization of the common intuition that centralization of decision-making is needed when interdependencies between activities are high. The qualifiers are: (i) the performance differential between centralized and decentralized decision-making is non-monotone in the importance of coordination, (ii) both of these common structures are dominated by asymmetric structures in sufficiently asymmetric environments and (iii) incentive alignment provides a natural alternative to centralization.

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1 Introduction

Many modern corporations seek to balance standardization and customization: standardization can facilitate functional excellence and cost economies, while customization to regions or customer groups can significantly increase demand for the firm’s products. As conditions change, such firms face the problem of coordinated adaptation, where customization inspires local adaptations, but standardization presses for coordinated responses to new conditions.

Coordinated adaptation can be especially difficult because three factors interact. First, specialization (by technology, region, customer group, or the like) means that decision-relevant information is distributed inside the organization, rather than being directly accessible to any single potential decision-maker. Second, much of that information is "soft," rather than communicable as hard facts. Finally, different members of the organization are likely to have conflicting preferences over possible courses of action.

This paper examines how the allocation of decision rights inside an organization influences the organization’s ability first to aggregate information from a variety of sources and then to use that information to construct a plan of action that achieves an appropriate balance between coordination (standardization) and adaptation (customization). By considering strategic behavior by the participants, we endogenize both the quality of decision-making, conditional on the information available to the decision-maker(s), and the accuracy of information transmission, as influenced by the allocation of decision rights and the underlying environment.

Motivated by the importance of soft information, communication is modeled as a cheap-talk game. While we build on the seminal work of Crawford and Sobel (1982), our solution differs in two respects. First, the realized incentive conflict between the sender and the receiver(s) is not constant, but instead is random and driven by the realization (and the beliefs over the realization) of the local conditions. Second, because the impact of information on the equilibrium decisions varies, the expected magnitude of the incentive conflict and thus the expected accuracy of communication are dependent on both the identity of the receiver(s) and the underlying environment.

Consider the following example. A firm produces two products. Each product is targeted at a specific customer group and is both developed and manufactured by a single division. For each product, there exists an ideal approach to design,
manufacturing and marketing that, other things constant, would best satisfy the
demands of the target customer group. However, cost-savings can be achieved both
upstream (for example by undertaking some standardization of the production process
and using some shared components) and downstream (for example by utilizing shared
distribution channels and common marketing).

Each division is headed by a local manager who knows the approach that would
optimize his division’s performance but is uninformed about the local conditions faced
by the other division. Each manager does value coordination to the extent that it
increases the performance of his division, but does not internalize the impact of his
choices on the other division’s performance. As a result, each local manager would like
the coordination to take place around his division’s preferred approach. In addition
to the local managers, there exists a central manager who would like to optimize
the overall firm performance, but by being more removed from the product-specific
conditions, has no direct access to information about the conditions faced by either
division.

To model this setting, we consider an organization that needs to perform two
activities ($i$ and $j$). The payoff to each activity depends on how well the local decision
d$_i$ matches both the local conditions $\theta_i$ and the outside decision $d_j$, captured by the
loss function

$$L_i = (1 - r_i) (\theta_i - d_i)^2 + r_i (d_j - d_i)^2,$$

where $r_i \in (0, 1)$ parameterizes the relative importance of coordination to activity
$i$. Associated with each activity is a local agent ($i$ and $j$, respectively), who is the
only actor having direct access to information about the locally optimal course of ac-
tion $\theta_i$. Agent $i$ aims at minimizing $L_i$ (rather than the overall loss function $L_i + L_j$),
but requires additional information about $\theta_j$ to make informed decisions. In addition
to the two local agents, there exists a central agent (manager, $M$). The manager aims
at minimizing $L_i + L_j$, but has to rely solely on information transmitted to him by
the local agents.\(^\text{1}\)

We analyze the relative performance of four distinct governance structures, sum-
marized in figure 1. Under decentralized authority, each local agent retains the deci-

\(^\text{1}\)The qualitative logic of the analysis continues to hold when some payoff-sharing between the
agents is feasible. Such issues of incentive alignment are discussed briefly in section 7 and analyzed
extensively in Rantakari (2006a).
sion right over his own activity. Under *centralized authority*, both decision rights are allocated to the manager. Under *partial centralization*, one of the decision rights is allocated to the manager, while the other remains in the hands of the local agent. Finally, under *directional authority*, both decision rights are allocated to one of the local agents.²

Under each governance structure, the local agents first learn their local conditions, after which they strategically communicate information \((m_i, m_j)\) to the decision-maker(s), followed by strategic decision-making. We show that this similarity in the structure of the game across governance structures translates into a similarity in the structure of the solutions. Indeed, a common solution covering all four governance structures and arbitrary (linear) degrees of ex ante incentive conflict is presented in Appendix A. On the other hand, our analysis shows that the equilibrium decisions and the quality and value of communication exhibit significant and systematic differences across the different governance structures. As a result, each governance structure arises as the preferred solution under specific environmental conditions.

²These four governance structures span the set of single-agent, unconditional decision-making structures that can arise in equilibrium. Governance structures where agent \(i\) would control \(d_j\) without also controlling \(d_i\) are never optimal in the present setting.
One implication of our results is a qualified formalization of the common intuition that centralization is needed when coordination is sufficiently important.\textsuperscript{3} Support for the common intuition follows from our result that centralized authority dominates decentralized authority whenever coordination is sufficiently important to either activity. Conversely, decentralized authority performs well only when coordination is unimportant to both parties: equilibrium decisions are then only mildly biased and communication, while inaccurate, is also unimportant.

Our results also qualify this common intuition on three fronts. First, the performance differential between centralized and decentralized authority is non-monotone in the importance of coordination. When coordination becomes the primary concern for at least one of the activities, the agents become increasingly able to coordinate their behavior under decentralized authority and the advantage of centralized authority is eroded. Indeed, the two solutions converge when one or both activities become fully dependent on coordination.

Second, in sufficiently asymmetric environments, centralized authority is in turn dominated by one or both of the asymmetric governance structures. Partial centralization can be optimal under two conditions: centralizing the activity with lower importance of coordination can be used to improve decision-making relative to decentralized authority while compromising the overall adaptiveness less than under centralized authority; alternatively, centralizing the activity with higher importance of coordination can be used to facilitate communication by increasing the bias in the equilibrium decisions. And directional authority can be optimal when one activity cares primarily about coordination, in which case both decision rights are allocated to the agent with lower importance of coordination.

Third, incentive alignment provides a natural alternative to centralization. Indeed, centralized authority is eliminated as an equilibrium outcome strictly before perfect preference alignment between the divisions is achieved. This outcome stands in contrast to partial centralization, where the role of the manager as a communication facilitator is eliminated only in the limit of perfect incentive alignment. This result illustrates how the quality of decision-making improves faster than the quality

\textsuperscript{3}The problem of coordinated adaptation and the relationship between the importance of coordination and the desirability of centralization has been a topic of discussion at least since Barnard (1938) and Simon (1947). This challenge of coordinated adaptation has later been discussed in alternative contexts by Richardson (1972), Williamson (1975,2002) and Langlois and Robertson (1993,1995), among others.
of communication as the incentive conflict between the participants is decreased.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and section 3 describes the model in detail. Section 4 derives the solution under decentralized authority and analyzes how its expected performance varies with the importance of coordination to the two activities. Drawing on the general solution in Appendix A, section 5 repeats the analysis for the alternative governance structures, focusing on how the solutions differ from the solution under decentralized authority. The results are then brought together in section 6 for comparative institutional analysis. Section 7 discusses some extensions, including the impact of incentive alignment on the relative performance of the analyzed governance structures. Section 8 concludes.

2 Related Literature

The present paper relates to a number of different literatures. In viewing the organizational problem as one of coordinated adaptation under distributed information, the paper is intellectually indebted to the analysis of decision-making, authority and adaptation by, among others, Barnard (1938), Simon (1947), Cyert and March (1963) and Williamson (1975), and similar considerations within the modern capabilities literature, especially Langlois and Robertson (1993,1995). However, these descriptive theories are significantly broader in scope and blend in their discussion incentive conflicts, bounded rationality and technological considerations. In contrast, our analysis focuses purely on the role of incentive conflicts in decision-making and communication.

Methodologically, we build on the seminal work of Crawford and Sobel (1982) (henceforth CS) on cheap talk. We maintain the assumption that the actors are unable to commit to a decision rule ex ante, but instead of examining one-way information transmission with a known, fixed preference bias, we focus on multiple senders with random preferences, where the use of a "principal" is a governance choice. In our focus on multiple senders, we are closer to Battaglini (2002), but instead of each

\footnote{The key technical advantage of our approach over the CS setting is that the most informative partition has an infinite number of elements, thus avoiding the integer problems present in the canonical CS example and making our setting much easier to work with.}
sender observing full, multidimensional information, the agents in our model observe only partial and independent information, making truth-telling impossible. In our introduction of random preferences and the possibility of full agreement between the sender and the receiver(s), the communication equilibrium is analogous to Stein (1989), who illustrates how the Federal Reserve with uncertain preferences is able to transmit its private information only partially.

A few recent papers are closely related to ours in their focus on the role of private information and communication in influencing the optimal organizational arrangements in terms of authority and decision rights. Dessein (2002) illustrates how, in the CS setting with an informed agent and an uninformed principal, delegation of the decision right is often preferred over the communication equilibrium because the welfare loss caused by self-interested communication is higher than the cost of biased decision-making. However, focusing on a single decision, the model remains silent on the importance of coordination.

Dessein and Santos (2006) examine a team-theoretic model that focuses on the limitations that the need for coordinated adaptation imposes on task specialization. They illustrate how organizations in their setting are characterized either by broad job descriptions and flexible adaptation, or by a high degree of specialization together with rigid ex ante guidelines on behavior. Their model is thus primarily about the tradeoff between ex ante and ex post coordination of activities. Also, by taking a team-theoretic approach and assuming a fixed quality of inter-personal communication, the model has no explicit role for authority, except in determining ex ante rules of behavior and in making a potential ex ante investment in the quality of communication channels. In contrast, our model is about different ways to achieve ex post coordination.

Dessein, Garicano and Gertner (2005) examine how the optimal allocation of decision rights between a functional manager (who learns the value of potential synergies) and two product managers (who learn the cost of synergy implementation in terms of compromised local adaptation) depends on the value of synergies, the value of local adaptation and the importance of incentives. Their model is thus primarily about "lines of authority" (whether the product managers should have the right to veto the synergy implementation decision by the functional manager) and the basic tradeoff is between incentive alignment and strong local incentives. Incentive alignment is needed to induce appropriate synergy implementation choices by the functional man-
ager and truthful communication or appropriate use of the veto power by the product managers, while strong local incentives are needed to induce effort provision.

Friebel and Raith (2006) examine a resource allocation problem. Two local agents exert effort to generate high-quality projects. The amount of resources available to them can be determined either ex ante (decentralization) or ex post by a central agent (centralization). The basic tradeoff is between better use of resources and diminished incentives for effort. Centralized ex post allocation of resources is more efficient as the allocation can be conditioned on the realized quality of the projects. However, to induce truthful communication by the agents about the quality of their projects, payoff-sharing has to be introduced, thus reducing the strength of local incentives and so the effort exerted by the agents to generate high-quality projects. In contrast to Dessein, Garicano and Gertner (2005) and Friebel and Raith (2006), we examine the two-fold question of (i) how the environment (the importance of coordination) alone influences the ability of the local agents to coordinate their decisions (even absent any payoff-sharing) and (ii) when is it appropriate to delegate any of the decision rights to a third, uninformed party with more aggregate preferences.

Finally, Alonso, Dessein and Matouschek (2006) (henceforth ADM) have independently developed a model that is very similar to ours. However, the focus and exposition of the two approaches are somewhat different. The focus of the comparative statics in ADM is on the degree of incentive alignment between the divisions, (symmetric) importance of coordination and the relative variance between the states. In contrast, we allow also for asymmetric importance of coordination. Our model thus replicates the results presented in ADM, but also extends the analysis in two dimensions. First, the asymmetric governance structures of partial centralization and directional authority are not analyzed in ADM, whereas they arise naturally as second-best governance structures when asymmetries in the importance of coordination are allowed. Second, the explicit separation of the importance of coordination to the two activities allows us to gain a better understanding of how the two activities interact, both in the decision-making and in the communication stages. In consequence, we focus more on the impact that asymmetries in the importance of coordination have on the relative performance of the different governance structures. The impacts of asymmetric variances and preference alignment are discussed only briefly in sections 6.1 and 7.3 (but see Rantakari (2006a,b) for various extensions).
3 The Model

3.1 Payoffs, actors and feasible governance structures

We examine the problem of coordinating the decision-making processes of two activities, \(i\) and \(j\). The ex post loss incurred in the performance of activity \(i\) (and borne by agent \(i\)) is given by

\[
L_i = (1 - r_i) (\theta_i - d_i)^2 + r_i (d_j - d_i)^2,
\]

where \(d_i\) and \(d_j\) stand for the decisions chosen for activities \(i\) and \(j\), and \(\theta_i \sim U [-\bar{\theta}_i, \bar{\theta}_i]\) indexes the locally optimal decision, with \(\theta_i\) and \(\theta_j\) independently distributed. The exogenous variable of interest in the analysis is \(r_i \in (0, 1)\), which captures the degree of interdependence of activity \(i\) on activity \(j\) and so the relative importance of coordination to activity \(i\). We will refer to \(r_i\) as the degree of dependency of activity \(i\). As \(r_i\) increases, the loss to activity \(i\) becomes increasingly dependent on how well the local decision \(d_i\) is coordinated with the outside decision \(d_j\) (relative to adapting to local conditions \(\theta_i\)). We will refer to \((\theta_i - d_i)^2\) as the adaptation component and to \((d_j - d_i)^2\) as the coordination component of the loss \(L_i\).

Associated with each activity is a local agent (\(i\) and \(j\), respectively). Only agent \(i\) has direct access to information about \(\theta_i\) and his ex post loss is given by \(L_i\). In addition to the local agents, there exists a central agent whom we will call the manager (indexed by \(M\)). The manager has no direct access to information about either \(\theta_i\) or \(\theta_j\) and her objective is to minimize \(L_i + L_j\).

Within this framework, we analyze four distinct governance structures, which were summarized in figure 1. Letting \(d^k\) denote the set of decision rights allocated to an agent, with \(k \in \{i, j, M\}\), the alternatives are: decentralized authority, where each local agent decides how their activity is to be performed (\(d^i = \{d_i\}, d^j = \{d_j\}, d^M = \{\emptyset\}\)); centralized authority, where both decision rights are allocated to the manager (\(d^i = \{\emptyset\}, d^j = \{\emptyset\}, d^M = \{d_i, d_j\}\)); partial centralization, where one decision right is centralized while the other is left local (\(d^i = \{d_i\}, d^j = \{\emptyset\}, d^M = \{d_j\}\)); and directional authority, where both decision rights are allocated to one of the local agents (\(d^i = \{d_i, d_j\}, d^j = \{\emptyset\}, d^M = \{\emptyset\}\)).
Throughout the paper, unless otherwise mentioned, we will use subscripts to index the activities and superscripts to index either the agent $k$ or the underlying governance structure $g \in \{\text{dec, cent, part}(j), \text{dir}(i)\}$. In the case of partial centralization, $j$ refers to the centralized activity, while in the case of directional authority, $i$ refers to the local agent gaining control over both activities. The exception to this notation are conditional expectations, where $E_k$ is used to denote the expectation by agent $k$ based on his current information, to avoid confusion with higher-order expectations. For example, $EL^\text{cent}_i$ is the equilibrium expected loss to activity $i$ under centralized authority.

### 3.2 Timing of events

The timing of events and the actions available to the agents are summarized in figure 2. At $t = 4$, decisions are made by the actors controlling the decision rights in the chosen governance structure. Let $m_i$ and $m_j$ be the messages sent by the local agents (regarding the realization of $\theta_i$ and $\theta_j$, respectively) at $t = 3$ to the decision-maker(s). Then, the local agent $i$ solves

$$\min_{d^i} E(L_i|\theta_i, m_i, m_j),$$

where $d^i$ is the (potentially empty) set of decision-rights held by agent $i$ under a given governance structure. Similarly, the manager solves

$$\min_{d^M} E(L_i + L_j|m_i, m_j),$$

where $d^M$ is the (potentially empty) set of decision-rights held by the manager under a given governance structure.

At $t = 3$, communication takes place, which is modeled as one round of simultaneous cheap talk.\footnote{Rantakari (2006b) discusses both sequential and continued communication. Sequential communication is never preferred when the variances are sufficiently symmetric. Further, the intended first-mover prefers to talk first over talking simultaneously only when sequential communication performs worse, independent of the underlying variances.} Foreseeing how equilibrium decisions are formed at $t = 4$ and how they are influenced by additional information, the local agents strategically send non-verifiable messages to the decision-maker(s) in an attempt to induce a more favorable
equilibrium outcome. No interim contracting on the decisions or renegotiation of the governance structure is allowed.

At $t = 2$, the local conditions $\theta_i$ and $\theta_j$ are learned by the respective local agents.\footnote{Imperfect information is discussed in section 7.2 and Rantakari (2006a) extends the framework to account for endogenous information acquisition.}

At $t = 1$, the governance structure is chosen to minimize the total expected loss $E(L_i + L_j)$. We follow the incomplete contracting tradition in assuming that no state- or message-contingent contracts on either decisions or decision rights are available at $t = 1$.

4 Decentralized Authority

We will begin the analysis by deriving the solution under decentralized authority, where each local agent retains control over his activity. We first solve for the equilibrium decisions given the information available to the agents. Having the equilibrium decisions, we then solve for the highest sustainable quality of communication, which is directly dependent on the equilibrium decisions. Having the equilibrium decisions and the quality of communication, we will then analyze the expected performance of decentralized authority relative to the first-best outcome (decisions that minimize $E(L_i + L_j)$ under perfect information). Section 5 repeats this exercise for the alternative governance structures and the comparative analysis of the governance structures is performed in section 6. Throughout sections 4 and 5, we analyze $E(L_i^\theta + L_j^\theta)$ under the assumption that $\overline{\theta}_i = \overline{\theta}_j$. The impact of asymmetric variances is discussed in section 6.1.
4.1 Equilibrium Decisions

At the decision-making stage, the information available to agent \( i \) consists of (i) the realization of his local state \( \theta_i \), (ii) message \( m_j \) received from agent \( j \), used by agent \( i \) to form beliefs over \( \theta_j \) (denoted \( E_i \theta_j \)), and (iii) message \( m_i \) sent to agent \( j \), used by agent \( j \) to form beliefs over the realization of \( \theta_i \) (\( E_j \theta_i \)). Given this information, agent \( i \) solves

\[
\min_{d_i} E_i \left( (1 - r_i) (\theta_i - d_i)^2 + r_i (d_j - d_i)^2 \right).
\]

Taking the first-order conditions and rearranging gives the reaction functions for the two decisions. These are given by

\[
d_i = (1 - r_i) \theta_i + r_i E_i d_j \quad \text{and} \quad d_j = (1 - r_j) \theta_j + r_j E_j d_i.
\]

Solving the reaction functions for the equilibrium decisions yields the following proposition:

Proposition 1 Equilibrium decisions under decentralized authority:

\[
d_i^{\text{dec}} = (1 - r_i) \theta_i + \frac{(1 - r_i) r_i}{(1 - r_i r_j)} E_i \theta_j + \frac{(1 - r_i) r_j}{(1 - r_i r_j)} E_j \theta_i.
\]

Proof. Special case of proposition 9. □

Note that the equilibrium decisions take the form \( d_i^{\text{dec}} = a_{i1} \theta_i + a_{i2} E_i \theta_j + a_{i3} E_j \theta_i \) (with \( a_{i1} + a_{i2} + a_{i3} = 1 \)). Indeed, this structure of the solution is the same under all analyzed governance structures, but with structure-specific coefficients \( a_{i1}^g, a_{i2}^g, \) and \( a_{i3}^g \). Also, observe that \( E_i \theta_j \) and \( E_j \theta_i \) will be based solely on the messages \( m_j \) and \( m_i \) exchanged at \( t = 3 \), and as a result the quality of decisions will depend on the accuracy of communication. However, before analyzing the accuracy of communication that the agents can sustain in equilibrium, let us first consider explicitly how the equilibrium responses of \( d_i \) and \( d_j \) to information about \( \theta_i \) are determined, to better understand the role of communication under decentralized authority.
4.1.1 Determination of the equilibrium coefficients

Having observed $\theta_i$ and absent any communication (so that $E_id_j = E_jd_i = 0$), agent $i$’s decision would be given by $d_i = (1 - r_i)\theta_i$. We will refer to the first coefficient of proposition 1, $a_{i1} = (1 - r_i)$, as the rate of direct adaptation.

By communicating information about the realization of $\theta_i$, agent $i$ is able to improve both the coordination between the decisions and the amount of adaptation he is able to achieve. Suppose agent $i$ has sent a message $m_i$ to agent $j$, inducing a belief $E_j\theta_i$. The message $m_i$ can be seen as triggering the following iterated best-response process. Given agent $i$’s first-order condition for $d_i$, agent $j$ believes that absent any response by him, agent $i$ will choose $d_i = (1 - r_i) E_j\theta_i$. Because agent $j$ puts weight $r_j$ on coordination, he will accommodate $d_i$ by $r_j (1 - r_i) E_j\theta_i$. Given that the message is informative, this improves the coordination between the decisions. Agent $i$, in turn, rationally expects this accommodation by agent $j$ and will increase the amount of adaptation by $r_i r_j (1 - r_i) E_j\theta_i$. Adaptation is thus improved due to the accommodation by the opponent. Because agent $i$ increases the amount of adaptation due to the accommodation by agent $j$, agent $j$ in turn increases the amount of accommodation and so forth. Solving this iterative process gives then the coefficients $a_{j2}$, which we will refer to as the rate of accommodation, and $a_{i3}$, which we will refer to as the rate of induced adaptation. The sum $a_{i1} + a_{i3} = a_{i1+i3}$ will be called simply the rate of adaptation. An equivalent process over the beliefs over $\theta_j$ determines the weights $a_{j1}, a_{i2}$ and $a_{j3}$.

Because of the strategic interaction between the decisions, the decisions will in expectation lie somewhere between agent $i$’s preferred decisions $d_i = d_j = \theta_i$ and agent $j$’s preferred decisions $d_i = d_j = \theta_j$. However, how far the equilibrium decisions are from each agent’s ideal point depends on how much weight each agent places on coordination and, as a result, on the rates of accommodation. As $r_i$ increases, any adaptation achieved by agent $i$ becomes increasingly dependent on accommodation by agent $j$. As a result, agent $j$ is able to decrease his rate of accommodation $a_{j2}$ and so move the equilibrium decisions in his favor. Similarly, as $r_j$ increases, the increased dependency of agent $j$ increases his rate of accommodation $a_{j2}$, allowing agent $i$ to move the equilibrium decisions towards his ideal decisions. Finally, along the diagonal $r_i = r_j = r$, each agent becomes increasingly dependent on the accommodation granted by the other, but at the same time each agent becomes more accommodating:
$a_{i2} = a_{j2}$ is increasing in $r$.

Not only do $a_{i2}$ and $a_{j2}$ play a large role in the determination of the equilibrium decisions, but as we will see below, as a measure of the incentive conflict between the two agents, these coefficients are the sole determinants of the quality of communication by the two agents under decentralized authority.

### 4.2 Equilibrium Communication

As discussed above, agent $i$ sends a message $m_i$ about the realization of $\theta_i$ to achieve accommodation by $d_j$, which in turn allows $i$ to achieve more adaptation. The discussion also made apparent the problem that the non-verifiability of information generates in the communication stage. In the decision-making stage, agent $i$ would like to have $d_i = d_j = \theta_i$ and so achieve both perfect adaptation and perfect coordination. On the other hand, agent $j$’s expected response to a message $m_i$ is given by $a_{j2}E_j(\theta_i|m_i)$, where the rate of accommodation $a_{j2}$ is strictly below one for all $r_j < 1$. As a result, if agent $j$ expects agent $i$ to tell the truth, agent $i$ will exaggerate the realized $\theta_i$ to induce a higher level of accommodation by agent $j$, making fully informative communication impossible.

However, as shown by Crawford and Sobel (1982), partially informative communication can still potentially be achieved. This partially informative communication is achieved by partitioning the state-space so that any message $m_i$ reveals only that $\theta_i$ belongs to some interval. Intuitively, such partitioning discretizes the response of the recipient so that the sender can be made to choose between an under-response from a lower message and an over-response from a higher message.

From this logic it is clear that for a partition to be incentive-compatible, it needs to be that when the realized state falls on the boundary between two elements (intervals) of the partition, the agent is indifferent between saying that the state belongs to either one of the intervals. This construction is illustrated in figure 3. Let the realized state be $\theta_i^M$, on the boundary between two intervals of a partition, $(\theta_i^L, \theta_i^M]$ and $(\theta_i^M, \theta_i^H]$. The agent now faces a choice between sending a message $m_i^L$, which reveals that $\theta_i \in (\theta_i^L, \theta_i^M]$, inducing a belief $E_j^L \theta_i$ and an expected response of $E_id_j^L$, and a message $m_i^H$, which reveals that $\theta_i \in (\theta_i^M, \theta_i^H]$, inducing a belief $E_j^H \theta_i$ and an expected response of $E_id_j^H$. Then, for incentive-compatibility, it needs to be that
\[ E_i L_i (\theta_i^M, d_i(., E_j^L \theta_i), d_j(., E_j^L \theta_i)) = E_i L_i (\theta_i^M, d_i(., E_j^H \theta_i), d_j(., E_j^H \theta_i)) \, . \]

Solving this indifference condition gives us the difference equation that defines the family of incentive-compatible partitions:

\[ |\theta_{i,k} - \theta_{i,k-1}| - |\theta_{i,k-1} - \theta_{i,k-2}| = \frac{4}{\varphi_{i}} |\theta_{i,k-1} - E_i \theta_j| , \]

where \(\theta_{i,k}\) are the cutoffs of the partition, with \(k\) increasing away from the expected preference intersection \(\theta_i = E_i \theta_j\), and

\[ \varphi_{i} = \frac{a_{i,j}}{1 - a_{i,j}} = \frac{r_j(1-r_i)}{(1-r_j)} , \]

which uniquely determines the rate at which the size of the intervals needs to grow to counter the increasing incentives for \(i\) to exaggerate the realized \(\theta_i\). In the case
Figure 4: Structure of the most informative partition

of simultaneous talk, \( E_i \theta_j = 0 \) before the messages are sent, which we will maintain from now on.

While \( \varphi_i^{dec} \) defines the relative size of two adjacent intervals, it does not yet define the absolute size of the intervals. Solving for the most informative partition, which minimizes the absolute size of the intervals given \( \varphi_i^{dec} \), gives the following proposition:

**Proposition 2** Equilibrium quality of communication:

The cutoffs of the finest incentive-compatible partition are characterized by

\[
|\theta_{i,n}| = \alpha \left( \varphi_i^{dec} \right)^{|n|} \overline{\theta}_i \quad \text{with} \quad n \in \{-\infty, ..., -1, 1, ... \infty\}, \quad \text{where}
\]

\[
\alpha \left( \varphi_i^{dec} \right) = \frac{\varphi_i^{dec}}{(1 + \sqrt{1 + \varphi_i^{dec}})} \in (0, 1) \quad \text{and} \quad \varphi_i^{dec} = \frac{a_j \beta}{1 - a_j} = \frac{r^j (1 - r_i)}{(1 - r_j)} \in (0, \infty).
\]

**Proof.** Special case of proposition 10. ■

This solution is illustrated in figure 4. Note that truth-telling is possible at the point of expected preference intersection: the expected response of the recipient matches the ideal response of the sender. The rate at which the accuracy of messages decreases in \(|\theta_i|\) is in turn uniquely determined by \( \varphi_i \), which we will refer to as the quality of communication (by agent \( i \) about \( \theta_i \)). As \( \alpha \left( \varphi_i^{dec} \right) \rightarrow 1 \), communication becomes fully informative, while \( \alpha \left( \varphi_i^{dec} \right) \rightarrow 0 \) implies binomial communication, which is always possible in the present setting due to the partial alignment of interests (i.e. the sender is always able to say whether his realized state is above or below the expected preference intersection).

The quality of communication by agent \( i \) is in turn solely determined by the rate of accommodation by the recipient, \( a_j \). This result should be no surprise since, as discussed in section 4.1.1, it is the rate of accommodation that measures under
decentralized authority the degree of conflict between the sender’s preferred outcome and the recipient’s equilibrium response. The higher the rate of accommodation, the smaller the incentive conflict and the higher the quality of communication that can be sustained. As the recipient becomes perfectly accommodating ($a_{j2} \to 1$), communication becomes perfect. Thus, in addition to influencing the equilibrium decisions, the relative importance of coordination to the two agents (and the resulting rates of accommodation) influence the ability of the agents to share their private information. Indeed, it is exactly because the equilibrium decisions vary with the importance of coordination that the quality of communication by each agent varies as well.

Figure 5 plots the quality of communication by agent $i$ as a function $r_i$ and $r_j$. Paralleling the rate of accommodation by agent $j$, the quality of communication by agent $i$ is increasing in $r_j$, decreasing in $r_i$ and increasing along the diagonal $r_i = r_j$. Thus, not only does an increase in $r_i$ tilt the equilibrium decisions in favor of agent $j$, but it also compromises the ability of agent $i$ to share his private information. Communication by agent $i$ is accurate only when he expects the equilibrium decisions to be close to his ideal decisions $d_i = d_j = \theta_i$. 

Figure 5: Quality of communication by agent $i$ under decentralized authority

Paralleling the rate of accommodation by agent $j$, the quality of communication by agent $i$ is increasing in $r_j$, decreasing in $r_i$ and increasing along the diagonal $r_i = r_j$. Thus, not only does an increase in $r_i$ tilt the equilibrium decisions in favor of agent $j$, but it also compromises the ability of agent $i$ to share his private information. Communication by agent $i$ is accurate only when he expects the equilibrium decisions to be close to his ideal decisions $d_i = d_j = \theta_i$. 

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4.3 Expected Losses

Having derived the equilibrium decisions and the equilibrium quality of communication, we can solve for the expected loss for each activity:

**Proposition 3 Expected Losses under Decentralized Authority:**

\[
EL_i^{\text{dec}} = \Lambda_i^{\text{dec}} \left( A(\varphi_i^{\text{dec}})\bar{\theta}_i^2 + A(\varphi_j^{\text{dec}})\bar{\theta}_j^2 \right) + \Lambda_i^{\text{dec}} B(\varphi_i^{\text{dec}})\bar{\theta}_i^2 + \Lambda_i^{\text{dec}} B(\varphi_j^{\text{dec}})\bar{\theta}_j^2, \quad \text{where}
\]

\[
\Lambda_i^{\text{dec}} = \frac{(1-r_i)r_j(1-r_j)^2}{(1-r_ir_j)^2}, \quad \Lambda_{i-j}^{\text{dec}} = r_i (1 - r_i), \quad \Lambda_{i-j}^{\text{dec}} = r_j (1 - r_j)^2,
\]

\[
A (\varphi_i^{\text{dec}})\bar{\theta}_i^2 = \frac{(1-a_i(\varphi_i^{\text{dec}}))(1+a_i(\varphi_i^{\text{dec}}))^2}{4(1-a_i(\varphi_i^{\text{dec}})^3)} \bar{\theta}_i^2 = E (E_{\text{receiver}} \theta_i)^2,
\]

\[
B (\varphi_i^{\text{dec}})\bar{\theta}_i^2 = \frac{(1-a_i(\varphi_i^{\text{dec}}))^3}{12(1-a_i(\varphi_i^{\text{dec}})^3)} \bar{\theta}_i^2 = E (\theta_i - E_{\text{receiver}} \theta_i)^2.
\]

**Proof.** Special case of proposition 11. ■

To understand the form of the expected loss, consider first the case when communication by both agents is perfect. In this case \(A(\cdot) = 1/3\) and \(B(\cdot) = 0\). Thus, \(\Lambda_i^{\text{dec}}\) measures the equilibrium expected loss to activity \(i\) under perfect information. We can see that \(\Lambda_i^{\text{dec}}\) is decreasing in \(r_j\). The more accommodating agent \(j\) is, the more the equilibrium decisions favor activity \(i\). Also, \(\Lambda_i^{\text{dec}} \to 0\) both when \(r_i \to 0\) and when \(r_i \to 1\). When coordination is unimportant, adaptation to local conditions is only mildly constrained. When coordination is very important, most gains are realized simply by accommodating what agent \(j\) is going to do.

Consider now the additional losses caused by inaccurate \(m_i\). First, as long as the rate of direct adaptation \(a_{i1}\) is positive, agent \(j\) is unable to perfectly predict \(d_i\). This inaccuracy leads to strategic uncertainty in the decision-making stage, which provides an additional source of coordination failures. Second, the component of induced adaptation in \(d_i\) is now based on an inaccurate message \(m_i\) instead of the true state \(\theta_i\), leading to additional failures in adaptation. Since \(A(\cdot) + B(\cdot) = 1/3\), the cost of inaccurate communication about \(\theta_i\) to activity \(i\) is given by \((\Lambda_{i-i}^{\text{dec}} - \Lambda_{i}^{\text{dec}}) B (\varphi_i^{\text{dec}})\). In similar fashion, the inaccuracy of \(m_j\) leads to strategic uncertainty as long as \(a_{j1}\)
is positive. However, agent $i$ also benefits from this inaccuracy since it reduces the expected cost of accommodation. The net effect is given by \((\Lambda_{i-j} - \Lambda_{i}^{dec}) B (\varphi_{j}^{dec})\).\(^7\)

To better understand how the expected total loss \(E (L_{i}^{dec} + L_{j}^{dec})\) varies with the environment, define the relative loss due to biased decisions by

\[
\frac{E(L_{i}^{dec} + L_{j}^{dec}|\alpha(.)=1) - E(L_{i}^{FB} + L_{j}^{FB})}{E(L_{i}^{FB} + L_{j}^{FB})}
\]

(i.e., the percentage increase in the expected total loss over the first-best outcome when decentralized decision-making is substituted for optimal decisions under perfect information). Similarly, define the relative loss due to strategic communication by

\[
\frac{E(L_{i}^{dec} + L_{j}^{dec}|\varphi^{dec}) - E(L_{i}^{dec} + L_{j}^{dec}|\alpha(.)=1)}{E(L_{i}^{FB} + L_{j}^{FB})}
\]

(i.e., the percentage increase in the expected total loss over the first-best outcome when strategic communication is substituted for perfect information under decentralized decision-making). Note that because the expected loss under the first-best outcome is also bound away from zero due to the technological tradeoff between adaptation and coordination, any losses must be measured in relation to this standard. The additional normalization by \(E (L_{i}^{FB} + L_{j}^{FB})\) is used to highlight differences in the limit behavior when the absolute loss converges to zero. Thus, it is worth noting that while the discussion below is framed in terms of relative loss, the absolute expected loss does converge to zero both when \(r_{i}, r_{j} \to 0\) and when \(r_{i}\) and/or \(r_{j} \to 1\).

The results are summarized in figure 6 and we will discuss each panel separately.

4.3.1 Relative loss due to biased decisions

Panel (i) of figure 6 plots the relative loss due to biased decisions. First, observe that the equilibrium decisions converge to first-best both when \(r_{i}\) and \(r_{j} \to 0\), and when \(r_{i}\) and/or \(r_{j} \to 1\). In the first case, each agent is able to make the individually optimal decision as no interdependencies are present. Similarly, as long as at least one

\(^7\)However, \(\Lambda_{i-j}^{dec} - \Lambda_{i}^{dec} - \Lambda_{j}^{dec} > 0\), so that more accurate communication is always beneficial in terms of the total expected loss. The complications that can arise from \(\Lambda_{j-i}^{dec} - \Lambda_{j}^{dec} < 0\) are discussed in section 7.1.
agent cares only about coordination, it is optimal to coordinate around the activity that places a positive value on adaptation. Thus, decision-making itself significantly compromises the performance of decentralized authority only when at least one agent faces an intermediate value for coordination and neither agent is extremely dependent on coordination.

The nature and cost of these biases, however, depend on the degree of asymmetry between the agents. Because each agent undervalues coordination, the decisions are in expectation too far apart. When the importance of coordination is relatively symmetric, this translates to a situation where each agent undertakes an excessive amount of adaptation. To return to our example of a firm with two divisions, each division, if allowed full autonomy, is tailoring their product and processes too closely to their individual needs and as a result fails to realize the cost-savings that could be achieved through better coordination of activities.

However, as discussed in section 4.1.1, when the importance of coordination is asymmetric, the less dependent agent is able to pull both decisions in his favor. While this outcome benefits the less dependent agent, it is extremely damaging to the more dependent agent: not only are the decisions under-coordinated, but they are now coordinated around agent i's preferred outcome. In terms of our example, a division that places only a little value on coordination will naturally make choices that are closely tailored to match its local needs, independent of what the other division

Figure 6: Expected performance of decentralized authority
is doing. As a result, the division that does benefit from coordination is forced to use as the basis for coordination decisions that are highly unsuitable to its local needs, which in turn significantly compromises any possibilities of valuable customization.

4.3.2 Relative loss due to strategic communication

Panel (ii) of figure 6 plots the relative loss due to biased decisions. Unlike the relative loss due to biased decisions, the relative loss due to strategic communication is almost everywhere increasing in the importance of coordination to either agent, in contrast with the quality of communication discussed in section 4.2. This result highlights the basic observation that accurate communication is not valuable in itself. It is valuable only to the extent that it improves decision-making. Thus, when both agents face a low importance of coordination, while communication is very inaccurate, it is also unimportant because each agent has direct access to their local information and adaptation is primarily direct (independent of accommodation). However, when coordination becomes more valuable, the inaccuracies in communication start taking their toll through the strategic uncertainty that gets introduced into the decision-making process. Even if information is shared and adjustments to this information are made, the final outputs don’t quite fit together as well as they could have. While the quality of communication is also increasing in the importance of coordination, it is not increasing fast enough to compensate for the increased cost of such coordination failures.

5 Alternative Governance Structures

As shown in Appendix A, the technical structure of the solution under the alternative governance structures is the same as under decentralized authority. In particular, the equilibrium decisions under governance structure $g$ take the form

$$d_m^i = a_{i_1}^g E_m \theta_i + a_{i_2}^g E_m \theta_j + a_{i_3}^g E_n \theta_l;$$

where $m$ and $n$ are the agents controlling the decision right for activities $i$ and $j$, 

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respectively, with \( a_{i1}^g + a_{i2}^g + a_{i3}^g = 1 \). Similarly, the indifference condition defining the communication equilibrium under governance structure \( g \) takes the form

\[
|\theta_{i,k} - \theta_{i,k-1}| - |\theta_{i,k-1} - \theta_{i,k-2}| = \frac{4}{\varphi_{i}^g} |\theta_{k-1}| ,
\]

and thus the most informative partition is characterized by

\[
|\theta_i^g| = \alpha(g_i^g)^{|n|} \bar{b}_i , \text{ where } \alpha(g_i^g) = \frac{\varphi_i^g}{(1+\sqrt{1+\varphi_i^g})} \in (0, 1) .
\]

As a result, we will present only the constants \( \varphi_i^g \). Finally, we can write the expected loss to activity \( i \) under governance structure \( g \) as

\[
EL_i^g = \Lambda_i^g \left( A(\varphi_i^g)\bar{b}_i^2 + A(\varphi_j^g)\bar{b}_j^2 \right) + \Lambda_{i-i}^g B(\varphi_i^g)\bar{b}_i^2 + \Lambda_{i-j}^g B(\varphi_j^g)\bar{b}_j^2 .
\]

As a result, we will present only the constants \( \Lambda_i^g, \Lambda_{i-i}^g \) and \( \Lambda_{i-j}^g \) when discussing the expected performance of governance structure \( g \).

While the structure of the solution is similar across the governance structures, the solutions themselves exhibit significant differences in their expected performance \( E \left( L_i^g + L_j^g \right) \). These differences arise for four interrelated reasons. First, the differences in the objective function(s) of the decision-maker(s) across the governance structures translate into systematic differences in the equilibrium decisions. Second, these differences in equilibrium decisions translate into differences in the equilibrium quality of communication by the two local agents. Third, whether agent \( i \) retains control of his activity or not directly impacts his qualitative motives for communication and thus his equilibrium quality of communication. Fourth, the differences in equilibrium decisions and the motives for communication translate into differences in the value of communication. For example, accurate communication is not valuable if the decision-maker(s) have no incentives to use the information communicated.

Because each governance structure provides a qualitatively different combination of equilibrium decisions and quality and value of communication, the relative performance of each governance structure varies systematically with the underlying environment. In consequence, each governance structure arises as the preferred (second-best) governance structure under specific environmental conditions. In the remainder of the
section, we will analyze the equilibrium outcomes under the alternative governance structures of centralized authority, partial centralization and directional authority. We will focus on the differences in the equilibrium decisions and the quality of communication, and the resulting reasons for and conditions under which each of them can arise as the preferred governance structure. The results of sections 4 and 5 are then brought together in section 6, where Figure 12 presents the mapping from \((r_i, r_j)\) to the preferred governance structure.

5.1 Centralized Authority

The outcome under centralized authority is summarized in the following proposition:

**Proposition 4 Equilibrium under centralized authority:**

The equilibrium decisions under centralized authority are given by

\[
d_{\text{cent}}^i = \frac{(1-r_i^2)E_{M\theta_i}+(r_i+r_j)(1-r_j)E_{M\theta_j}}{(1+r_i)(1+r_j)-(r_i+r_j)^2}
\]

and the quality of communication is determined by

\[
\varphi_{\text{cent}}^i = \frac{(1-r_i)(a_{11+13}^\text{cent})^2+r_i(a_{11+13}^\text{cent}-a_{22}^\text{cent})^2}{(1-r_i)a_{11+13}^\text{cent}(1-a_{11+13}^\text{cent})-r_i(a_{11+13}^\text{cent}-a_{22}^\text{cent})^2} = \frac{(1-r_i)(1+r_i)^2+r_i(1-r_j)^2}{(1-r_j)(r_j+2r_ir_j+r_j^2)}.
\]

The coefficients for the expected loss are

\[
\Lambda_{i\text{cent}} = \frac{(1-r_i)(1-r_j)^2(r_i+2r_ir_j+r_j^2)}{(1+r_i)(1+r_j)-(r_i+r_j)^2}, \quad \Lambda_{i\text{cent}}^{i-i} = (1-r_i) \quad \text{and} \quad \Lambda_{i\text{cent}}^{i-j} = 0
\]

**Proof.** Special case of propositions 9, 10 and 11. □

The decisions are, by assumption, optimal conditional on the information available to the manager. The manager, however, has to rely solely on information communicated to her by the local agents and as a result the coefficients of direct and induced accommodation are pooled together in the equilibrium decisions. This difference in the information structure, in turn, alters the incentives for communication.
Recall that under decentralized authority, agent $i$ communicated to induce accommodation by agent $j$, which both improved coordination and allowed agent $i$ to achieve more adaptation. As a result, the quality of communication was uniquely determined by the rate of accommodation $a^\text{dec}_{j2}$. Under centralized authority, sending a message $m_i$ induces adaptation by the manager (with $d_i$) at the rate $a^\text{cent}_{i1+i3}$ and accommodation (with $d_j$) at the rate $a^\text{cent}_{j2}$. This change in the qualitative motives for communication generates two primary differences compared to decentralized authority.

First, from the perspective of the adaptation component, it is the rate of adaptation $a^\text{cent}_{i1+i3}$ instead of the rate of accommodation $a^\text{cent}_{j2}$ that measures the degree of incentive conflict between the sender (agent $i$) and the receiver (manager). Intuitively, in the case of decentralized authority, the ability of agent $i$ to achieve adaptation was constrained by the amount of accommodation by agent $j$. In the case of centralized authority, it is constrained because the agent no longer controls the decision and so has to persuade the manager to undertake any adaptation to $\theta_i$.

Second, the coordination component further limits the incentives to exaggerate under centralized authority. This is because $a^\text{cent}_{i1+i3} - a^\text{cent}_{j2} > 0$, so that the decisions diverge in $|m_i|$. The constraining influence of the coordination component on agent $i$’s incentive to exaggerate is strongest when $r_i$ is intermediate and $r_j$ is low to intermediate, so that the decisions do diverge significantly with the messages sent and the sender cares about it. For low $r_i$, agent $i$ doesn’t care about coordination, while when either $r_i$ or $r_j$ is large, the two decisions remain quite close together, independent of $|m_i|$. This logic explains the additional term $r_i \left(a^\text{cent}_{i1+i3} - a^\text{cent}_{j2}\right)^2$ that enters the equation defining the quality of communication under centralized authority.

These differences in the motives for communication are reflected in the quality of communication by agent $i$, summarized in figure 7. In particular, we can see that while the quality of communication under centralized authority is always higher than under decentralized authority, this difference is decreasing in the importance of coordination to either agent. In particular, the quality of communication is decreasing in $r_i = r_j$ under centralized authority while it is increasing in $r_i = r_j$ under decentralized authority. Under centralized authority, as the importance of coordination increases, the manager becomes less adaptive. This reduction in $a^\text{cent}_{i1+i3}$ increases the incentive conflict between the manager and agent $i$ and thus worsens the quality of communication. In contrast, under decentralized authority, as the importance of coordination increases, the local agents become increasingly accommodating. This increase in $a^\text{cent}_{j2}$...
in turn leads to an increase in the quality of communication. Indeed, the two solutions converge as $r_j = r_i \rightarrow 1$.

Figure 8 decomposes the expected total loss $E(L_{i}^{cent} + L_{j}^{cent})$ into the relative loss due to biased decisions and the relative loss due to strategic communication. By assumption, the relative loss due to biased decisions is zero. Also, since the manager controls both decisions, there is no strategic uncertainty in the decision-making stage. As a result, $\Lambda_{i-j}^{cent} = 0$. However, because the manager is initially uninformed, any inaccuracies in communication now translate directly into fundamental uncertainty about the realized states of the world in the decision-making stage: $E(\theta_i - E_M\theta_i|m_i)^2 > 0$ whenever $\alpha(\varphi_i^{cent}) < 1$. As a result, while the quality of communication by both local agents is higher under centralized authority, the value of more accurate communication about either state is higher as well. In terms of the expected losses, we can see this by noting that

$$\left(\Lambda_{i-i}^{cent} - \Lambda_{i}^{cent} - \Lambda_{j}^{cent}\right) B(\varphi_i^*) > \left(\Lambda_{i-i}^{dec} + \Lambda_{j-i}^{dec} - \Lambda_{i}^{dec} - \Lambda_{j}^{dec}\right) B(\varphi_i^*) \quad \forall r_i, r_j < 1.$$ 

That is, the net cost of fundamental uncertainty under centralized authority is always higher than the net cost of the same level of strategic uncertainty under decentralized authority. This result is reflected in panel (ii), which shows that despite the higher quality of communication, the relative loss due to strategic communication is almost everywhere higher than under decentralized authority. Even if the quality of commu-
nication by both agents tends to perfect as \( r_i, r_j \to 0 \), it is not increasing fast enough to compensate for the increasing value of communication.

As a result, decentralized authority dominates centralized authority whenever the importance of coordination is sufficiently low to both activities. Conversely, centralized authority dominates decentralized authority whenever the importance of coordination is sufficiently high to either activity. However, this advantage derives solely from the elimination of the bias in the equilibrium decisions, not from the quality of information itself. As a consequence, the relative advantage of decentralized authority is largest when at least one activity faces an intermediate value for coordination. The two solutions converge when \( r_i \) and/or \( r_j \to 1 \), so that the local agents become increasingly able to coordinate their decisions under decentralized authority.

Figure 8: Expected performance of centralized authority
5.2 Partial Centralization

Under partial centralization, only one of the decisions is centralized. Thus, we need to look separately at the outcomes for the agent retaining control and for the agent losing control. We will discuss the case where activity \( j \) is centralized. The outcome under partial centralization is summarized in the following proposition:

**Proposition 5 Equilibrium under partial centralization (of \( j \)):**

The equilibrium decisions are given by

\[
d_{i}^{\text{part}(j)} = (1 - r_{i}) \theta_{i} + \frac{r_{i}(1 - r_{i})E_{i}E_{Mj} + r_{i}(r_{j} - r_{i})(1 - r_{i})E_{Mj}}{(1 - r_{i}) - r_{i}(r_{j} + r_{i})} \quad \text{and} \quad d_{j}^{\text{part}(j)} = \frac{(1 - r_{j})E_{Mj} + (r_{j} - r_{i})(1 - r_{i})E_{Mj}}{(1 + r_{i}) - r_{i}(r_{j} + r_{i})}.
\]

The quality of communication by the agent retaining control is given by

\[
\varphi_{i}^{\text{part}(j)} = \frac{a_{i}^{\text{part}(j)}}{1 - a_{j}^{\text{part}(j)}} = \frac{(r_{i} - r_{j})(1 - r_{i})}{(1 - r_{j})},
\]

while the quality of communication by the agent losing control is given by

\[
\varphi_{j}^{\text{part}(j)} = \frac{(1 - r_{j})\left(a_{i}^{\text{part}(j)} + r_{j}\left(a_{i}^{\text{part}(j)} - a_{j}^{\text{part}(j)}\right)\right)^{2}}{(1 - r_{j})\left(a_{i}^{\text{part}(j)} + r_{j}\left(a_{i}^{\text{part}(j)} - a_{j}^{\text{part}(j)}\right)\right) - r_{j}\left(a_{i}^{\text{part}(j)} - a_{j}^{\text{part}(j)}\right)} = \frac{1 - 2r_{i}r_{j} + r_{i}^{2}r_{j}}{(1 - r_{i})r_{j}(1 + r_{j})}.
\]

The coefficients for the expected loss are

\[
\Lambda_{i}^{\text{part}(j)} = \frac{(1 - r_{i})r_{j}(1 - r_{j})^{2}}{(1 - r_{i}r_{j} + r_{i}(1 - r_{j}))^{2}}, \quad \Lambda_{i-i}^{\text{part}(j)} = r_{i}(1 - r_{i}), \quad \Lambda_{i-j}^{\text{part}(j)} = 0, \quad \Lambda_{j}^{\text{part}(j)} = \frac{(1 - r_{j})^{2}(1 - r_{j})(r_{i} + 2r_{j}r_{j} + r_{j}^{2})}{(1 - r_{i}r_{j} + r_{i}(1 - r_{j}))^{2}}.
\]

The coefficients for the expected loss are

\[
\Lambda_{j}^{\text{part}(j)} = \frac{(1 - r_{j})^{2}(1 - r_{j})(r_{i} + 2r_{j}r_{j} + r_{j}^{2})}{(1 - r_{i}r_{j} + r_{i}(1 - r_{j}))^{2}}, \quad \Lambda_{j-j}^{\text{part}(j)} = (1 - r_{j}) \quad \text{and} \quad \Lambda_{j-i}^{\text{part}(j)} = r_{j}(1 - r_{i})^{2}.
\]

**Proof.** Special case of propositions 9, 10 and 11. ■

Figure 9 decomposes the expected total loss \( E\left(L_{i}^{\text{part}(j)} + L_{j}^{\text{part}(j)}\right) \) into the relative loss due to biased decisions and the relative loss due to strategic communication. Consider first the relative loss due to biased decisions. Since agent \( i \) undervalues
coordinated, the equilibrium decisions remain under-coordinated. Also, the equilibrium decisions are always biased in favor of the agent retaining control. To see this result, note that agent $i$ places a weight $r_i$ on coordination while the manager places a weight $r_i + r_j$ on coordination. In consequence, the manager is relatively more accommodating, which in turn allows agent $i$ to pull the equilibrium decisions in his favor. This outcome stands in contrast to decentralized authority, where the equilibrium decisions favored the less dependent agent.

From the perspective of expected performance, the most important result is that as $r_j \to 0$, the equilibrium decisions under partial centralization converge to optimal decisions. Because the biases in decision-making arise from the local agents not internalizing the value of coordination to the opponent, as $r_j \to 0$, $d_i$ becomes optimal conditional on $d_j$. Thus, to achieve appropriate decision-making at the limit, it is sufficient to centralize $d_j$. In general, centralizing the less dependent activity can be used to improve decision-making relative to the decentralized outcome, given a sufficient initial asymmetry.\footnote{Because centralization causes a discrete change in the valuation of coordination by the decision-maker and so in the equilibrium rate of accommodation, the initial asymmetry needs to be sufficiently large as not to tilt the bias in the opposite direction following centralization.} Equivalently, centralizing the more dependent activity will bias the equilibrium decisions even further in favor of the less dependent activity, leading to an even higher relative loss due to biased decisions than decentralized

Figure 9: Expected performance of partial centralization (of $j$)
To understand the relative loss due to strategic communication, note that it is now a mix of strategic and fundamental uncertainty. Let us first consider the quality of communication by the two agents, summarized in figure 10. Note that the problem faced by agent $i$ (retaining control) is analogous to the problem he faces under decentralized authority, in that he sends $m_i$ to induce accommodation, but now by the manager. The only difference is that the manager is more accommodating compared to agent $j$, and as a result, the quality of communication by agent $i$ is always higher than under decentralized authority. Further, the quality of communication can also be better than under centralized authority. This happens when $r_j$ is large, so that the rate of accommodation is large and the constraining effect of the coordination component under centralized authority would be small.

Similarly, the problem faced by agent $j$ (losing control) is analogous to the problem he faces under centralized authority, in that he sends $m_j$ to induce both adaptation by $d_j$ and accommodation by $d_i$ (which in turn produces additional adaptation). The differences are two-fold. First, because of the constraining role played by the agent

Figure 10: Quality of communication under partial centralization

...
retaining control, the manager is less responsive to j’s messages, increasing the incentive conflict between the manager and agent j. Second, because of the reduced coordination between the decisions, the constraint on exaggeration provided by the coordination component is stronger. As a consequence, the quality of communication by agent j under partial centralization can either dominate his quality of communication under both decentralized and centralized authority, or be dominated by both.

It dominates both when \( r_j \) is large, so that the amount of adaptation or accommodation achieved by agent j would approach zero under all governance structures and the stronger constraint provided by the coordination component dominates the outcome.

In terms of the total expected loss, partial centralization dominates both symmetric governance structures under two different conditions. First, as mentioned above, centralizing activity \( j \) when \( r_j \) is low is sufficient to eliminate most of the bias in the equilibrium decisions under decentralized authority. At the same time, not centralizing activity \( i \) prevents the introduction of unnecessary fundamental uncertainty into the decision-making process. Even if the quality of communication by both agents is lower than under centralized authority, the overall value of accurate communication is lower as well. This containment of the loss due to strategic communication relative to centralized authority can dominate the cost of the remaining bias in the equilibrium decisions and as a result, partial centralization performs better than both centralized and decentralized authority when \( r_j \) is small enough and \( r_i \) is large enough.

Second, centralizing activity \( j \) when \( r_j \) is large enough will also yield a lower expected total loss than centralized or decentralized authority. Recall that when \( r_j \) is large, the primary source of loss under both decentralized and centralized authority is strategic communication. Under decentralized authority, it is the strategic uncertainty faced by agent \( j \) in the decision-making stage and under centralized authority, it is the fundamental uncertainty faced by the manager over \( \theta_i \) – both caused by inaccurate \( m_i \). Now, as discussed above, centralizing only activity \( j \) will increase the bias in equilibrium decisions relative to decentralized authority. But it is exactly this increase in the bias that improves the quality of communication by agent \( i \), reducing the loss due to strategic communication. Further, the quality of communication by agent \( j \) is also better or only slightly compromised relative to centralized authority. When \( r_j \) is large enough, this improvement in communication dominates the loss caused by the increased bias in the decisions.
5.3 Directional Authority

Under directional authority, one of the local agents controls both decisions. Assuming that both decisions are allocated to agent $i$, the following proposition summarizes the outcome:

**Proposition 6 Equilibrium under directional authority (by $i$):**

The equilibrium decisions are given by

$$d_{\text{dir}}^i = d_{\text{dir}}^j = \theta_i,$$

while the quality of communication is indeterminate, as all information sent by agent $j$ is ignored by agent $i$. The coefficients for the expected loss are

$$\Lambda_{i}^{\text{dir}(i)} = \Lambda_{1-i}^{\text{dir}(i)} = \Lambda_{i-j}^{\text{dir}(i)} = 0,$$

$$\Lambda_{j}^{\text{dir}(i)} = \frac{(1-r_j)}{3} \quad \text{and} \quad \Lambda_{j-j}^{\text{dir}(i)} = \Lambda_{j-i}^{\text{dir}(i)} = 0.$$

**Proof.** Special case of propositions 9, 10 and 11. ■

Figure 11 decomposes the expected total loss $E\left( L_{\text{dir}}^i + L_{\text{dir}}^j \right)$ into the relative loss due to biased decisions and the relative loss due to strategic communication. Recall that agent $i$ aims at minimizing only $L_i$. As a result, if given control of both decisions, he will set both decisions as to optimize his local performance and ignores any messages sent to him by agent $j$. Thus, the decisions are both highly biased and excessively coordinated unless $r_j$ is very high.\(^9\) However, there is no loss due to strategic communication, as no adaptation to $\theta_j$ is attempted. In the region of very high $r_j$, this avoidance of informational losses dominates losses caused by biased decisions, and as a result directional authority arises as the preferred governance structure. Indeed, as $r_j \to 1$, full efficiency is achieved under directional authority while all other governance structures continue to suffer a first-order loss due to strategic communication.

\(^9\)We have capped the relative loss due to biased decisions at 30% to keep the figures comparable.
It might be surprising that we are granting authority to the less dependent agent. However, the result is immediate when we recall that coordination in the model is achieved by accommodating adaptation, and so the locus of coordination should reflect the relative importance of adaptation. As $r_j \to 1$, only adaptation to $\theta_i$ matters, and as a result it is more efficient to allow agent $i$ to control both decisions than to rely on strategic communication as the coordination device.

## 6 Relative Performance

Figure 12 summarizes the mapping from the relative dependency of the two activities to the second-best governance structure. Having already discussed the reasons for the performance differentials through sections 4 and 5, we will now provide only a short summary of the results. First, decentralized authority is the preferred governance structure only when both activities face a low importance of coordination. The equilibrium decisions are only mildly biased and communication, while inaccurate, is also unimportant since each agent has direct access to local information. In contrast, centralized authority is the preferred governance structure when the importance of coordination is intermediate to high and not too asymmetric. The advantage of centralized authority is largest in the region of intermediate and symmetric importance.
of coordination, where the decisions would exhibit significant under-coordination under decentralized authority and the asymmetric governance structures would do even worse. As $r_i$ and/or $r_j \to 1$, the solutions under centralized and decentralized authority converge.

While centralized authority dominates decentralized authority in all regions but low importance of coordination, centralized authority is in turn dominated by at least one of the asymmetric governance structures when the activities face a sufficiently asymmetric importance of coordination. First, centralizing only the less dependent activity can be used to improve decision-making relative to decentralized authority while limiting losses due to strategic communication relative to centralized authority. Second, centralizing only the more dependent activity can be used to worsen decision-making relative to decentralized authority to generate a communication outcome that dominates both centralized and decentralized authority. Third, directional authority by the less dependent agent can dominate all three alternatives when the other activity is highly dependent as it fully avoids losses due to strategic communication.
6.1 Asymmetric variances

So far, we have discussed the results only in the setting of symmetric overall importance of adaptation, as measured by the relative variance of the two states. The case of asymmetric variances is straightforward: if $\bar{\theta}_i$ increases, the likelihood of control by agent $i$ of any activity increases, and the likelihood of control of $d_j$ by the manager increases. These results follow for two reasons. First, as illustrated in the previous sections, the relative quality of communication is constant. As a consequence, the absolute quality of communication is decreasing in the overall variance of states, which makes any garbling of information more damaging. Second, any biases in decision-making going against activity $i$ become more damaging in expectation. As a consequence, the choice of governance structure becomes skewed in agent $i$’s favor when $\bar{\theta}_i > \bar{\theta}_j$.\footnote{Detailed results available from the author on request.}

7 Some Caveats and Extensions

We have made a number of simplifying assumptions in our analysis to allow us to focus in a simple and tractable way on the interaction between decision-making and communication. We have abstracted from at least the following issues: (1) the decisions made need also be implemented, (2) decision-making and communication need not be simultaneous, (3) information is rarely free and need not be non-verifiable, (4) interim renegotiation (of both decisions and decision rights) is often possible and (5) performance measures can be used to manage the incentive conflicts present. Many of these extensions are discussed in detail in Rantakari (2006a,b). In the remainder of this section, we will briefly discuss three such extensions: (i) incentives to listen, (ii) imperfect information and (iii) incentive alignment.

Three significant assumptions, however, remain unexamined: absence of interim contracting on decisions, interim reallocation of decision rights and problems of implementation. For interim contracting, while the informational asymmetries in the interim stage will prevent efficient bargaining over the decisions, the degree to which such bargaining could improve the performance of the different governance structures...
(in particular, decentralized authority) remains unexamined. Also, the relative performance of the governance structures differs in $|\theta_i - \theta_j|$, which generates the possibility of both selective intervention and conditional delegation. Enriching the analysis to account for these additional behavioral alternatives appears a promising avenue for future research.

Problems of implementation when the decision-maker and the implementer are two separate agents, on the other hand, have been extensively discussed. In the present setting ensuring compliance is likely to involve (i) costly monitoring by the decision-maker, (ii) efficiency wage based on compliance and (iii) an increase in global (with a corresponding decrease in local) incentives faced by the implementer. However, two aspects of the present framework make further analysis of the implementation problem warranted. First, the degree of incentive conflict and so the gains from non-compliance depend on the importance of coordination, thus making the cost of ensuring compliance environment-specific. Second, centralized authority can actually benefit from implementation problems. This result follows because the fourth margin that the manager will use to ensure compliance is to bias the decisions in favor of the implementer, thus decreasing the attractiveness of non-compliance. The initial bias will have only a second-order impact on the quality of decisions but will have a first-order impact on the quality of communication.

7.1 Incentives to talk and listen

In the analysis we focused on the most informative incentive-compatible communication equilibrium. From a social perspective, this is the desired outcome. As a result, the manager always wants to receive any incoming messages. Also, the sender prefers as accurate communication as possible. However, when the local agent retains control of his activity (decentralization or partial centralization), he might prefer not to listen to incoming messages. The result is summarized in the following proposition:

**Proposition 7 Incentives to Listen:**

*Under decentralized authority, agent $i$ would prefer not to listen to incoming messages when*
\[ r_j \geq r_j = \frac{1-\sqrt{1-r_j}}{r_i}. \]

Under partial centralization, agent \( i \) would prefer not to listen to incoming messages when

\[ r_j \geq r'_j = \frac{(1+r_i)(1-\sqrt{1-r_i})}{r_i} - r_i. \]

Further, \( r'_j \leq r_j \).

**Proof.** See Appendix B. ■

The reason for this result is that while listening to informative messages helps to reduce coordination failures, this reduction happens through the process of accommodation. Because accommodation induces additional adaptation, the cost of this accommodation can exceed the gains from improved coordination. By not listening, the agent commits not to accommodate, which in turn limits the amount of adaptation undertaken by the sender. This reduction in adaptation, in turn, limits the cost of the coordination failure that results from not listening. As a result, communication can break down if coordination is sufficiently important to the sender relative to the receiver, in which case the sender’s total adaptation is closely tied to the accommodation that the receiver would grant upon receiving the message. For the same reason, the agent retaining control under partial centralization is more likely to prefer not to listen compared to decentralized authority, as the manager puts more weight on coordination.

Note, however, that the selection of the babbling equilibrium relies on the ability of the receiver to commit to not listening. If the sender sends an informative message that he expects to be read, then the induced component of adaptation will be present in his decision and it is better for the receiver to read the message and accommodate. Similarly, if the sender sends an informative message that he expects not to be read, then the receiver would prefer to read the message because responding to the information helps to improve coordination without the added cost of accommodation generated by the component of induced adaptation. However, if the agents are able to take ex ante actions that will credibly signal inability to understand any messages, a
situation of strategic ignorance can arise where one or both directions of communication are fully uninformative and the relative performance of the governance structure is significantly compromised.\(^\text{11}\)

### 7.2 Noisy information

The results presented in the analysis are robust to noisy information. In particular, if agent \(i\) observes a signal \(s_i\) that is equal to \(\theta_i\) with probability \(p_i\) but is a random draw from \(U[\bar{\theta}_i, \tilde{\theta}_i]\) with probability \((1 - p_i)\), the expected loss under governance structure \(g\) is given by the following proposition:

**Proposition 8 Expected loss under noisy information:**

\[
EL_i^g = \Lambda_i^g \left( A (\varphi_i^g) (p_i \bar{\theta}_i)^2 + A (\varphi_j^g) (p_j \bar{\theta}_j)^2 \right) + \Lambda_{i-j}^g B (\varphi_i^g) (p_i \bar{\theta}_i)^2 + \Lambda_{i-j}^g B (\varphi_j^g) (p_j \bar{\theta}_j)^2 + \frac{(1 - p_i^2)}{3} (1 - p_i^2) \bar{\theta}_i^2.
\]

**Proof.** See Appendix B  ■

The model is thus equivalent to a perfect-information model with the states distributed on \(U[\bar{\theta}_i, \tilde{\theta}_i]\), where \(\tilde{\theta}_i = p_i \bar{\theta}_i\), plus a common extra cost term reflecting the quality of primary information. Intuitively, because all the participants are rational, they discount information in the same way. Conflict exists only over how that information should translate into final decisions. As a result, the relative incentive conflict between the actors is unchanged when the primary information is noisy. As Rantakari (2006a) shows, this is no longer the case if the communication process itself is noisy, as that does affect the degree of incentive conflict between the agents. Also, if \(p_i\) are endogenous, differences among the governance structures arise because of the differing incentives to acquire information.

\(^{11}\)Babbling is an equilibrium in any cheap talk game. However, in typically the most informative equilibrium is also Pareto superior and so a natural focal equilibrium. In our setting, when the conditions of the proposition are met, no Pareto superior communication equilibrium exists.
7.3 Incentive alignment

Even if interim recontracting were not possible, payoff sharing can potentially be used to improve the outcome under any governance structure. We will here briefly discuss the case where the degree of incentive alignment is exogenously limited. Endogenous degree of incentive alignment is analyzed in Rantakari (2006a).

Consider a situation where agent $i$ retains a share $s_i$ of the payoff to activity $i$ and gains a share $1 - s_j$ of the payoff to activity $j$. That is, agent $i$’s objective function becomes $U_i = s_i L_i + (1 - s_j) L_j$. The role of incentive alignment in communication and decision-making is immediate. As $s_i \to 1 - s_j$, the interests of the local agents become perfectly aligned and so first-best is achieved under all governance structures. However, the rates of convergence do vary among the governance structures. The main conclusion here revolves around the rate at which the use of the manager (centralized authority and partial centralization) is eliminated with increased incentive alignment.

To illustrate the results, figure 13 gives two examples how the choice of governance structure changes when we increase the degree of incentive alignment. To understand these results, recall that there are essentially three different uses for the manager. First, centralizing the less dependent activity in the asymmetric settings helped to eliminate the strong bias in the equilibrium decisions under decentralized authority (the manager as a controller). Second, centralized authority was used to solve the mutual under-coordination problem between the agents (the manager as a coordinator). Third, centralizing the more dependent activity in the asymmetric settings helped to improve the quality of communication by increasing the bias in equilibrium decisions (the manager as a facilitator). Note that this order reflects the relative improvement in decision-making over the improvement in communication.

As the degree of incentive alignment increases, the agents become more willing to cooperate and coordinate among themselves. Thus, not surprisingly, the use of the manager as a controller is eliminated first. Similarly, centralized authority disappears fully before perfect incentive alignment is achieved. In contrast, the use of the manager as a facilitator initially increases in the degree of incentive alignment. This trend is eventually reversed but the facilitating role of the manager is fully eliminated only as $s_i = s_j \to 0.5$. In the limit, the choice of an equilibrium governance structure is made between decentralized and directional authority.

The conclusion that follows from these results is that the quality of decision-
making improves faster than the quality of communication in the degree of incentive alignment. This conclusion is thus highly analogous to Dessein (2002), who shows that in the CS framework, delegation is always preferred over the cheap-talk solution when the bias between the agent and the principal is sufficiently small. Note also that this result was already implicitly present in two of our earlier results. First, when $r_i, r_j \rightarrow 0$, the quality of decision-making under decentralized authority improved faster than the quality of communication under centralized authority, making decentralized authority the preferred governance structure for the region of low importance of coordination. Second, when $r_j \rightarrow 1$, it was better to choose directional authority by agent $i$ to avoid the limit losses due to inaccurate communication that were present under all the other governance structures.

8 Conclusion

We have investigated how the allocation of decision rights inside an organization can be used to influence decision-making and facilitate communication among strategic agents when decisions need to balance coordination and adaptation and when infor-
information is both soft and distributed.

Decentralized authority, where each local agent retains control over his activity, is the preferred method of organization when coordination is unimportant to both parties. The equilibrium decisions are then only mildly biased and communication, while inaccurate, is also unimportant because each local agent has direct access to their local information.

Centralized authority, where both decision rights are allocated to the manager, is the preferred method of organization when the importance of coordination is intermediate to high and not too asymmetric. By making socially optimal decisions, the manager is able to eliminate the biases in the equilibrium decisions under decentralized authority. However, because the manager needs to rely solely on information communicated to him by the local agents, informational losses tend to remain higher than under decentralized authority. In other words, the quality of adaptation remains limited. The advantage of centralized authority is largest in the region of intermediate importance of coordination, where the under-coordination problem under decentralized authority is particularly damaging. As the importance of coordination grows further, the local agents become increasingly willing to coordinate their decisions under decentralized authority and as a result the decisional advantage of centralized authority is eroded.

Partial centralization, where one of the decisions is centralized while the other is left under local control, is the preferred governance structure under two very different asymmetric environments. First, centralizing the less dependent activity is sufficient to eliminate most of the bias in the equilibrium decisions under decentralized authority whenever the less dependent activity is sufficiently independent. Similarly, not centralizing the more dependent activity avoids introducing any additional fundamental uncertainty to the decision-making process, which limits the total informational losses and so compensates for the remaining bias in the decisions. Second, centralizing the more dependent activity helps to improve the information flows between the agents by increasing the bias in the equilibrium decisions. When the dependent activity is sufficiently dependent, the gains from this increased accuracy of communication will outweigh the losses from the increased bias in the equilibrium decisions.

An alternative solution in the case of high dependency of one activity is to simply allocate both decision rights to the less dependent agent (directional authority). Even if the agent gaining control completely neglects the adaptive needs of the other activ-
ity, the allocation does eliminate all strategic uncertainty from the decision-making process. When the agent losing control cares primarily about coordination, the resulting gains from the full elimination of strategic uncertainty and the associated coordination failures will outweigh the further increase in the bias in the equilibrium decisions.

Finally, we illustrated how incentive alignment provides a natural substitute for managerial interference. In particular, the use of both centralized authority and partial centralization for the purposes of control is eliminated as an equilibrium outcome strictly before perfect incentive alignment is achieved between the local agents. This outcome stands in contrast to the use of partial centralization for the purposes of communication facilitation, which is eliminated as an equilibrium outcome only in the limit of perfect incentive alignment. Indeed, the use of partial centralization for the purposes of communication facilitation is initially increasing in the degree of incentive alignment. These results illustrate how the quality of decision-making improves faster than the quality of communication as the preferences of the agents become more aligned.

While the exact results are based on a number of restrictive assumptions, the qualitative logic of the results is robust to a number of extensions. Further insights can be obtained by considering issues of endogenous incentive alignment, timing of communication and decisions, and different modes of communication. These, together with additional extensions, are discussed in Rantakari (2006a,b).
References


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A The General Solution

We will derive here the general solution to the framework analyzed. We will first derive the equilibrium decisions, then the equilibrium quality of communication and finally the expected losses.

A.1 Equilibrium Decisions

In the final stage of the game, the actors holding the decision rights choose decisions that maximize their payoffs conditional on their current information. Let $m$ and $n$ denote the identity of the decision-maker for activities $i$ and $j$, respectively, and let $\omega_m$ denote the set of information held by agent $m$ when deciding. Also, let $E_m(.)$ denote $E(.)|\omega_m$ and $(\beta_i^m, \beta_j^m)$ denote the weights that agent $m$ places on activities $i$ and $j$, respectively. Then, $m$ solves:

$$
\min_{d_i} E_m \left[ \beta_i^m ((1 - r_i) (\theta_i - d_i)^2 + r_i (d_j - d_i)^2) + \beta_j^m ((1 - r_j) (\theta_j - d_j)^2 + r_j (d_j - d_i)^2) \right],
$$

and symmetrically for $n$. For example, under centralized authority, $\beta_i^m = \beta_j^m = 1$, and under decentralized authority, $\beta_i^m = s_i, \beta_j^m = (1 - s_j)$, where $s_i$ is the share of $L_i$ retained by agent $i$ and $(1 - s_j)$ the share gained of $L_j$. The first-order conditions are then

$$
d_i^m = \frac{\beta_i^m (1-r_i) E_m \theta_i + (\beta_i^m r_i + \beta_j^m r_j) E_m d_j}{\beta_i^m + \beta_j^m r_j}, \quad d_j^m = \frac{\beta_j^m (1-r_j) E_m \theta_j + (\beta_j^m r_j + \beta_i^m r_i) E_m d_i}{\beta_j^m + \beta_i^m r_i}.
$$

Define for notational simplicity

$$
a_1 = \frac{\beta_i^m (1-r_i)}{\beta_i^m + \beta_j^m r_j}, \quad a_2 = \frac{\beta_i^m r_i + \beta_j^m r_j}{\beta_i^m + \beta_j^m r_j},
$$

$$
b_1 = \frac{\beta_j^m (1-r_j)}{\beta_j^m + \beta_i^m r_i}, \quad b_2 = \frac{\beta_j^m r_j + \beta_i^m r_i}{\beta_j^m + \beta_i^m r_i},
$$

which gives us the first-order conditions

$$
d_i^m = a_1 E_m \theta_i + a_2 E_m d_j \quad \text{and} \quad d_j^m = b_1 E_n \theta_j + b_2 E_n d_i.
$$

The equilibrium decisions are then given by the intersection of the two reaction functions and summarized in the following proposition:
Proposition 9 *Equilibrium Decisions:*

Let $m$ and $n$ be the identity of the decision-maker for activities $i$ and $j$, respectively. Then,

$$
d_{mn}^{im} = \frac{a_1(1-b_2a_2)E_m\theta_i + a_2b_1E_mE_n\theta_j + a_2b_2a_1E_mE_n\theta_i}{1-b_2a_2}.
$$

**Proof.** See Appendix B.

---

A.2 *Equilibrium Communication*

In the communication stage, the agents send non-verifiable messages about their local conditions to the decision-maker(s). Given the decisions and objective functions from above, the indifference condition for agent $i$ is given by

$$
E_i\left(s_iL_i\left(d_i(.,m^L),d_j(.,m^L)\right) + (1-s_j)L_j\left(.,d_i(.,m^L),d_j(.,m^L)\right)\right)\left[\tilde{\theta},m^L\right] = E_i\left(s_iL_i\left(d_i(.,m^H),d_j(.,m^H)\right) + (1-s_j)L_j\left(.,d_i(.,m^H),d_j(.,m^H)\right)\right)\left[\tilde{\theta},m^H\right].
$$

Rearranging the indifference condition and solving the difference equation yields the following proposition:

Proposition 10 *Equilibrium quality of communication:* Under any governance structure, we can write the indifference condition defining the family of incentive-compatible partitions as

$$
|\theta_i^k - \theta_i^{k-1}| - |\theta_i^{k-1} - \theta_i^{k-2}| = +4\frac{1}{\varphi_i(r_i,r_j,s_i,s_j,g)} |\theta_i^{k-1} - E_i\theta_j|,
$$

with $g$ indexing the governance structure and $k$ increasing away from the expected preference intersection ($\theta_i = E_i\theta_j$). The constant $\varphi_i(.)$ is given by

$$
\varphi_i(r_i,r_j,s_i,s_j,g)
$$

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\[
\frac{a_1 \left[ \beta_i (1-r_i) (1-I_{m=i}(1-b_2 a_2)) + (\beta_i r_i + \beta_j r_j)(b_1 - I_{m=i}(1-b_2 a_2)) + \beta_j (1-r_j) b_2^2 \right]}{\beta_i (1-r_i) a_2 b_1 (1-I_{m=i}(1-b_2 a_2)) - (\beta_i r_i + \beta_j r_j) a_1 (b_1 - I_{m=i}(1-b_2 a_2)) b_1 - \beta_j (1-r_j) b_2^2 a_1} \in (0, \infty),
\]

where \( I_{m=i} \in \{1, 0\} \) is an indicator function for whether the sender (agent \( i \)) retains control over his own activity (\( d_i \)). The cutoffs of the finest incentive-compatible partition are then given by

\[
\theta_n - E_i \theta_j = \alpha (\varphi_i)^n (\bar{\theta} - E_i \theta_j) \text{ for } \theta_n > E_i \theta_j \ (n \in \{1, \ldots, \infty\}),
\]

\[
\theta_n - E_i \theta_j = -\alpha (\varphi_i)^{|n|} (\bar{\theta} + E_i \theta_j) \text{ for } \theta_n < E_i \theta_j \ (n \in \{-\infty, \ldots, -1\}),
\]

where \( \alpha (\varphi_i) = \frac{\varphi_i}{(1+\sqrt{1+\varphi_i})^2} \in (0, 1) \),

with \(|n| = 1\) indexing the largest interior cutoff and \(|n| \to \infty\) implying \( \theta_{|n|} \to E_i \theta_j \).

**Proof.** See Appendix B  \( \blacksquare \)

### A.3 Expected Losses

Having derived the equilibrium decisions and the resulting quality of communication, the expected losses follow from a simple substitution of these solutions into the payoff functions. However, to illustrate the impact of communication, let us look at the outcomes component-wise. For the adaptation component, we have that

\[
E (\theta_i - d_i)^2 = \frac{a_2^2}{(1-b_2 a_2)^2} \left( (A) \left( b_1 \right)^2 E (E_m \theta_i)^2 + (B) b_2 a_1 (b_2 a_1 + 2 b_1) E (E_m \theta_i - E_n E_m \theta_i)^2 \right)
\]

\[
+ (C) EV ar_m \theta_i + \frac{1}{(1-b_2 a_2)^2} (a_2 b_1)^2 (D) E (E_m E_n \theta_j)^2.
\]

Here, (C) captures the interim uncertainty over the appropriate course of action faced by the decision-maker, constituting the *fundamental uncertainty* over \( \theta_i \) remaining at this stage, and (A) captures the baseline loss caused by the technological limitations of adaptation under the equilibrium decisions. The components of interest are (B) and (D). (B) captures the cost of the fact that when communication is noisy, the induced component of adaptation is based on an inaccurate message. (D) gives
the cost of accommodating $d_j$. Because coordination is achieved by accommodating the adaptive process of the outside decision, such accommodation comes at the cost of worse adaptation. In a similar fashion, we can write the coordination component as

$$E(d_j - d_i)^2 = \frac{1}{(1-b_2a_2)^2} (b_1a_1)^2 \left( E(E_mE_n\theta_j)^2 + E(E_nE_m\theta_i)^2 \right)^{(A)}$$

$$+ b_1^2 E(E_n\theta_j - E_mE_n\theta_j)^2 + a_1^2 E(E_m\theta_i - E_nE_m\theta_i)^2. \quad (B)$$

$$E(E_nE_m\theta_i)^2 + E(E_mE_n\theta_j)^2 \quad (C)$$

Here, (A) captures the baseline divergence in decisions, caused by the need for adaptation. The components of interest are (B) and (C), comprising the strategic uncertainty remaining in the interim stage, caused by the inaccurate communication and the associated incomplete ability to predict exactly what the opponent is going to do. To bring the two together in a simple form, we need to rearrange $(\theta_i - d_i)^2$ based on the common knowledge components, which gives

$$Var_m\theta_i + a_2^2 (E_m\theta_i - E_nE_m\theta_i)^2 + \frac{(a_2b_1)^2}{(1-b_2a_2)^2} \left( (E_nE_m\theta_i)^2 + (E_mE_n\theta_j)^2 \right).$$

Then, adding up $(1 - r_i) (\theta_i - d_i)^2 + r_i (d_j - d_i)^2$ gives us

$$\frac{b_1^2((1-r_i)a_2^2 + r_i a_1^2)}{(1-b_2a_2)^2} \left( (E_nE_m\theta_i)^2 + (E_mE_n\theta_j)^2 \right) + (1 - r_i) Var_m\theta_i$$

$$+ (((1-r_i)a_2^2 + r_i a_1^2) (E_m\theta_i - E_nE_m\theta_i)^2 + r_i b_1^2 (E_n\theta_j - E_mE_n\theta_j)^2.$$

Now, whether any of the three last terms are present in any given situation depends on the identity of the decision-makers and the underlying accuracy of information. For example, under perfect primary information and decentralized authority, $Var_m\theta_i = 0$, but both the beliefs of the two agents will be different. Similarly, under centralized authority, $Var_m\theta_i > 0$, but the last two terms are not present because the manager knows what he knows. What remains is to solve the expectations, which then gives the following proposition:

**Proposition 11 Expected Losses:**

*In the case of simultaneous communication, we can write the expected loss to activity $i$ as*
\[ EL_i = \Lambda_i^g \left( A(\varphi_i) \bar{\theta}_i^2 + A(\varphi_j) \bar{\theta}_j^2 \right) + \Lambda_{i-i}^g B(\varphi_i) \bar{\theta}_i^2 + \Lambda_{i-j}^g B(\varphi_j) \bar{\theta}_j^2, \]

where \( \Lambda_k^g \) is shorthand for \( \Lambda_k^g (r_i, r_j, s_i, s_j, g) \), constants that depend only on the underlying environment, the amount of payoff sharing present and the governance structure chosen, and

\[ A(x) \equiv \frac{(1-\alpha(x))(1+\alpha(x))^2}{4(1-\alpha(x)^2)}, \quad B(x) \equiv \frac{(1-\alpha(x))^3}{12(1-\alpha(x)^3)}; \]

with \( A(x) + B(x) = \frac{1}{3} \) and \( \frac{\partial A(x)}{\partial x} > 0 \)

**Proof.** See Appendix B ■

**B  Proofs and Derivations**

**B.1  Proposition 7**

Recall that

\[ EL_i = \Lambda_i^g \left( A(\varphi_i) \bar{\theta}_i^2 + A(\varphi_j) \bar{\theta}_j^2 \right) + \Lambda_{i-i}^g B(\varphi_i) \bar{\theta}_i^2 + \Lambda_{i-j}^g B(\varphi_j) \bar{\theta}_j^2 \]

and

\[ A(x) + B(x) = \frac{1}{3}. \]

Thus, \( \frac{\partial A(x)}{x} = -\frac{\partial B(x)}{x} \). As a result,

\[ \frac{\partial EL_i}{\partial \varphi_j} > 0 \quad \text{iff} \quad \Lambda_i^g > \Lambda_{i-j}^g. \]

For decentralized authority, we have

\[ \Lambda_i^g = \frac{(1-r_i)r_i(1-r_j)}{(1-r_i r_j)^2} \quad \text{and} \quad \Lambda_{i-j}^g = r_i (1 - r_j)^2. \]

Rearranging the constants gives
\[ \Lambda_i^q > \Lambda_{i-j}^q \quad \text{iff} \quad r_j \geq r_i \frac{1 - r_i}{r_i} \]

The condition for partial centralization follows equivalently.

**B.2 Proposition 8**

Recall from A.2 that the expected loss under any governance structure simplifies to

\[
\frac{\delta_i^2 ((1 - r_i)a_i^2 + r_i a_i^2)}{(1 - b_2a_2)^2} \left( (E\eta_i E_m \theta_i)^2 + (E_m E_i \theta_j)^2 \right) + (1 - r_i) \text{Var}_i \theta_i + ((1 - r_i) a_i^2 + r_i a_i^2) (E_m \theta_i - E_n E_m \theta_i)^2 + r_i b_i^2 (E_m \theta_j - E_m E_n \theta_j)^2.
\]

First, note that given the signal \( s_i \), agent \( i \)'s belief about the realized \( \theta_i \) is given by \( p_i s_i \). In consequence, agent \( i \)'s belief about the realization of \( \theta_i \) is distributed \( U [-p_i \bar{\theta}_i, p_i \bar{\theta}_i] \). As a result, the communication solution presented goes directly through but now in the space of beliefs. The only component that directly depends on the quality of primary information is \( \text{Var}_i \theta_i \). When \( m = i \),

\[
E (\theta_i - E_i \theta_i)^2 = p_i E (\theta_i - p_i \theta_i)^2 + (1 - p_i) E (\theta_i - p_i \theta_i)^2 = (1 - p_i^2) \frac{\bar{\theta}_i^2}{3}.
\]

When \( m \neq i \),

\[
E (\theta_i - E_m \theta_i)^2 = E (\theta_i - p_i E_m s_i)^2 = p_i E (\theta_i - p_i E_m \theta_i)^2 + (1 - p_i) E (\theta_i - p_i E_m \theta_i)^2.
\]

Now, note first that \( E (\theta_i - p_i E_m \theta_i)^2 = E \theta_i^2 + p_i^2 E (E_m \theta_i)^2 \). Second, note that

\[
E (\theta_i - p_i E_m \theta_i)^2 = E (\theta_i - E_m \theta_i)^2 + (1 - p_i)^2 E (E_m \theta_i)^2
\]

Adding the two components together and adding and subtracting \( p_i^2 E (\theta_i - E_m \theta_i)^2 \) gives

\[
p_i (E (\theta_i - E_m \theta_i)^2 + (1 - p_i)^2 E (E_m \theta_i)^2) + (1 - p_i) (E \theta_i^2 + p_i^2 E (E_m \theta_i)^2)
\]

\[
+ p_i^2 E (\theta_i - E_m \theta_i)^2 - p_i^2 E (\theta_i - E_m \theta_i)^2
\]

\[
(1 - p_i^2) E \theta_i^2 + (1 - p_i) p_i^2 E (E_m \theta_i)^2 - p_i^2 (1 - p_i) E (E_m \theta_i)^2 + p_i^2 E (\theta_i - E_m \theta_i)^2
\]

and noting that \( E (E_m \theta_i)^2 = E (E \theta_i)^2 \), this simplifies to
\[ E (p_i \theta_i - p_i E_m \theta_i)^2 + (1 - p_i^2) \frac{\sigma^2_i}{\lambda_i}. \]

First component is equivalent to the fundamental uncertainty that would be present if the state was distributed on \( U [-p_i \bar{\theta}_i, p_i \bar{\theta}_i] \) and the second component gives the additional loss due to inaccurate primary information.

### B.3 Proposition 9

The first-order conditions are given by

\[ d^m_i = a_1 E_m \theta_i + a_2 E_m d_j \quad \text{and} \quad d^m_j = b_1 E_n \theta_j + b_2 E_n d_i, \]

where

\[ E_m d_j = b_1 E_m E_n \theta_j + b_2 E_m E_n d_i \quad \text{and} \quad E_n d_i = a_1 E_n E_m \theta_i + a_2 E_n E_m d_j. \]

Note that since \( E_n d_i \) and \( E_m d_j \) are based solely on the messages \( m_i, m_j \) sent, "what you think that I know" and so all higher-order beliefs (which are equal) are common knowledge.\(^{12}\) Thus, we can write by repeated substitution

\[ d^m_i = a_1 E_m \theta_i + a_2 (b_1 E_m E_n \theta_j + b_2 (a_1 E_n E_m \theta_i + a_2 (b_1 E_m E_n \theta_j + b_2 (...)))), \]

which, after rearranging, simplifies to

\[ d^m_i = \frac{a_1 (1 - b_2 a_2) E_m \theta_i + a_2 b_1 E_m E_n \theta_j + a_2 b_2 a_1 E_n E_m \theta_i}{1 - b_2 a_2}. \]

### B.4 Proposition 10

The linearity of the solution follows directly from the quadratic form of the payoffs and the linearity of the equilibrium decisions in all information. In particular, we can write the objective function of the sender (agent \( i \)) as:

\(^{12}\)Rantakari (2006a) discusses the case where the message can be misinterpreted with probability \( p \), in which case nothing is common knowledge.
\[
\min_{m_i} E_i \left[ \beta_i \left( (1 - r_i) (\theta_i - d_i (., m_i))^2 + (\beta_i r_i + \beta_j r_j) (d_i (., m_i) - d_j (., m_i))^2 \right) \right] \\
+ \beta_j (1 - r_j) (\theta_j - d_j (., m_i))^2
\]

where \( m_i \) stands for the message sent, and our task is to solve for an incentive-compatible partition of the message space. Let \( I_{m=i} \in \{0,1\} \) be an indicator function for whether the sender retains control for his own activity. Then we can write the indifference condition component-wise (dropping constant components):\(^{13}\)

\[
\Delta (\theta_i - d_i (., m_i))^2 = a_1^2 (1 - I_{m=i} (1 - b_2 a_2))^2 (\theta_k + \theta_{k-2} - 2 \theta_{k-1}) - 2 a_1 a_2 b_1 (1 - I_{m=i} (1 - b_2 a_2)) (\theta_{k-1} - E_i \theta_j)
\]

\[
\Delta (\theta_j - d_j (., m_i))^2 = (b_2 a_1)^2 (\theta_k + \theta_{k-2} - 2 \theta_{k-1}) + 2 (b_2 a_1)^2 (\theta_{k-1} - E_i \theta_j)
\]

\[
\Delta (d_i (., m_i) - d_j (., m_i))^2 = a_1^2 (b_1 - I_{m=i} (1 - b_2 a_2))^2 (\theta_k + \theta_{k-2} - 2 \theta_{k-1}) + 2 a_1^2 (b_1 - I_{m=i} (1 - b_2 a_2)) b_1 (\theta_{k-1} - E_i \theta_j)
\]

substituting back and rearranging gives then:

\[
\theta_k = 2 \theta_{k-2} - \theta_{k-1} + 4 \left[ \frac{\beta_i (1-r_i)a_2 b_1 (1-I_{m=i}(1-b_2 a_2)) - \beta_i r_i + \beta_j r_j) a_1 (b_1 - I_{m=i}(1-b_2 a_2)) b_1 - \beta_j (1-r_j) b_2 a_1}{a_1 [\beta_i (1-r_i)(1-I_{m=i}(1-b_2 a_2))^2 + (\beta_i r_i + \beta_j r_j) (b_1 - I_{m=i}(1-b_2 a_2))^2 + \beta_j (1-r_j) b_2^2]} \right] (\theta_i - E_i \theta_j)
\]

or

\[
\theta_n = 2 \theta_{n-2} - \theta_{n-1} + 4 \frac{\varphi}{\varphi^2} (\theta_{n-1} - E_i \theta_j)
\]

The general solution to the difference equation with the above structure is given by:

\[
\theta_n - E_i \theta_j = \frac{\varphi}{4 \sqrt{1+\varphi}} \left( \left( \frac{\varphi}{1-\sqrt{1+\varphi}} \right)^{i} - \left( \frac{\varphi}{1+\sqrt{1+\varphi}} \right)^{i} \right) (\theta_1 - E_i \theta_j)
\]

let \( l \) and \( k \) be the two different cutoffs. Then,

\[
\frac{\theta_l - E_i \theta_j}{\theta_k - E_i \theta_j} = \frac{\varphi}{4 \sqrt{1+\varphi}} \left( \left( \frac{\varphi}{1-\sqrt{1+\varphi}} \right)^{i} - \left( \frac{\varphi}{1+\sqrt{1+\varphi}} \right)^{i} \right) = \frac{\varphi}{4 \sqrt{1+\varphi}} \left( \frac{1+\sqrt{1+\varphi}}{1-\sqrt{1+\varphi}} \right)^{2l} \left( \frac{1-\sqrt{1+\varphi}}{1+\sqrt{1+\varphi}} \right)^{2k}
\]

\(^{13}\)We make the assumption that \( E_i \theta_j = E_i E_n \theta_j = E_i E_m \theta_j \) - that is, agent \( i \) doesn’t have better information about \( \theta_j \) than the person deciding \( d_j \). In the simultaneous one-round case, there is no prior information and as a result this condition is trivially satisfied.
define \( k = n \) and \( l = n - x \), where \( x \) is the distance between the two cutoffs, with \( x = 1 \) implying adjacent cutoffs. This substitution allows us to write the equation as

\[
\frac{\theta_{n-x}-E_i\theta_j}{\theta_{n}-E_i\theta_j} = \frac{(1+\sqrt{1+\varphi})^{2(n-x)}-(1-\sqrt{1+\varphi})^{2(n-x)}}{(1+\sqrt{1+\varphi})^{2n}-(1-\sqrt{1+\varphi})^{2n}}
\]

Now, solving for the most informative partition, which minimizes the absolute size of the intervals conditional on the indifference condition between two adjacent intervals given by \( \varphi \), we let \( n \to \infty \). Observe that:

\[
\frac{(1-\sqrt{1+\varphi})^{2y}}{\varphi^y} = \left(\frac{2-2\sqrt{1+\varphi}+\varphi}{\varphi^y}\right)|_{y \to \infty} \to 0
\]

as long as \( \varphi > 0 \). Therefore, the above rearranges to:

\[
\frac{\theta_{n-x}-E_i\theta_j}{\theta_{n}-E_i\theta_j} = \left(\frac{\varphi}{1+\sqrt{1+\varphi}}\right)^x = \alpha (\varphi)^x
\]

Letting \( \theta_n = \bar{\theta} \) gives the full characterization of the partition. The solution going backwards (\( \theta_n < E_i\theta_j \)) follows similarly.

### B.5 Proposition 11

To solve the expectations we need to use to communication equilibrium (we continue to let \( \alpha \) to stand for \( \alpha_i \) to simplify):

(a) probabilities

The probability of \( \theta_i \in [\theta_{k-1}, \theta_k] \) is simply:

\[
\left(\frac{\bar{\theta}-E_i\theta_j}{2\bar{\theta}}\right) \alpha^{i-1} (1 - \alpha) \quad \theta_i > E_i\theta_j
\]

\[
\left(\frac{E_i\theta_j+\bar{\theta}}{2\bar{\theta}}\right) \alpha^{i-1} (1 - \alpha) \quad \theta_i < E_i\theta_j
\]

where \( i = 1 \) indexes the furthest partition.

(b) cutoffs

We can write the cutoffs:
\[ \theta_i = E_i \theta_j + \alpha^i (\overline{\theta} - E_i \theta_j), \]

where \( i = 0 \rightarrow \theta_i = \overline{\theta} \) and symmetrically downwards

(c) conditional expectations and variances

From the cutoffs it follows immediately that

\[ E \theta_i = E_i \theta_j + \frac{1}{2} \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} - E_i \theta_j \right) \quad \theta_i > E_i \theta_j \]
\[ E \theta_i = E_i \theta_j - \frac{1}{2} \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} + E_i \theta_j \right) \quad \theta_i < E_i \theta_j \]

(d) ex ante expectations and variances

First, note that since learning is a random walk, \( EE \theta_i = 0 \). The two components that do matter are \( E (E_j \theta_i)^2 \) and \( EV ar(\theta_i) \)

Evaluating \( E (E_j \theta_i)^2 : \)

\[ E (E_j \theta_i)^2 = \sum_{i=1}^{\infty} \left( \frac{\overline{\theta} - E_i \theta_j}{2 \theta} \right) \alpha^{i-1} (1 - \alpha) \left( E_i \theta_j + \frac{1}{2} \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} - E_i \theta_j \right) \right)^2 \]
\[ + \sum_{i=1}^{\infty} \left( \frac{\overline{\theta} + E_i \theta_j}{2 \theta} \right) \alpha^{i-1} (1 - \alpha) \left( E_i \theta_j - \frac{1}{2} \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} + E_i \theta_j \right) \right)^2 \]
\[ E (E_j \theta_i)^2 = \frac{1}{4} \left( \left( 1 + \frac{\alpha (1 - \alpha)}{(1 - \alpha^3)} \right) \overline{\theta}^2 - \left( 1 - \frac{3 \alpha (1 - \alpha)}{(1 - \alpha^3)} \right) (E_i \theta_j)^2 \right) \]

Similarly for \( E (\theta_i - E_j \theta_i)^2 : \)

\[ E (\theta_i - E_j \theta_i)^2 = \sum_{i=1}^{\infty} \left( \frac{\overline{\theta} - E_i \theta_j}{2 \theta} \right) \alpha^{i-1} (1 - \alpha) \frac{1}{12} \left( \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} - E_i \theta_j \right) \right)^2 \]
\[ + \sum_{i=1}^{\infty} \left( \frac{\overline{\theta} + E_i \theta_j}{2 \theta} \right) \alpha^{i-1} (1 - \alpha) \frac{1}{12} \left( \alpha^{i-1} (1 + \alpha) \left( \overline{\theta} + E_i \theta_j \right) \right)^2 \]
\[ EV ar_j \theta_i = \frac{1}{12} \left( \frac{(1 - \alpha)^3}{(1 - \alpha^3)} \right) \left( \overline{\theta}^2 + 3 (E_i \theta_j)^2 \right) \]

in the case of simultaneous one-round communication, \( E_i \theta_j = E_j \theta_i = 0 \), and we get the functions \( A(x) \) and \( B(x) \) of the proposition

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